

Correlation functions and event shapes in gauge theories

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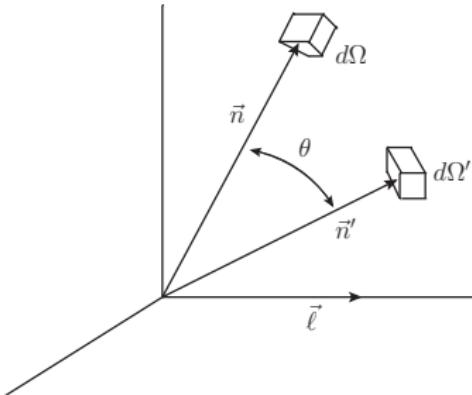
arXiv:2001.10806; to appear

Outline

- Event shapes as weighted cross sections
- Event shapes as integrated correlation functions
- Correlator of four currents at one loop in QCD
- Charge-charge correlation at LO
- Conformal anomaly for pseudo-scalar operators and the Adler axial anomaly

Part I: Event shapes and correlation functions

- Classes of IR safe observables, e.g. energy-energy correlation (EEC)
- Usual amplitude approach relies on intricate cancellations of IR divergences and complicated phase space integrals
- Alternative approach based on correlation functions
 - proposal for conformal colliders Hofman&Maldacena 2008
 - developed and applied to $\mathcal{N} = 4$ SYM at:
NLO Belitsky, Hoenegger, Korchemsky, E.S., Zhiboedov 2013
NNLO Henn, E.S., Yan, Zhiboedov 2019
 - $\mathcal{N} = 4$ supersymmetry has been extremely useful: all the event shapes derive from a 4-point correlator of 1/2-BPS scalars (known to 3 loops)
- Subject of this talk: Can we do without supersymmetry, e.g. in QCD?
We consider charge-charge correlations (QQC) for simplicity



Event shapes as weighted cross sections

- Definition of QQC

$$\sigma_w(q) = \sigma_{\text{tot}}^{-1} \int d^4x e^{iqx} \sum_X \langle 0 | \bar{P} \cdot J(x) | X \rangle w(X) \langle X | P \cdot J(0) | 0 \rangle$$

e-m current $J_\mu(x)$; polarization vector P^μ ; final state $|X\rangle$

- charge weight factor

$$w_Q(k_1, \dots, k_\ell) = \sum_{i=1}^{\ell} Q_i \delta^{(2)}(\Omega_{\vec{k}_i} - \Omega_{\vec{n}})$$

- charge operator

$$\mathcal{Q}(\vec{n})|X\rangle = w_Q(X)|X\rangle, \quad \mathcal{Q}(\vec{n}) = \int_0^\infty dt \lim_{r \rightarrow \infty} r^2 n^i J_0(t, r\vec{n})$$

- charge correlations

$$\langle \mathcal{Q}(\vec{n}) \rangle = \sigma_{\text{tot}}^{-1} \int d^4x e^{iqx} \langle 0 | \bar{P} \cdot J(x) \mathcal{Q}(\vec{n}) P \cdot J(0) | 0 \rangle_W,$$

$$\langle \mathcal{Q}(\vec{n}) \mathcal{Q}(\vec{n}') \rangle = \sigma_{\text{tot}}^{-1} \int d^4x e^{iqx} \langle 0 | \bar{P} \cdot J(x) \mathcal{Q}(\vec{n}) \mathcal{Q}(\vec{n}') P \cdot J(0) | 0 \rangle_W, \quad \text{etc.}$$

Event shapes as integrated correlation functions

- Lorentz covariant definition of detectors

$$x^\mu = x_+ n^\mu + x_- \bar{n}^\mu, \quad n^2 = \bar{n}^2 = 0$$

$$\mathcal{Q}(n) = \int_{-\infty}^{\infty} dx_- (n\bar{n}) \lim_{x_+ \rightarrow \infty} x_+^2 J_+(x_+ n + x_- \bar{n}), \quad J_+ \equiv \frac{\bar{n}^\mu J_\mu(x)}{(n\bar{n})}$$

- Charge-charge correlation (averaged over the orientation of the detector plane)

$$\begin{aligned} \langle \mathcal{Q}(n) \mathcal{Q}(n') \rangle &= \sigma_{\text{tot}}^{-1} \int d^4x e^{iqx} \langle 0 | J^\mu(x) \mathcal{Q}(n) \mathcal{Q}(n') J_\mu(0) | 0 \rangle_W \\ &= \frac{F(\zeta)}{4\pi^2(nn')^2}, \quad \zeta = \frac{q^2(nn')}{2(qn)(qn')} \xrightarrow{\text{rest frame}} \frac{1}{2}(1 - \cos\theta) \end{aligned}$$

- 3-step procedure:

- compute the 4-point correlator $\langle 0 | J^\mu(x_1) J^\nu(x_2) J^\lambda(x_3) J^\rho(x_4) | 0 \rangle$
- detector limit and time integration
- Fourier integral

Correlation function of four vector currents at one loop

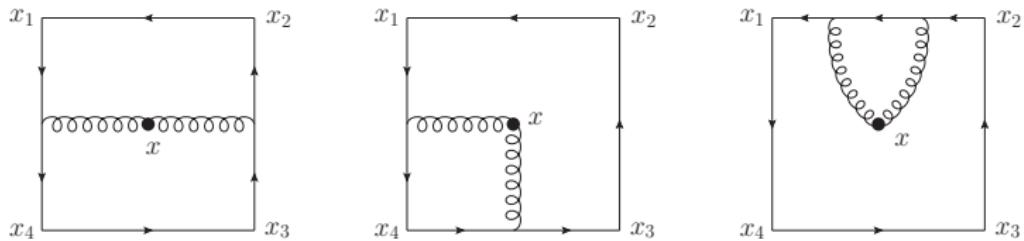
- QCD/QED Lagrangian in 2-component spinor notation

$$L = -\frac{1}{2} \text{tr} F_{\mu\nu} F^{\mu\nu} + \text{tr} \bar{\Psi} i\gamma^\mu \mathcal{D}_\mu \Psi, \quad \Psi = (\chi_\alpha, \bar{\psi}^{\dot{\alpha}})$$

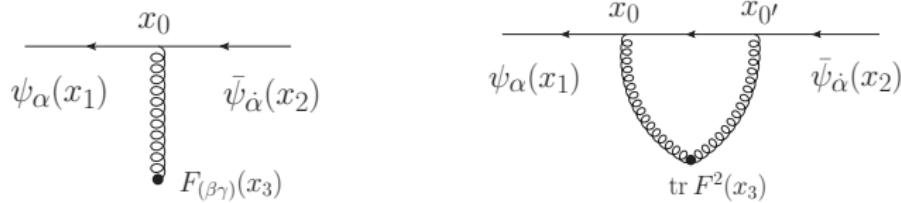
- Loop correction at order g^2 via Lagrangian insertion

$$\langle J_\mu(x_1) \dots J_\rho(x_4) \rangle_{\text{1-loop}} = -\frac{i}{g^2} \int d^4x \langle L_{\text{YM}}(x) J_\mu(x_1) \dots J_\rho(x_4) \rangle_{\text{Born}}$$

- Feynman graphs



made of rational 3-point blocks



Result for the four-point correlation function

- Conformal 4-point function (complicated!)

$$\langle J_\mu(x_1) J_\nu(x_2) J_\lambda(x_3) J_\rho(x_4) \rangle_{\text{1-loop}} =$$

$$-\frac{4g^2 N_c C_F}{(2\pi)^{10}} \left[R_c(x) \Phi^{(1)}(u, v) + R_u(x) \log(u) + R_v(x) \log(v) + R_r(x) \right] + R'_r(x)$$

where

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

$$\Phi^{(1)} = \frac{1}{\bar{z}-z} \left[2\text{Li}_2\left(\frac{z}{z-1}\right) - 2\text{Li}_2\left(\frac{\bar{z}}{\bar{z}-1}\right) - \log\left(\frac{z\bar{z}}{(1-z)(1-\bar{z})}\right) \log\left(\frac{1-z}{1-\bar{z}}\right) \right]$$

and R are rational conformal tensors over a basis of 1024 structures !

- After contracting the indices μ, ρ and taking the detector limits at points 2,3, the expression drastically simplifies:

Results (ctd)

- detector limit

$$\lim_{x_{2+}, x_{3+} \rightarrow \infty} x_{2+}^2 x_{3+}^2 \langle J^\mu(x_1) J_+(x_2) J_+(x_3) J_\mu(x_4) \rangle \\ = -\frac{g^2 N_c C_F}{2^4 \pi^{10}} \frac{(n\bar{n})(n'\bar{n}')}{x_{14}^8 (nn')^3} \left\{ \frac{v^4}{u^2} [r_{uu} \partial_{uu}^2 + r_{uv} \partial_{uv}^2 + r_{vv} \partial_{vv}^2] \Phi^{(1)}(u, v) + \frac{v^2}{u^3} r_r \right\}$$

where the r are polynomials in u, v and $\gamma = \frac{2(nx_{14})(n'x_{14})}{x_{14}^2(nn')}$

- Analytic continuation to Wightman via Mellin transform

$$\Phi^{(1)}(u, v) = -\frac{1}{4} \int_{\delta-i\infty}^{\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} [\Gamma(-j_1)\Gamma(-j_2)\Gamma(j_1+j_2+1)]^2 u^{j_1} v^{j_2},$$

with Wightman prescription

$$u = \frac{(x_{21-} + i\epsilon)(x_{34-} - i\epsilon)}{(x_{31-} + i\epsilon)(x_{24-} - i\epsilon)}, \quad v = -\frac{x_{14}^2(nn')}{2(x_{31-} - i\epsilon)(x_{24-} + i\epsilon)} \\ x_{21-} = x_{2-}(n\bar{n}) - (nx_1), \quad \text{etc.}$$

Results (end)

- Detector time integration

$$\langle J^\mu(x) \mathcal{Q}(n) \mathcal{Q}(n') J_\mu(0) \rangle = \frac{N_c C_F g^2}{2\pi^8 x^6 \gamma^3 (nn')^2} \left[\text{Li}_2 \left(1 - \frac{1}{\gamma} \right) + \frac{3}{2} \log(\gamma) - \frac{\pi^2}{6} + \frac{\gamma}{2} + \frac{7}{4} \right]$$

- Fourier integral $x \rightarrow q$: Final result

$$\begin{aligned} F_{\text{QQC}}^{\text{QCD}}(\zeta) &= 4\pi^2 (nn')^2 \sigma_0^{-1} \int d^4 x e^{iqx} \langle J^\mu(x) \mathcal{Q} \mathcal{Q} J_\mu(0) \rangle \\ &= \frac{C_F g^2}{4\pi^2} \frac{2 \log(1-\zeta) + \zeta(2+\zeta)}{\zeta(1-\zeta)} + O(g^4) \end{aligned}$$

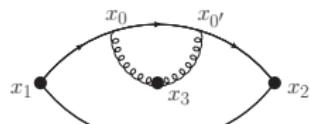
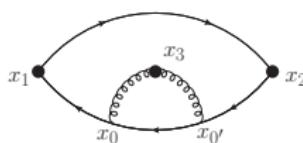
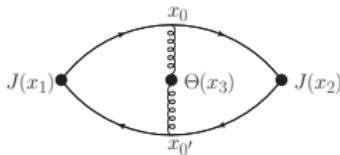
- differs slightly from the $\mathcal{N} = 4$ SYM result
 - contact terms $C_1 \delta(\xi) + C_2 \delta(1-\xi)$ not considered
- How to proceed to two loops???
 - Need EEC because QQC is IR divergent
 - Conformal fixed point?
 - Learn to take the detector limit of Feynman diagrams?

Part II: Conformal anomaly for pseudoscalar operators

- Spin-off of the calculation of the correlator: 3-point function of 2 vector currents $V_\mu = \text{tr } \bar{\Psi} \gamma_\mu \Psi$ and the topological term $\Theta = \text{tr } F_{\mu\nu} \tilde{F}^{\mu\nu}$

$$\langle V_\mu(x_1) V_\nu(x_2) \Theta(x_3) \rangle_{\text{Born}} = -\frac{24g^2 C_F}{(2\pi)^8} \frac{\epsilon_{\mu\nu\lambda\rho} x_{13}^\lambda x_{23}^\rho}{x_{12}^4 x_{13}^4 x_{23}^4}$$

- Feynman diagrams made of rational blocks



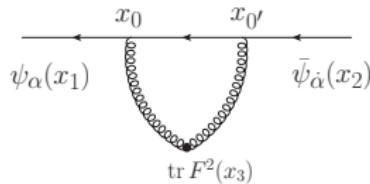
- Question: Is it conformal? Answer: NO!
 - Proof by contradiction: if conformal, we can choose a frame $x_1 = 0, x_2 = \infty \Rightarrow$ too many indices left
 - There exists no parity-odd rank-two conformal tensor in 4 dimensions!

Anomalous conformal Ward identity

- Conformal variation with deformed measure in $D = 4 - 2\epsilon$ dimensions

$$\begin{aligned} & \mathbb{K}^\lambda \langle J_\mu(x_1) J_\nu(x_2) \Theta(x_3) \rangle_{\text{Born}} \\ &= 4 \lim_{\epsilon \rightarrow 0} \epsilon \int d^D x_0 x_0^\lambda \langle L_{\text{QCD}}(x_0) J_\mu(x_1) J_\nu(x_2) \Theta(x_3) \rangle_{\text{Born}} \end{aligned}$$

- Chiral insertion into the fermion propagator $\Pi_{\alpha\dot{\alpha}}$



$$\begin{aligned} [\mathbb{K}_{\gamma\dot{\gamma}} \Pi_{\alpha\dot{\alpha}}]_{\text{anom}} &\sim \lim_{\epsilon \rightarrow 0} \epsilon (\partial_1 \tilde{\partial}_3 \partial_2)_{\alpha\dot{\alpha}} \int d^D x_0 \frac{(x_0)_{\gamma\dot{\gamma}}}{x_{10}^2 x_{20}^2} \left[\frac{i\pi^2}{\epsilon} \delta^{(4)}(x_{30}) + \mathcal{O}(\epsilon^0) \right] \\ &\sim \frac{(x_{13})_{\alpha\dot{\gamma}} (x_{23})_{\gamma\dot{\alpha}}}{x_{13}^4 x_{23}^4} \text{ coming from } \frac{1}{x^4} \rightarrow \frac{i\pi^2}{\epsilon} \delta^{(4)}(x) + \mathcal{O}(\epsilon^0) \end{aligned}$$

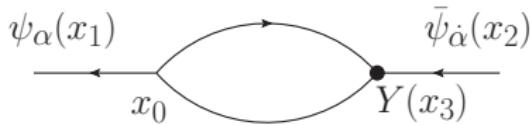
- No anomaly if $L_{YM} = \text{tr} F^{\mu\nu} F_{\mu\nu}$ is inserted

Conformal anomaly of the Yukawa vertex

- Chiral and antichiral Yukawa vertices

$$Y = g \varphi \psi^\alpha \psi_\alpha , \quad \bar{Y} = g \bar{\varphi} \bar{\psi}^\dot{\alpha} \bar{\psi}^{\dot{\alpha}}$$

- Chiral insertion into the fermion propagator



$$\langle \psi_\alpha(x_1) \bar{\psi}_{\dot{\alpha}}(x_2) Y(x_3) \rangle_{g^2} = \frac{4g^2}{(2\pi)^6} \frac{(x_{23})_{\alpha\dot{\alpha}}}{x_{13}^4 x_{23}^4} \quad \text{not conformal!}$$

- Conformal anomaly cancellation in the $\mathcal{N} = 4$ SYM on-shell Lagrangian

$$L_{\mathcal{N}=4} = \text{tr} \left\{ -\frac{1}{2} F_{\alpha\beta} F^{\alpha\beta} + \sqrt{2} g \psi^{\alpha A} [\phi_{AB}, \psi_\alpha^B] - \frac{1}{8} g^2 [\phi^{AB}, \phi^{CD}] [\phi_{AB}, \phi_{CD}] \right\}$$

Chiral anomaly and conformal anomaly

- Correlator of two vector and one axial currents at Born level $O(g^0)$

$$\langle V_\mu(x_1) V_\nu(x_2) A_\lambda(x_3) \rangle_{\text{Born}} = \frac{1}{(2\pi^2)^3} \frac{\epsilon_{\mu' \nu' \rho \lambda} I_\mu^{\mu'}(x_{13}) I_\nu^{\nu'}(x_{23}) Z^\rho(x_3|x_1, x_2)}{x_{12}^4 x_{13}^2 x_{23}^2}$$

- Axial current not conserved because of the Adler anomaly

$$\partial^\lambda A_\lambda = \frac{g^2}{8\pi^2} \Theta \Rightarrow \text{anomalous dimension } \gamma = -\frac{3C_F g^4}{2^7 \pi^4} + O(g^6)$$

- Quantum correction to the 3-point function

$$\langle V_\mu(x_1) V_\nu(x_2) A_\lambda(x_3) \rangle_{\text{loop}} = \frac{C(g)}{(2\pi^2)^3} \frac{\epsilon_{\mu' \nu' \rho \lambda} I_\mu^{\mu'}(x_{13}) I_\nu^{\nu'}(x_{23}) Z^\rho(x_3|x_1, x_2)}{(x_{12}^2)^{2-\gamma/2} (x_{13}^2)^{1+\gamma/2} (x_{23}^2)^{1+\gamma/2}} + \frac{\beta(g)}{g} ???$$

- Divergence of the axial current (not conformal!) matches exactly our earlier result

$$\partial_{x_3}^\lambda \langle V_\mu(x_1) V_\nu(x_2) A_\lambda(x_3) \rangle_{\text{loop}} = \frac{g^2}{8\pi^2} \langle V_\mu(x_1) V_\nu(x_2) \Theta(x_3) \rangle + O(g^6)$$