

# Transport model approach for clusters in heavy-ion collision dynamics

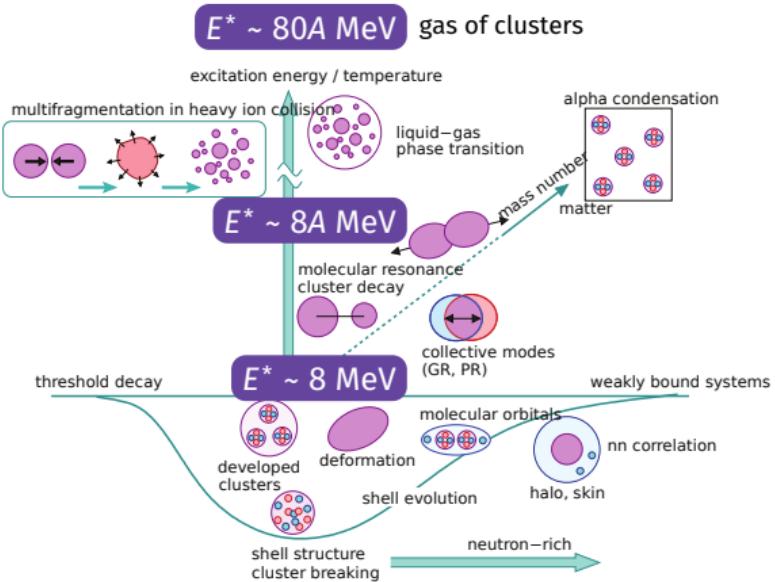
Akira Ono

Tohoku University

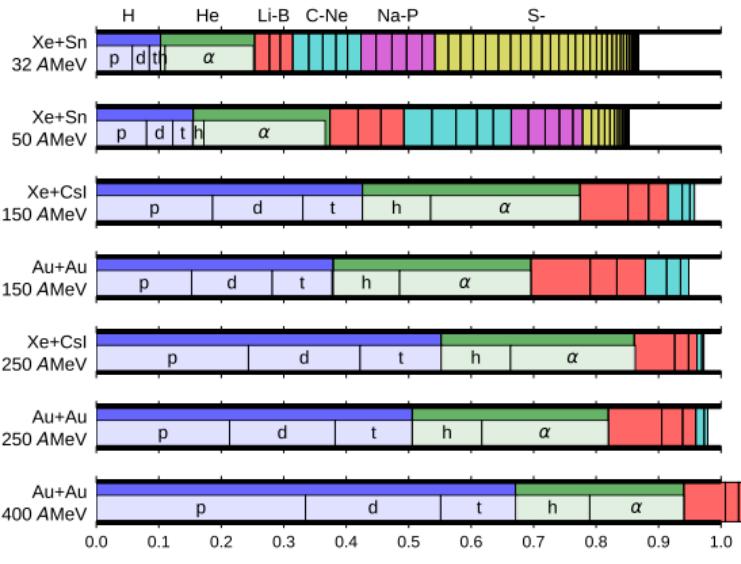
IWM-EC 2021: International Workshop on Multi-facets of EOS and Clustering,  
23-26 November 2021, GANIL, Caen

- Cluster correlations in transport models, e.g. AMD
- Some comparisons with experimental data (e.g. S $\pi$ RIT data)  
to discuss implications of cluster observables

# Clustering phenomena in excited states of nuclear many-body systems



Fraction of protons in heavy-ion collisions ( $b \approx 0$ )



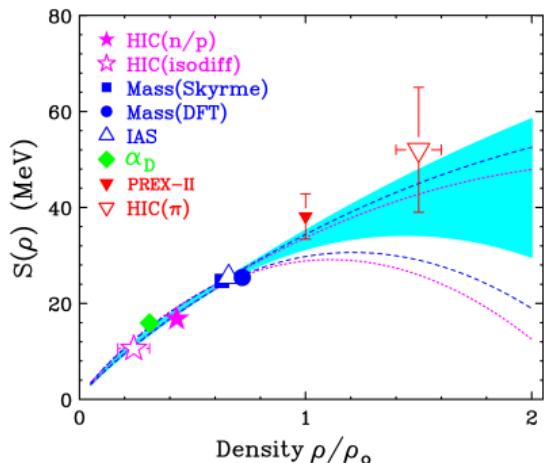
Z<sub>i</sub>Y<sub>i</sub> / Z<sub>tot</sub>

INDRA: Hudan et al., PRC67 (2003) 064613.  
FOPI: Reisdorf et al., NPA 848 (2010) 366.

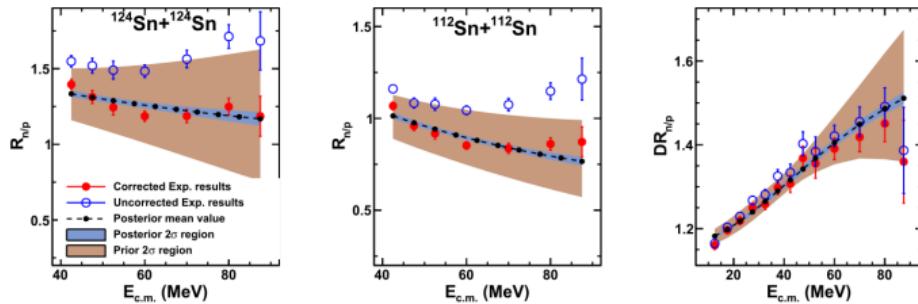
Figure in Ono, PPNP 105 (2019) 139.

This has been a challenge to transport models.

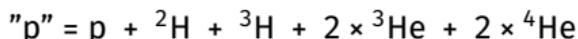
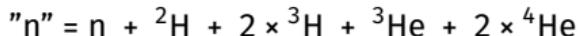
# Symmetry energy constraints and clusters



Lynch, Tsang, arXiv:2106.10119.



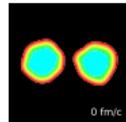
★ HIC(n/p) is a result from “coalescence-invariant (CI)” neutron and proton spectra from Sn + Sn central collisions at 120A MeV.  
[Morfouace et al., PLB 799 (2019) 135045.]



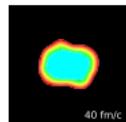
“CI” spectra may (or not?) be affected by cluster formation if many clusters are formed, e.g., through energy conservation.

We have to understand cluster formation, in transport model approaches.

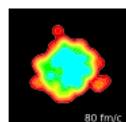
# Cluster recognition and cluster dynamics in transport approaches



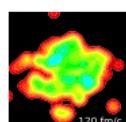
0 fm/c



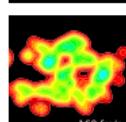
40 fm/c



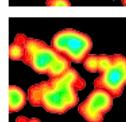
80 fm/c



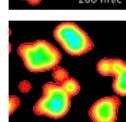
120 fm/c



160 fm/c



200 fm/c



240 fm/c

- Calculate  $e^{-iHt/\hbar} |\psi(t = 0)\rangle$ , and find clusters.
- Cluster correlation in dynamics – Find whether a nucleon moves together with some other nucleons for a while.
  - Clusters will not necessarily be emitted.
  - Correlations affect the time evolution.
- SACA, FRIGA [Le Fèvre et al., PRC 100 (2019) 034904.], for QMD.
- Coalescence prescription, to predict clusters in BUU.
- At large  $t$  (e.g.  $t = 200\text{--}1000 \text{ fm}/c$ ), there is nothing controversial in finding clusters and fragments.
  - ..., if the state at this time is predicted by the transport model reasonably well.
  - The decay of excited fragments should be calculated by a statistical decay code.

## Transport Model Evaluation Project (TMEP)

- Realistic HIC (flow etc.) [J. Xu et al., PRC 93 (2016) 044609]
- Collision term ( $NN \leftrightarrow NN$ ) in a box [Y.X. Zhang et al., PRC 97 (2018) 034625]
- Collision term ( $NN \leftrightarrow N\Delta, \Delta \leftrightarrow N\pi$ ) in a box [Ono et al., PRC 100 (2019) 044617]
- Prediction of pions for the  $S\pi RIT$  systems [G. Jhang et al., PLB 813 (2021) 136016]
- Mean-field dynamics in a box [Colonna et al., PRC 104 (2021) 024603]

Cluster correlations (and other correlations) have not been studied in TMEP so far.

## Existing codes with cluster correlations

- (QMD models) ~ correlations as in the classical mechanics
- BUU with clusters as elementary degrees of freedom
- AMD with clusters (made by nucleon wave packets)

# Transport with clusters (pBUU)

## BUU with clusters

Danielewicz and Bertsch, NPA 533 (1991) 712.

Coupled equations for  $f_n(\mathbf{r}, \mathbf{p}, t)$ ,  $f_p(\mathbf{r}, \mathbf{p}, t)$ ,  $f_d(\mathbf{r}, \mathbf{p}, t)$ ,  $f_t(\mathbf{r}, \mathbf{p}, t)$ ,  $f_h(\mathbf{r}, \mathbf{p}, t)$  are solved by the test particle method.

$$\frac{\partial f_n}{\partial t} + \mathbf{v} \cdot \frac{\partial f_n}{\partial \mathbf{r}} - \frac{\partial U_n}{\partial \mathbf{r}} \cdot \frac{\partial f_n}{\partial \mathbf{p}} = I_n^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_p}{\partial t} + \mathbf{v} \cdot \frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial U_p}{\partial \mathbf{r}} \cdot \frac{\partial f_p}{\partial \mathbf{p}} = I_p^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

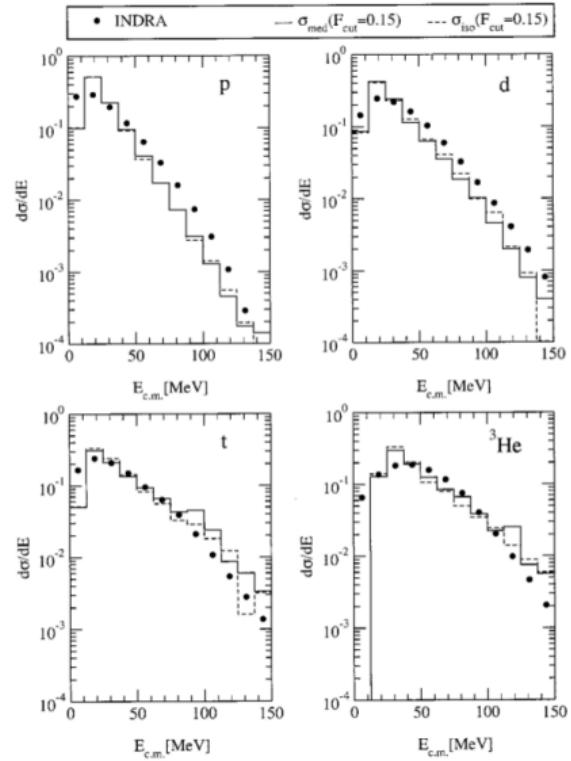
$$\frac{\partial f_d}{\partial t} + \mathbf{v} \cdot \frac{\partial f_d}{\partial \mathbf{r}} - \frac{\partial U_d}{\partial \mathbf{r}} \cdot \frac{\partial f_d}{\partial \mathbf{p}} = I_d^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_t}{\partial t} + \mathbf{v} \cdot \frac{\partial f_t}{\partial \mathbf{r}} - \frac{\partial U_t}{\partial \mathbf{r}} \cdot \frac{\partial f_t}{\partial \mathbf{p}} = I_t^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_h}{\partial t} + \mathbf{v} \cdot \frac{\partial f_h}{\partial \mathbf{r}} - \frac{\partial U_h}{\partial \mathbf{r}} \cdot \frac{\partial f_h}{\partial \mathbf{p}} = I_h^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

Similar transport models for relativistic collisions:

Oliinychenko et al., PRC 99 (2019) 044907. KJ Sun et al., arXiv:2106.12742 [nucl-th].



*Renormalized cluster spectra*

Xe + Sn at 50A MeV

Kuhrts et al., PRC63(2001)034605.

# Transport with clusters (pBUU)

## BUU with clusters

Danielewicz and Bertsch, NPA 533 (1991) 712.

Coupled equations for  $f_n(\mathbf{r}, \mathbf{p}, t)$ ,  $f_p(\mathbf{r}, \mathbf{p}, t)$ ,  $f_d(\mathbf{r}, \mathbf{p}, t)$ ,  $f_t(\mathbf{r}, \mathbf{p}, t)$ ,  $f_h(\mathbf{r}, \mathbf{p}, t)$  are solved by the test particle method.

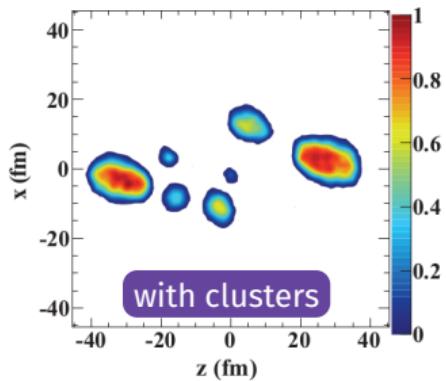
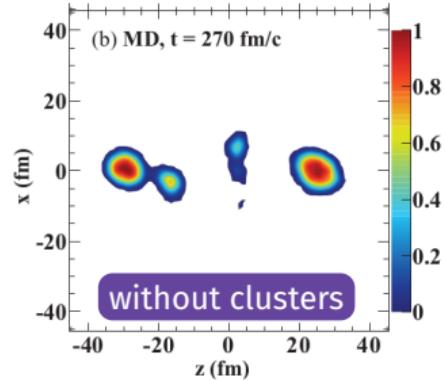
$$\frac{\partial f_n}{\partial t} + \mathbf{v} \cdot \frac{\partial f_n}{\partial \mathbf{r}} - \frac{\partial U_n}{\partial \mathbf{r}} \cdot \frac{\partial f_n}{\partial \mathbf{p}} = I_n^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_p}{\partial t} + \mathbf{v} \cdot \frac{\partial f_p}{\partial \mathbf{r}} - \frac{\partial U_p}{\partial \mathbf{r}} \cdot \frac{\partial f_p}{\partial \mathbf{p}} = I_p^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_d}{\partial t} + \mathbf{v} \cdot \frac{\partial f_d}{\partial \mathbf{r}} - \frac{\partial U_d}{\partial \mathbf{r}} \cdot \frac{\partial f_d}{\partial \mathbf{p}} = I_d^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_t}{\partial t} + \mathbf{v} \cdot \frac{\partial f_t}{\partial \mathbf{r}} - \frac{\partial U_t}{\partial \mathbf{r}} \cdot \frac{\partial f_t}{\partial \mathbf{p}} = I_t^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$

$$\frac{\partial f_h}{\partial t} + \mathbf{v} \cdot \frac{\partial f_h}{\partial \mathbf{r}} - \frac{\partial U_h}{\partial \mathbf{r}} \cdot \frac{\partial f_h}{\partial \mathbf{p}} = I_h^{\text{coll}}[f_n, f_p, f_d, f_t, f_h]$$



Similar transport models for relativistic collisions:

Oliinchenko et al., PRC 99 (2019) 044907. KJ Sun et al., arXiv:2106.12742 [nucl-th].

Isospin diffusion and fragmentation  
Sn + Sn at 50A MeV

Coupland et al., PRC 84 (2011) 054603

# Antisymmetrized Molecular Dynamics (very basic version)



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij} \left[ \exp\left\{-v\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{v}}\right)^2\right\} \chi_{a_i}(j) \right]$$

$$\mathbf{Z}_i = \sqrt{v}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{v}}\mathbf{K}_i$$

$v$  : Width parameter =  $(2.5 \text{ fm})^{-2}$

$\chi_{a_i}$  : Spin-isospin states =  $p \uparrow, p \downarrow, n \uparrow, n \downarrow$

Equation of motion for the wave packet centroids  $Z$

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, H\}_{\text{PB}} + (\text{NN collisions})$$

$\{\mathbf{Z}_i, H\}_{\text{PB}}$ : Motion in the mean field

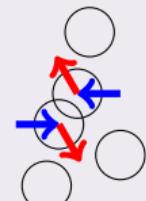
$$H = \frac{\langle \Phi(Z) | H | \Phi(Z) \rangle}{\langle \Phi(Z) | \Phi(Z) \rangle} + (\text{c.m. correction})$$

$H$ : Effective interaction (e.g. Skyrme force)

NN collisions

$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \psi_f | V | \psi_i \rangle|^2 \delta(E_f - E_i)$$

- $|V|^2$  or  $\sigma_{NN}$  (in medium)
- Pauli blocking

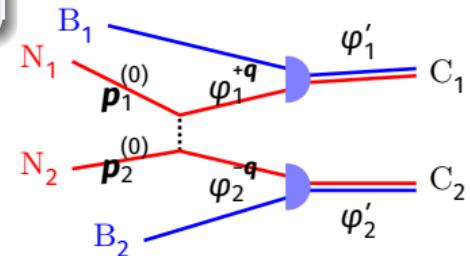


Ono, Horiuchi et al., Prog. Theor. Phys. 87 (1992) 1185.

# NN collisions with cluster correlations



- $N_1, N_2$ : Colliding nucleons
- $B_1, B_2$ : Spectator nucleons/clusters
- $C_1, C_2$ :  $N, (2N), (3N), (4N)$  (up to a cluster)



## Transition probability

$$W(\text{NBNB} \rightarrow \text{CC}) = \frac{2\pi}{\hbar} |\langle \text{CC} | V | \text{NBNB} \rangle|^2 \delta(E_f - E_i)$$

$$vd\sigma \propto |\langle \varphi'_1 | \varphi_1^{+q} \rangle|^2 |\langle \varphi'_2 | \varphi_2^{-q} \rangle|^2 |M|^2 \delta(E_f - E_i) p_{\text{rel}}^2 d\Omega$$

$|M|^2 = |\langle \text{NN} | V | \text{NN} \rangle|^2$ : Matrix elements of NN scattering  
 $\Leftarrow (d\sigma/d\Omega)_{\text{NN}}$  in free space (or in medium)

$$\mathbf{p}_{\text{rel}} = \frac{1}{2} (\mathbf{p}_1 - \mathbf{p}_2) = p_{\text{rel}} \hat{\Omega}$$

$$\mathbf{p}_1 = \mathbf{p}_1^{(0)} + \mathbf{q}$$

$$\mathbf{p}_2 = \mathbf{p}_2^{(0)} - \mathbf{q}$$

$$\varphi_1^{+q} = \exp(+i\mathbf{q} \cdot \mathbf{r}_{N_1}) \varphi_1^{(0)}$$

$$\varphi_2^{-q} = \exp(-i\mathbf{q} \cdot \mathbf{r}_{N_2}) \varphi_2^{(0)}$$

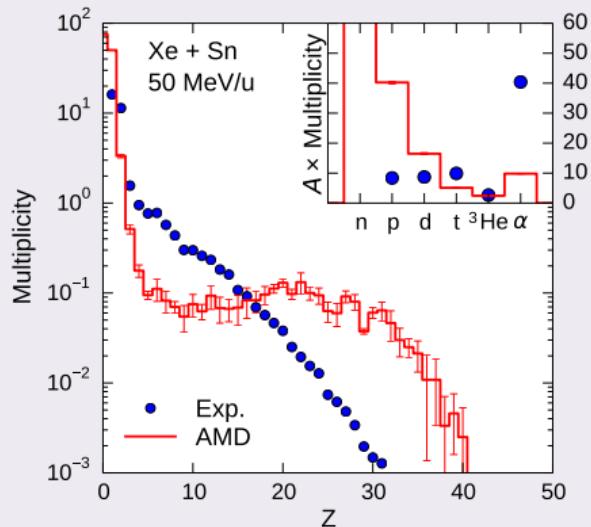
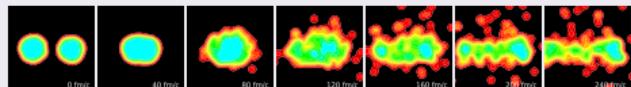
Ono, J. Phys. Conf. Ser. 420 (2013) 012103.

Ikeno, Ono et al., PRC 93 (2016) 044612.

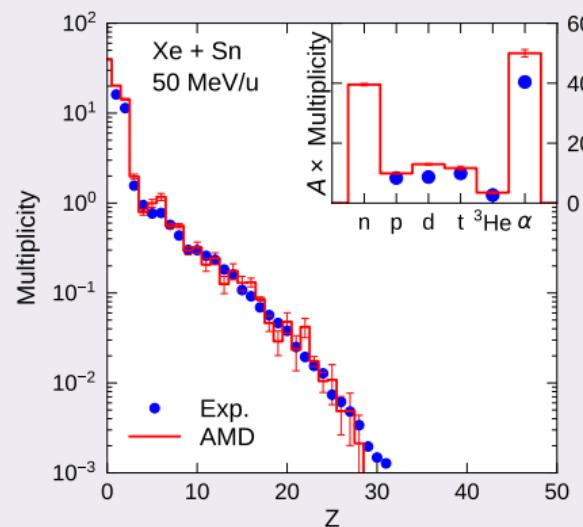
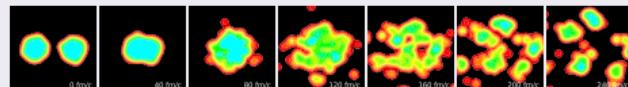
Ono, JPS Conference Proceedings 32 (2020) 010076.

# Effect of cluster correlations: central Xe + Sn at 50 MeV/u

## Without clusters

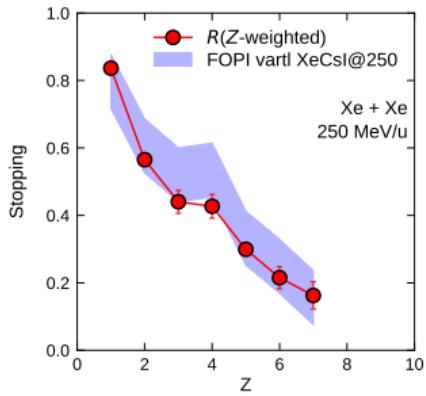
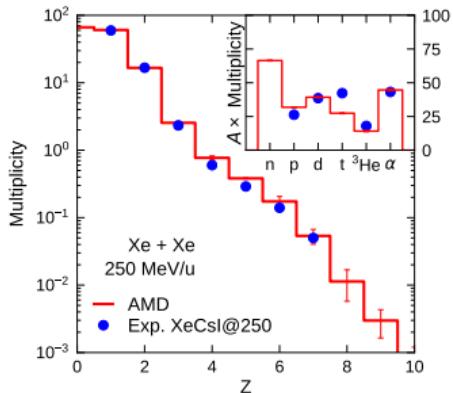


## With clusters



INDRA data: Hudan et al., PRC67 (2003) 064613.

# Comparison for Xe + CsI (or Xe) @250A MeV



Central collisions at 250 MeV/nucleon

- AMD:  $^{129}\text{Xe} + ^{129}\text{Xe}$
- FOPI: Xe + CsI [Reisdorf et al., NPA 848 (2010) 366]

Parameters have been adjusted to reasonably reproduce stopping in collisions at 50–250 MeV/nucleon. [Ono, JPS Conference Proceedings 32 (2020) 010076]



$$vd\sigma \propto |\langle \varphi'_1 | \varphi_1^{+q} \rangle|^2 |\langle \varphi'_2 | \varphi_2^{-q} \rangle|^2 |M|^2 \delta(E_f - E_i) p_{\text{rel}}^2 dp_{\text{rel}} d\Omega$$

$$\Rightarrow P(C_1, C_2, p_{\text{rel}}, \Omega) \times \left| M(p_{\text{rel}}^{(0)}, p_{\text{rel}}, \Omega) \right|^2 \times \frac{p_{\text{rel}}^2 d\Omega}{\partial E_f / \partial p_{\text{rel}}}$$

- In-medium  $NN$  cross sections (or matrix elements)
  - Some in-medium effects are automatically included in addition to the matrix element.
- Condition to switch on/off cluster formation

# Finding a cluster (or clusters) in a many-body state (in general)

One proton( $\uparrow$ ) and one neutron( $\uparrow$ ) (both described by Gaussian wave packets)

$$\left| \begin{array}{c} p \\ n \end{array} \right\rangle = c_d \left| \begin{array}{c} d \\ n \end{array} \right\rangle + c' \left| \begin{array}{c} p \\ n \end{array} \right\rangle + \dots$$

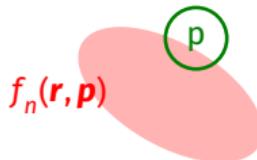
$$\varphi_p(\mathbf{r}_1) \varphi_n(\mathbf{r}_2) = \left[ c_d \underline{\psi_d(\mathbf{r})} + (\text{continuum states}) \right] \varphi_{\text{cm}}(\mathbf{R}), \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$$

$$P_d(\mathbf{P}) = \left| \langle e^{i\mathbf{P}\cdot\mathbf{R}/\hbar} \underline{\psi_d} | \varphi_p \varphi_n \rangle \right|^2 = \int \frac{d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{p}}{(2\pi\hbar)^3} \underline{\rho_d^W(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{p})} f_p(\mathbf{r}_1, \frac{1}{2}\mathbf{P} + \mathbf{p}) f_n(\mathbf{r}_2, \frac{1}{2}\mathbf{P} - \mathbf{p})$$

One proton( $\uparrow$ ) and many neutrons( $\uparrow$ )

$$P_d(\mathbf{P}) \stackrel{??}{=} \int \frac{d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{p}}{(2\pi\hbar)^3} \underline{\rho_d^W(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{p})} f_p(\mathbf{r}_1, \frac{1}{2}\mathbf{P} + \mathbf{p}) f_n(\mathbf{r}_2, \frac{1}{2}\mathbf{P} - \mathbf{p})$$

LW Chen, CM Ko, BA Li, NPA 729 (2003) 809. Mattiello et al., PRC 55 (1997) 1443.



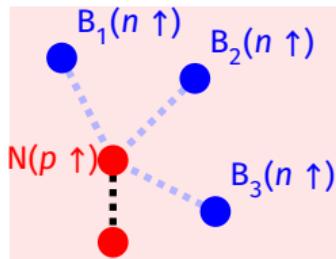
This is valid only in the dilute limit. In general, e.g, the “probability”

$$N_d = \int \frac{d\mathbf{P}}{(2\pi\hbar)^3} P_d(\mathbf{P}) \text{ can be } > 1.$$

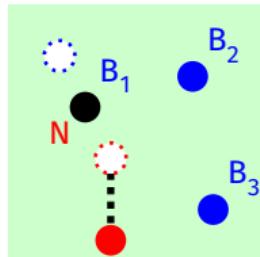
- Identifying clusters in a many-body system is a fundamental problem.
- Another question is how the identified clusters are propagated.

# Construction of Final States in AMD/Cluster

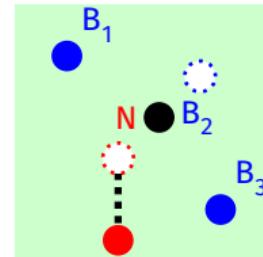
Clusters (in the final states) are assumed to have  $(0s)^N$  configuration.



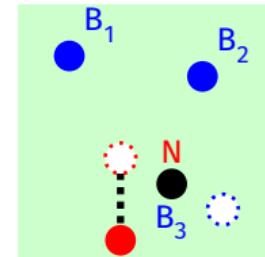
$$|\Phi^q\rangle \\ \text{After } p^{(0)} \rightarrow p^{(0)} + q$$



$$|\Phi'_1\rangle \\ N + B_1 \rightarrow C_1$$



$$|\Phi'_2\rangle \\ N + B_2 \rightarrow C_2$$



$$|\Phi'_3\rangle \\ N + B_3 \rightarrow C_3$$

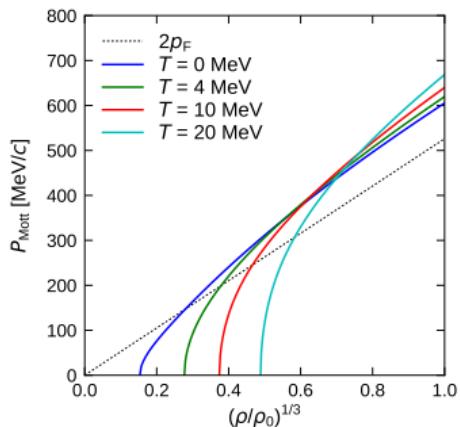
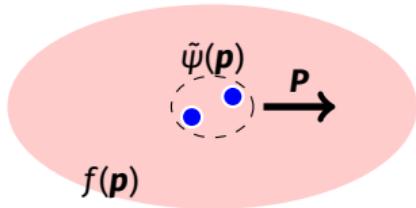
Final states are not orthogonal:  $N_{ij} \equiv \langle \Phi'_i | \Phi'_j \rangle \neq \delta_{ij}$

The probability of cluster formation with one of B's:

$$\hat{P} = \sum_{ij} |\Phi'_i\rangle N_{ij}^{-1} \langle \Phi'_j|, \quad P = \langle \Phi^q | \hat{P} | \Phi^q \rangle \quad \neq \sum_i |\langle \Phi'_i | \Phi^q \rangle|^2$$

$$\begin{cases} P & \Rightarrow \text{Choose one of the candidates and make a cluster.} \\ 1 - P & \Rightarrow \text{Don't make a cluster (with any n↑).} \end{cases}$$

# A cluster in medium (theory)



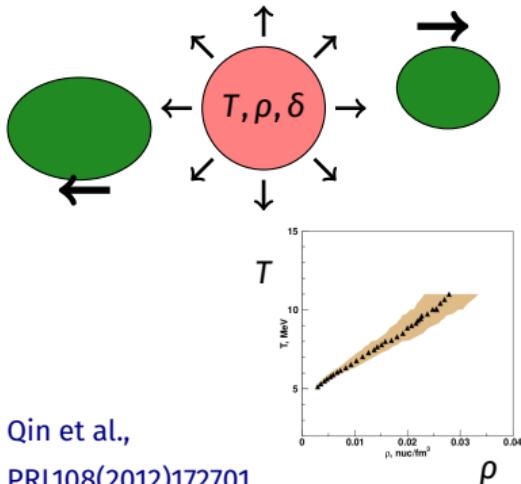
Equation for a deuteron in uncorrelated medium

$$\begin{aligned} & \left[ e\left(\frac{1}{2}\mathbf{P} + \mathbf{p}\right) + e\left(\frac{1}{2}\mathbf{P} - \mathbf{p}\right) \right] \tilde{\psi}(\mathbf{p}) \\ & + \left[ 1 - f\left(\frac{1}{2}\mathbf{P} + \mathbf{p}\right) - f\left(\frac{1}{2}\mathbf{P} - \mathbf{p}\right) \right] \int \frac{d\mathbf{p}'}{(2\pi)^3} \langle \mathbf{p} | v | \mathbf{p}' \rangle \tilde{\psi}(\mathbf{p}') \\ & = E \tilde{\psi}(\mathbf{p}) \end{aligned}$$

- A bound deuteron cannot exist inside the Fermi sphere, except at very low densities.
- A deuteron can exist if its momentum is high enough.
- When there is no bound solution, is it still possible that nucleons are correlated in the continuum?
- What happens when the medium is correlated?

Formula from Röpke, NPA867 (2011) 66.

# Clusters in low-density medium (experiments)



Qin et al.,  
PRL108(2012)172701

Ar, Zn + Sn @ 47A MeV

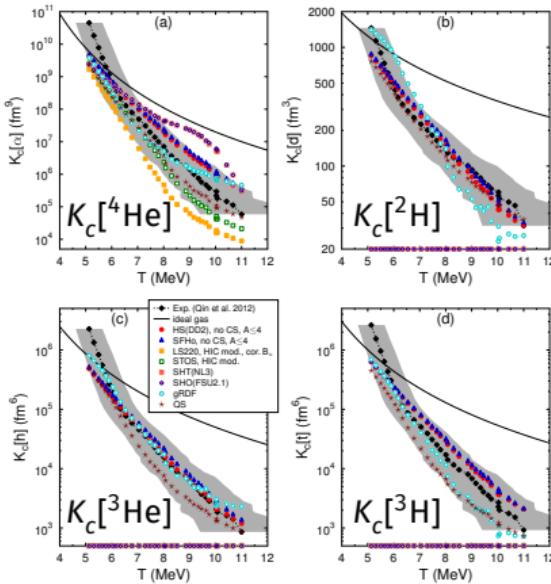
## Assumption

The particles emitted at the same velocity  $v_{\text{surf}}$  were emitted at the same time from the same source characterized by  $(T, \rho, \delta)$ .

**Equilibrium Constants**

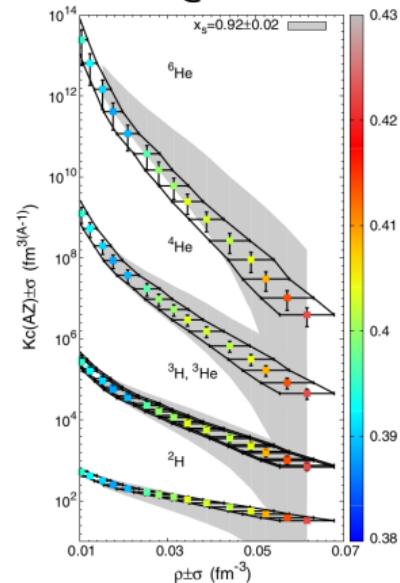
$$K_c(N, Z) = \frac{\rho(N, Z)}{\rho_p^Z \rho_n^N} \quad \text{for cluster } (N, Z)$$

Hempel et al., PRC 91 (2015) 045805.

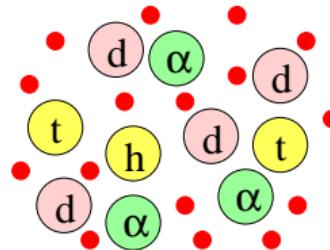
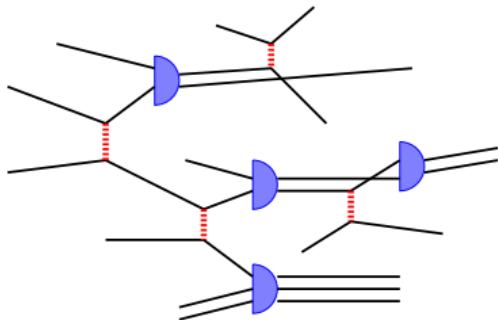


Paris, Bougault et al.,  
PRL 125 (2020) 012701.

Xe + Sn @ 32A MeV



## Some more details of AMD with clusters



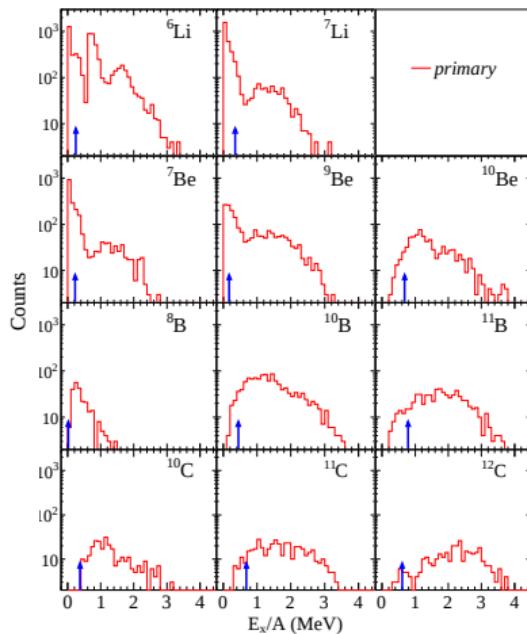
- A cluster in AMD is still composed of nucleon wave packets. The many-body state is a Slater determinant of nucleons.
- Clusters are not only created but also broken.
- Clusters are in medium, so the existence and the properties may be modified. (Cluster formation may be turned off depending on a phase-space density.)
- For a collision, there are many possible configurations ( $C_1, C_2$ ) of cluster formation. Non-orthogonality is treated suitably.
- Correlations to bind several clusters (like  $\alpha + t = {}^7\text{Li}$ ) are also important and taken into account.

- $n + p + X \leftrightarrow d + X'$
- $d + n + X \leftrightarrow t + X'$
- $d + p + X \leftrightarrow h + X'$
- $t + p + X \leftrightarrow \alpha + X'$
- $h + n + X \leftrightarrow \alpha + X'$
- $d + d + X \leftrightarrow \alpha + X'$
- $2n + p + X \leftrightarrow t + X'$
- $n + 2p + X \leftrightarrow h + X'$
- $d + n + p + X \leftrightarrow \alpha + X'$
- $2n + 2p + X \leftrightarrow \alpha + X'$
- $d + d \leftrightarrow p + t$
- $d + d \leftrightarrow n + h$
- $p + t \leftrightarrow n + h$
- $d + t \leftrightarrow n + \alpha$
- $d + h \leftrightarrow p + \alpha$
- $d + t \leftrightarrow 2n + h$
- $d + h \leftrightarrow 2p + t$
- $d + \alpha \leftrightarrow t + h$

# Production of light nuclei

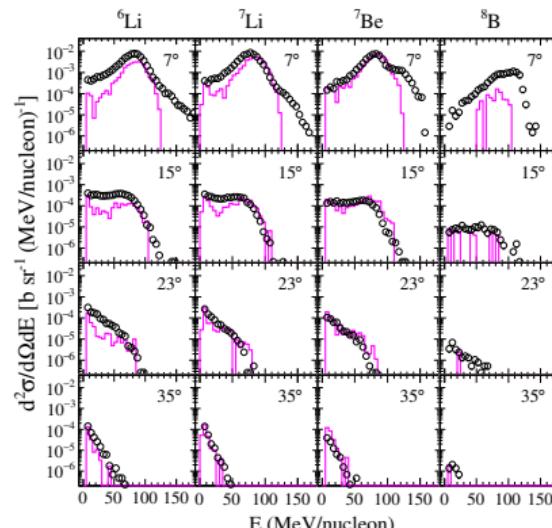
$^{12}\text{C} + ^{12}\text{C}$  at 95 MeV/nucleon

Tian et al., PRC 97 (2018) 034610.



Excitation energy per nucleon

Some light nuclei are emitted at large angles ( $\theta_{\text{lab}} > 20^\circ$ ) almost in their ground states, at  $t = 300 \text{ fm}/c$ .

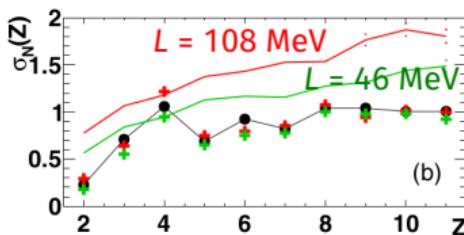
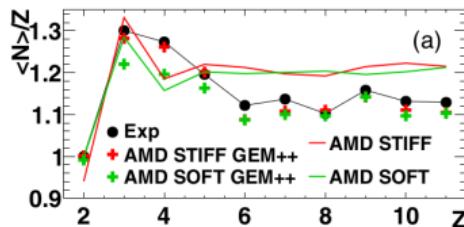


Exp. Data:

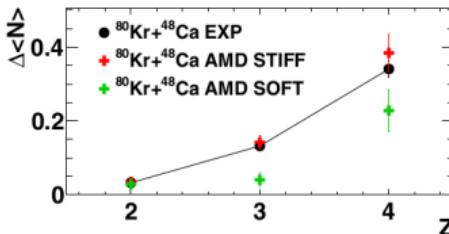
Dudouet et al., PRC 88 (2013) 024606.,  
<http://hadrontherapy-data.in2p3.fr/>.

# Isospin drift (+ isospin diffusion + isospin fractionation)

Fragments in  $v_{cm} < v < v_{QP}$

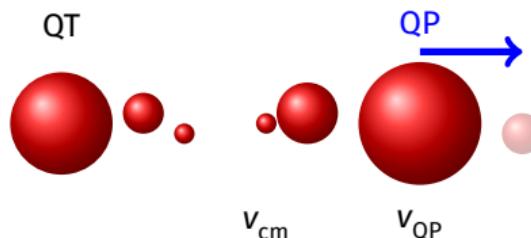


$(v_{cm} < v < v_{QP}) - (v > v_{QP})$



Piantelli et al. (FAZIA Collaboration), PRC 103 (2021) 014603.

$^{80}\text{Kr} + ^{48}\text{Ca}$  at 35 MeV/nucleon (INFN-LNS)



- In the width of the isotope distribution  $\sigma_N(Z)$ , clear dependence on the stiffness of the symmetry energy is found, for the excited fragments produced in AMD.
  - After statistical decays calculated by GEMINI++, the sensitivity in  $\sigma_N(Z)$  becomes weaker.
- For the final He, Li and Be fragments (emitted backward in the QP frame), the yields of neutron-rich isotopes are sensitive to the stiffness of the symmetry energy.

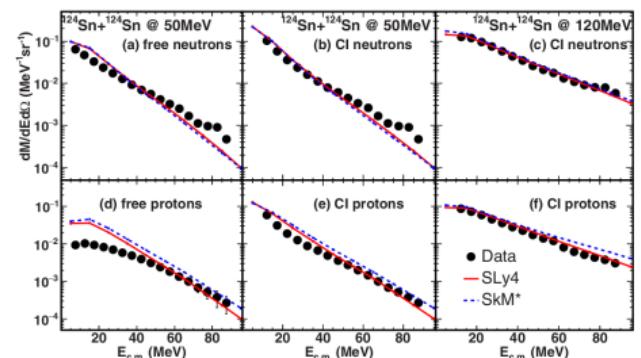
$L = 108 \text{ MeV}$  (strong isospin drift) is more favored than  $L = 46 \text{ MeV}$ .

# Spectra and ratios of emitted nucleons and clusters in central collisions

MSU data:

Coupland et al., PRC 96 (2016) 011601(R).

$\text{Sn} + \text{Sn}$ ,  $E/A = 50$  and  $120$  MeV,  $b \lesssim 3$  fm



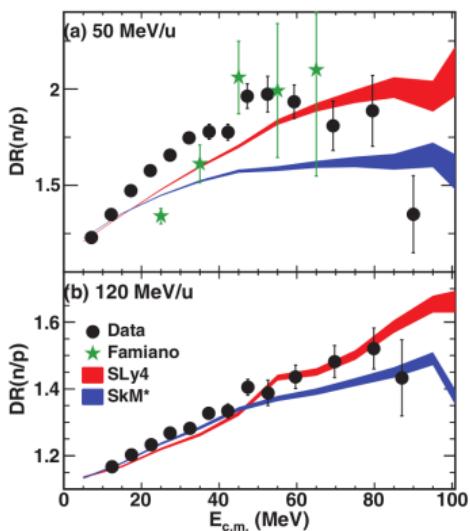
CI = (free nucleons) + (nucleons in clusters)

Bayesian analyses at  $E/A = 120$  MeV by Morfouace et al., PLB 799 (2019) 135045.

- $(m_n^* - m_p^*)/m_N = -0.05^{+0.09}_{-0.09} (\rho_n - \rho_p)/\rho$
- $S(\rho_s) = 16.8^{+1.2}_{-1.2} \text{ MeV at } \rho_s/\rho_0 = 0.43^{+0.05}_{-0.05}$

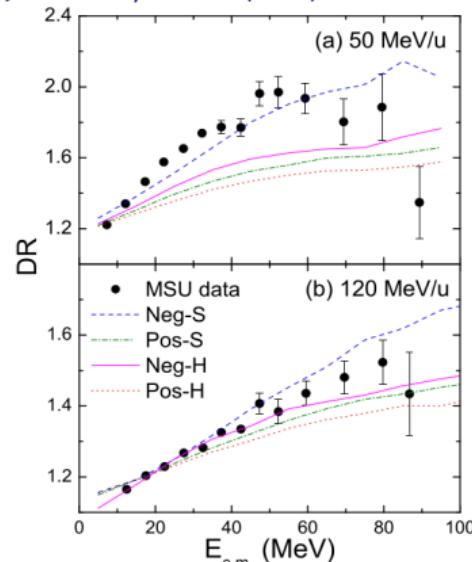
$$\text{DR} = \frac{\text{"n/p" in } ^{124}\text{Sn} + ^{124}\text{Sn}}{\text{"n/p" in } ^{112}\text{Sn} + ^{112}\text{Sn}}$$

compared with ImQMD



DR compared with an IQMD

J. Su et al., PRC 94 (2016) 034619.

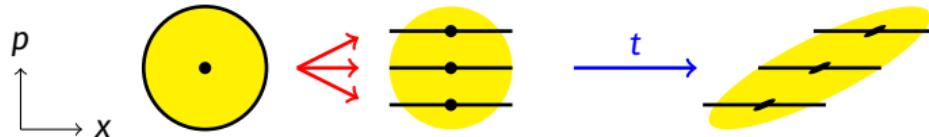


Similar result in

YX Zhang et al., PLB 732 (2014) 186.

# Transition from a wave packet to a plane wave

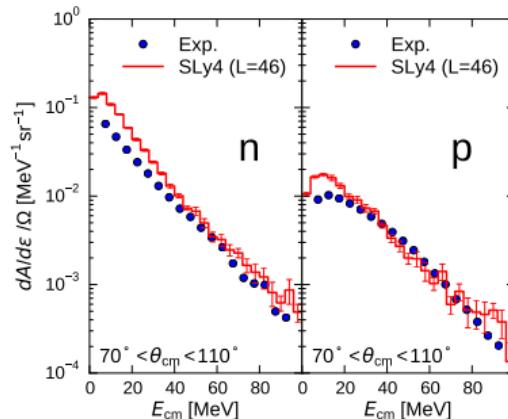
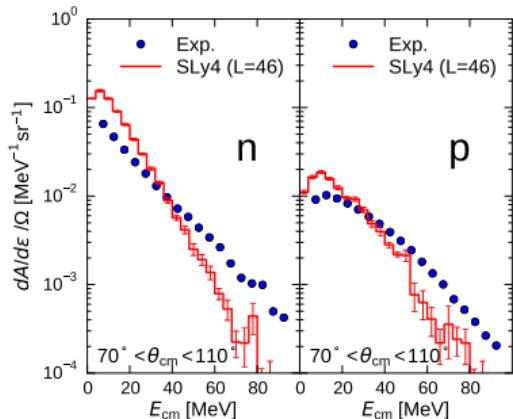
Each wave packet has a momentum width. For example, it is an important part of the Fermi motion of nucleons in a nucleus.



$$f(\mathbf{p}) = N e^{-(\mathbf{p}-\mathbf{P})^2/2\Delta p^2}$$

$$\mathbf{p} = \mathbf{P} + \Delta \mathbf{p}$$

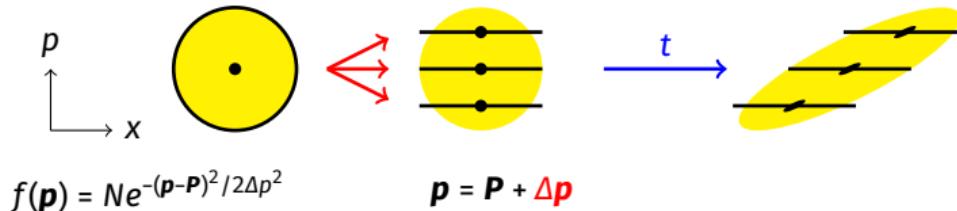
⇒ Improvement of nucleon spectra



$^{124}\text{Sn} + ^{124}\text{Sn}$ ,  $E/A = 50$  MeV/u,  $b \approx 0$  Data: Coupland et al., PRC 94 (2016) 011601(R).

## Transition from a wave packet to a plane wave

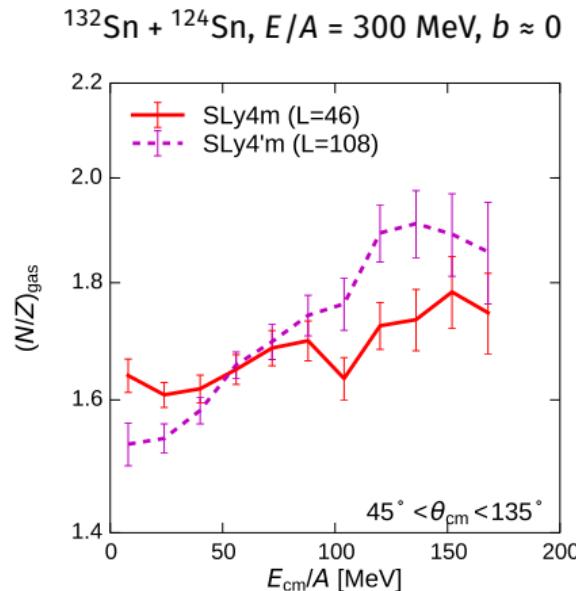
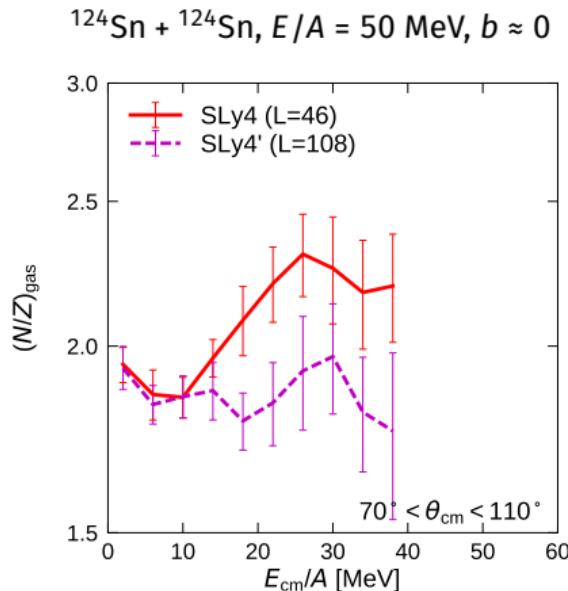
Each wave packet has a momentum width. For example, it is an important part of the Fermi motion of nucleons in a nucleus.



The momentum fluctuation  $\Delta p$  is given to a wave packet when it is '**emitted**', following [Ono and Horiuchi, PRC53 \(1996\) 845](#) [a simple version of wp splitting].

- For a formed cluster, the momentum fluctuation is given to its center-of-mass motion.
- Total momentum and energy conservation.
- A particle is regarded as '**emitted**' when there is no other particles around it in phase space within the radius  $(\Delta r, \Delta v) = (3.5 \text{ fm}, 0.25c)$ .
- Consistency with the method of the zero-point energy correction.

# $N/Z$ Ratio at 50 and 300 MeV/u calculated by AMD with clusters



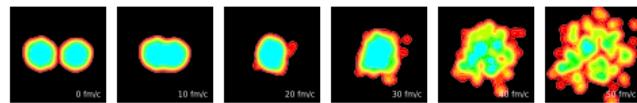
Low density effect       $\leftrightarrow$       High density effect

$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(\epsilon) + Y_d(\epsilon) + 2Y_t(\epsilon) + Y_h(\epsilon) + 2Y_\alpha(\epsilon)}{Y_p(\epsilon) + Y_d(\epsilon) + Y_t(\epsilon) + 2Y_h(\epsilon) + 2Y_\alpha(\epsilon)}, \quad \epsilon = E_{\text{cm}}/A$$

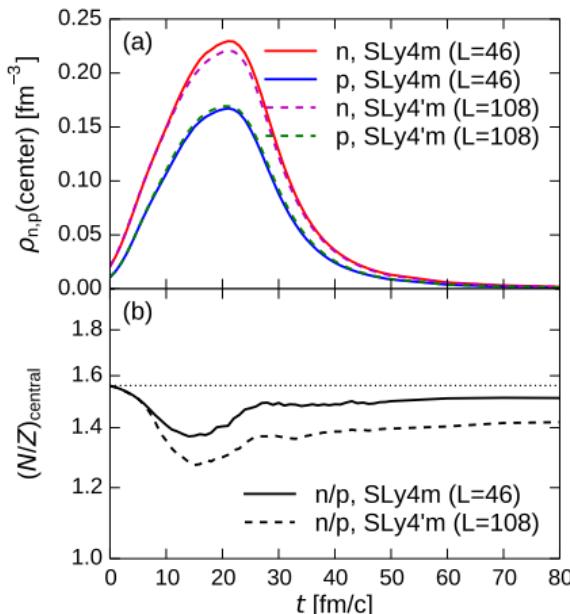
For 50A MeV, the  $(N/Z)_{\text{gas}}$  ratio in the AMD calculation seems to be higher than in QMD calculations.

However, the nucleon spectra (before taking ratio) need to be understood better in calculations to draw a definite conclusion.

# Compression and expansion in collisions at 300 MeV/nucleon



$^{132}\text{Sn} + ^{124}\text{Sn}$ ,  $E/A = 300 \text{ MeV}$ ,  $b \sim 0$



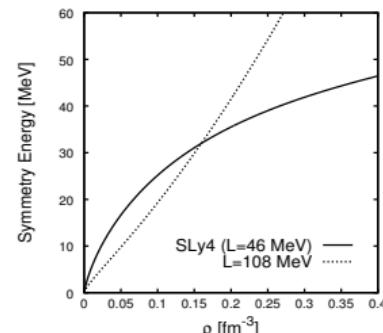
“central”: Inside of the sphere which includes 25% of total nucleons.

## Nuclear EOS (at $T = 0$ )

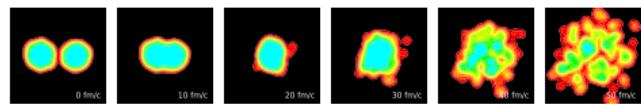
$$(E/A)(\rho_p, \rho_n) = (E/A)_0(\rho) + S(\rho)\delta^2 + \dots$$

$$\rho = \rho_p + \rho_n, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

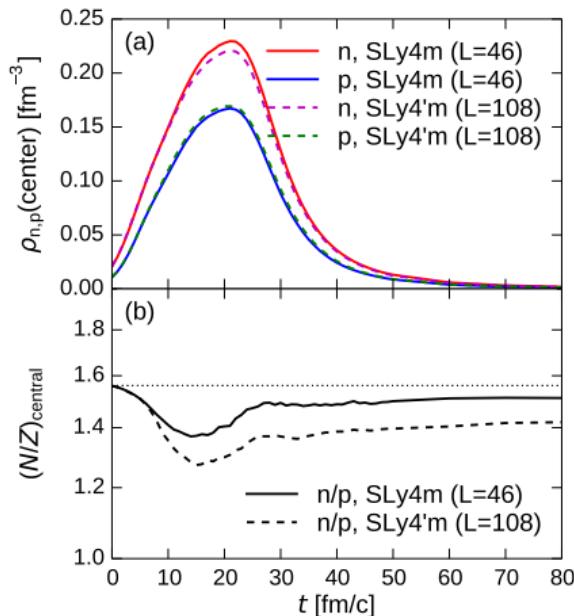
- $S_0 = S(\rho_0)$
- $L = 3\rho_0(dS/d\rho)_{\rho=\rho_0}$



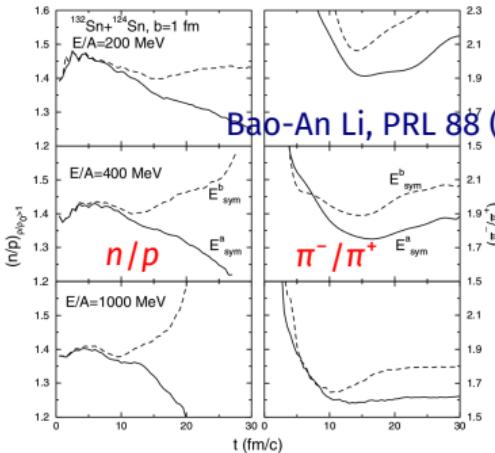
# Compression and expansion in collisions at 300 MeV/nucleon



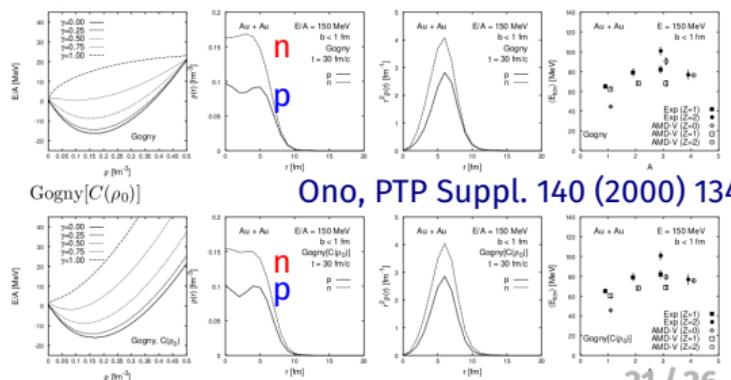
$^{132}\text{Sn} + ^{124}\text{Sn}$ ,  $E/A = 300 \text{ MeV}$ ,  $b \sim 0$



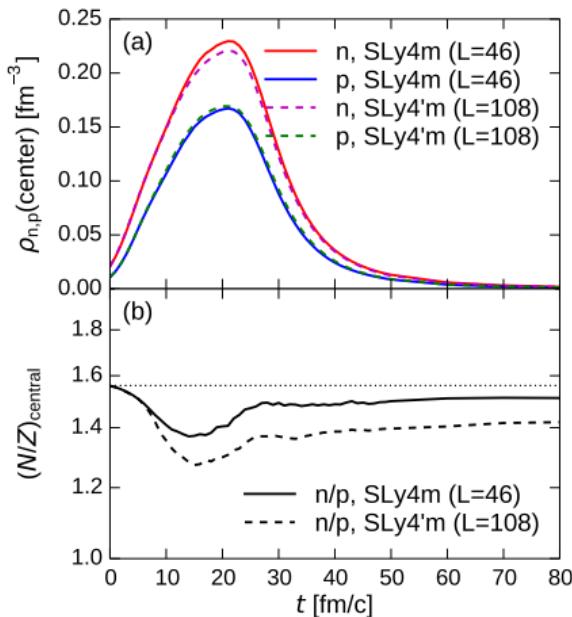
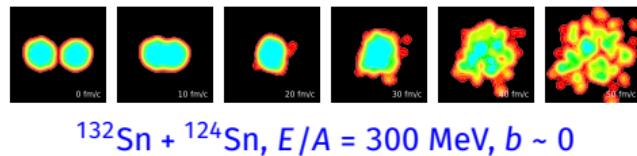
“central”: Inside of the sphere which includes 25% of total nucleons.



Gogny



# Compression and expansion in collisions at 300 MeV/nucleon

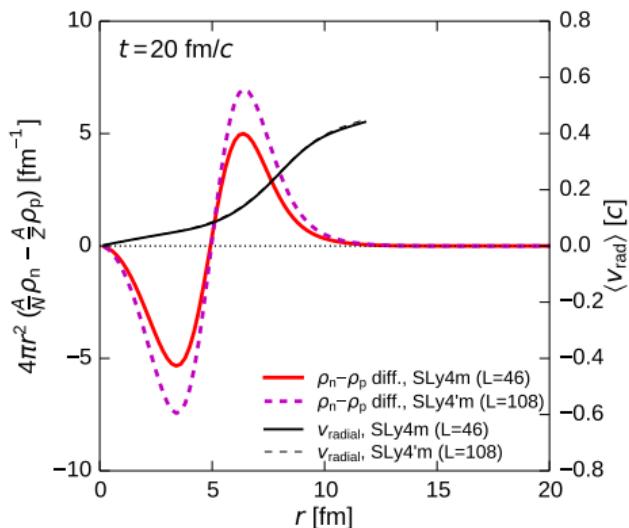


“central”: Inside of the sphere which includes 25% of total nucleons.

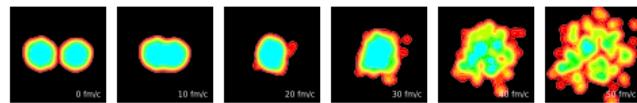
- Neutron-proton density diff. (fn of  $r$ )

$$4\pi r^2 \left[ \frac{A}{N} \rho_n(r) - \frac{A}{Z} \rho_p(r) \right]$$

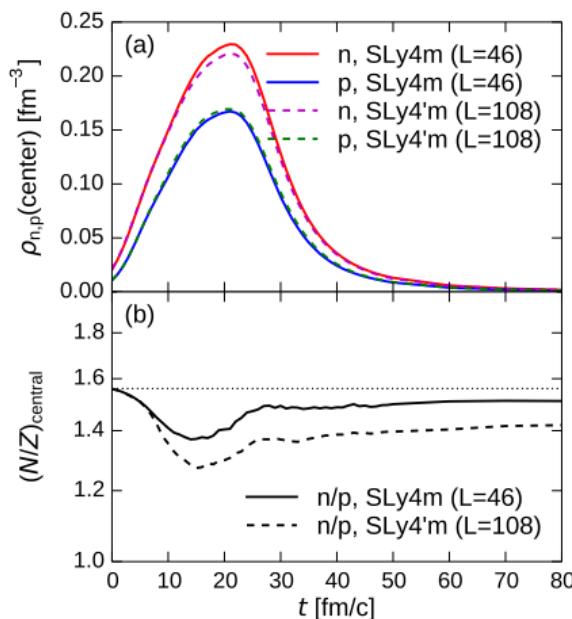
- Radial expansion velocity  $v_{\text{rad}}(r)$



# Compression and expansion in collisions at 300 MeV/nucleon



$^{132}\text{Sn} + ^{124}\text{Sn}$ ,  $E/A = 300 \text{ MeV}$ ,  $b \sim 0$

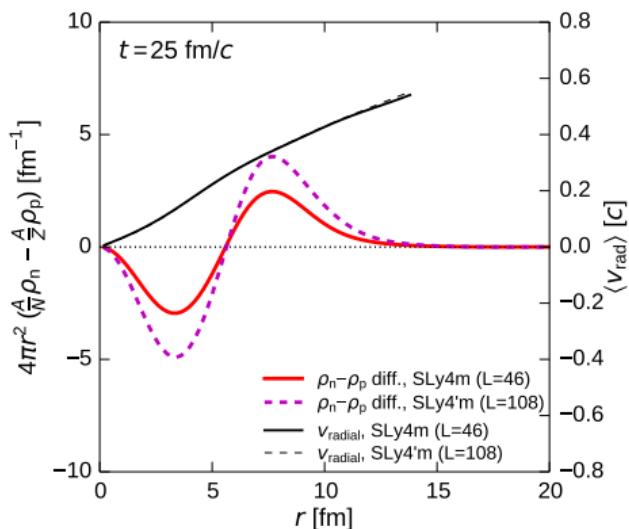


“central”: Inside of the sphere which includes 25% of total nucleons.

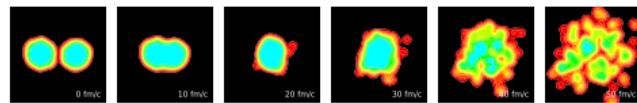
- Neutron-proton density diff. (fn of  $r$ )

$$4\pi r^2 \left[ \frac{A}{N} \rho_n(r) - \frac{A}{Z} \rho_p(r) \right]$$

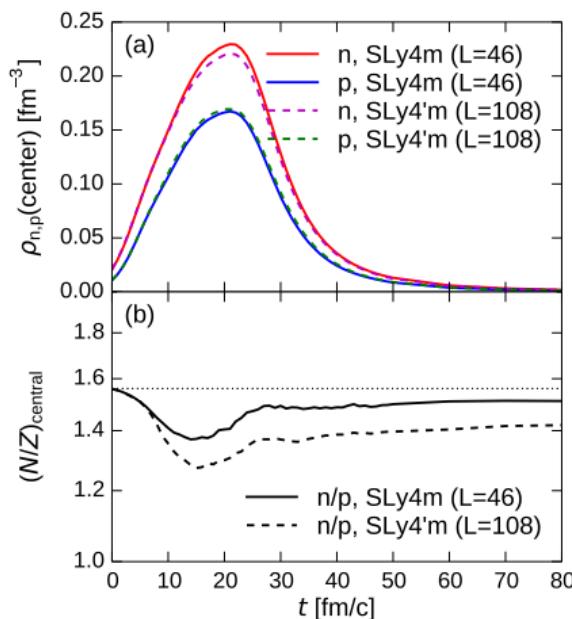
- Radial expansion velocity  $v_{\text{rad}}(r)$



# Compression and expansion in collisions at 300 MeV/nucleon



$^{132}\text{Sn} + ^{124}\text{Sn}$ ,  $E/A = 300 \text{ MeV}$ ,  $b \sim 0$

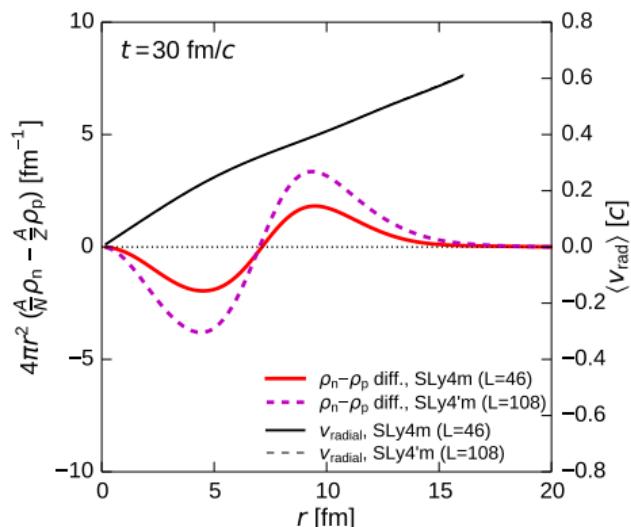


“central”: Inside of the sphere which includes 25% of total nucleons.

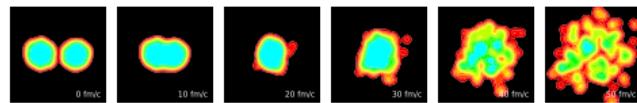
- Neutron-proton density diff. (fn of  $r$ )

$$4\pi r^2 \left[ \frac{A}{N} \rho_n(r) - \frac{A}{Z} \rho_p(r) \right]$$

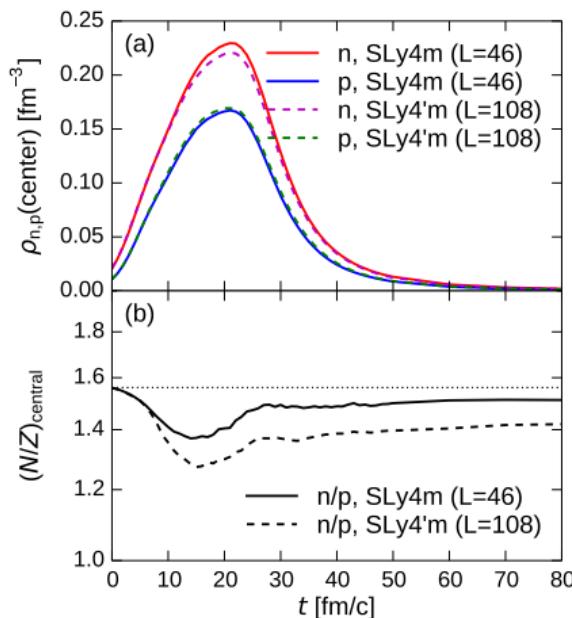
- Radial expansion velocity  $v_{\text{rad}}(r)$



# Compression and expansion in collisions at 300 MeV/nucleon



$^{132}\text{Sn} + ^{124}\text{Sn}$ ,  $E/A = 300 \text{ MeV}$ ,  $b \sim 0$

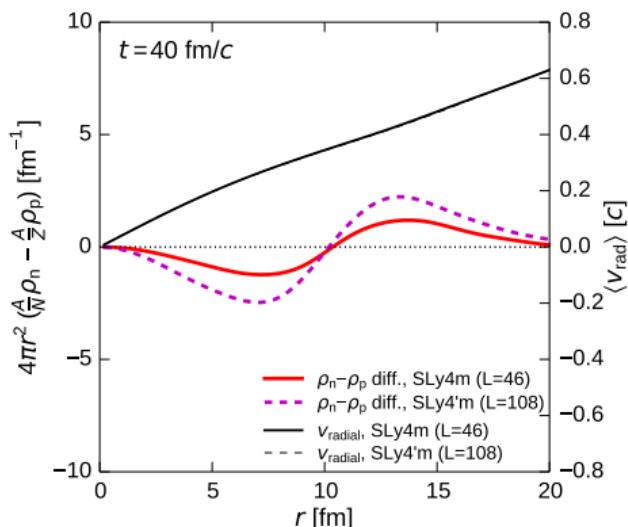


“central”: Inside of the sphere which includes 25% of total nucleons.

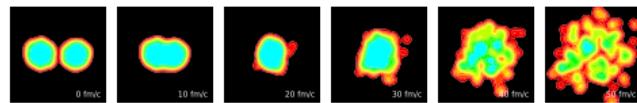
- Neutron-proton density diff. (fn of  $r$ )

$$4\pi r^2 \left[ \frac{A}{N} \rho_n(r) - \frac{A}{Z} \rho_p(r) \right]$$

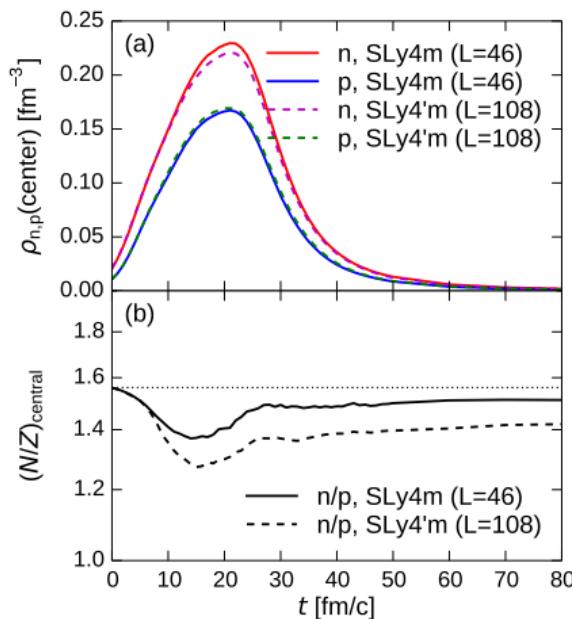
- Radial expansion velocity  $v_{\text{rad}}(r)$



# Compression and expansion in collisions at 300 MeV/nucleon



$^{132}\text{Sn} + ^{124}\text{Sn}$ ,  $E/A = 300 \text{ MeV}$ ,  $b \sim 0$

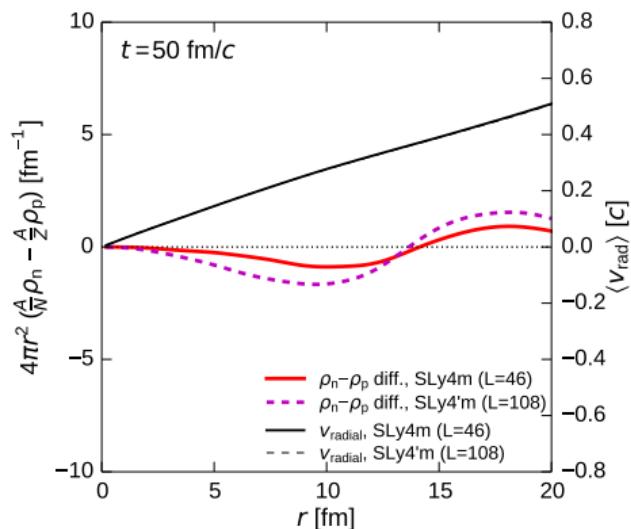


“central”: Inside of the sphere which includes 25% of total nucleons.

- Neutron-proton density diff. (fn of  $r$ )

$$4\pi r^2 \left[ \frac{A}{N} \rho_n(r) - \frac{A}{Z} \rho_p(r) \right]$$

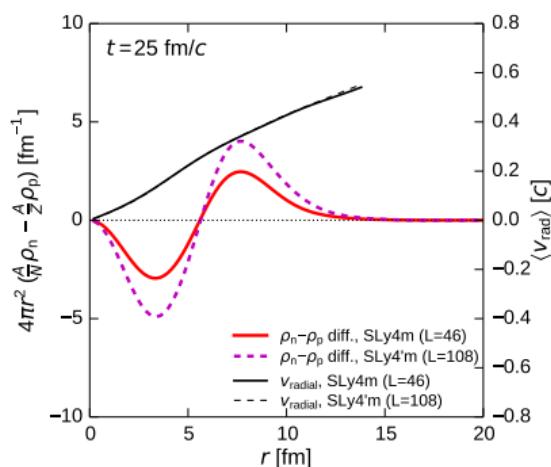
- Radial expansion velocity  $v_{\text{rad}}(r)$



# $N/Z$ Spectrum Ratio – an observable

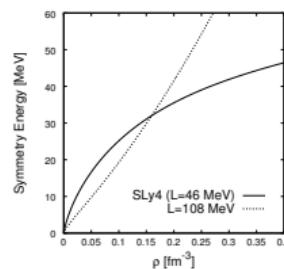
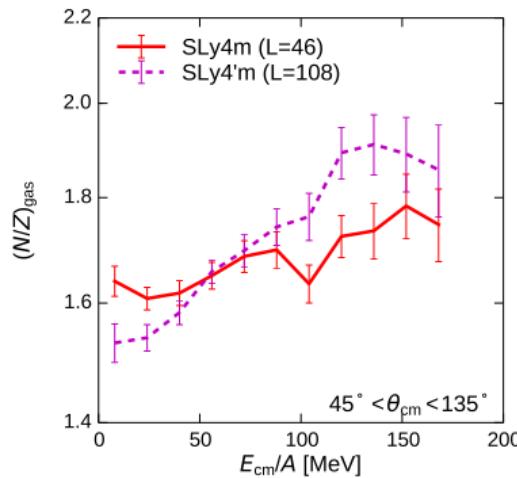
A. Ono, EPJ Web of Conferences 117 (2016) 07003.

$\rho_n - \rho_p$  at the compression stage



↔  
similar

$N/Z$  of the spectrum of emitted particles



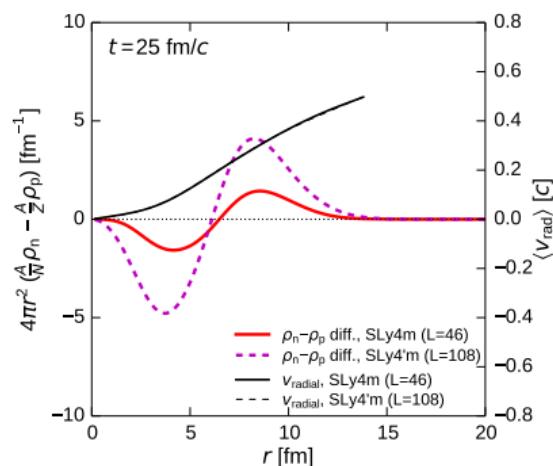
$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(\epsilon) + Y_d(\epsilon) + 2Y_t(\epsilon) + Y_h(\epsilon) + 2Y_a(\epsilon)}{Y_p(\epsilon) + Y_d(\epsilon) + Y_t(\epsilon) + 2Y_h(\epsilon) + 2Y_a(\epsilon)}, \quad \epsilon = E_{\text{cm}}/A$$

Caution: The similarity strongly depends on the dynamics with cluster correlations.

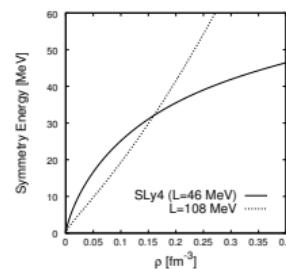
# $N/Z$ Spectrum Ratio – an observable

A. Ono, EPJ Web of Conferences 117 (2016) 07003.

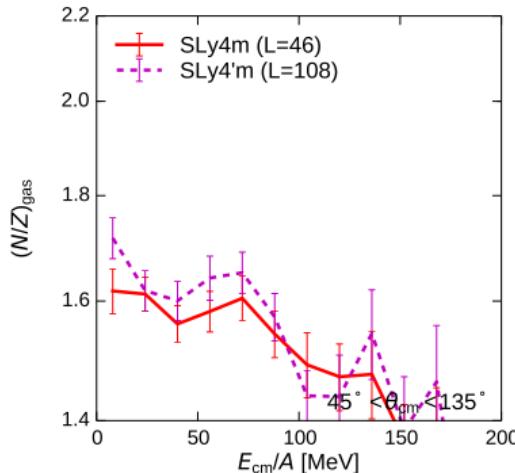
$\rho_n - \rho_p$  at the compression stage



$\Leftrightarrow$   
 Not similar  
 when cluster  
 is turned off.



$N/Z$  of the spectrum of emitted particles



$$\left(\frac{N}{Z}\right)_{\text{gas}} = \frac{Y_n(\epsilon) + Y_d(\epsilon) + 2Y_t(\epsilon) + Y_h(\epsilon) + 2Y_a(\epsilon)}{Y_p(\epsilon) + Y_d(\epsilon) + Y_t(\epsilon) + 2Y_h(\epsilon) + 2Y_a(\epsilon)}, \quad \epsilon = E_{\text{cm}}/A$$

Caution: The similarity strongly depends on the dynamics with cluster correlations.

M. Kaneko, Murakami, Isobe, Kurata-Nishimura, Ono, Ikeda et al. ( $\pi\pi$ RIT), PLB 822 (2021) 136681.

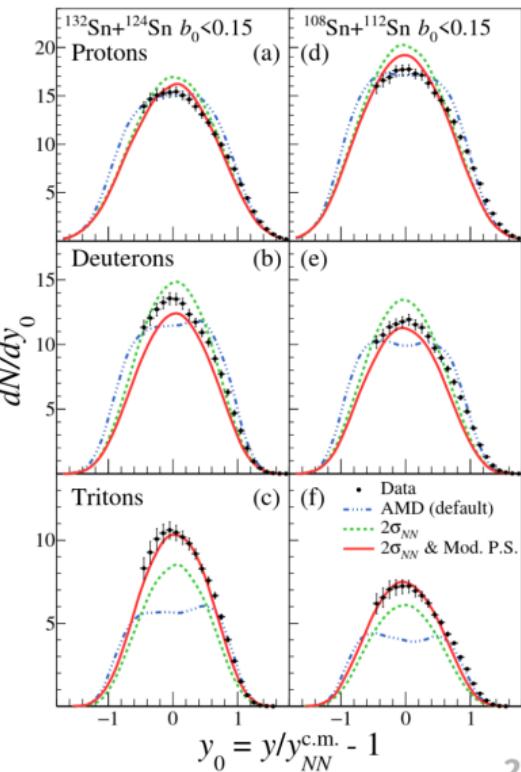
## Rapidity distributions

for  $p$ ,  $d$  and  $t$ , in the central collisions of

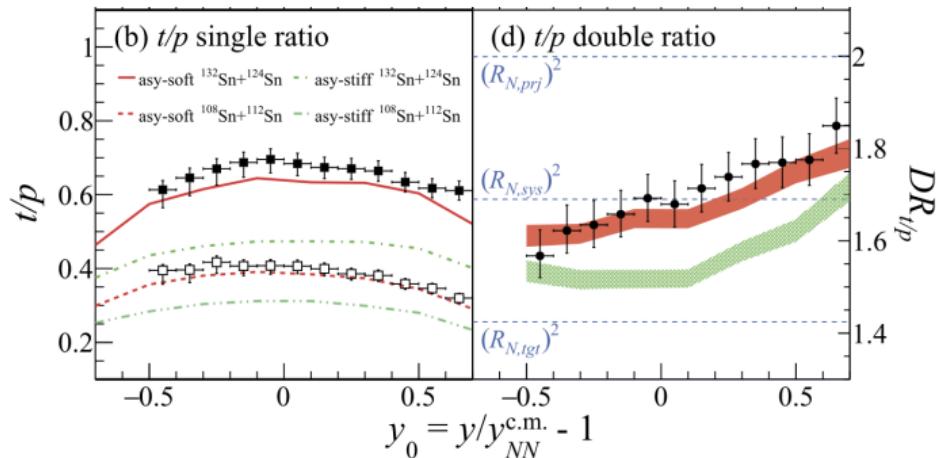
- $^{132}\text{Sn} + ^{124}\text{Sn}$  at 270 MeV/nucleon
- $^{108}\text{Sn} + ^{112}\text{Sn}$  at 270 MeV/nucleon

- Black points:  $\pi\pi$ RIT data
- Lines: AMD calculations (the asy-soft symmetry energy  $L = 46$  MeV) with different model parameters.
  - Understanding of stopping needs more investigation.
  - Understanding of triton production needs more investigation.
  - The results in the next page don't depend much of the model parameters.

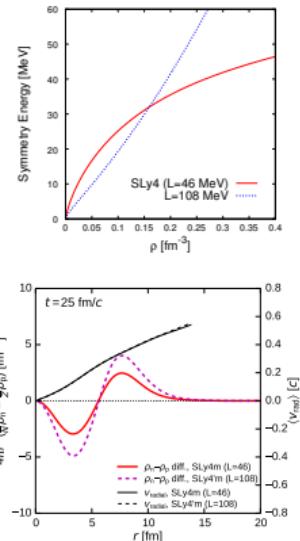
neutron rich vs. neutron deficient



# $t/p$ ratio and its implication on the symmetry energy



$$\frac{t/p \text{ in } ^{132}\text{Sn} + ^{124}\text{Sn}}{t/p \text{ in } ^{108}\text{Sn} + ^{112}\text{Sn}} = DR(t/p): t/p \text{ double ratio}$$

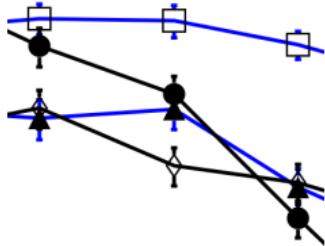
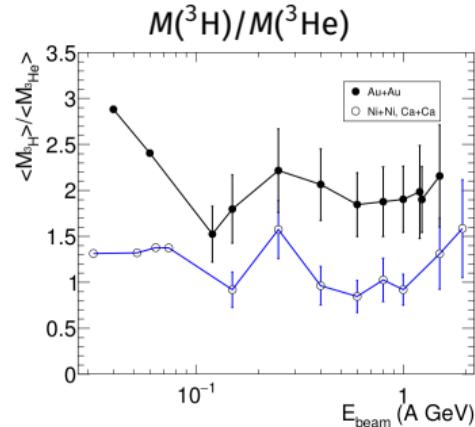
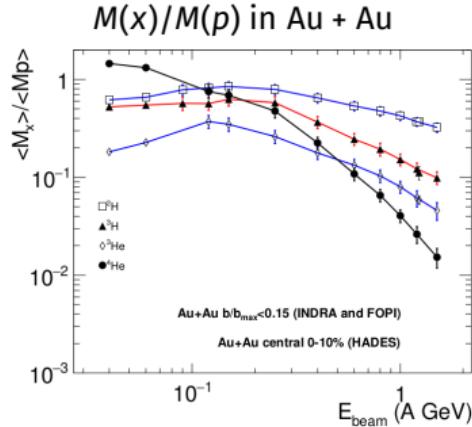
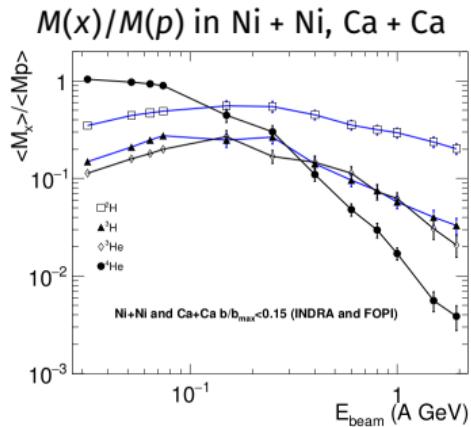


- The value of  $DR(t/p)$  is close to  $(R_{N,\text{sys}})^2 = [(82 + 74)/(58 + 62)]^2 = 1.69$  calculated for the total systems.
- The S $\pi$ RIT data (black points) is more consistent with the AMD calculation with **the asy-soft symmetry energy ( $L=46$  MeV)** rather than **the asy-stiff symmetry energy ( $L=108$  MeV)**.
- The moderate rapidity dependence of the  $t/p$  double ratio implies a partial isospin mixing.
- $t/{}^3\text{He}$  should also be a better probe.
- Stopping and flow need to be understood better systematically.

# Cluster yields at various incident energies

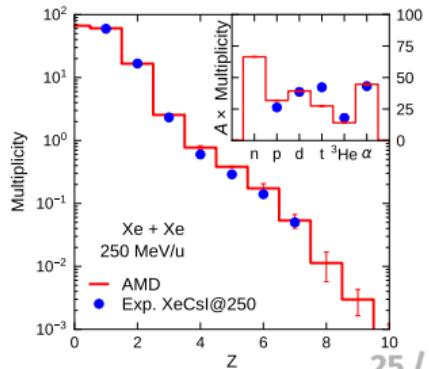
Bougault et al., Symmetry 2021, 13, 1406.

Data sets from INDRA and FOPI



Is there any special mechanism for the triton production around 250A MeV  
(beyond the current transport model descriptions)?

Does it depend on  $N/Z$  of the system?



# Summary

- Understanding cluster correlations is essential in heavy-ion collisions.
- Theoretically, description of cluster correlations by transport models is not straightforward, but some models are working practically.
- In central collisions, the cluster correlations affect the composition and the expansion dynamics dynamics, in AMD calculations.
- At  $E/A = 250\text{--}300 \text{ MeV}$ ,  $\rho_n/\rho_p$  in the high density region is correlated with the triton yield (or  $t/p$  ratio), and the S $\pi$ RIT data suggests  $(\rho_n/\rho_p)_{\text{high } \rho} \approx (N/Z)_{\text{system}}$ .
- Some problems need to be solved.
  - Consistent description of stopping and flow.
  - Is there anything special to produce extra tritons?
  - How should clusters be suppressed in medium?
  - ...

