Comparison of heavy ion transport simulations for mean-field dynamics

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Comparison of heavy-ion transport simulations

- The interpretation of experimental signals (from heavy ion reactions) by transport theories is often affected by model dependence.
  This weakens considerably the constraints extracted for the nuclear EoS

➡ *Transport Model Evaluation Project* (TMEP):

- Started at ECT* (Trento) in **2004**:

-> heavy ion collisions in the AGeV regime *E.E. Kolomeitsev et al.*, J.Phys. G31, S741 (2005)

- New boost from **2014** onwards:

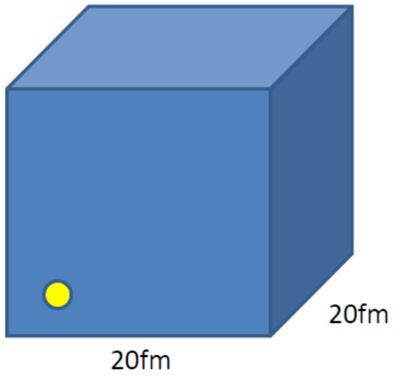
-> heavy ion collisions at 100 and 400 AMeV *J.Xu et al.*, PRC93, 044609 (2016)

➡ Nuclear dynamics under controlled conditions: **box calculations**

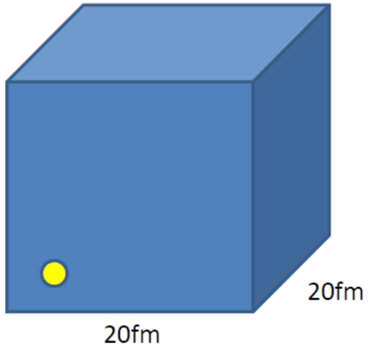
-> collision integral *Y.Zhang et al.*, PRC97, 034625 (2018)

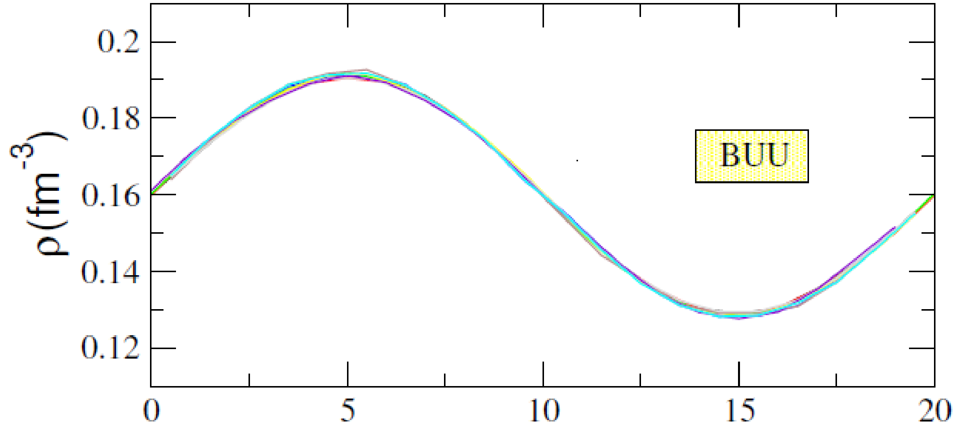
-> pion production *A.Ono et al.*, PRC100, 044617 (2019)

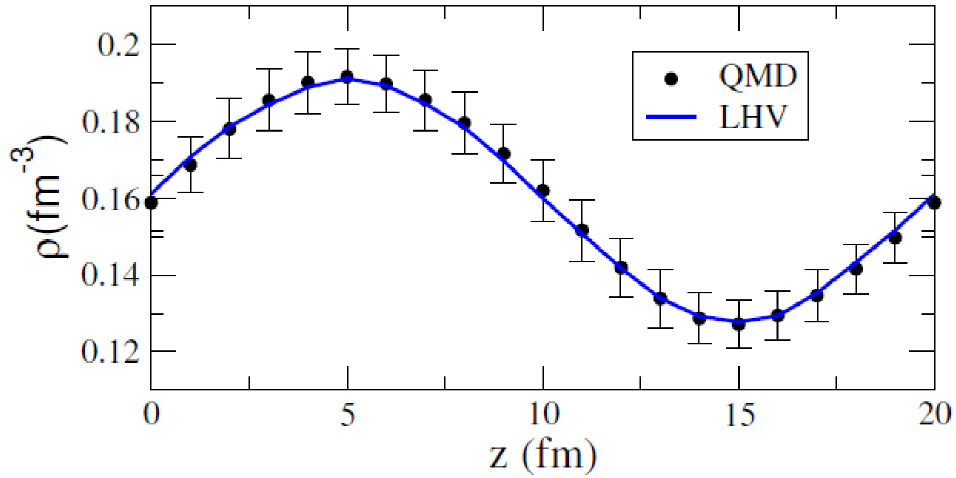
-> **mean-field dynamics** *M.Colonna et al.*, PRC104, 024603 (2021)



Box simulations: test of mean-field dynamics







- Sinusoidal perturbation:
  $\rho(z,t=t_0) = \rho_0 + a_\rho \sin(kz)$

$k = 2\pi/L$, $L = 20$ fm $a_\rho = 0.2\,\rho_0$

- *Fermi sphere defined as a function of the local density*

Symmetric matter zero temperature

- Only mean-field potential
- No surface terms
- Compressibility K = 500 MeV

Simulations:
200 runs for QMD-like codes
10 runs for BUU-like codes, with $N_{TP} = 100$

Transport models: BUU vs. QMD

BUU-like

$$\left(\frac{\partial}{\partial t} + \vec{\nabla}_p \epsilon \cdot \vec{\nabla}_r - \vec{\nabla}_r \epsilon \cdot \vec{\nabla}_p\right) f(\vec{r}, \vec{p}; t) = I_{\text{coll}}(\vec{r}, \vec{p}; t)$$

$$f(\vec{r}, \vec{p}; t) = \frac{(2\pi)^3}{4N_{\text{TP}}} \sum_{i=1}^{AN_{\text{TP}}} G(\vec{r} - \vec{R}_i(t))\, \tilde{G}(\vec{p} - \vec{P}_i(t))$$

Test particle method

QMD-like

$$\Psi(\vec{r}_1, \ldots, \vec{r}_A; t) = \prod_{i=1}^{A} \phi_i(\vec{r}_i; t),$$

$$\phi_i(\vec{r}_i; t) = \frac{1}{[2\pi(\Delta x)^2]^{\frac{3}{4}}} \exp\left[-\frac{[\vec{r}_i - \vec{R}_i(t)]^2}{4(\Delta x)^2}\right] e^{(i/\hbar)\vec{P}_i(t)\cdot\vec{r}_i}$$

- *Fluctuations*

$$\frac{d\vec{R}_i}{dt} = \vec{\nabla}_{P_i} \epsilon \qquad \text{and} \qquad \frac{d\vec{P}_i}{dt} = -\vec{\nabla}_{R_i} \epsilon$$

- ε: single-particle energy

Ex.

$$\epsilon = \frac{\vec{p}^2}{2M} + U(\rho) + M$$

$$U(\rho) = a(\rho/\rho_0) + b(\rho/\rho_0)^\sigma$$

$$\frac{\partial U}{\partial Z_i} \approx \int d^3r\, U(\rho) \frac{\partial G(\vec{r} - \vec{R}_i)}{\partial Z_i} = \frac{\partial H_{pot}}{\partial Z_i}$$

List of codes involved in the project

Type	Acronym	Code Correspondents	Rel/Non-Rel	Particle profiles	$(\Delta x)^2$ [fm^2][a] or l [fm][b]
BUU	BUU-VM[c]	S. Mallik	non-rel	triangle	1
	DJBUU	Y. Kim	cov	$[1-(\lvert\vec{r}\rvert/\Delta x)^2]^3$	6.25
	GiBUU	J. Weil	cov	Gaussian	1
	IBUU[d]	J. Xu	rel	triangle	1
	LHV	R. Wang	rel	triangle	2
	pBUU	P. Danielewicz	cov	trapezoid	0.92
	RVUU	Z. Zhang	cov	point	0
	SMASH	A. Sorensen	cov	triangle	2
	SMF	M. Colonna	non-rel	triangle	2
QMD	ImQMD[e]	Y. X. Zhang	rel	Gaussian	2
	IQMD-BNU	J. Su	rel	Gaussian	2
	IQMD-IMP[f]	Z. Q. Feng	rel	Gaussian	2
	TuQMD	D. Cozma	rel	Gaussian	2
	UrQMD	Y. J. Wang	rel	Gaussian	2

- 9 BUU-like codes

- 5 QMD-like codes

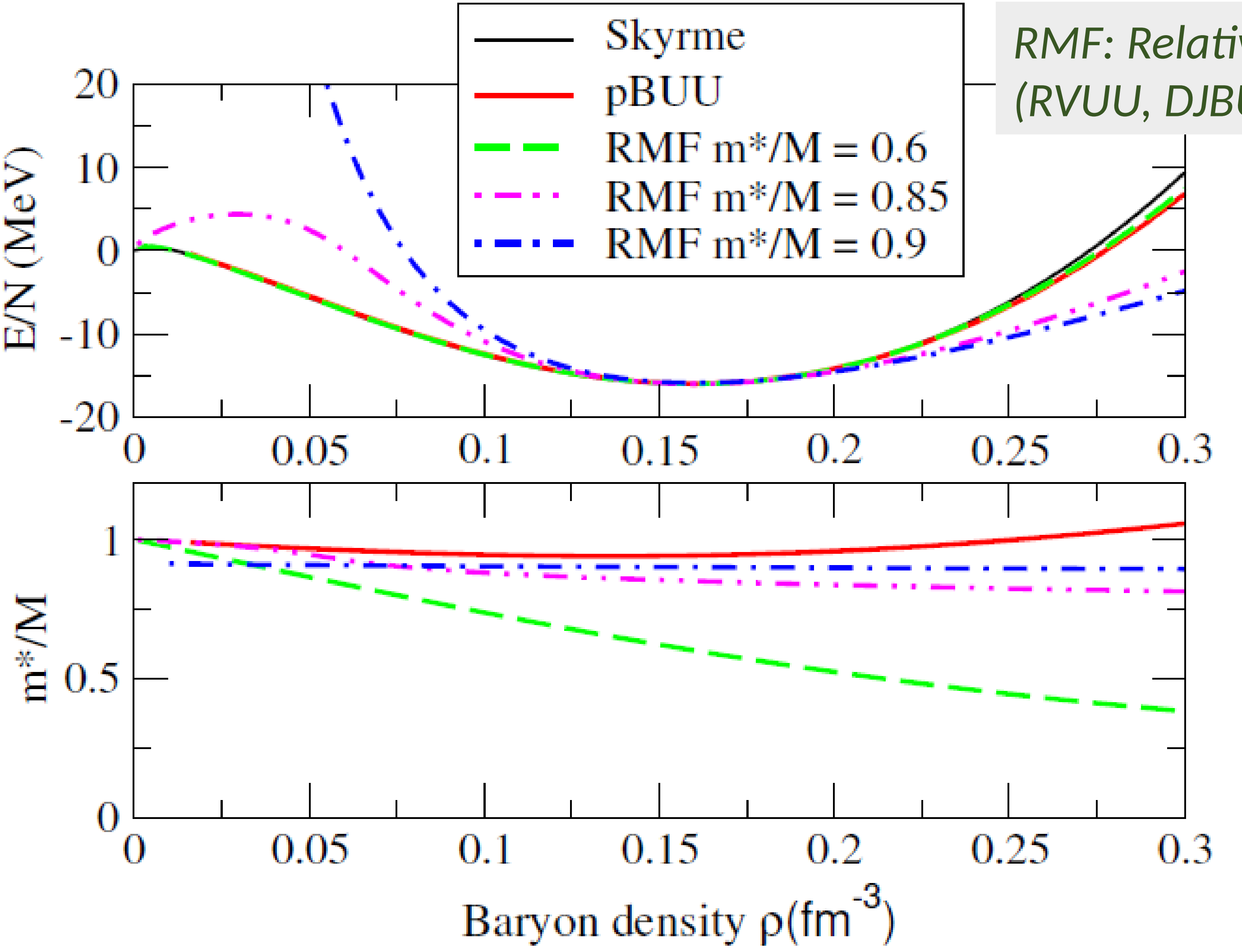
Non-rel: non relativistic -- rel: only relativistic kinematics – cov: full covariant formulation

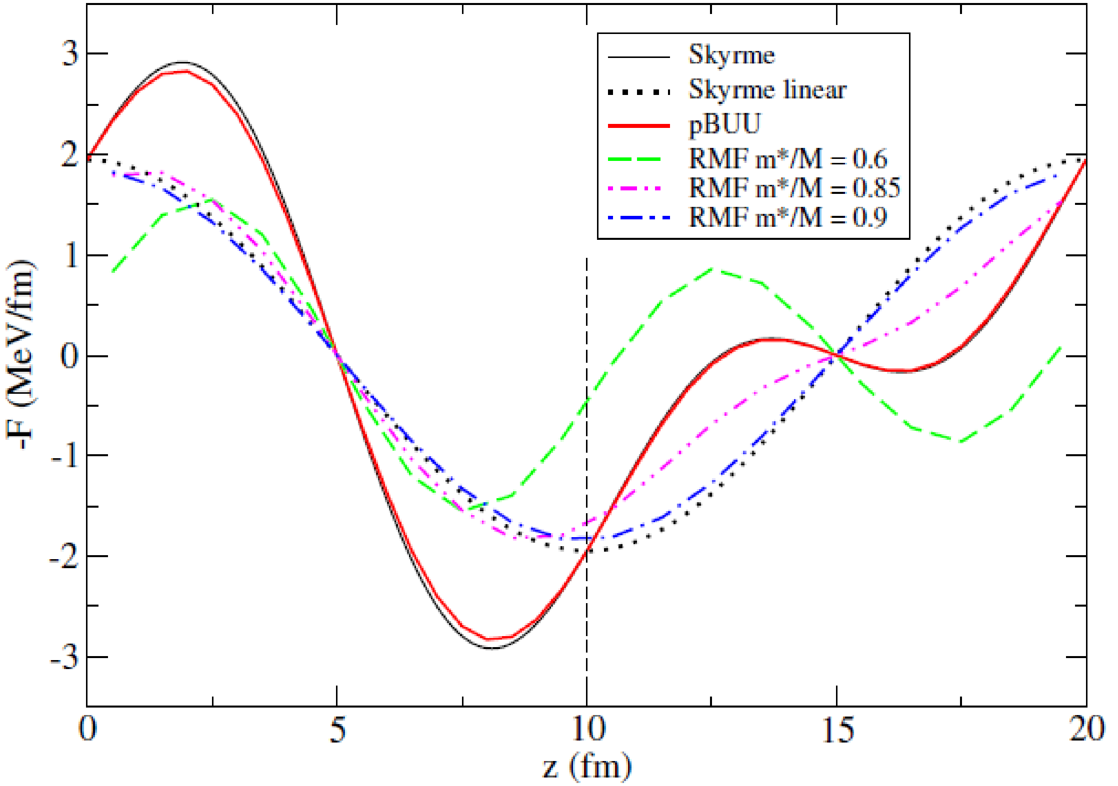
Energy per nucleon and gradients

Potential Energy *Ex:*

$$H_{pot} = \int d^3r\ [\frac{a}{2}\ (\rho^2/\rho_0) + \frac{b}{\sigma+1}\ (\rho^{\sigma+1}/\rho_0^{\sigma})]$$

RMF: Relativistic Mean Field (RVUU, DJBUU)





$-F = \partial U/\partial z$

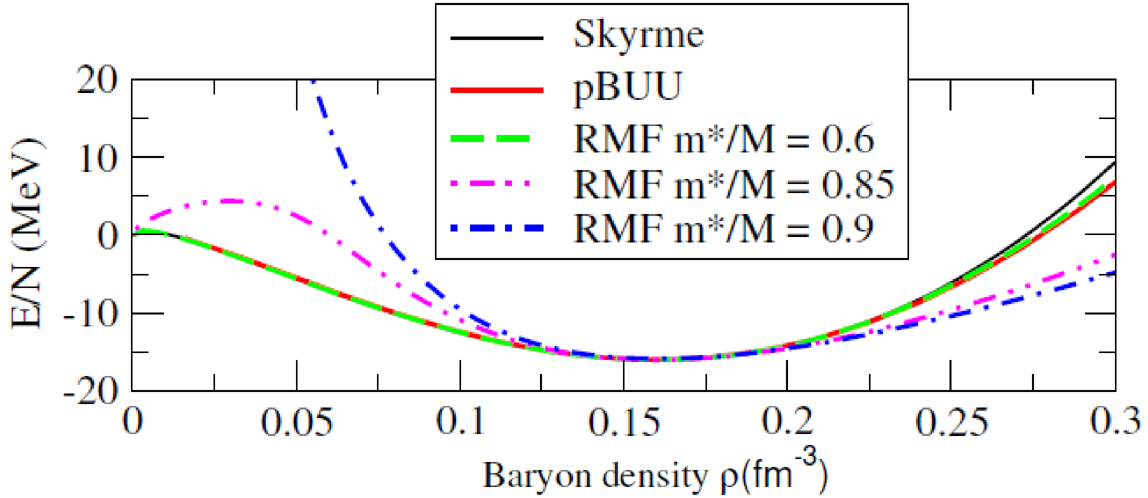
Type	m^*/M	$\tilde{F}_0$	s	M/E_F^*	v_s
"non-rel"	1	1.259	1.073	1	0.301
"rel"	1	1.308	1.079	0.963	0.291
"cov"					
SMASH	1	1.471	1.099	0.963	0.297
pBUU	0.942	1.208	1.067	1.017	0.304
RVUU	0.6	-0.956	-	1.510	-
DJBUU	0.6	0.496	1.005	1.510	0.425
RVUU	0.7	-0.207	-	1.326	-
DJBUU	0.7	0.704	1.017	1.326	0.378
RVUU	0.8	0.437	1.003	1.180	0.332
DJBUU	0.8	0.915	1.036	1.180	0.343
RVUU	0.85	0.728	1.019	1.117	0.319
DJBUU	0.85	1.022	1.047	1.117	0.328
RVUU	0.9	1.002	1.044	1.061	0.311
DJBUU	0.9	1.130	1.058	1.061	0.315

RMF

Landau theory of Fermi liquids:

***Zero-sound** velocity* frequency of density oscillations

Effective interactions with similar EoS may have different sound velocity !



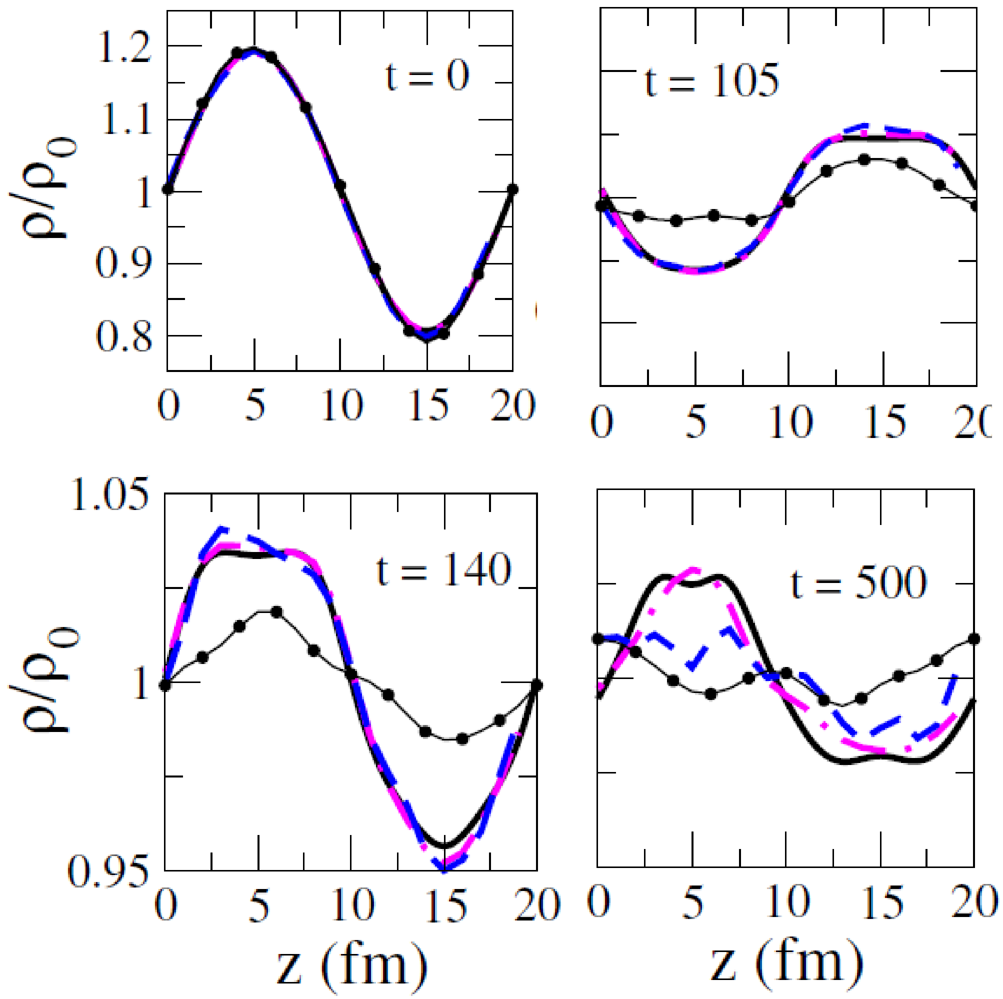
Time evolution of density profile

----- Exact solution (Deformed Fermi Sphere – *A.Ono*)

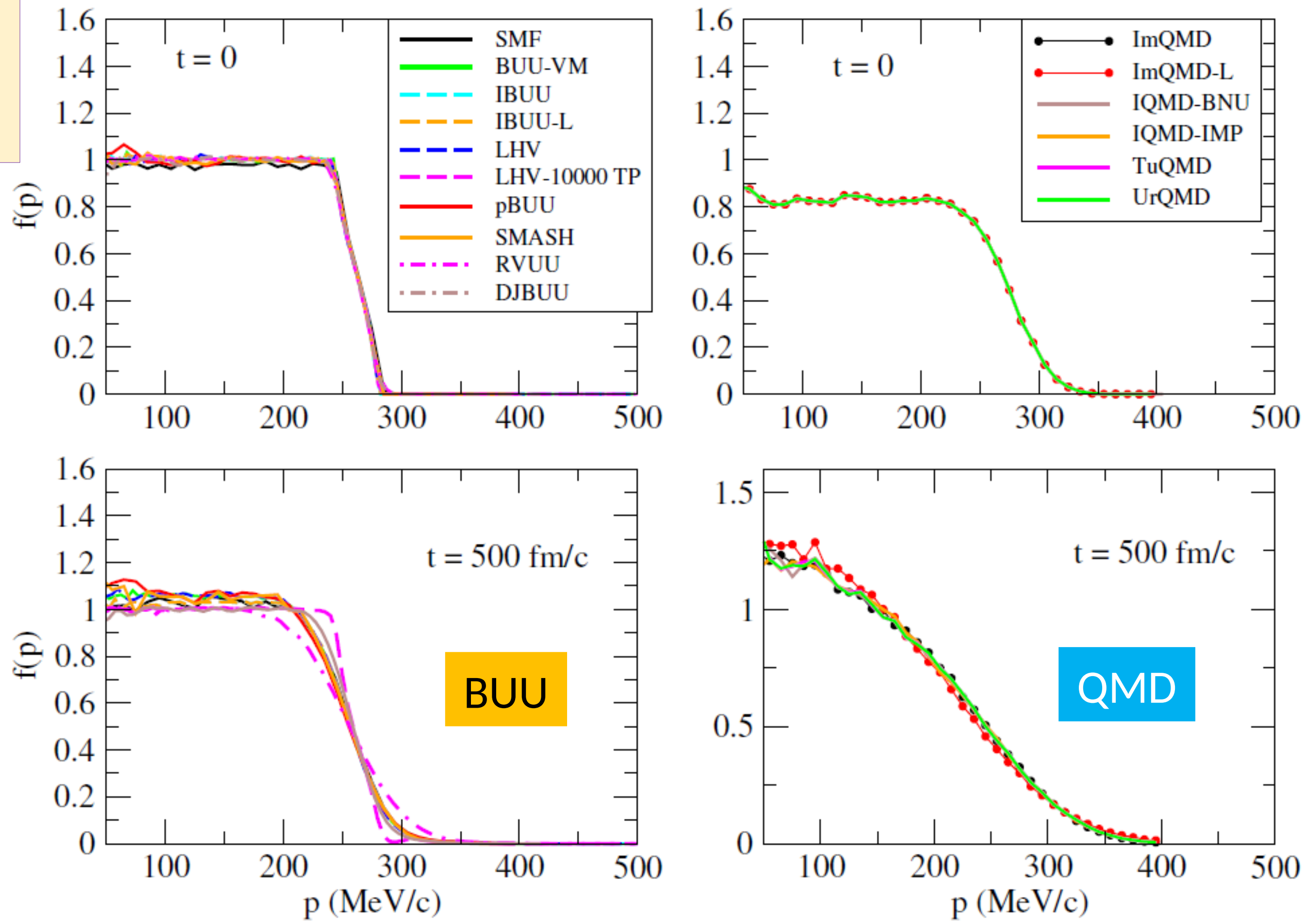
- - - LHV (BUU-Like) 100 TP

- - - LHV 2500 TP

• ImQMD



Momentum distribution

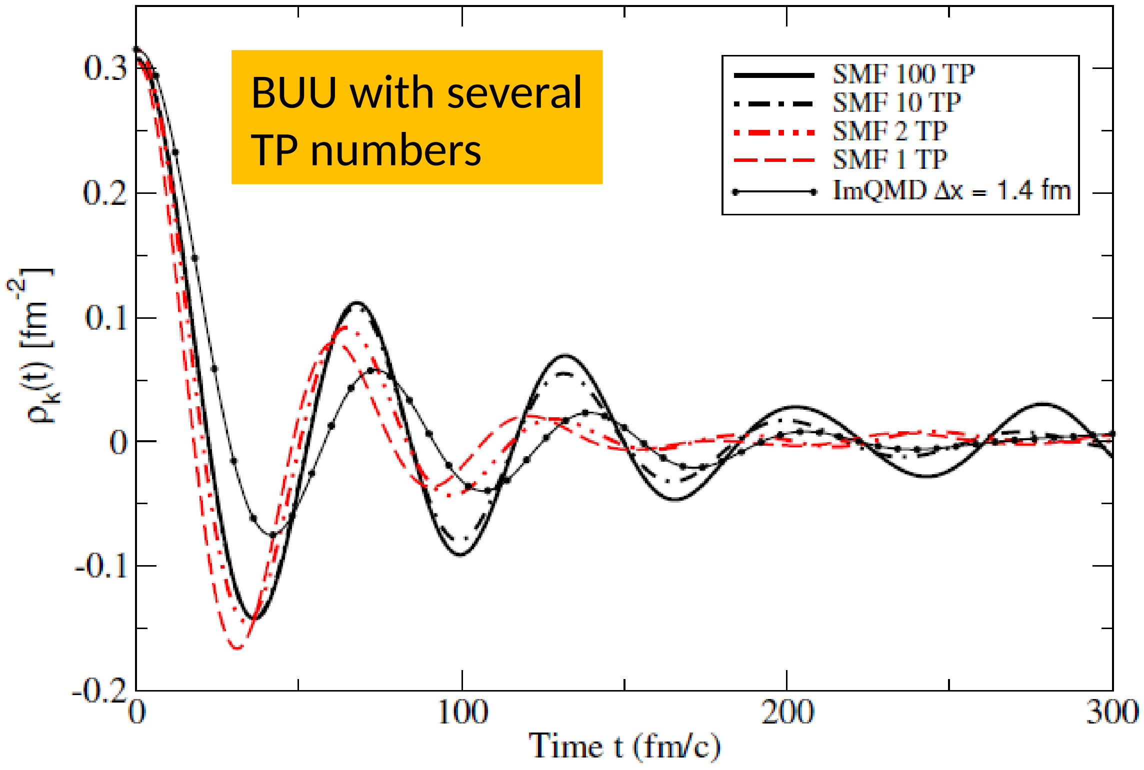


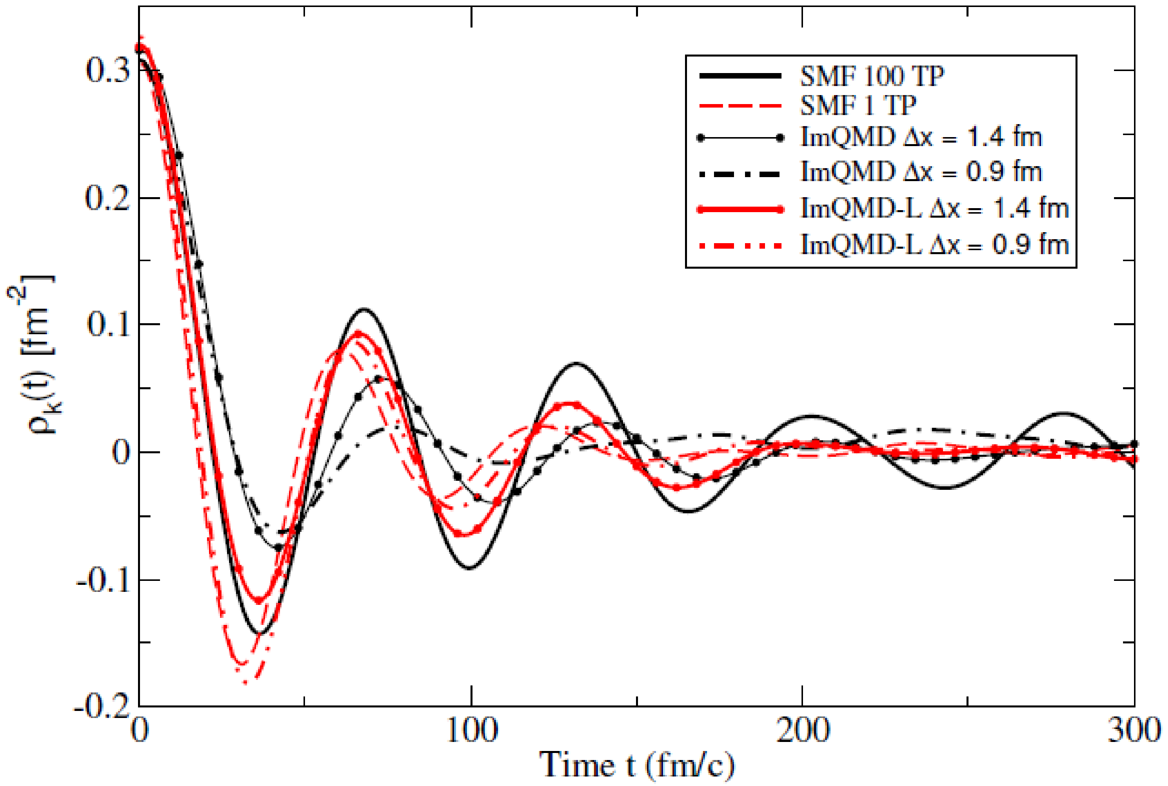
- ❑ The fluctuations in QMD generates more damping effects
- ❑ QMD tends to approach a Boltzmann distribution

Frequency and damping of density oscillations

Strength function: $\rho_k(t) = \int_0^{L_z} dz\, \rho(z,t)\sin(kz)$

QMD: standard vs. Lattice formulation
(more precise evaluation of the force F)



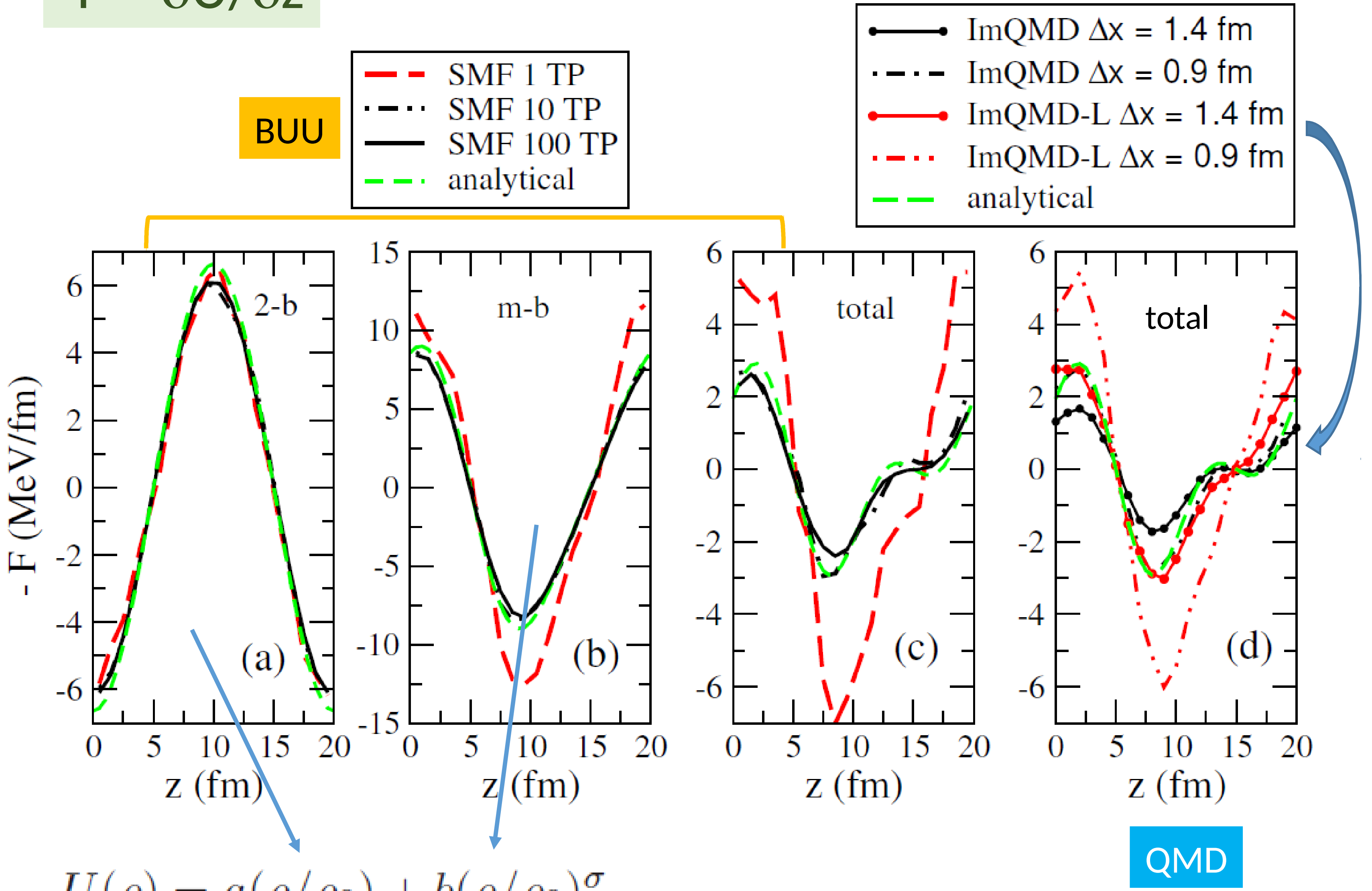


- ❑ Fluctuations act on the damping, but also on the oscillation frequency
- ❑ Good agreement between BUU (1 TP) and QMD with Lattice

Mean-field gradients in numerical simulations

$$\frac{\partial U}{\partial Z_i} \approx \int d^3r\, U(\rho)\, \frac{\partial G(\vec{r}-\vec{R}_i)}{\partial Z_i} = \frac{\partial H_{pot}}{\partial Z_i}$$

-F = ∂U/∂z



$$U(\rho) = a(\rho/\rho_0) + b(\rho/\rho_0)^\sigma$$

Gradients in QMD:

➢ **Standard QMD:**

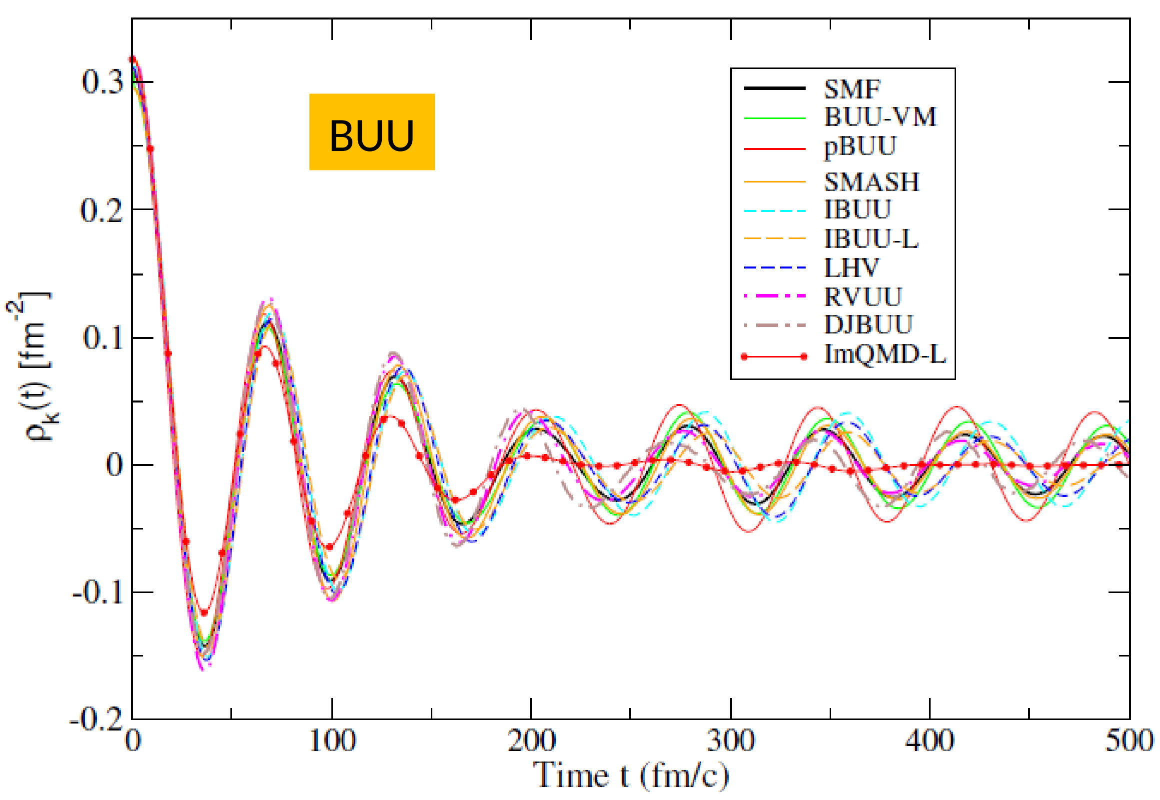
$$H_{pot}^{2body,QMD} = \frac{a}{2\rho_0}\sum_i \tilde{\rho}_i$$ *exact*

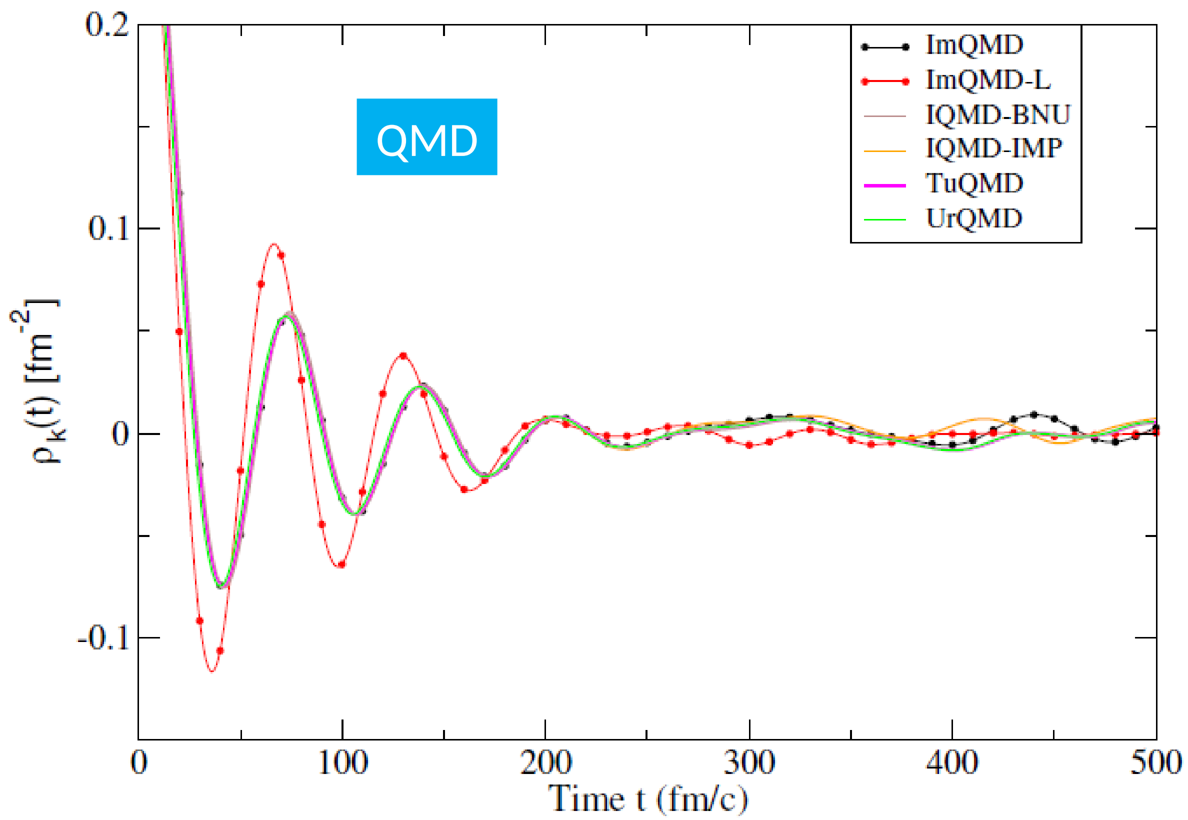
$$H_{pot}^{3body,QMD} = \frac{b}{(\sigma+1)\rho_0^\sigma}\sum_i \tilde{\rho}_i^\sigma$$ *approx.*

→ strength of the many-body term underestimated

➢ **Lattice formulation**: tuning the Gaussian width, the analytical expectation for the mean-field gradients is well reproduced

Results of all codes: I

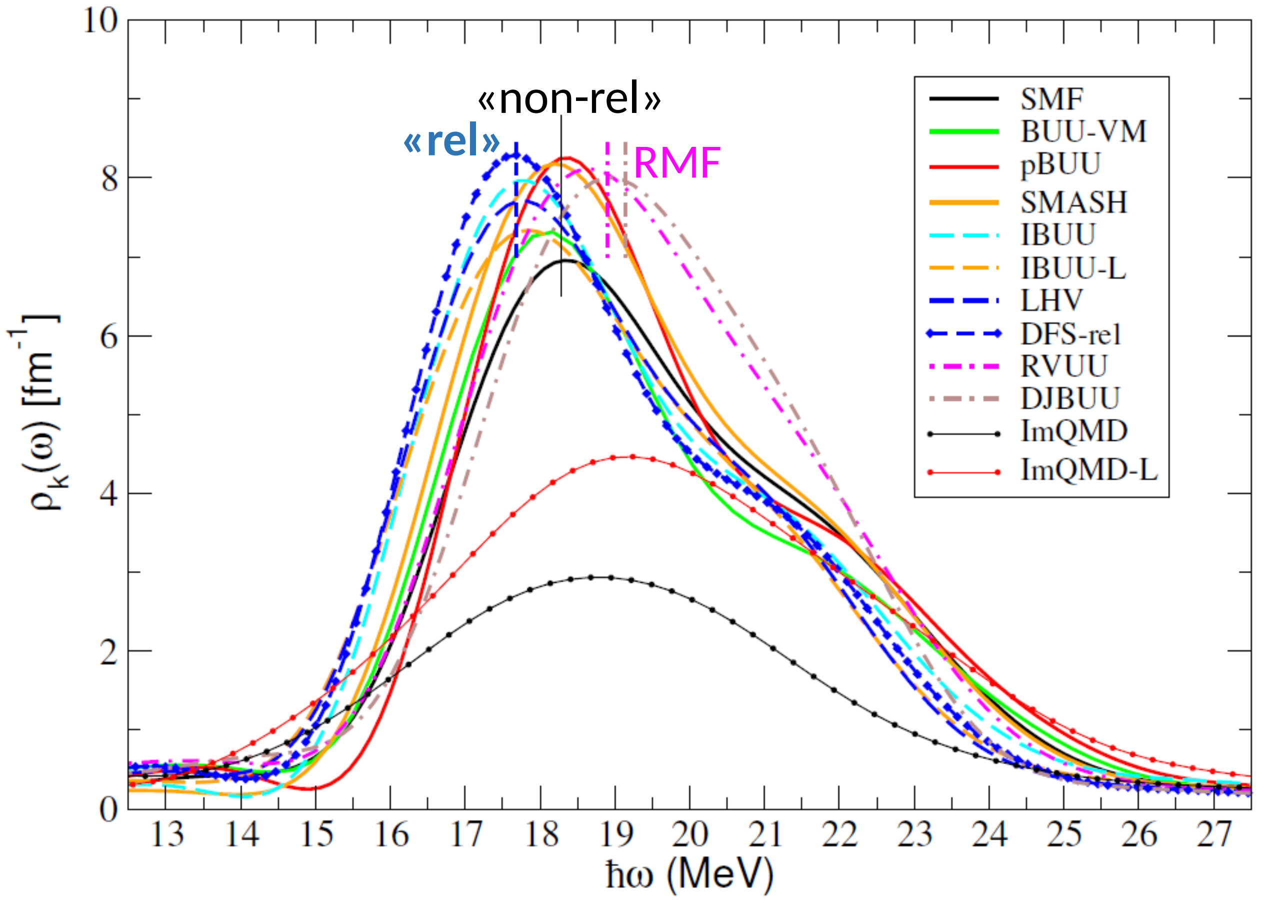




- Good agreement: small differences between codes, compatible with zero-sound analysis (details of mean-field (or Kin.) implementation)
- Excellent agreement between QMD codes. ImQMD-L agrees with BUU (frequency), but damping effects are larger

Results of all codes: II

Response function: $\rho_k(\omega) = \int_{t_{in}}^{t_{fin}} dt\, \rho_k(t)\, \cos(\omega(t - t_{in}))$



- Differences between **BUU** codes are compatible with different treatment of kinematics and/or mean-field
- **QMD** codes:
  - frequency affected by less repulsive many-body term (can be cured with Lattice method)
  - large damping effects

Conclusions

- The details of the effective interaction are important to correctly describe transport dynamics (propagation of density fluctuations investigated here)
  - Not univocally determined by the EoS

- The presence of fluctuations induces larger damping effects (see QMD-like models)
  QMD: The width of the Gaussian packet can be tuned to give oscillation frequency compatible with BUU

Project carried out within the
TMEP Collaboration
(**T**ransport **M**odel **E**valuation **P**roject)

(about 30 participants)

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