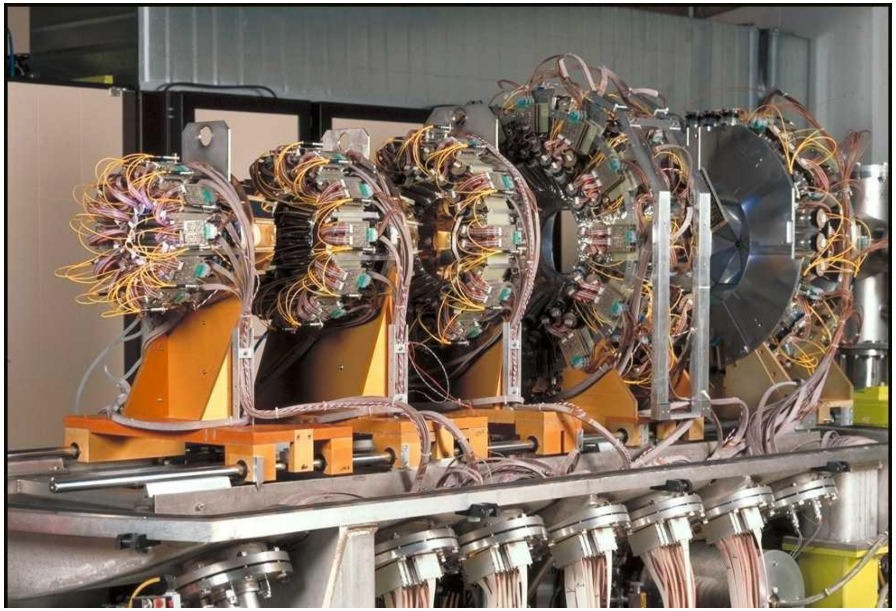
Negative heat capacity for hot nuclei: confirmation with formulation from the microcanonical ensemble

quasi-fused systems from central 129Xe+natSn collisions 32-50 AMeV

INDRA Coll.



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IWM-EC 2021

Negative heat capacity for hot nuclei: confirmation with formulation from the microcanonical ensemble

An important challenge of H.I. collisions at int. energies was the identification and characterization of the L.G. phase transition in hot nuclei.
Huge progress has been made even if some points can be deeper investigated
Liquid-Gas phase transition in nuclei, BB and J.D. Frankland, PPNP 105 (2019) 82-138

This was notably the case for the experimental observation of negative microcanonical heat capacity related to the consequences of local convexity of the entropy for finite systems.
About 20 years ago MULTICS and INDRA Coll. highlighted this signal.
The method was proposed by P. Chomaz and F. Gulminelli. It is based on the fact that for a given total thermal energy, the average partial energy stored in a part of the system is a good microcanonical thermometer, while the associated fluctuations can be used to construct heat capacity => a single temperature is used.

On the other side it was also shown, from a very complete simulation,the necessity to impose a limitation of Tfrag to be able to well reproduce exp. data and consequently the necessity to use 2 temperatures: Tmicro and Tfrag.
Exact microcanonical formulae using 2 temperatures were proposed by Al. and Ad. Raduta => these are used

both methods need information at freeze out

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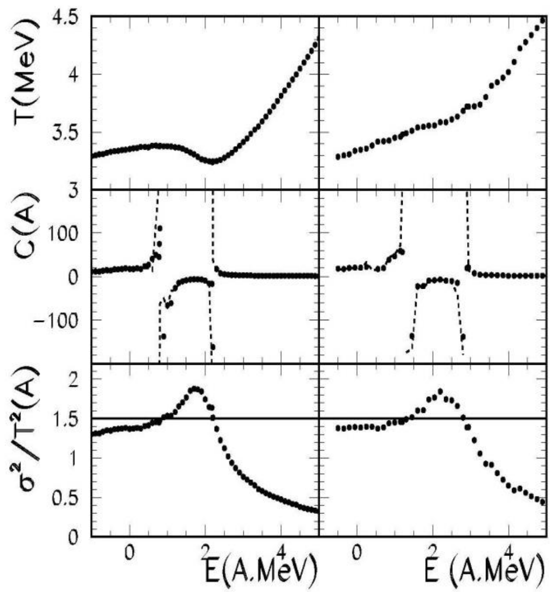
Microcanonical negative heat capacity: a robust signal

Microcanonical lattice gas model (216 particles)

Ph. Chomaz, V. Duflot and F. Gulminelli

PRL 85 (2000) 3587

cst Pressure - cst <Volume>



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Microcanonical heat capacity using partial energy fluctuations

Method proposed by P. Chomaz et al. (NPA 647 (1999) 153 and applied on quasi-fused systems from central Xe+Sn collisions 32-50 AMeV- INDRA

same T for both internal excitation and thermal motion of products at F.O.

E_k calculated by subtracting the potential part
$\Rightarrow$ Kinetic energy fluctuations at F.O. reflect the configurational energy fluctuations

Estimator of the microcanonical T obtained from the kinetic equ. of state:

$\langle E_k \rangle = \langle \sum a_i \rangle T^2 + \langle 3/2(M-1) \rangle T$

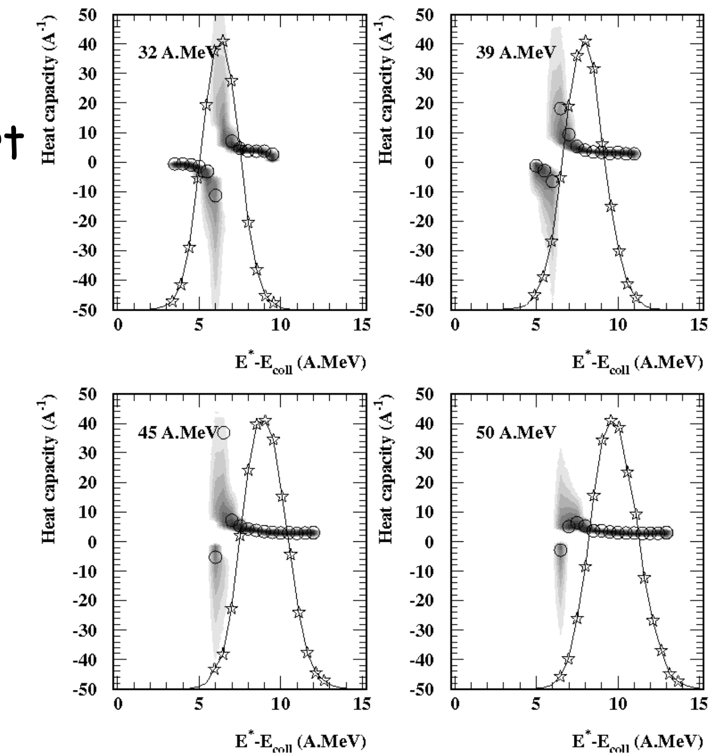
- $\langle\rangle$ average on events with same E*
- M: mult. at F.O.
- a_i : level density parameter

3 equations

$C_k = \delta \langle E_k/A \rangle / \delta T$

$A\sigma_k^2 \approx T^2 \, C_k C_{pot}/(C_k + C_{pot})$ – Gaussian app.

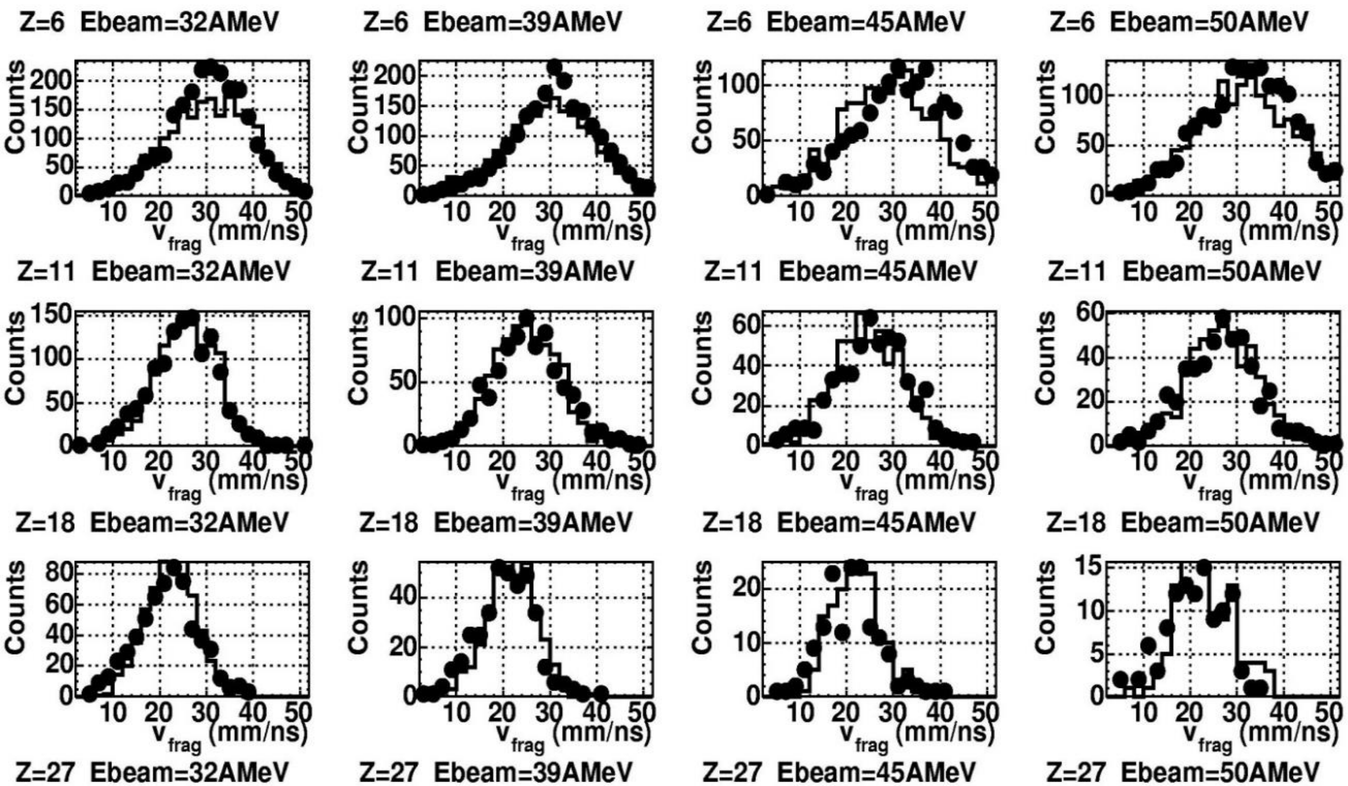
$(C/A)_{micro} \approx C_k + C_{pot} \approx C_k^2 / (C_k - A\sigma_k^2/T^2)$



N. Le Neindre et al. Proc. Bormio 2000

Comparison data-very complete simulation (asympt. values)

QF: Xe + Sn 32-50AMeV



With a single temperature (Tfrag=Tkin), Coulomb effects + collective energies + thermal kinetic energy (directed at random) + fragment decays are responsible for ~ 60-70% of the observed widths. By introducing a limiting temperature for fragments (9 MeV), thermal kinetic energy largely increases, due to energy conservation, which produces the missing percentage for the widths of final velocity distributions.

S. Piantelli et al., NPA 809 (2008) 111

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Microcanonical formulation with Tmicro and Tfrag

Al. and Ad. Raduta PRC 61 (2000) 034611 and NPA 703 (2002) 876

Fermi gas formula for the level density of fragments completed with exp (-E/Tlim) andTlim is fixed at 9 MeV

Statistical weight of a configuration:Wc
n fixed to 1 to ensure coherence with simulations (energy + lin. moment. conser.)

Second derivative of entropy or heat capacity (C) are calculated for ensemble states

These 2 quantities only depend on 2 values to be estimated at F.O.
Mc (total multiplicity) and K –total thermal kinetic energy)
The average over the ensemble states are assimilated to an average over « event ensembles » sorted in E* bins of 0.5 AMeV

$$W_C(A,Z,E,V) = \frac{1}{M_C!}\chi V^{M_C}\prod_{n=1}^{M_C}\left(\frac{\rho_n(\epsilon_n)}{h^3}(mA_n)^{3/2}\right) \times \frac{2\pi}{\Gamma(3/2(M_C-n))}\frac{1}{\sqrt{(\det I)}}\frac{(2\pi K)^{(3/2)(M_C-n)-1}}{(mA)^{3/2}}, \qquad (2)$$

$$T = \left(\frac{\partial S}{\partial E}\right)^{-1} = \left(\frac{1}{\sum_C W_C}\sum_C W_C(3/2M_C-5/2)/K\right)^{-1} \qquad (3)$$

$$= \langle(3/2M_C-5/2)/K\rangle^{-1}. \qquad (4)$$

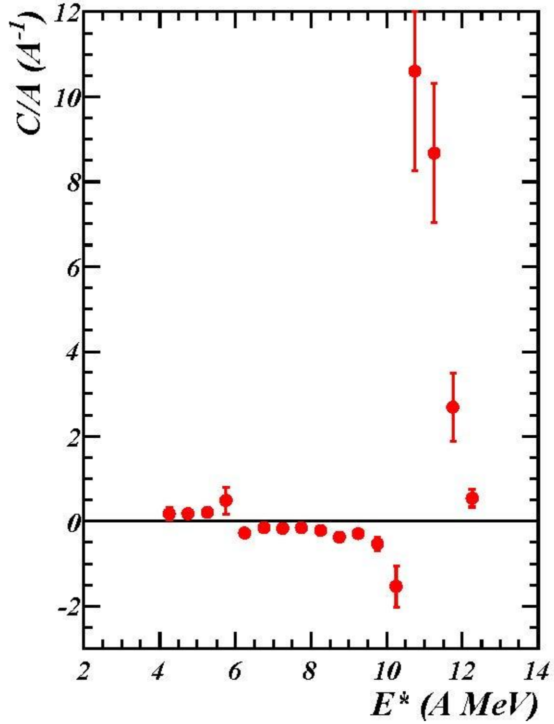
The notation $\langle\rangle$ refers for the average over the ensemble states. The heat capacity of the system C is related to the second derivative of the entropy by the equation $\partial^2 S/\partial E^2 = -1/CT^2$. Thus, one can evaluate the second derivative of the system entropy versus E (Eq. (5)) or alternatively the heat capacity C (Eq. (6))

$$\frac{\partial^2 S}{\partial E^2} = \langle\frac{(3/2M_C-5/2)(3/2M_C-7/2)}{K^2}\rangle - \langle\frac{(3/2M_C-5/2)}{K}\rangle^2 \qquad (5)$$

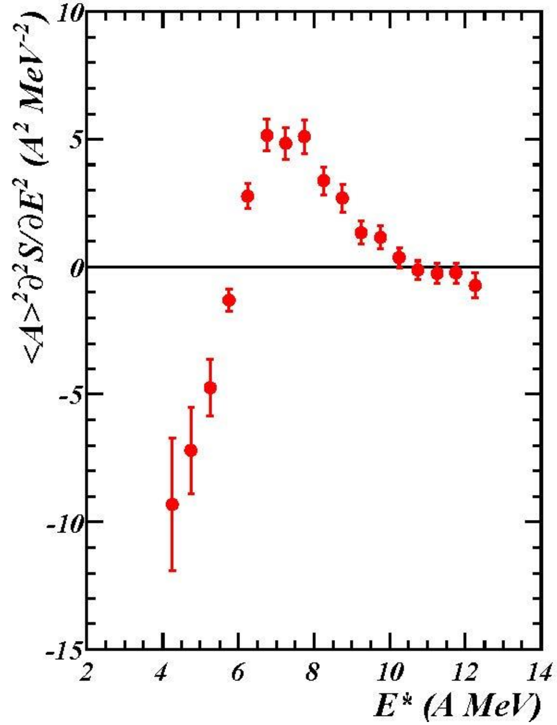
$$C = \left(1 - T^2\langle\frac{(3/2M_C-5/2)(3/2M_C-7/2)}{K^2}\rangle\right)^{-1} \qquad (6)$$

These two quantities only depend on two parameters M_C and K which must be estimated at freeze-out.

Heat capacity and second derivative of entropy versus E*



$\delta^2 S/\delta E^2 = -1/CT^2$



BB et al. – INDRA Coll. Eur. Phys. J. A. 56 (2020) 101

The two methods: comparison of results

E* domain of negative heat capacity: method 1 < 4.0±1.0 – 6.0±1.0 AMeV

method 2 6.0±1.0 – 10.0±1.0 AMeV

For the 2 methods: - same shape event sorting

- ≠degrees of completeness ≥ 80% (met.1),

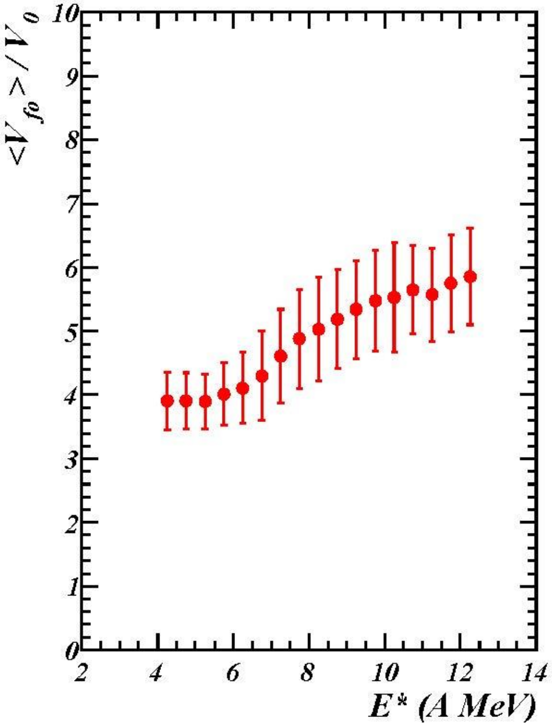
  ≥ 93% (met.2)

- ≠reconstructions of F.O. properties
- ≠ average F.O. volumes fixed at $3V_0$ (met.1),

  variation with E* from ≈ 4 to $6V_0$ (met.2)

⇒ direct consequence for met.1: an increase of E_{Coul} and a decrease and a distortion with E* of $\langle E_k \rangle$

=> Verification: F.O. data of met.2 applied to met.1



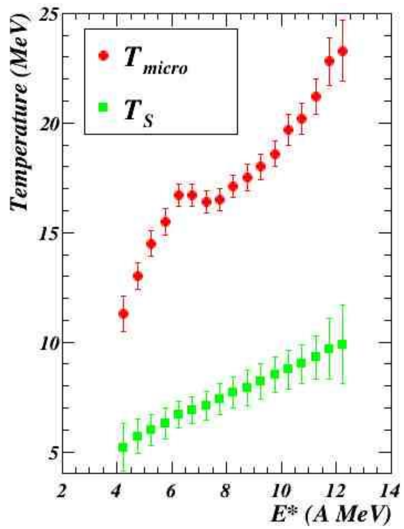
F.O. data of met.2 applied to met.1

$\langle E_k \rangle = \langle \sum a_i \rangle T_s^2 + (E_{kin\_tot} / T_{kin\_sim}) T_s$
(from simulation)

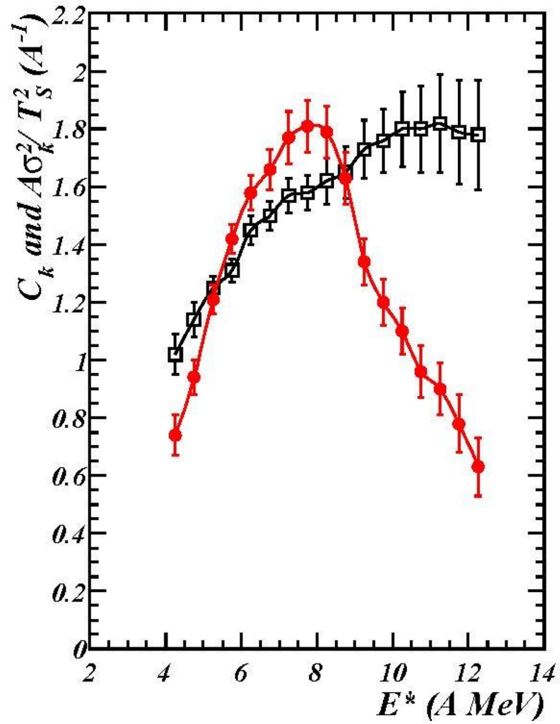
$(C/A)_{micro} \approx C_k^2 / (C_k - A\sigma_k^2 / T_s^2)$

heat capacity becomes negative when the normalized kinetic energy fluctuations overcome the kinetic heat capacity
$C_k = \delta \langle E_k/A \rangle / \delta T_s$

Tfrag ≈ 4 to 7MeV
when E* 4. -> 12.5AMeV



E* => 5.5±1.0 – 9.0±1.0 AMeV



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Conclusion

Heat capacity measurements have been revisited for hot nuclei with A around 200 in the coexistence region of the phase transition.

Microcanonical formulae without approximation and data reconstructed at freeze-out with the help of a very complete simulation have been used. Two temperatures, one associated with internal excitation of fragments and the other with thermal motion, mandatory to reproduce experimental data, have been used.

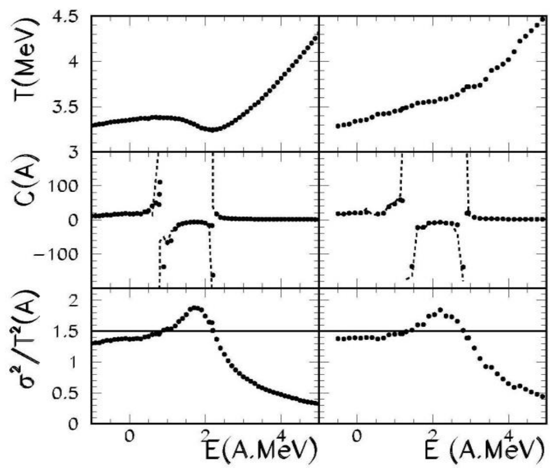
These measurements fully confirm the presence of a thermal excitation energy region of negative heat capacity expected for a first order phase transition in finite systems like hot nuclei.

As compared to previous measurements the difference of E* domain observed must be mainly attributed to the different reconstructions at F.O. and especially to the volumes.

Microcanonical negative heat capacity: a robust signal

Microcanonical lattice gas model (216 particles)
Ph. Chomaz, V. Duflot and F. Gulminelli
PRL 85 (2000) 3587

cst Pressure - cst <Volume>

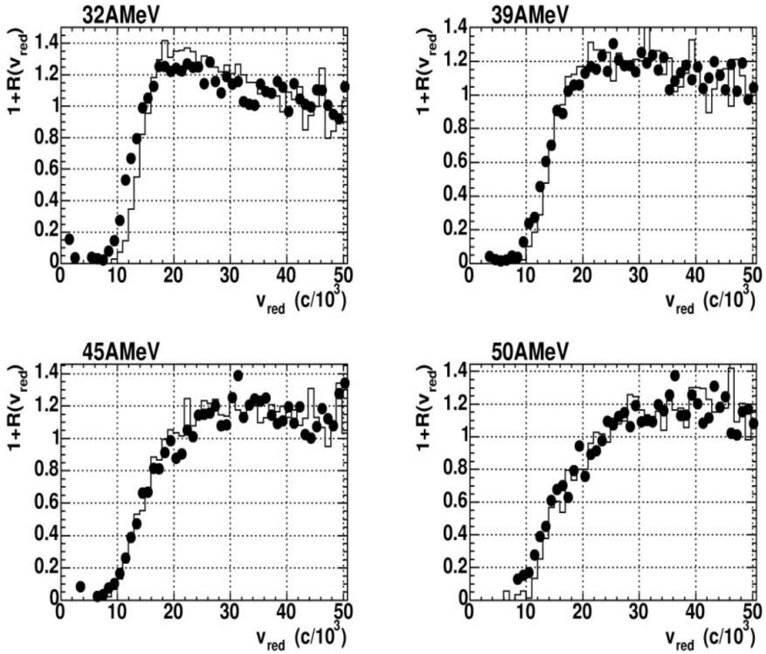


It is important to stress that negative heat capacity cannot be observed at constant volume. For the LG transition in hot nuclei the volume is not fixed but multiplicity and partition dependent => theo. descr. with stat. ensemble for which the volume can fluctuate from event to event around an average value

Comparison data-simulation (asymptotic values)

QF: Xe + Sn 32-50AMeV

frag.-frag. correlations



Good agreement between data and simulations

⇒ retained method and parameters relevant to correctly describe freeze-out topologies

S. Piantelli et al., NPA 809 (2008) 111

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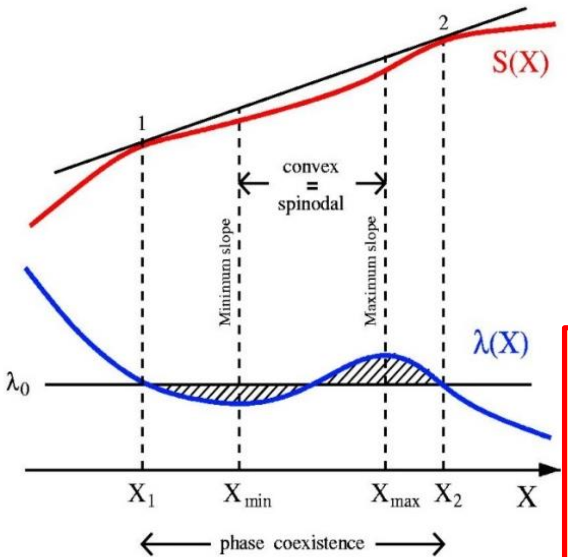
Finite syst. and first order phase transition

Ph. Chomaz et al., Phys. Rep. 389 (2004) 263

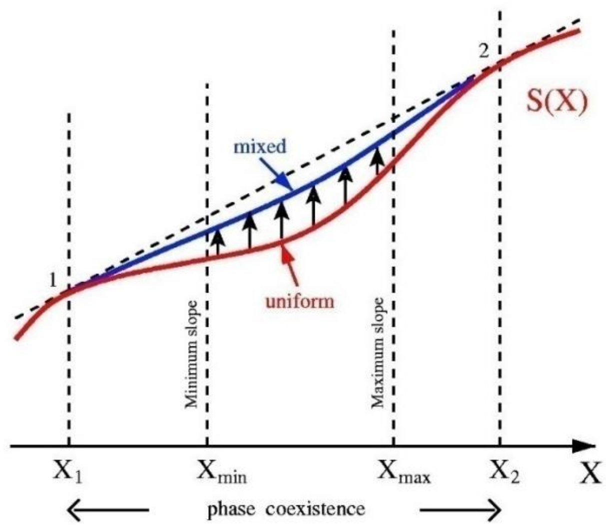
X extensive variable
(E, N, V)

Conjugate
intensive variable
$\lambda(\mathbf{X}) = \partial S / \partial X$
(1/T, - μ/T, P/T)

NEGATIVE HEAT CAPACITY

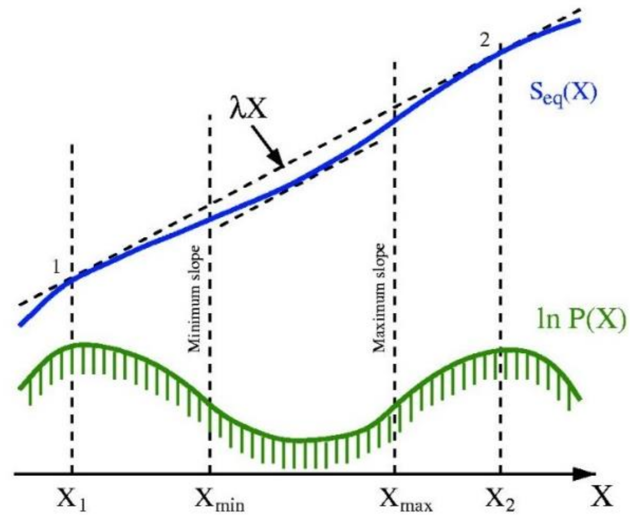


μ canonical sampling
(Fixed value of X)



SPINODAL INSTABILITY

BIMODALITY



Canonical-Gaussian sampling

$P(X) \sim \exp(S(X) - \lambda X)$

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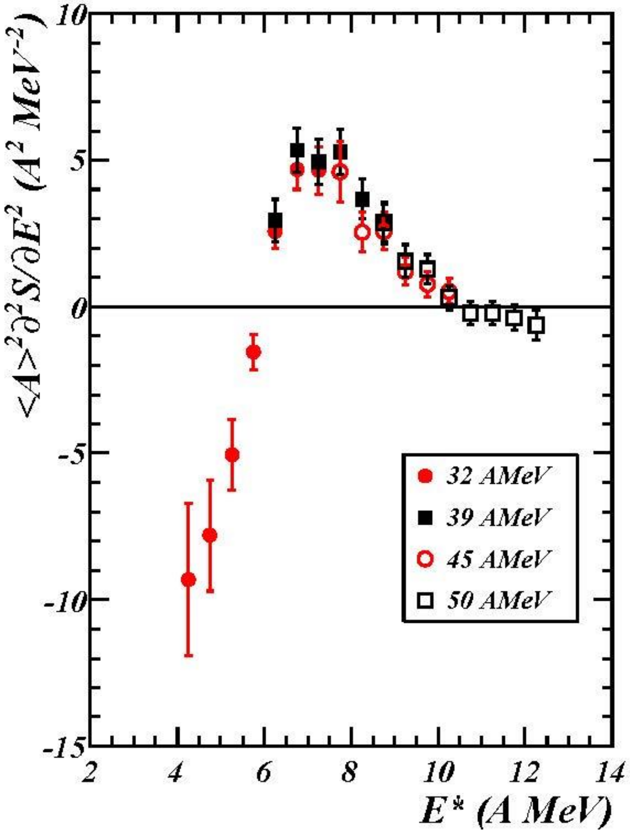
Results for the different incident energies



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Selection of hot sources/nuclei produced by quasi-fused systems

Xe + Sn   incident energies 32, 39, 45 and 50 AMeV

$Z_{source} \geq 93\%$ of Z_{system} $P_{source} \geq 80\%$ of P_{system}

$(\text{Flow angle})_{cm} \geq 60°$

A_{source} reconstructed event by event in the cm
with all fragments
twice the LCP emitted in the range 60°-120°
and the number of neutrons (not detected) calculated
to keep the N/Z of the system

S. Piantelli et al., NPA 809 (2008) 111

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A very complete simulation

S. Piantelli et al. PLB 627 (2005) 18, NPA 809 (2008) 111

from complete experimental events Ztot ≥ 93% to strongly reduce underestimation of the Coulomb repulsion

- **built event by event from all the available experimental information (LCP spectra, average and standard deviation of frag. velocity spectra and calorimetry)**
- **F.O. partitions are built by dressing fragments with particles**
- **F.O. topology built up at random from spherical nuclei**
- **Excited fragments and particles at F.O. undergo propagation (Coulomb+ thermal kin. E) during which fragments evaporate particles**

A very complete simulation

4 free parameters to recover the data:

- percentage of particles evaporated from primary frag. (80 -> 40%)
- radial collective energy (0.7 -> 1.7 AMeV)
- minim. distance between the surfaces of products at F.O. (1.5fm)
- limiting temperature for fragments (vanishing of the level density at high E*- 1/Tfrag = 1/Tkin + 1/Tlim (Tlim=9MeV)
  S.E. Koonin and J. Randrup A474 1987,173)
  Tfrag (≈ 4 to 7MeV when E* covers the range 4 -> 12.5AMeV)

For each event (80 < Zs < 100) we deduce:

- tot. excit. energy E^*_{tot}, E_{coll}, tot.therm. energy $E^* = E^*_{tot} - E_{coll}$
- at F.O.: total M, volume,total thermal kinetic E, volume, energy sharing between internal excitation energy and tot. thermal kinetic E

Calorimetry

(1) S.E.Koonin et al., NPA 474(1987)173

$$\frac{1}{T_{frag}} = \frac{3}{2\langle K^{fo}\rangle} + \frac{1}{T_{lim}}$$

T_{lim}= limiting T for fragments (independent of A)

exp

$$\sum_{k=1}^{M_{cp}} K_{cp}^{k} + \Delta B_{cp} + M_n^{fo}\langle K^{fo}\rangle + M_n^{evap}\theta_{frag} + \Delta B_n =$$

(2)

$$= \left(M^{fo}-1\right)\langle K^{fo}\rangle + \sum_{k=1}^{nfrag} a_k\theta_{frag}^2 + \Delta B_{fo} + V_{Coul}^{fo} + \sum_{k=1}^{M_{fo}}\left(\frac{r_k}{R_0}\right)^2 A_k E_0$$

level density parameter

$$a_k = \frac{A_k}{10}\,\mathrm{MeV}^{-1}$$

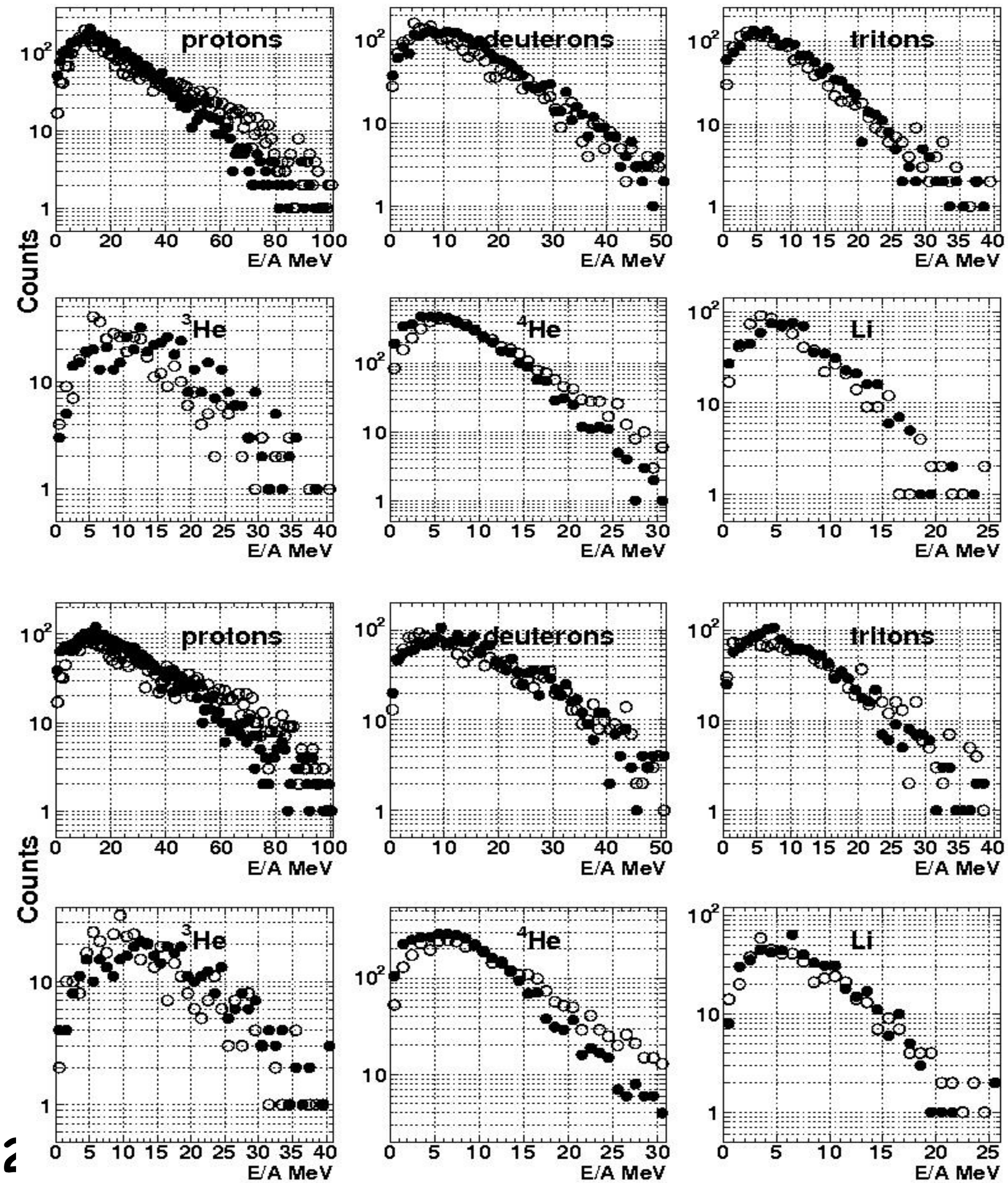
$$\frac{2\langle K^{fo}\rangle}{3} = T_{kin}$$

ensemble average

$$\langle\theta_{frag}\rangle = T_{frag}$$

Inserting (1) in (2) we obtain a third degree equation in $\langle K^{fo}\rangle$=> we find the energy sharing between E* of fragments and thermal kinetic energy

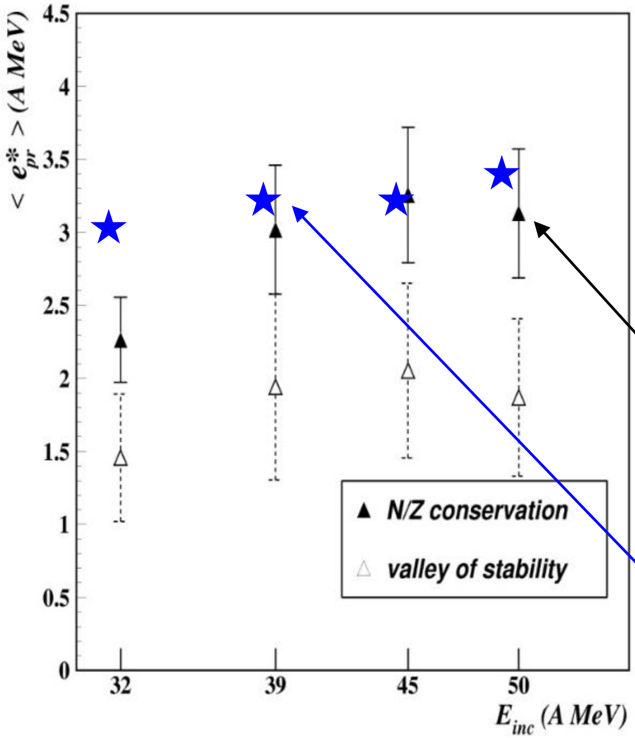
Comparison with the experimental observables: energy spectra for charged particles



39AMeV

50AMeV

E* of primary fragments



QF: Xe + Sn 32-50AMeV

<E*> limited to ~ 3-3.5 MeV

deduced from fragment-particule correlations
S. Hudan et al., PRC 67 (2003) 064613

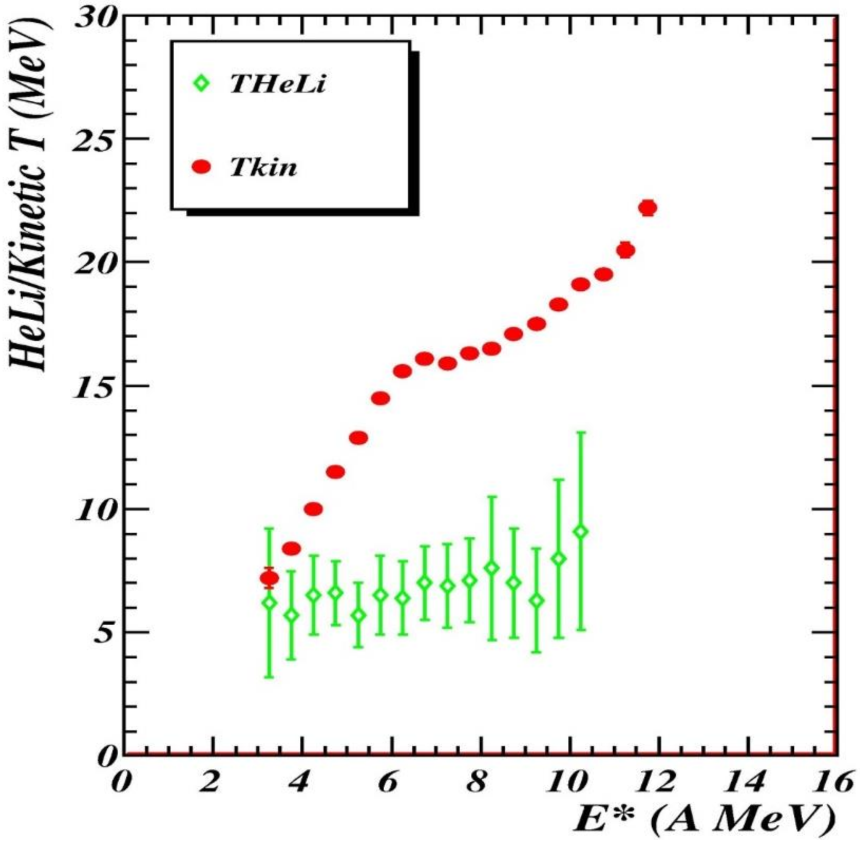
simulation

and 20% (60%) of measured LCP are emitted at the freeze out stage for multifragmenting hot nuclei with an excitation energy of 5.5 (9.0) AMeV

S.Piantelli et al., NPA 809 (2008) 111

Caloric curve deduced from THeLi

it mainly reflects Tfrag up to E*= 10 MeV/A



BB et al. – INDRA Coll. PLB 723 (2013) 140