



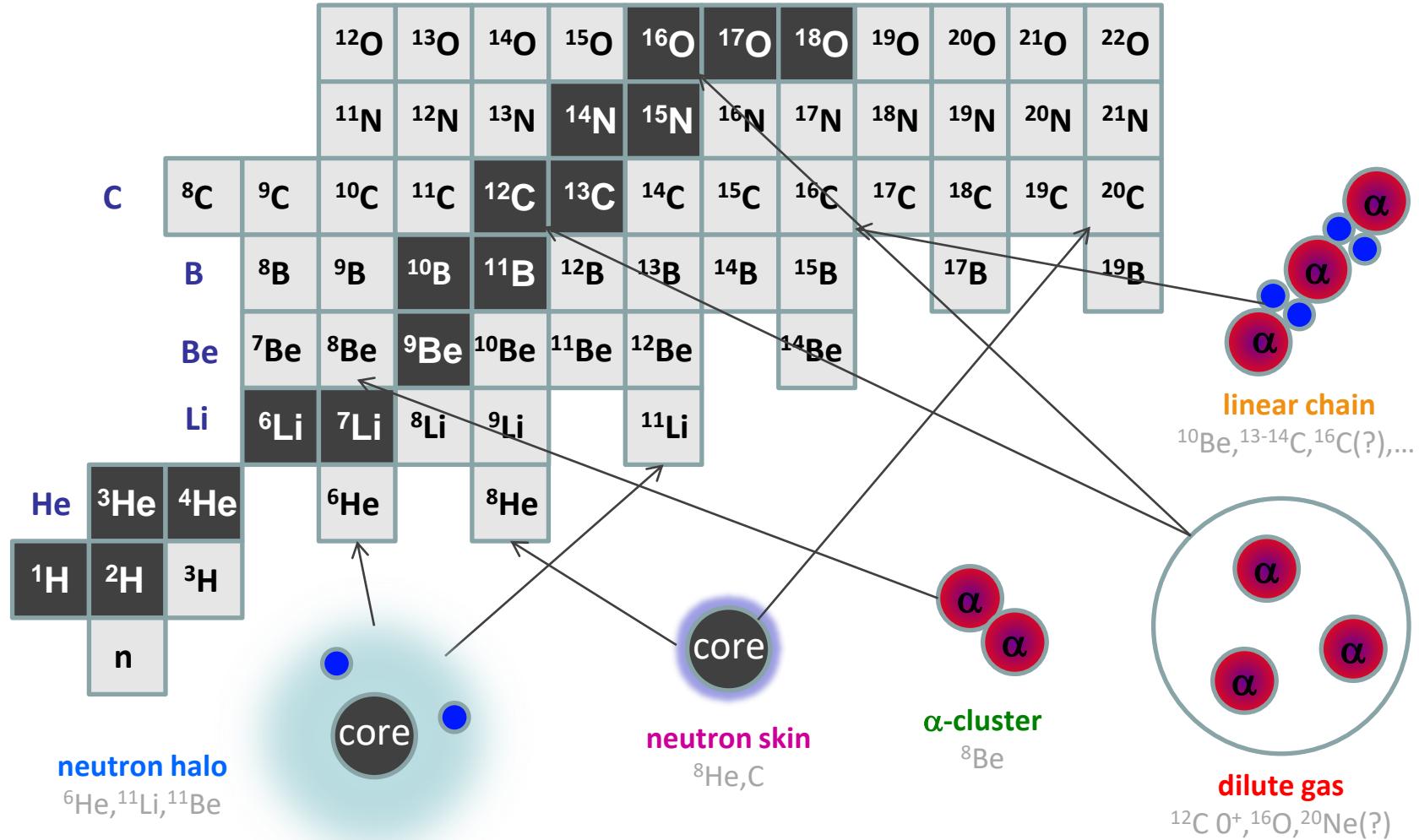
# Clustering effects in nuclear reactions at low and medium energies

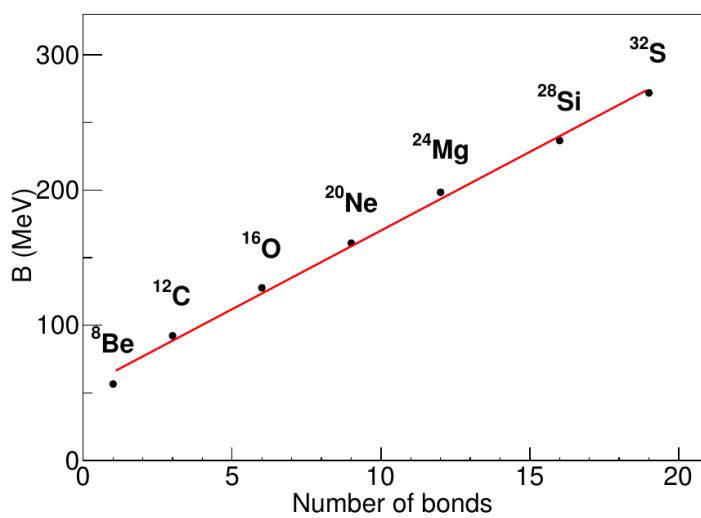
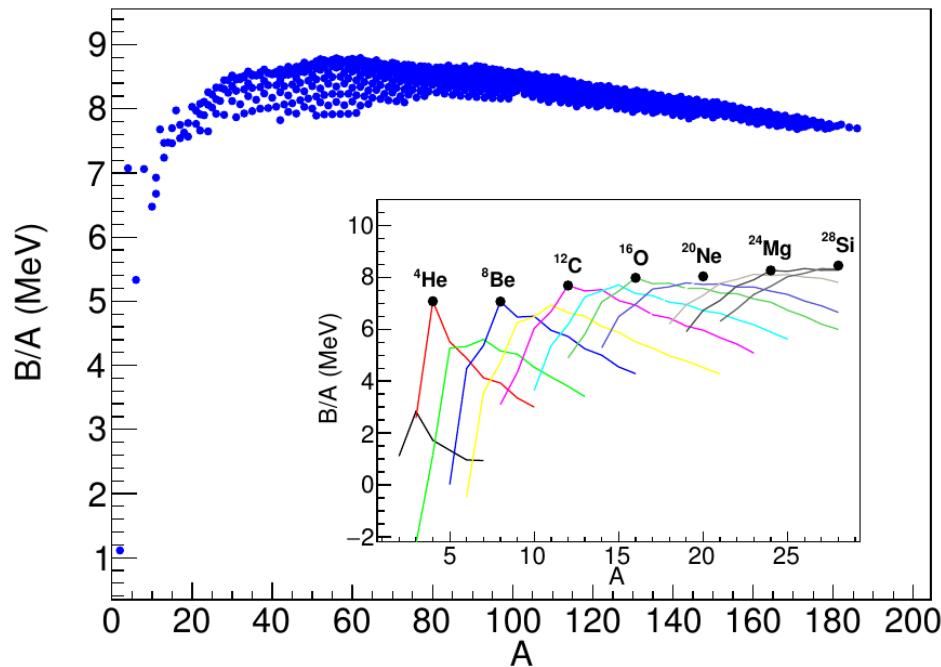
**Ivano Lombardo**  
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**Complexity of nuclear force** → deviation from the **sphericity**: axial deformation (collective behaviours), spatial re-organization of nucleons in bounded **sub-units (cluster model)**.



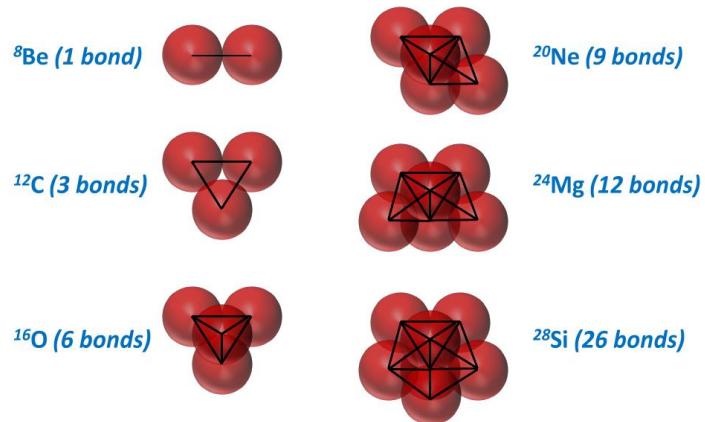


Binding energy of nuclei → self-conjugated nuclei are exceptionally bound → *long range* correlations → **clustering phenomena**.

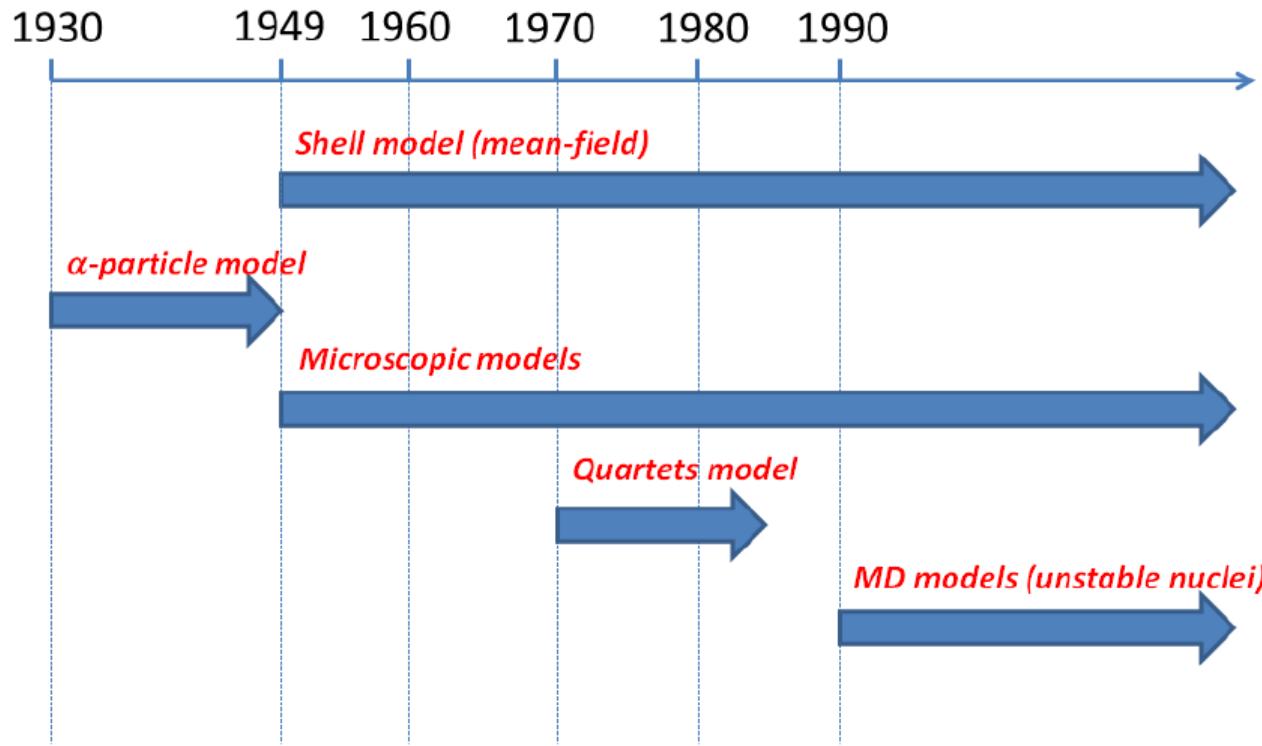
1930's : *Hasfstad and Teller* point out the existence of a linear correlation between the number of  $\alpha$ - $\alpha$  bonds and the binding energy of self-conjugated nuclei:

*L. Hafstad and E. Teller, Phys. Rev. 54, 681 (1938)*

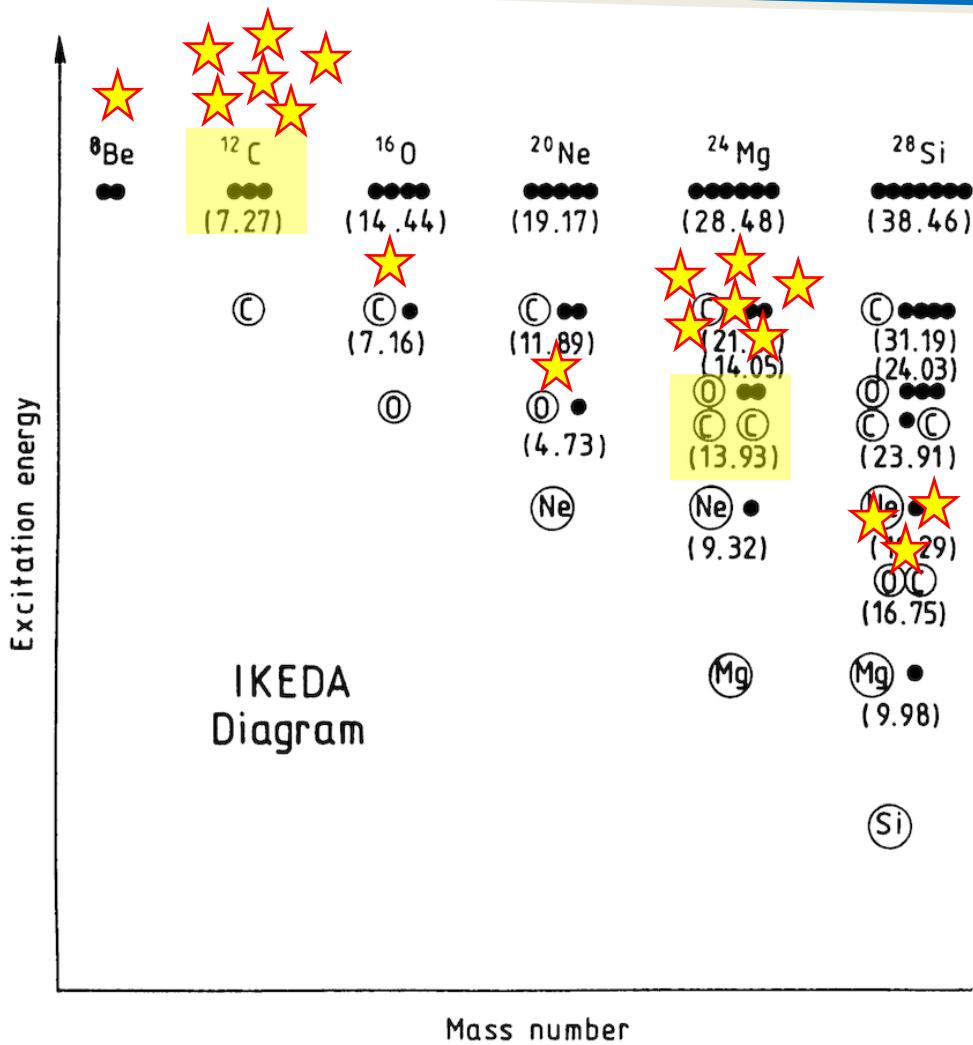
The beginning of the  $\alpha$ -cluster model.



# Historical evolution of cluster models of nuclei



- **$\alpha$ -particle model**: spatial disposition of clusters in self-conjugated systems and rotational/vibrational excitations (sensitive to the  $\alpha$ - $\alpha$  interaction)
- **Shell model**: mean-field and residual interaction → clusters emerge
- **Microscopic models (RGM, GCM, OCM)**: detailed treatment of the Pauli principle and inter-cluster motion
- **Quartet models**: excitation of quartets of nucleons and their mutual interaction
- **MD models (AMD, FMD)**: resolution of the Schrödinger equation by using gaussian wave packets
- **Algebraic model (ACM)**: group symmetries of peculiar geometrical configurations



The **Ikeda rule**:  $\alpha$ -cluster effects should appear close to the  **$\text{N}\alpha$  (or cluster+ $\alpha$ ) decay thresholds**



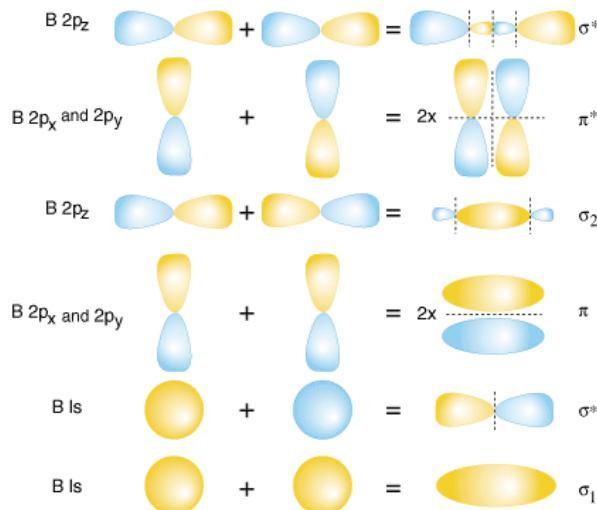
Enormous effects of  $\alpha$ -clustering in nuclear astrophysics!

Modified Ikeda diagram → non self conjugated nuclei

A **qualitative** (but still useful) rule

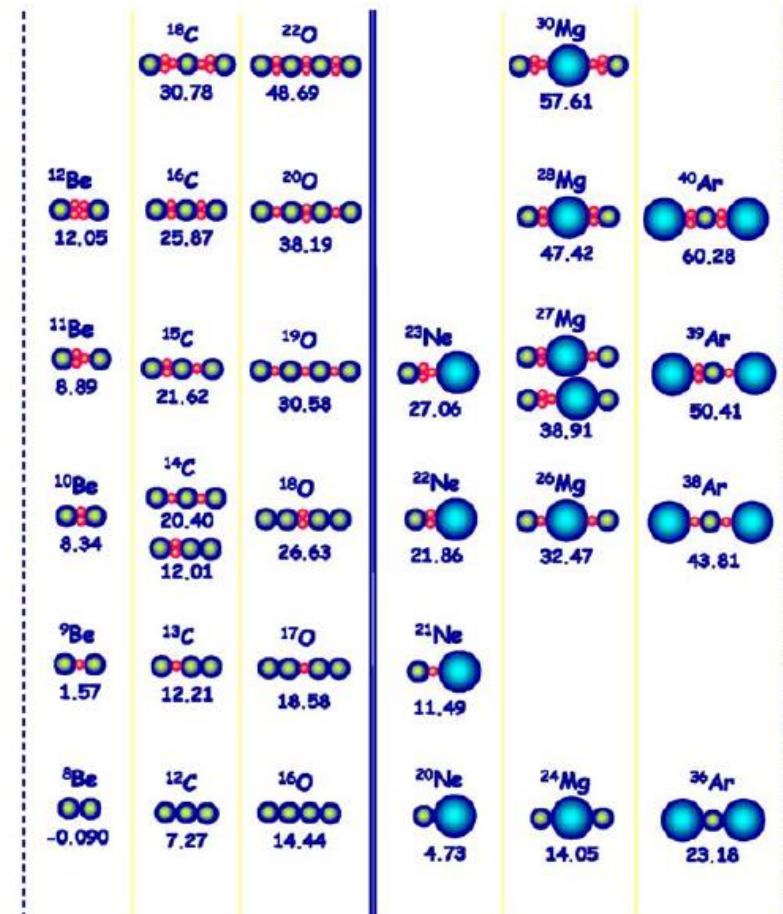
Even if the occurrence of **clustering** in non-self-conjugated nuclei has been reported many decades ago (see, e.g. Rosenfeld & Blatt-Weisskopf) → renewed interest in more recent times

Some ideas taken from **physical chemistry**  
→ extra neutrons acting as **covalence particles**

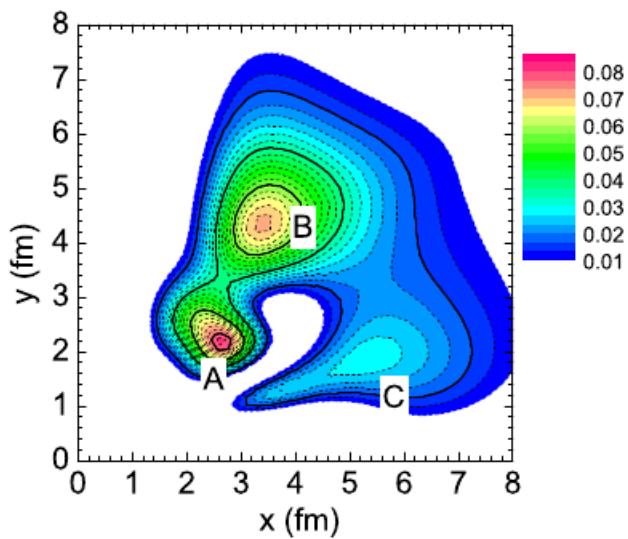


**Energy** considerations → molecular states close to **Nα+n** disintegration thresholds

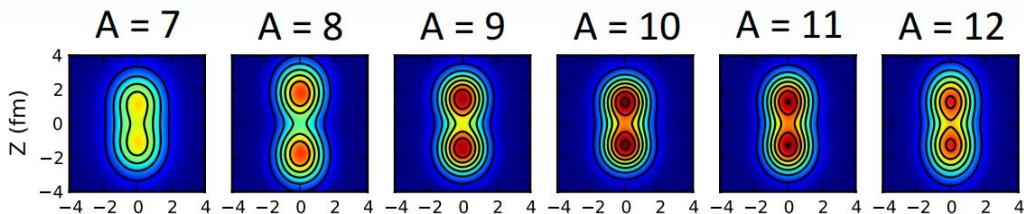
Signals of clusterization also in **p-rich** nuclei



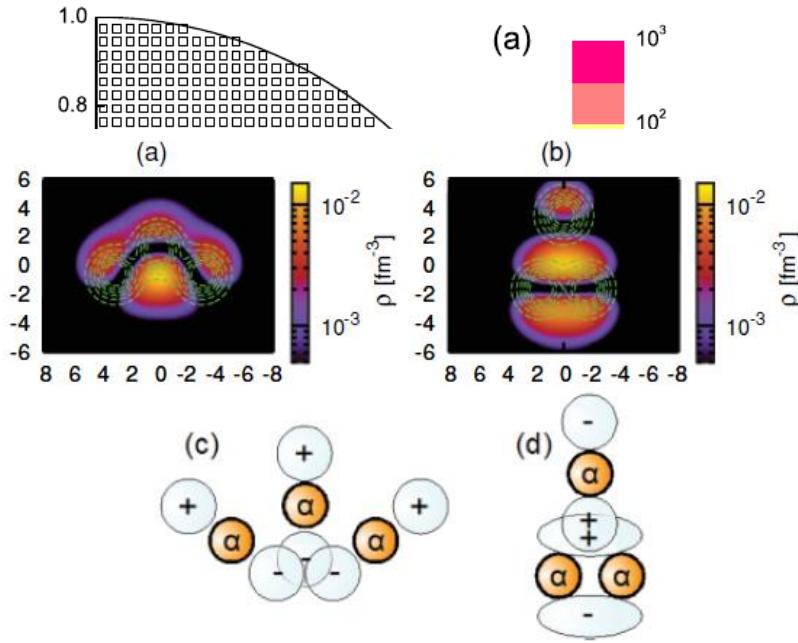
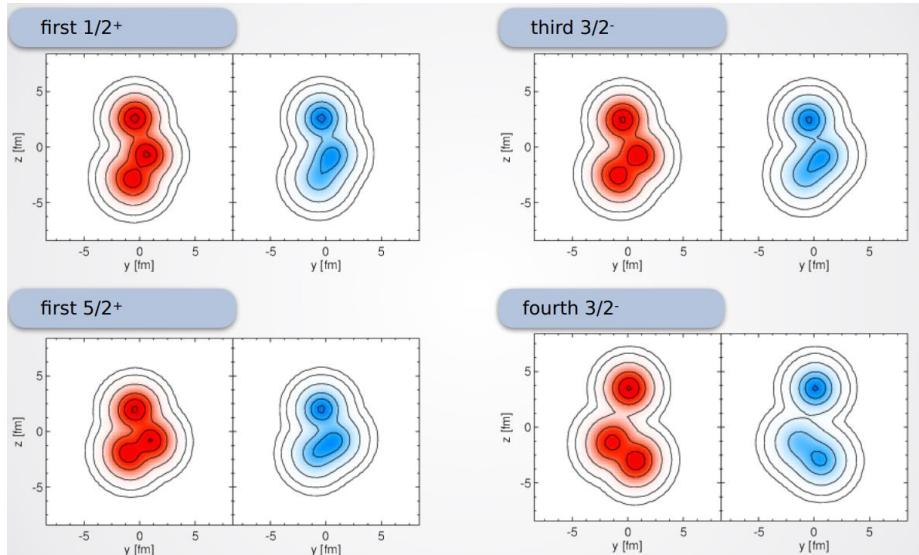
«**Modified** Ikeda Diagram»



## GS of Be isotopes from MCSM, Otsuka. Also BE2

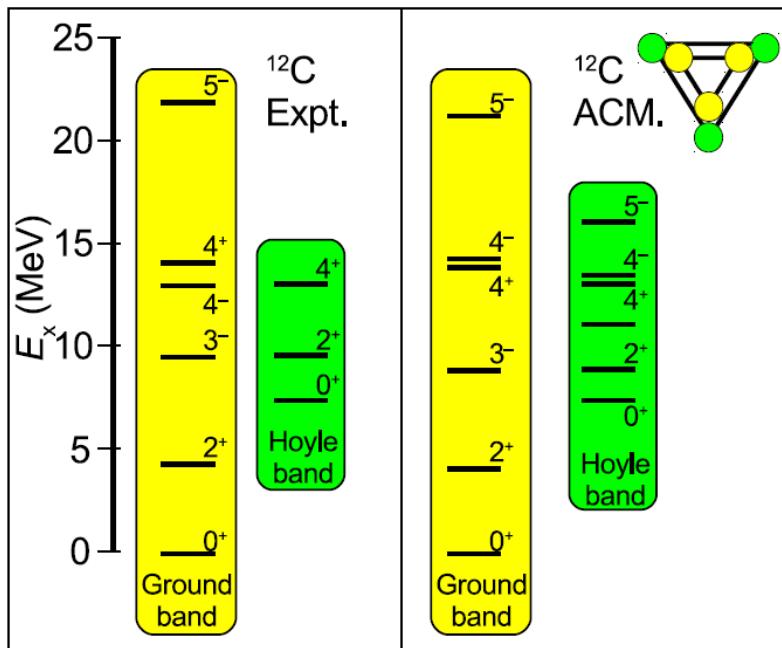


## $^{11}\text{C}$ cluster structure, from Neff (FMD)

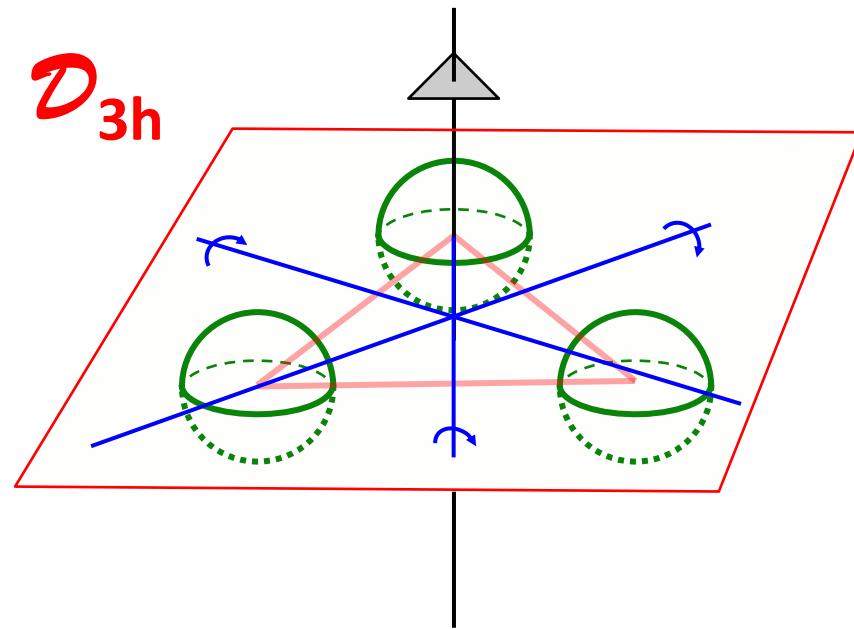


$^{13}\text{C}$  structure of  $3/2^-$  excited states  
above the  $\alpha$  threshold (GCM)  
Kimura & Furutachi

Recently → applications of *point group symmetries* to clusterized nuclei, as  $^{12}\text{C}$



R. Bijker & F. Iachello, Phys. Rev. C 61, 067305 (2000).  
D. M. Lambarri et al, Phys. Rev. Lett. 113 (2014) 012502



- 3-fold principal symmetry axis ( $\mathcal{C}_3$ )
- 3 two-fold axes perpendicular to  $\mathcal{C}_3$
- Horizontal mirror plane

**Symmetry** considerations → important  
benchmark for *microscopic calculations*

Probe of the geometry of  $^{12}\text{C}$  → *nuclear fluorescence* experiments with *linearly polarized  $\gamma$  beams*, see e.g. L. Fortunato, PRC 99 (2019) 031302(R)

**Symmetry** considerations apply  
for *other self-conjugate* nuclei!

And for *non self-conjugate* nuclei?

To investigate *clustering phenomena* in nuclei → nuclear spectroscopy ( $E_x$ ,  $J$ ,  $T$ ,  $\pi$ ,  $C^2S$ )

$E_x$  = excitation energy;

$J$  = spin;

$T$  = isospin;

$\pi$  = parity;

$C^2S$  = spectroscopic factors and/or  $\Gamma$ , partial widths

**Experimental techniques** to study the spectroscopy of nuclei:

- *Compound nucleus reactions («formation») [VdG, Tandem]*

The excited state is populated in the intermediate stage of the reaction (compound nucleus) → the cross section reflects the properties of the compound nucleus spectroscopy → R-matrix theory.

*low energy reactions: RES, (p,a) (n,a) (d,a) etc*

- *Direct reactions and correlations («production») [VdG, Tandem, Cyclotron, RIB facilities]*

Few-step reactions involving the excitation of a nucleus and/or the transfer of one or more nucleons

Suited to study states above or below the emission threshold:

→ Dalitz plot, multi-particle correlation techniques, angular correlations and angular distributions.

*breakup reactions* (ex:  $^{12}\text{C}$  breakup from [M. Freer et al., Phys. Rev. Lett. 82 \(1999\) 1383](#)).

*transfer reactions* (ex:  $^{11}\text{B}({}^3\text{He},\text{d})^{12}\text{C}^*$  from [O.S. Kirsebom et al., Phys. Rev. Lett. 108 \(2012\) 202501](#)).

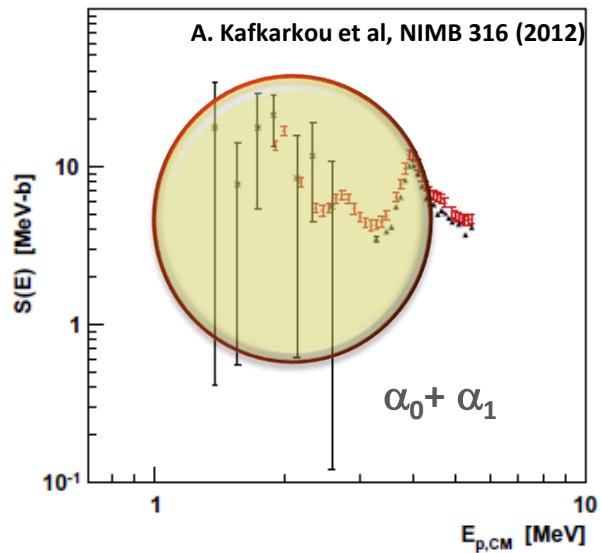
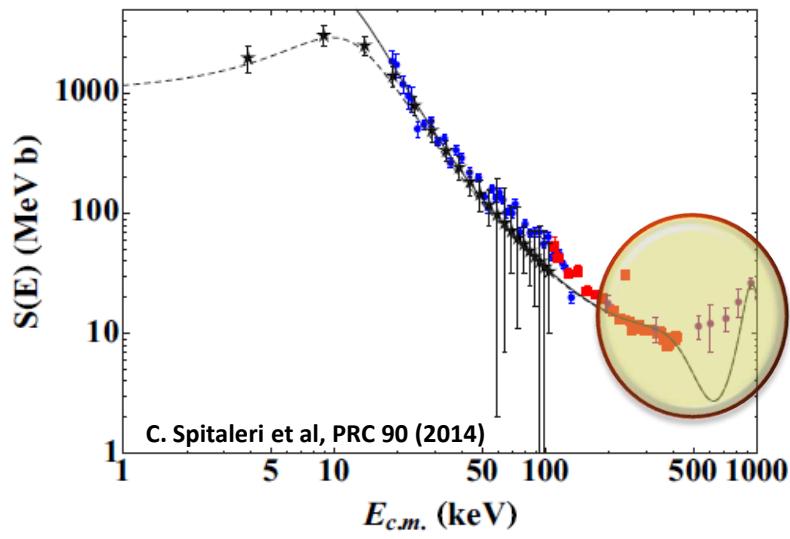
*inelastic scattering* (ex:  $^{12}\text{C}(^{12}\text{C}, {}^{12}\text{C}^*)^{12}\text{C}$  from [M. Itoh et al., Phys. Rev. Lett. 113 \(2014\) 102501](#)).

# Clustering in light systems: Some examples from experiments

# The structure of $^{11}\text{C}$ with the $^{10}\text{B}(\text{p},\alpha)^7\text{Be}$ reaction

**Importance** of the  $^{10}\text{B}(\text{p},\alpha)^{7}\text{Be}$  reaction at **low energy**:

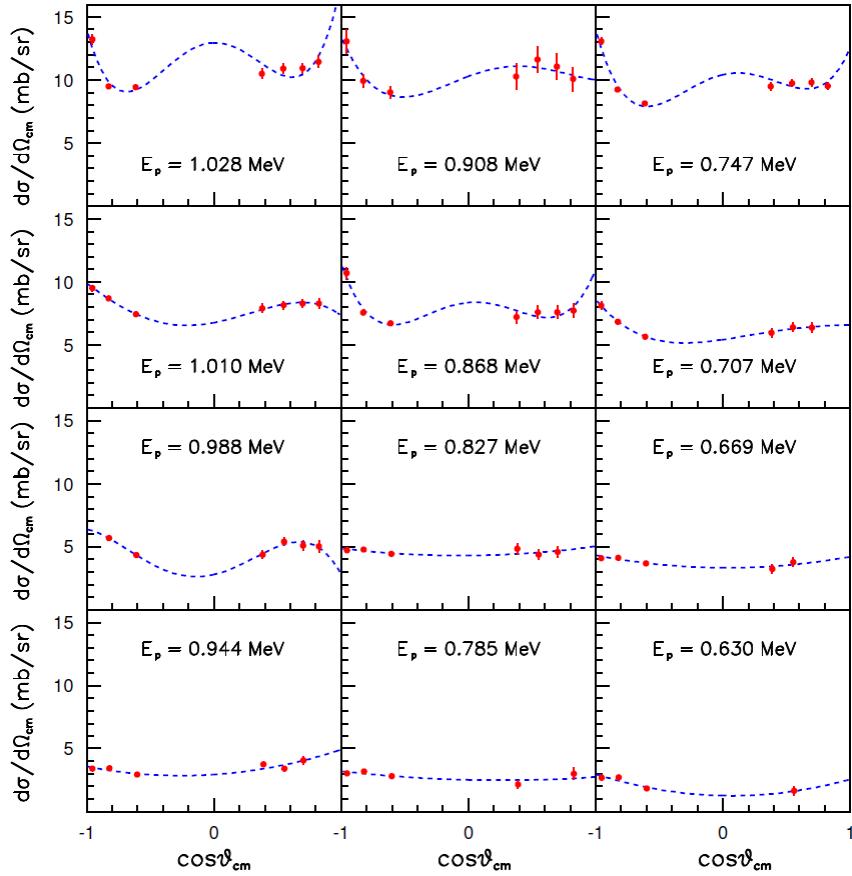
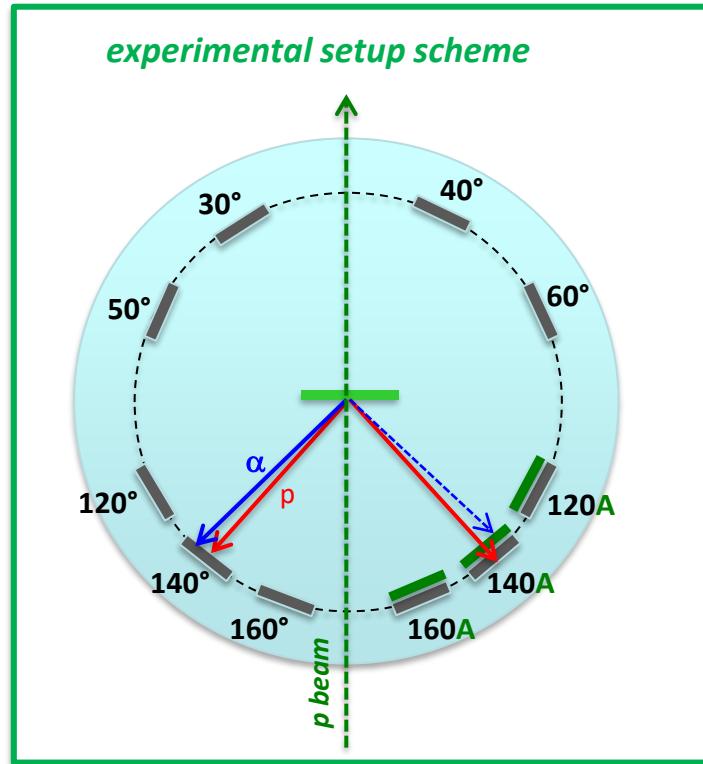
- *Structure* of the compound nucleus,  $^{11}\text{C}$  (*clustering, IAS ...*)
- *Astrophysical* importance →  $^{10}\text{B}$  *destruction* in stars
- *Applied physics* context → *a-neutronic fusion*

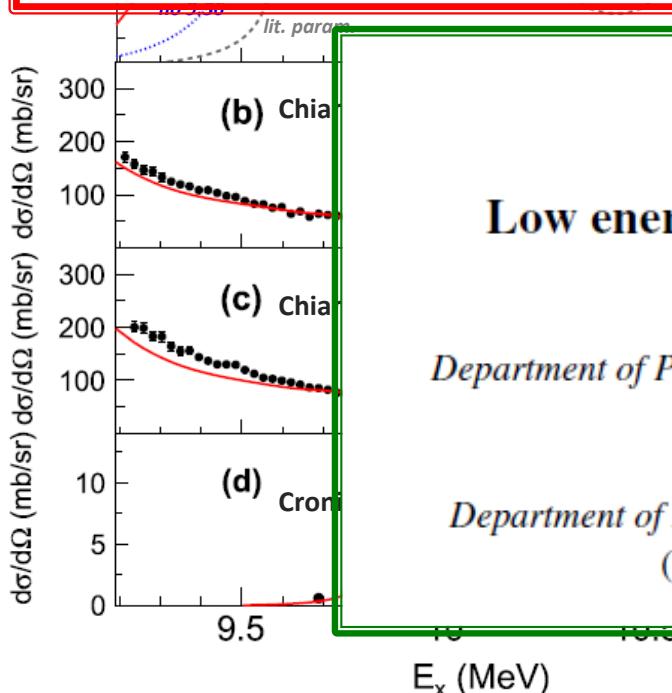


- **Problems** in the description of the **0.5 – 1.5 MeV** region with **known states**
- Very **large error bars** from the (indirect) **thick target yield** measurement  
[Roughton et al, ADNDT 23 (1979)]

**New experiment in Naples with the TTT3 tandem: 0.6 – 1.1 MeV region:**

- 20 – 40 keV steps
- Self supporting  $^{10}\text{B}$  target ( $40 \mu\text{g}/\text{cm}^2$ )  
+ formvar
- Forward angles: 30, 40, 50, 60° → OK
- Backward angles (120, 140, 160°):  
→ inverse absorber technique  
(3  $\mu\text{m}$  Al foil to highly degrade/stop  $\alpha_0$ )



A new study of  $^{10}\text{B}(\text{p},\alpha)^7\text{Be}$  reaction at low energiesA. Caciolli<sup>1,a</sup>, R. Depalo<sup>1</sup>, C. Broggini<sup>2</sup>, M. La Cognata<sup>3</sup>, L. Lamia<sup>4</sup>, R. Menegazzo<sup>2</sup>, L. Mou<sup>5</sup>, S.M.R. Puglia<sup>3</sup>, V. Rigato<sup>5</sup>, S. Romano<sup>3,4</sup>, C. Rossi Alvarez<sup>5</sup>, M.L. Sergi<sup>3</sup>, C. Spitaleri<sup>3,4</sup>, and A. Tumino<sup>3,6</sup>

PHYSICAL REVIEW C 95, 044617 (2017)

Low energy measurements of the  $^{10}\text{B}(\text{p},\alpha)^7\text{Be}$  reaction  
Notre-Dame experimentM. Wiescher,<sup>\*</sup> R. J. deBoer, and J. Görres

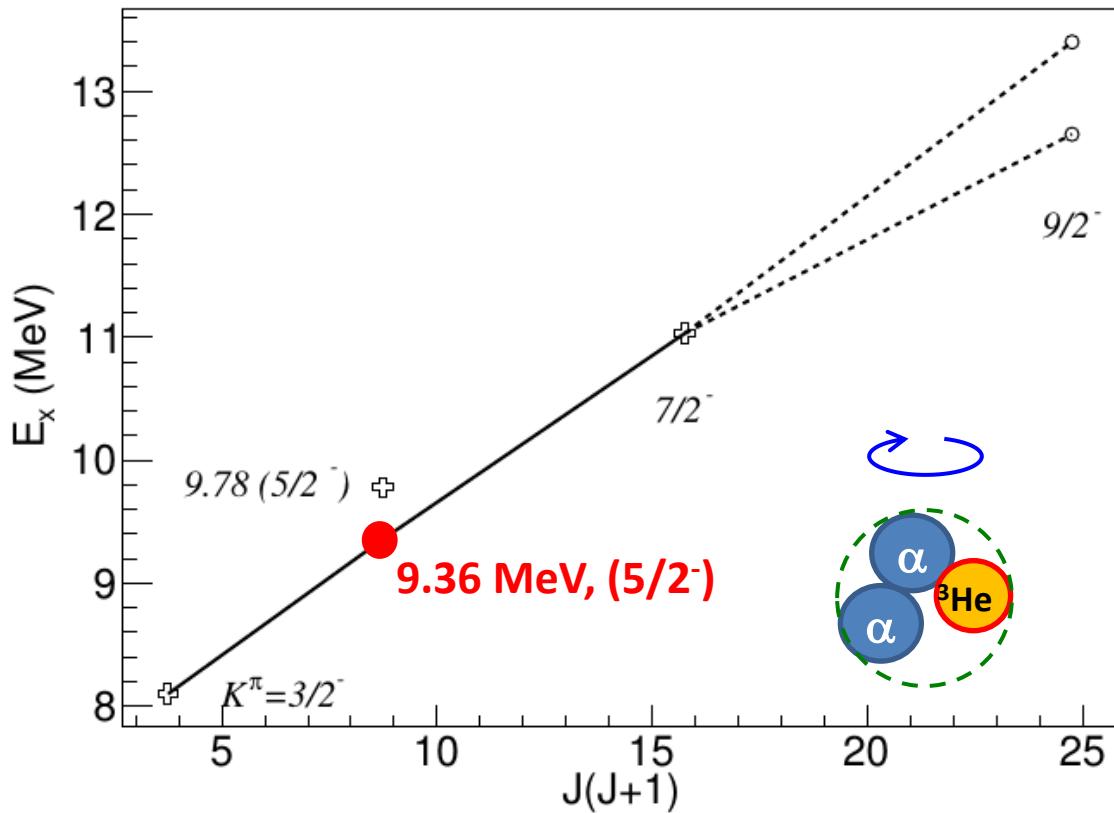
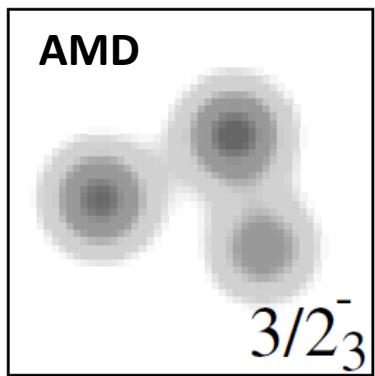
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA

R. E. Azuma

Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada

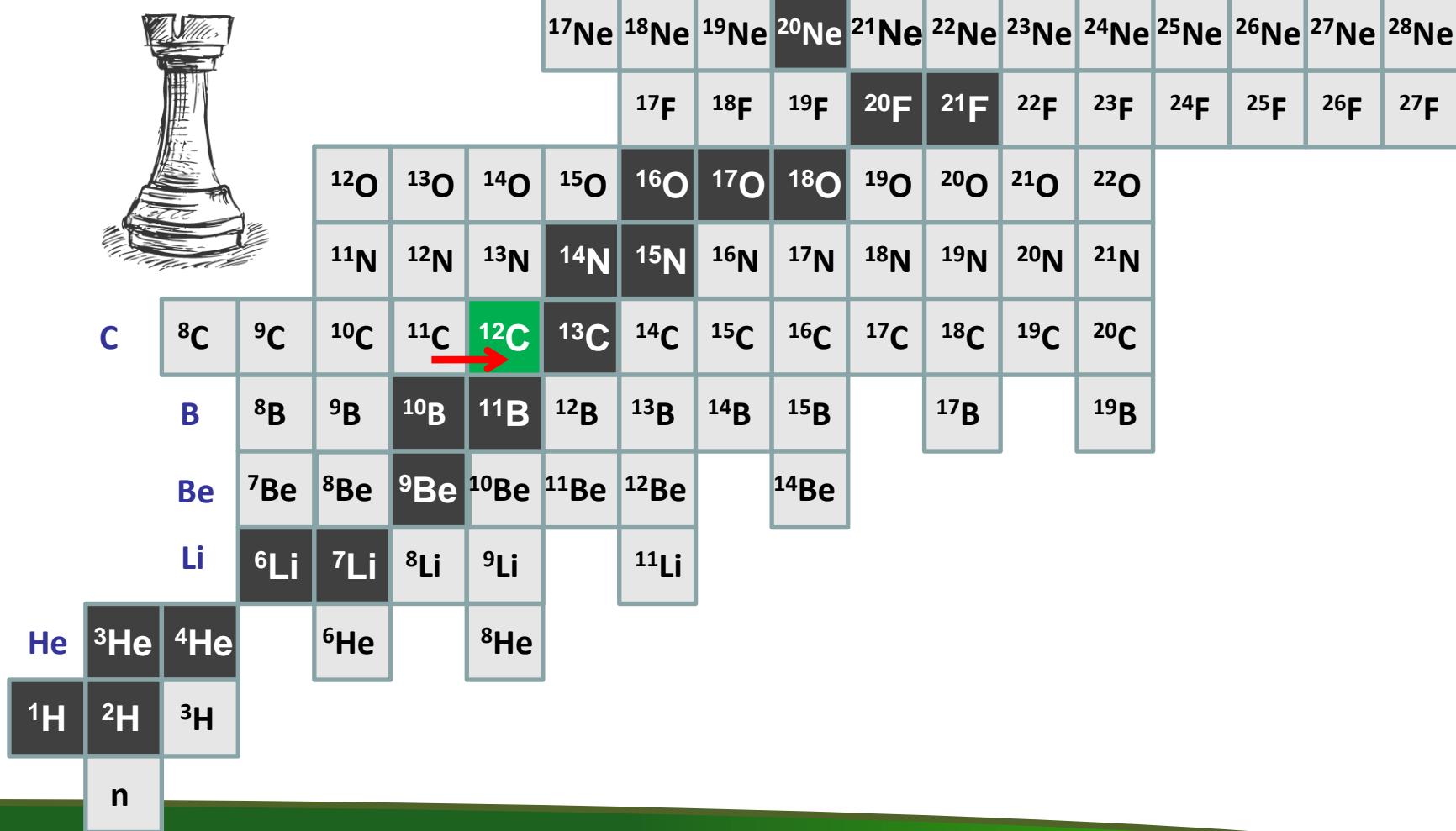
(Received 20 January 2017; published 26 April 2017)

045109



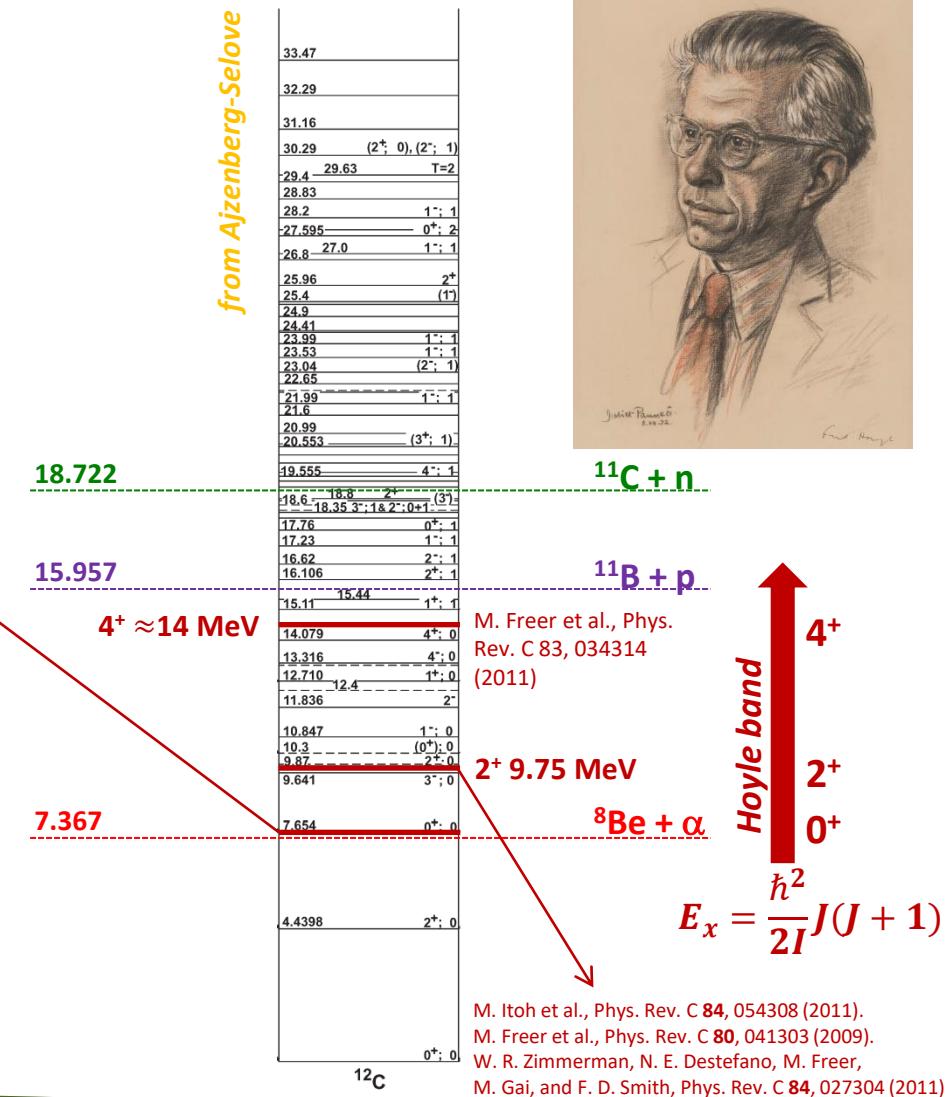
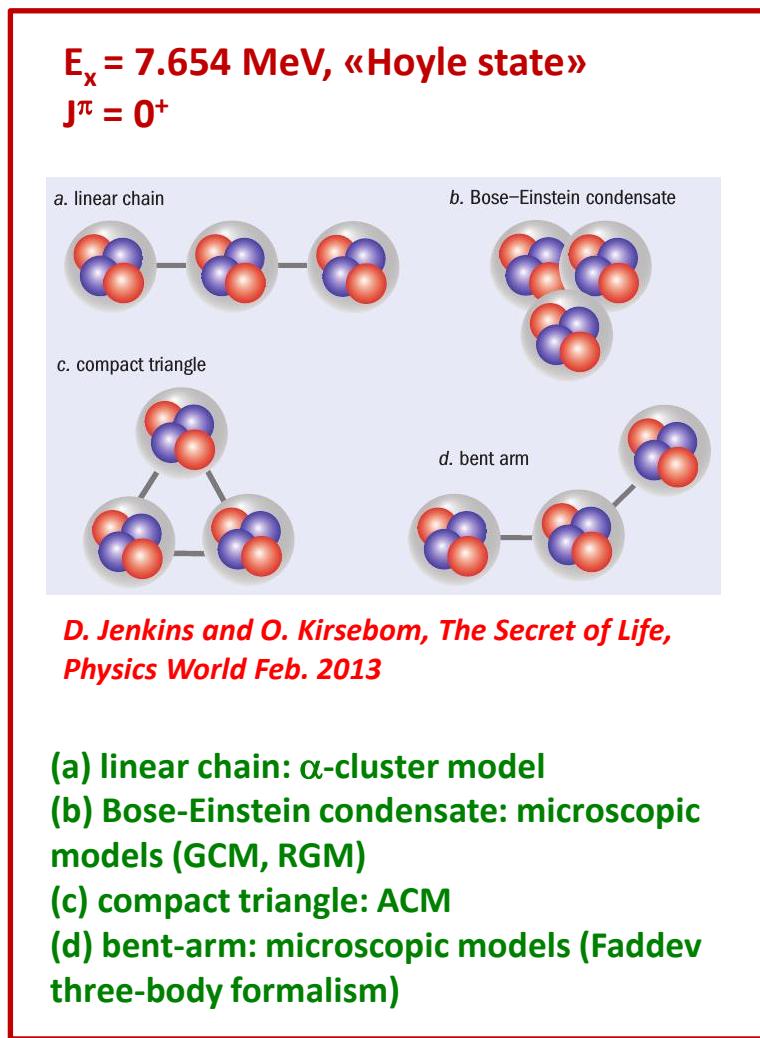
Rotational band  $K^\pi = 3/2^-$  built on the 8.10 MeV state → ambiguity in the assignment of the  $9/2^-$  member ( $\approx 13$  MeV) → best fit with our 9.36 MeV  $5/2^-$ .

# Clustering in $^{12}\text{C}$ : the decay path of the Hoyle state

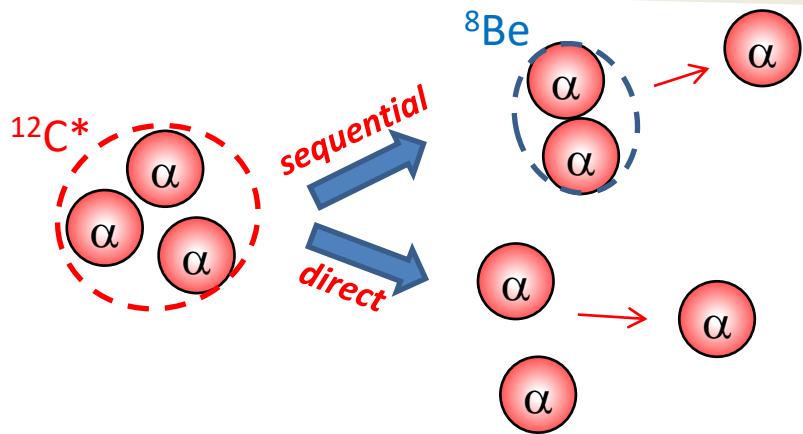


## The structure of $^{12}\text{C}$

**Cluster state** of  $^{12}\text{C}$  located at 7.654 MeV ( $0^+$ ) → a pronounced cluster nature  
 → not well understood properties → challenging open question in nuclear physics

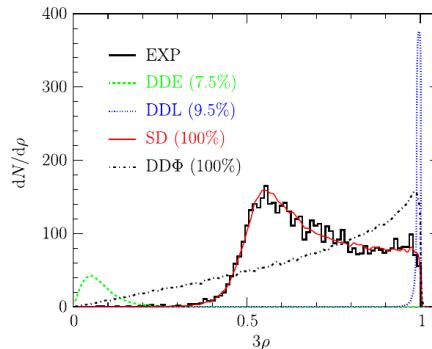


# The Hoyle state in $^{12}\text{C}$ : the non-resonant decay branch



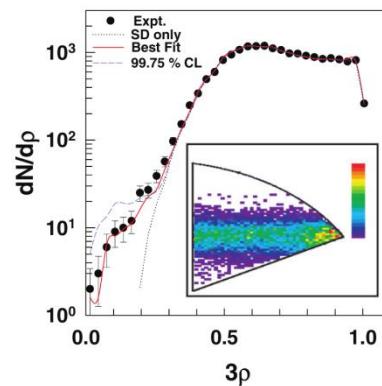
Experiment	$\Gamma_{3\alpha}/\Gamma (\%)$
[1] M. Freer <i>et al.</i>	$< 4$
[2] Ad.R. Raduta <i>et al.</i>	$17.0 \pm 5.0$
[3] J. Manfredi <i>et al.</i>	$3.9 \pm 1.1$
[4] O.S. Kirsebom <i>et al.</i>	$< 0.5$
[5] T.K. Rana <i>et al.</i>	$0.91 \pm 0.14$
[6] M. Itoh <i>et al.</i>	$< 0.2$
[7] L. Morelli <i>et al.</i>	$1.1 \pm 0.8$

[1] M. Freer *et al.*, Phys. Rev. C 49, R1751 (1994). [2] Ad. R. Raduta *et al.*, Phys. Lett. B 705, 65 (2011). [3] J. Manfredi *et al.*, Phys. Rev. C 85, 037603 (2012). [4] O. S. Kirsebom *et al.*, Phys. Rev. Lett. 108, 202501 (2012). [5] T. K. Rana *et al.*, Phys. Rev. C 88, 021601(R) (2013). [6] M. Itoh *et al.*, Phys. Rev. Lett 113, 102501 (2014). [7] L. Morelli *et al.*, J. Phys. G: Nucl. Part. Phys. 43, 045110 (2016).



O.S. Kirsebom *et al.*

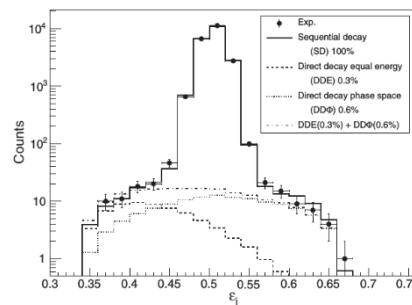
$\approx 5000$  events  $\rightarrow$  no evidence of direct decays  $\rightarrow$  upper limit of 0.5% to direct branching ratios.



T.K. Rana *et al.*

$\approx 20000$  events  $\rightarrow$  non-vanishing direct decay branching ratio

$$\frac{\Gamma_{3\alpha}}{\Gamma} = 0.91 \pm 0.14$$



M. Itoh *et al.*

$\approx 20000$  events  $\rightarrow$  direct decay under the sensitivity of the experimental sensitivity  $\rightarrow < 0.2\%$

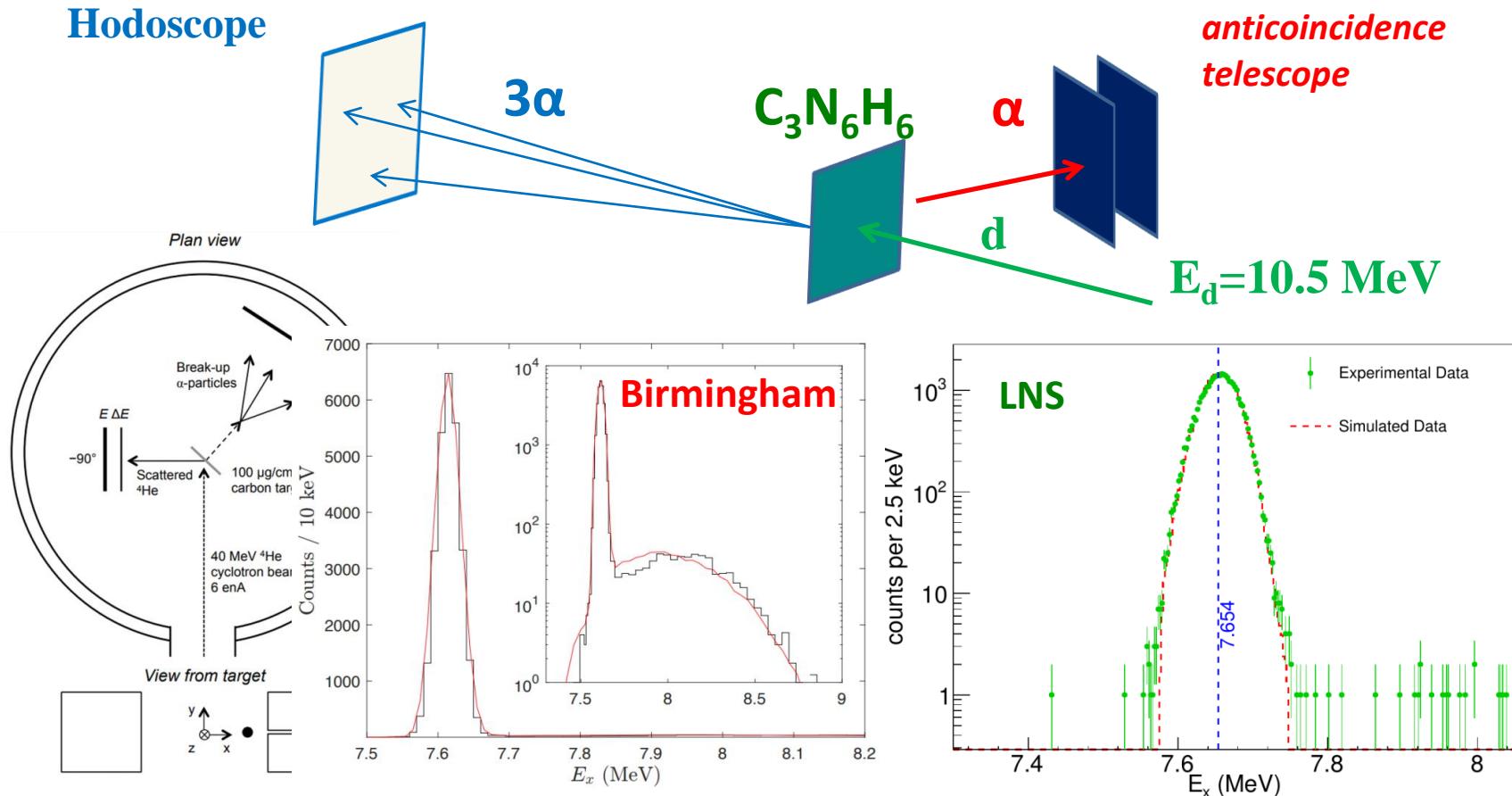
# High-precision probe of the Hoyle state direct decay width



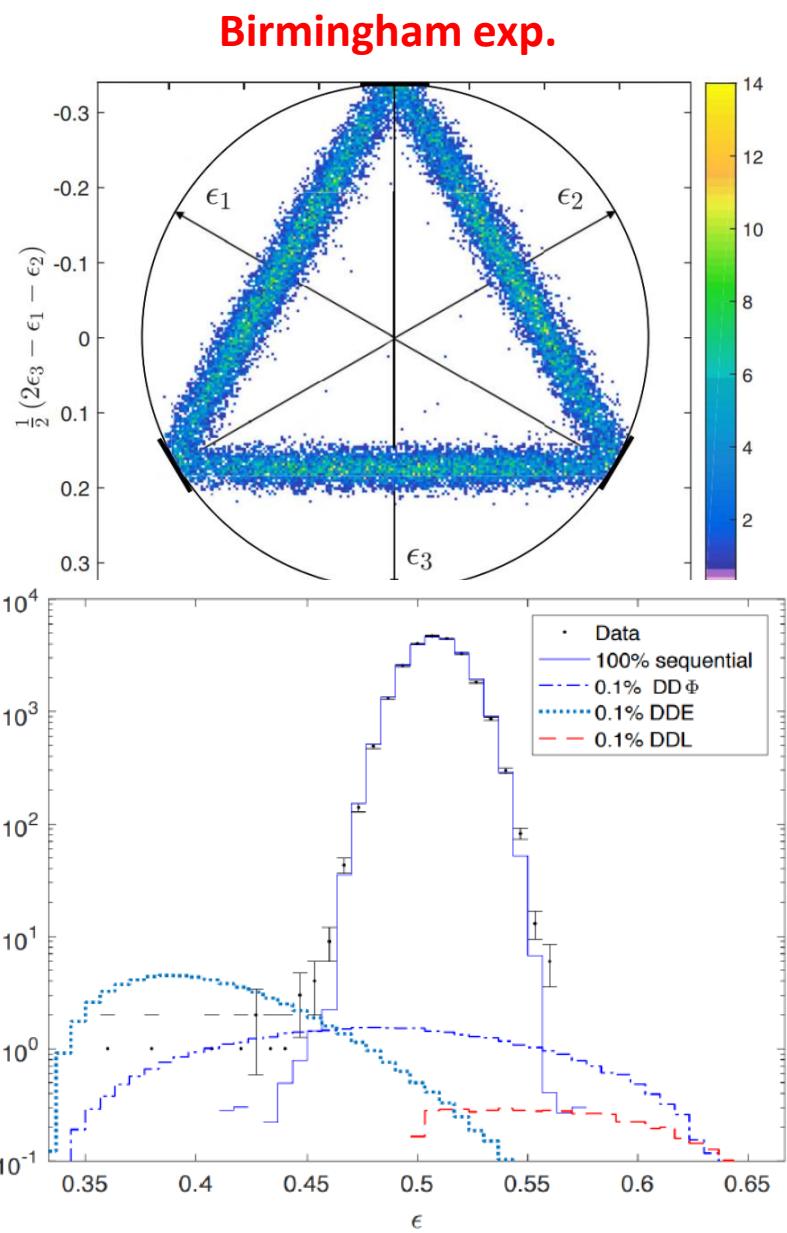
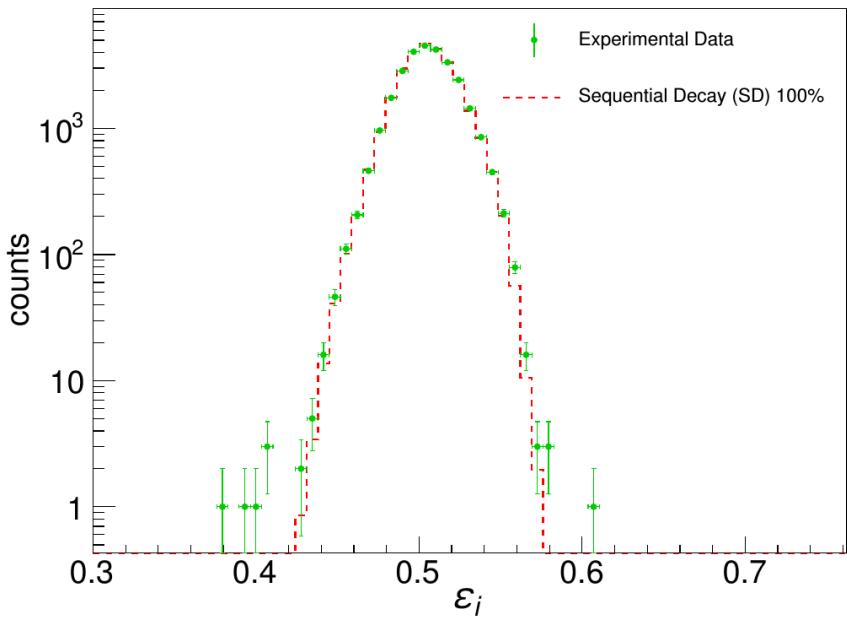
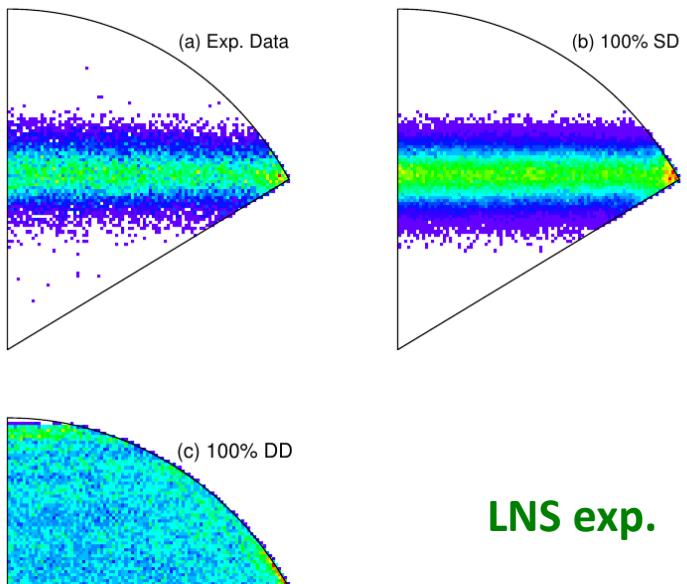
## $^{14}\text{N}(\text{d},\alpha_2)^{12}\text{C}(7.654)$ at LNS, Catania

- *d beam* at  $E_d = 10.5 \text{ MeV}$ ;
- $\text{C}_3\text{N}_6\text{H}_6$  target (**40  $\mu\text{g}/\text{cm}^2$** ) + **5/10  $\mu\text{g}/\text{cm}^2$  backing C**;
- $I_d \approx 4 \text{ enA}$ ;
- $\approx 30 \text{ days}$  of beam time;
- anti-coincidence telescope + hodoscope in KC

D. Dell'Aquila et al, PRL 119 (2017) 132501



# High-precision probe of the Hoyle state direct decay width



Faddeev **3-body calculations** by Ishikawa (2014):  $DD < 1\%$

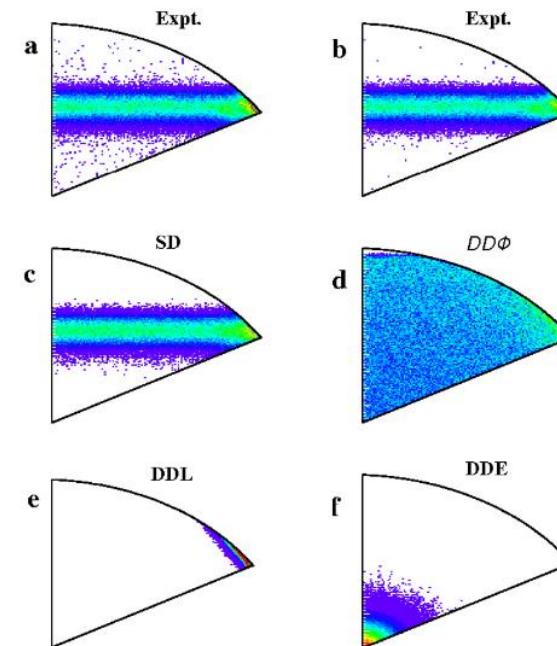
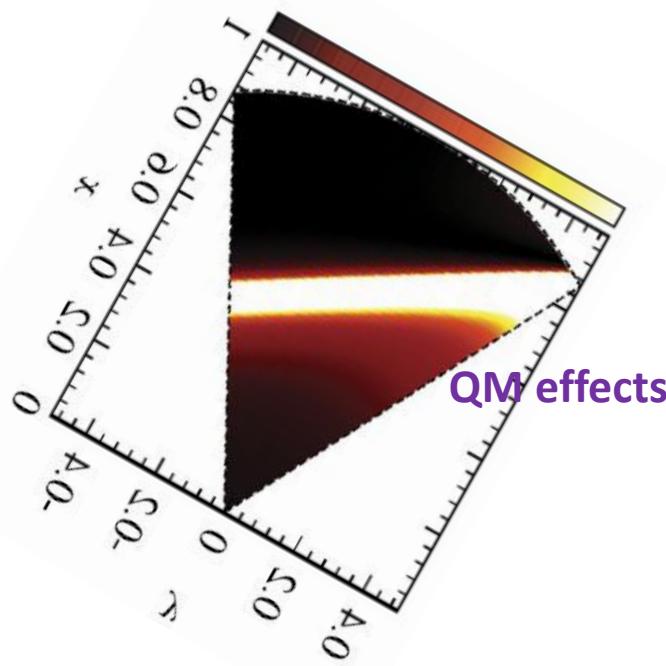
Simple **phase-space** calculations+WKB penetrability by Smith et al (PRL, 2017):

$$BR \approx \frac{\varphi_{3body}}{\varphi_{2body}} \times P_{WKB} = 0.0018 \times 0.333 \approx 0.0006$$

**Semi-classical calculations** by Zheng et al (Phys. Lett. B, 2018)

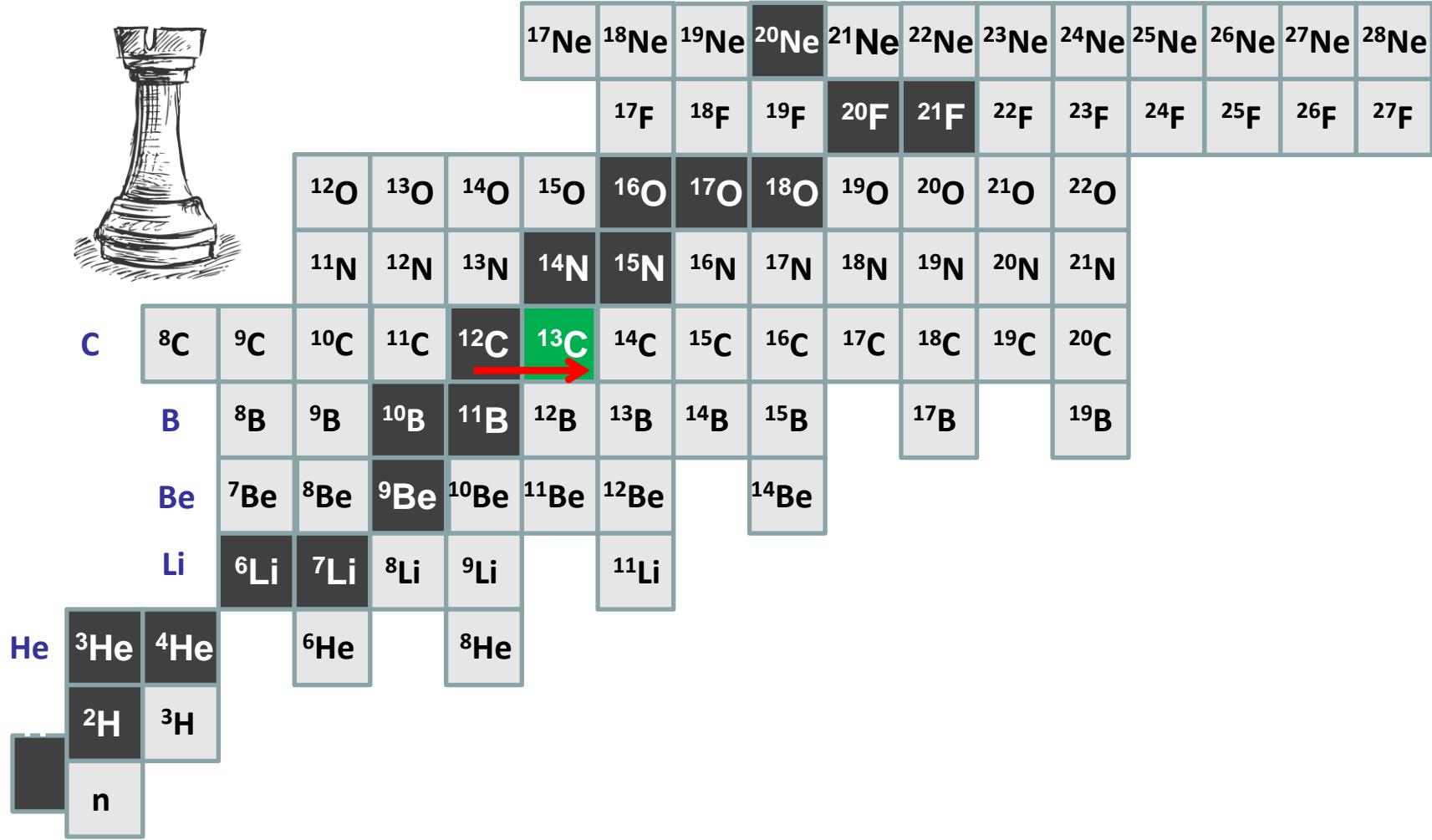
**R-matrix calculations** by Refsgaard et al (Phys. Lett. B, 2018)

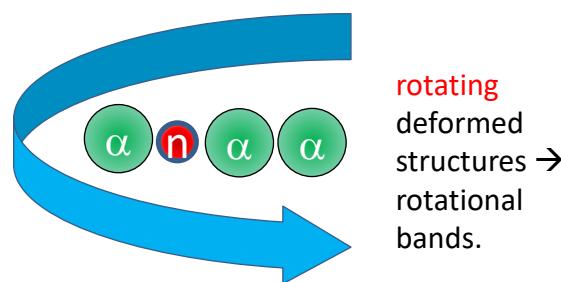
<0.019% 95% CL



T.K. Rana et al, Phys. Lett. B 793 (2019)

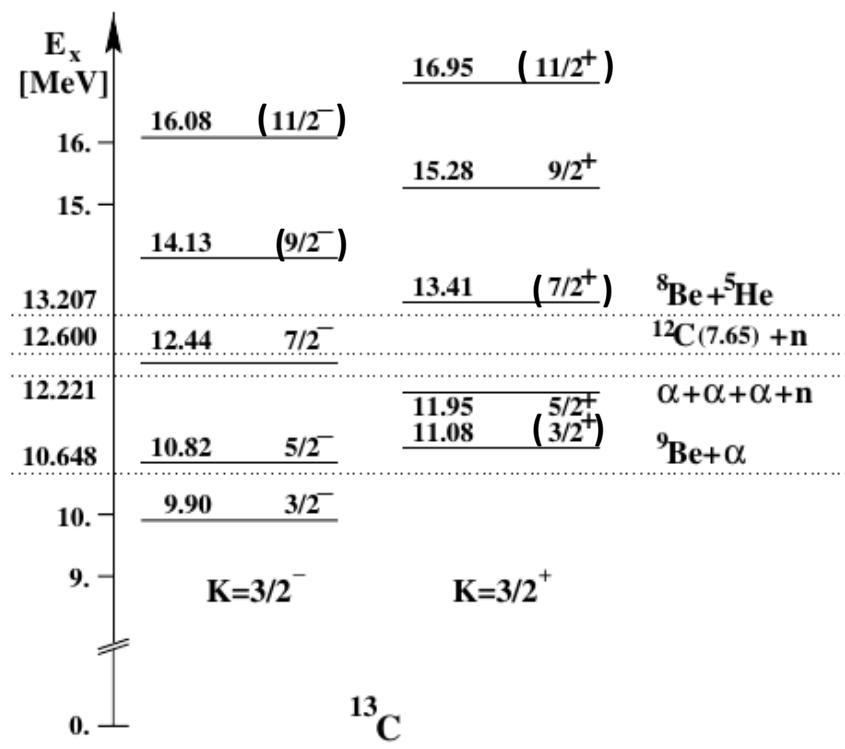
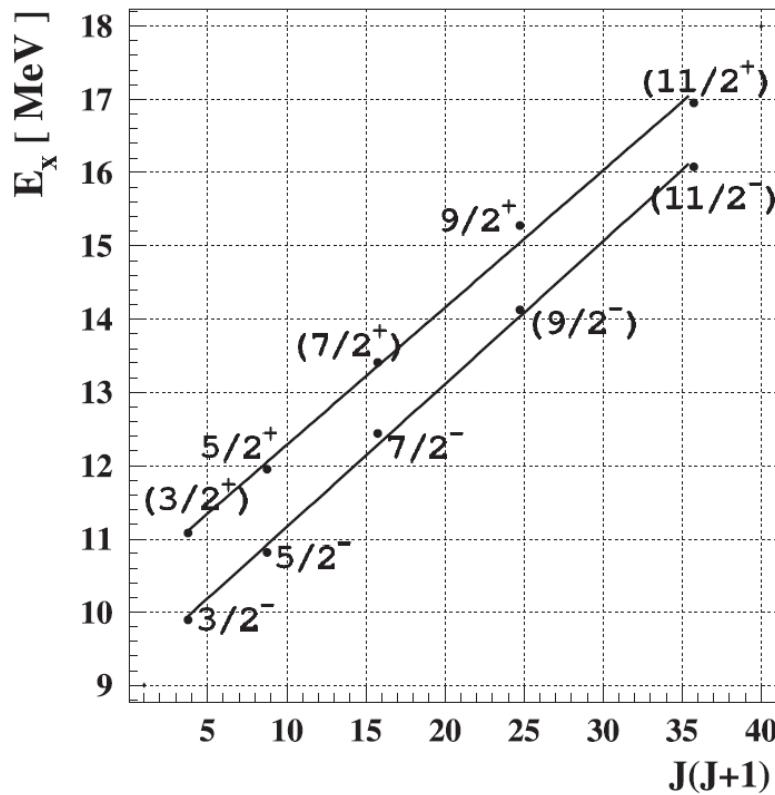
# Clustering in $^{13}\text{C}$ with $\alpha + ^9\text{Be}$ reactions





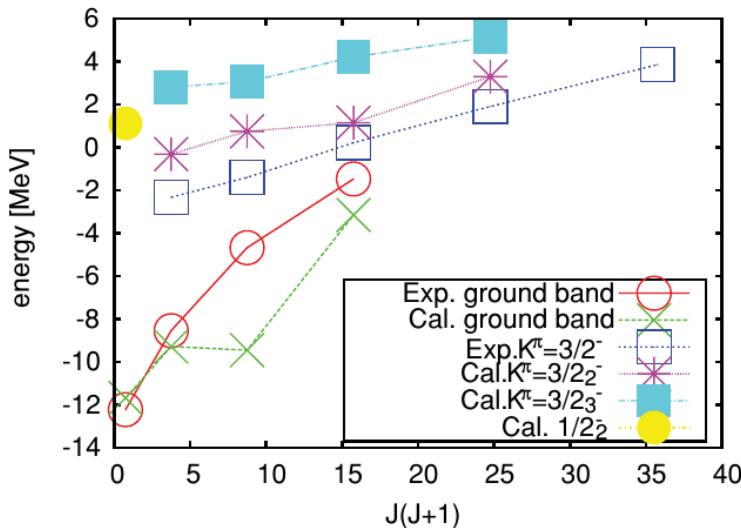
Near and above the  $\alpha$ -threshold  $\rightarrow$  different  $\alpha$ -cluster configurations proposed for  $^{13}\text{C}$   $\rightarrow$  theoretical works:

M. Milin and W. Von Oertzen EPJ A 14 (2002) 295  
proposed parity doublet band of  $^9\text{Be}_{\text{gs}} + \alpha$  cluster prolate configuration  $\rightarrow J^\pi$  assignments based on the rotational bands ( $K=3/2^\pm$ ).



M. Milin and W. Von Oertzen EPJ A 14 (2002) 295

Near and above the  $\alpha$ -threshold  $\rightarrow$  different  $\alpha$ -cluster configurations proposed for <sup>13</sup>C



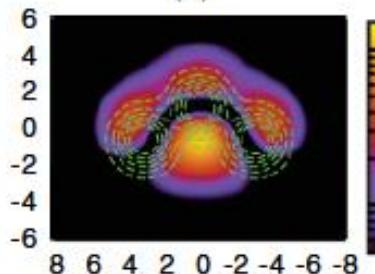
“By adding the valence neutron in this ( $\pi$ ) orbital, the  $\alpha$ - $\alpha$  **distance shrinks** and the **binding energy** of the system **increases**. As a consequence, the **stability** of the **bent linear-chain** configuration is **achieved**.”

N. Furutachi and M. Kimura, Phys. Rev. C 83 (2011)

Microscopic GCM  $3\alpha+n$  model  $\rightarrow$  proposed two new rotational bands ( $K=3/2_{-2}$  and  $K=3/2_{-3}$ ).

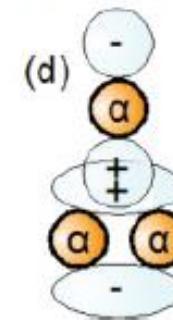
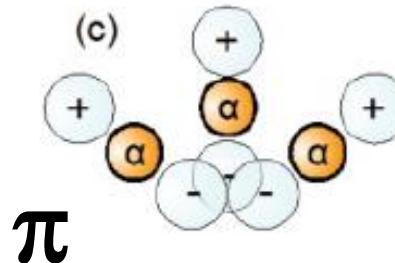
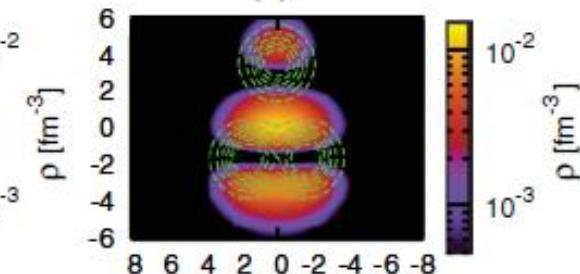
$3/2_{-2}^{-}$

(a)

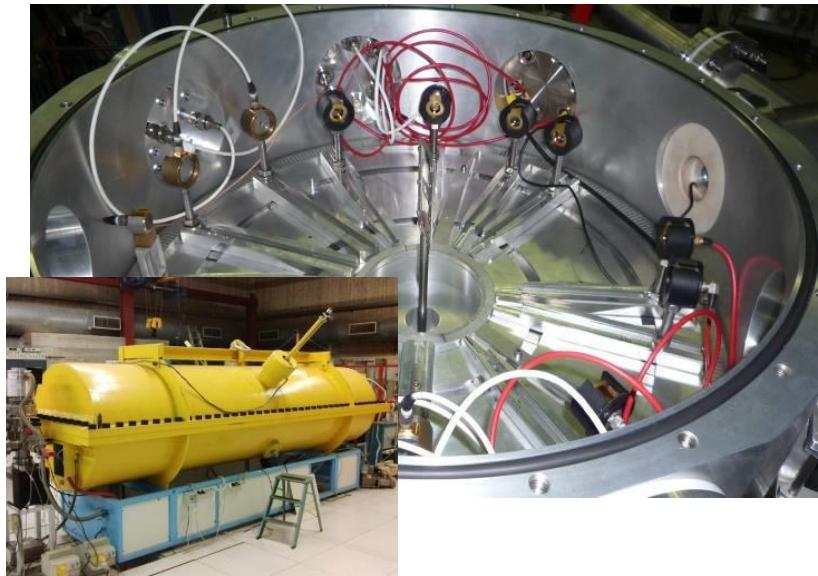
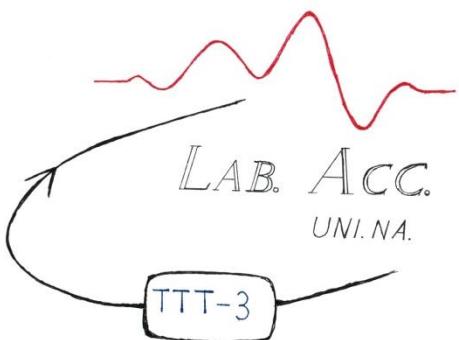


$3/2_{-3}^{-}$

(b)



Spur.



**Thin  ${}^9\text{Be}$  target** → self supporting,  $122 \mu\text{g}/\text{cm}^2$

**C** and **O** contamination

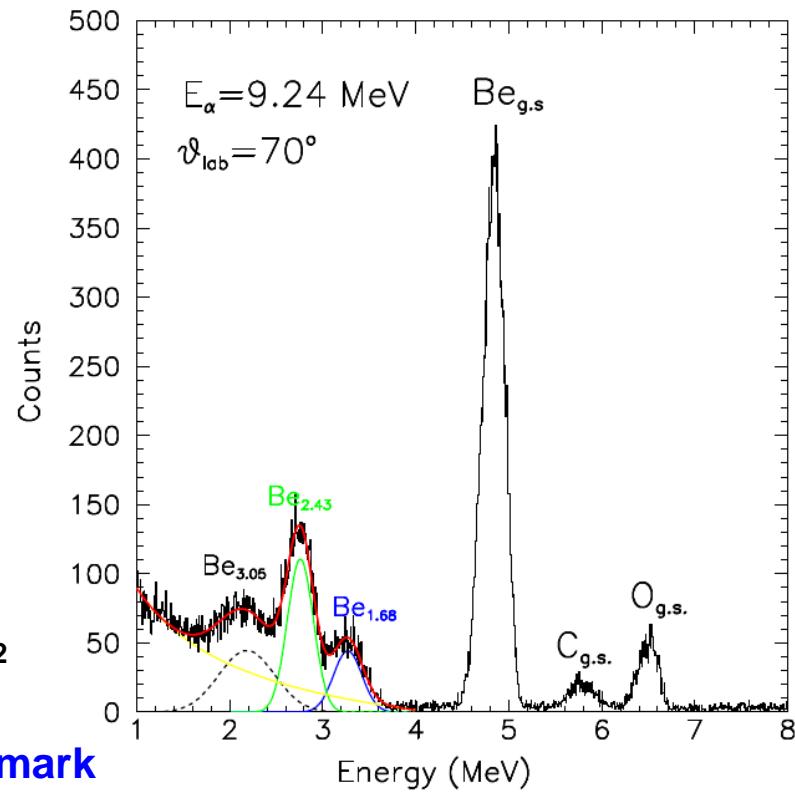
**Thin target absolute cross sections** → **benchmark**  
with a dedicated **thick target experiment**

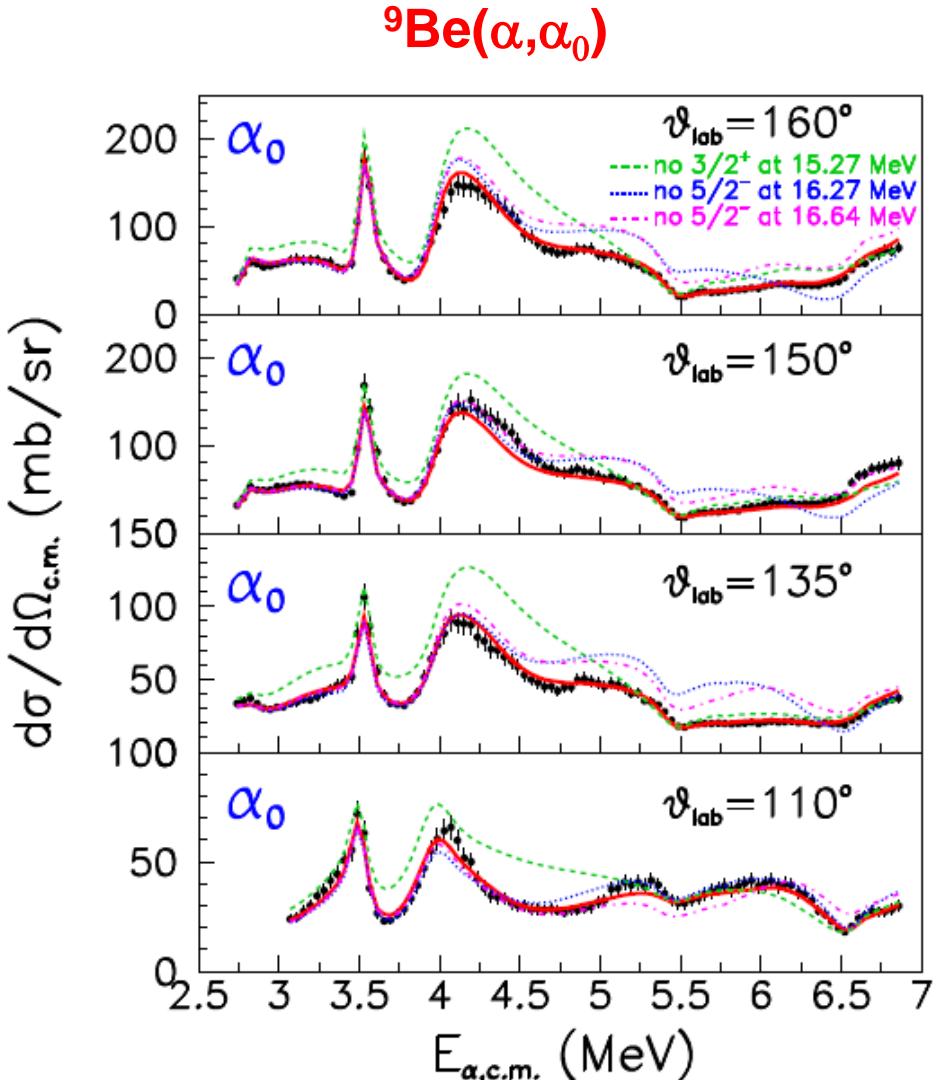
I. Lombardo et al, NIM B 302 (2013) 19-23

**Beam:**  ${}^4\text{He}^{++} \rightarrow E_{\text{Lab}}$ : **3.6 – 10 MeV**

(**60 keV step, 110 energy changes, energy spread  $\leq 3 \text{ keV}$** ) TTT-3MV of Lab. Acceleratore – Naples, Italy

**Silicon detectors:** energy resolution  $\approx 5.0 \%$

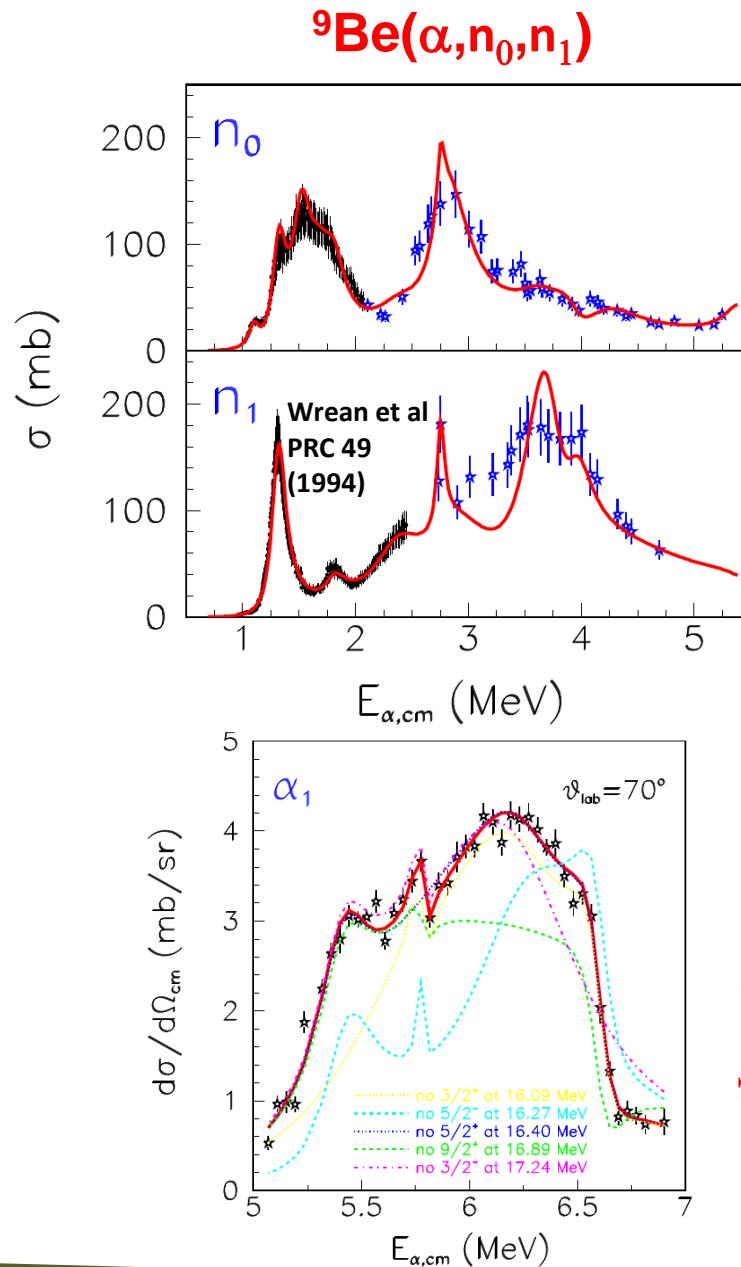


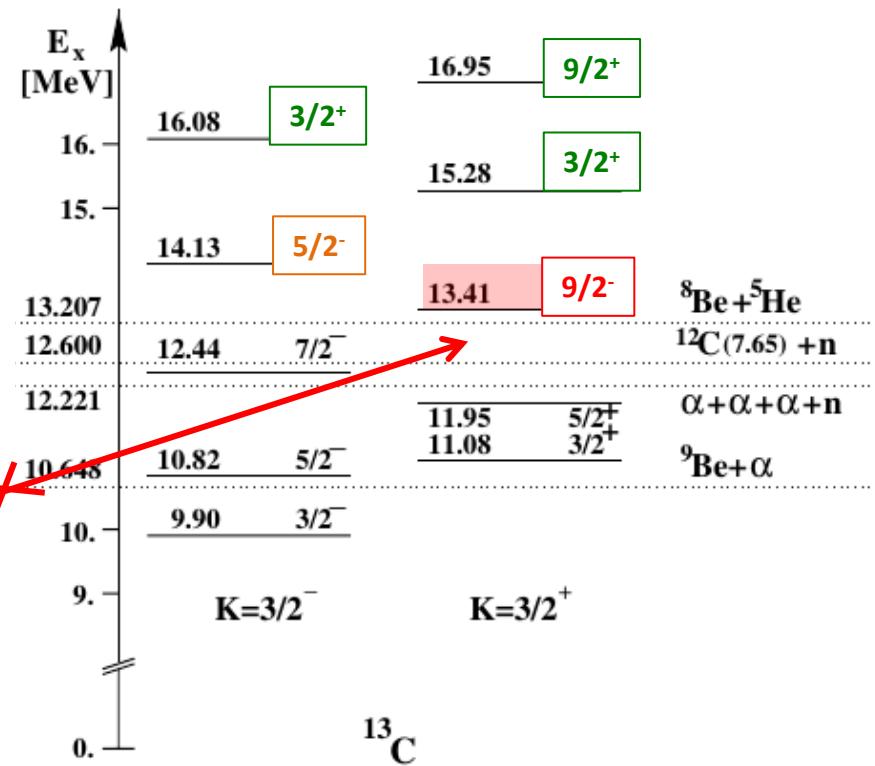
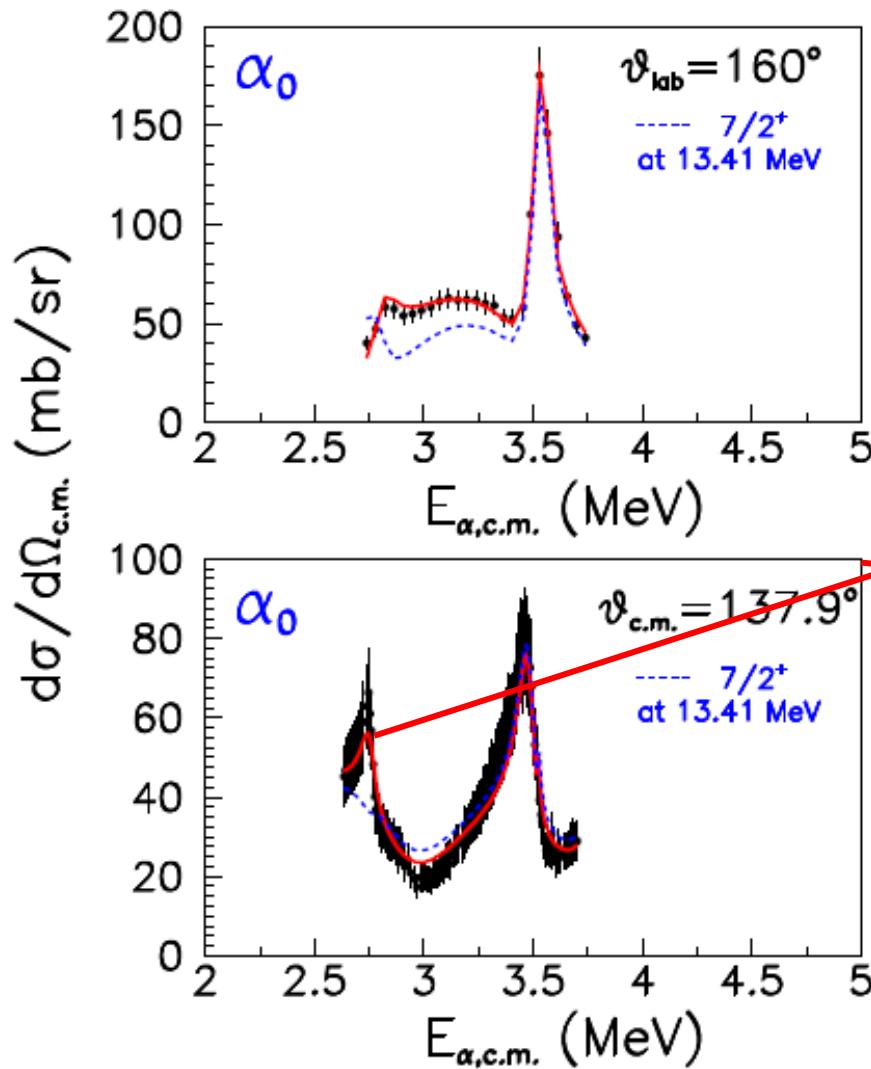


AZURE R-Matrix Program

AZURE2

R. Azuma et al, Phys. Rev. C 81, 045805 (2010)



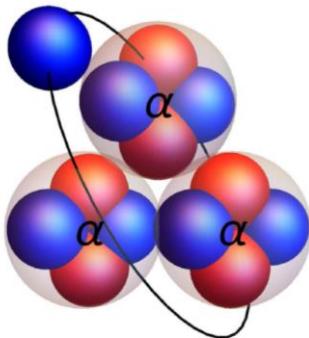
«*Differential diagnosis*» on some «*critical*»  $J^\pi$  assignments

# $^{13}\text{C}$ structure via $\alpha + {}^9\text{Be}$ : R-matrix calculations

literature

our data from the Napoli exp.

$E_x^{\text{lit}}$	$J_x^{\pi}$	$E_x$	$J_x^{\pi}$	$\Gamma$	$\Gamma_{\alpha_0}$	$\Gamma_{\alpha_1}$	$\Gamma_n$
11.75	$3/2^-$	11.75	$3/2^-$	116(27)	3(1)	—	113(26)
11.97	$5/2^+$	11.97	$5/2^+$	152(38)	65(17)	—	87(21)
12.13	$5/2^-$	12.17	$5/2^-$	199(28)	28(7)	—	171(21)
12.14	$1/2^+$	12.33	$1/2^+$	230(37)	40(7)	—	190(30)
12.44	$7/2^-$	12.45	$7/2^-$	222(36)	16(2)	—	206(34)
13.28	$3/2^-$	13.05	$3/2^-$	546(112)	153(29)	—	393(83)
13.41	$9/2^-$	13.41	$9/2^-$	84(27)	21(7)	—	63(20)
13.57	$7/2^-$	13.49	$7/2^-$	417(116)	114(51)	—	303(65)
13.76	$(3/2, 5/2)^+$	13.63	$5/2^+$	743(51)	623(30)	—	120(21)
14.13	$3/2^-$	14.13	$5/2^-$	94(12)	94(12)	—	—
		14.17	$7/2^+$	6(1)	6(1)	—	—
14.39	$(1/2, 5/2)^-$	14.28	$7/2^-$	392(93)	185(51)	—	207(42)
14.58	$(7/2^+, 9/2^+)$	14.36	$9/2^+$	322(62)	70(16)	—	252(46)

FIG. 6. Molecularlike picture of <sup>13</sup>C.

Very recently, Bijker and Iachello → description of <sup>13</sup>C spectrum  
with *symmetry considerations*

R. Bijker & F. Iachello, Phys. Rev. Lett. 122 (2019) 162501

$\mathcal{D}'_{3h}$   
double group

$$\begin{aligned} \Omega = E_{1/2}^{(+)}: \quad K^P = 1/2^+ \quad \text{and} \\ K = 3n \pm \frac{1}{2} \quad P = (-)^n \\ \Omega = E_{1/2}^{(-)}: \quad K^P = 1/2^- \quad \text{and} \\ K = 3n \pm \frac{1}{2} \quad P = (-)^{n+1} \\ \Omega = E_{3/2}: \quad K^P = (3n - \frac{3}{2})^\pm \quad n = 1, 2, 3... \end{aligned}$$

For rotational bands:

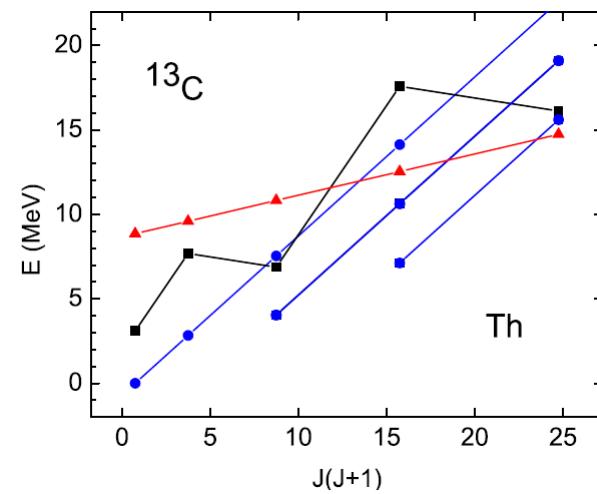
$$E_{\text{rot}}(\Omega, K, J) = \varepsilon_\Omega + A_\Omega[J(J+1) + b_\Omega K^2 + a_\Omega g_\Omega(J)]$$

where  $\varepsilon_\Omega$  is the intrinsic energy [8],  $A_\Omega = \hbar^2/2\mathfrak{J}$  is the inertial parameter,  $b_\Omega$  is a Coriolis term, and  $a_\Omega$  is the so-called decoupling parameter with  $g_\Omega(J) = \delta_{K,1/2}(-1)^{J+1/2}(J+1/2)$ . The latter term applies only to representations  $\Omega \equiv E_{1/2}^{(\pm)}$  and  $K^P = 1/2^\pm$ .

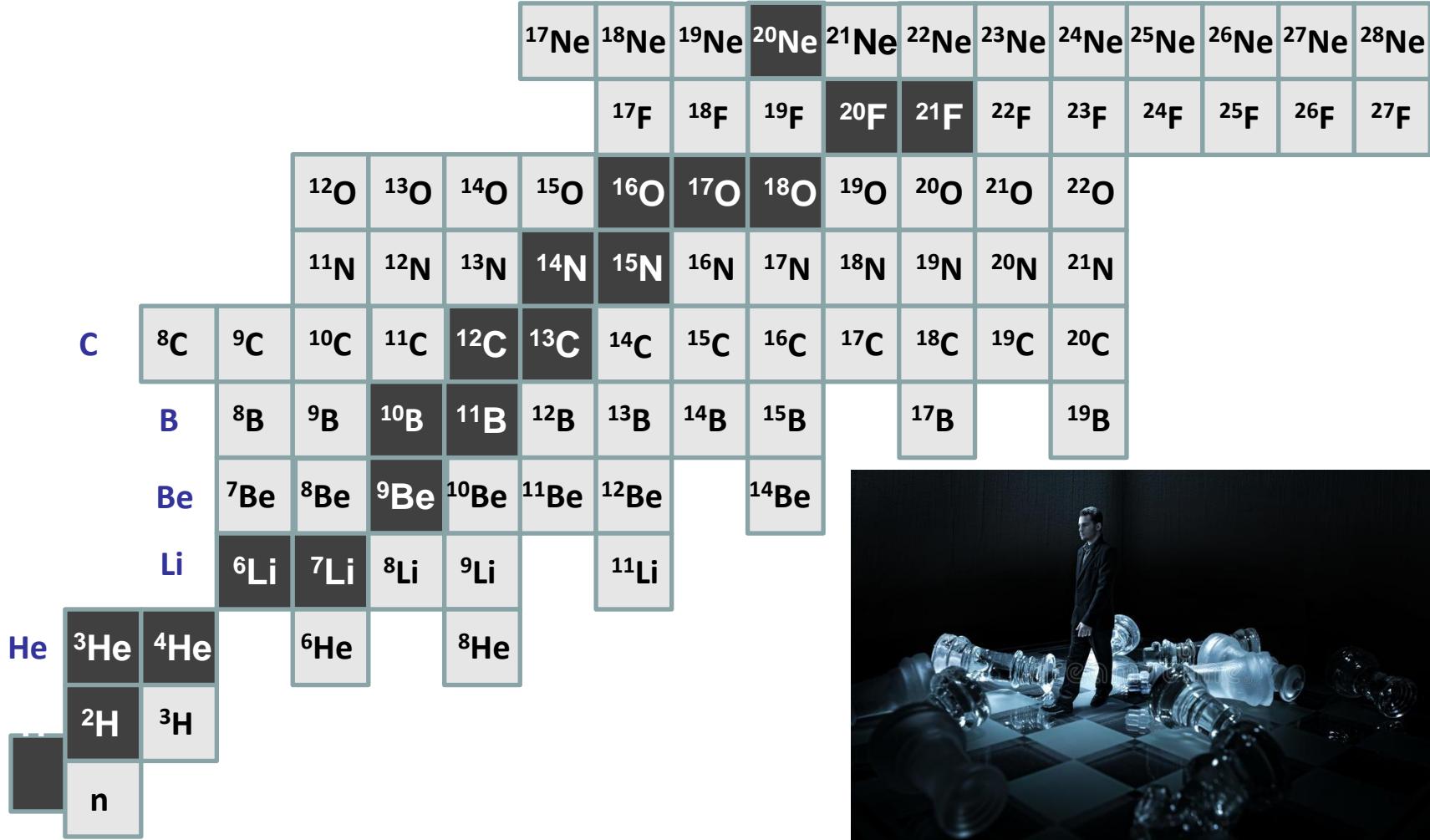
Symmetry → *recurrences* in the rotational bands

e.g.  $1/2^-$  coupled to  $5/2^+$  and  $7/2^+$ , etc...

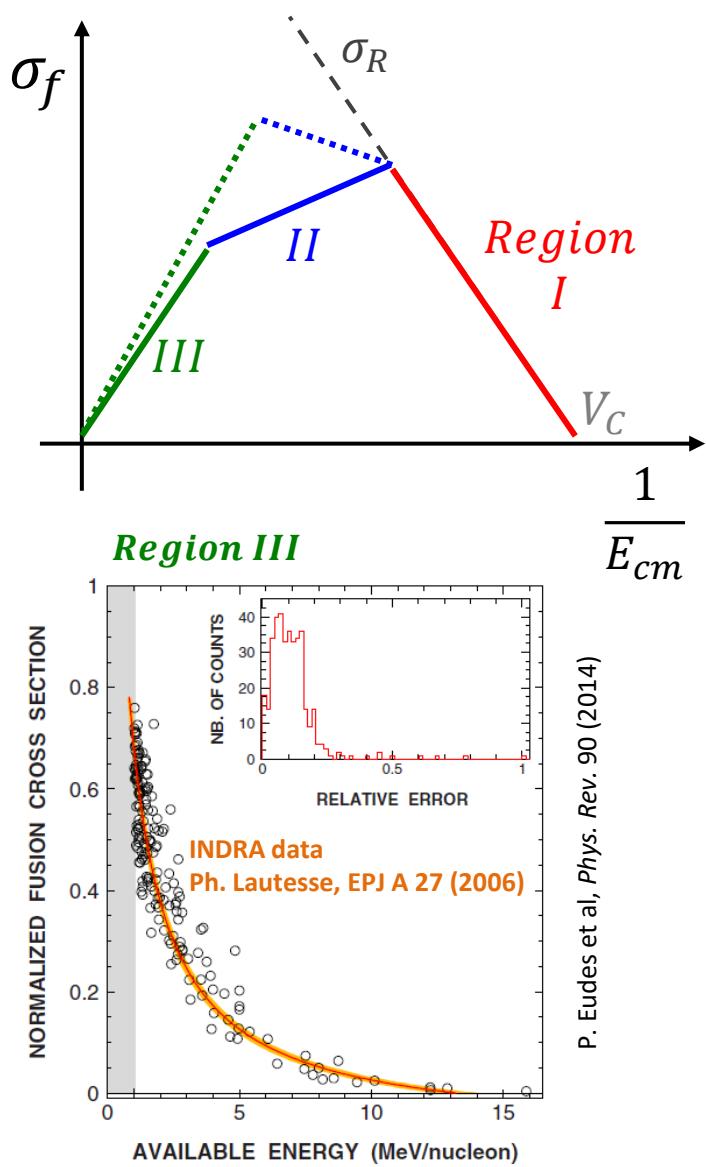
e.g.  $1/2^+$  coupled to  $5/2^-$  and  $7/2^-$ , etc...



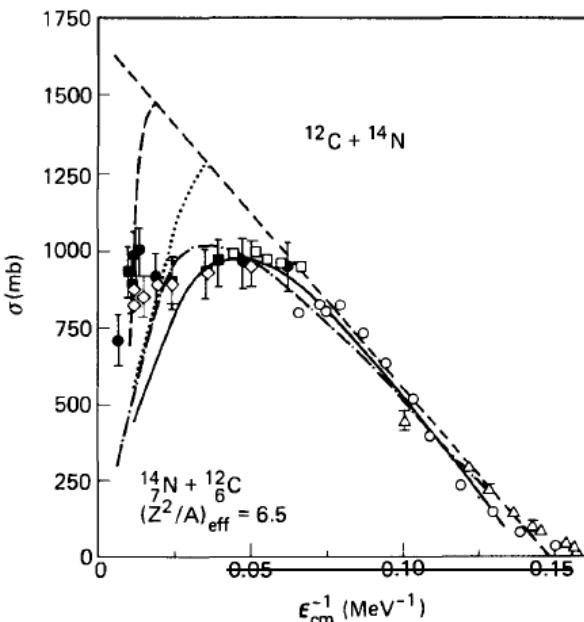
# Clustering and heavy ion fusion?



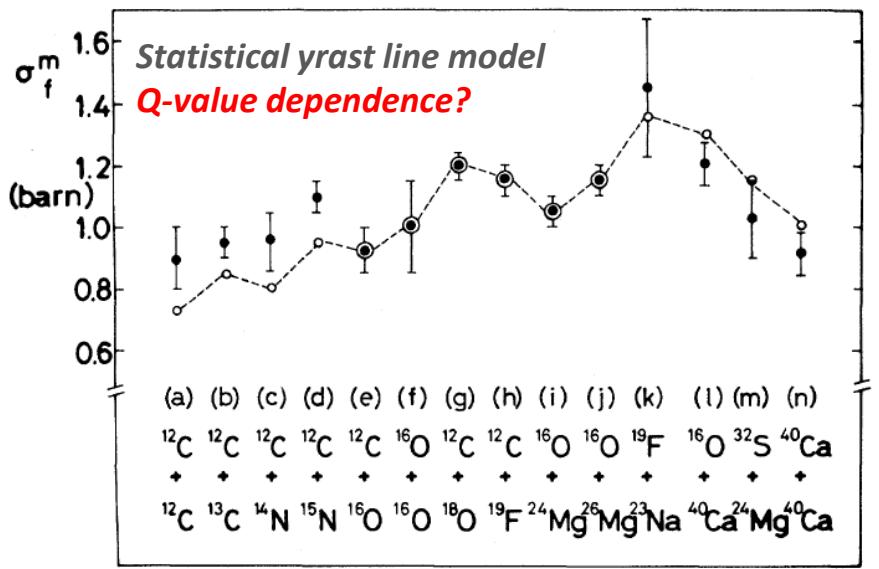
## Fusion **above** barrier: why so **abandoned?**



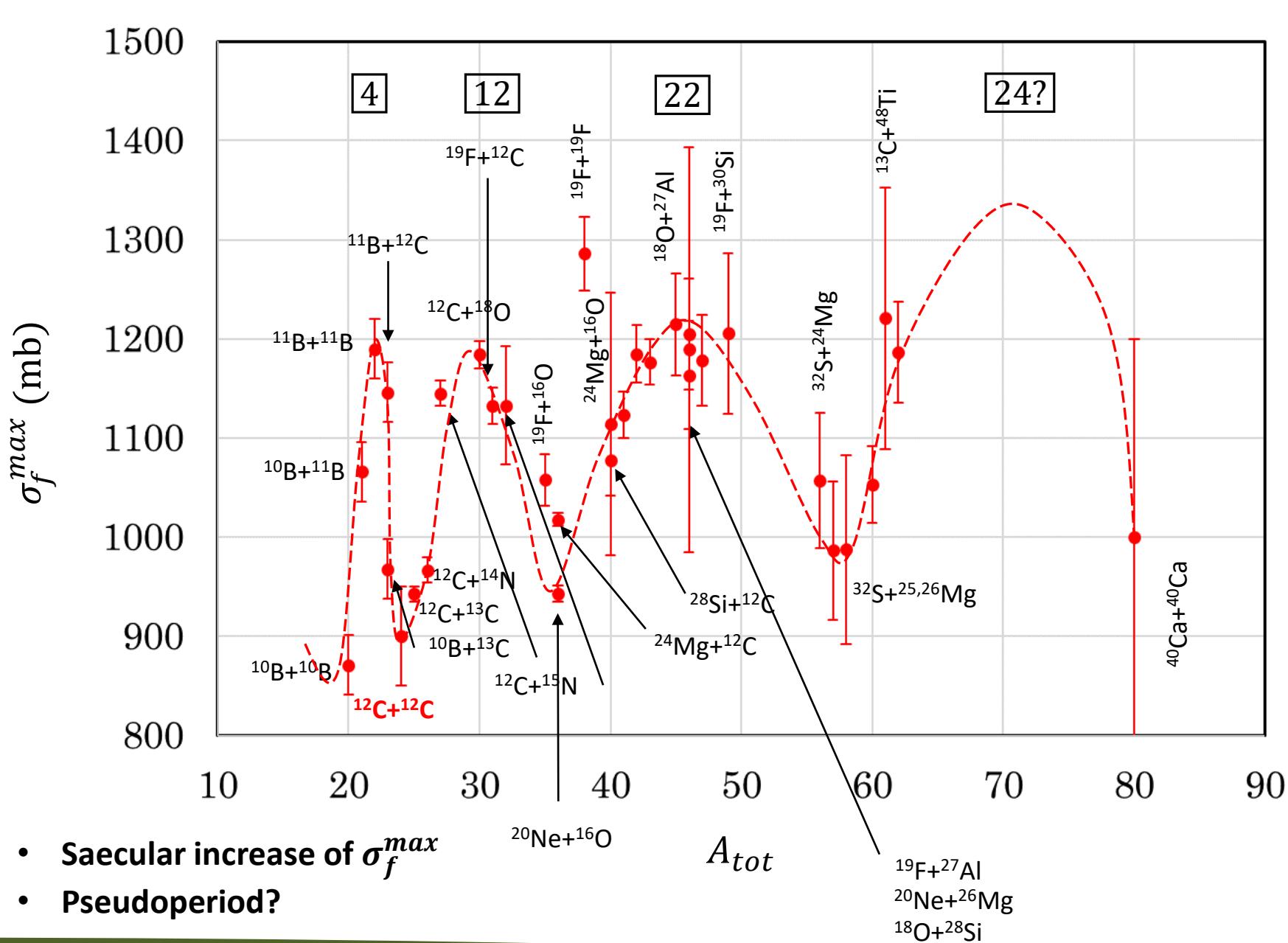
Le Sech & Ngo, *Physique Nucléaire*, Dunod

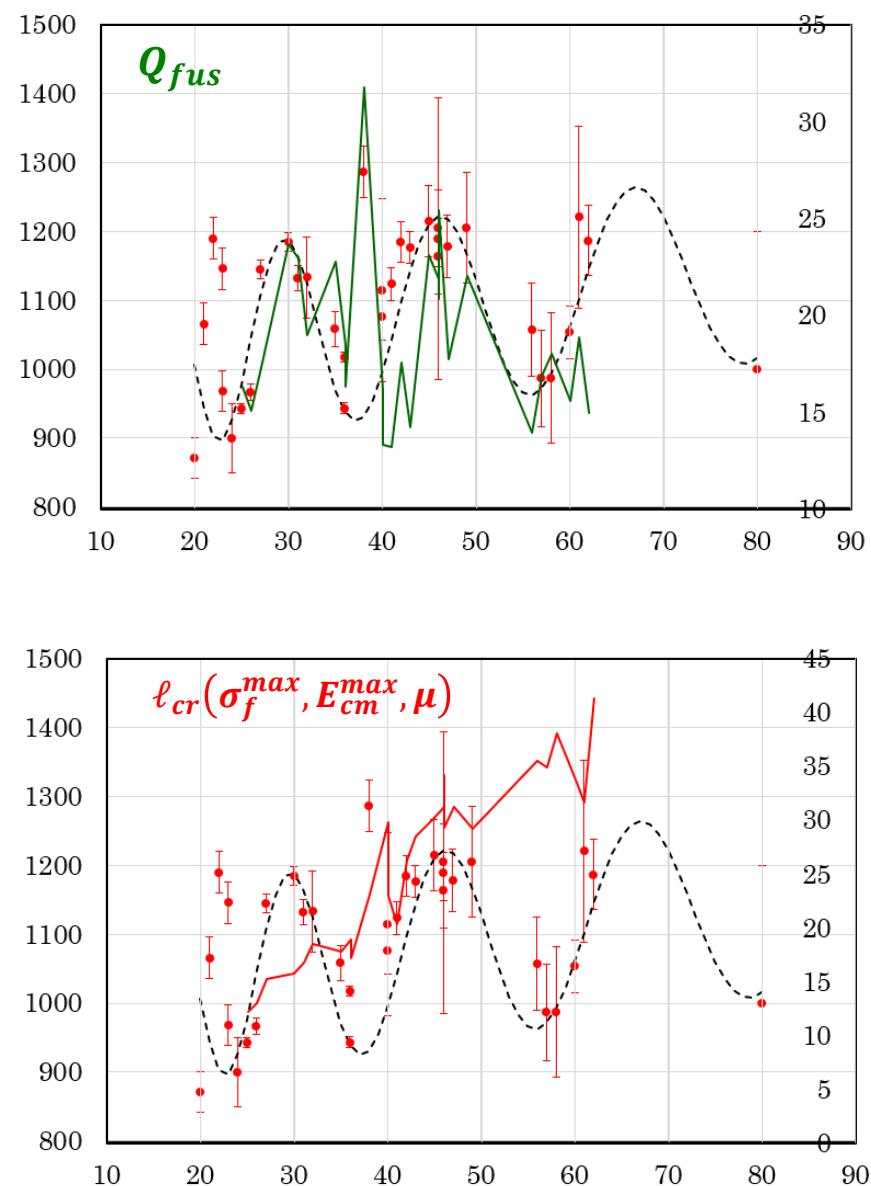
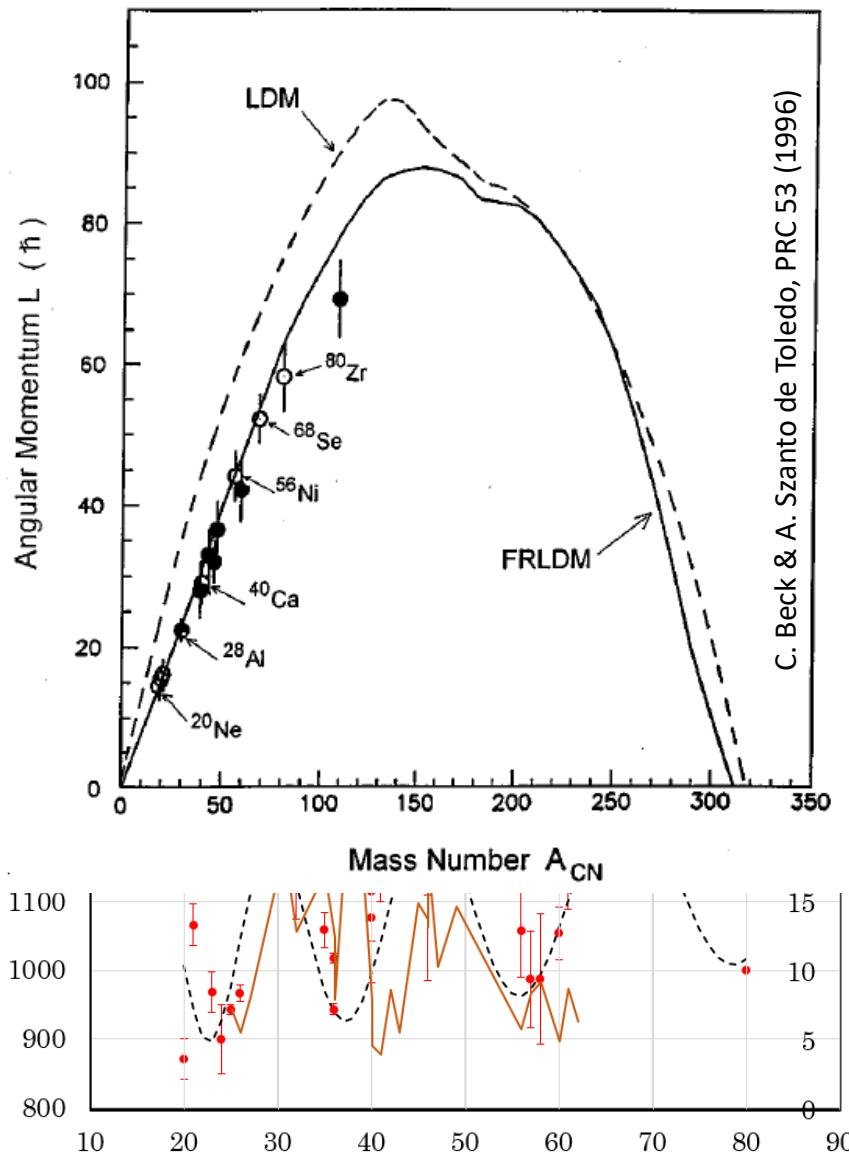


P. Frobrich, *Phys. Rep.* 116 (1984)



S.M. Lee et al, *Phys. Rev. Lett.* 45 (1980)





Local minima → early onset of ICF? Cluster transfer?

- $\alpha$ -clustering is a peculiar aspect of structure of *light nuclei*.
- Strong connections with other scientific fields (chemistry, mathematics) and other physics fields (astrophysics, molecular physics, particle physics). Some reactions are also useful in applied physics (ion beam analysis, nuclear fusion technology, ...).
- The «*carbon family*» → particularly interesting for *trimeric* structure
- Low energy reactions and resonant elastic scattering studies can shed light on the structure of  $^{11}\text{C}$  and  $^{13}\text{C}$
- $^{11}\text{C} \rightarrow p + ^{10}\text{B}$  reactions at low energies: a comprehensive *R-matrix fit* of data → a  $5/2^-$  state candidate to the *Hoyle-like band*
- $^{12}\text{C} \rightarrow$  the Hoyle state is still a debated topic (BEC, QM effects). Race towards measurements of all its properties →  $\Gamma_{rad}$  (n.t.), H.I. (n.n.t.)
- $^{13}\text{C} \rightarrow \alpha + ^9\text{Be}$  reaction and scattering at low energies: a comprehensive *R-matrix fit* of data → strong *revision of the level scheme* at large excitation energies
- Curious effects on maxima in complete fusion cross section → clustering?