The Mass of the lightest Higgs boson as fundamental parameter of the MSSM - Introducing the m_hMSSM -

Dirk Zerwas
On behalf of Gilbert Moultaka, Jean-Loic Kneur, Rima El-Kosseifi
L2C, CPPM, IJCLab and DMLab

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Introduction

- fermion ↔ boson
- has "no" problems with radiative corrections (quadrat. div.)
- has a light Higgs Boson (<140GeV)
- interesting pheno at the TeV scale

spin-0	spin-1/2	spin-1
Squarks: $\tilde{q}_R$, $\tilde{q}_L$	q	
	Gluino: g	$\tilde{g}$
Sleptons: $\tilde{\ell}_R$, $\tilde{\ell}_L$	ℓ	
h,H,A	Neutralino $\chi_{i=1-4}$	Z, γ
$H^{\pm}$	Charginos: $\chi^{\pm}_{i=1-2}$	$W^{\pm}$

3 neutral Higgs bosons: h, A, H
1 charged Higgs boson: $H^{\pm}$
and supersymmetric particles

Many different models:
- (p)MSSM (minimal supersymmetric extension of the standard model)
- mSUGRA
- GMSB
- AMB
- NMSSM

R-Parity
- Production of SUSY particles in pairs
- (Cascade-) decays to the lightest SUSY particle
- LSP stable, neutral and weakly interacting: neutralino (χ_1)
- less than half of the particles observed
- Great hope for discovery due to LHC CM increase to 13.6TeV (you have heard that before?)

MSSM LowScale

Higgs sector
$\tan\beta(Q = M_{Z^0})$
$m^2_{H_u}(Q = M_{\mathrm{EWSB}})$
$m^2_{H_d}(Q = M_{\mathrm{EWSB}})$
$\mu(Q = M_{\mathrm{EWSB}})$
m_{A^0}

Gauge sector
$M_1(Q = M_{\mathrm{EWSB}})$
$M_2(Q = M_{\mathrm{EWSB}})$
$M_3(Q = M_{\mathrm{EWSB}})$

trilinear couplings
$A_t(Q = M_{\mathrm{EWSB}})$
$A_b(Q = M_{\mathrm{EWSB}})$
$A_\tau(Q = M_{\mathrm{EWSB}})$

$SU(2)_L$ doublets
$m_{\tilde{q}1_L}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{q}2_L}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{q}3_L}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{\ell}1_L}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{\ell}2_L}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{\ell}3_L}(Q = M_{\mathrm{EWSB}})$

$SU(2)_L$ singlets
$m_{\tilde{u}_R}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{d}_R}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{c}_R}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{s}_R}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{t}_R}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{b}_R}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{e}_R}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{\mu}_R}(Q = M_{\mathrm{EWSB}})$
$m_{\tilde{\tau}_R}(Q = M_{\mathrm{EWSB}})$

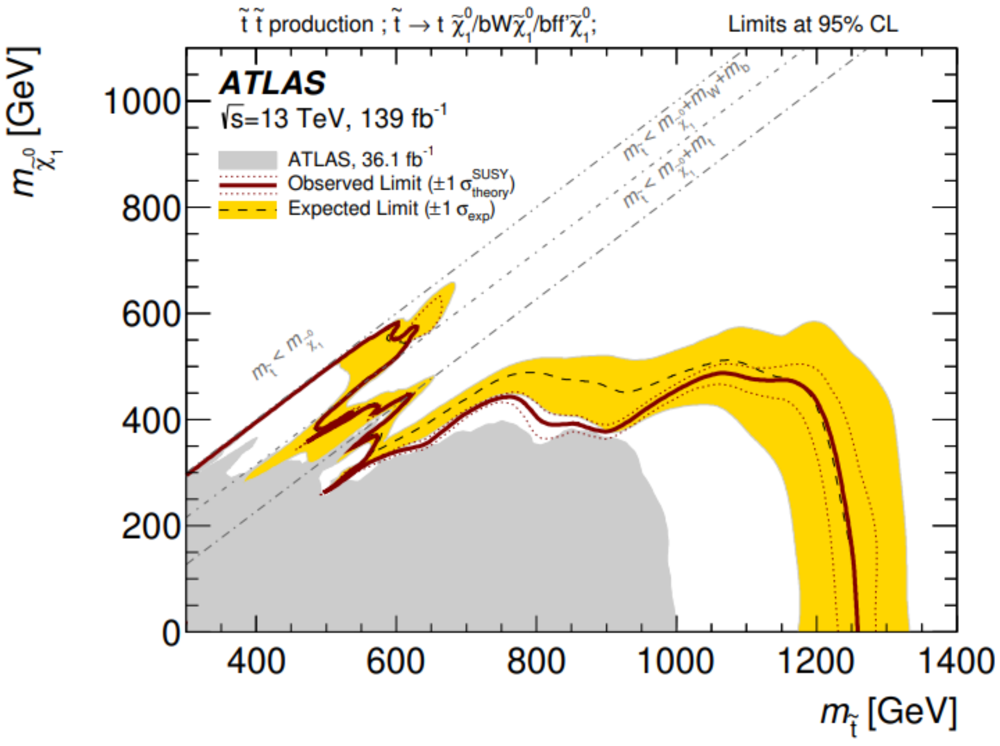
Study motivated by: A.Djouadi, L.Maiani, G.Moreau, A.Polosa, J.Quevillon and V.~Riquer, Habemus MSSM Eur. Phys. J. C 73 (2013), 2650, [arXiv:1307.5205 [hep-ph]].

Goal
- m_hMSSM: replace 1 parameter with a measured quantity:
  - A_t replaced by m_h
- Inversion algorithm instead of a scan

The Stop Cliff

An example point:

- **Heavy squarks and sleptons**
- **Light LSP (Bino)**



Stop sector:

- **Lightest stop at detection mass limit**
- **At=3610 GeV**

EW	2.0 TeV
$m^2_{H_d}$	3.65740418 TeV^2
$m^2_{H_u}$	-0.213361994 TeV^2
sign(μ)	+
A_t	3.610 TeV
$m_{\tilde{t}_R}$	1.27 TeV
$m_{\tilde{q}3_L}$	3 TeV
M_1	300 GeV
M_2	2 TeV
M_3	3 TeV
$A_b,\ A_\tau$	0 GeV
$\tan\beta$	10
$m_{\tilde{e}_L} = m_{\tilde{\mu}_L} = m_{\tilde{\tau}_L} = m_{\tilde{e}_R} = m_{\tilde{\mu}_R} = m_{\tilde{\tau}_R}$	2 TeV
$m_{\tilde{q}1_L} = m_{\tilde{q}2_L} = m_{\tilde{u}_R} = m_{\tilde{c}_R} = m_{\tilde{d}_R} = m_{\tilde{s}_R} = m_{\tilde{b}_R}$	3 TeV
m_h	125.012 GeV
$m_{\tilde{t}_1}$	1306 GeV
$m_{\tilde{\chi}_1^0}$	294 GeV

Higgs mass:

- **Experimental error ~ 0.15GeV**
- **Typical non-parametric error: 2GeV**
- **GeV level rounding: 125GeV**

Concept

A_t replaced by m_h

- **Need to invert Higgs mass dependence on A_t**

$$M_s^2(p^2) = \begin{pmatrix} \overline{m}_{11}^2 - \Pi_{11}(p^2) + \frac{t_1}{v_1} & \overline{m}_{12}^2 - \Pi_{12}(p^2) \\ \overline{m}_{12}^2 - \Pi_{12}(p^2) & \overline{m}_{22}^2 - \Pi_{22}(p^2) + \frac{t_2}{v_2} \end{pmatrix}$$

$$\begin{aligned} \overline{m}_{11}^2 &= \overline{m}_Z^2 \cos^2\beta + \overline{m}_A^2 \sin^2\beta, \\ \overline{m}_{22}^2 &= \overline{m}_Z^2 \sin^2\beta + \overline{m}_A^2 \cos^2\beta, \\ \overline{m}_{12}^2 &= -\frac{1}{2}(\overline{m}_Z^2 + \overline{m}_A^2) \sin 2\beta. \end{aligned}$$

EWSB determines iteratively:

- **Higgs mass parameter mu**
- **Pseudoscalar running mass**

$$\begin{aligned} \overline{m}_A^2(M_{EWSB}) &= \frac{1}{\cos 2\beta}\left(\hat{m}_{H_u}^2 - \hat{m}_{H_d}^2\right) - \overline{m}_Z^2, \\ \mu^2(M_{EWSB}) &= \frac{1}{2}\Big(\left(\hat{m}_{H_u}^2 \tan\beta - \hat{m}_{H_d}^2 \cot\beta\right) \tan 2\beta - \overline{m}_Z^2\Big). \end{aligned}$$

Add the determination of A_t:

- **A_t determined from a pole mass**
- **Need all radiative corrections**
- **Can only be implemented post-EWSB**

Important:

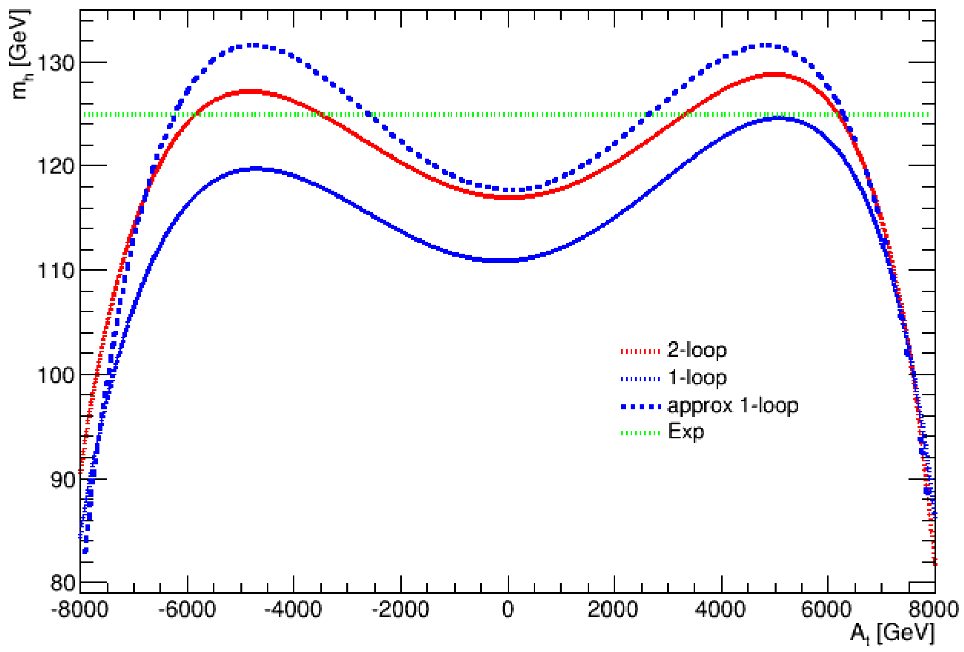
- **A spectrum calculation is iterative:**
  - **RGE: high scale, low scale, Z scale**
  - **EWSB**

Proof of Concept - Approximate 1-loop

$$m_h^2 = \overline{m}_h^2 + \frac{3g_2^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12M_S^4}\right]$$

$$M_S^2 = \sqrt{(m_{\tilde{q}3_L}^2 + (\frac{1}{2} - \frac{2}{3}s_W^2)m_Z^2\cos 2\beta + m_t^2)} \cdot \sqrt{(m_{\tilde{t}_R}^2 + \frac{2}{3}s_W^2 m_Z^2 \cos 2\beta + m_t^2)}$$

$$X_t = A_t - \mu \cot\beta$$



Approximate 1-loop:

- A_t^4
- Local minimum: $A_t = \mu$ cotanβ
- Particularity of the benchmark: larger corrections

1-loop and 2-loop (with the Pietro Slavich terms):

- Structure preserved
- Non-negligeable contribution from 2-loop

		Stop cliff
s1	A_t [TeV]	-5.44
s2	A_t [TeV]	-3.61
s3	A_t [TeV]	2.87
s4	A_t [TeV]	6.36

Inversion with approximate 1-loop:

- Invertible analytically
- 4 solutions for the stop cliff
- But: A_t is off by 30%

Proof of Concept – 1-loop

Start from Eigenvalue equation:

$$m_{h,H}^4 - m_{h,H}^2((M_s^2)_{11} + (M_s^2)_{22}) + (M_s^2)_{11}(M_s^2)_{22} - ((M_s^2)_{12})^2 = 0,$$

Identify the terms depending on A_t, eg in couplings h-stop-stop:

$$g_{s_2 t_1 t_1} = c_t^2\, g_{s_2 \tilde{t}_L \tilde{t}_L} + 2 c_t s_t\, g_{s_2 \tilde{t}_L \tilde{t}_R} + s_t^2\, g_{s_2 \tilde{t}_R \tilde{t}_R}$$

$$g_{s_2 t_2 t_2} = s_t^2\, g_{s_2 \tilde{t}_L \tilde{t}_L} - 2 c_t s_t\, g_{s_2 \tilde{t}_L \tilde{t}_R} + c_t^2\, g_{s_2 \tilde{t}_R \tilde{t}_R}$$

$$g_{s_2 t_1 t_2} = s_t c_t (g_{s_2 \tilde{t}_R \tilde{t}_R} - g_{s_2 \tilde{t}_L \tilde{t}_L}) + (c_t^2 - s_t^2)\, g_{s_2 \tilde{t}_L \tilde{t}_R}$$

$$g_{s_2 \tilde{t}_L \tilde{t}_R} = \frac{y_t}{\sqrt{2}} A_t$$

Eg in the one-loop scalar function (log term ignored)

$$A_0(m_{\tilde{t}_i}) = m_{\tilde{t}_i}^2 \left(1 - \ln\left(\frac{m_{\tilde{t}_i}^2}{Q^2}\right)\right)$$

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2}\left(M^2 \mp \sqrt{a_s A_t^2 + b_s A_t + c_s}\right)$$

$$M^2 = m_{\tilde{q}3_L}^2 + m_{\tilde{t}_R}^2 + 2 m_t^2 + \frac{1}{2} m_Z^2 \cos 2\beta,$$

$$a_s = 4 m_t^2,$$

$$b_s = -8 m_t^2 \mu \cot\beta,$$

$$c_s = \left(m_{\tilde{q}3_L}^2 - m_{\tilde{t}_R}^2 + \left(\frac{1}{2} - \frac{4}{3} s_W^2\right) m_Z^2 \cos 2\beta\right)^2 + 4 m_t^2 \mu^2 \cot^2\beta.$$

Rewrite tadpoles and self-energies (0: log dependence):

$$\frac{t_1}{v_1} = t_1^{(s)} \sqrt{a_s A_t^2 + b_s A_t + c_s} + t_1^{(0)}$$

$$\frac{t_2}{v_2} = t_2^{(1s)} A_t \sqrt{a_s A_t^2 + b_s A_t + c_s} + t_2^{(1)} A_t + t_2^{(s)} \sqrt{a_s A_t^2 + b_s A_t + c_s} + t_2^{(0)}$$

$$\Pi_{11} = \pi_{11}^{(s)} \sqrt{a_s A_t^2 + b_s A_t + c_s} + \pi_{11}^{(0)}$$

$$\Pi_{12} = \pi_{12}^{(1)} A_t + \pi_{12}^{(0)}$$

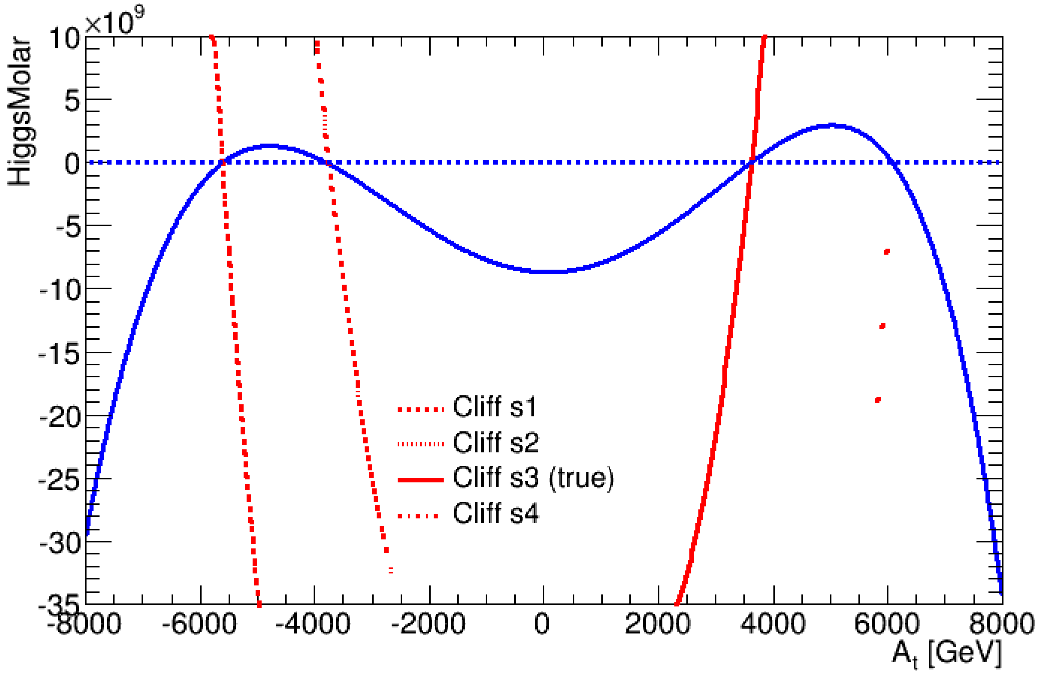
$$\Pi_{22} = \pi_{22}^{(2)} A_t^2 + \pi_{22}^{(1)} A_t + \pi_{22}^{(s)} \sqrt{a_s A_t^2 + b_s A_t + c_s} + \pi_{22}^{(0)}$$

Proof of Concept

Leading to a new function:

$$\mathrm{HiggsMolar}(A_t) = C_3 A_t^3 + C_2 A_t^2 + C_1 A_t + C_0 + (R_2 A_t^2 + R_1 A_t + R_0)\sqrt{a_s A_t^2 + b_s A_t + c_s} = 0$$

m_h^2 enters C_2, C_1, C_0, R_1, R_0



$$C_0[A_t] = c_s(\pi_{11}^{(s)} - t_1^{(s)})(\pi_{22}^{(s)} - t_2^{(s)}) + (m_h^2 + \pi_{11}^{(0)} - t_1^{(0)} - \overline{m}_{11}^2)(m_h^2 + \pi_{22}^{(0)} - t_2^{(0)} - \overline{m}_{22}^2) - (\pi_{12}^{(0)} - \overline{m}_{12}^2)^2,$$

$$C_1[A_t] = (\pi_{11}^{(s)} - t_1^{(s)})(b_s(\pi_{22}^{(s)} - t_2^{(s)}) - c_s t_2^{(1s)}) + (\pi_{22}^{(1)} - t_2^{(1)})(m_h^2 + \pi_{11}^{(0)} - t_1^{(0)} - \overline{m}_{11}^2) + 2\pi_{12}^{(1)}(\overline{m}_{12}^2 - \pi_{12}^{(0)}),$$

$$C_2[A_t] = (\pi_{11}^{(s)} - t_1^{(s)})(a_s(\pi_{22}^{(s)} - t_2^{(s)}) - b_s t_2^{(1s)}) + \pi_{22}^{(2)}(m_h^2 + \pi_{11}^{(0)} - t_1^{(0)} - \overline{m}_{11}^2) - (\pi_{12}^{(1)})^2,$$

$$C_3[A_t] = a_s(t_1^{(s)} - \pi_{11}^{(s)})t_2^{(1s)},$$

$$R_0[A_t] = (\pi_{11}^{(s)} - t_1^{(s)})(m_h^2 + \pi_{22}^{(0)} - t_2^{(0)} - \overline{m}_{22}^2) + (\pi_{22}^{(s)} - t_2^{(s)})(m_h^2 + \pi_{11}^{(0)} - t_1^{(0)} - \overline{m}_{11}^2),$$

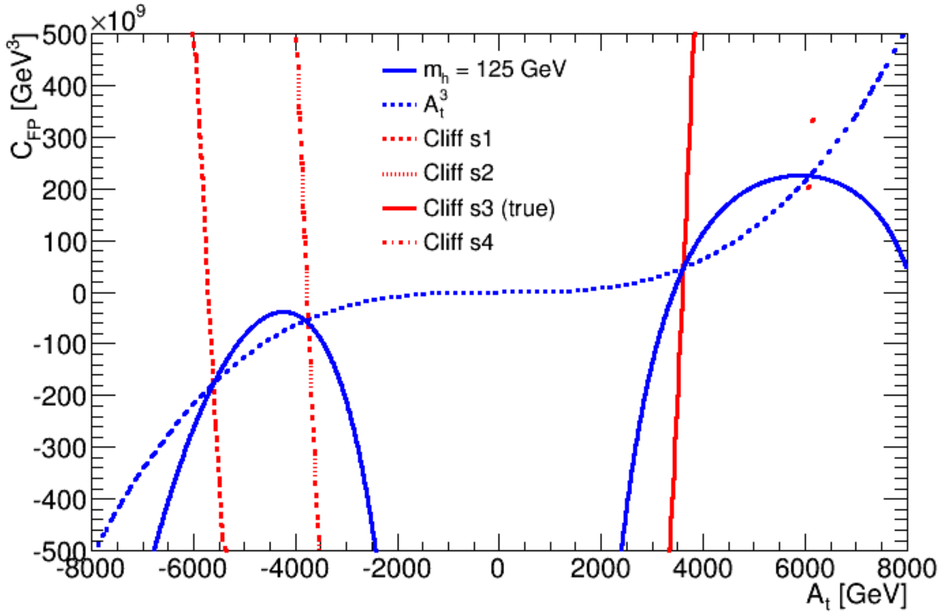
$$R_1[A_t] = (t_1^{(s)} - \pi_{11}^{(s)})(t_2^{(1)} - \pi_{22}^{(1)}) + (t_1^{(0)} - \pi_{11}^{(0)} + \overline{m}_{11}^2 - m_h^2)t_2^{(1s)},$$

$$R_2[A_t] = \pi_{22}^{(2)}(\pi_{11}^{(s)} - t_1^{(s)}),$$

HiggsMolar:

- **Exact 1-loop**
- **Similar form as the approximate 1-loop**
- **Zeros correspond to m_h=125GeV**
- **Four solutions**
- **A_t^3 but 4 solutions:**
  - **Function valid only in vicinity of solution**
  - **EWSB modifies the "pseudo-constants" as function of A_t**

Proof of Concept 1-loop and 2-loop



Solve for A_t:

- Not possible analytically in general
- Step through the function in steps on 1MeV (just kidding)

Transform Molar to a fixedPoint problem:

$$\mathrm{C_{FP}}(A_t) = -\frac{1}{C_3}[C_2A_t^2 + C_1A_t + C_0 + (R_2A_t^2 + R_1A_t + R_0)\sqrt{a_sA_t^2 + b_sA_t + c_s}],$$

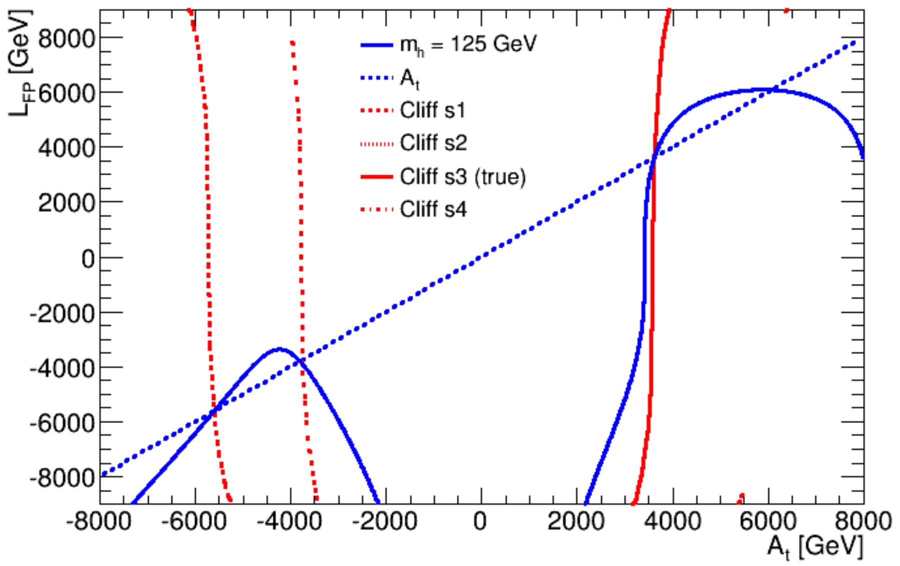
$$A_t = \sqrt[3]{\mathrm{C_{FP}}(A_t)}.$$

$$\mathrm{L_{FP}}(A_t) \equiv \sqrt[3]{\mathrm{C_{FP}}(A_t)},$$

C_{FP} and LFP:

- Strong local dependence guides convergence
- But to converge need |LFP'| < 1 (against repulsive FPs etc)
- Define convergence parameter and function:

$$\mathrm{L_{FP}}_\tau(A_t) = \frac{1}{\tau}(\mathrm{L_{FP}}(A_t) - A_t) + A_t.$$



2-loop (and higher orders):

- Enter in the mass matrix
- Soft dependence

Remnants, log(A_t) and 2-loop:

- Taken into account exactly in the EWSB iterations
- EWSB iterations are standard also in "standard" MSSM!

Proof of Concept – Full Algorithm

Full Algorithm:

1. Stabilize top yukawa
2. Use approximate 1-loop inversion as first guess
3. EWSB: add fixed point iteration
4. Adapt tau locally

$$\mathrm{L_{FP}}'_\tau(A_t) = 1 + \frac{\mathrm{L_{FP}}'(A_t) - 1}{\tau},$$

stop cliff	s1	s2	s3	s4
A_t [GeV]	-5617.3	-3796.1	3609.7	6082.5
m_h [GeV]	125.012	125.012	125.012	125.012

Results promising:

- 0.1 permil precision reached on At (1permil set)
- m_h excellent (too good for practical purposes)
- A_t precision better than requested: effect of iterations

EWSB:

- Not uniquely defined (see SLHA)
- m_{Hu}, m_{Hd}, sign(μ)
- $m_A(Q)$, μ
- m_A, μ
- Use reduced RGE precision

EWSB	stop cliff	s1	s2	s3	s4
$m^2_{H_d}$, $m^2_{H_u}$, sign(μ)	A_t [GeV]	-5617.8	-3795.0	3610.5	6085.9
	m_h [GeV]	125.012	125.012	125.012	125.012
$m^2_A(Q)$, μ	A_t [GeV]	-5606.9	-3795.1	3610.7	6090.1
	m_h [GeV]	125.012	125.012	125.012	125.012
m_A, μ	A_t [GeV]	-5607.2	-3794.7	3610.7	6089.9
	m_h [GeV]	125.012	125.012	125.012	125.012

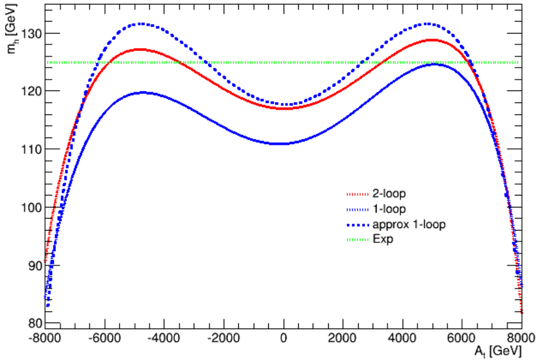
It works:

- Similar precision achieved in all cases
- Stop condition is on μ for 2 cases

Technical remark:

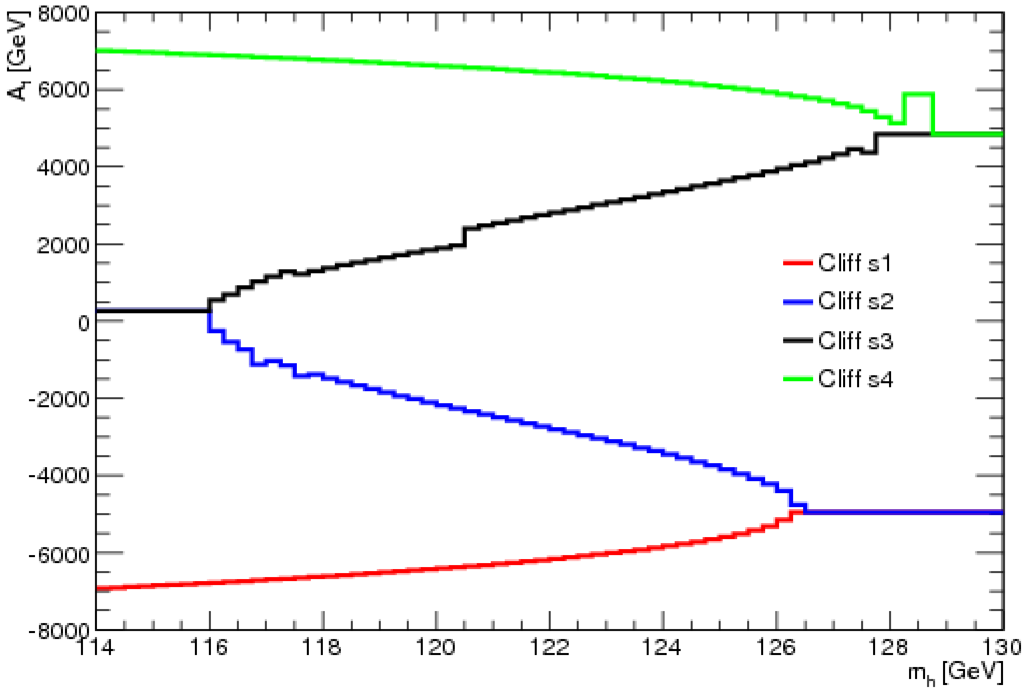
- C++ Inheritance made the extensions easy (in spite of Diamond inheritance)
- Works also for Highscale models
- Overhead: 1xRGE, doubles EWSB

Proof of Concept: Beyond the benchmark point



Proof of complete Inversion in more than 1 point:

- Stepping through mh
- Specifying s1, s2, s3, s4
- Necessitates a stepper function applied regularly to identify the local minima and maxima in m_h
- Close to extremal FP is complemented by a standard Bisection algorithm



It works (better than expected):

- Regions are separated
- continuous
- Small steps corresponding to changes in pseudoscalar mass and μ leads to a <2GeV deviation in mh
- 3 points (of 256) not converged

Inversion works: from $m_h(A_t)$ to $A_t(m_h)$

Conclusions

Proof of concept:

- m_h as fundamental parameter of the MSSM: doable
- Correct to all orders
- Stop cliff benchmark: works
- 1d scan: works

Future work:

- Improve the algorithm
- FullFledged Algorithm for all configurations