Recent progress in the calculation of the Higgs trilinear coupling in models with extended scalar sectors

Based on JB, Kanemura, PLB 796 (2019) 38-46 & EPJC 80 (2020) 3, 227
and JB, Kanemura, Shimoda, JHEP 03 (2021) 297

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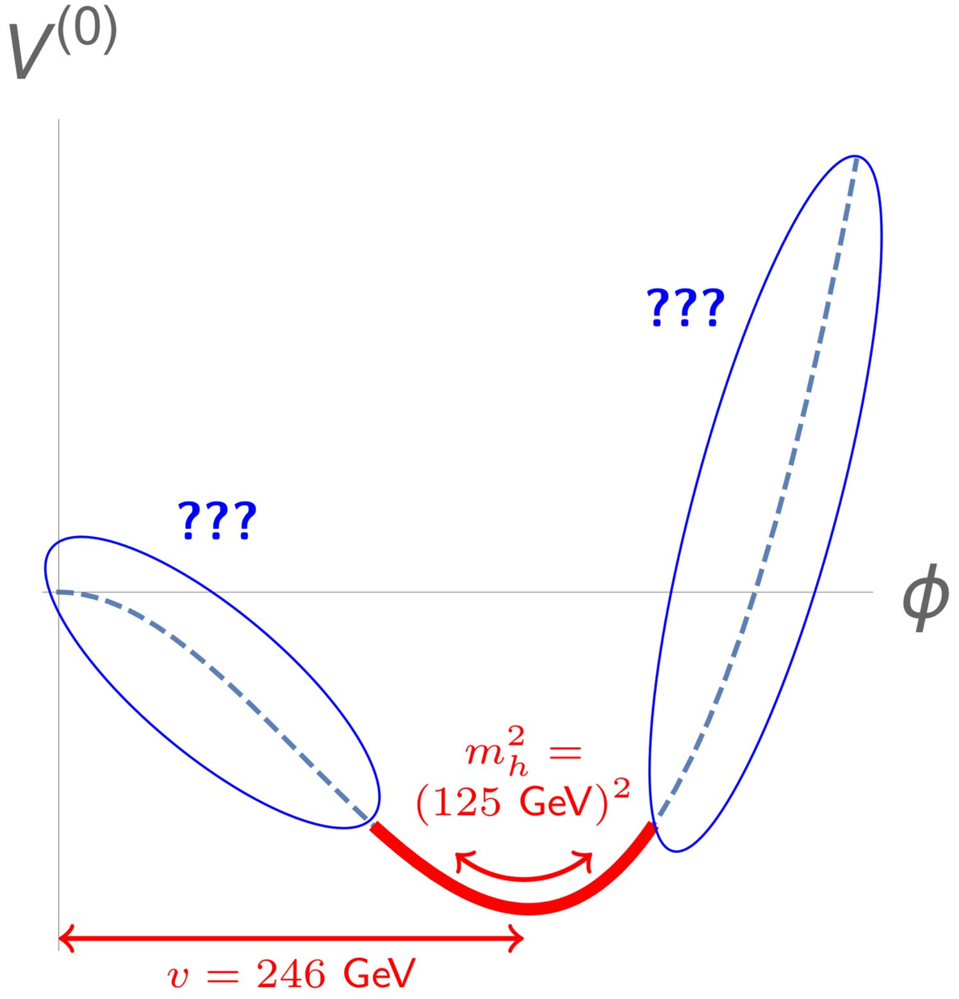
IRN Terascale meeting at LPC-Clermont
Clermont-Ferrand, France | November 22-24, 2021

Why investigate λ_{hhh}?

Probing the shape of the Higgs potential

- Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:
  → the location of the EW minimum:
  v = 246 GeV
  → the curvature of the potential around the EW minimum:
  m_h = 125 GeV
  However we still don't know the **shape** of the potential, away from EW minimum → depends on λ_{hhh}



- **λ_{hhh} determines the nature of the EWPT**!
  ⇒ O(20%) deviation of λ_{hhh} from its SM prediction needed to have a strongly first-order EWPT → necessary for EWBG [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]

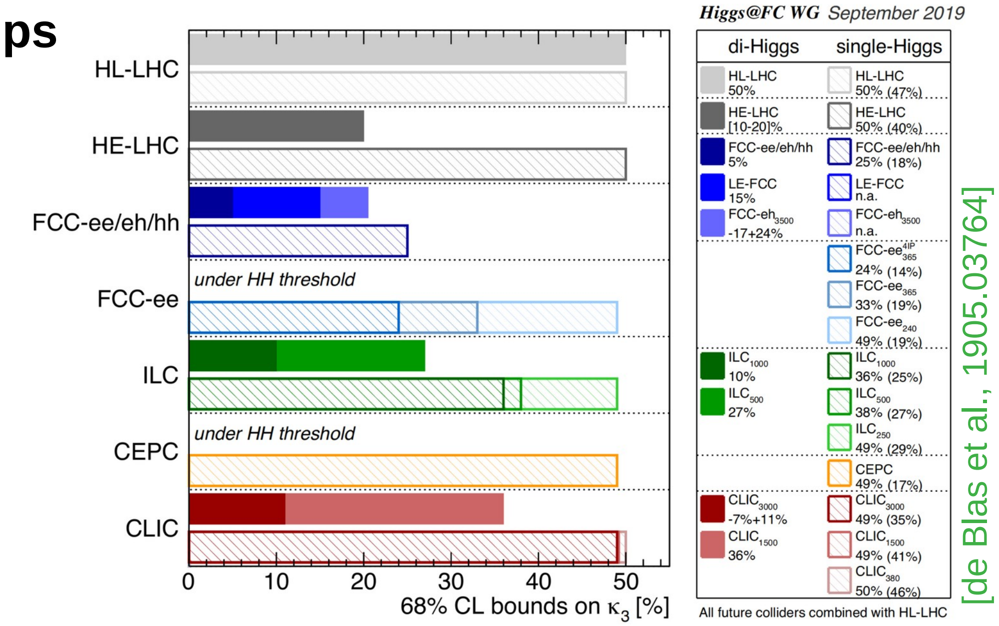
Distinguish aligned scenarios <u>with or without decoupling</u>

- Aligned scenarios already seem to be favoured → Higgs couplings are SM-like at tree-level
- Non-aligned scenarios (e.g. in 2HDMs) could be almost entirely excluded in the close future using **synergy** of **HL-LHC** and **ILC**!
  → **Alignment through decoupling**? or **alignment without decoupling**?

- If alignment without decoupling, Higgs couplings like λ_{hhh} can still exhibit large deviations from SM predictions because of **non-decoupling effects from BSM loops**

- Current best limit (at 95% CL):
  $\mathbf{-1.0 < \lambda_{hhh} /(\lambda_{hhh})^{SM} < 6.6}$ [ATLAS-CONF-2021-052]

- Improvement at future colliders:
  - **HL-LHC**: $\lambda_{hhh} /(\lambda_{hhh})^{SM}$ within ~ 50-100%;
  - At lepton colliders – **ILC**, **CLIC** – within some tens of %;
  - At a **100-TeV hadron collider**, down to 5-7%



[de Blas et al., 1905.03764]

see also [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], *etc.*

Non-decoupling effects at one loop

The Two-Higgs-Doublet Model

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge ½
- CP-conserving 2HDM, with softly-broken Z_2 symmetry ($\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$) to avoid tree-level FCNCs

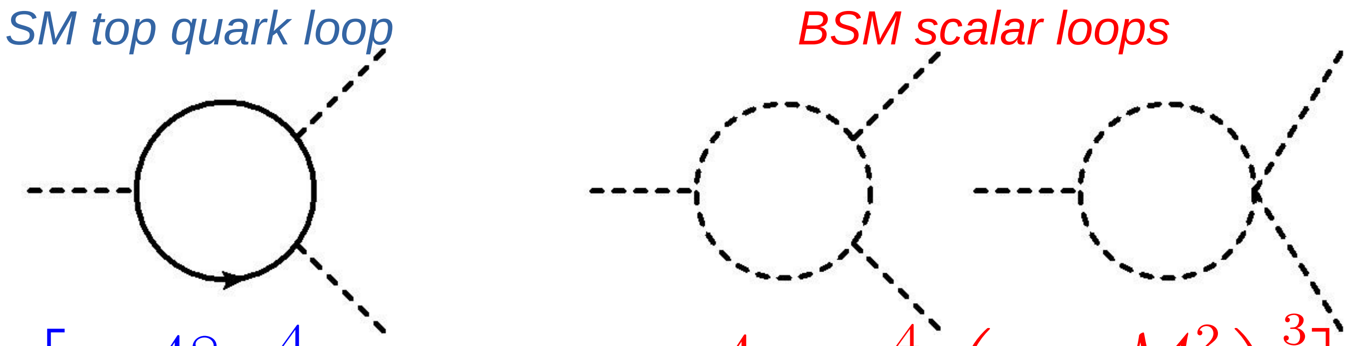
$$V^{(0)}_{\rm 2HDM} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

- 7 free parameters in scalar sector: m_3, λ_i (i=1,..,5), $\tan\beta \equiv v_2/v_1$
- Mass eigenstates: h, H: CP-even Higgses, A: CP-odd Higgs, $H^\pm$: charged Higgs
- λ_i (i=1,..,5) traded for mass eigenvalues m_h, m_H, m_A, $m_{H\pm}$ and CP-even Higgs mixing angle α
- m_3 replaced by a Z_2 *soft-breaking mass scale*

$$M^2 = \frac{2m_3^2}{s_{2\beta}}$$

One-loop λ_{hhh} and non-decoupling effects

- Leading one-loop corrections to λ_{hhh} in models with extended sectors (e.g. 2HDM):



$$\delta^{(1)}\lambda_{hhh} \supset \frac{1}{16\pi^2}\left[-\frac{48m_t^4}{v^3} + \sum_\Phi \frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3\right]$$

$\mathcal{M}$: **BSM mass scale**, e.g. soft breaking scale M of Z_2 symmetry in 2HDM
n_Φ : # of d.o.f of field Φ

- Size of new effects depends on how the BSM scalars acquire their mass: $m_\Phi^2 \sim \mathcal{M}^2 + \tilde{\lambda}v^2$

$$\left(1-\frac{\mathcal{M}^2}{m_\Phi^2}\right)^3 \longrightarrow \begin{cases} 0, \text{ for } \mathcal{M}^2 \gg \tilde{\lambda}v^2 \\ 1, \text{ for } \mathcal{M}^2 \ll \tilde{\lambda}v^2 \end{cases}$$

→ **Huge BSM effects possible!**

One-loop λ_{hhh} and non-decoupling effects

- Leading one-loop c

$\delta^{(1)}\lambda_{hhh} \supset$

$\mathcal{M}$: **BSM mass**
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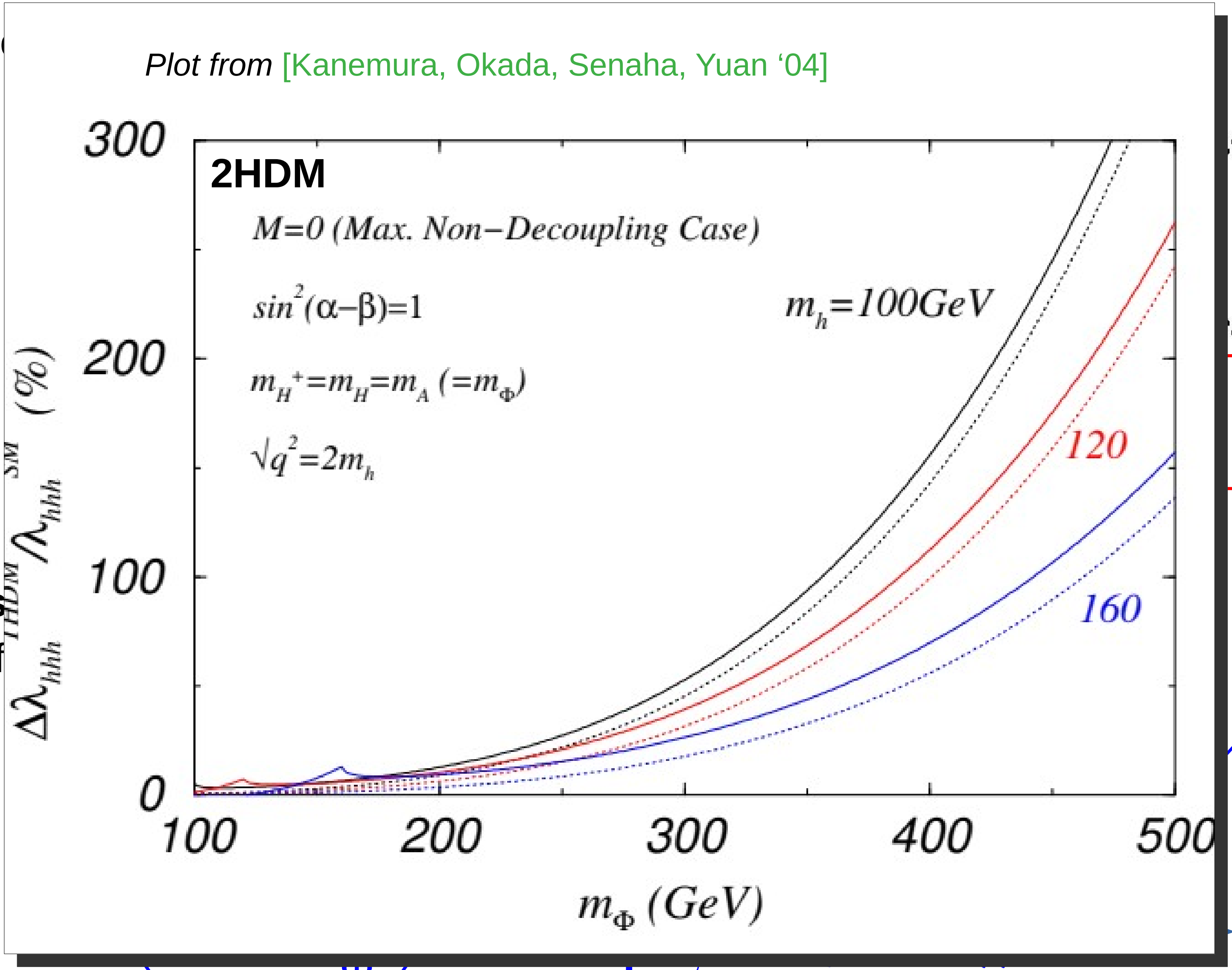
- Size of new effects

$\mathcal{M}^2 + \tilde{\lambda}v^2$



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One-loop λ_{hhh} and non-decoupling effects

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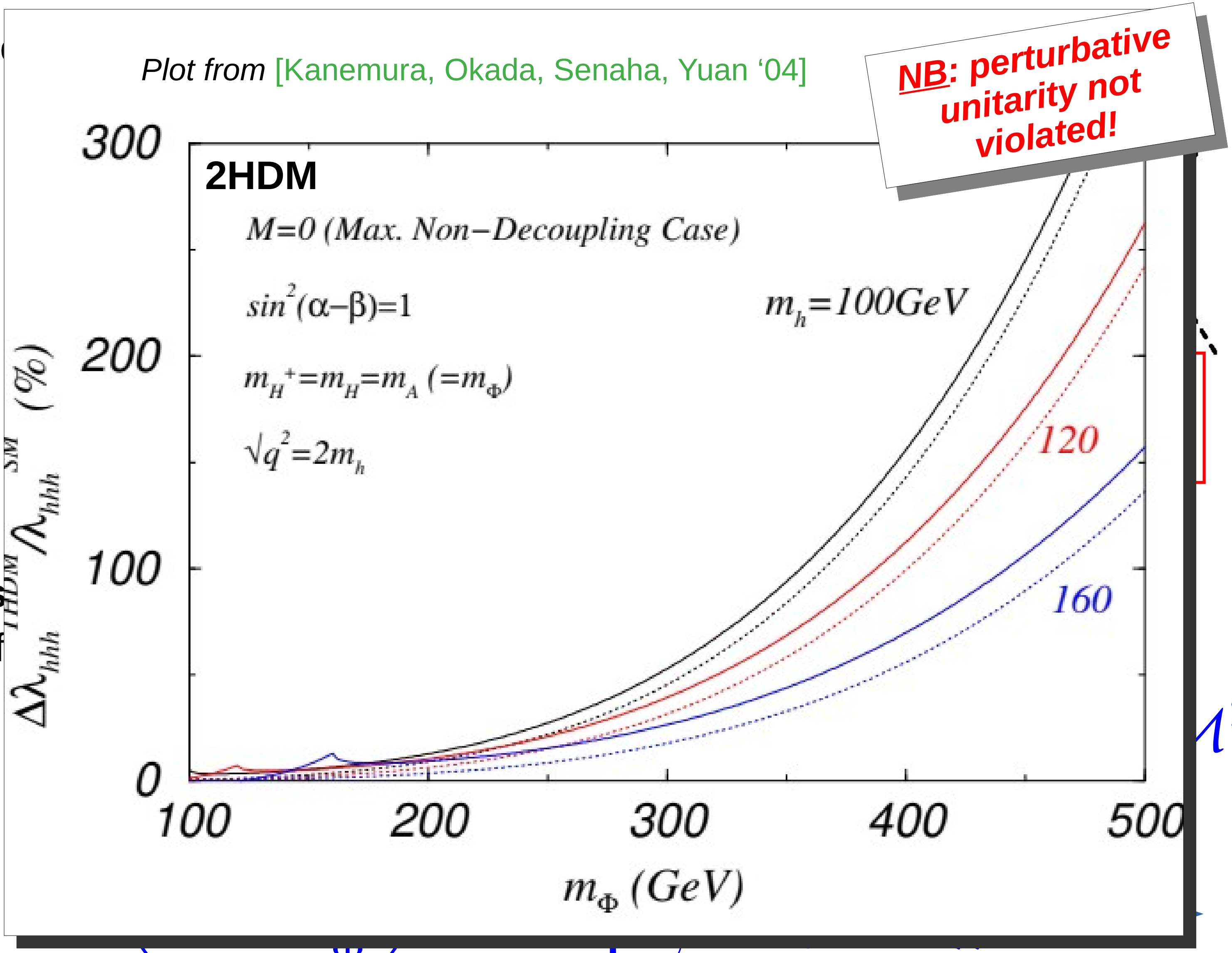
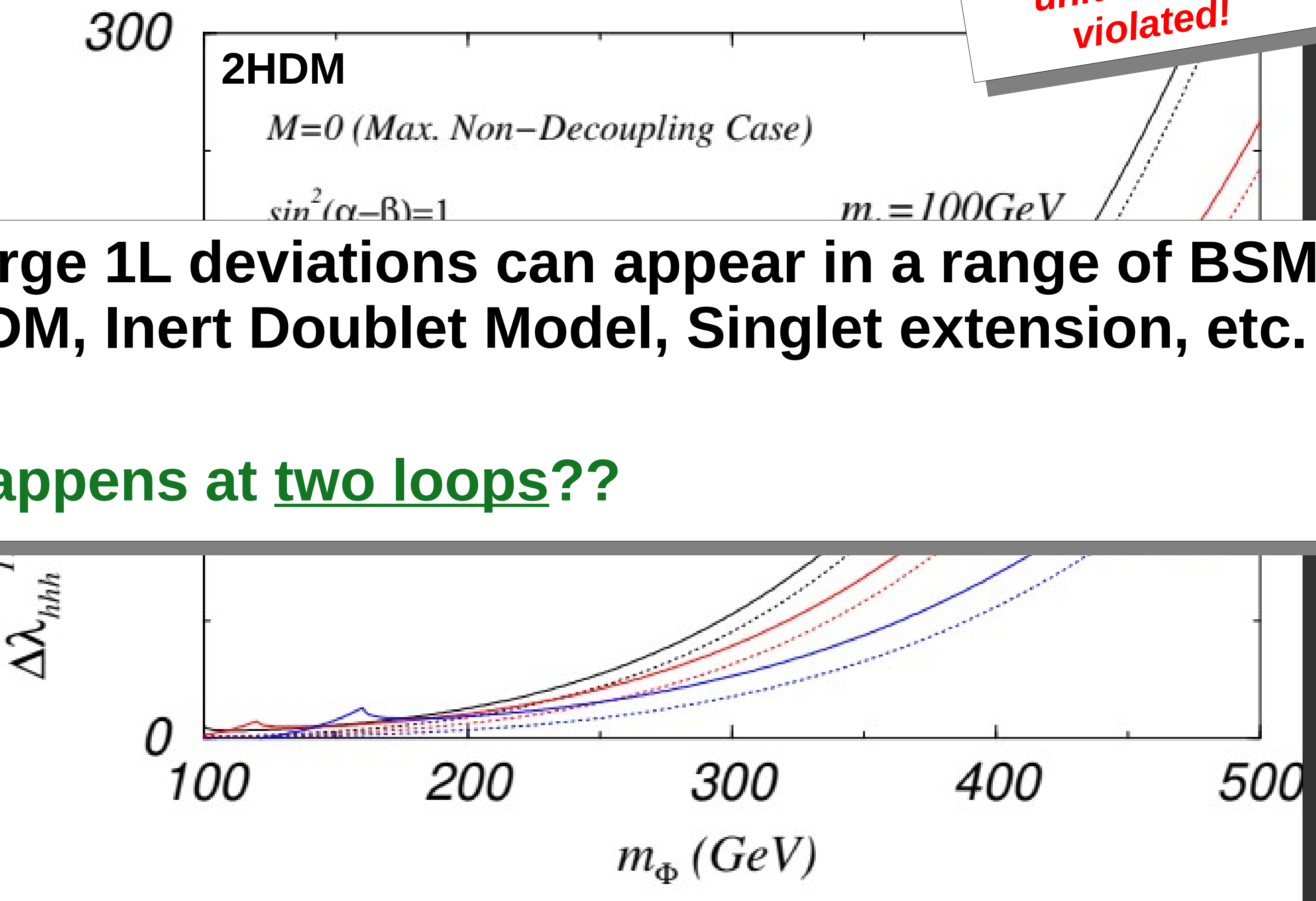


Huge BSM effects possible!

One-loop λ_{hhh} and non-decoupling effects

- Leading one-loop c

Plot from [Kanemura, Okada, Senaha, Yuan ‘04]



NB: perturbative unitarity not violated!

δ

- **Such large 1L deviations can appear in a range of BSM models: e.g. 2HDM, Inert Doublet Model, Singlet extension, etc.**

- **What happens at two loops??**

$\mathcal{M}$

n_Φ : # of d.o.f of

- Size of new effects

$\Lambda^2 + \tilde{\lambda}v^2$

Huge BSM effects possible!

Our calculations

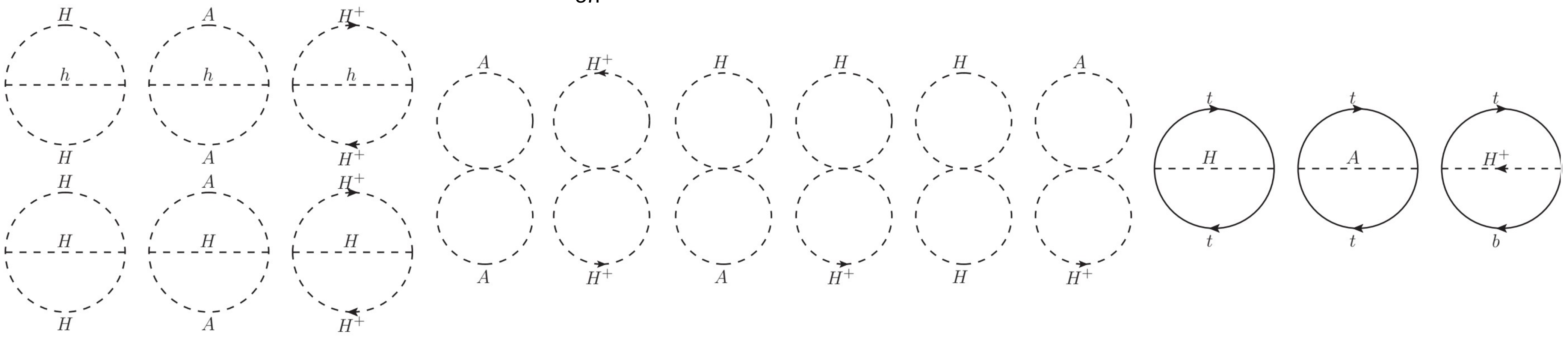
Our setup

- We want to know **how large** the two-loop corrections to λ_{hhh} can become:
  - *Effective Higgs trilinear coupling* (i.e. neglect subleading effects from ext. momentum, but corresponds to κ_λ, used by experimentalists)

$$\lambda_{hhh} \equiv \left.\frac{\partial^3 V_{\text{eff}}}{\partial h^3}\right|_{\text{min.}}$$

  - *Dominant two-loop corrections to V_{eff}* = diagrams involving **heavy BSM scalars and top quark**

e.g. for 2HDM



  - *Aligned scenarios* → no mixing + compatible with experimental results
  - Results expressed in terms of physical (OS) parameters *(details in backup)*

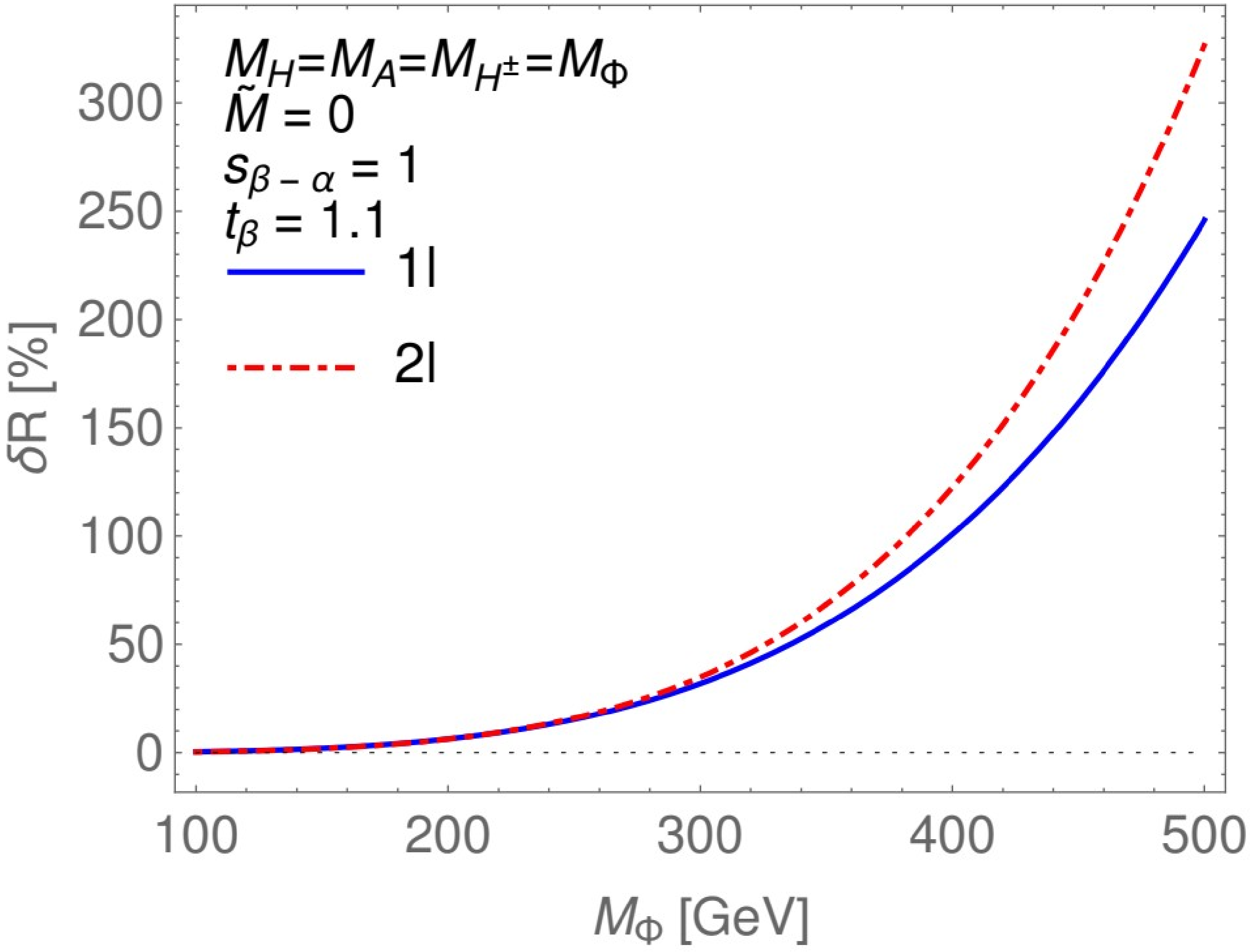
Numerical results

$$\delta R \equiv \frac{\hat{\lambda}_{hhh}^{\mathrm{BSM}} - \hat{\lambda}_{hhh}^{\mathrm{SM}}}{\hat{\lambda}_{hhh}^{\mathrm{SM}}}$$

BSM deviation of λ_{hhh} in an aligned 2HDM

Taking degenerate BSM scalar masses: $M_\Phi = M_H = M_A = M_{H^+}$

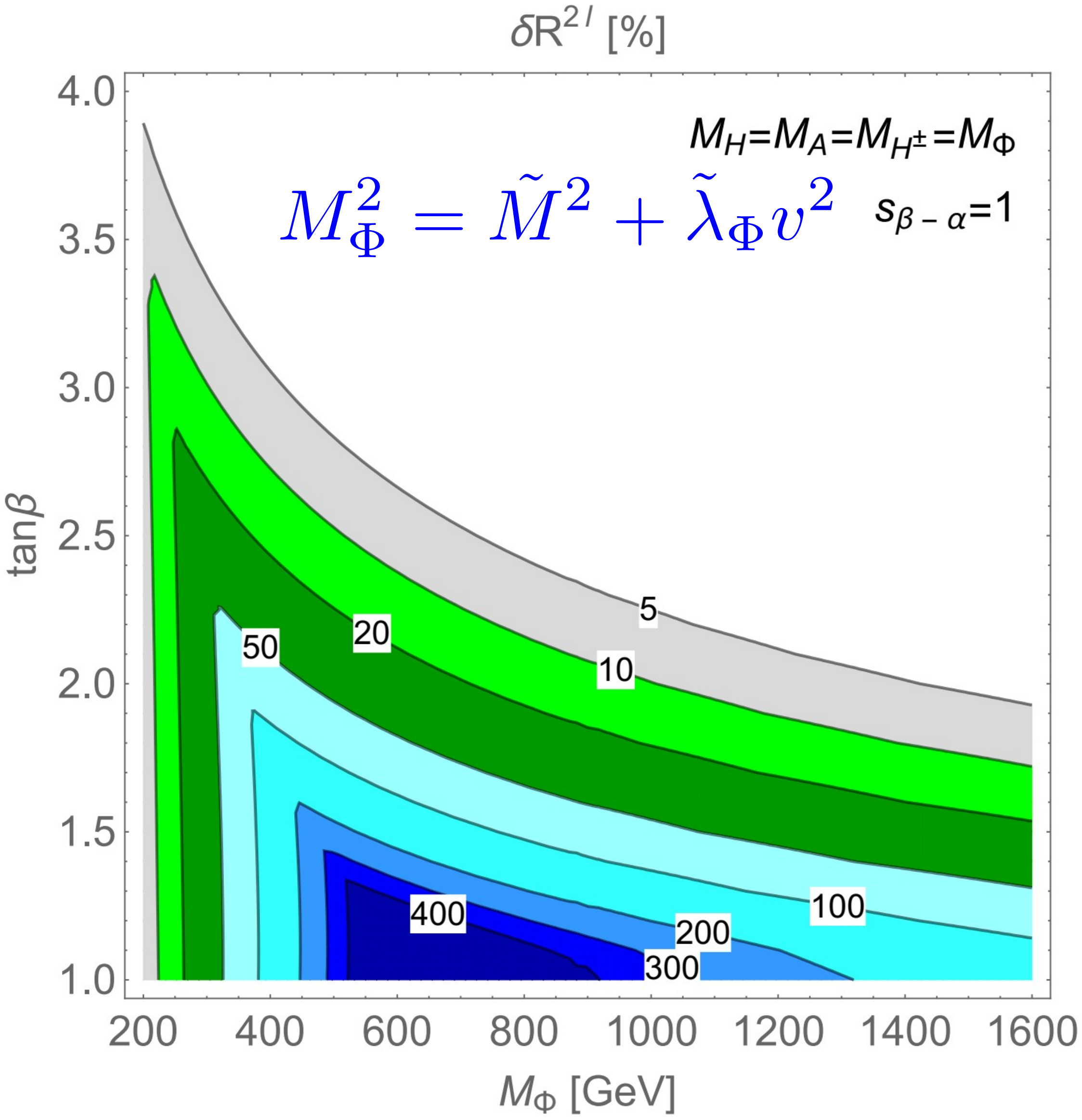
[JB, Kanemura ‘19]



- $\tilde{M} = 0$ → maximal non-decoupling effects
- $\delta^{(2)}\lambda_{hhh}$ typically 10-20% of $\delta^{(1)}\lambda_{hhh}$ for most of mass range, at most 30%

Maximal BSM deviation in an aligned 2HDM scenario

[JB, Kanemura '19]



- Maximal δR (1l+2l) allowed while fulfilling perturbative unitarity [Kanemura, Kubota, Takasugi '93]
- Max. deviations for low tanβ and M_Φ~600-800 GeV →heavy BSM scalars acquiring their mass from Higgs VEV **only**
  - 1 loop: up to ~300% deviation at most
  - 2 loops: additional 100% (for same points)
- For increasing tanβ, unitarity constraints become more stringent → smaller δR
- **Blue region:** probed at **HL-LHC** (50% accuracy on λ_{hhh})
- **Green region:** probed at lepton colliders, e.g. **ILC** (50% accuracy at 250 GeV; 27% at 500 GeV; 10% at 1 TeV)

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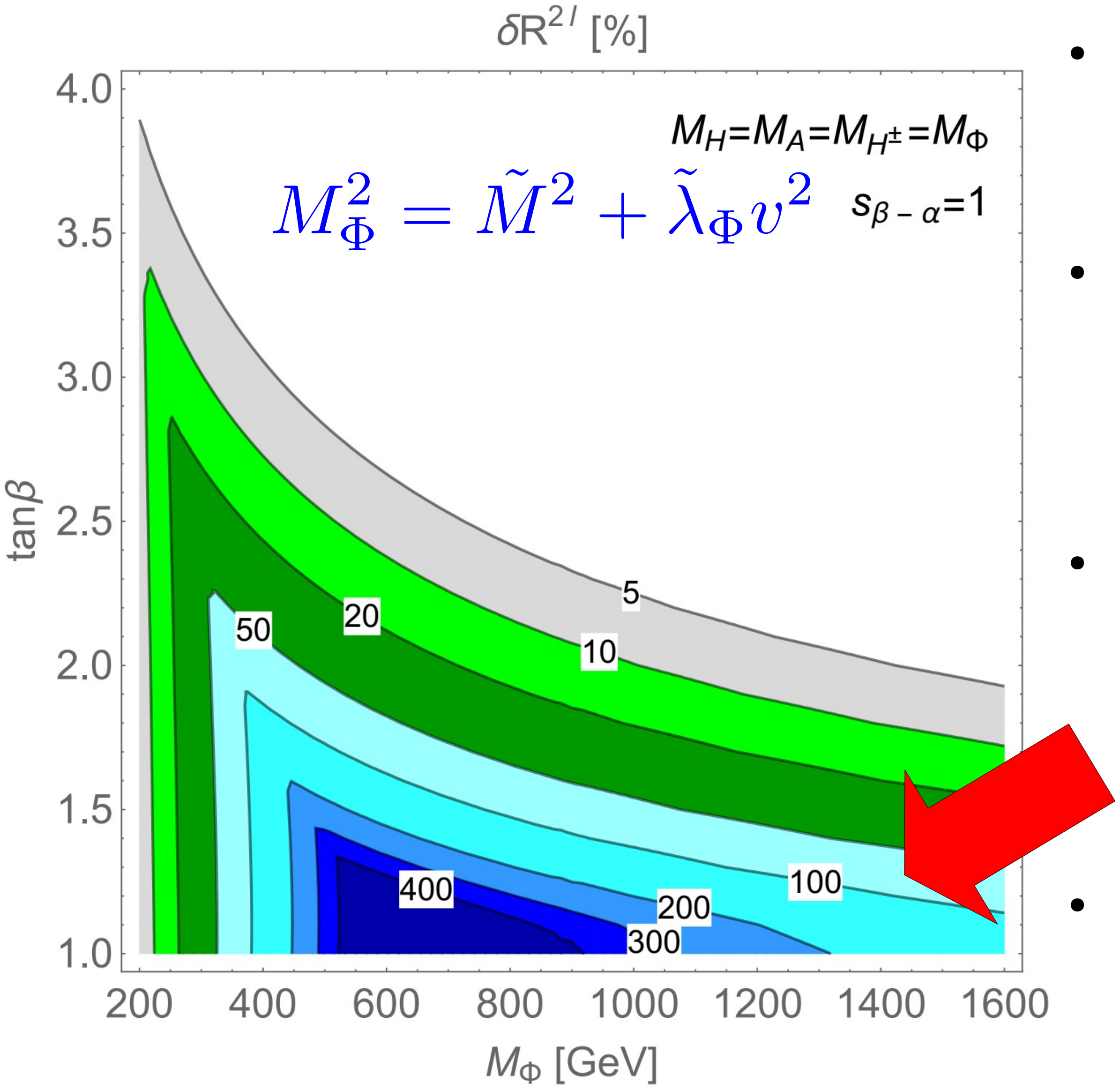

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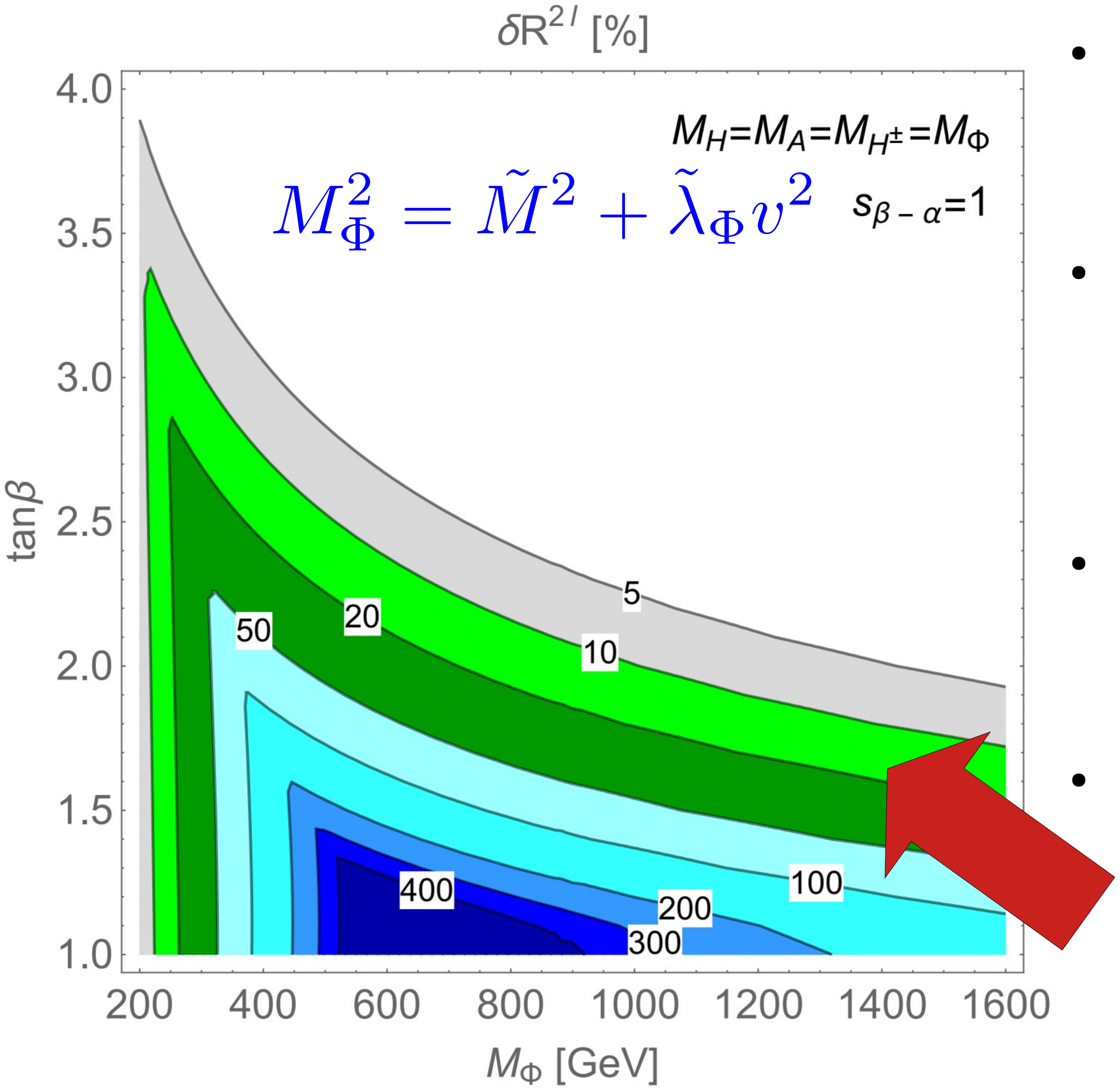

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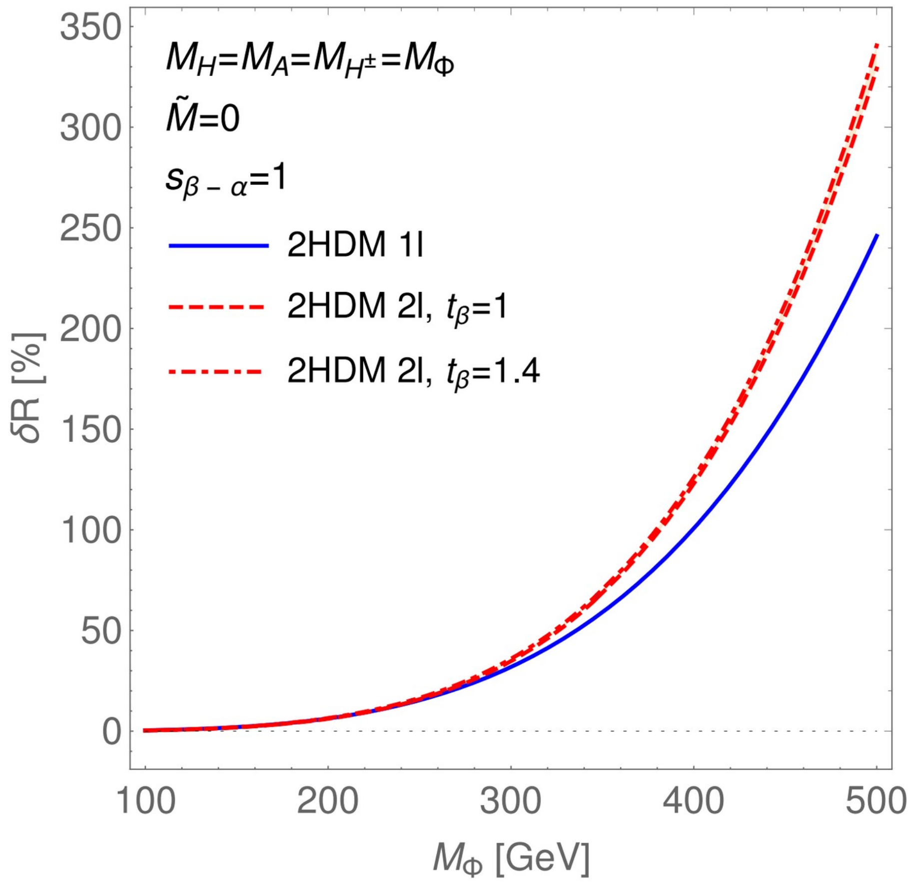
λ_{hhh} at two loops in more models

[JB, Kanemura '19]

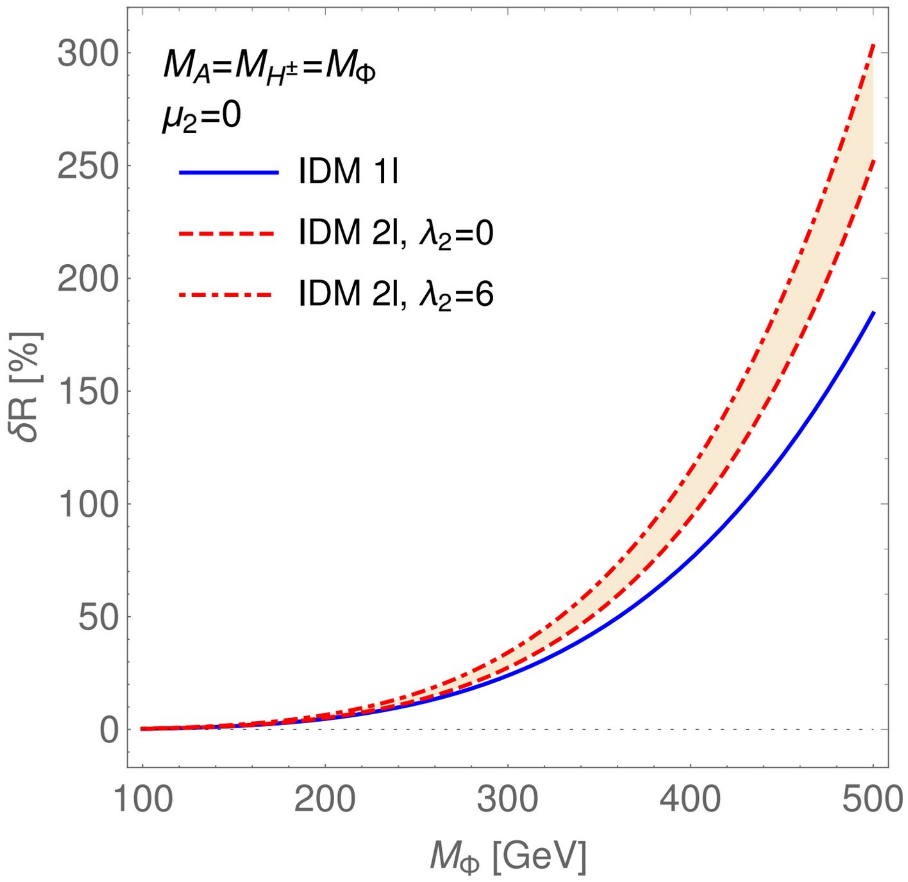
- Calculations in several other models: *IDM*, *singlet extension of SM*
- Each model contains a **new parameter appearing from two loops** → **can large enhancements occur?**

Aligned 2HDM → tanβ



tanβ constrained by perturbative unitarity
→ only small effects

IDM → λ_2 (quartic coupling of inert doublet)



λ_2 is less contrained → **enhancement is possible**
(but 2L effects remains <u>well smaller</u> than 1L ones)

Theories with classical scale invariance (CSI)

Classical scale invariance

- CSI: forbid mass-dimensionful parameters at classical (= tree) level

  → tree-level potential: $V^{(0)} = \Lambda_{ijkl}\varphi_i\varphi_j\varphi_k\varphi_l$

- However broken **explicitly** at loop level

- EW symmetry breaking: (c.f. [Coleman, Weinberg ‘73], [Gildener, Weinberg ‘76])
  - Must occur along a flat direction of $V^{(0)}$ (= Higgs/scalon direction)
  - EW sym. broken à la Coleman-Weinberg along flat direction
  - EW scale generated by dimensional transmutation

- Here: **CSI assumed around EW scale, for phenomenology**
  - Higgs (scalon) automatically aligned at tree level → compatible with current exp. results
  - BSM states can’t be decoupled (no BSM mass term!)
  - CSI scenarios: **alignment with decoupling**

One-loop effective potential and λ_{hhh}

- Only source of mass = coupling to Higgs and its VEV: $m_i^2(h) = m_i^2 \times \left(1+\frac{h}{v}\right)^2$

- Greatly simplifies the one-loop potential along Higgs (scalon) direction:

$$V^{(1)} = A(v+h)^4 + B(v+h)^4 \log\frac{(v+h)^2}{Q^2}$$

with

$$A \equiv \frac{1}{64\pi^2 v^4}\left\{\mathrm{tr}\left[M_S^4\left(\log\frac{M_S^2}{v^2}-\frac{3}{2}\right)\right] - 4\mathrm{tr}\left[M_f^4\left(\log\frac{M_f^2}{v^2}-\frac{3}{2}\right)\right] + 3\mathrm{tr}\left[M_V^4\left(\log\frac{M_V^2}{v^2}-\frac{5}{6}\right)\right]\right\}$$

$$B \equiv \frac{1}{64\pi^2 v^4}\left(\mathrm{tr}\left[M_S^4\right] - 4\mathrm{tr}\left[M_f^4\right] + 3\mathrm{tr}\left[M_V^4\right]\right)$$

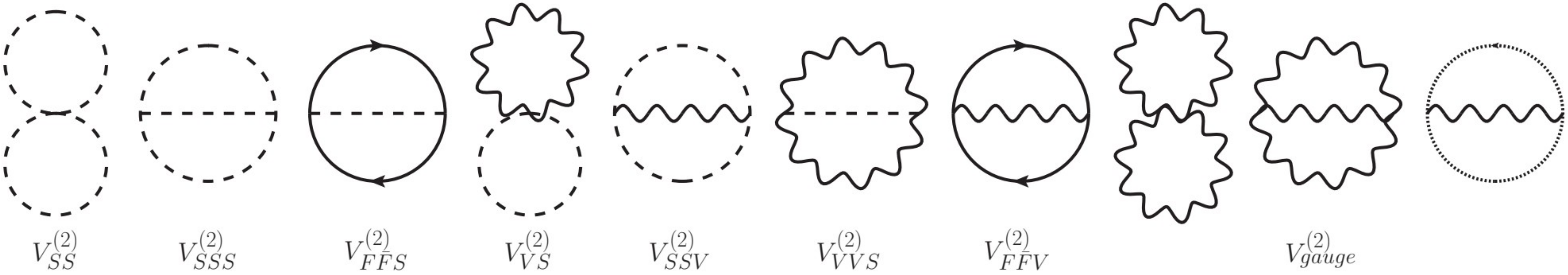
- Taking successive derivatives of the potential
  - 1st derivative = tadpole equation → fix A in terms of v and B
  - 2nd derivative = Higgs (effective potential) mass $[M_h^2]_{V_\text{eff}}$ → fix B in terms of v and M_h
  - 3rd derivative = λ_{hhh} but $V^{(1)}$ is **entirely determined** by A, B →

$$\lambda_{hhh} = \frac{5[M_h^2]_{V_\text{eff}}}{v} = \frac{5}{3}\lambda_{hhh}^{\text{SM,tree}}$$

Universal one-loop result in CSI theories!

Effective potential at two loops

- Form of V_{eff} changes at two loops:



- New type of contribution:

new log^2 term!

$$V_{\text{eff}} = A(v+h)^4 + B(v+h)^4 \log\frac{(v+h)^2}{Q^2} + C(v+h)^4 \log^2\frac{(v+h)^2}{Q^2}$$

λ_{hhh} at two loops in CSI models

[JB, Kanemura, Shimoda '20]

- Follow same procedure as at one loop:
  - ➔ Eliminate A with tadpole equation, B with Higgs mass
  - ➔ Still, **C remains!**
- One finds: $$\lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min}} = \frac{5[M_h^2]_{V_{\text{eff}}}}{v} + 32Cv$$
  - ➔ Deviation in λ_{hhh} depends on log^2 term in V_{eff}
  - ➔ **Universality found at one loop is lost at two loops!**

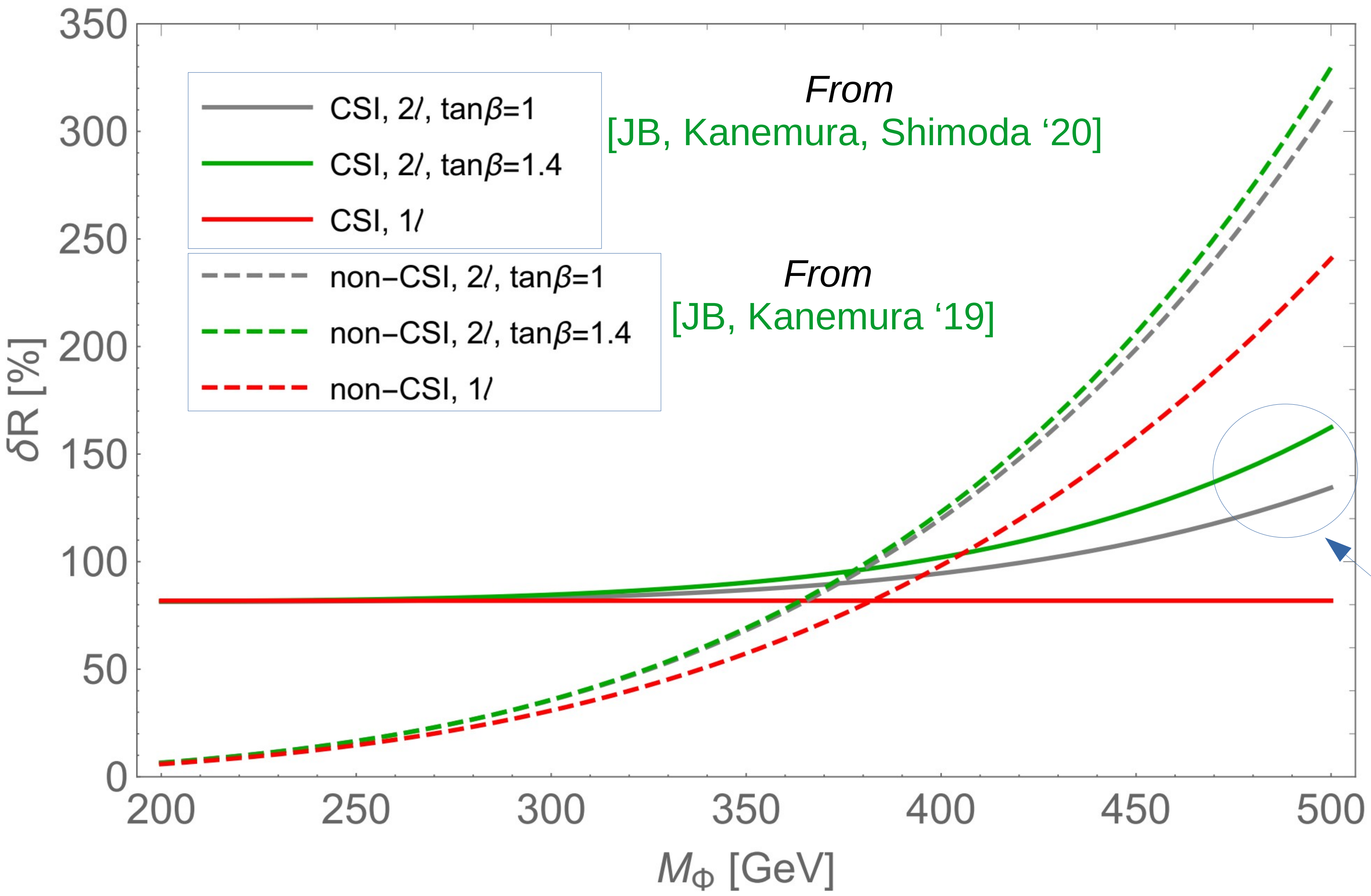
Numerical results

$$\delta R \equiv \frac{\hat{\lambda}_{hhh}^{\text{CSI-2HDM}} - \hat{\lambda}_{hhh}^{\text{SM}}}{\hat{\lambda}_{hhh}^{\text{SM}}}$$

Comparing λ_{hhh} in 2HDM scenarios with or without CSI

Taking degenerate BSM masses: $M_\Phi = M_H = M_A = M_{H^+}$

[JB, Kanemura, Shimoda '20]

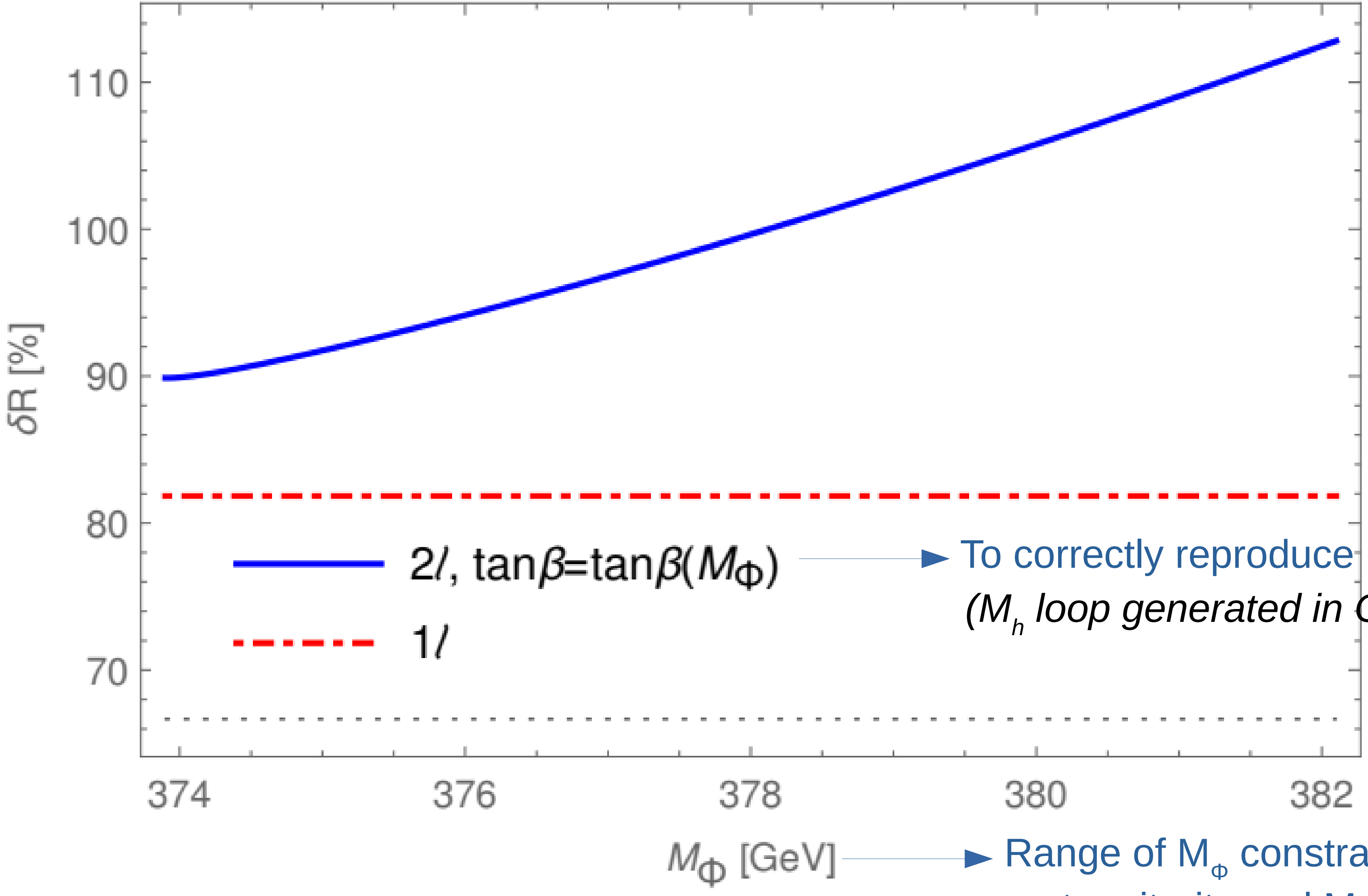


We can now distinguish CSI scenarios with different values of tanβ or M_Φ!

Allowed range of BSM deviations in a CSI-2HDM

Perturbative unitarity and M_h strongly constrain the allowed range of BSM parameters!

[JB, Kanemura, Shimoda '20]



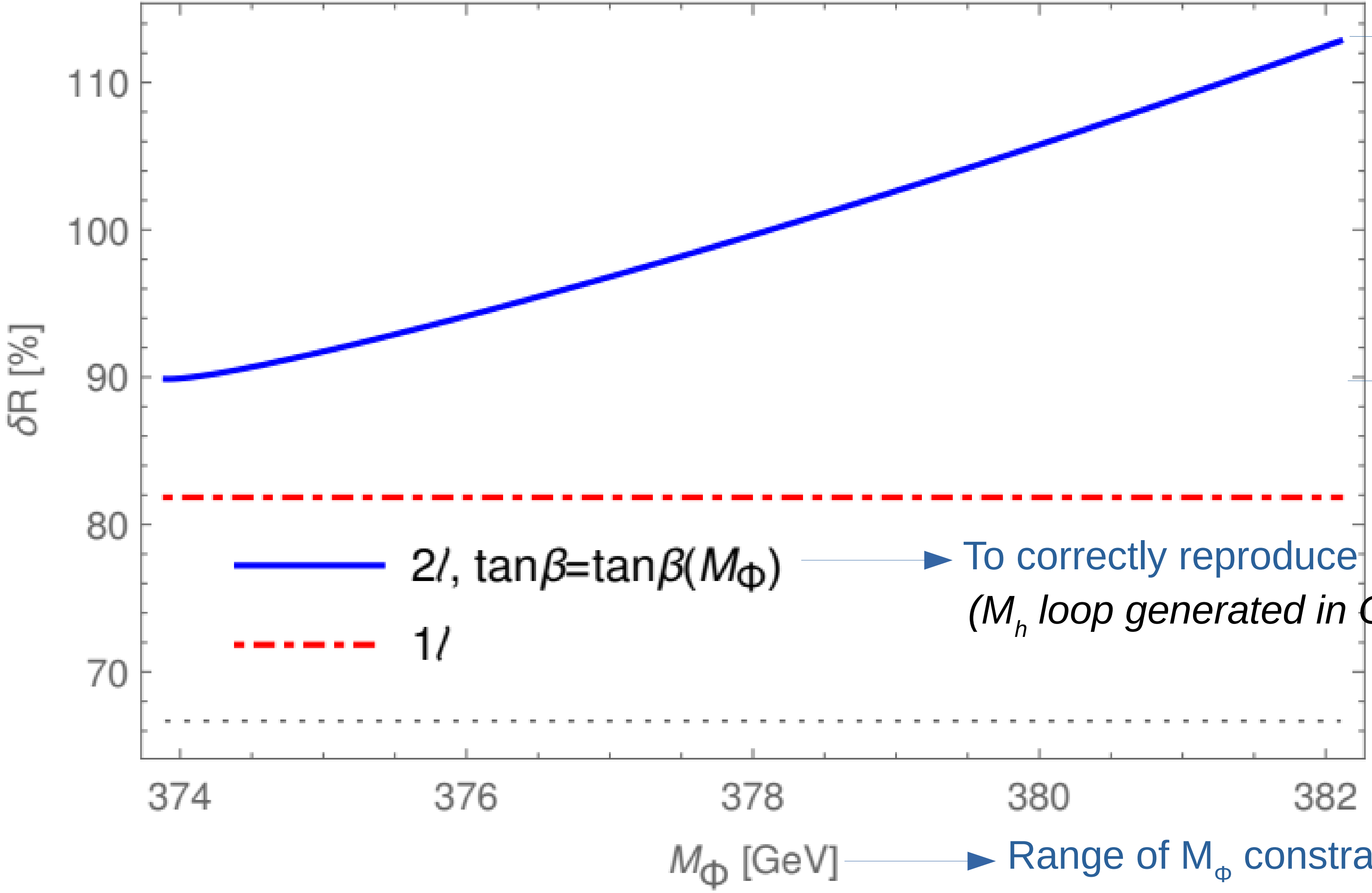
► To correctly reproduce M_h=125 GeV *(M_h loop generated in CSI theories)*

► Range of M_Φ constrained by pert. unitarity and M_h

Allowed range of BSM deviations in a CSI-2HDM

Perturbative unitarity and M_h strongly constrain the allowed range of BSM parameters!

[JB, Kanemura, Shimoda '20]



Could also be obtained in a non-CSI 2HDM [JB, Kanemura '19]

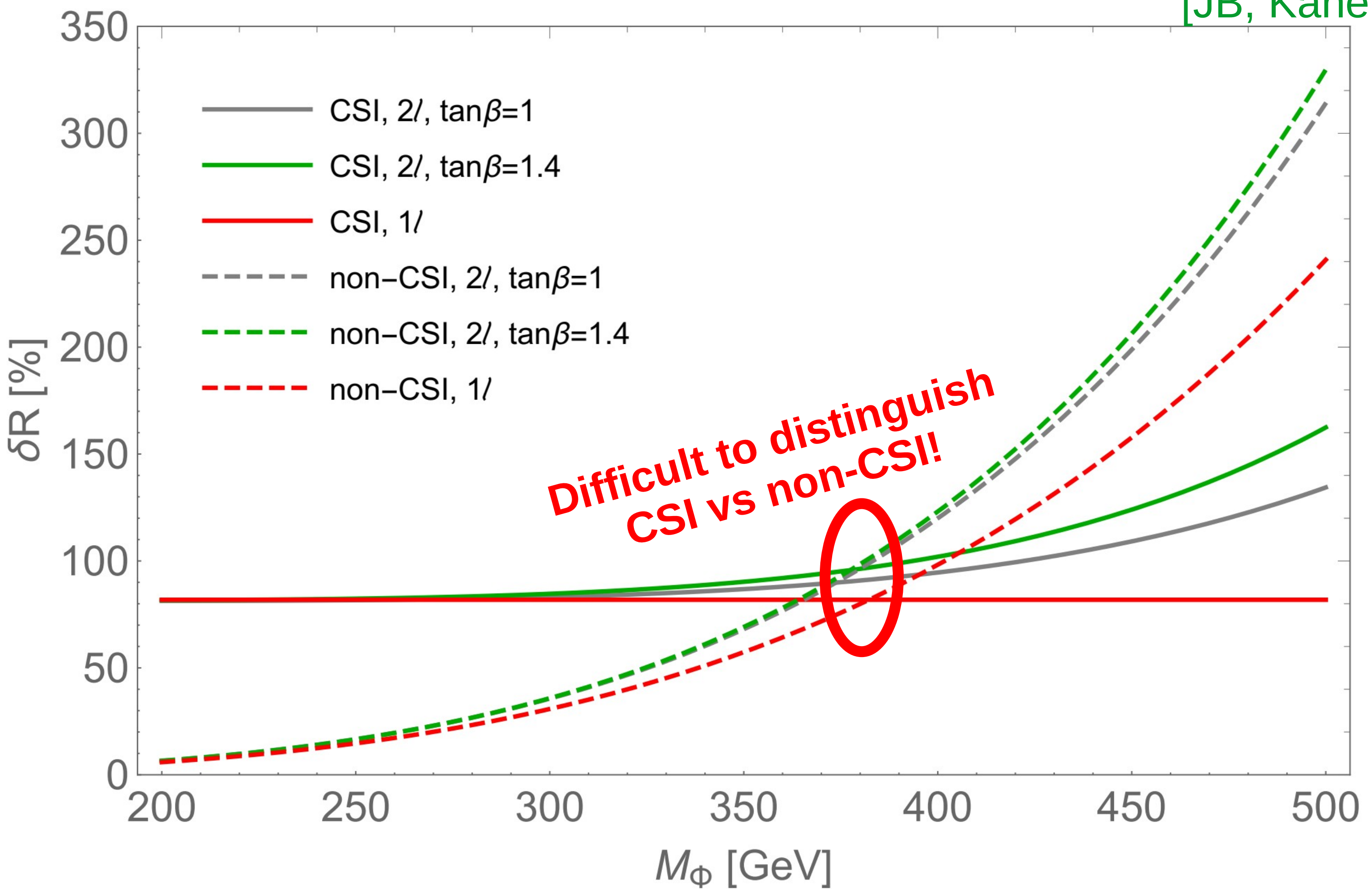
► To correctly reproduce M_h=125 GeV *(M_h loop generated in CSI theories)*

► Range of M_Φ constrained by pert. unitarity and M_h

Comparing λ_{hhh} in 2HDM scenarios with or without CSI

Taking once again degenerate BSM masses: $M_\Phi = M_H = M_A = M_{H^+}$

[JB, Kanemura, Shimoda '20]



- Separating models with or without CSI difficult with only λ_{hhh} , but possible with **synergy** of λ_{hhh} and either collider or GW signals (see e.g. [Hashino, Kakizaki, Kanemura, Matsui '16])

Summary

Explicit two-loop calculation of λ_{hhh} in theories with extended scalar sectors

⇒ Size of the two-loop corrections remain **well below** that of the one-loop corrections – typically to 10-20% of one-loop contributions (max. ~ 30%)

⇒ Non-decoupling effects found at one loop are **not drastically changed**

⇒ Computations beyond one loop will be **necessary** given the expected accuracy of the measurement of λ_{hhh} at future colliders

⇒ Precise calculation of Higgs couplings (λ_{hhh} , etc.) can allow distinguishing aligned scenarios **with or without decoupling**, by accessing **non-decoupling effects**!

⇒ Matching level of accuracy now achieved for results in CSI theories → two-loop corrections allow **distinguishing different scenarios with CSI**

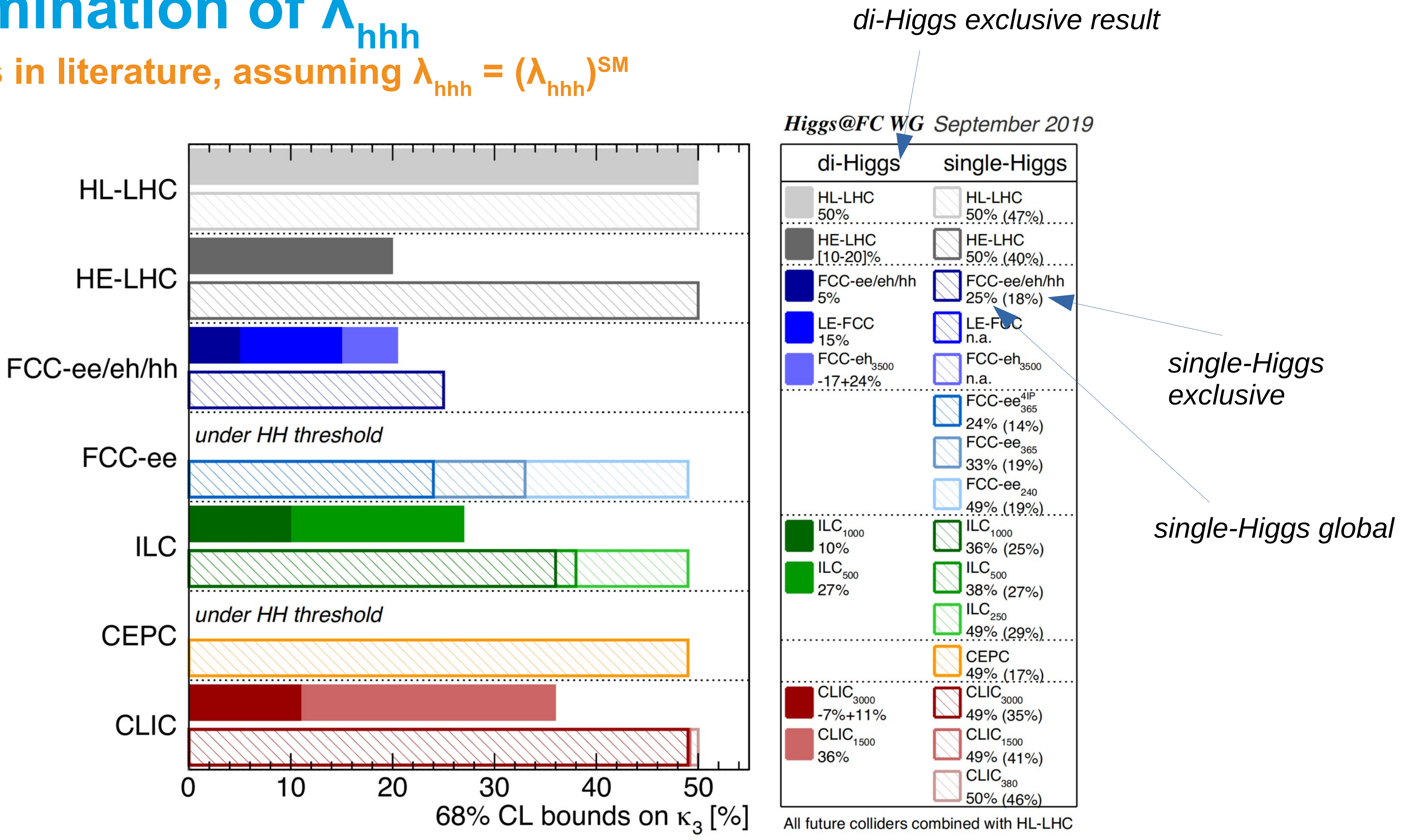
Thank you for your attention!

Backup slides

Future determination of λ_{hhh}

Expected sensitivities in literature, assuming $\lambda_{hhh} = (\lambda_{hhh})^{SM}$

Plot taken from
[de Blas et al., 1905.03764]



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Future determination of λ_{hhh}

Higgs production cross-sections (here double Higgs production) depend on λ_{hhh}

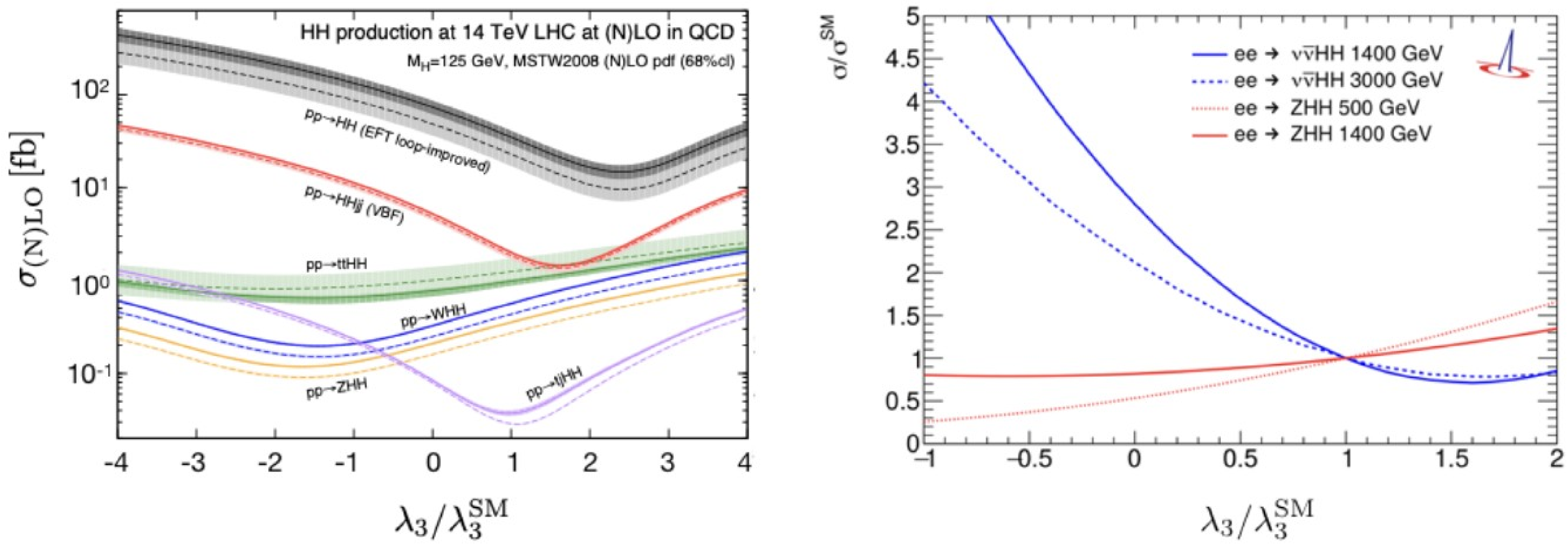
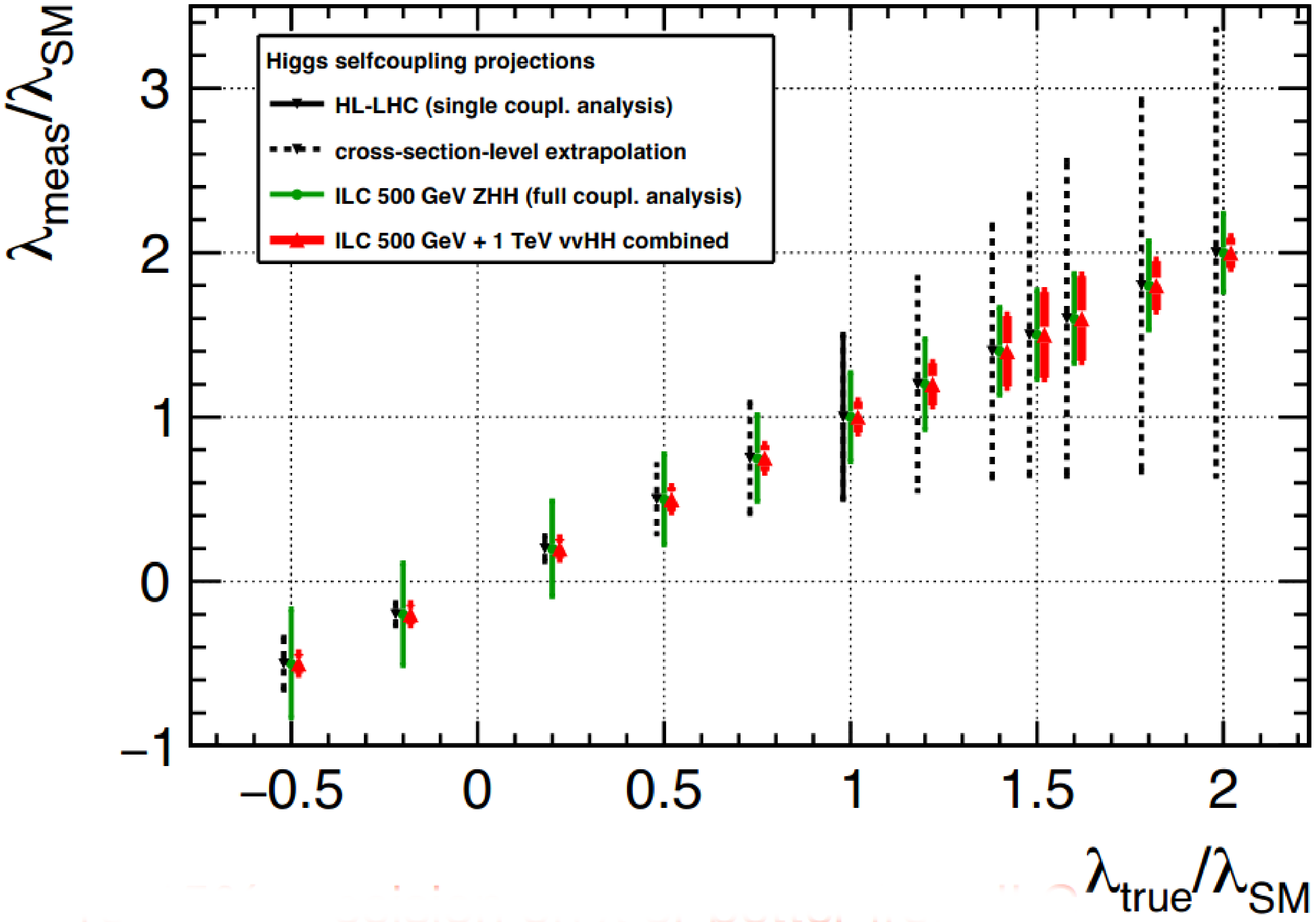


Figure 10. Double Higgs production at hadron (left) [65] and lepton (right) [66] colliders as a function of the modified Higgs cubic self-coupling. See Table 18 for the SM rates. At lepton colliders, the production cross sections do depend on the polarisation but this dependence drops out in the ratios to the SM rates (beam spectrum and QED ISR effects have been included).

Plots taken from
[de Blas et al., 1905.03764]

Future determination of λ_{hhh}

Achieved accuracy actually depends on the value of λ_{hhh}



[J. List et al. ‘21], *see also talk by G. Weiglein on Tuesday*

See also [Dürig, DESY-THESIS-2016-027]

The Two-Higgs-Doublet Model

- 2 $SU(2)_L$ doublets $\Phi_{1,2}$ of hypercharge ½

- CP-conserving 2HDM, with softly-broken Z_2 symmetry ($\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$) to avoid tree-level FCNCs

$$V^{(0)}_{\text{2HDM}} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

- m_1,m_2 eliminated with tadpole equations, and $v_1^2 + v_2^2 = v^2 = (246\ \text{GeV})^2$

- 7 free parameters in scalar sector: m_3, λ_i (i=1,..,5), $\tan\beta \equiv v_2/v_1$

- Mass eigenstates: h, H: CP-even Higgses, A: CP-odd Higgs, $H^\pm$: charged Higgs, α: CP-even Higgs mixing angle

- λ_i (i=1,..,5) traded for mass eigenvalues m_h, m_H, m_A, $m_{H\pm}$ and angle α
- m_3 replaced by a Z_2 soft-breaking mass scale

$$M^2 = \frac{2m_3^2}{s_{2\beta}}$$

$\overline{\text{MS}}$ to OS scheme conversion

- V_{eff}: we use expressions in MS scheme hence results for λ_{hhh} also in $\overline{\text{MS}}$ scheme

- We include finite counterterms to express the Higgs trilinear coupling in terms of physical quantities

$$\underbrace{m_X^2}_{\overline{\text{MS}}} = \underbrace{M_X^2}_{\text{pole}} - \mathfrak{Re}[\Pi^{\text{fin.}}_{XX}(p^2 = M_X^2)], \qquad v^2 = \underbrace{(\sqrt{2}G_F)^{-1}}_{\equiv v^2_{\text{OS}}} + \frac{3M_t^2}{16\pi^2}\left(2\log\frac{M_t^2}{Q^2} - 1\right) + \cdots$$

- Also we include finite WFR effects → OS scheme

$$\underbrace{\hat{\lambda}_{hhh}}_{\text{OS}} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{finite WFR}} \underbrace{\lambda_{hhh}}_{\overline{\text{MS}}} = -\underbrace{\Gamma_{hhh}(0,0,0)}_{\text{3-pt. func.}}$$

Theoretical and experimental constraints in [JB, Kanemura, Shimoda '20]

- **Perturbative unitarity**: we constrain parameters entering only at two loops
  → tree-level perturbative unitarity suffices [Kanemura, Kubota, Takasugi '93]

- EW vacuum must be **true minimum of V_{eff}**, i.e. check that

$$\underbrace{V_{\text{eff}}(v+h=0)}_{=0} - V_{\text{eff}}(h=0) > 0 \quad \Rightarrow \quad V_{\text{eff}}(h=0) < 0$$

- M_h, generated at loop level, must be **125 GeV**
  → imposes a relation between SM parameters, M_H, M_A, M_H^+, tanβ, e.g. we can extract:

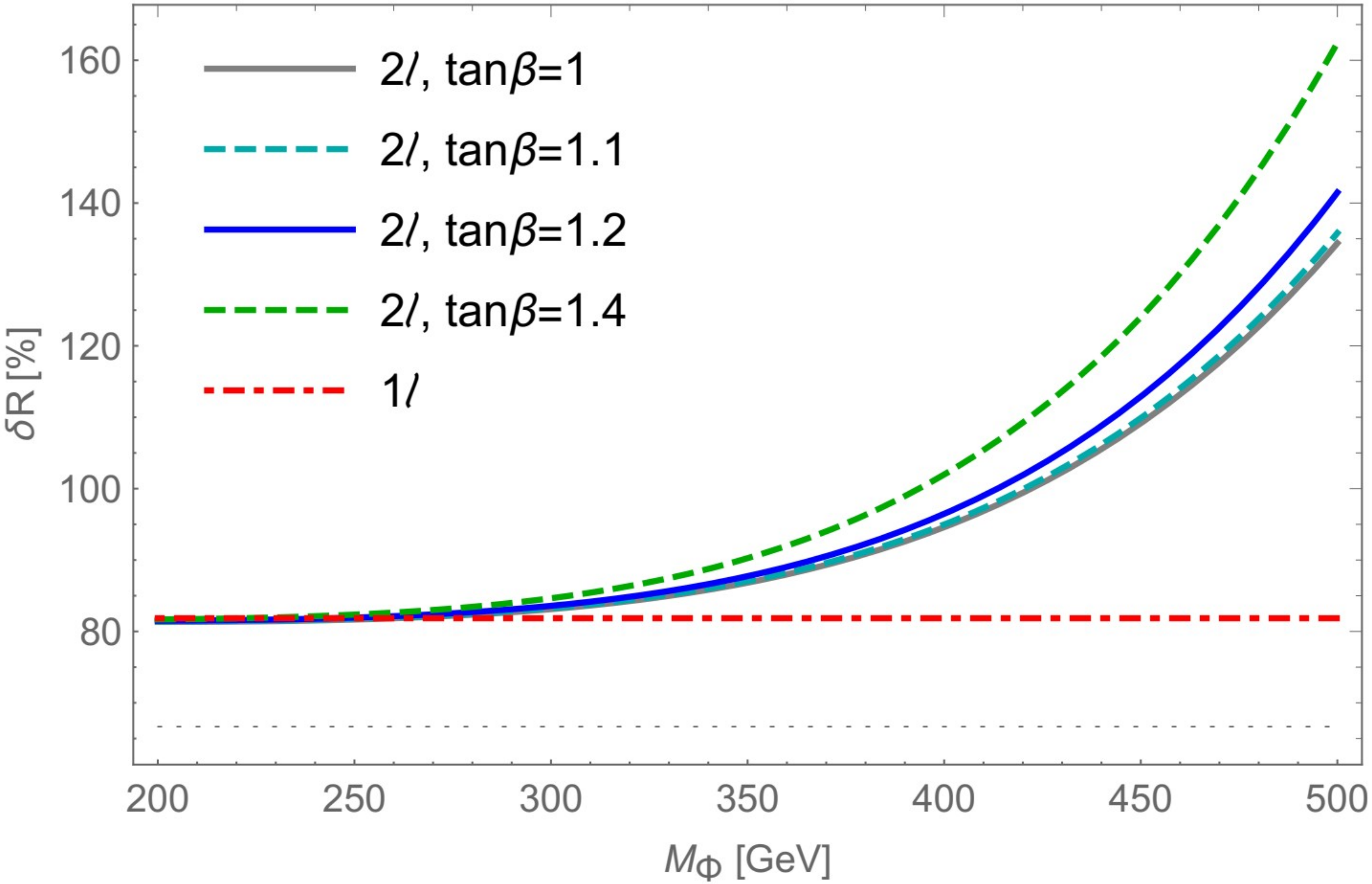
$$[M_h^2]_{V_{\text{eff}}} = \left.\frac{\partial^2 V_{\text{eff}}}{\partial h^2}\right|_{\text{min}} \quad \Rightarrow \quad \tan\beta = \tan\beta(\underbrace{M_h, M_t, \cdots}_{\text{measured SM values}}, \underbrace{M_H, M_A, M_{H^\pm}}_{\text{BSM inputs}})$$

- Limits from **collider searches** with HiggsBounds and HiggsSignals

No constraints

Taking degenerate BSM masses: $M_\Phi = M_H = M_A = M_{H^+}$

[JB, Kanemura, Shimoda ‘20]



Unitarity and constraint from M_h in the CSI-2HDM

[JB, Kanemura, Shimoda ‘20]