Asymmetry in flavour changing electromagnetic transitions of vector–like quarks

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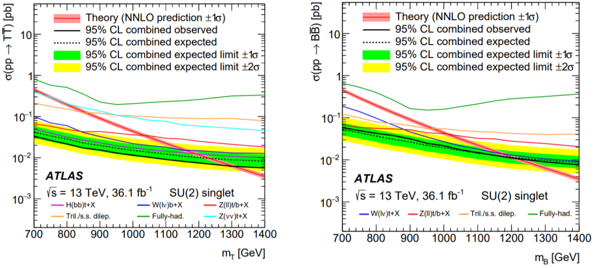
Introduction

- Vector–like quarks (VLQs) have been of interest for a plethora of experimentally motivated reasons
- VLQs have come under increased investigation due to appreciably improving global fits to several flavour physics and precision EW measurements.
- They naturally arise near the EW scale in many new physics models: minimal super–symmetric model, Left–Right symmetry models, top colour assisted technicolour and two Higgs doublets with four generations of quarks

- Simplest VLQ: isosinglet with both left– and right–handed components transforming as singlets under $SU(2)_L$ with $I_3 = 0$ that can either be up– (t') or down–type (b').

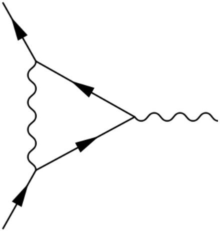


- Indirect signals of VLQs can be considered due to the loop–level contributions of the exotic quarks in SM particle processes.
- Precision flavour measurements place strong limits on the new heavy quarks and set the lowest mass scale and maximum mixing for these states.

Vector–like quark radiative decays

- The addition of VLQ singlets breaks the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and enables tree–level flavour changing neutral (FCN) vertices.
- The resulting radiative flavour changing decays of these particles through the electromagnetic transition dipole moment are a key means study their properties and search for them at the LHC etc.
- Interested in computing the size and the *CP*–violation induced these decays
- Based on Ref. [1].

CP-violation in the quark sector



Electromagnetic dipole moments (EDMs) generated at various loop levels.

CP violation in radiative decays

$q_i \rightarrow q_f + \gamma_{\pm}$, $q_i \rightarrow q_f + g_{\pm}$

SM: CP negligible → New Physics

If CP is violated in the radiative decay, an asymmetry between two circularly polarised photons can be generated $\gamma_+ \neq \gamma_-$

Circular polarisation of photons potentially provides a crucial way to measure CP violation in the extended quark sector and new physics.

Transition dipole moment

The matrix elements can be written in terms of a generic vertex function $\Gamma^{\mu}_{\mathrm{fi}}$ like

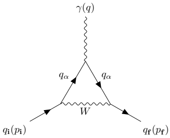
$$i\mathcal{M}(q_{\mathrm{i}} \to q_{\mathrm{f}} + \gamma_{\pm}) = \bar{u}(p_{\mathrm{f}})\Gamma^{\mu}_{\mathrm{fi}}(q^2)u(p_{\mathrm{i}})\varepsilon^{*}_{\pm,\mu}(q)\,. \tag{1}$$

By requiring $q^2 = 0$ and choosing the Lorenz gauge $q \cdot \varepsilon_{p} = 0$, the anapole does not contribute to $\Gamma^{\mu}_{\mathrm{fi}}$ but only $f^{\mathrm{E}}_{\mathrm{fi}}$ and $f^{\mathrm{M}}_{\mathrm{fi}}$ which are the electric and magnetic transition dipole moments of $q_{\mathrm{i}} \to q_{\mathrm{f}}\gamma$ respectively.
We can rewrite the generic vertex function as

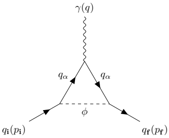
$$\Gamma^{\mu}_{\mathrm{fi}}(q^2) \;=\; i\sigma^{\mu\nu}q_{\nu}[f^{\mathrm{L}}_{\mathrm{fi}}(q^2)P_L + f^{\mathrm{R}}_{\mathrm{fi}}(q^2)P_R]\,, \tag{2}$$

where $f^{\mathrm{L}}_{\mathrm{fi}}$ and $f^{\mathrm{R}}_{\mathrm{fi}}$ are the left and right chiral form factors.

Feynman diagrams



(1)



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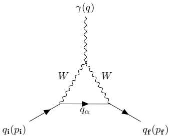

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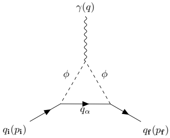

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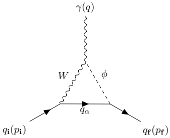

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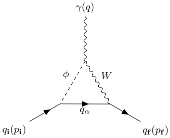

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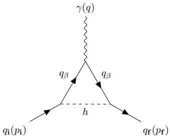

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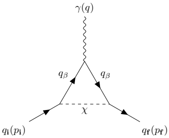

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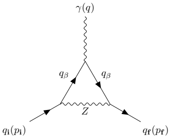
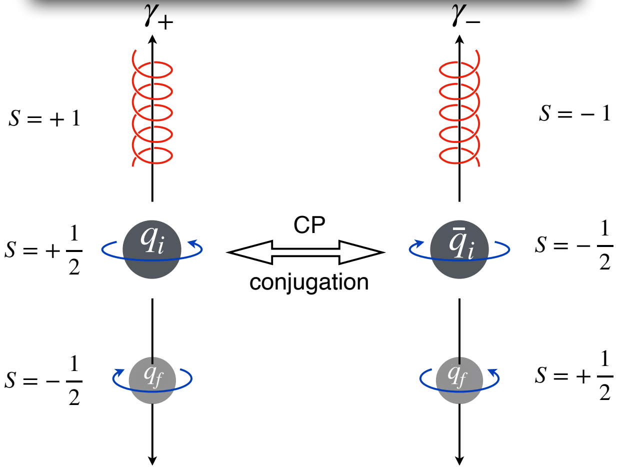
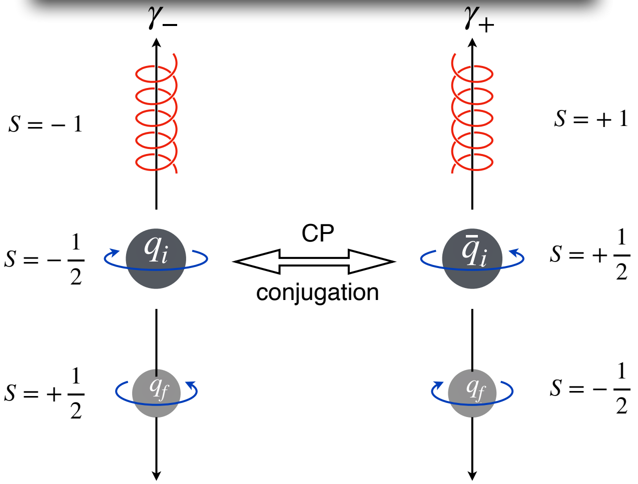


(9)

$$\Delta_{CP,+} = \frac{\Gamma(q_i \to q_f + \gamma_+) - \Gamma(\bar{q}_i \to \bar{q}_f + \gamma_-)}{\Gamma(q_i \to q_f + \gamma) + \Gamma(\bar{q}_i \to \bar{q}_f + \gamma)}$$



$$\Delta_{CP,-} = \frac{\Gamma(q_i \to q_f + \gamma_-) - \Gamma(\bar{q}_i \to \bar{q}_f + \gamma_+)}{\Gamma(q_i \to q_f + \gamma) + \Gamma(\bar{q}_i \to \bar{q}_f + \gamma)}$$



Vector isosinglet up–type quark

- We now consider implications of a t' quark by analysing a global fit that uses complementary flavour physics observables to constrain the quark mixing matrix [5]

- The new physics effects of the t' are mainly through charged current interactions which involve quark mixing via V and neutral current and Higgs interactions that mix through the matrix $X^u = VV^\dagger$.

- The extended CKM matrix V in such a setup comprises four SM and five new physics parameters.

Quark mixing

Parameter	SM	$m_{t'} = 800$ GeV	$m_{t'} = 1200$ GeV
λ	0.226 ± 0.001	0.226 ± 0.001	0.226 ± 0.001
A	0.780 ± 0.015	0.770 ± 0.019	0.769 ± 0.019
C	0.39 ± 0.01	0.44 ± 0.02	0.43 ± 0.02
δ_{13}	1.21 ± 0.08	1.13 ± 0.11	1.15 ± 0.09
P	-	0.40 ± 0.26	0.30 ± 0.21
Q	-	0.04 ± 0.06	0.03 ± 0.05
R	-	0.45 ± 0.25	0.36 ± 0.22
δ_{41}	-	0.55 ± 0.45	0.76 ± 0.42
δ_{42}	-	0.52 ± 3.26	0.96 ± 1.21
$\chi^2/$ d.o.f.	$71.15/60$	$63.35/59$	$63.60/59$

Table: Best fit parameters for the quark mixing matrix with a vector–like quark isosinglet t' included. These are shown for two benchmark quark masses $m_{t'} = 800\ \mathrm{GeV}$ and $m_{t'} = 1200\ \mathrm{GeV}$ outlined in Table 4 of Ref. [5].

Results

Decay channel	$m_{t'} = 800$ GeV		
	$\mathcal{B}$	$\Delta_{CP,+}$	$\Delta_{CP,-}$
$t' \to u\gamma$	$(1.2^{+7.0}_{-1.1} \times 10^{-8}$	$(-0.6^{+2.3}_{-4.0}) \times 10^{-14}$	$(-0.5^{+2.5}_{-6.0}) \times 10^{-5}$
$t' \to c\gamma$	$(1.0^{+2.4}_{-1.0}) \times 10^{-8}$	$(0.1^{+3.5}_{-8.0}) \times 10^{-8}$	$0.02^{+1.40}_{-2.63} \times 10^{-3}$
$t' \to t\gamma$	$(3.36^{+0.03}_{-0.06}) \times 10^{-6}$	$(-0.04^{+2.09}_{-1.00}) \times 10^{-6}$	$(-0.09^{+2.9}_{-1.9}) \times 10^{-6}$
$t' \to ug$	$(0.5^{+2.0}_{-0.4}) \times 10^{-7}$	$(0.7^{+7.0}_{-1.3}) \times 10^{-13}$	$(1.1^{+9.0}_{-1.8}) \times 10^{-4}$
$t' \to cg$	$(0.4^{+2.1}_{-0.4}) \times 10^{-7}$	$(-0.01^{+0.2}_{-1}) \times 10^{-6}$	$(-0.02^{+0.40}_{-1.82}) \times 10^{-2}$
$t' \to tg$	$3.937^{+0.077}_{-0.109} \times 10^{-5}$	$(-0.09^{+4.05}_{-2.90}) \times 10^{-6}$	$(0.11^{+8.07}_{-6.00}) \times 10^{-7}$
$t \to u\gamma$	$(0.29^{+1.1}_{-0.28}) \times 10^{-11}$	$(0.7^{+4.0}_{-0.7}) \times 10^{-12}$	$(-0.60^{+0.70}_{-2.80}) \times 10^{-2}$
$t \to c\gamma$	$(0.7^{+5.0}_{-0.7}) \times 10^{-12}$	$(-0.04^{+0.8}_{-1.1}) \times 10^{-5}$	$0.04^{+0.90}_{-0.92}$
$t \to ug$	$(0.7^{+2.4}_{-0.6}) \times 10^{-10}$	$(1.4^{+8.0}_{-1.5}) \times 10^{-12}$	$-0.012^{+0.011}_{-0.070}$
$t \to cg$	$(0.13^{+2.00}_{-0.11}) \times 10^{-10}$	$(-0.1^{+1.3}_{-0.9}) \times 10^{-5}$	$0.1^{+0.7}_{-1.1}$

Decay channel	$m_{t'} = 1200$ GeV		
	$\mathcal{B}$	$\Delta_{CP,+}$	$\Delta_{CP,-}$
$t' \to u\gamma$	$(0.8^{+0.6}_{-0.7}) \times 10^{-8}$	$(-0.22^{+0.27}_{-1.6} \times 10^{-14})$	$(0.1^{+2.0}_{-2.3}) \times 10^{-5}$
$t' \to c\gamma$	$(0.6^{+5.0}_{-0.5}) \times 10^{-8}$	$(0.7^{+9.0}_{-5.0}) \times 10^{-9}$	$(0.3^{+3.0}_{-2.7}) \times 10^{-4}$
$t' \to t\gamma$	$(2.71^{+0.01}_{-0.06}) \times 10^{-6}$	$(-0.8^{+4.0}_{-8.0}) \times 10^{-7}$	$(-0.2^{+1.2}_{-1.4}) \times 10^{-6}$
$t' \to ug$	$(0.26^{+1.7}_{-0.19}) \times 10^{-7}$	$(0.3^{+2.2}_{-0.4}) \times 10^{-13}$	$(0.5^{+2.3}_{-0.7}) \times 10^{-4}$
$t' \to cg$	$(0.21^{+1.1}_{-0.14}) \times 10^{-7}$	$(-0.1^{+0.8}_{-1.3}) \times 10^{-7}$	$(-0.3^{+2.4}_{-2.3}) \times 10^{-3}$
$t' \to tg$	$(4.21^{+0.54}_{-1.08}) \times 10^{-5}$	$(-1.0^{+7.0}_{-8.0}) \times 10^{-7}$	$(0.2^{+4.0}_{-2.3}) \times 10^{-7}$
$t \to u\gamma$	$(1.0^{+4.0}_{-1.0}) \times 10^{-12}$	$(0.8^{+2.7}_{-1.2}) \times 10^{-12}$	$-0.006^{+0.011}_{-0.018}$
$t \to c\gamma$	$(0.22^{+2.7}_{-0.22}) \times 10^{-12}$	$(-0.2^{+1.2}_{-0.7}) \times 10^{-5}$	$0.2^{+0.6}_{-1.0}$
$t \to ug$	$(0.24^{+1.30}_{-0.21}) \times 10^{-10}$	$(0.15^{+1.3}_{-0.34}) \times 10^{-11}$	$-0.014^{+0.03}_{-0.11}$
$t \to cg$	$(0.5^{+6.0}_{-0.4}) \times 10^{-11}$	$(-0.6^{+1.8}_{-0.4}) \times 10^{-5}$	$0.61^{+0.28}_{-1.70}$

Resolving the CKM unitarity problem

- It has been pointed out that hints of deviations from unitarity where $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 < 1$ at 1–2σ in the first row of the CKM matrix may be explained by the presence of t' [3, 4].
- The fit from the study considered in this section explored how the stringent experimental constraints arising from CP Violation from meson mixing
- The mixing matrix once again contains angles θ_{12}, θ_{13} and θ_{23} and one CP violating phase δ_{13} along with three new mixing angles θ_{14}, θ_{24} and θ_{34} and two new CP violating phases δ_{14} and δ_{24}.
- Concluded that $m_{t'} < 7$ TeV.

Quark mixing

Their best fit resulted in

$$\theta_{12} = 0.2265, \qquad \theta_{13} = 0.003818, \qquad \theta_{23} = 0.03988,$$
$$\theta_{14} = 0.03951, \qquad \theta_{24} = 0.002078, \qquad \theta_{34} = 0.01271,$$
$$\delta_{13} = 0.0396\pi, \qquad \delta_{14} = 1.818\pi, \qquad \delta_{24} = 0.728\pi, \tag{3}$$

with $m_{t'} = 1.5$ TeV

Results

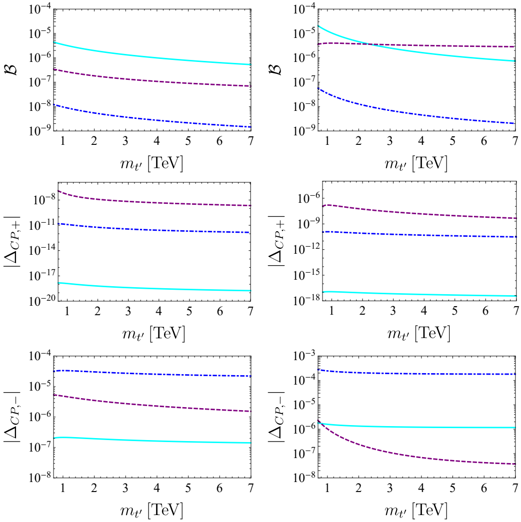


Figure: The left (right) panels are for a final state photon (gluon). The cyan, blue and purple lines correspond to u, c and t final state respectively.

Results continued...

Correlated top quark decays

$$\begin{aligned}
&\mathcal{B}(t \to u\gamma) \simeq 3 \times 10^{-12}, && \mathcal{B}(t \to c\gamma) \simeq 9 \times 10^{-14}, \\
&|\Delta_{CP,+}(t \to u\gamma)| \simeq 1.4 \times 10^{-12}, && |\Delta_{CP,+}(t \to c\gamma)| \simeq 1.8 \times 10^{-6}, \\
&|\Delta_{CP,-}(t \to u\gamma)| \simeq 9.3 \times 10^{-3}, && |\Delta_{CP,-}(t \to c\gamma)| \simeq 0.14,
\end{aligned} \tag{4}$$

$$\begin{aligned}
&\mathcal{B}(t \to ug) \simeq 6.4 \times 10^{-11}, && \mathcal{B}(t \to cg) \simeq 6.7 \times 10^{-12}, \\
&|\Delta_{CP,+}(t \to ug)| \simeq 2.4 \times 10^{-12}, && |\Delta_{CP,+}(t \to cg)| \simeq 9 \times 10^{-7}, \\
&|\Delta_{CP,-}(t \to ug)| \simeq 0.024, && |\Delta_{CP,-}(t \to cg)| \simeq 0.11,
\end{aligned} \tag{5}$$

Vector isosinglet down–type quark

- Here we consider a model and global fit shown in Ref. [2] where the SM is extended by the addition of a VLQ isosinglet down–type quark b'.

- The mixing matrix once again contains angles θ_{12}, θ_{13} and θ_{23} and one CP violating phase δ_{13} along with three new mixing angles θ_{14}, θ_{24} and θ_{34} and two new CP violating phases δ_{14} and δ_{24}.

- The authors consider constraints from several flavour observables [2] and compare with measurements and perform a χ^2 fit

- They redo this process with predictions for a b' VLQ and obtain values for the various mixing angles and phases mentioned above

Quark mixing

Parameter	SM	$m_{b'} = 800$ GeV	$m_{b'} = 1200$ GeV
θ_{12}	0.2273 ± 0.0007	0.2271 ± 0.0008	0.2270 ± 0.0008
θ_{13}	0.0035 ± 0.0001	0.0038 ± 0.0001	0.0038 ± 0.0001
θ_{23}	0.039 ± 0.0007	0.0391 ± 0.0007	0.0391 ± 0.0007
δ_{13}	1.10 ± 0.1	1.04 ± 0.08	1.04 ± 0.08
θ_{14}	-	0.0151 ± 0.0154	0.0147 ± 0.0149
θ_{24}	-	0.0031 ± 0.0039	0.0029 ± 0.0036
θ_{34}	-	0.0133 ± 0.0130	0.0123 ± 0.0122
δ_{14}	-	0.11 ± 0.22	0.11 ± 0.23
δ_{24}	-	3.23 ± 0.24	3.23 ± 0.27
$\chi^2/$ d.o.f.	$71.15/60$	$70.99/63$	$70.96/63$

Results

Decay channel	$m_{b'} = 800$ GeV		
	$\mathcal{B}$	$\Delta_{CP,+}$	$\Delta_{CP,-}$
$b' \to d\gamma$	$(3.5^{+2.3}_{-3.5} \times 10^{-6}$	$(-0.15^{+0.16}_{-2.60}) \times 10^{-12}$	$(-0.33^{+0.3}_{-0.6}) \times 10^{-2}$
$b' \to s\gamma$	$(0.007^{+4.0}_{-0.010}) \times 10^{-6}$	$(2.1^{+8.0}_{-2.1}) \times 10^{-9}$	$0.12^{+0.5}_{-0.18}$
$b' \to b\gamma$	$(1.0^{+1.3}_{-1.0}) \times 10^{-5}$	$(0.14^{+5.00}_{-0.15}) \times 10^{-7}$	$0.12^{+4.0}_{-0.13} \times 10^{-2}$
$b' \to dg$	$(1.3^{+0.9}_{-1.3}) \times 10^{-5}$	$(-0.05^{+1.8}_{-1.2}) \times 10^{-11}$	$-0.01^{+0.4}_{-0.27}$
$b' \to sg$	$(0.08^{+1.0}_{-0.008}) \times 10^{-6}$	$(0.3^{+1.6}_{-1.7}) \times 10^{-8}$	$0.2^{+0.8}_{-1.0}$
$b' \to bg$	$0.7^{+1.0}_{-0.7} \times 10^{-4}$	$(0.2^{+7.0}_{-0.4}) \times 10^{-7}$	$(0.20^{+6.00}_{-0.34}) \times 10^{-2}$

Decay channel	$m_{b'} = 1200$ GeV		
	$\mathcal{B}$	$\Delta_{CP,+}$	$\Delta_{CP,-}$
$b' \to d\gamma$	$(2.6^{+1.7}_{-2.6}) \times 10^{-6}$	$(-0.10^{+0.10}_{-1.4} \times 10^{-12})$	$(-0.22^{+0.21}_{-2.80}) \times 10^{-2}$
$b' \to s\gamma$	$(0.003^{+4.0}_{-0.032}) \times 10^{-6}$	$(0.25^{+1.2}_{-0.25} \times 10^{-8}$	$0.15^{+0.7}_{-0.18}$
$b' \to b\gamma$	$(6.0^{+9.0}_{-6.0}) \times 10^{-6}$	$(0.10^{+8.0}_{-0.11}) \times 10^{-7}$	$(0.08^{+7.0}_{-0.09}) \times 10^{-2}$
$b' \to dg$	$(8.0^{+0.80}_{-1.2}) \times 10^{-6}$	$(-0.05^{+0.028}_{-1.2}) \times 10^{-11}$	$-0.011^{+0.008}_{-0.35}$
$b' \to sg$	$(0.05^{+9.0}_{-0.05}) \times 10^{-6}$	$(0.3^{+1.1}_{-1.5}) \times 10^{-8}$	$(0.2^{+0.8}_{-1.2})$
$b' \to bg$	$(5.0^{+8.0}_{-5.0}) \times 10^{-5}$	$(0.17^{+5.0}_{-0.16}) \times 10^{-7}$	$(0.16^{+6.0}_{-0.2}) \times 10^{-2}$

Conclusion

- Analytical calculation of the electromagnetic transition dipole moment of a VLQ quark isosinglet
- Studied radiative decays via branching ratios and CP asymmetries
- Several numerical studies for several global fits favouring inclusion of t' and b' vector isosinglets to the SM.
- For t', we find phenomenological interesting results with branching ratios up to $\simeq 10^{-5}$ and CP asymmetries $\geq 10^{-2}$.
- For b', we find branching ratios $\simeq 10^{-4}$ and CP asymmetries up to order unity.
- Radiative decays of VLQs in such scenarios provide a tantalising probe for new physics and provide clean experimental signatures.
- Results can be applied to a variety of VLQ parameter scans and be easily extended to consider more exotic scenarios such as doublet and triplet VLQs in the future.

Thank You!

References

S. Balaji, [arXiv:2110.05473 [hep-ph]].

A. K. Alok, S. Banerjee, D. Kumar and S. Uma Sankar, Nucl. Phys. B **906** (2016), 321-341 doi:10.1016/j.nuclphysb.2016.03.012 [arXiv:1402.1023 [hep-ph]].

B. Belfatto and Z. Berezhiani, JHEP **10** (2021), 079 doi:10.1007/JHEP10(2021)079 [arXiv:2103.05549 [hep-ph]].

G. C. Branco, J. T. Penedo, P. M. F. Pereira, M. N. Rebelo and J. I. Silva-Marcos, JHEP **07** (2021), 099 doi:10.1007/JHEP07(2021)099 [arXiv:2103.13409 [hep-ph]].

A. K. Alok, S. Banerjee, D. Kumar, S. U. Sankar and D. London, Phys. Rev. D **92** (2015), 013002 doi:10.1103/PhysRevD.92.013002 [arXiv:1504.00517 [hep-ph]].

L. Wolfenstein, Phys. Rev. Lett. **51** (1983), 1945 doi:10.1103/PhysRevLett.51.1945

Backup

Since the Lorentz invariant amplitude can be written

$$i\mathcal{M}(q_\mathrm{i} \to q_\mathrm{f} + \gamma) = i\bar{u}(p_\beta)\sigma^{\mu\nu}(A_\gamma + B_\gamma\gamma_5)q_\nu u(p_t)\varepsilon^*_{\pm,\mu}(q)\,, \tag{6}$$

we may express the decay width in terms of the vector and axial form factors A and B

$$\begin{aligned}\Gamma(q_\mathrm{i} \to q_\mathrm{f}\gamma) &= \frac{1}{\pi}\left(\frac{m_\mathrm{i}^2 - m_\mathrm{f}^2}{2m_\mathrm{i}}\right)^3\left(|A_\gamma|^2 + |B_\gamma|^2\right)\,,\\ \Gamma(q_\mathrm{i} \to q_\mathrm{f} g) &= \frac{C_F}{\pi}\left(\frac{m_\mathrm{i}^2 - m_\mathrm{f}^2}{2m_\mathrm{i}}\right)^3\left(|A_g|^2 + |B_g|^2\right)\,,\end{aligned} \tag{7}$$

where $C_F = 4/3$ is the standard colour factor. Comparing the vector and axial form factors with their chiral counterparts we get

$$A = \frac{f_\mathrm{fi}^\mathrm{L} + f_\mathrm{fi}^\mathrm{R}}{2}, \qquad B = \frac{f_\mathrm{fi}^\mathrm{R} - f_\mathrm{fi}^\mathrm{L}}{2}.$$

The branching ratio is $\mathcal{B}(q_\mathrm{i} \to q_\mathrm{f}\gamma) = \Gamma(q_\mathrm{i} \to q_\mathrm{f}\gamma)/\Gamma_{q_\mathrm{i}}$ for the radiative process.

Interactions

We have the following charged and neutral current interactions

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\bar{u}_\alpha \gamma^\mu V_{\alpha\beta} d_\beta W_\mu^+ + \mathrm{h.c.}, \tag{8}$$

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_w}\bar{q}_\alpha \gamma^\mu (c_L + c_R) q_\beta Z_\mu + \mathrm{h.c.}, \tag{9}$$

where $c_L = \pm X^q_{\alpha\beta} - 2Q_q \sin^2\theta_w \delta_{\alpha\beta}$ and $c_R = -2Q_q \sin^2\theta_w \delta_{\alpha\beta}$ and $q = u_i, d_i$. $Q_u = 2/3$ and $Q_d = -1/3$.

$$X^u_{\alpha\beta} = (VV^\dagger)_{\alpha\beta}\,, \qquad X^d_{\alpha\beta} = (V^\dagger V)_{\alpha\beta}. \tag{10}$$

And with the charged Goldstone mode

$$\mathcal{L}_\phi = -\frac{g}{\sqrt{2}m_W}\bar{u}_\alpha V_{\alpha\beta}(m_\alpha - m_\beta) d_\beta \phi^+ + \mathrm{h.c.} \tag{11}$$

Interactions continued...

The neutral Goldstone mode

$$\mathcal{L}_\chi = \frac{ig}{2m_W}\left[\bar{u}_\alpha X^u_{\alpha\beta}(m_\alpha - m_\beta)u_\beta - \bar{d}_\alpha X^d_{\alpha\beta}(m_\alpha - m_\beta)d_\beta\right]\chi + \mathrm{h.c.} \quad (12)$$

and finally the Higgs

$$\mathcal{L}_h = \frac{g}{2m_W}\left[\bar{u}_\alpha X^u_{\alpha\beta}(m_\alpha + m_\beta)u_\beta + \bar{d}_\alpha X^d_{\alpha\beta}(m_\alpha + m_\beta)d_\beta\right] + \mathrm{h.c.} \quad (13)$$

From here, we may derive the Feynman rules and calculate amplitudes for the decay processes of interest.

Wolfenstein parametrisation

If we consider a t' VLQ only, V is the left–hand 4×3 submatrix of the 4×4 unitary $\mathcal{V}_L^\dagger$. Here it is best to choose a parametrisation of $\mathcal{V}_L^\dagger$ such that the new matrix elements V_{41}, V_{42} and V_{43} take simple forms. Using the Hou–Soni–Steger parametrisation we have the elements of V given by

$$\begin{aligned} V_{12} &= \lambda\,, & V_{23} &= A\lambda^2\,, & V_{13} &= A\lambda^3 C e^{i\delta_{13}} \\ V_{41} &= -P\lambda^3 e^{i\delta_{41}}\,, & V_{42} &= -Q\lambda^2 e^{i\delta_{42}}\,, & V_{43} &= -r\lambda. \end{aligned} \tag{14}$$

There are four SM parameters λ, A, C, δ_{13} and five new physics parameters P, Q, r, δ_{41}, δ_{42}. Of the remaining six CKM matrix elements, V_{11}, V_{21} and V_{22} retain their SM parametrisations

$$V_{11} = 1 - \frac{\lambda^2}{2}\,, \qquad V_{21} = -\lambda\,, \qquad V_{22} = 1 - \frac{\lambda^2}{2} \tag{15}$$

and the third row is given by

$$V_{31} = A\lambda^3(1 - Ce^{i\delta_{13}}) - Pr\lambda^4 e^{i\delta_{41}} + \frac{1}{2}AC\lambda^5 e^{i\delta_{13}},$$

$$V_{32} = -A\lambda^2 - Qr\lambda^3 e^{i\delta_{42}} + A\lambda^4\left(\frac{1}{2} - Ce^{i\delta_{13}}\right), \qquad V_{33} = 1 - \frac{1}{2}r^2\lambda^2.$$