Bottom-up reconstruction scenarios for MSSM parameters at the LHC

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1. Issues and strategies at LHC

2. Reconstructing SUSY model (MSSM) parameters:
an alternative bottom-up approach

Motivations: reconstruct SUSY basic parameters for "minimal" set of identified sparticles, within different scenarios (e.g. in SUSY if GUT scale universality assumptions or not)

3. Summary

1. LHC: General issues and Strategies

Very optimistic SUSY scenario: all MSSM sparticles +Higgses found; fit mSUGRA model; find sthing like 'SPS1a'

Real life probably harder...

Recent years, focus shifted from "discovering SUSY and measuring its parameters" to gradual questions:

- How to discover SUSY-like (weakly interacting theory with partners at TeV) or/and non SUSY-like (strongly interact. EFT)

To begin, already experimental and theoretical issues:

- Trying to tell signal from backgrounds (a challenge both for TH and EXP)
- Tune MC to (signal free) data, see if any deviation from SM,..

Here much more focused aim:

How to measure basic parameters accurately enough to extract underlying SUSY symmetry breaking scale pattern?

Beware the “LHC inverse problem”
i.e. discrete ambiguities (potentially many) in reconstructing basic MSSM parameters (Arkani-Hamed, Kane, Thaler, Wang ’05)

However, ambiguity levels clearly reduced if using most sophisticated analysis, both experimental and theoretical:
-Efforts to calculate all signals at NLO accuracy
-Global fits, new observables (e.g. “footprints” in signature space (Arkani-Hamed et al))
-Low energy constraints, interplay with ILC and dark matter

MSSM basic parameter reconstruction

Very lively debate now on what will be most efficient approach: standard “top-down” versus bottom-up; “blind” analysis; fewer observable based, etc.

- Up to now mostly "top-down" approach:

GUT scale Lagrangian $\longrightarrow$ RG evolution $\longrightarrow$ Electroweak Symmetry Breaking (low scale) $\longrightarrow$ Spectrum determination (diagonalization+ rad. corr.)

Fit model parameters (e.g mSUGRA) to data set (masses, cross-sections, etc)

+ Pb if too much parameters: hardly fitting general MSSM (22 parameters) even if all sparticle masses, x-sections known..

(but recent progress made e.g. SFitter $\longrightarrow$ see next talk by D. Zerwas)

2.1 Top-down reconstruction (some recent developments)

SFITTER (Lafaye, Plehn, Rauch, D.Zerwas) takes LHC measurements:

kinematic edges (from long gluino/squark decay chains), masses, mass differences, cross sections, BRs) +Indirect constraints $(g-2)_\mu$; BR($b \to s\gamma$), DM Ωh^2

Compare to th predictions (Spectrum calculators: SoftSUSY, SuSPECT, ISASUSY; Cross sections and BRs: Prospino2, MsmLib, SUSYHit (HDecay + SDecay))

Find best fits using different techniques (Gradient search (Minuit), Markov Chains techniques, Simulated Annealing (Fittino [Bechtle, Desch, Wienemann])

From kinematic edge +other “SPS1a” data ($\sim$ 15 sparticle mass input):

	only stat errors	(+th errors)
$\tan\beta$	9.8 $\pm$ 2.3	(4.5)
M_1	101.5 $\pm$ 4.6	(7.8)
M_2	191.7 $\pm$ 4.8	(7.8)
M_3	575.7 $\pm$ 7.7	(14.5)
μ	350.9 $\pm$ 7.3	(14.5)
$M_{\tilde{q}_R}$	506.2 $\pm$ 11.7	(17.5)

2.2 An alternative “Bottom-up” approach

From physical masses to basic (Lagrangian) parameters

(at EWSB scale; then RG evolve up to high (GUT) scale

•Analytic, if possible

•Some tree-level inversions worked out in the past

(Moultaka, JLK ’98); extended by Kalinowski et al, P. Zerwas et al +many

(but mainly ILC context)

•Transparent, exhibit explicit correlations $\longrightarrow$ useful guide to more elaborated analysis

•New: incorporating as much as possible of the radiative corrections

•Delineate results valid in a general vs. constrained MSSM (i.e. with GUT scale universality relations)

-Limited scope yet: not related with MC, only mass input,..

Experimental assumptions and strategy

At LHC, can determine quite accurately some masses from "kinematical endpoints" analysis of (2-body) cascade decays

$$\tilde{g} \to \tilde{q}_L q \to \chi_2^0 q_f q \to \tilde{l}_R l q_f q \to \chi_1^0 l_f l q_f q$$

$\to$quite precise $m_{\tilde{g}}, m_{N_2}, m_{N_1}, m_{\tilde{q}_L}, m_{\tilde{l}_R}, m_{b_1}$ determination from "kinematical endpoints" analysis

(Allanach et al '01, Gjelsen, Miller, Osland '05)

+eventually M_h, + eventually other (independent) $\tilde{q}$ decay

NB $\tilde{q} = \tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}$ or $\tilde{b}_1, \tilde{b}_2$ (could be $\tilde{t}_1, \tilde{t}_2$ too but not for SPS1a)

No way to distinguish experimentally $\tilde{q}$ (similar B.R., no $\tilde{q}$ charge/flavor tagging at LHC)

Above sparticle mass set defines our "minimal" input different gradually optimistic assumptions on the amount of sparticle mass measurements at the LHC, from gluino cascade and other decays

scenarios (+th assumptions)	measured mass	expected LHC accuracy (GeV)	decay or process
(minimal):	$m_{\tilde{g}}$,	7.2	$\tilde{g}$ cascade decay
S_1(MSSM),	$m_{\tilde{\chi}_1^0}$,	3.7	" "
S_2(universality)	$m_{\tilde{\chi}_2^0}$.	3.6	" "
S_4,	$m_{\tilde{q}_L}$,	3.7	" "
S'_4 (universality)	$m_{\tilde{l}_R}$	6.0	" "
$S_3 = S_1$ +:	$m_{\tilde{\chi}_4^0}$	5.1	$\tilde{q}_L \to \tilde{\chi}_4^0 +$ .. cascade
S_5,	$m_{\tilde{b}_1}$,	7.5	$\tilde{g}$ cascade decay
S'_5 (universality)	$m_{\tilde{b}_2}$	7.9	" "
$S_6 = S_2 + S'_4 + S'_5$ +:	m_h	0.25 (exp)–2 (th)	$h \to \gamma\gamma$ (mainly)

(Accuracies from Weiglein et al ’04 report +Gjelsen, Miller, Osland ’05)

Bottom-up MSSM reconstruction at LHC

-Three naturally separated sectors (at tree level):

-gauginos/Higgsinos M_1, M_2, μ, $\tan\beta$

-squarks/sleptons μ, $\tan\beta$, $\tilde{m}_{q_L}$, $\tilde{m}_{q_R}$, $\tilde{m}_{e_L}$,..

-Higgs parameter sector μ, $\tan\beta$, M_{H_u}, M_{H_d}, M_A

NB μ, $\tan\beta$ common to all sectors! (and very crucial parameters)

For each sector there are simple analytical inversions (at tree-level): linear or quadratic eqs.

Strategy crucially depend on available input masses...(but also the case for standard top-down approach)

Proceed "step by step", in the 3 sectors, rather than global “all at once" fit

Concrete example: Gaugino/Higgsino sector

- Consider the Neutralino mass matrix:

$$M_N = \begin{pmatrix} M_1 & 0 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta \\ 0 & M_2 & m_Z c_W \cos\beta & -m_Z c_W \sin\beta \\ -m_Z s_W \cos\beta & m_Z c_W \cos\beta & 0 & -\mu \\ m_Z s_W \sin\beta & -m_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}$$

Trick: use the 4 invariants (under diagonalization) (Moultaka,JLK '98):

$$TrM_N, \quad \frac{(TrM_N)^2}{2} - \frac{Tr(M_N^2)}{2}, \quad (TrM_N)^3 + \cdots ; \quad DetM_N$$

give (rather simple) equations; flexible input/output choice (e.g. only 2 $\tilde{M}_{N_i}$ input!)

$$P_{ij}^2 + (\mu^2 + m_Z^2 - M_1 M_2 + (M_1 + M_2) S_{ij} - S_{ij}^2) P_{ij} + \mu m_Z^2 (c_W^2 M_1 + s_W^2 M_2) \sin 2\beta - \mu^2 M_1 M_2 = 0$$

$$(M_1 + M_2 - S_{ij}) P_{ij}^2 + (\mu^2 (M_1 + M_2) + m_Z^2 (c_W^2 M_1 + s_W^2 M_2 - \mu \sin 2\beta)) P_{ij}$$
$$+ \mu (m_Z^2 (c_W^2 M_1 + s_W^2 M_2) \sin 2\beta - \mu M_1 M_2) S_{ij} = 0$$

$S_{ij} \equiv \tilde{M}_{N_i} + \tilde{M}_{N_j}$, $P_{ij} \equiv \tilde{M}_{N_i} \tilde{M}_{N_j}$ where $i, j = 1, ..4$

Incorporating Radiative Corrections

•R.C gives highly non-linear dependence on parameters $\rightarrow$ "brute force" inversion untractable

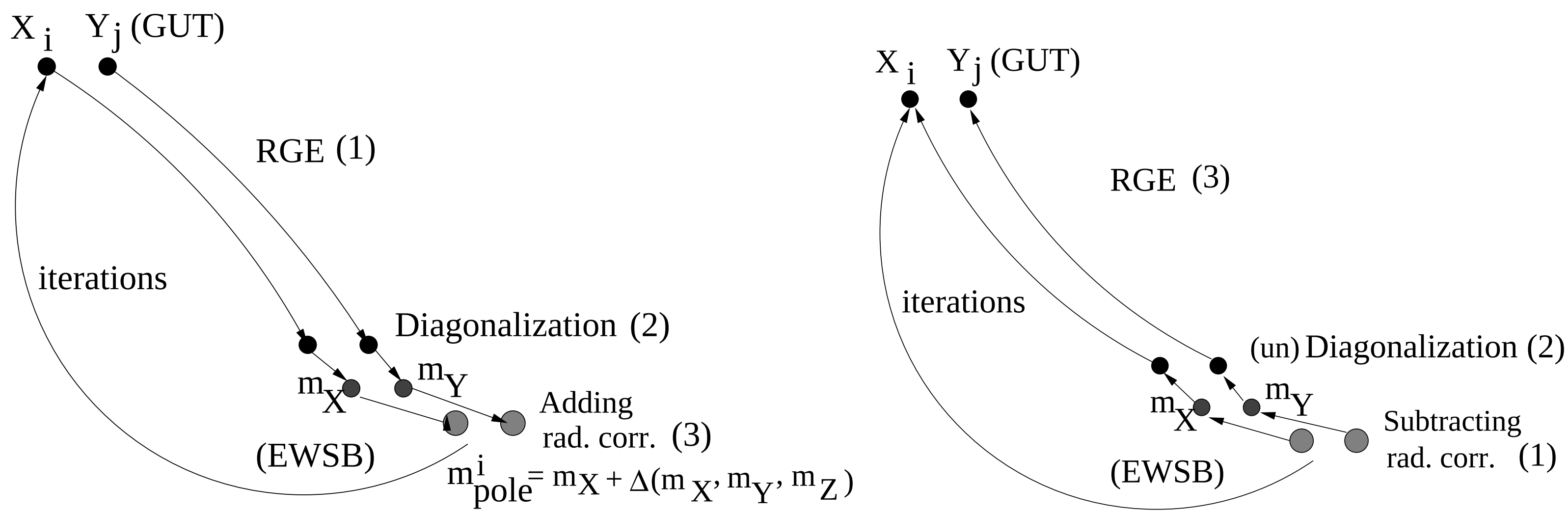
To very good approximation, keeps tree-level form: e.g $\mu \rightarrow \mu + \Delta\mu$, $M_1 \rightarrow M_1 + \Delta M_1$,.. (where $\Delta\mu$, ΔM_1, ΔM_2 depend on other sector: squarks, sleptons, ..)

$\rightarrow$ preserves analytic form of inversion

•Leading R.C. for $\tilde{g}$ involve $\tilde{q}$ of cascade (and vice-versa): $\rightarrow$ known!

•Once some parameters determined, eventually assume universality (SUGRA) relations **within** loops (should be good approximation in many cases)

General R.C. picture, RGE, etc



Top-down (left) versus bottom-up (right) mappings and their similarities.

1 (3): X_i, Y_j,.. running parameters: RGE GUT $\leftrightarrow$ EWSB

2: X_i and Y_j may mix: diag. $\rightarrow$ running masses m_X, m_Y

3 (1): R. C. linking running to pole masses m^i_{pole} added (subtracted) may depend on extra unknown parameters Z_k, m_k: $\rightarrow$ Specific assumptions +iterations needed.

“Fit" strategy

-Solve these analytical (tree-level) equations for various input/output choices;

-vary mass input within errors (uniform “flat prior” or Gaussian distributed)

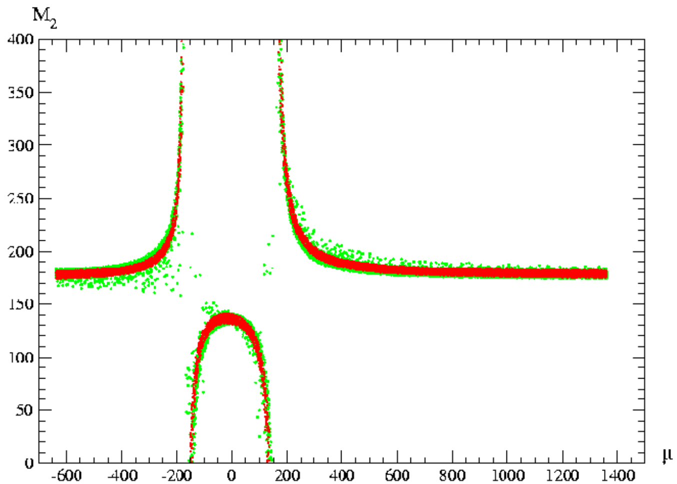
-determine allowed contours, or χ^2, for output basic MSSM parameters within different TH asumptions

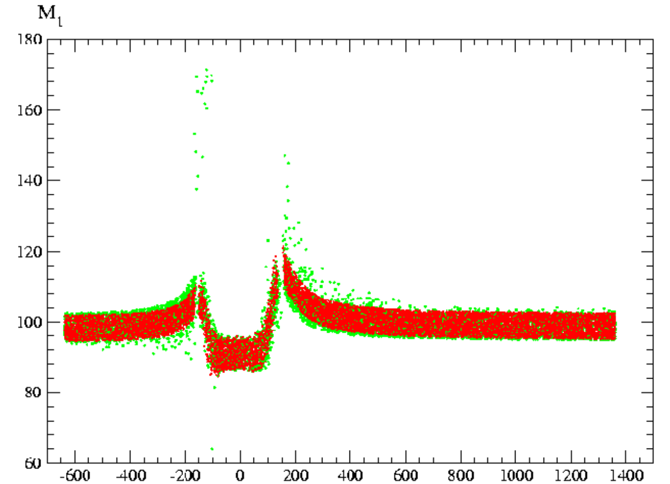
-A bit simple-minded w.r.t. sophisticated M.C.+ MINUIT χ^2 minimization..

but very easy +fast! +exhibit clearly correlations/ambiguities (multi solutions)

•We compare with MINUIT top-down fits with same input at different stages

Scenario S1: non-univ. M_1, M_2 from m_{N_1} m_{N_2}



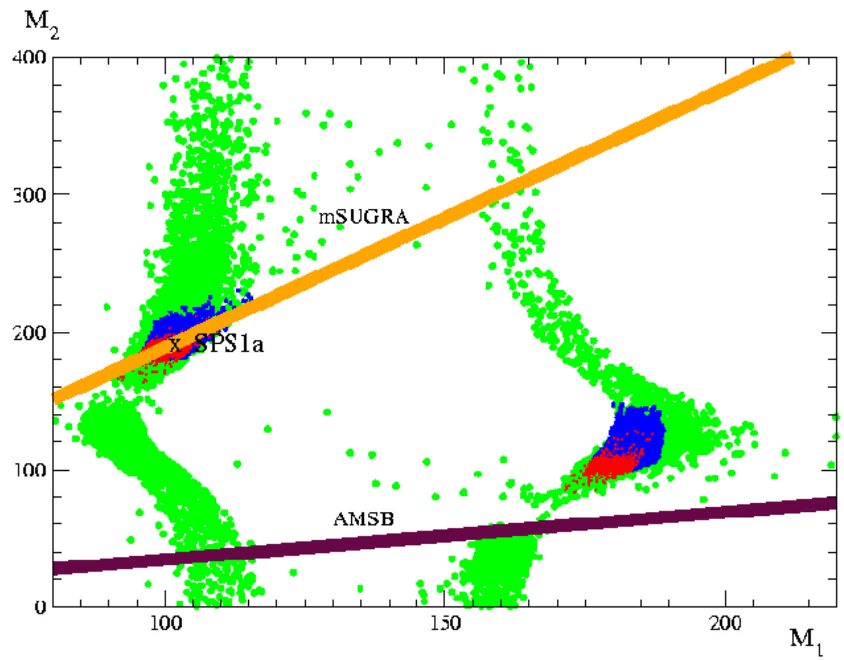


M_2 (left) and M_1 (right) ($M_1 < M_2$ case) as functions of μ. Spreading of points is due to $1 < \tan\beta < 50$ (green) plus m_{N_1}, m_{N_2} variation within SPS1a accuracy (red).

- Exhibit clear correlations
- Very simple solutions:

$$M_2 \sim m_{N_2} + m_W^2 \frac{(m_{N_2} + \mu \sin 2\beta)}{\mu^2 - m_{N_2}^2} + \mathcal{O}(m_W^4), \quad M_1 \sim m_{N_1} + ..$$

- Very good (few $\%$) $M_{1,2}$ determination except near "pole" μ regions (but eliminated if e.g. using $\mu(EWSB) \sim 300 - 400$ GeV, or 3 χ^0 input)

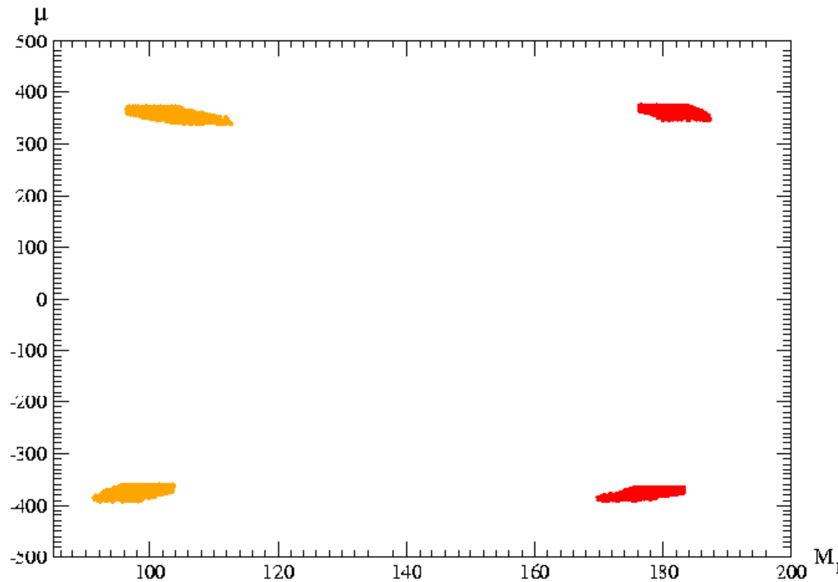


M_1, M_2 from m_{N_1}, m_{N_2} in Non-univ MSSM: 1) green: $0 \lesssim \mu \lesssim 1$ TeV, $0 \lesssim \tan\beta \lesssim 50$; 2) blue: $\Delta \tan\beta = 10$, $\Delta\mu = 100$ GeV; 3) red: +m_{N_4} input, $1 < \tan\beta < 50$, $\Delta\mu \sim 30$ GeV.

Notice two-fold ambiguity: $M_1 < M_2$ (mSUGRA-like) or $M_2 < M_1$! consistent with SFITTER results (next talk)

Still, may “favor" different scenarios (mSUGRA, AMSB) rather simply at low-energy

what if 3 neutralino mass input?



M_1, μ from m_{N_1}, m_{N_2} m_{N_4} in Non-univ MSSM.

Extra eq. is:

$\mu^2 = M_1 M_2 - m_Z^2 - (P_{124} + S_{124}(M_1 + M_2 - S_{124}))$

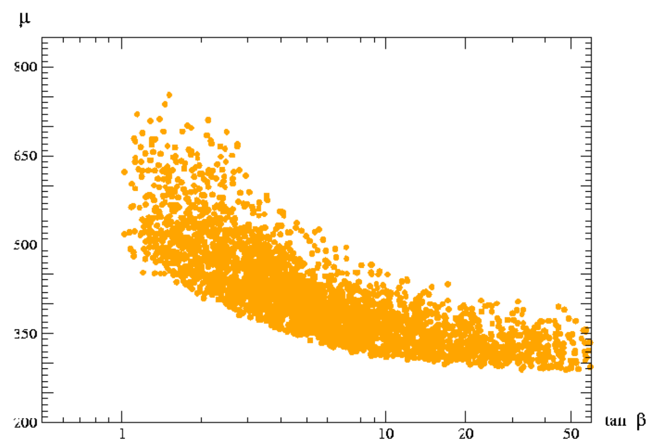
$S_{124} = m_{N_1} + m_{N_2} + m_{N_4}; \qquad P_{124} = m_{N_1} m_{N_2} + perm.$

But multi-fold ambiguities! ($M_1 \leftrightarrow M_2 \leftrightarrow |\mu|$)

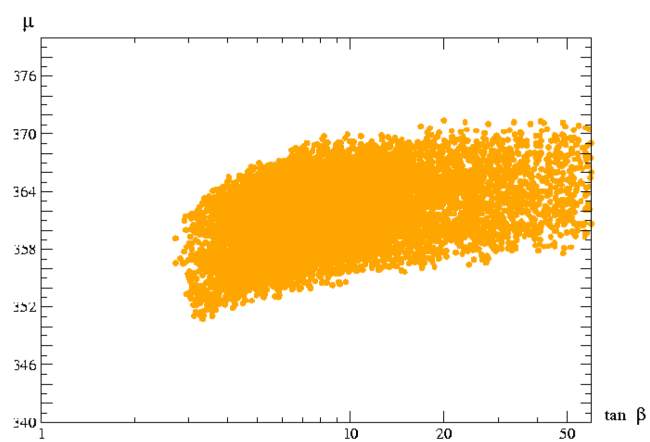
Clearly need extra TH/EXP info to discriminate (e.g. relic density, etc)

Scenario S2: μ, $\tan\beta$ determination for gaugino M_i universality

IF $M_1 = M_2 = M_3$ (GUT): same Eqs. $\rightarrow \mu, \tan\beta$ from M_1,M_2:



Assuming third M_{N_4} measurement:



Squark, slepton parameter (first two generations)

$$
\begin{aligned}
m^2_{\tilde{u}_1} &= m^2_{\tilde{u}_L} + (\frac{1}{2} - \frac{2}{3}s^2_W)m^2_Z \cos 2\beta \\
m^2_{\tilde{e}_2} &= m^2_{\tilde{e}_R} - s^2_W m^2_Z \cos 2\beta
\end{aligned}
$$

- linear combination to eliminate the $\tan\beta$ dependence ("sum rule"):

$$
s^2_W m^2_{\tilde{u}1} + (\frac{1}{2} - \frac{2}{3}s^2_W)m^2_{\tilde{e}2} = s^2_W m^2_{\tilde{u}_L} + (\frac{1}{2} - \frac{2}{3}s^2_W)m^2_{\tilde{e}_R}
$$

simple to work out RG evolution:

$$
\begin{aligned}
&\frac{d}{dt}\left[s^2_W m^2_{\tilde{u}_L} + (\tfrac{1}{2} - \tfrac{2}{3}s^2_W)m^2_{\tilde{e}_R}\right] \\
&= s^2_W \frac{d\tilde{m}^2_{u_L}}{dt} + (\tfrac{1}{2} - \tfrac{2}{3}s^2_W)\frac{dm^2_{\tilde{e}_R}}{dt} + \frac{ds^2_W}{dt}(\tilde{m}^2_{u_L} - \tfrac{2}{3}\tilde{m}^2_{e_R}) \qquad (1)
\end{aligned}
$$

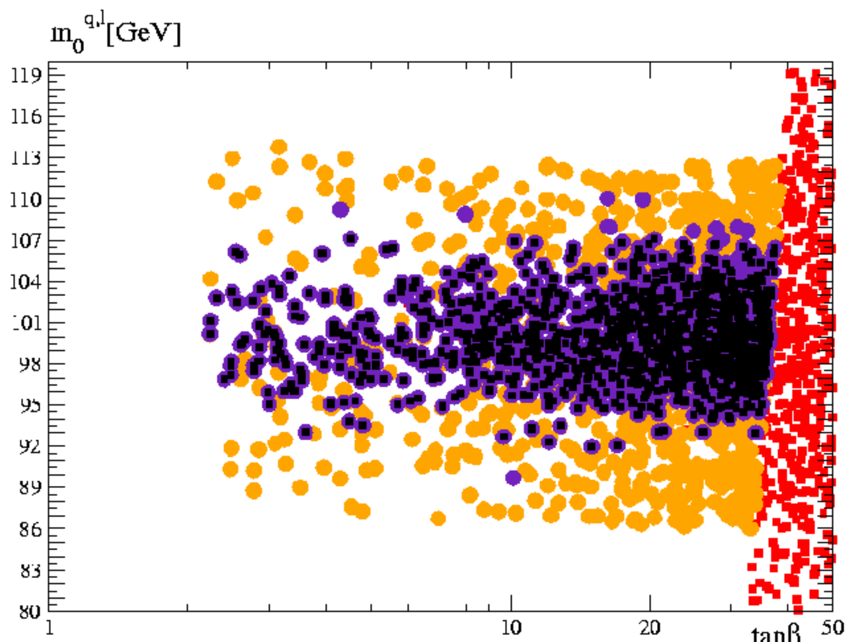
where $t \equiv \ln Q$ and

$$
\frac{ds^2_W}{dt} = (\frac{3}{5}g^2_1 + g^2_2)^{-1} \left(\frac{3}{5}c^2_W \frac{dg^2_1}{dt} - s^2_W \frac{dg^2_2}{dt}\right)
$$

NB this RGE (one-loop) only depend on gaugino M_i and gauge cplings!

$$84\,(86)\ \text{GeV} \lesssim m_0^{q,l} \lesssim 116\,(112)\ \text{GeV}$$

for linear (quad.) error combination, *independently of* $\tan\beta$.



Constraints (from Gaussian scan) on $m_0^{q,l}, \tan\beta$: orange: 2-σ from combined $m_{\tilde{e}_R}, m_{\tilde{u}_1}$; indigo: 2-$\sigma$ from separating $m_{\tilde{u}_1}$ relation; black: 1-σ from $m_{\tilde{u}_1}$. Red: excluded by tachyon $\tilde{\tau}_1$.

Sbottom sector 1: scalar non-universality

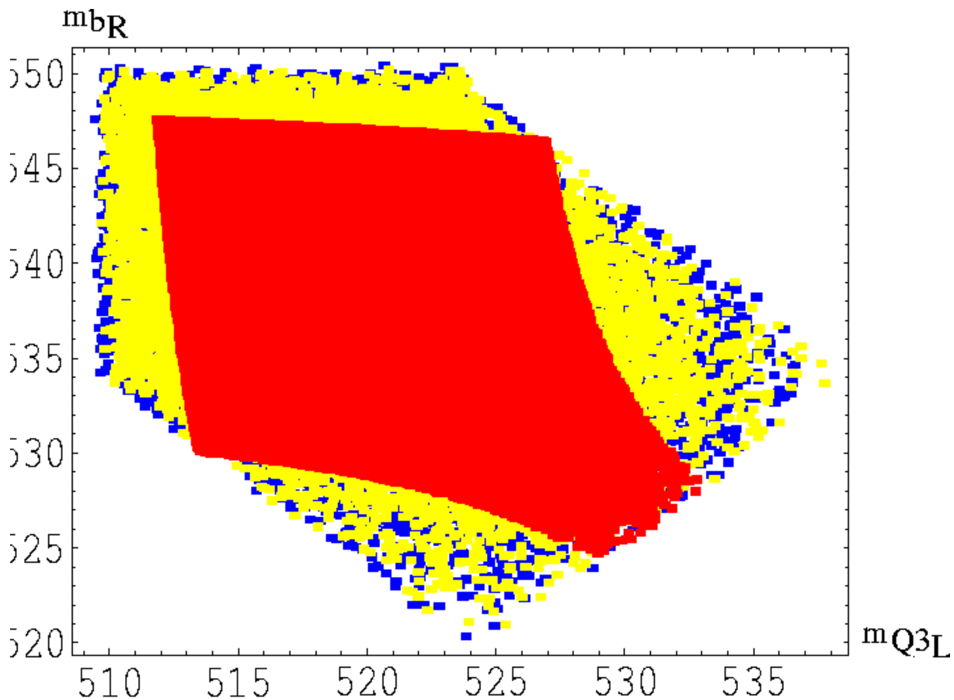
$\tilde{b}_1, \tilde{b}_2$ involved in $\tilde{g}$ decay (though more difficult for $\tilde{b}_2$)

Simple inverted relations to determine m_{Q3_L}, m_{b_R}

$$m_{Q3_L(b_R)} = \left[\frac{S + (-)D}{2}\right]^{1/2}$$

$$\begin{aligned} S &= m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2} + \frac{m_Z^2}{2}\cos 2\beta - 2m_b^2 \\ D &= -Y + \left[(m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1} - 2m_b X_b)(m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1} + 2m_b X_b)\right]^{1/2} \end{aligned}$$

$Y = (-\frac{1}{2} + \frac{2}{3}s_W^2)m_Z^2 \cos 2\beta$, $\qquad X_b = A_b - \mu \tan\beta$



Red: $\tan\beta \sim 9.73$, $\mu \sim 357$ GeV ($A_b = 0$). Yellow: $3 \lesssim \tan\beta \lesssim 35$, $\Delta\mu \sim 10$ GeV, $-100GeV < A_b < 100GeV$; blue: $3 \lesssim \tan\beta \lesssim 35$, $\Delta\mu \sim 200$ GeV, $-1TeV < A_b < 1TeV$.

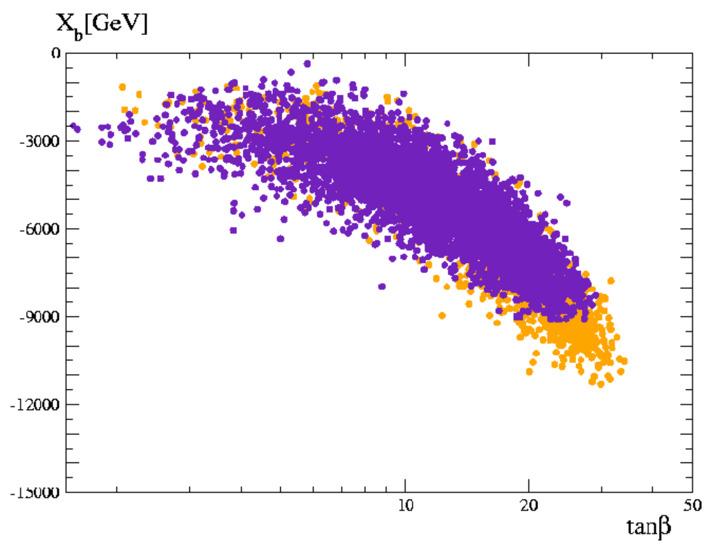
Sbottom sector 2: scalar universality

Relate m_{Q3_L}, m_{b_R} to $m_0^{q,l}$ constraints:

$$m_{Q3_L}(Q_{EWSB}) \sim 498 \pm 1.2 \pm 7\text{GeV}, \quad m_{b_R}(Q_{EWSB}) \sim 521 \pm 1.8 \pm 6\text{GeV}$$

(NB dominant error from RGE via M_3 uncertainty)

$$2\, m_b\, X_b = -\left[(m^2_{\tilde{b}_2} - m^2_{\tilde{b}_1})^2 - (m^2_{\tilde{Q}3_L} - m^2_{\tilde{b}_R} + Y)^2 \right]^{1/2}$$



Constraints on $\tan\beta, X_b = A_b - \mu \tan\beta$ from Gaussian scan: indigo: one-σ (68% C.L.); orange: two-σ (95% C.L.).

Determination in Higgs sector parameters

In general MSSM: running m_A value:

$$\bar{m}_A^2(Q) = m_{H_d}^2 + m_{H_u}^2 + 2\mu^2 = \frac{\bar{m}_h^2(m_Z^2 - \bar{m}_h^2)}{m_Z^2 \cos^2 2\beta - \bar{m}_h^2} + \text{Rad. Corr.}$$

$$m_h^2 = m_h^{2,tree} + \frac{3 g m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \frac{X_t^2}{M_S^2} - \frac{X_t^4}{12 M_S^4}\right]$$

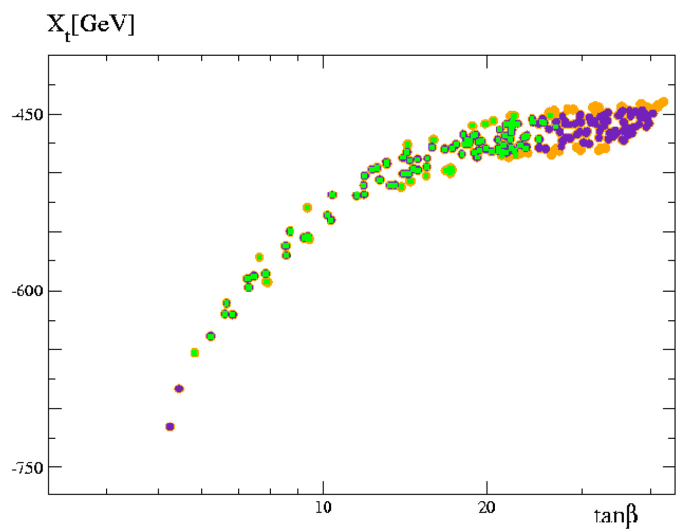
where $X_t = A_t - \mu \cot\beta$, $\quad M_S^2 \simeq m_{\tilde{t}_1} m_{\tilde{t}_2}$

NB we use more elaborated 1(2)-loop m_h R.C. (Heinemeyer, Hollik, Weiglein '99)

-general MSSM: poorly constrained IF nothing know on stop sector, or m_A... (not expected for SPS1a)

-IF universality: $m_0^{q,l} \equiv m_{H_u}(Q_{GUT}) = m_{H_d}(Q_{GUT}) \to m_A(Q)$ determined

$\to X_t, \tan\beta$ constraints



Constraints on $\tan\beta$ and $X_t \equiv A_t - \mu\tan\beta$: indigo: 1-$\sigma$; orange: 2-$\sigma$; green: 1-$\sigma$ if adding sbottom mass measurements.

Renormalization Group “bottom-up" evolution

- Once parameters determined at Q_{EWSB} scale, evolve them to GUT scale

RGE evidently invertible, but to evolve MSSM parameters from EWSB scale UP to GUT scale, while matching low-energy (gauge+yukawa) data is not staightforward.

+ Care to be taken: $Tr[Ym^2] \neq 0$ may increase error propagations

NB public bottom-up RGE option now installed in (new) SuSpect ver $\geq$ 2.40

Bottom-up RG evolution with error propagations

par.	input(GeV)	GUT output	$\Delta M_3 = \mp 1\%$	$\Delta m_{H_u} = \mp 1\%$	$\Delta m_{Q3_L} = \mp 1\%$
M_1	101.5	250.004	negl.	negl.	negl.
M_2	191.6	249.998	" "	" "	" "
M_3	586.6	249.999	± 2.2	" "	" "
$m^2_{H_d}$	$(179.9)^2$	$(100.004)^2$	$(100.6)^2$– $(99.4)^2$	$(100.7)^2$– $(99.2)^2$	$(101.2)^2$– $(98.7)^2$
$m^2_{H_u}$	$-(358.1)^2$	$(100.017)^2$	$(132.6)^2$– $(48.4)^2$	$(64.9)^2$– $(124.4)^2$	$(63.7)^2$– $(126.4)^2$
(μ)	356.9	353			
m_{e_R}	136	99.998	100–99.9	98.4–101.6	96.8–103.1
m_{Q1_L}	545.8	100.001	121–72	99.7–100.3	99.1–100.8
m_{Q3_L}	497	100.005	131–52	94.6–104.6	55.2–130.4
m_{u_R}	527.8	99.997	121–72	101–99	101.8–98.1
m_{t_R}	421.5	100.006	140–14	90.6–107.5	81.9–115.3
m_{b_R}	522.4	99.997	122–72	99.4–100.6	98.5–101.5
$-A_t$	494.5	100.009	$111 - -89$	" "	" "
$-A_b$	795.2	100.002	$106 - -94$	" "	" "

Combining all determination from bottom-up approach

Assumptions	Parameter	Constraint (GeV)	SPS1a
gen. MSSM	$M_1(Q_{EWSB})^\star$	∼95–115	101.5
" "	$M_2(Q_{EWSB})^\star$	∼175–225	191.6
" "	$M_3(Q_{EWSB})$	∼580–595	586.6
" "	$(\frac{3}{8}m^2_{u_L} + \frac{m^2_{e_R}}{4})^{1/2}(Q_{GUT})$	∼68–89	~ 79
" "	$m_{Q3_L}(Q_{EWSB})^\star$	∼488–518	497
" "	$m_{b_R}(Q_{EWSB})^\star$	∼510–540	522
" "	$\mu(Q_{EWSB})^\star$	∼280–750	357
+ m_{N_4}	$\mu(Q_{EWSB})$	∼350–372	357
$\tilde{q}, \tilde{l}$-universality	$m_0^{q,l}(Q_{GUT})$	∼90–112	100
M_i-universality	$M_i(Q_{GUT})$	∼ 245–255	250
$\tilde{b}_1, \tilde{b}_2$ +universality	$\tan\beta(Q_{EWSB})$	∼3–28	9.74
mSUGRA	m_0	∼90-112	100
	$m_{1/2}$	∼245–255	250
	$-A_0$	∼ -100-350	100
	$\tan\beta(m_Z)$	∼ 5.5–28	10

Comparison with standard top-down χ^2 fits

Combined constraints on mSUGRA basic parameters from top-down fit with minuit of $\tilde{g}$ decay + M_h measurements.

Assumptions	Parameter	Constraint (GeV)	SPS1a value
mSUGRA	m_0	99.96 $\pm$ 11.2	100
2-loop RGE + $\tilde{q}$ R.C. + 2-l m_h		(99.95 $\pm$ 11.7)	
(1-loop RGE+ no $\tilde{q}$ R.C. +m_h approx.)	$m_{1/2}$	250.0 $\pm$ 3.7	250
		(249.5 $\pm$ 4.7)	
	A_0	-104.2 ± 379	-100
		(-100.6 ± 136)	
	$\tan\beta(m_Z)$	9.9 $^{+9.4}_{-4.7}$	10
		(9.96 ± 4.11)	

Conclusion

-Quite simple-minded approach but clear handle on possible obstacles in bottom-up approach

May suggest new strategies/discriminating variables, not automatically foreseen by global fit (e.g. combination observables in squark/slepton sector, etc)

-Compare reasonably well with more standard top-down fitting approaches

could be linked with other tools (SFITTER, ...) as "guidelines"

-likely to help solving part of the dicrete ambiguities (LHC inverse pb) (needs further dedicated studies)

May help also to distinghish from other BSM (specific SUSY spectrum properties)