Perturbative QCD Calculation Of Heavy Quarkonium-gluon/hadron Cross Sections

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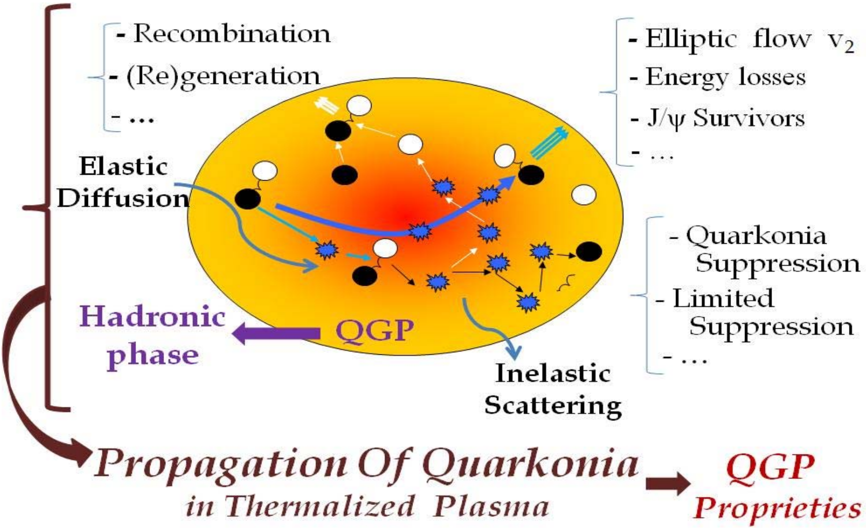
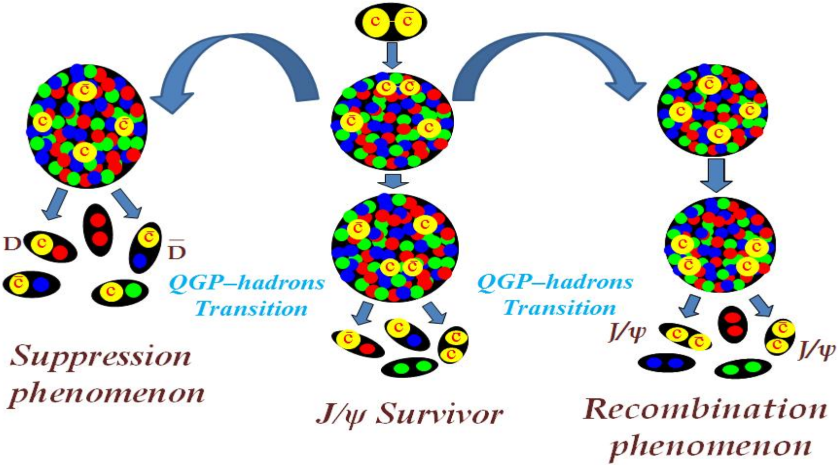
Rencontre " Théorie LHC France", 2009

Outline

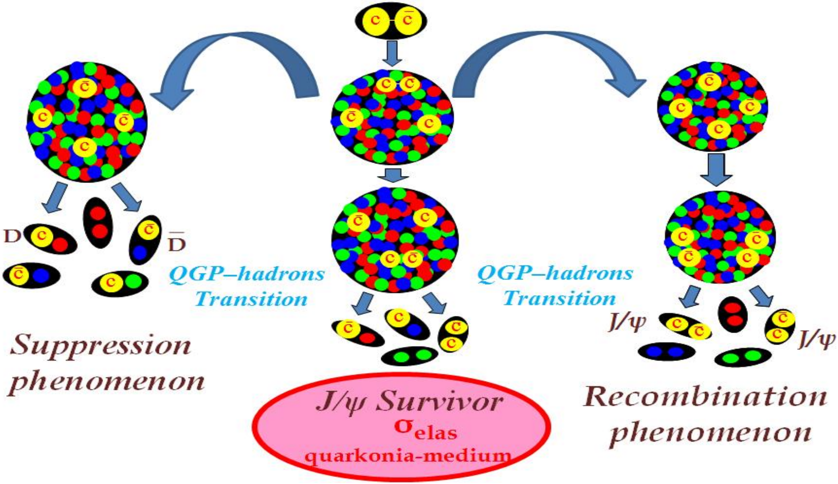
Global Project



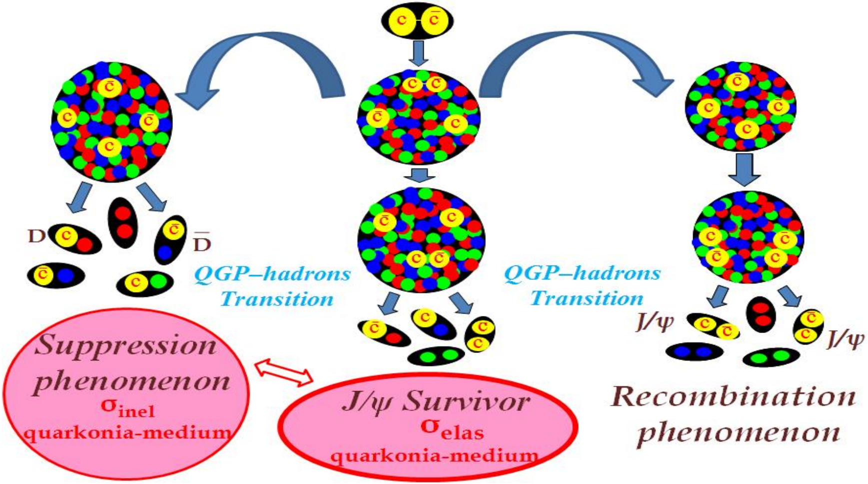
Propagation of Quarkonia



Propagation of Quarkonia



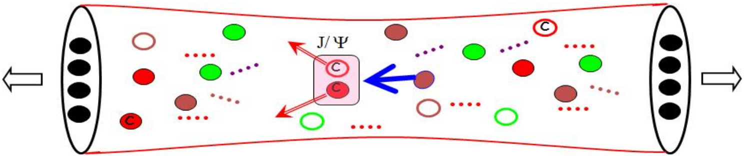
Propagation of Quarkonia



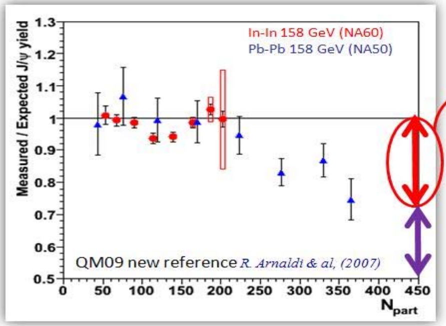
J/ψ Physics $\Leftrightarrow$ QGP physics

⋆ Relevant questions and Comments on J/ψ physics

1. What are the effects of Dynamical quarks ?
2. What is the survival probability of J/ψ in QGP?
   Asakawa and Hatsuda: J/ψ will survive in QGP up to 1.6 Tc ?
3. Dissociation effects until Tc ?
4. Suppression in hadronic phase ...
   J/ψ suppression can occur through dissociation in later stages of HIC.
5. ...



Motivation for σ_{elas} calculation

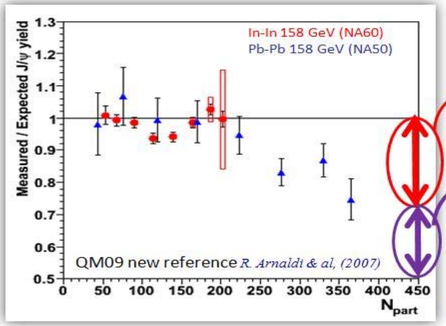


- Suppressed (studied with σ_{inel})

+ *J/ψ Suppression studied for 20 years,*
... But No consistent results ....

+ *Recent Results of RHIC* ... No significant additional suppression expected has been observed for J/ψ by increasing energy...

Motivation for σ_{elas} calculation



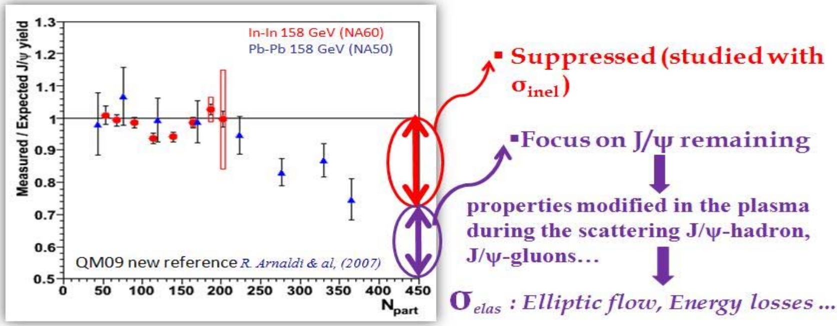
- **Suppressed (studied with σ_{inel})**
- **Focus on J/ψ remaining**

properties modified in the plasma during the scattering J/ψ-hadron, J/ψ-gluons…

σ_{elas} : Elliptic flow, Energy losses …

+ *J/ψ Suppression studied for 20 years,*
  … But No consistent results ….
+ *Recent Results of RHIC* … No significant additional suppression expected has been observed for J/ψ by increasing energy…

Motivation for σ_{elas} calculation



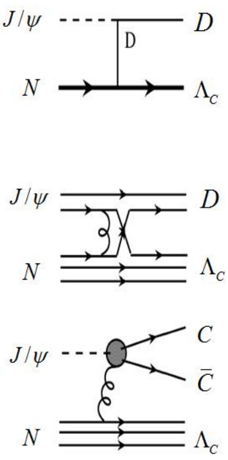
+ *J/ψ Suppression studied for 20 years,*
  … But No consistent results ….
+ *Recent Results of RHIC* … No significant additional suppression expected has been observed for J/ψ by increasing energy…
+ *Large fluctuations over the value of* σ_{elas}

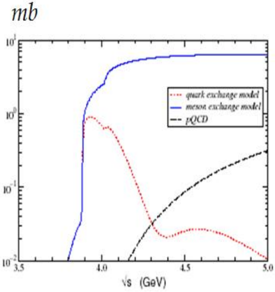
Formalisms for σ_{inel} calculation

1. 1. Effective model
2. 2. Quark exchange model

   [Povh and Hüfner, Zi-wei Lin 02. A. Sibirtsev and al 01 ...]
3. 3. LO pert QCD

   [Bhanot and Peskin 79]



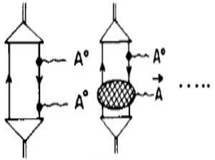


S.Lee (05), Voloshin, R. Rapp (03).

Formalisms for σ_{elas} calculation

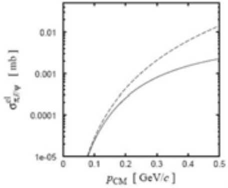
1] ◇ Historical Calculation of Q-h σ_{elas}

- 1) - Bhanot and Peskin formalism (79)
  - a) From OPE (operator product expansion)
  - b) Binding energy $= \epsilon_0 \gg$ LQCD



Interaction entre champ-dipole

- 2)- Kharzeev and Fujii (99), Povh and Hüfner
  - a) Short-distance QCD calculation
  - b) Optic theorem ...



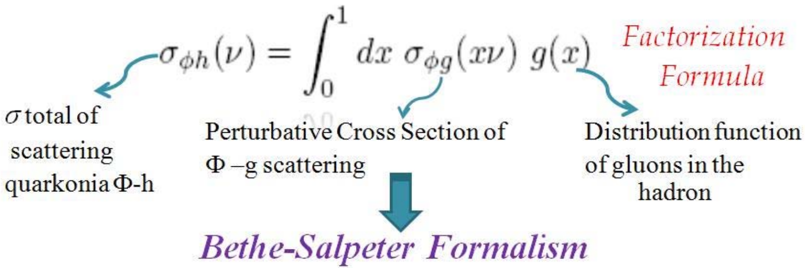
Kharzeev and Fujii (99)

Formalisms for σ_{elas} calculation

2] ◇ Rederivation of Peskin formula using Bethe-Salpeter

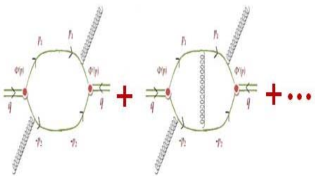
- 3) - Other equivalent method in pQCD 1^{st} order based on :

Factorization Theorem + Bethe-Salpeter Amplitudes

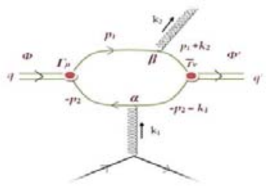


σ_{inel} **calculation :** Y. Oh, S. Kim, S. H. Lee, (2002)

Elastic Process

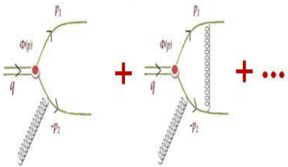


Compton diffusion . . . Most important for us

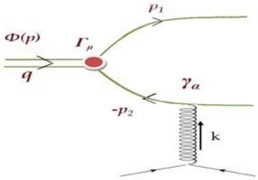


Möller scattering (el.)

Inelastic Process

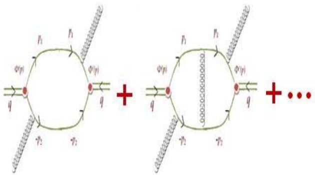


Gluon dissociation . . . well Known



. . . Not well Unknown

Elastic Process



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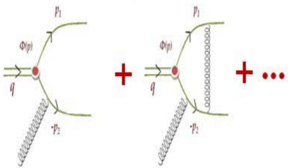

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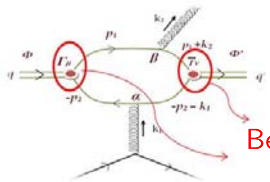
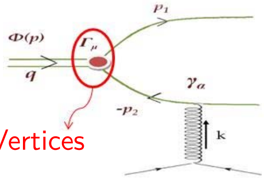


Bethe-Salpeter Vertices



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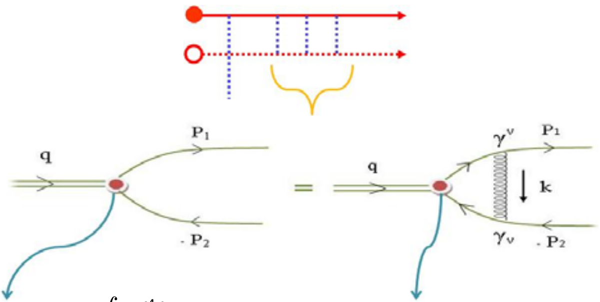
Bethe-Salpeter Equation

- Resume Bound state by Bethe-Salpeter Equation

$$\mathcal{M} = V + VGV + VGVG + \cdots + (VG)V^n + \cdots = \frac{V}{1 - VG};$$ M: amplitude, V: Kernel, G: propagator



$$\Gamma_\mu(p_1, -p_2) = iC_{color} \int \frac{d^4k}{(2\pi)^4} g^2 V(k)\gamma^\nu \triangle(p_1 + k)\Gamma_\mu(p_1 + k, -p_2 + k)\triangle(-p_2 + k)\gamma_\nu$$

Y. Oh, S. Kim, S. H. Lee, (2002)

Bethe-Salpeter Vertices

1. **Case of ϕ in the rest frame:** 1^{st} order on relative velocity v

$$\Gamma_\mu\left(\frac{q}{2}+p,\frac{-q}{2}+p\right) = -\left(\epsilon - \frac{\vec{p}_1^{\,2}}{m}\right)\left(\frac{M_\phi}{N_c}\right)^{\frac{1}{2}} \psi(\vec{p})\, \frac{I+\gamma_0+\not{v}}{2}\, \gamma_\mu\, \frac{I-\gamma_0+\not{v}}{2}$$

Normalized wave function for the bound state.

Projection on positive and negative energy parties

Vector vertice

Bethe-Salpeter Vertices

1. **Case of ϕ in the rest frame:** 1^{st} order on relative velocity v

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Normalized wave function for the bound state.

Projection on positive and negative energy parties

Vector vertice

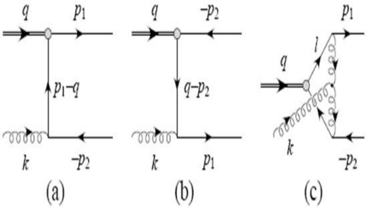
2. **Case of ϕ in NR mvt, with $v_\phi \neq 0$:** $m^2 g^2 \ll |q|^2 \ll M_\phi^2$

$$\Gamma_\mu\left(\frac{q}{2}+p,\frac{-q}{2}+p\right) = -\left(\epsilon-\frac{\vec{p}_1{}^2}{m}\right)\left(\frac{M_\phi}{N_c}\right)^{\frac{1}{2}}\psi(\vec{p})\,\frac{I+\gamma_0-v\not{\phi}-\not{v}}{2}\,\gamma_\mu\,\frac{I-\gamma_0+v\not{\phi}-\not{v}}{2}$$

Very Important consequences

with : $\vec{v}_\phi$(CM velocity) : any order, $\vec{v}$ (relative velocity) : 1^{st} order, $\vec{v}_\phi\vec{v}$: 0 order.

J/ψ-g Gluon Dissociation Process



$\Updownarrow$

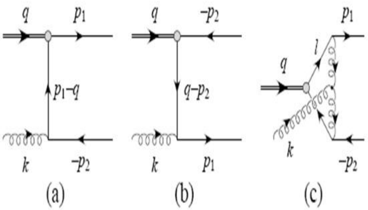
$$\overline{|\mathcal{M}|}^2 = \frac{4g^2 m^2 M_\phi k_0^2}{3N_c} \, |\vec{\nabla}\psi(\vec{p})|^2$$

$\Downarrow$

$$\sigma_{\phi g} = \frac{128 g^2}{3N_c} \, a_0^2 \, \frac{(\lambda/\epsilon_0 - 1)^{3/2}}{(\lambda/\epsilon_0)^5}$$

$$with : \lambda = \frac{q.k}{M_\phi} = \frac{s - M_\phi^2}{2M_\phi}$$

J/ψ-g Gluon Dissociation Process



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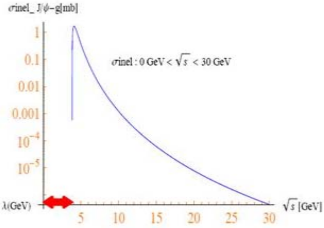
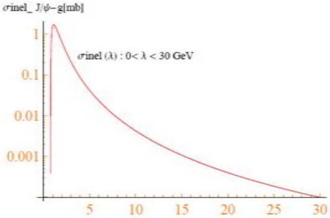
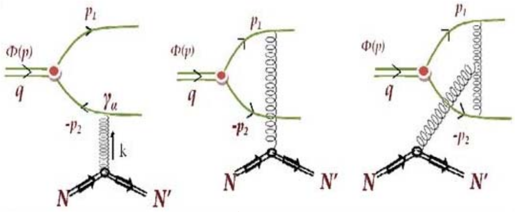




m = 1.95 GeV, ϵ_0 = 0.78 GeV.

Y. Oh, S. Kim, S. H. Lee, (2002)

J/ψ-N Hadron Dissociation Process



$\Updownarrow$

$$\sigma_{\phi N}(\nu) = \int_0^1 dx\ \sigma_{\phi g}(x\nu)\ g(x), \quad with: \ \nu = \frac{p.q}{M_\phi}$$

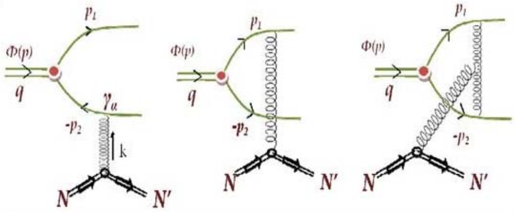
$\sigma_{\phi g}(x\nu)$: J/ψ-g Gluon dissociation process

$$g(x) = 0.5(\eta + 1)\frac{(1-x)^\eta}{x}, \ \ \eta = 5\ (BP)$$

Gluon distribution function in the proton

♣ Factorization Theorem

J/ψ-N Hadron Dissociation Process



⇕

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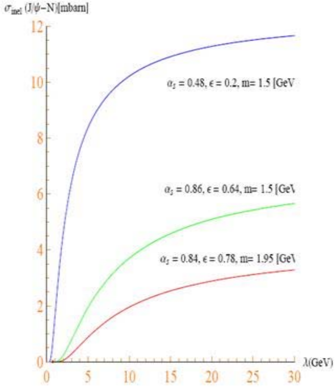
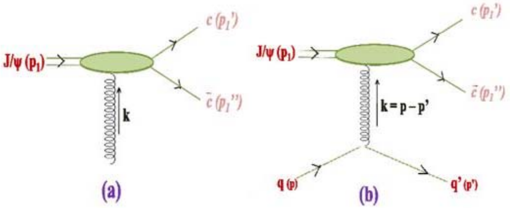


Dependance σ_{inel} vs ϵ_0, m, g.

♣ **Factorization Theorem**

Y. Oh, S. Kim, S. H. Lee, (2002)

J/ψ-q Quark Dissociation Process



$\Updownarrow$

$$d\sigma^{(b)}_{J/\psi q} = d\sigma^{(a)}_{J/\psi g}\ n(\omega)\ d\omega, \text{ with : } n(\omega) = \frac{2\alpha}{\pi} \ln\left(\frac{\epsilon_0 M_\phi c^2}{m_q \hbar\omega}\right) \frac{d\omega}{\omega}$$

$n(\omega)$: Spectral distribution of equivalent gluons.

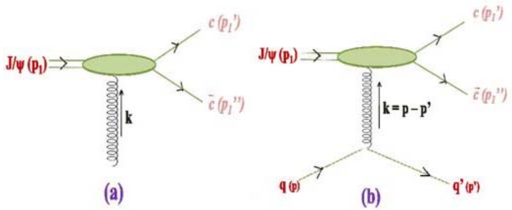
ω : energy of gluon in the rest frame of J/ψ.

$\Downarrow$

$$\sigma^{(b)}_{J/\psi q} = \frac{2731\alpha_s a_0}{M-\epsilon_0} \int_{\epsilon_0}^{\epsilon} \frac{(\omega/\epsilon_0 - 1)^{3/2}}{(\omega/\epsilon_0)^5} \ln\left(\frac{\epsilon M}{m_q \omega}\right) \frac{d\omega}{\omega}$$

♣ **Weizsäcker-Williams** method of g equivalent

J/ψ-q Quark Dissociation Process



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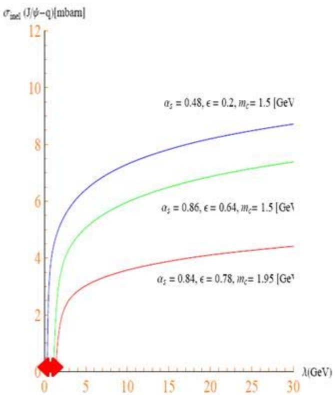


$$\Longrightarrow \sigma^{(b)}_{J/\psi - q} \propto \alpha^2 a_0^2.$$

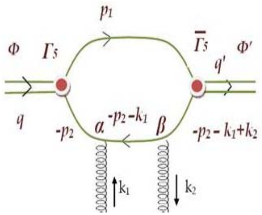
♣ **Weizsäcker-Williams** method of g equivalent

$J/\psi(\eta_c)$-g Compton Diffusion Process

♠ **Case of pseudo-scalar η_c and vector J/ψ vertice**

♡ 2 gluons exchanged "LO"

$\Rightarrow$ 6 diagrams (bb||, bbx, tt||, ttx, tb, bt)



$J/\psi(\eta_c)$-g Compton Diffusion Process

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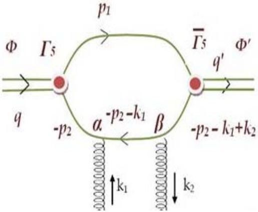
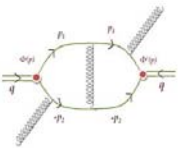
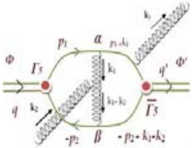


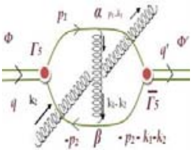
♡ 3, . . . gluons exchanged "SNLO"



$\Rightarrow$ 4 diagrams (btx, bb||, tt||, tbx)



$\Rightarrow$ 7 diagrams (gluon emitted in each fermionic and gluonic line).



$\Rightarrow$ 1 diagram

η_c-g Compton Diffusion Process

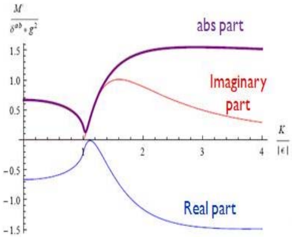
♠ Case of pseudo-scalar vertice η_c

♡ 2 gluons exchanged Diagrams "LO"

♦ Soft gluons ($k \approx mg^4$) • coulombic state

$$|\mathcal{M}(k, k' \approx mg^4)| \approx 2\alpha g^2 \frac{\delta^{ab}}{2N_c} \varepsilon_{\lambda 1}(\mathbf{k\ 1}).\varepsilon_{\lambda 2}(\mathbf{k\ 2})$$

√ Opening of the imaginary part for $K > |\epsilon|$
√ Opening of the inelastic channel for $K = |\epsilon|$



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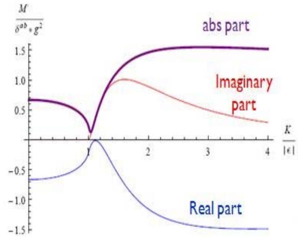
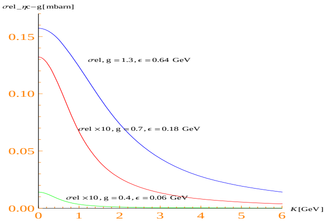


♦ Hard gluons ($k \approx mg^2$) • coulombic state

$$|\mathcal{M}(k, k' \approx mg^4)| \approx \frac{4g^2\delta_{ij}\delta^{ab}}{N_c} \frac{1}{\left(1 + \left(\frac{a_0|k-k'|}{4}\right)^2\right)^2}$$

⇓ Form Factor

$$\sigma_{elas} \approx \frac{g^4}{\pi m^2 N_c^2} \times \frac{1 + \frac{K^2}{8m|\epsilon|} + \frac{1}{2}\left(\frac{K^2}{8m|\epsilon|}\right)^2}{\left(1 + \frac{K^2}{8m|\epsilon|}\right)^3}$$



J/ψ-g Compton Diffusion Process

♠ Case of vector vertice J/ψ

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♦ Soft gluons ($k \approx mg^4$) • coulombic state

$$|\mathcal{M}(k, k' \approx mg^4)| \approx 4\alpha \frac{g^2}{N_c} \delta^{ab} g^{km} g^{\mu\nu} \; \varepsilon_{\lambda 1}(\mathbf{k}\ \mathbf{1}).\varepsilon_{\lambda 2}(\mathbf{k}\ \mathbf{2})$$

√ Opening of the imaginary part for $K > |\epsilon|$
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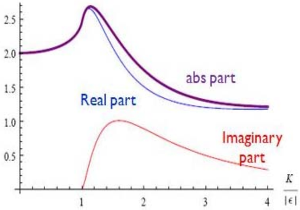

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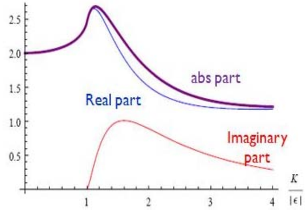
$$|\mathcal{M}(k, k' \approx mg^4)| \approx 4\alpha \frac{g^2}{N_c} \delta^{ab} g^{km} g^{\mu\nu} \;\; \varepsilon_{\lambda 1}(\mathbf{k}\ \mathbf{1}).\varepsilon_{\lambda 2}(\mathbf{k}\ \mathbf{2})$$

√ Opening of the imaginary part for $K > |\epsilon|$
√ Opening of the inelastic channel for $K = |\epsilon|$



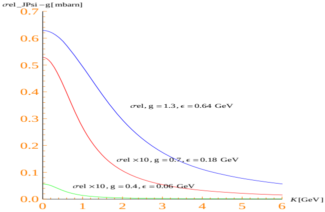
♦ Hard gluons ($k \approx mg^2$) • coulombic state

$$|\mathcal{M}(k, k' \approx mg^4)| \approx -4 \frac{g^2}{N_c} \delta^{ab} g^{km} g^{\mu\nu} \frac{1}{\left(1 + \left(\frac{a_0 |k - k'|}{4}\right)^2\right)^2}$$

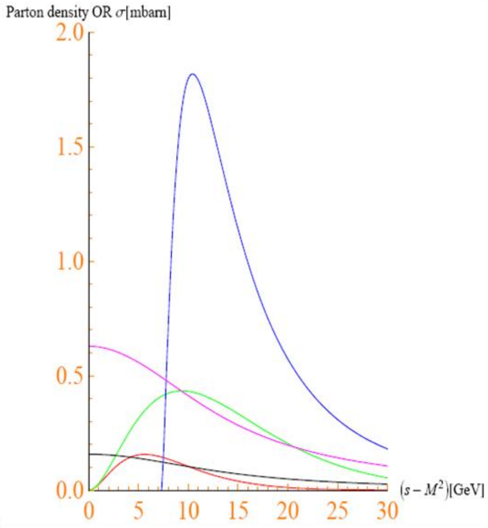
Form Factor

⇓

$$\sigma_{elas} \approx \frac{12 g^4}{\pi m^2 N_c^2} \times \frac{1 + \frac{K^2}{8m|\epsilon|} + \frac{1}{2}\left(\frac{K^2}{8m|\epsilon|}\right)^2}{\left(1 + \frac{K^2}{8m|\epsilon|}\right)^3}$$



$J/\psi - g$: Gluon Dissociation vs Compton Diffusion . . . "LO" Diagrams



$$R = \int d^3k \; n_{mb}(k) \; \sigma_{inel/elas}(s(k))$$

$$\hookrightarrow \; R = \int_{M^2}^{+\infty} ds \; \tilde{n}_{mb}(s) \; \sigma_{inel/elas}(s)$$

σ Inelastic $J/\psi - g$

Parton density at LHC energy

Parton density at RHIC energy

σ Elastic $J/\psi - g$

σ Elastic $\eta_c - g$

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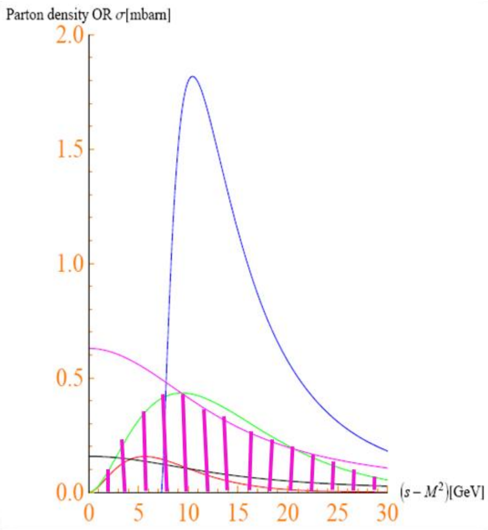

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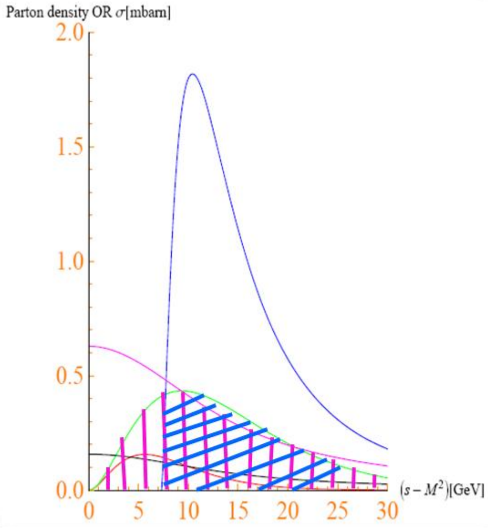

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Energy loss and Transport Coefficient

♠ Energy loss given by Bjorken

$$\frac{dE}{dt} = \int d^3q \; n_{mb}(\vec{q})[flux] \; \int dt \frac{\sigma_{elas}}{dt} \underbrace{(E' - E)}_{\frac{t}{2M_\phi}\left(\frac{E_{cell}}{M_\phi} + \underbrace{\frac{\vec{p}_{cell}\cdot\vec{q}}{\| \vec{q} \|^2}}_{Energy\ loss\ term}\right)}, \quad with : [flux] := \frac{\sqrt{(p.q)^2 - M^2 m^2}}{Ee}$$

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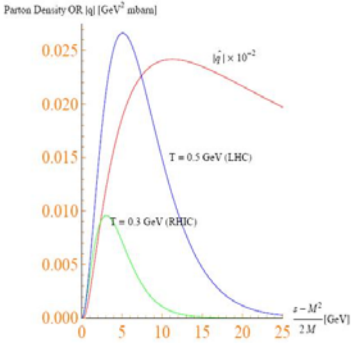
♠ Transport Coefficient

$$\hat{q}(s) = \int \frac{d\sigma_{elas}}{dt} t \; dt$$

$$\Updownarrow$$

$$\hat{q}(s) \propto -\frac{32\pi\alpha^2}{3}\left(\frac{\sqrt{s} - M}{\sqrt{s} + M}\right)^2, \; \mu < \frac{1}{a_0} \, and \, \frac{1}{a_0} > \sqrt{s} - M$$

$$\hat{q}(s) \propto -\frac{64\pi\alpha^2}{3a_0^2(\sqrt{s} + M)^2}, \; \mu < \frac{1}{a_0} < \sqrt{s} - M$$



Summary and Outlook

♠ Summary

1. Global project : revisiting quarkonium cross sections
2. Develop a calculating method of elastic cross section of bound state in the plasma.
3. Interests of the study of elastic cross section . . .

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♠ Outlook

1. Implement this σ elastic in hydrodynamic model to study the propagation of quarkonia in the plasma.
2. Extract information from studying quarkonia during their "travel" in the plasma : elliptic flow, energy losses . . .
3. Finish our calculation for "SNLO" . . .

Summary and Outlook