$t\bar{t}$ Hadro-Production at NNLO in QCD

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Plan of the Talk

- General introduction
- $t\bar{t}$ inclusive cross section: status
- NNLO corrections:
  - Method
  - Results

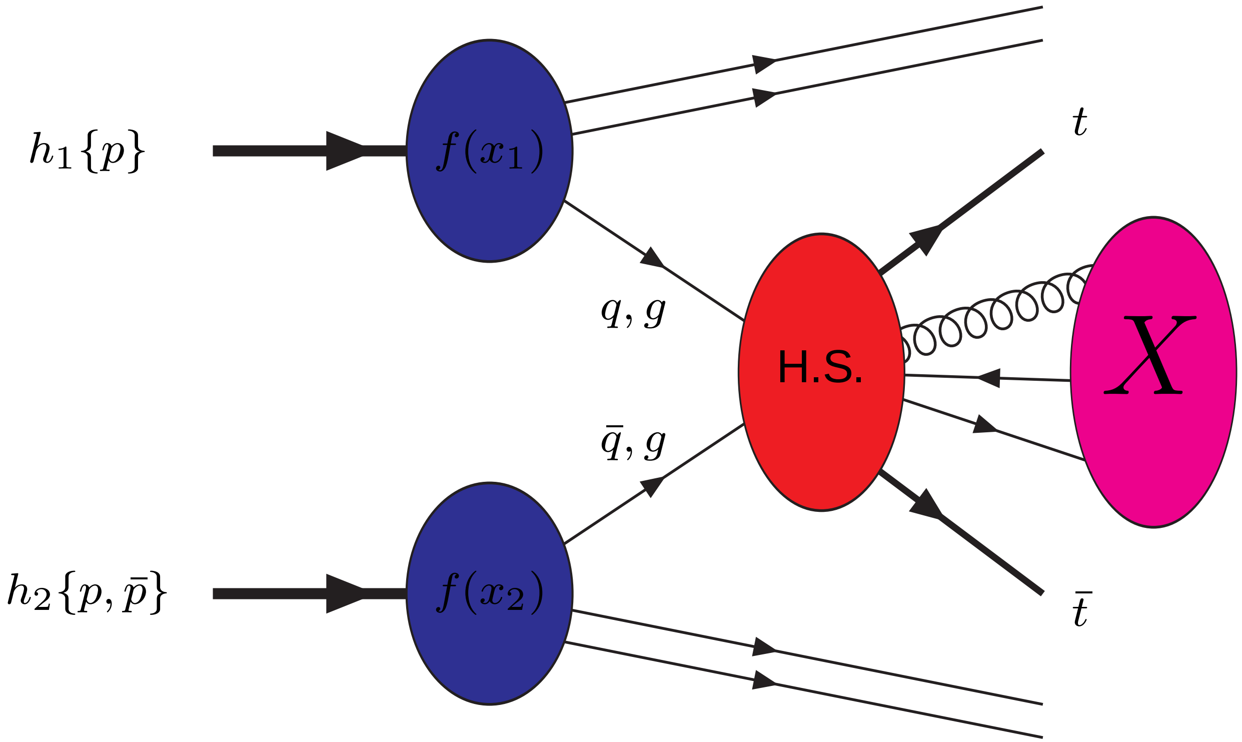
    R. B., A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, JHEP **0807** (2008) 129.
    R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, JHEP **0908** (2009) 067.
- Conclusions and Outlook

Top Quark

- With a mass of $m_t = 173.1 \pm 1.3$ GeV, the TOP quark is the heaviest elementary particle produced so far at colliders.
- Because of its mass, top quark is going to play a unique role in understanding the EW symmetry breaking $\Rightarrow$ Heavy-Quark physics crucial at the LHC.
- To date the Top quark could be produced and studied only at the Tevatron, where it was discovered in 1995.
  - the mass is measured at better than 1%
  - the total cross section $\sigma_{t\bar{t}}$ is measured at the 12% level (D0 arXiv:0903.5525: $\sigma_{t\bar{t}} = 8.18^{+0.98}_{-0.87}$ pb)
  - In total, $\mathcal{O}(10^3)$ $t\bar{t}$ pairs were produced at Tevatron since the top discovery.
- At the LHC the situation is quite different:
  - Even in the first low-luminosity phase we are expected to see millions of $t\bar{t}$ pairs per year!
  - With LHC at full speed, $\sigma_{t\bar{t}}$ is expected to be measured at better than 5%!!
- $\Longrightarrow$ top-quark physics will become “precision” physics.

Top-Anti Top Pair Production

According to the factorization theorem, the process $h_1 + h_2 \to t\bar{t} + X$ can be sketched as in the figure:

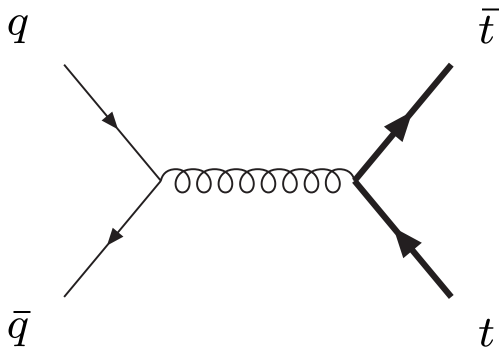


$$\sigma^{t\bar{t}}_{h_1,h_2} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{h_1,i}(x_1,\mu_F) f_{h_2,j}(x_2,\mu_F) \hat{\sigma}_{ij}\left(\hat{s}, m_t, \alpha_s(\mu_R), \mu_F, \mu_R\right)$$

$$s = \left(p_{h_1} + p_{h_2}\right)^2 \ , \ \hat{s} = x_1 x_2 s$$

The Partonic Cross Section: Tree-Level

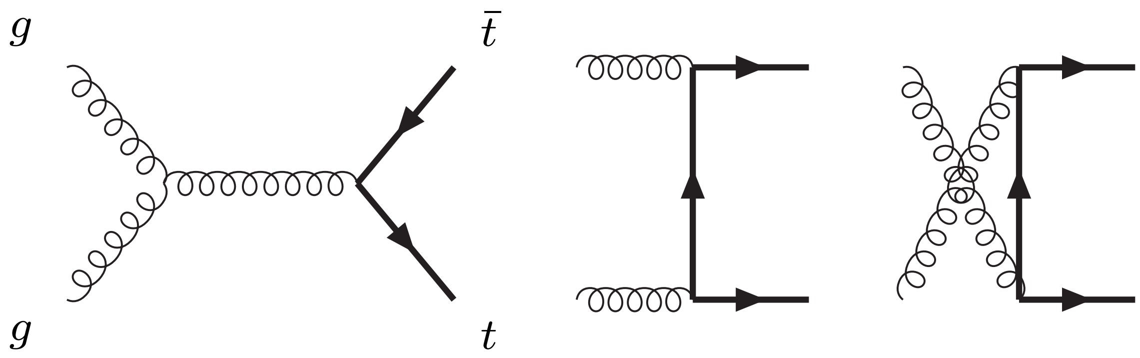
$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



Dominant at Tevatron

$\sim 85\%$

$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



Dominant at LHC

$\sim 90\%$

$\sigma_{t\bar{t}}^{LO}(LHC, m_t = 171\,\text{GeV}) = 583\,\text{pb} \pm 30\%$

$\sigma_{t\bar{t}}^{LO}(Tev, m_t = 171\,\text{GeV}) = 5.92\,\text{pb} \pm 44\%$

The Partonic Cross Section: NLO

Fixed Order

- The NLO QCD corrections are quite sizable: + 25% at Tevatron and +50% at LHC. Reduction of the th error to $\pm 15\%$.

  Nason, Dawson, Ellis '88-'90; Beenakker, Kuijf, van Neerven, Smith '89-'91; Mangano, Nason, Ridolfi '92; Frixione et al. '95; Czakon and Mitov '08.

- Mixed NLO QCD-EW corrections are small: - 1% at Tevatron and -0.5% at LHC.

  Beenakker *et al.* '94 Bernreuther, Fuecker, and Si '05-'08
  Kühn, Scharf, and Uwer '05-'06; Moretti, Nolten, and Ross '06.

All-order Soft-Gluon Resummation

- Leading-Logs (LL)

  Laenen et al. '92-'95; Berger and Contopanagos '95-'96; Catani et al. '96.

- Next-to-Leading-Logs (NLL)

  Kidonakis and Sterman '97; B., Catani, Mangano, and Nason '98-'03.

- Next-to-Next-to-Leading-Logs (NNLL) under study ...

  Moch and Uwer '08; Beneke et al. '09; Czakon et al. '09.

The effect of NLL is to enhance the NLO cross section of +4% and to reduce the dependence on $\mu_{F/R}$ (to $\sim 2/3$ at the Tevatron).

Best "Conservative" Theoretical Prediction

TEVATRON

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm Tev}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 7.61~^{+0.30(3.9\%)}_{-0.53(6.9\%)}~({\rm scales})~^{+0.53(7\%)}_{-0.36(4.8\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm Tev}, m_t = 171~{\rm GeV}, {\rm MRST2006nnlo}) = 7.93~^{+0.34(4.3\%)}_{-0.56(7.1\%)}~({\rm scales})~^{+0.24(3.1\%)}_{-0.20(2.5\%)}~({\rm PDFs})~~{\rm pb}\,.$$

$$\sigma_{t\bar{t}}^{\rm NLO}({\rm Tev}, m_t = 171~~{\rm GeV}, {\rm CTEQ6.5}) = 7.35~^{+0.38(5.1\%)}_{-0.80(10.9\%)}~({\rm scales})~^{+0.49(6.6\%)}_{-0.34(4.6\%)}~({\rm PDFs})~~{\rm pb}$$

LHC

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 908~^{+82(9.0\%)}_{-85(9.3\%)}~({\rm scales})~^{+30(3.3\%)}_{-29(3.2\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm MRST2006nnlo}) = 961~^{+89(9.2\%)}_{-91(9.4\%)}~({\rm scales})~^{+11(1.1\%)}_{-12(1.2\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO}({\rm LHC}, m_t = 171~~{\rm GeV}, {\rm CTEQ6.5}) = 875~^{+102(11.6\%)}_{-100(11.5\%)}~({\rm scales})~^{+30(3.4\%)}_{-29(3.3\%)}~({\rm PDFs})~~{\rm pb}$$

M. Cacciari, S. Frixione, M. Mangano, P. Nason, and G. Ridolfi, JHEP 0809:127,2008.

Measurement Requirements

.. this is to be compared with the experimental requirements for $\sigma_{t\bar{t}}$:

- Tevatron $\Delta\sigma/\sigma \sim 12\% \Longrightarrow$ ok!
- LHC (14 TeV, high luminosity) $\Delta\sigma/\sigma \sim 5\% \ll$ current theoretical prediction!!

Moch and Uwer presented an "approximated" NNLO result for $\sigma_{t\bar{t}}$ including

- scale dependence at NNLO
- NNLL soft-gluon contributions
- Coulomb corrections

This drastically reduces the uncertainty (factorization/renormalization scale dependence) to the level predicted for LHC: $\boxed{\sim 4 - 6\%}$.

This result is an "approximated" NNLO result.
Nevertheless, it indicates that a COMPLETE NNLO computation is indeed needed in order to match the experimental precision.

Next-to-Next-to-Leading Order

The NNLO calculation of the top-quark pair hadro-production requires several ingredients:

- Virtual Corrections
  - two-loop matrix elements for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
  - interference of one-loop diagrams

Körner et al. ’05-’08; Anastasiou and Aybat ’08

- Real Corrections
  - one-loop matrix elements for the hadronic production of $t\bar{t}$ + 1 parton
  - tree-level matrix elements for the hadronic production of $t\bar{t}$ + 2 partons

Dittmaier, Uwer and Weinzierl ’07-’08

Both matrix elements known for $t\bar{t} + j$ calculation, BUT subtraction up to 1 unresolved parton, while in a complete NNLO computation of $\sigma_{t\bar{t}}$ we need subtraction terms with up to 2 unresolved partons.

$\Longrightarrow$ Need an extension of the subtraction methods at the NNLO.

Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times0)} + \mathcal{A}_2^{(1\times1)}$$

$$\begin{aligned} \mathcal{A}_2^{(2\times0)} \quad = \quad & N_c C_F \Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l \left(N_c D_l + \frac{E_l}{N_c}\right) \\ & + N_h \left(N_c D_h + \frac{E_h}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\Big] \end{aligned}$$

10 different color coefficients

- The whole $\mathcal{A}_2^{(2\times0)}$ is known numerically (evaluation too slow)

  Czakon '08.

- The coefficients D_i, E_i, F_i, and A are known now analytically

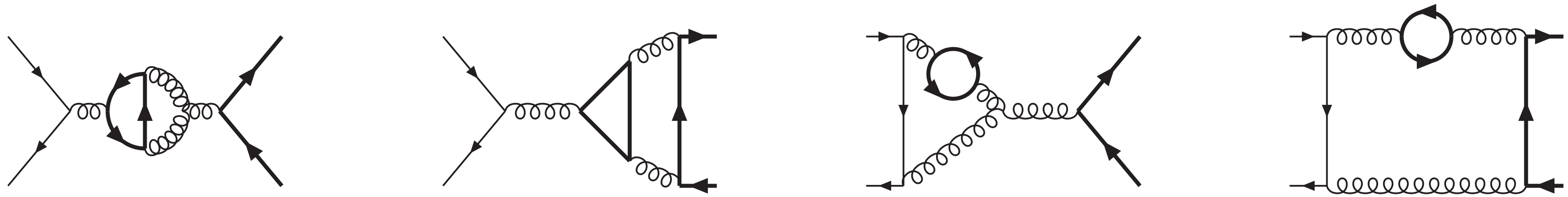
  B., Ferroglia, Gehrmann, Maitre, and Studerus '08-'09

- The poles of $\mathcal{A}_2^{(2\times0)}$ (and therefore of B and C) are known analytically

  Ferroglia, Neubert, Pecjak, and Li Yang '09

Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

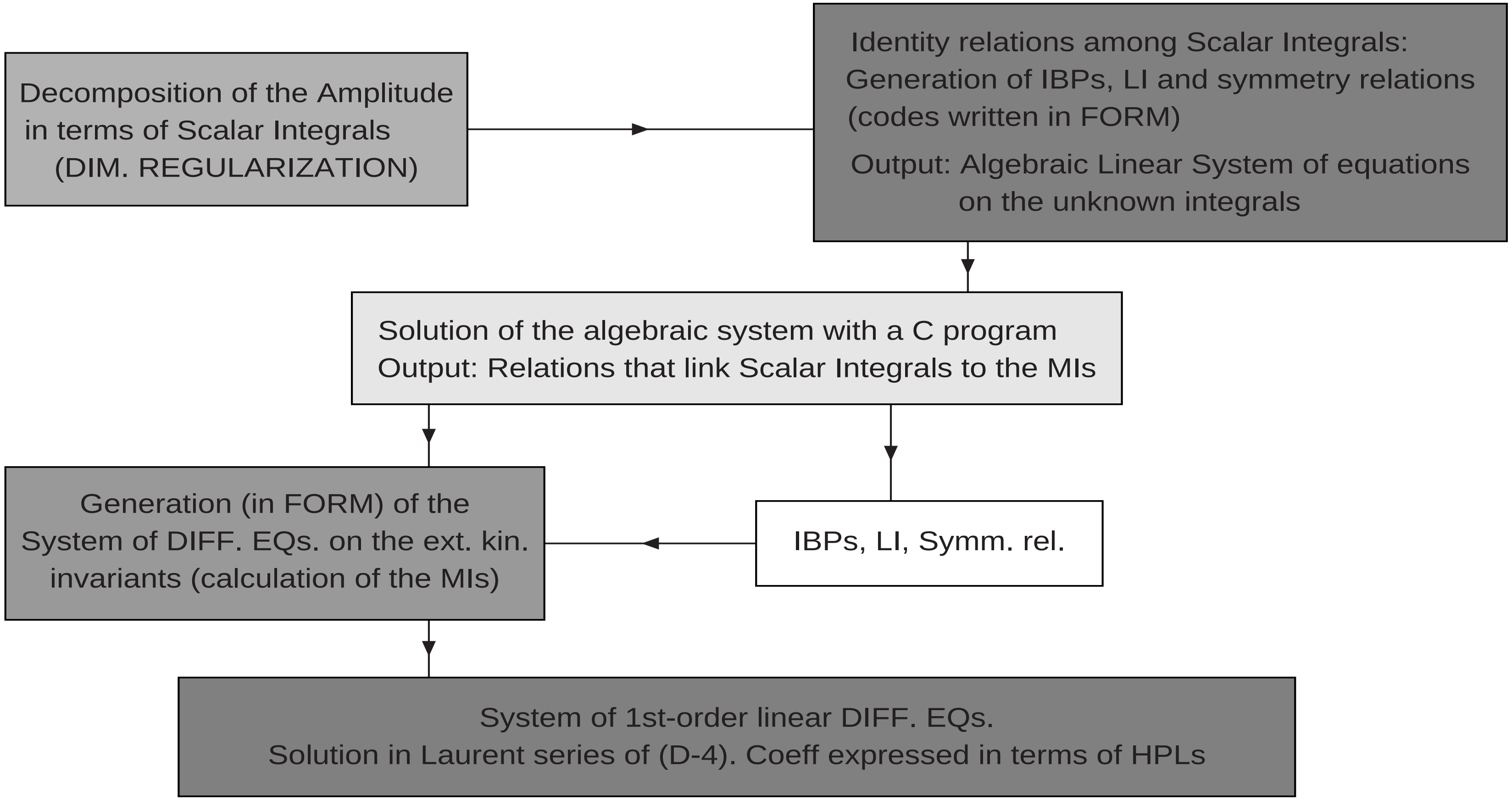
- $\boxed{D_i, E_i, F_i}$ come from the corrections involving a closed (light or heavy) fermionic loop:



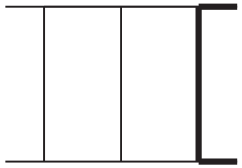
- $\boxed{A}$ the leading-color coefficient, comes from the planar diagrams:

- The calculation is carried out analytically using:
  - Laporta Algorithm for the reduction of the dimensionally-regularized scalar integrals (in terms of which we express the $|\mathcal{M}|^2$) to the Master Integrals (MIs)
  - Differential Equations Method for the analytic solution of the MIs

Laporta Algorithm and Diff. Equations



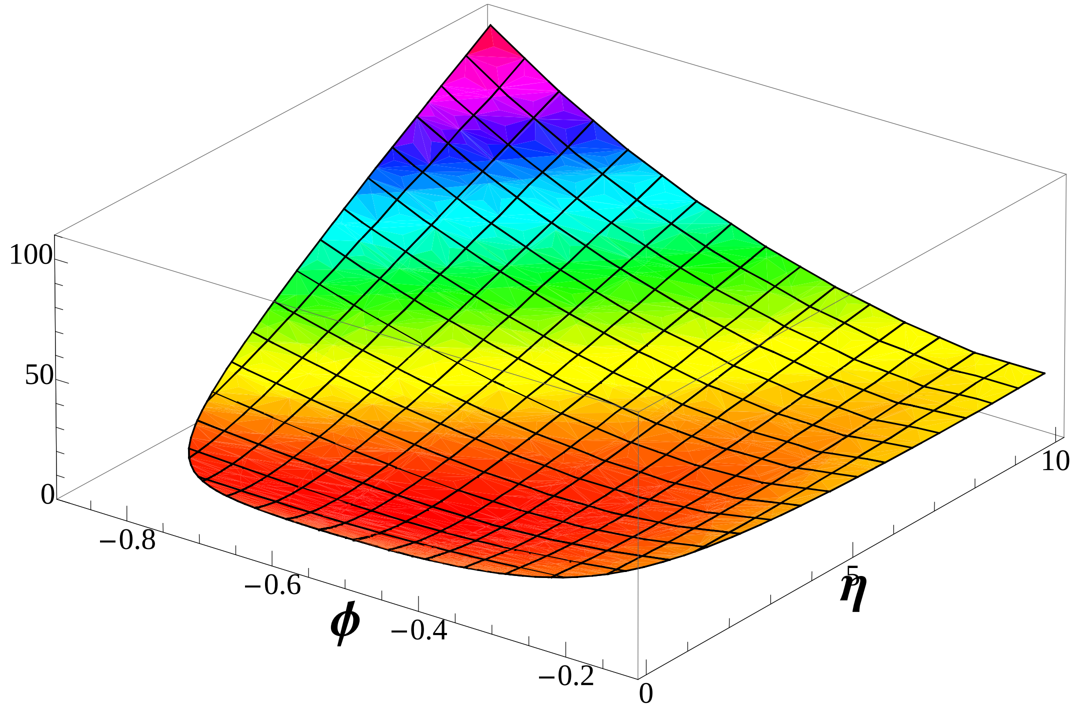
Example



$$= \frac{1}{m^6} \sum_{i=-4}^{-1} A_i^i + \mathcal{O}(^0)$$

$$
\begin{aligned}
A_{-4} &= \frac{x^2}{24(1-x)^4(1+y)}\,, \\
A_{-3} &= \frac{x^2}{96(1-x)^4(1+y)}\Big[-10G(-1;y) + 3G(0;x) - 6G(1;x)\Big]\,, \\
A_{-2} &= \frac{x^2}{48(1-x)^4(1+y)}\Big[-5\zeta(2) - 6G(-1;y)G(0;x) + 12G(-1;y)G(1;x) + 8G(-1,-1;y)\Big]\,, \\
A_{-1} &= \frac{x^2}{48(1-x)^4(1+y)}\Big[-13\zeta(3) + 38\zeta(2)G(-1;y) + 9\zeta(2)G(0;x) + 6\zeta(2)G(1;x) - 24\zeta(2)G\left(-1/y;x\right) \\
&+24G(0;x)G(-1,-1;y) - 24G(1;x)G(-1,-1;y) - 12G\left(-1/y;x\right)G(-1,-1;y) \\
&-12G(-y;x)G(-1,-1;y) - 6G(0;x)G(0,-1;y) + 6G\left(-1/y;x\right)G(0,-1;y) + 6G(-y;x)G(0,-1;y) \\
&+12G(-1;y)G(1,0;x) - 24G(-1;y)G(1,1;x) - 6G(-1;y)G\left(-1/y,0;x\right) + 12G(-1;y)G\left(-1/y,1;x\right) \\
&-6G(-1;y)G(-y,0;x) + 12G(-1;y)G(-y,1;x) + 16G(-1,-1,-1;y) - 12G(-1,0,-1;y) \\
&-12G(0,-1,-1;y) + 6G(0,0,-1;y) + 6G(1,0,0;x) - 12G(1,0,1;x) - 12G(1,1,0;x) + 24G(1,1,1;x) \\
&-6G\left(-1/y,0,0;x\right) + 12G\left(-1/y,0,1;x\right) + 6G\left(-1/y,1,0;x\right) - 12G\left(-1/y,1,1;x\right) + 6G(-y,1,0;x) \\
&-12G(-y,1,1;x)\Big]
\end{aligned}
$$

Coefficient A



$$\eta = \frac{s}{4m^2} - 1\,, \quad \phi = -\frac{t-m^2}{s}\,, \quad \frac{1}{2}\left(1 - \sqrt{\frac{\eta}{1+\eta}}\right) \le \phi \le \frac{1}{2}\left(1 + \sqrt{\frac{\eta}{1+\eta}}\right).$$

Numerical evaluation with GiNaC C++ routines.

Vollinga and Weinzierl '04

Two-Loop Corrections to $gg \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times0)} = \; & (N_c^2-1)\Big(N_c^3 A + N_c B + \frac{1}{N_c}C + \frac{1}{N_c^3}D + N_c^2 N_l E_l + N_c^2 N_h E_h \\ & + N_l F_l + N_h F_h + \frac{N_l}{N_c^2}G_l + \frac{N_h}{N_c^2}G_h + N_c N_l^2 H_l + N_c N_h^2 H_h \\ & + N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh}\Big)\end{aligned}$$

16 different color coefficients

- No numeric results for $\mathcal{A}_2^{(2\times0)}$
- The poles of $\mathcal{A}_2^{(2\times0)}$ are known analytically

Ferroglia, Neubert, Pecjak, and Li Yang ’09

- The coefficients A, E_l–I_l can be evaluated analitically as for the $q\bar{q}$ channel

B., Ferroglia, Gehrmann, and Studerus, in preparation.

Summary and Outlook

- Top physics plays a big role in the LHC program. LHC will be a heavy-quark factory and the properties of the top quark will become soon “precision” physics
- The total cross section for the production of $t\bar{t}$ pairs (which is a standard candle in hadronic collisions for a SM check) is suppose to be measured at the 5% accuracy level (2013?). The thoretical predictions are still far from this goal $\Longrightarrow$ need of a complete NNLO calculation that could match the needed accuracy (see approximated studies)
- Among the ingredients of the NNLO calculation, we are addressing the calculation of the two-loop matrix elements. Their numerical evaluation is still too slow. The analytic evaluation, instead, is fast and totally under control
- Our goal is to complete, in the near future,the evaluation of the two-loop matrix elements and to address the problem of the subtraction terms at NNLO for the real part