Black Hole radiation
from
Bose Einstein Condensates

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arXiv:0905.3634 (BEC)

:0903.2224

w/ sub + superlum

quartic disp. models

Motivations.

1. Compute the fluxes of phonons from "first principles" (i.e. without relying on the gravitational "**analogy**")

(BH)

by the sonic horizon in a

emitted by in a stationary

1 D condensate.

∞-order eq. (Visser et al.)

The analogy (Unruh '81)

* In the <u>hydrodynamical</u> <u>approx.</u>
  (i.e. for long wave lengths)
  sound waves obey:

  • no vorticity
  • no dissipation
  • $P = P(\rho)$.

$$\left[(i\partial_t + i\partial_x v) \frac{\rho_0}{c^2} (i\partial_t + iv\partial_x) + \frac{1}{\rho_0} \partial_x \rho_0 \partial_x \right] \phi = 0$$

$v(t,x)$: velocity of the flow

$\rho_0(t,x)$: density , $c^2(t,x)$: sound speed

* this eq. is identical to $\Box \phi = 0$
  in the 4D acoustic metric

$P = P(\rho)$
no dissipation
no vorticity,

$$\sqrt{-g}\, g^{\mu\nu}_{ac} = \frac{\rho_0}{c^2} \begin{pmatrix} -1 & \vec{v} \\ \vec{v} & c^2 - v^2 \end{pmatrix}$$

$\delta\vec{v} = \vec{\nabla}\phi$, $\vec{\nabla} \times \delta\vec{v}$ $\delta\vec{v} = \vec{\nabla}\phi$,

$P = P(\rho)$

⇒ a sonic horizon (ie $v^2 = c^2$)

will act as a black hole.

⇒ an acoustic black hole should emit a steady and (near) thermal flux of phonons.

(~ Hawking radiation).

⇒ Experimenting Hawking radiation in the lab. ?

(Unruh '81)

Statio. BH metric in

regular coordinate systems.

NB

* $$ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \frac{dr^2}{(1-r_s/r)} + r^2 d\Omega^2$$

$$r_s = 2GM/c^2$$: Schwarzschild radius

is singular for $r \to r_s$.

Instead

* $$ds^2 = -d\tau^2 + (dr + \underline{v(r)}\, d\tau)^2 + r^2 d\Omega^2$$

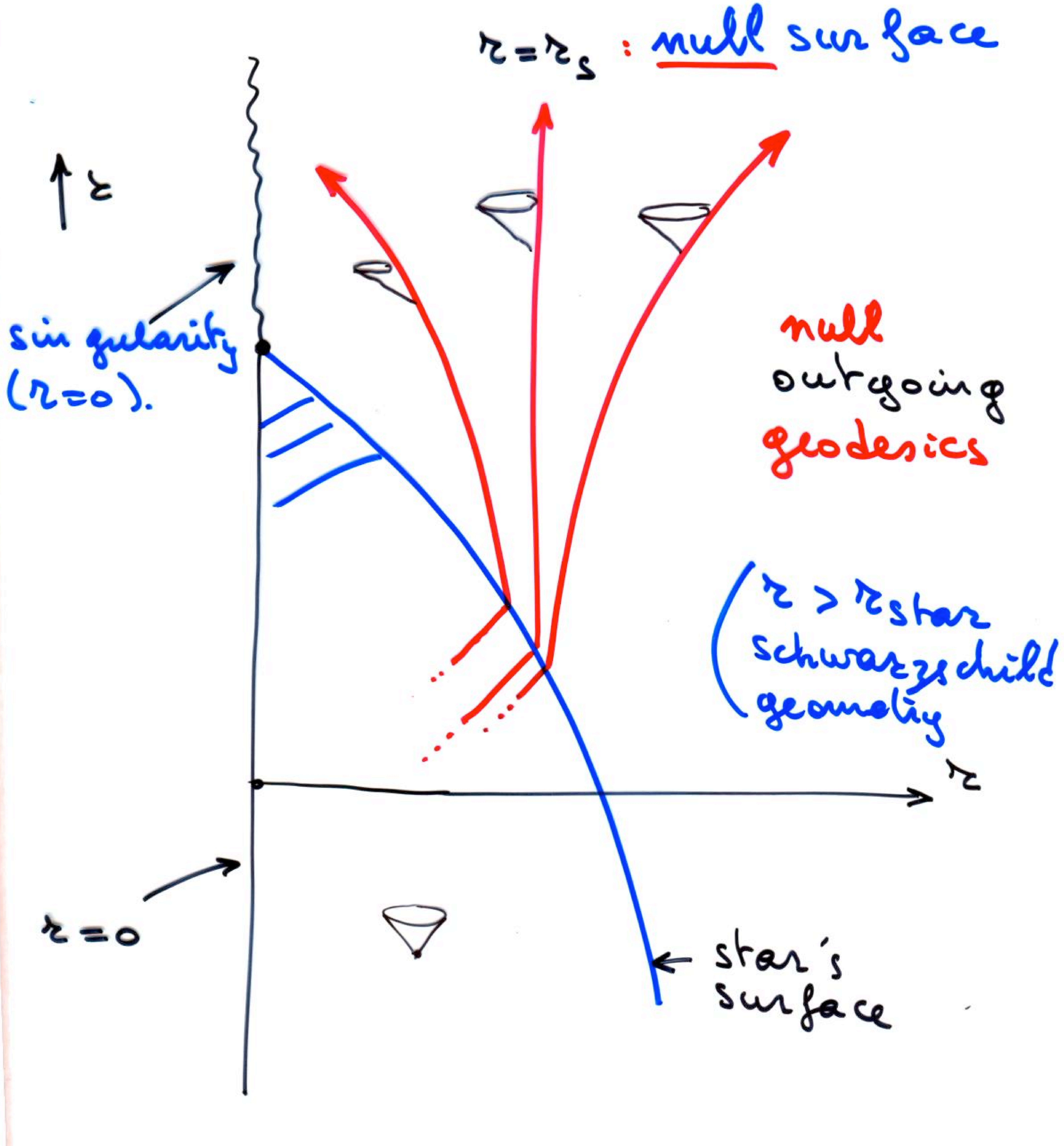
$$\begin{cases} v(r) = -\left(\frac{r_s}{r}\right)^{1/2} \\ d\tau = dt + f(r)\, dr \end{cases}$$

is REGULAR on the future horizon

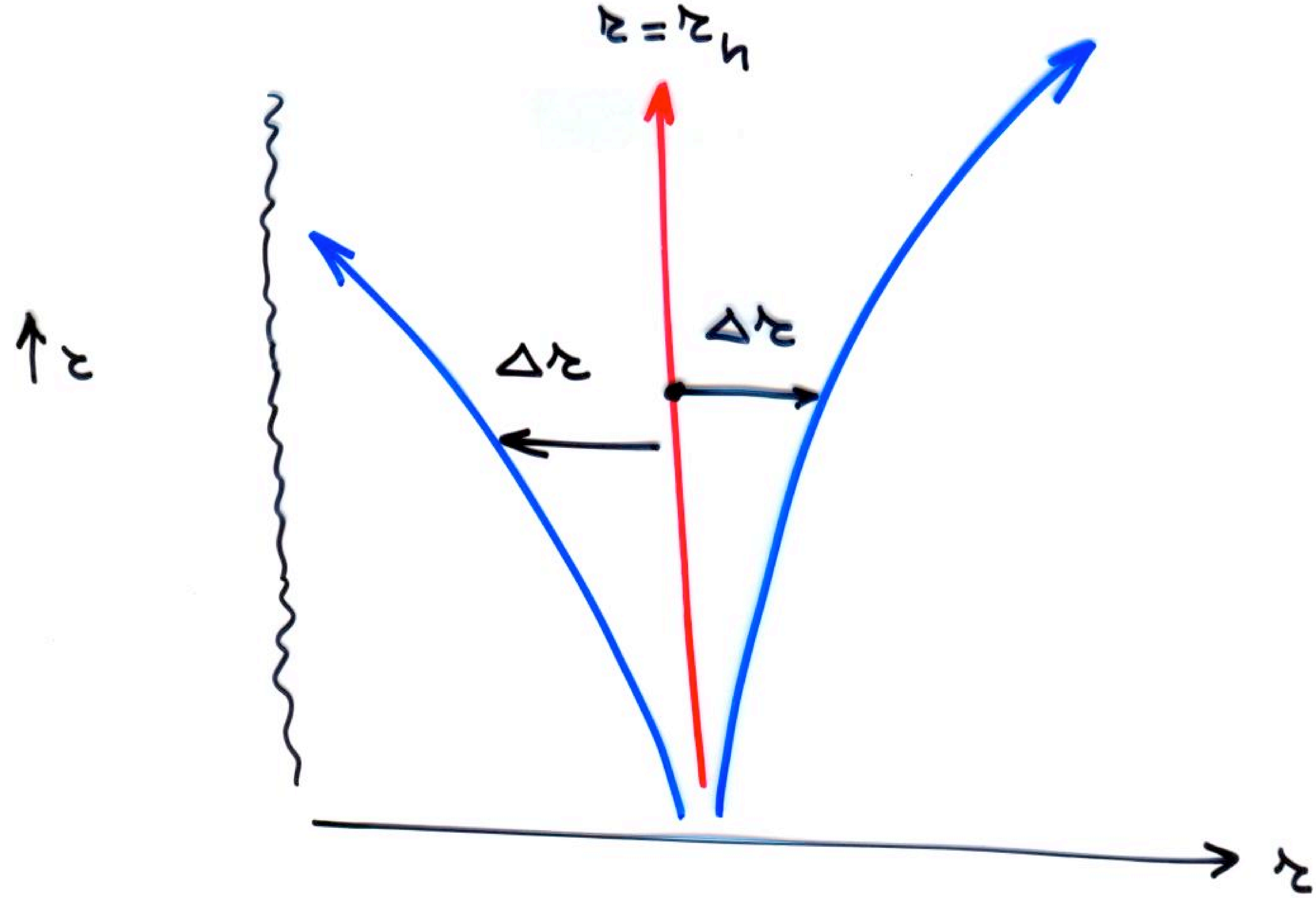
(Painlevé - Gullstrand). P.G.

The space-time geometry.

(in the regular (τ, r) system).



The null outgoing geodesics.



$$\boxed{\Delta r(t) = \Delta r_0 \, e^{\kappa \Delta t}} \quad ; \quad \Delta r_0 \lessgtr 0.$$

$$* \quad \kappa = \left.\frac{\partial v}{\partial r}\right|_{v=-1} = \frac{1}{2 r_s} \qquad \text{"surface gravity".}$$

$$\boxed{p_r = i\partial_r = p_r^0 \, e^{-\kappa \Delta t}}$$

$\boxed{\text{exponential red shift}}$!

NB

$$ds^2 = -dt^2 + (dr + v(r)\, dt)^2$$

in BOTH

1. Schwarzschild sp-time in PG coordinates, $v(r) = -\left(\frac{r_S}{r}\right)^{1/2}$

2. the acoustic line element in the lab coordinates t, r for a 1D flow $v(r)$

$\Rightarrow$ when $|v(r)| = c = 1$, the near horizon acoustic geometry is identical to Schwarzschild with $\boxed{\kappa = \partial_r v \big|_{v=-1.}}$

(1) at short wave lengths,

because of dispersion,

the near horizon propagation

is modified.

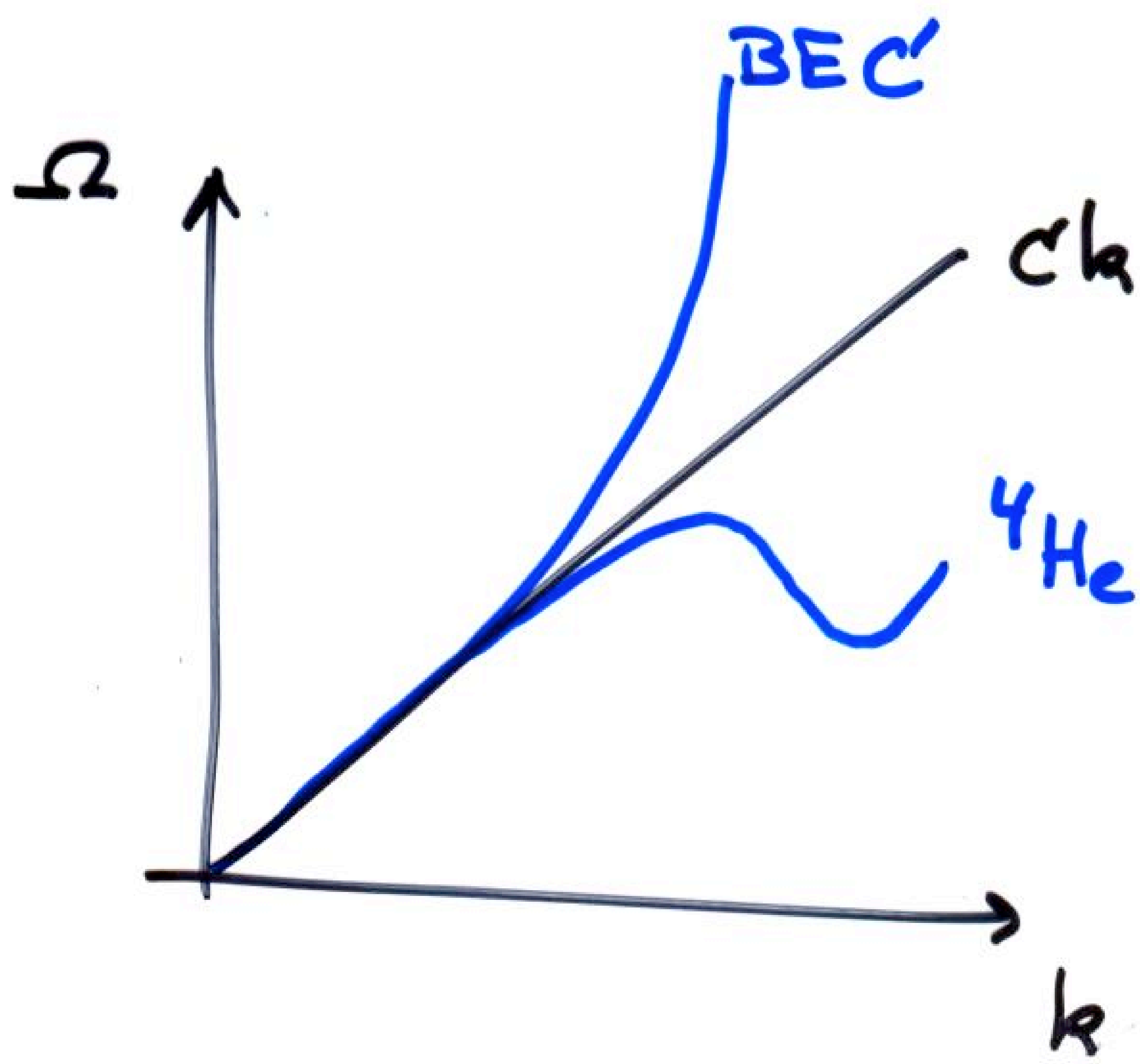
Jacobson '91

Unruh '94

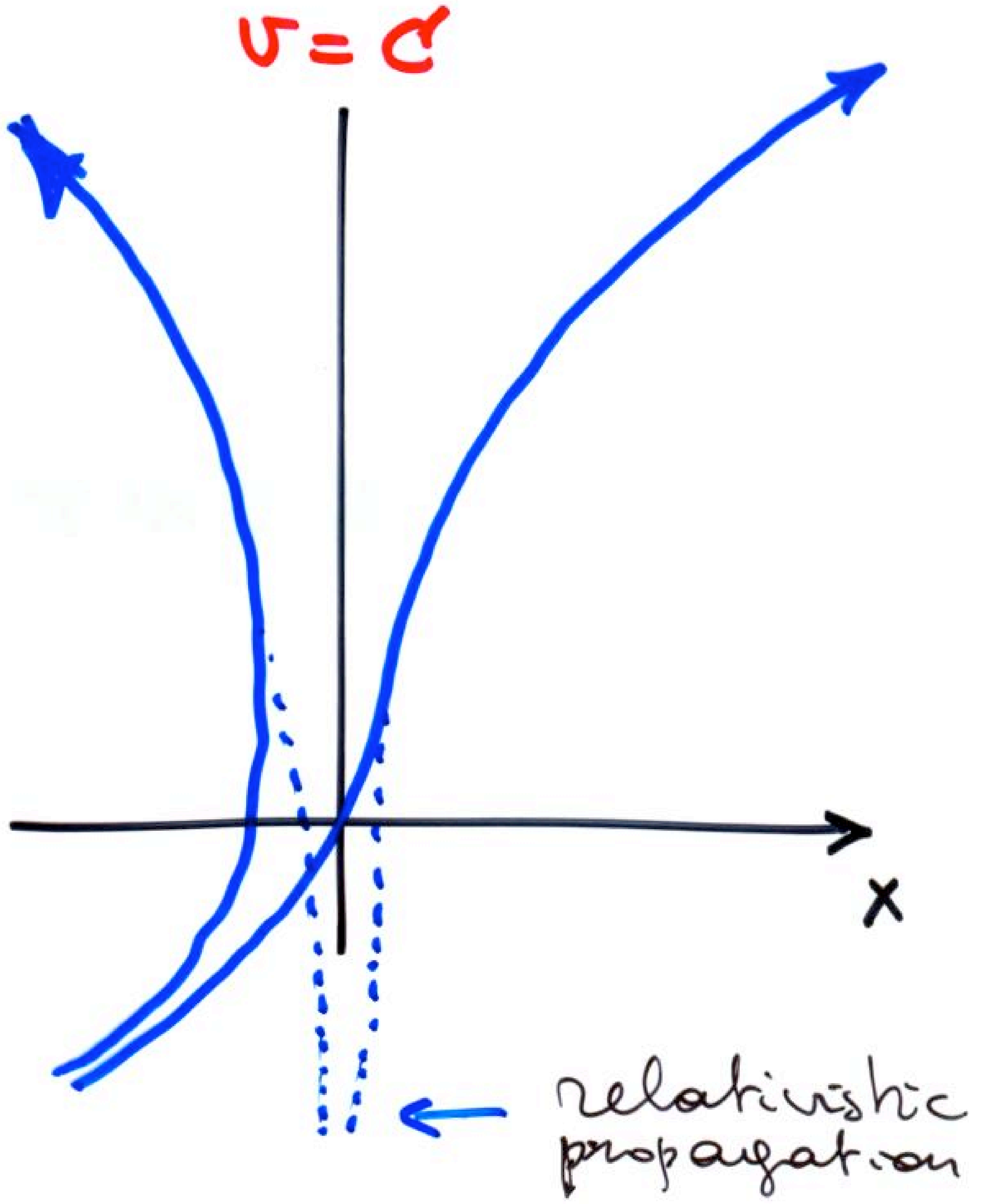
dispersion

e.g. in a B.E.C. :

$$\Omega^2 = c^2 k^2 + \left(\frac{\hbar}{2m}\right)^2 k^4$$



in a BEC :



The early propag. (near v = c) is radically modified.

Question : Will this modify the properties of Hawking radiation ?

N.B. : This question is the same as that raised when exploring Lorentz violations in QG.

⇒ <u>Lesson 0</u> (from Ac BHs.)

Cond-mat models offer a rational framework to <u>compute</u> H.R. when LI is broken in the U.V. :

i.e.:

<u>Given</u> $\Omega^2 = c^2 k^2 \left(1 \pm (k^2/\Lambda^2)^\alpha\right)$

how $n_\omega^{(\Lambda)}$ differs from $n_\omega^{(0)}$?

standard HR.

1

There exist 2 different domains

1) Study detailed properties of Hawking radiation in cond-mat models. (e.g. in a BEC).

$\Rightarrow$ no gravity, no speculations.

2) Phenomenology of Quantum Gravity. (more conjectural)

* aplied to inflationary spectra CMB (since 2000).

Lesson 1 (the robustness).

* when including dispersion, and when $(\kappa/\Lambda) \ll 1$,

I. the spectrum is robust:

$$\bar{n}_\omega^{(\Lambda)} = |\beta_\omega^\Lambda|^2 = \bar{n}_\omega^{(0)} \left(1 + \text{small deviations}\right)$$

II. the correlation pattern between Hawking quanta + partners is also robust.

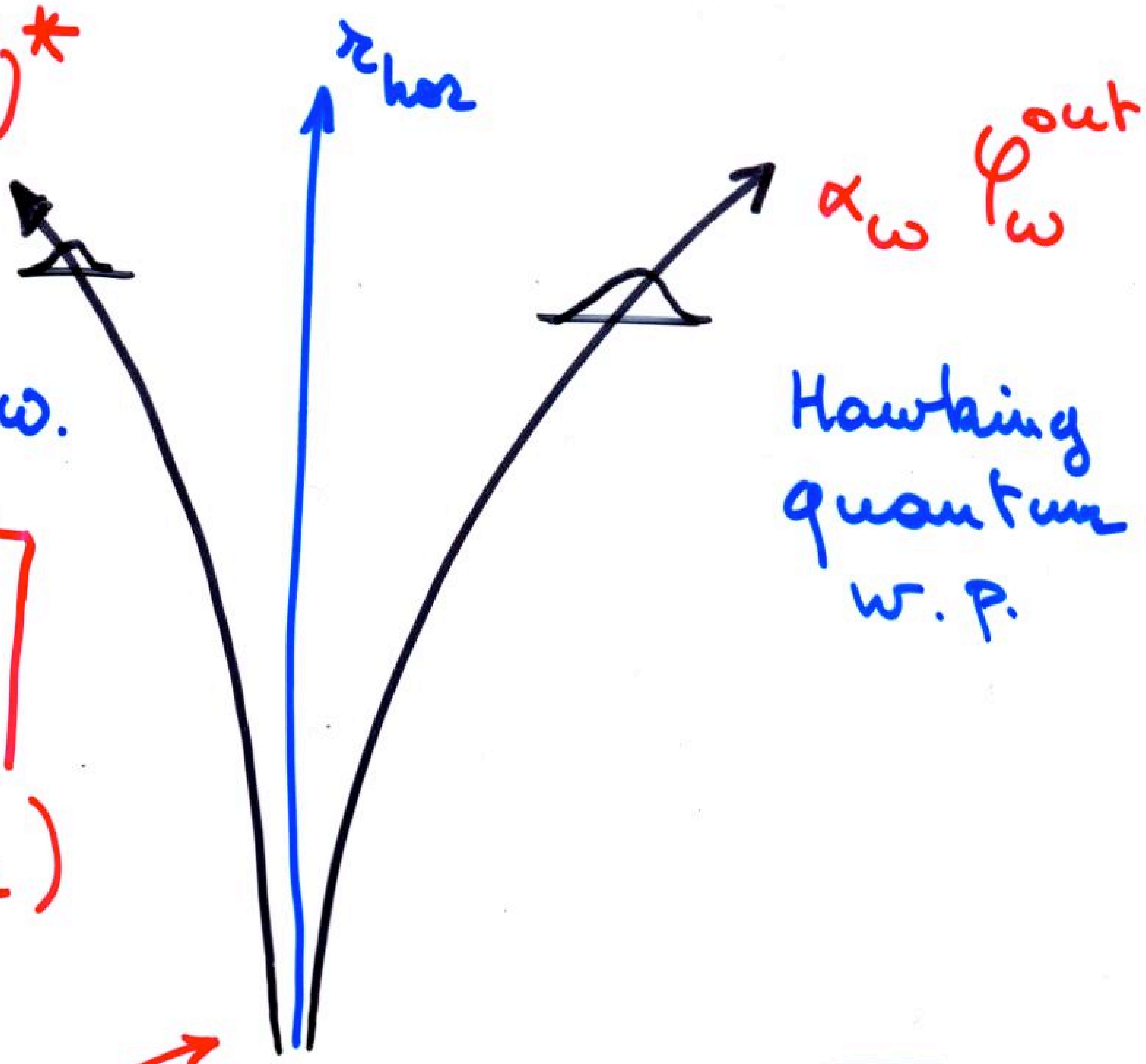
Even though the near horizon propagation is completely modified!

('94 Unruh (PDE), '95 Brout et al, '96 C+J. (ODE)

Space time pattern of the in wave paquet: $\int f_\omega \, \varphi^{in}_\omega(r)\, e^{-i\omega t} d\omega$

(unit, positive norm)

$\omega = i\partial_t$: Killing freq.



partner of negative ω.

Hawking quantum w.p.

I $\boxed{\Delta r_p = -\Delta r_H \text{ at all } t.}$

$(\Delta r = r - r_{hor})$

II. bottom-less focusing $\boxed{\Delta r \sim e^{\kappa \Delta t}}$

because Lorentz invariance.

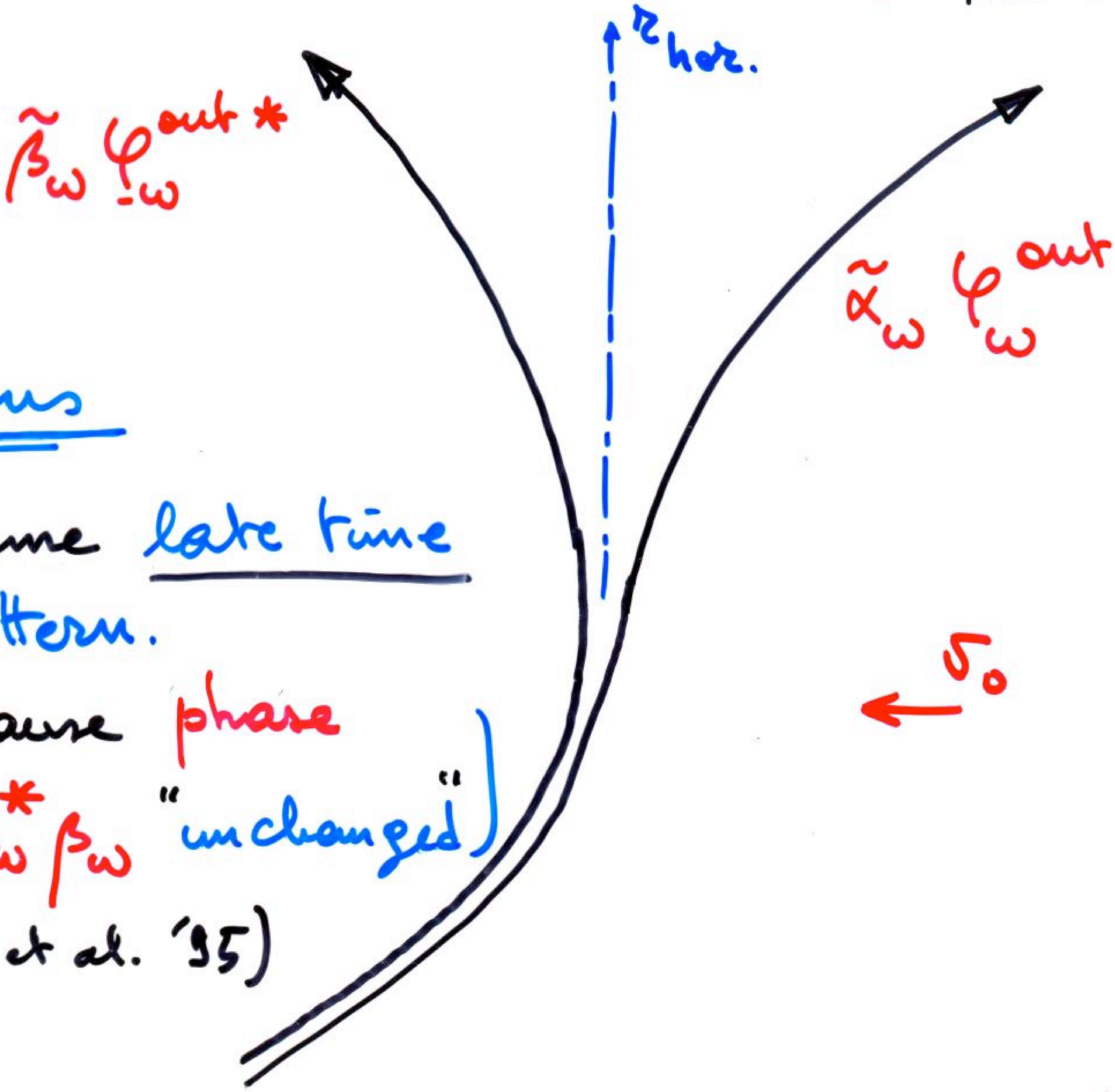
(No $\underbrace{\text{scale}}_{\text{UV}}$ to stop it)

Space time pattern of φ^{in}_ω

with superluminal dispersion.

$$\left(\Omega^2 = k^2 + k^4/\Lambda^2 \quad \text{e.g.} \right)$$

↑ UV scale



Lessons

I. Same late time pattern.

(because phase of $\alpha^*_\omega \beta_\omega$ "unchanged")

(Brout et al. '95)

II. End of focusing at early time when $\kappa \Delta r \sim \dfrac{\omega}{\Lambda}$, ie $\Omega \sim \Lambda$.

$\Rightarrow$ both $\cdot \; |\beta_\omega|^2 = \bar{n}_\omega$

$\cdot \; \alpha_\omega^* \beta_\omega \in \mathbb{C}$

hardly affected by breaking LI in the U.V.

$\Downarrow$

Lesson 1 $\begin{cases} \text{from Ac BH} \\ \text{for Q.G.} \end{cases}$:

* Even though we do not know the UV behavior of QG, Hawking rad. is (most probably) as derived using QFT in curved space-t.

. ——— .

NB : $\Rightarrow$ HR cannot be used to $\overbrace{\text{discriminate}}^{\text{easily}}$ $\neq$ models of Q.G.

Lesson 2 from Ac BH.

* For all dispersion relations:

$$\Omega^2 = F_\Lambda^2(k^2),$$

but the SR ones : $\Omega^2 = c^2k^2 + m^2$,

the blue shifting stops

when $k \sim \Lambda$.

⇒ the relativistic case is

an isolated and singular case.

(as far as how stationarity is achieved).

Stationarity of HR.

* in relativistic settings, stationarity is achieved by using $\Omega_{FF} \sim \omega e^{\kappa \Delta \tau}$:
  unusual unbounded growth

* in dispersive theories, stationarity is achieved as in the Golden Rule, as usual

- $\Omega_{FF} \sim \Omega$: fixed
- mode density $\sim \Delta t \Rightarrow$ constant rate.

Conclusion

* the analogy can be used to get a first approximation.

But first principles should be used to determine the

1. validity of the analogy.
2. deviations (due to dispersion)

(⚠ fibers)

⇒ 1st Aim:

Derive the phonon fluxes from the Bogoliubov - de Gennes equation.

(without any further approx.)

2^d Aim

* Explain the origin of
  the correlation pattern
  which has been "numerically observed"
  by Carusotto et al. '08.

* Establish why it is not
  washed out by thermal noise,
  but reinforced!

3^d Aim

Quantify the observable effects
to guide experimenting b.h. radiation

(... Technion group) (Cornell ?)

june '09

Plan

* Motivations

* Study Bogo-de Gennes eq. in
<u>static</u> 1D condensates
with 1 sonic horizon

* <u>Complete</u> set of modes
  ⇒ critical frequency $\omega_{max} \neq 1$
  ⇒ 3 x 3 Bogoliubov trsf.
    ⇒ 3 fluxes and
    3 types of long distance
    <u>correlations</u>.

* Numerical analysis.

BEC - Dilute gases.

- Non Relat. 2nd Q.zed treatment.

. $$\hat{H} = \frac{\hbar^2}{2m} \vec{\nabla}\hat{\Psi} . \vec{\nabla}\hat{\Psi}^{+} + V_{ext} |\hat{\Psi}|^2 + g |\hat{\Psi}|^4$$

. $$[\hat{\Psi}(t,\vec{x}), \hat{\Psi}^{+}(t,\vec{y})] = \delta^3(\vec{x}-\vec{y})$$

. $\hat{\Psi}(t,\vec{x})$ destroys an atom at $t,\vec{x}$.

. $$\left.\begin{cases} V_{ext} = V_{ext}(t,\vec{x}) & \text{ext. pot.} \\ g = g(t,\vec{x}) & \text{coupling} \end{cases}\right\} \text{can be tuned.}$$

Condensation.

at low temperature, a large fraction of atoms condensates:

- $$\hat{\Psi}(t,x) = \Psi_0(t,x) + \hat{\chi}(t,x)$$
  $$= \Psi_0(t,x)\left(\hat{1} + \hat{\phi}(t,x)\right)$$

- expansion in ϕ:

  0^{th} order : the condensate w. f. Ψ_0 (the background). (GP eq.)

  1^{th} order : linear mode eq. (Bog. - de Gennes)

  higher orders : interactions (neglected here)

The condensate $\varphi_0(t,x)$

- obeys : $i\partial_t \varphi_0 = (\hat{T} + V + g\rho_0)\,\varphi_0$

  with $\rho_0 \doteq |\varphi_0(t,x)|^2$. (GP eq.)

- focus on statio - 1D condensates.

$\Rightarrow \varphi_0(t,x) = e^{-i\omega_0 t} \times \sqrt{\rho_0(x)}\, e^{iW_0(x)}$

$$\Rightarrow \begin{cases} \rho_0(x)\, v_0(x) = c^{st.}, \quad v_0 \doteq \dfrac{k_0}{m} \doteq \dfrac{\partial_x W_0}{m} \\ \omega_0 = \dfrac{k_0^2}{2m}(x) + \underline{V_{ext}}(x) + \underline{g}(x)\,\rho_0(x). \end{cases}$$

---

charact. by

2 functions : $\boxed{v_0(x)}$ and $\boxed{c^2(x) \doteq g\,\rho_0/m.}$

The perturbations.

$$\hat{\chi}(t,x) = \psi_0(t,x)\ \hat{\phi}(t,x)$$

$$i\partial_t \hat{\chi} = (T + V + 2g|\psi_0|^2)\hat{\chi} + g\,\psi_0^2\,\hat{\chi}^\dagger$$

↑ BdG eq.

can be rewritten as:

$$\boxed{i(\partial_t + v_0\partial_x)\hat{\phi} = T_\rho\,\hat{\phi} + mc^2(\hat{\phi} + \hat{\phi}^\dagger)}$$

$$T_\rho = \frac{-1}{2m}\,\frac{1}{\rho_0}\,\partial_x\,\rho_0\,\partial_x \qquad \text{(self adj.)}$$

$$(\hbar = 1)$$

The (stationary) mode equation.

- $\hat{\phi}_\omega = \hat{a}_\omega \, e^{-i\omega t} \phi_\omega(x) + \hat{a}^\dagger_\omega \left(e^{-i\omega t} \varphi_\omega(x) \right)^*$

- $\hat{a}_\omega$ destroys a phonon.

- $(\phi_\omega, \varphi_\omega)$ obeys the 2x2 system

$$\left.\begin{aligned} \left[(\omega + iv\partial_x) - T_\rho - mc^2 \right] \phi_\omega &= mc^2 \varphi_\omega \\ \left[-(\omega + iv\partial_x) - T_\rho - mc^2 \right] \varphi_\omega &= mc^2 \phi_\omega . \end{aligned}\right\}$$

eliminate φ_ω.

$$\left[-(\quad) - T_\rho \cdots \right] \frac{1}{c^2_{(x)}} \left[(\quad) - T_\rho \cdots \right] \phi_\omega = m^2 c^2 \phi_\omega$$

- 4^{th} order ODE in x, (2^d order in t.).
- non trivial ordering of c^2, ρ_0, v with ∂_x.

The properties of the mode eq.

1°) when gradients of c^2, ρ_0, v_0 neglected (WKB) $\rightarrow$

$$\boxed{(\omega - v_0 k_\omega(x))^2 = c^2 k_\omega^2 + \left(\frac{k_\omega^2}{2m}\right)^2}$$

- quartic disp. relation.

- fixes $\begin{cases} k_\omega(x) & : \text{wave vector} \\ \boxed{\Omega(\omega,x) \doteq \omega - v_0(x)\, k_\omega(x)} & \end{cases}$

(comoving freq.)

in terms of
- $v_0(x)$, $c^2(x)$: backgrd.
- ω : conserved (Killing) freq.

2°) possesses a well-defined

$\left\{ \begin{array}{l} \text{dispersion-less} \\ \text{hydrodynamical} \end{array} \right.$ limit : $m \to \infty$.

$$\boxed{\left[(\omega + i v_0 \partial_x) \frac{1}{c^2} (\omega + i v_0 \partial_x) + v_0 \partial_x \frac{1}{v_0} \partial_x \right] \phi_\omega = 0.} \quad (2)$$

⚠ (1). NOT that of a 2D relat. field.
(in PG coord)

$\Rightarrow$ right and left movers
do not decouple in the IR.

$\Rightarrow$ "strong" back-scattering.
(in the general case)

$\Box \phi = 0$?

• (2) is the "Euler" mode eq.

Background profiles : $\underline{v_0(x)}$, $\underline{c(x)}$

and BH / WH geometries.

ex:
$$\boxed{c(x) + v_0(x) = c_0 D \, \text{th}\left(\frac{\kappa x}{D c_0}\right)}$$
$$= 0 \quad (x = 0)$$

near the sonic horizon :

$$\boxed{c + v_0 = \kappa x + O(\underline{\kappa x})^3 D^{-2}}$$

Trajectories ?

$\Rightarrow$ Disp. relation (in the hydro. approx.)

$$\boxed{\omega - v_0 k = c k} \quad + \quad \frac{dk}{dt} = -\frac{\partial \omega}{\partial x} = -\kappa k$$

(HJ eq.)

$$\Rightarrow \begin{cases} k(t) = k_0 \, e^{-\kappa t} \\ x(t) = x_0 \, e^{\kappa t} \end{cases}$$ for all ω !

$\Rightarrow$ Analoguous to the near horizon propag. of a relativistic field.

Remarks

1. With (arbitrary) dispersion, and $c = c^{st}$,

$$\boxed{k_\omega = k_0 \, e^{-\kappa t}}$$

still exact in the n.h. region ('05)

2. because left and right movers mix when including dispersion

$$c(x) = c_0 + (1-q)\,(c+v)(x)$$

$$v(x) = -c_0 + q\,(c+v)(x)$$

$q = 1$: only v varies ("like" in PG)

$q = 0$: " $c(x)$ " (like in ?)

($q \sim \frac{2}{3}$ in the 'Technion' black hole)

Asymptotic modes and critical frequency $\boxed{\omega_{max}.}$

- For $x \to \pm\infty$, $c, v \to \underbrace{c_\pm, v_\pm}_{\text{constant!}}$

$\Rightarrow$ modes : $e^{-i\omega t} e^{i k_\omega^\pm x}$

$$(\omega - v_\pm k_\omega)^2 = c_\pm^2 k_\omega^2 + \frac{k_\omega^4}{4m^2} = \Omega_\pm^2(k)$$

- Subsonic side : $c_+ > |v_+|$
  - 2 real sol : $k_+^u > 0$, $k_+^v < 0$.
  - + 2 $\mathbb{C}$ sol : $k_{gr}^+, k_{dec}^+ = (k_{gr}^+)^*$

- u-modes $\Leftrightarrow$ "right movers" w.r.t the fluid.

- v- modes $\Leftrightarrow$ left movers

Supersonic side : $c_- < |v_-|$

- for $\omega > \omega_{max}$

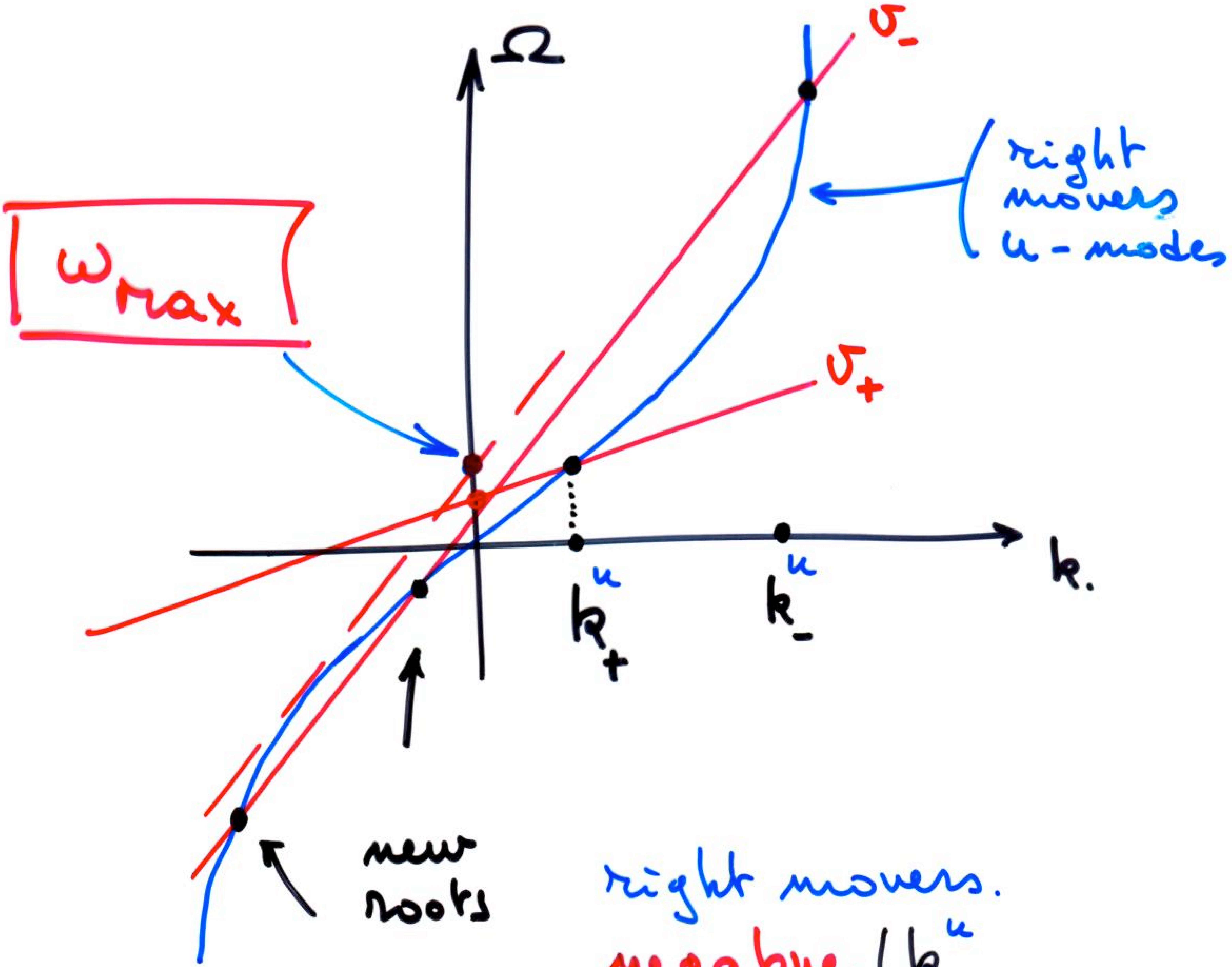
  as in the subsonic regime:
  2 real + 2 $\mathbb{C}$ roots k_ω

- for $0 < \omega < \omega_{max}$

  4 real roots :

  - k_-^u , k_-^v : 'evolved' k_+^u , k_+^v
  - 2 roots $k^u < 0$ (right movers)

$$\omega - v k = \pm \Omega = \pm \sqrt{c^2 k^2 + \frac{k^4}{4m^2}}$$



right movers.

negative $\begin{cases} k^u \\ \Omega \end{cases}$

(• negative Ω $\Rightarrow$ negative norm.)

not in TJ '91, '93. no in '9?, yes in CJ.
Unruh '94? before bumps '95

Rôles of ω_{max}.

$$\omega_{max} = \left(\frac{\hbar}{m}\right) f(D) \;, \quad \text{where}$$

$$-D = \frac{(c_- + v_-)}{c_0} \;, \left(v_- < 0, |v_-| > c_-\right)$$

1 • $D \ll 1$, $\omega_{max} \sim \frac{\hbar}{m} D^{3/2} \ll \frac{\hbar}{m}$.

I. ω_{max} cuts off phonon production:

$$\underline{\bar{n}(\omega) \equiv 0} \;, \quad \omega > \omega_{max}.$$

II ω_{max} governs

- leading deviations w r t standard HR.
- <u>range</u> of robustness.

JM + RP + RG. '09

Quantization (3 steps)

in statio, 1D, *single* horizon,
asymptotic. flat condensates.

1. in each asympt. region :

$$\hat{\phi} = \int_{-\infty}^{+\infty} dk \left[\left(\hat{a}_k \, e^{ikx} \, e^{-i\omega_k^\pm t} \right) \phi_k + (\ )^\dagger \varphi_k \right]$$

$\omega_k^\pm = v_\pm k + \Omega_k$: (single valued)

$|\phi_k|^2 - |\varphi_k|^2 = 1$

2. in each region separately

→ ω - representation $\hat{a}_k \to \hat{a}_\omega$

using $k^\pm(\omega)$: $\underbrace{\text{multiple valued.}}_{\omega\text{ - dep}}$

3. Merge these asympt. modes to construct

→ in/out mode basis.

→ Bogoliubov trsf. relating them.

In the ω-representation:

$$\hat{\phi} = \int_0^\infty d\omega \left(e^{-i\omega t} \hat{\phi}_\omega(x) + e^{i\omega t} \hat{\varphi}_\omega^{+}(x) \right)$$

1. When $\underline{\omega > \omega_{max}}$: $\underline{\underline{2\text{-mode}}}$ sector

$$\hat{\phi}_\omega = \hat{a}_\omega^{u,in} \phi_\omega^{u,in}(x) + \hat{a}_\omega^{v,in} \phi_\omega^{v,in}(x)$$

$=$ same with $\underline{\underline{in}} \to \underline{\underline{out}}$.

with

$$\phi_\omega^{u,in} = \underline{T_\omega} \phi_\omega^{u,out} + \underline{R_\omega} \phi_\omega^{v,out}$$

$$\boxed{1 = |T_\omega|^2 + |R_\omega|^2}$$

<u>Elastic</u> scatt. due to $\left\{ \begin{array}{l} \partial_x v \neq 0 \\ \partial_x c \neq 0. \end{array} \right.$

<u><u>NB</u></u>: <u>in</u> (out) modes (when stationary) well defined in the asympt. regions $c = c_\pm$

When $\omega < \omega_{max}$: 3-mode sector

$$\hat{\phi}_\omega = \hat{a}^u_\omega \phi^u_\omega + \hat{a}^v_\omega \phi^v_\omega + \hat{a}^{u\dagger}_{-\omega} (\varphi^u_{-\omega})^*$$

(both in in and out basis.)

$\boxed{\omega > 0}$

$\Rightarrow$ 3×3 Bogoliubov transformation.

Enlarged Bog. transf.
(in $\to$ out)

$$\phi^{u,in}_{\omega} = \alpha_{\omega}\, \phi^{u}_{\omega} + \beta_{-\omega} (\varphi^{u}_{-\omega})^{*} + A_{\omega}\, \phi^{v}_{\omega}$$

$$\phi^{v,in}_{\omega} = \alpha^{v}_{\omega}\, \phi^{v}_{\omega} + \tilde{B}_{\omega} (\varphi^{u}_{-\omega})^{*} + \tilde{A}_{\omega}\, \phi^{u}_{\omega}$$

$$\phi^{u,in}_{-\omega} = \alpha_{-\omega}\, \phi^{u}_{-\omega} + \beta_{\omega} (\varphi^{u}_{\omega})^{*} + B_{\omega} (\varphi^{v}_{\omega})$$

Mean occupation number
in in-vacuum.

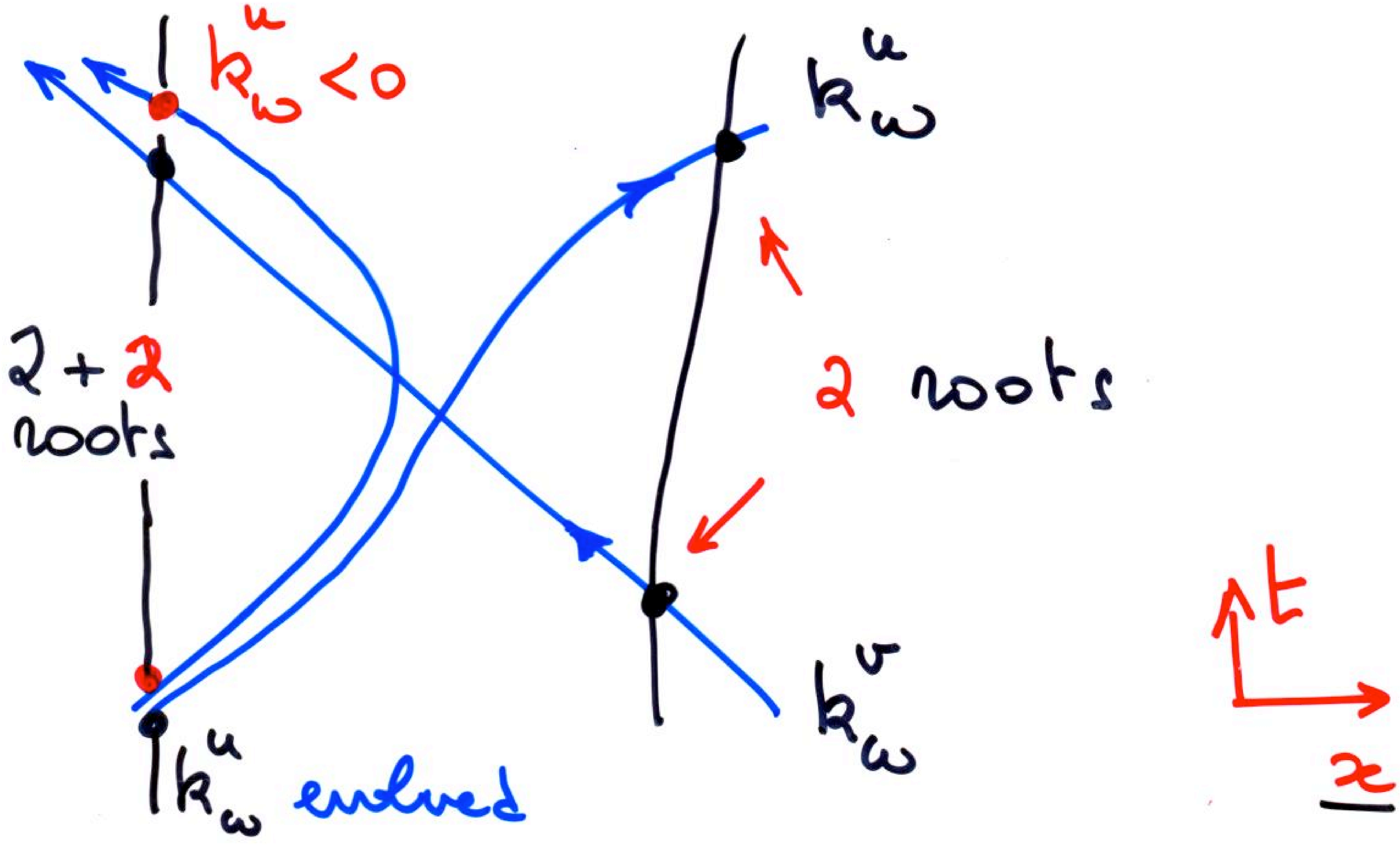
$$\begin{cases} \bar{n}^{u}_{\omega} = |\beta_{\omega}|^{2} \\ \bar{n}^{v}_{\omega} = |B_{\omega}|^{2} \\ \bar{n}^{u}_{-\omega} = \bar{n}^{u}_{\omega} + \bar{n}^{v}_{\omega} \end{cases} \qquad (0 < \omega < \omega_{max}.)$$

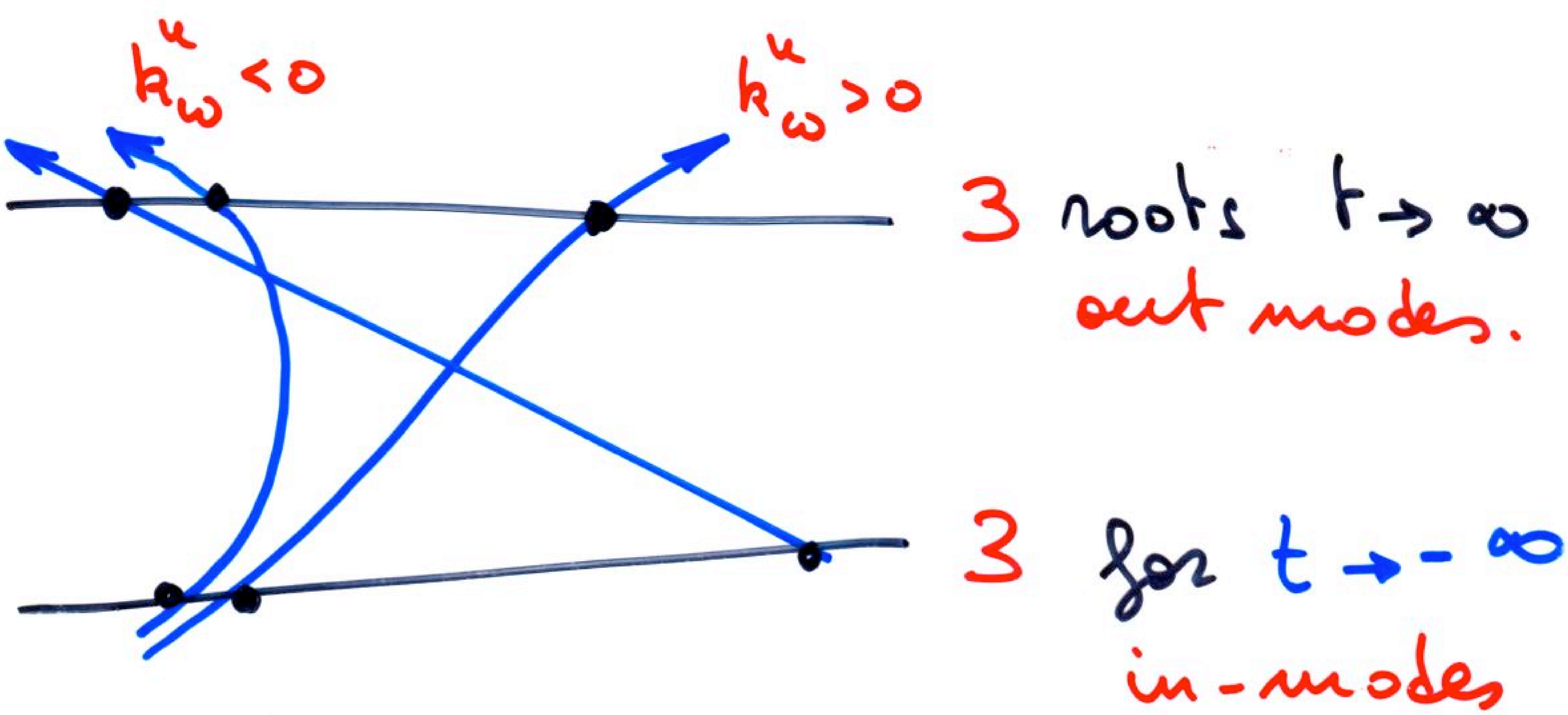
$(\bar{n}^{i} \equiv 0$
$\omega > \omega_{max})$

4 roots in supersonic flow

+ 2 roots in sub

= 3 initial + 3 final





Remarks

0°) This 3×3 Bog. transf.
IS the general case.

valid
- for any dispersive theory
- in any dimension (with symetries)

for
- stationary
- single horizon
- asymptotically "flat"

backgrounds.

Remarks.

1°) . $\bar{n}^u_\omega < \bar{n}^u_{-\omega}$

because 2 pair prod. <u>channels</u>

The partner of a u-quantum of freq. $-\omega$ can be

either a u-quant. of freq ω

or a v- " " " ω.

• <u>N.B.</u>

$$\boxed{\bar{n}^u_\omega = \bar{n}^u_{-\omega}}$$

only when no u-v mixing.

• Only 1 case: 2D massless relat. field

• could be good approximation.

(when u-v mixing small)

2°) $|\beta_\omega|^2$ can differ from $|\beta_{-\omega}|^2$
(because 3×3)

$$\beta_\omega = -\left(\varphi^{*u,out}_{\omega}, \phi^{u,in}_{-\omega}\right)$$

$$\beta_{-\omega} = -\left(\varphi^{*u,out}_{-\omega}, \phi^{u,in}_{\omega}\right)$$

and $\bar{n}_\omega = |\beta_\omega|^2 = \langle 0_{in}| a^\dagger_\omega a_\omega |0, in\rangle$

$\Rightarrow |\beta_{-\omega}|^2 =$ which observable ?

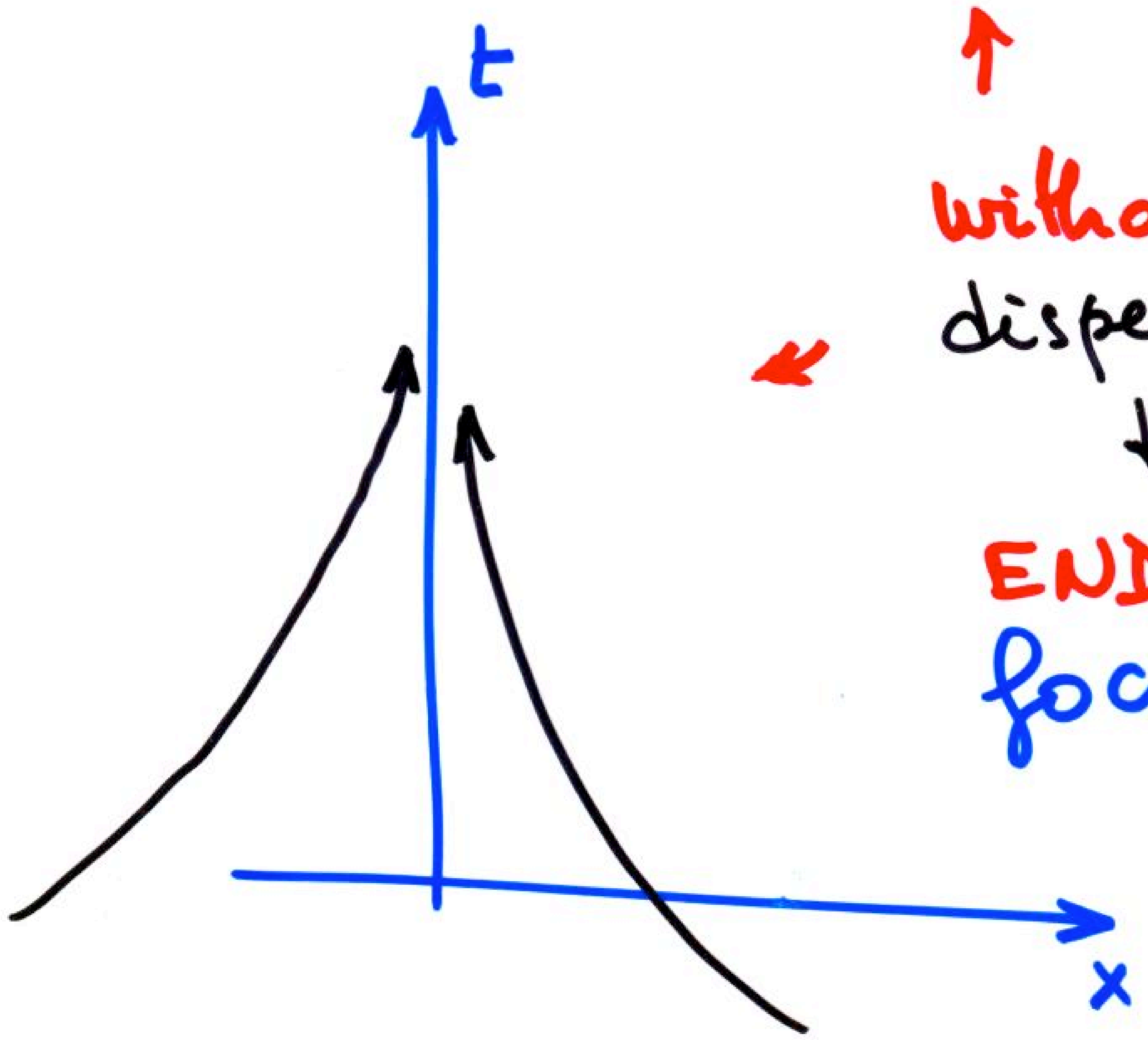
White Hole vs BH fluxes.

- under $\begin{cases} v_0 \to -v_0 > 0 & \text{: flow to right} \\ c(x) \text{ unchanged} \end{cases}$

one obtains a WH (for left movers)

i.e. $\frac{dk}{dt} = +k\kappa$: $k = k_0\, e^{+\kappa t}$

$x = x_0\, e^{-\kappa t}$



without dispersion.

$\Downarrow$

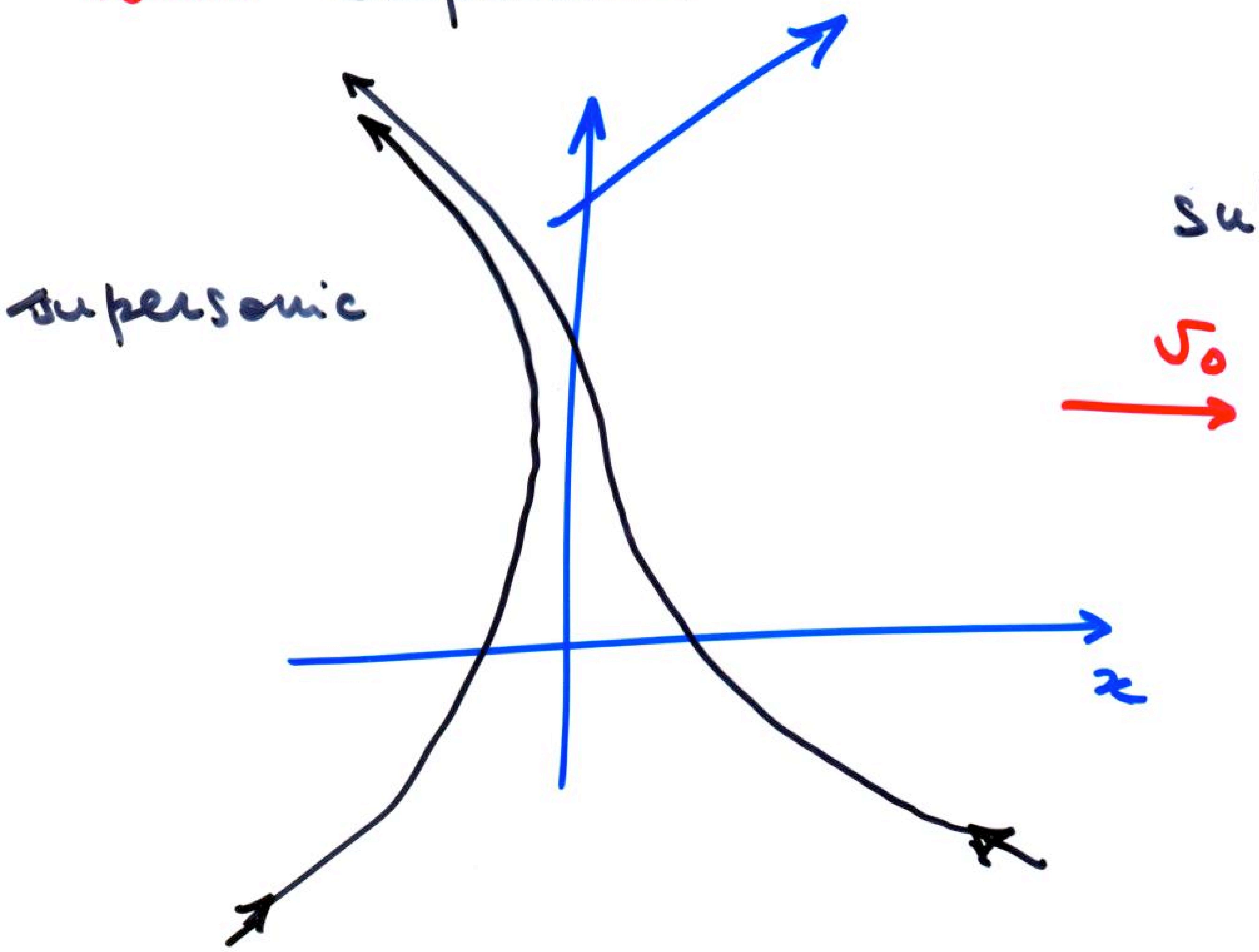
ENDLESS focussing

$\Downarrow$

NO radiation

(BEC)

With dispersion



- the focusing stops when $k \sim \Lambda \sim \hbar/m$!
- Pairs are produced and are radiated away
- right movers are also produced

WH fluxes vs BH fluxes.

- BdG eq. is invariant under $\begin{cases} v_0 \to -v_0 \\ k \to -k \quad (\partial_x \to -\partial_x) \end{cases}$

- $\underline{\text{in}} \begin{cases} \text{left} \\ \text{right} \end{cases} \leftrightarrow \underline{\text{out}} \begin{cases} \text{right movers.} \\ \text{left movers.} \end{cases}$

$$\Rightarrow \begin{cases} \boxed{\bar{n}_{-\omega}^{WH} = \bar{n}_{-\omega}^{BH}} \qquad (1) \\ \bar{n}_{\omega}^{WH} = |\beta_{-\omega}|^2 \neq \bar{n}_{\omega}^{BH} \\ \bar{n}_{\omega}^{v\,WH} = |\tilde{B}_{\omega}|^2 \neq \bar{n}_{\omega}^{v\,BH} \end{cases}$$

(1) is time reversal symmetry:

$$|\langle 0, in | 0, out \rangle|^2 = (\bar{n}_{-\omega} + 1)^{-1}$$

Lessons

(Single)

1. WH, in dispersive theories,

- are STABLE (BdG eq.)
- emit a steady radiation
- their spectra $\bar{n}^i_\omega$ are in 1 to 1 relation with BH ones

* when $|B^v_\omega|^2 \ll |\beta^u_\omega|^2$,

$$\bar{n}^{WH}_\omega \simeq \bar{n}^{BH}_\omega$$

(weak u-v mixing)

2. But $\bar{k}^{WH}_{fin} \sim \Lambda = (l_{healing})^{-1}$

whereas $\bar{k}^{BH}_{fin} \sim \bar{\omega} \sim \frac{\kappa}{2\pi}$

$\Rightarrow$ Experimenting

White Hole

radiation ?

(RP '01)

- through fluxes ?
- correlations ?

1

in BEC, T_0, the <u>condensate temperature</u> is never 0.

- $kT_0 \sim \frac{\hbar c}{\xi} \sim mc^2$ $\qquad \left(\xi \doteq \frac{\hbar}{2mc} \right)$ "healing length"

- <u>BAD NEW</u>

$$\partial_x (c+v)\Big|_{hor} = \kappa < 1/\xi$$

$$\Rightarrow \boxed{T_{Hawking} < T_0} \qquad \text{"always"}$$

$\Rightarrow$ Hawking radiation <u>hidden</u> ?

• In the in-vacuum :

$$\boxed{\bar{n}_\omega = |\beta_\omega|^2}$$

: mean occ. number of out phonons.

• in a non-vacuum state :

$$\boxed{\begin{aligned}\bar{n}_\omega = \bar{n}_\omega^{\,in} &+ |A_\omega|^2 \left(\bar{n}_\omega^{\,v\,in} - \bar{n}_\omega^{\,in}\right) \\ &+ |\beta_\omega|^2 \left(1 + \bar{n}_\omega^{\,in} + \bar{n}_{-\omega}^{\,in}\right)\end{aligned}}$$

where $\bar{n}_\omega^{\,in} \doteq \mathrm{Tr}\left[\rho_{in} \; a_\omega^{in\,+} a_\omega^{in}\right]$

$|A_\omega|^2$ term : elastic scattering $v \leftrightarrow u$

$|\beta_\omega|^2$ term : spontaneous + <u>stimulated</u> emission.

Conclusion : for $\frac{\omega}{\kappa} > 0,2$

$\bar{n}_\omega^{in}$ dominates / hides

Hawk. rad.

$\Rightarrow$ marginally possible detection.

$\Rightarrow$ look for other observables

What about

<u>long distance correlations</u> ?

(rather than fluxes)

BEC : (Balbinot et al '07
Carusotto et al '08

non-lin. opt. (Leonhardt + RP '04

The <u>origin</u> of the correlations.

HR is pair production.

$$|0, in\rangle = e^{\beta/\alpha \, a^\dagger_\omega a^\dagger_{-\omega}} |0, out\rangle$$

$$= |0, out\rangle + \frac{\beta}{\alpha} |1_\omega, 1_{-\omega}\rangle + \cdots$$

$\Rightarrow$ EPR correlations. between <u>u modes</u> across the horizon

$$\boxed{\text{What about their space-time properties?}}$$

These correlations show up through various means:

• ——— •

<u>NB</u>: Similar correlations are found for the Unruh effect.

0°] wave packets of in modes
(simple, naive, but correct)

1°] Stimulated emission.

Wald, Bekenstein '75
'76.

2°] Correlation functions :

$$G_2 = \langle \hat{T}_{\mu\nu}(x)\ \hat{T}_{\alpha\beta}(x') \rangle$$ Carlitz, Willey '87

(has a bump on the partner's trajectory)
(as in inflation Campo-RP 2003)

3°] Y. Aharonov "post selection".

$$\bar{T}_{\mu\nu}(x) \doteq \langle \hat{T}_{\mu\nu}(x)\ \hat{\Pi}_{\underline{final}} \rangle$$

"conditional mean value"

(Massar, RP '95.

$\in \mathbb{C}$!

Relativistic propagation.

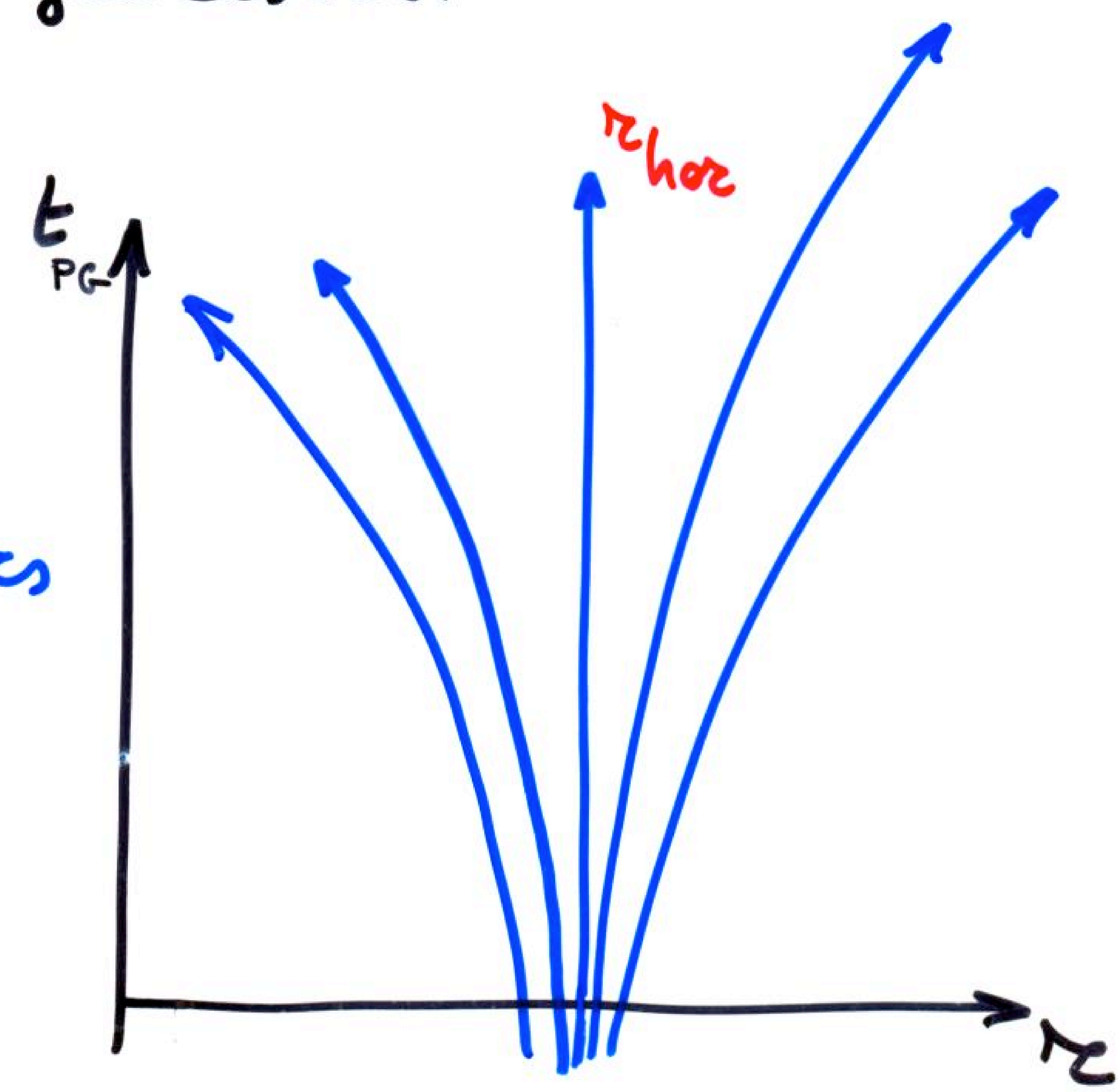
Statio. BH metric in PG coordin.

$$ds^2 = -dt_{PG}^2 + (dr - v(r)\, dt_{PG})^2$$

$v(r_{hor.}) = -1$ (on the horizon).

The characteristics of $\Box \phi = 0$

are null geodesics.



outgoing null geodesics

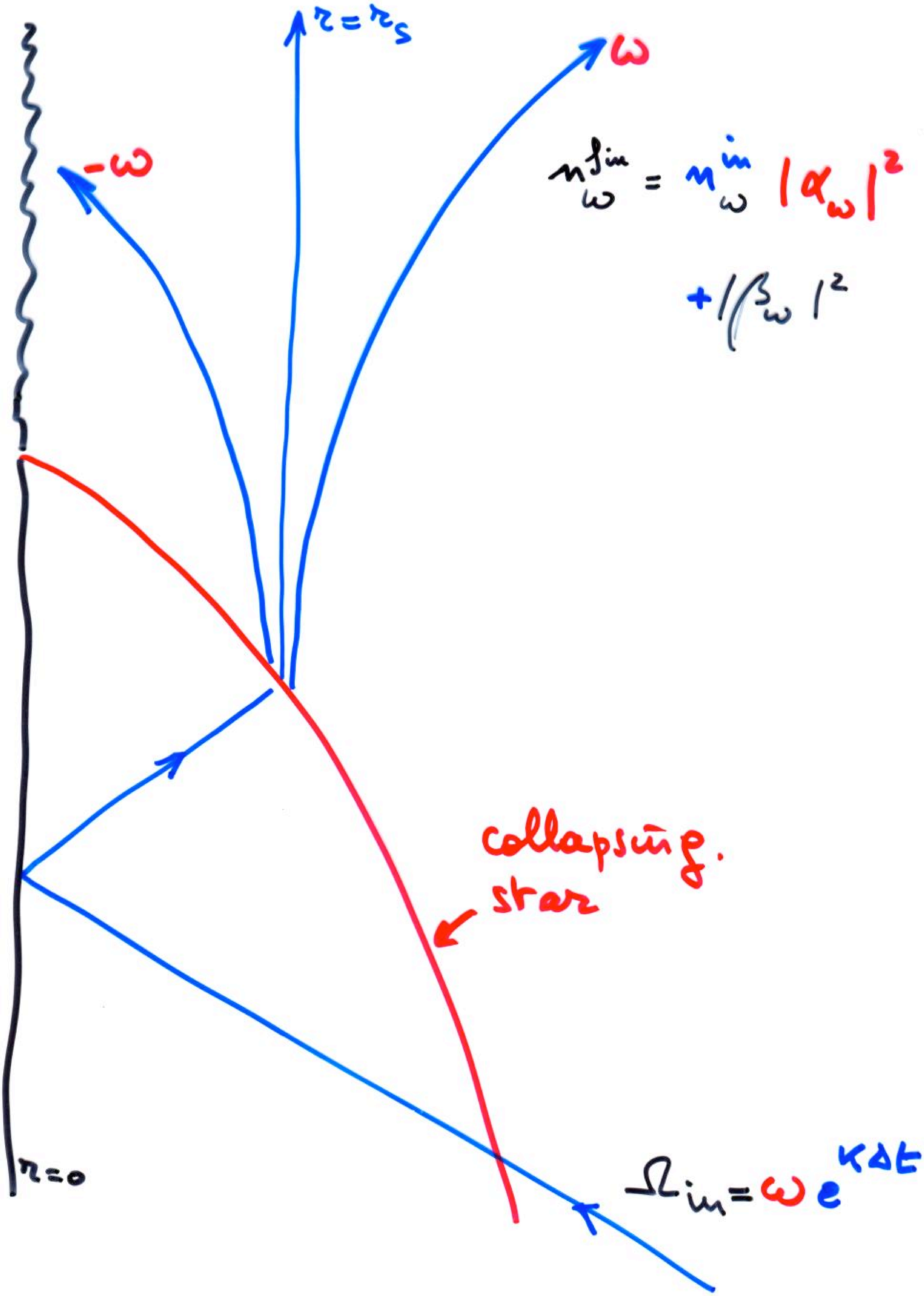
Two classes separated by r_{hor}.

Stimulated emission.

'75
76

$$n^{fin}_{\omega} = n^{in}_{\omega} |\alpha_\omega|^2 + |\beta_\omega|^2$$

$$n^{fin}_{-\omega} = n^{in}_{\omega} |\beta_\omega|^2 + |\beta_\omega|^2$$



Dispersive propagation. (BEC)

$$\Omega^2 = c^2 k^2 + \frac{k^4}{\Lambda^2}$$

Is the space-time pattern modified ?

completely

YES : the early UV pattern is modified

No : the late IR pattern is hardly modified

→ robust as well.

Fluxes versus correlations from 2-point functions.

$$G^{in}(x,x') \doteq \langle 0,in| \{\hat\phi(x), \hat\phi(x')\}_+ |0,in\rangle$$
$$= \int_{-\infty}^{\infty} d\omega\, e^{-i\omega\Delta t}\, G^{in}_\omega(x,x')$$

$$G^{in}_\omega(x,x') = \frac{Re}{2}\left[\varphi^{in}_\omega(x)\, \varphi^{in\,*}_\omega(x') + \left\{\begin{matrix}\omega\to-\omega \\ *\end{matrix}\right\} \right]$$

Flux. take $x = x' \to \infty$

$$\Rightarrow G^{in}_\omega = \underbrace{\tfrac{1}{2}(|\alpha_\omega|^2 + |\beta_\omega|^2)}_{\bar n_\omega + 1/2 \;:\; \text{power.}} |\phi^{out}_\omega|^2$$

long distance **Correlations** : $x = -x' \to \infty$ (spread $= 1/\kappa$)

$$G^{in}_\omega = \frac{Re}{2}\Big[\underbrace{\alpha_\omega \beta_\omega}_{=(\mathrm{sh}\,\omega/2T)^{-1}}\, \phi^{out}_\omega(x)\, \phi^{out}_{-\omega}(-x) \Big]$$

Remark

The flux is 'diagonal' whereas L.D. correlations are 'interfering':

$$\bar{n}_\omega \sim \underline{\langle 2 | a^\dagger_\omega a_\omega | 2 \rangle} \sim |\beta_\omega|^2$$

$$G_\omega(x,-x) \sim \underline{\langle 0 | a_\omega a_{-\omega} | 2 \rangle} \sim \alpha_\omega \beta_\omega$$

(in a thermal bath $\quad \mathrm{Tr}\left[e^{-\beta H} \hat{a}_\omega \hat{a}_{\omega'} \right] \equiv 0.$)

$\Rightarrow$ <u>no L.D.C.</u> !

In BEC :

$$\mathrm{Re}(\hat{\phi}) \doteq \hat{\chi} = \frac{\delta\hat{\rho}}{2\rho_0}$$: density fluctuation

consider :

$$G(t,x;0,x') \doteq \mathrm{Tr}\left[\hat{\rho}_{in}\{\hat{\chi}(t,x),\hat{\chi}(x')\}_+\right]$$
$$\doteq \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\, G_\omega(x;x')$$

focus on long distance correl.
(far away from horizon)

$$\Rightarrow \chi_\omega^{out,i} \to \chi_\omega^{as,i}(x) = e^{ik_\omega^i x}\,\chi_\omega^i$$

$i = (u, v, *)$ $\hookrightarrow$ 3 plane waves.

$\uparrow$ $-\omega < 0$

$\underline{3}$ waves $\Rightarrow$ $\underline{3}$ types of correlations.

* $x \gg c/\kappa$: subsonic $\to$ only $\underset{\omega>0}{u}$.

  $-x' \gg c/\kappa$: supers. $\to$ $\underset{\omega>0}{v}$ and $\underset{\omega<0}{u}$

$$\Rightarrow \quad G_\omega(x,x') = \chi_\omega^{as,u}(x) \times$$

ly
ssibility

$$\left[\underline{A_\omega}\left(\chi_\omega^{as,v}(x')\right)^* + \underline{B_\omega}\, \chi_{-\omega}^{as,u}(x')\right]$$

* when both $-x, -x' \gg c/\kappa$:

$$G_\omega(x,x') = \underline{C_\omega}\, \chi_\omega^{as,v}(x)\ \ \chi_{-\omega}^{as,u}(x')$$

$\to$ 3 coefficients $A_\omega, B_\omega, C_\omega$

The B_ω coefficient.

$$B_\omega \doteq \mathrm{Tr}\left[\hat{\rho}_{in}\; a^{out,u}_{\omega}\; a^{out,u}_{-\omega}\right] \in \mathbb{C}$$

$$= \bar{n}^{in\,u}_{\omega}\, \alpha_\omega \beta^*_{-\omega} + \bar{n}^{in\,v}_{\omega}\, A_\omega B^*_\omega$$

$$+ \left(\bar{n}^{in\,u}_{-\omega} \underline{+1}\right) \beta^*_\omega\, \alpha_{-\omega}$$

- The first 2 are absent in the in-vacuum.

- The last one is <u>amplified</u> when $\bar{n}^{in,u}_{-\omega} \neq 0$. (<u>No</u> phase shift.)

- When u-v mixing small $\Leftrightarrow$ $|A_\omega B_\omega| \ll 1$

$\Rightarrow$ $\alpha_\omega \beta^*_{-\omega} = \beta^*_\omega\, \alpha_{-\omega}$ $\Leftarrow$ (double amplific.)

<u>1st lesson</u> In a thermal state,

unlike fluxes which receive additional contributions due to $\bar{n}_\omega^{in} \neq 0$,

<u>correlations</u> get amplified by a <u>multiplicative</u> factor $\in \mathbb{R}^+$ containing $\bar{n}_\omega^{in} \neq 0$.

$\Rightarrow$ The vacuum pattern is amplified <u>without</u> being altered!!

(numerically observed by Carusotto et al '08).

The space-time pattern.

* computing $G(t,x;t,x')$ (equal t)

  the $\int^{\omega_{max}} d\omega$ gives constructive interfer.

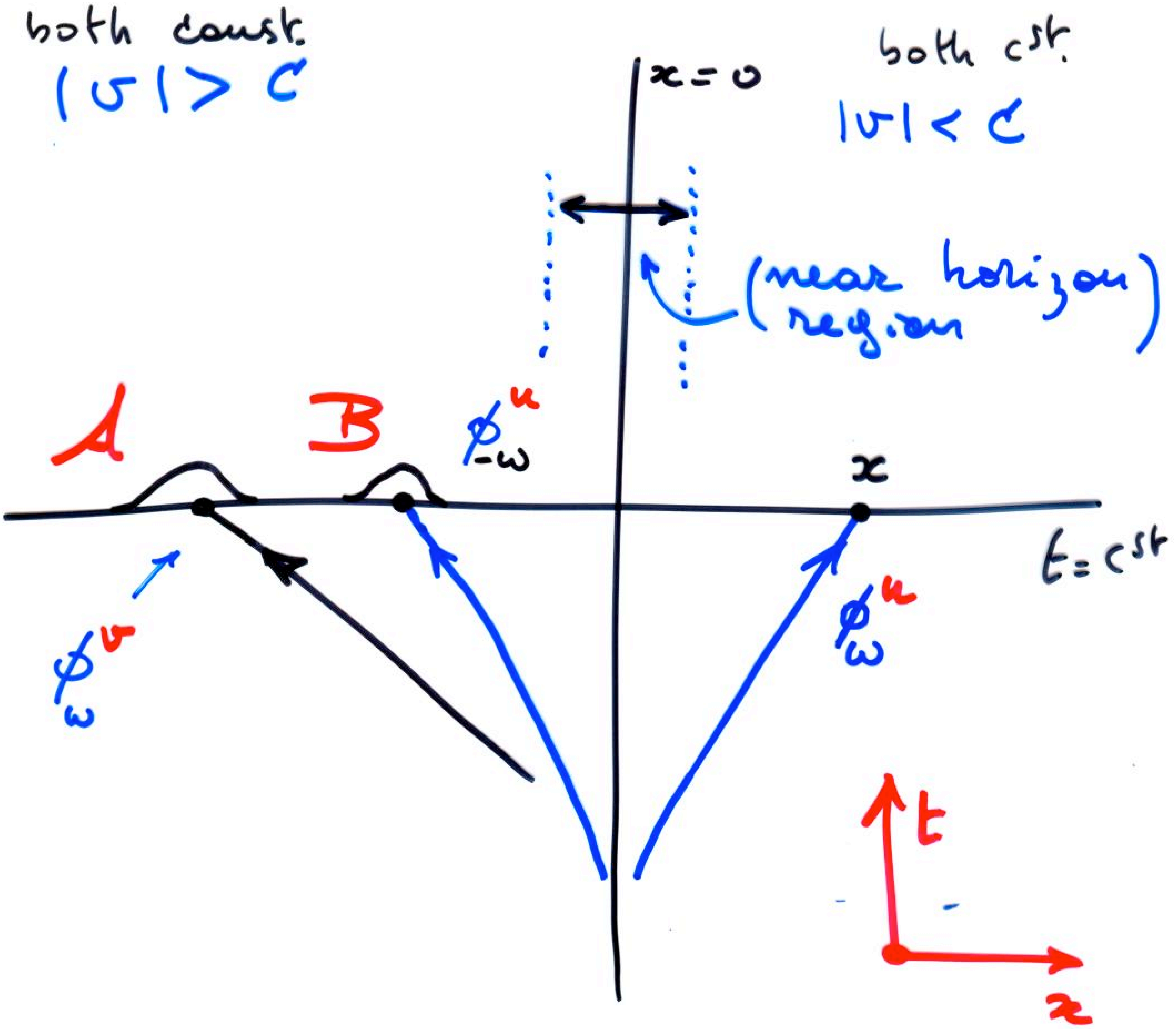
  along 'correlated' characteristics.

  (MTW '71)

  i.e. fixing $\boxed{x \gg c/\kappa}$, $G(x')$

  has two bumps where x'

  hits one of these 2 charact.



This is exactly what Carusotto et àl numerically obtained.

⚠ <u>weakness</u> of the signal ?

Amplification ? by $c^2 \to 0$. Cornell '09.

N.B.

- The correlation pattern is very robust. (too robust?)

- Found even when
  - $\kappa \to \infty$
  - $\kappa \to 0$

$\Rightarrow$ still a "signature" of Hawking radiation?

$\Rightarrow$ Reconsider: what would be a reliable signature of Hawking radiation?