$b \to X_s\gamma$ @ N^{2}LO$^{(*)}$ and feasibility of $b \to X_c \ell \bar{\nu}$ @ N^{3}LO

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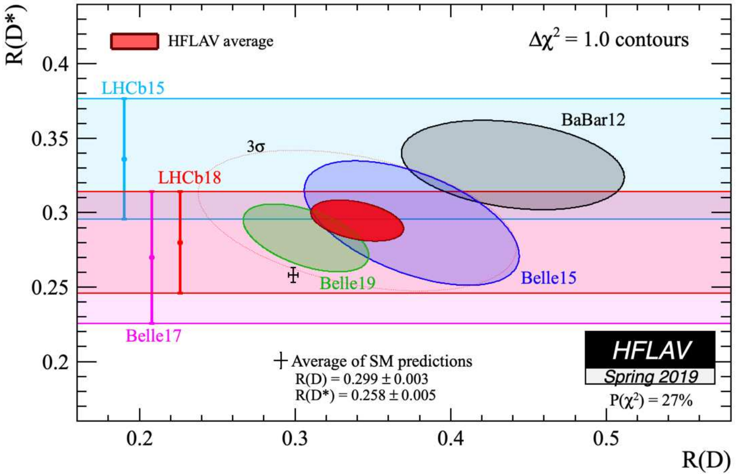
University of Warsaw

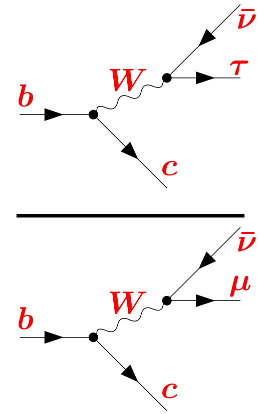
(*) In collaboration with Abdur Rehman and Matthias Steinhauser [arXiv:2002.01548],

as well as Mateusz Czaja, Tobias Huber and Go Mishima

1. Introduction
2. The radiative decay
   - *(i)* $\mathcal{O}(\alpha_s^2)$ contributions to $\hat{G}_{17}$ and $\hat{G}_{27}$
   - *(ii)* Non-perturbative effects in $\bar{B} \to X_s\gamma$
   - *(iii)* Updated SM predictions for $\mathcal{B}_{s\gamma}$ and R_γ
3. The semileptonic decay
   - *(i)* Motivation for $\mathcal{O}(\alpha_s^3)$
   - *(ii)* Challenges
4. Summary

$R(D)$ and $R(D^*)$ "anomalies" [https://hflav.web.cern.ch] (3.1σ)

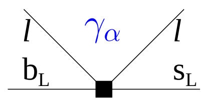


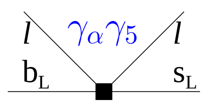


$$R(D^{(*)}) = \mathcal{B}(B \to D^{(*)}\tau\bar{\nu})/\mathcal{B}(B \to D^{(*)}\mu\bar{\nu})$$

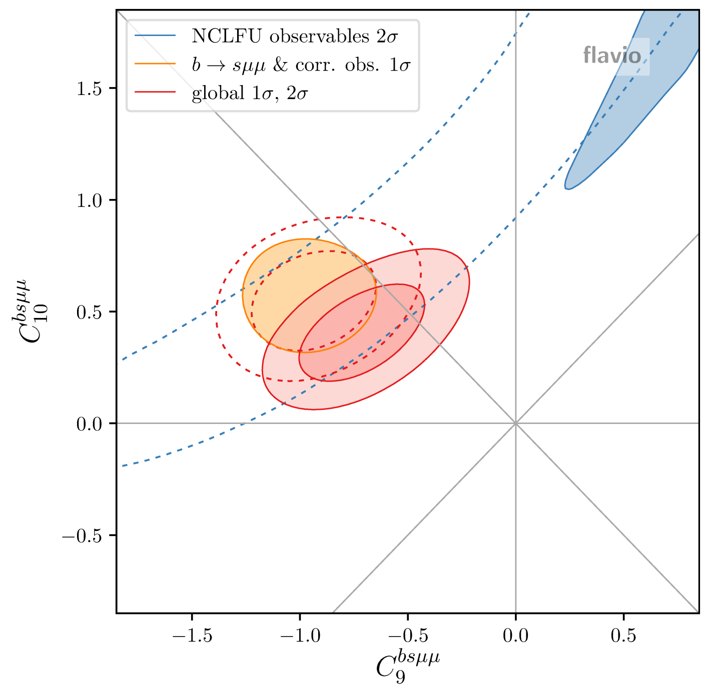
$b \to s\ell^+\ell^-$ "anomalies" $(> 5\sigma)$

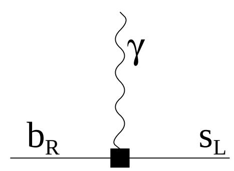
[see, e.g., J. Aebischer *et al.*, arXiv:1903.10434]

$Q_9^\ell =$ 

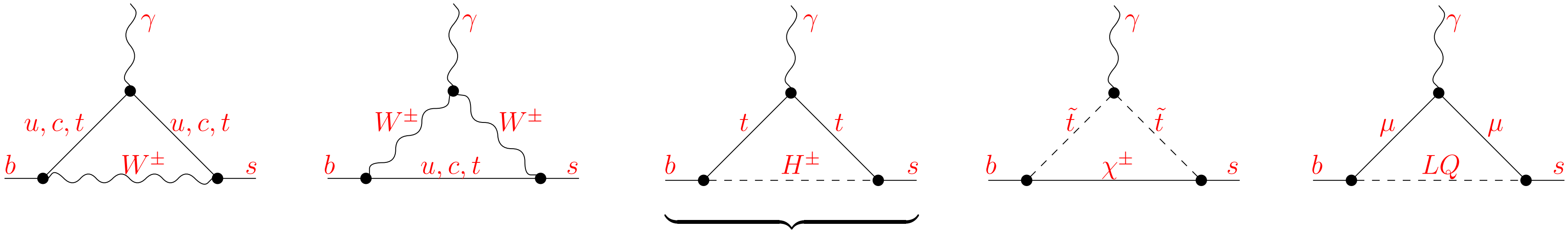
$Q_{10}^\ell =$ 

$\ell = e$ or μ



C_7, the Wilson coefficient of $Q_7 =$  is an important input in the fits.

Sample Leading-Order (LO) contributions to C_7 in the SM and beyond:



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in the 2HDM-II

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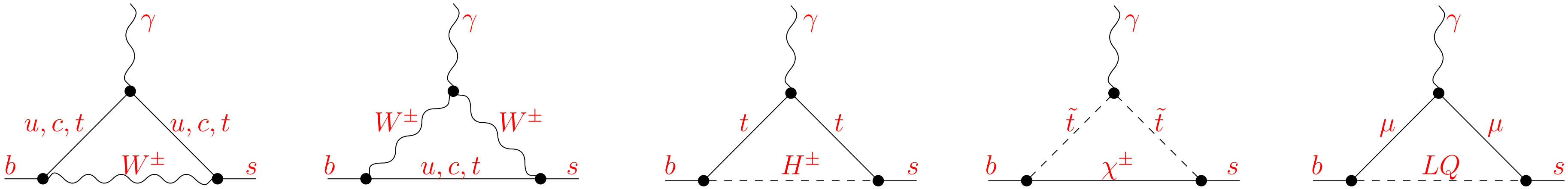

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The strongest experimental constraint on C_7 comes from $\mathcal{B}_{s\gamma}$ —
— the CP- and isospin-averaged BR of $\underset{(\bar{B}^0, B^-)}{\bar{B}} \to X_s\gamma$ and $\underset{(B^0, B^+)}{B} \to X_{\bar{s}}\gamma$.

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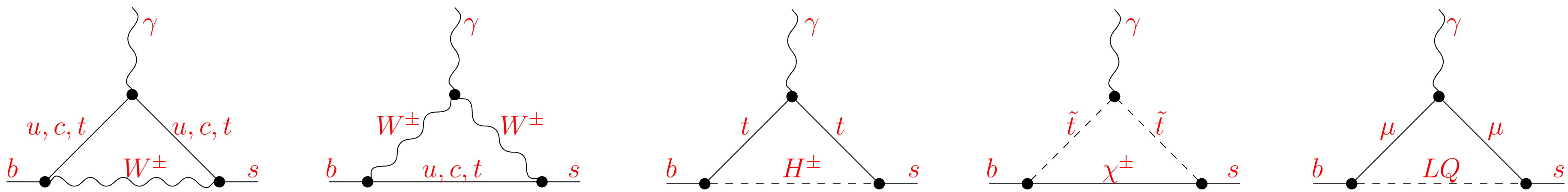

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HFLAV 2022, PDG 2022: $\mathcal{B}_{s\gamma}^{\rm exp} = \underset{(\pm 5.4\%)}{(3.49 \pm 0.19)} \times 10^{-4}$ **for** $E_\gamma > E_0 = 1.6\,\mathrm{GeV} \simeq \frac{m_b}{3}$,

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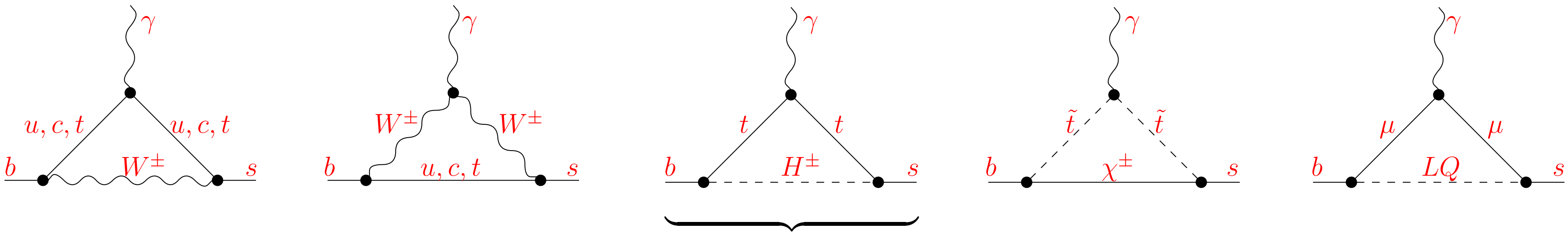

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With the full BELLE-II dataset, a $\pm 2.6\%$ **uncertainty in the world average for** $\mathcal{B}_{s\gamma}^{\text{exp}}$ **is expected.**

SM calculations must be improved to reach a similar precision.

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$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \mathcal{B}(\bar{B} \to X_c e\bar{\nu})_{\rm exp} \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\rm em}}{\pi\, C} \Big[\underbrace{P(E_0)}_{\substack{\text{pert.} \\ \sim 96\%}} + \underbrace{N(E_0)}_{\substack{\text{non-pert.} \\ \sim 4\%}} \Big]$$

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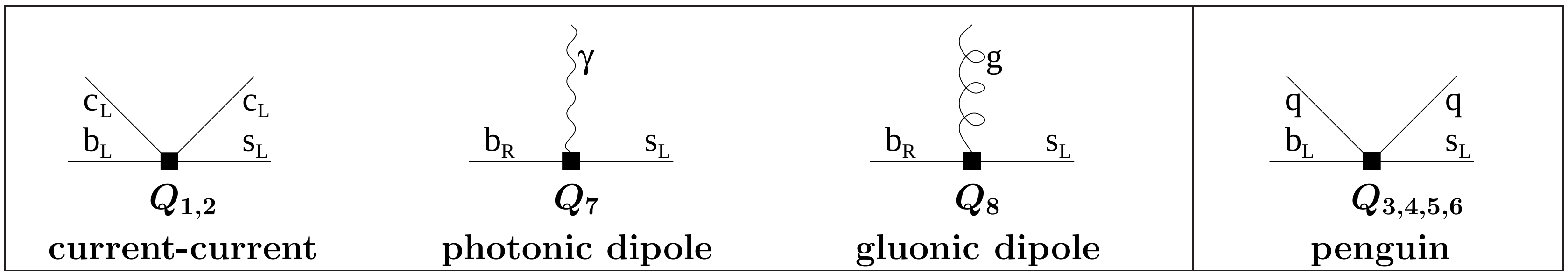

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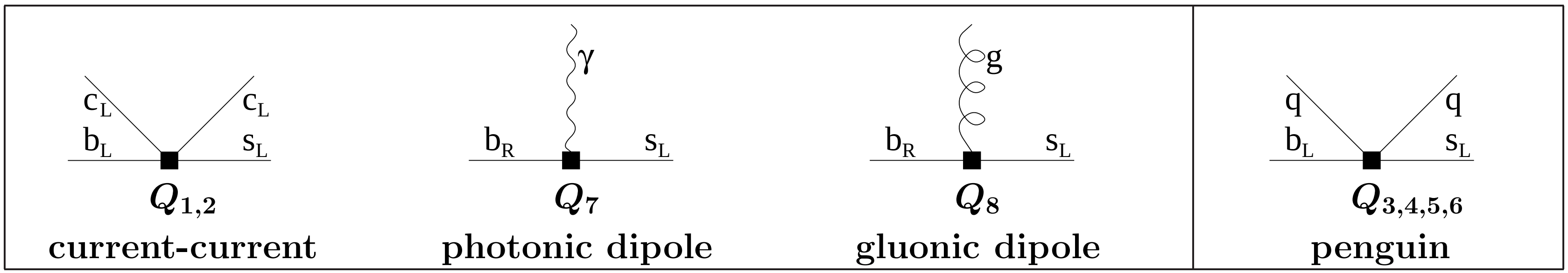

$Q_{1,2}$ current-current | Q_7 photonic dipole | Q_8 gluonic dipole | $Q_{3,4,5,6}$ penguin

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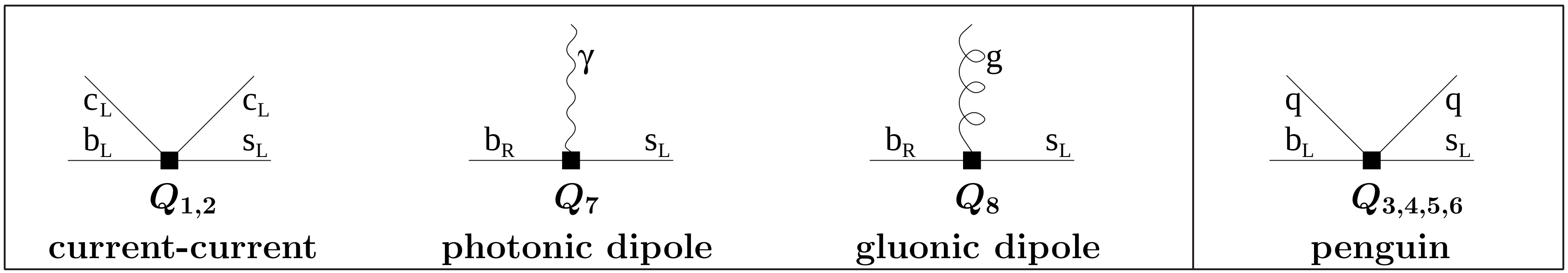

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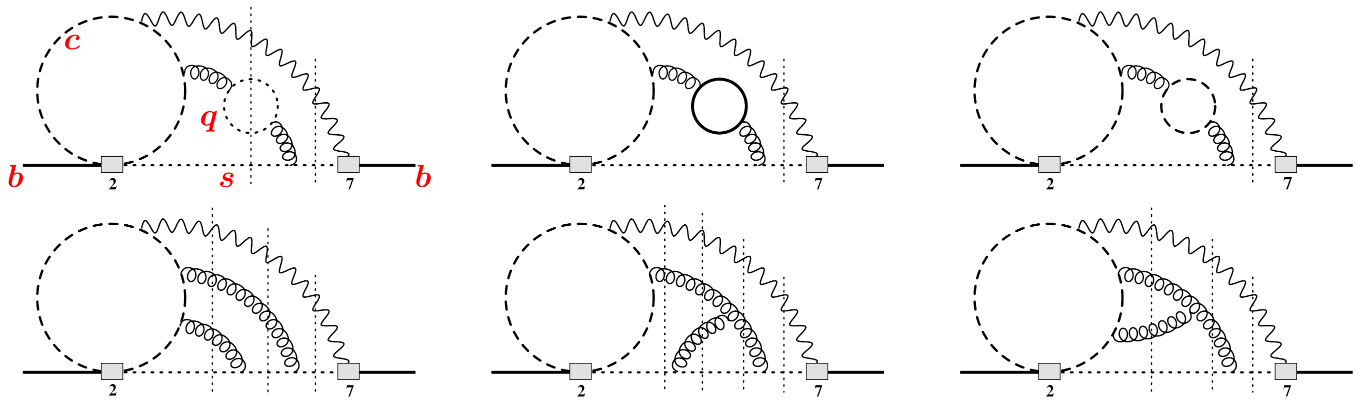
NLO ($\mathcal{O}(\alpha_s)$) – last missing pieces being evaluated by Tobias Huber and Lars-Thorben Moos [arXiv:1912.07916]

Most important @ NNLO ($\mathcal{O}(\alpha_s^2)$): $\underset{\text{known}}{\hat{G}_{77}}$, $\underset{\text{interpolated}}{\hat{G}_{17}, \hat{G}_{27}}$

between the $m_c \gg m_b$ and $m_c = 0$ limits [arXiv:1503.01791]

$\Rightarrow$ $\pm 3\%$ uncertainty in $\mathcal{B}_{s\gamma}^{\rm SM}$

Sample diagrams contributing to $\hat{G}_{27}$ @ NNLO:



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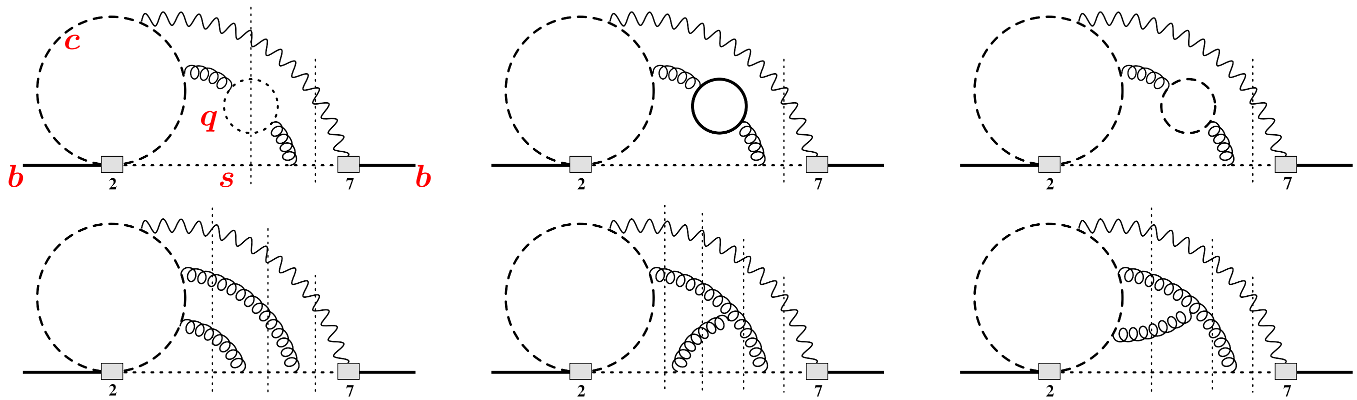

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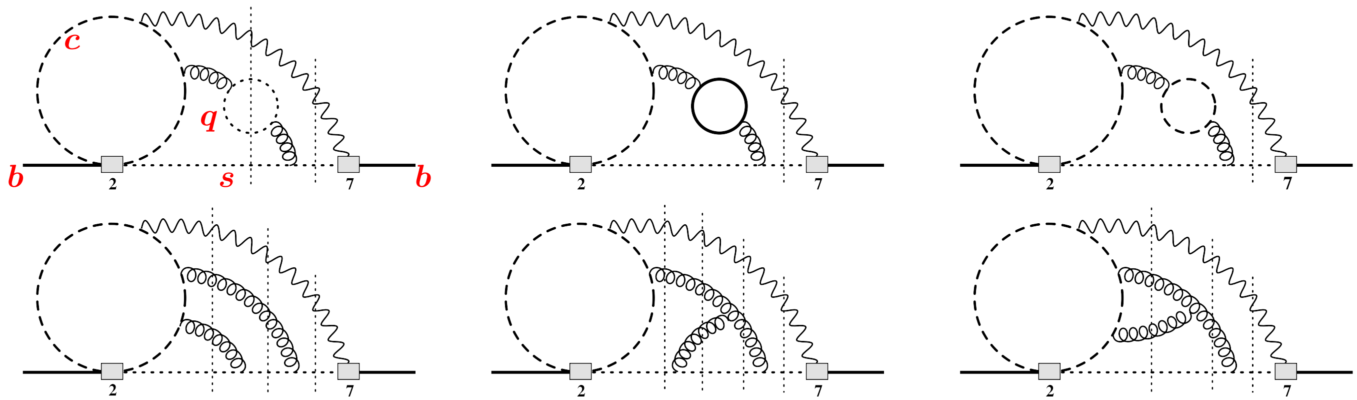

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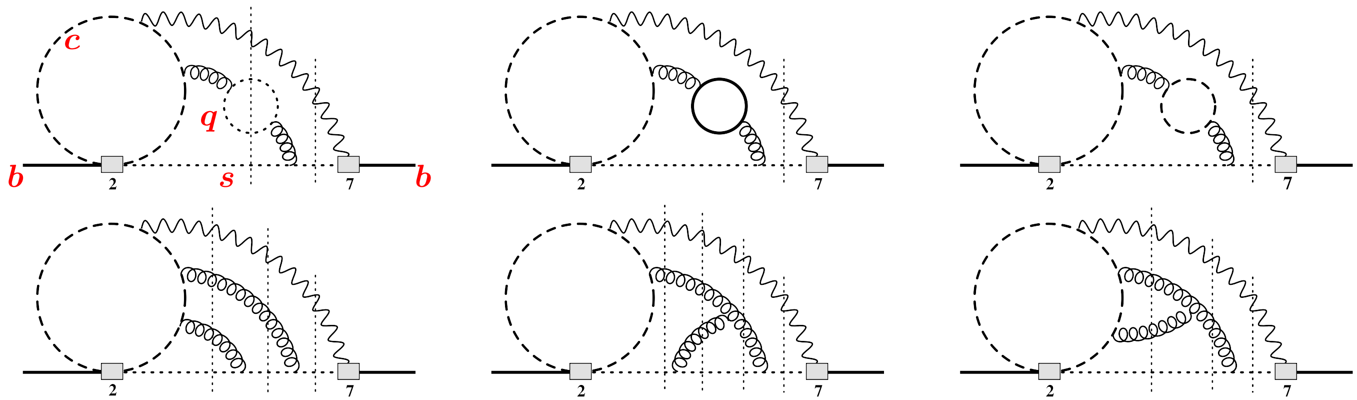

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$$\frac{d}{dz} M_k(z,\epsilon) = \sum_l R_{kl}(z,\epsilon)\, M_l(z,\epsilon), \qquad (*)$$

where R_{nk} are rational functions of their arguments.

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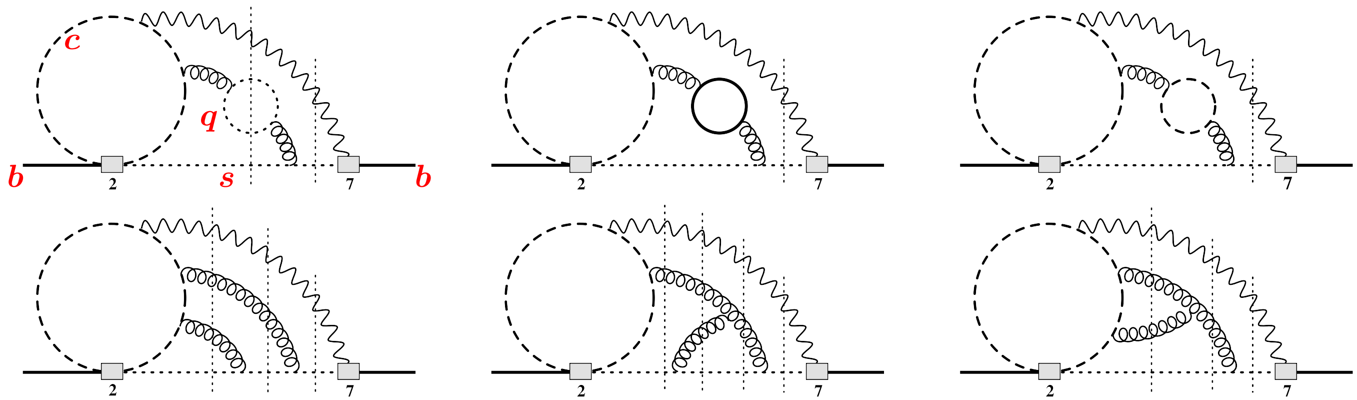

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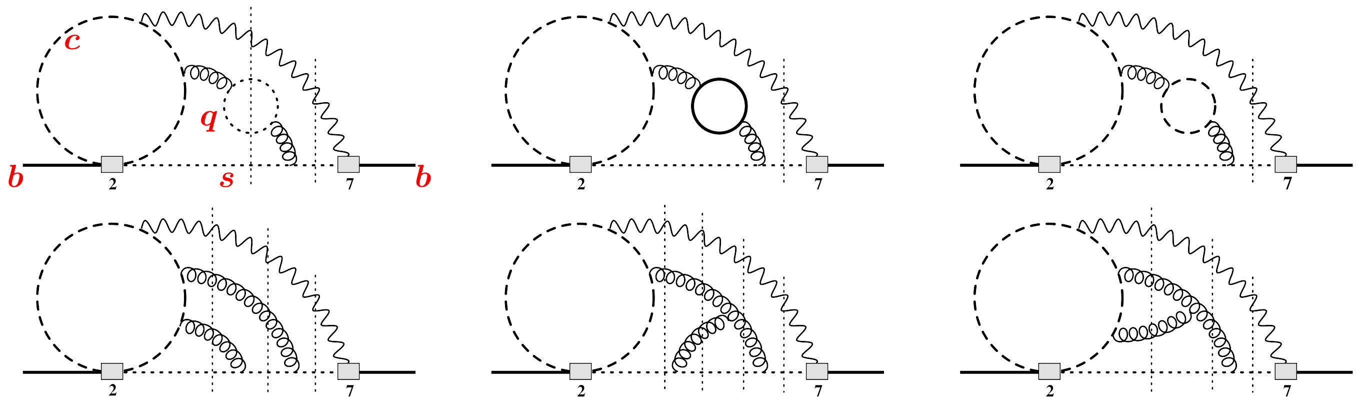

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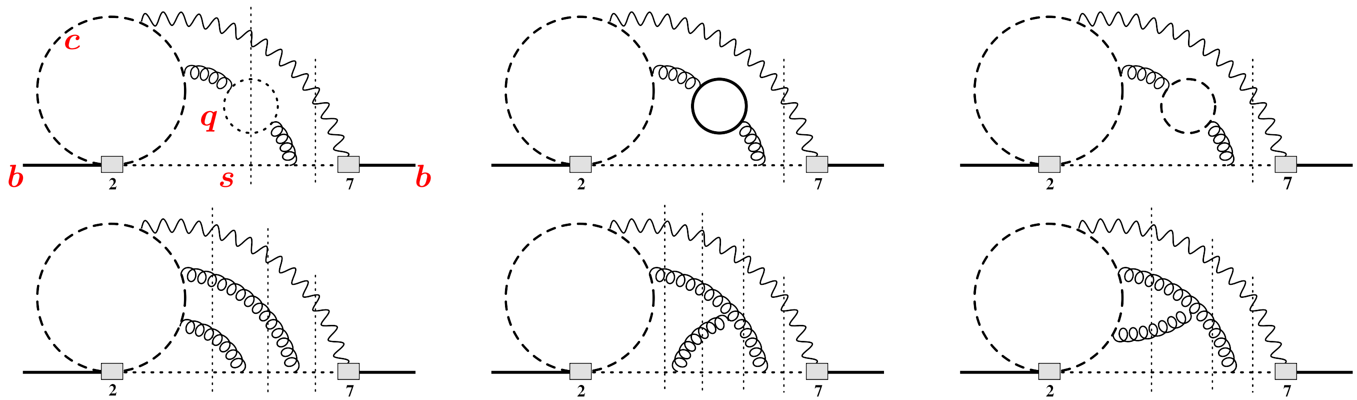

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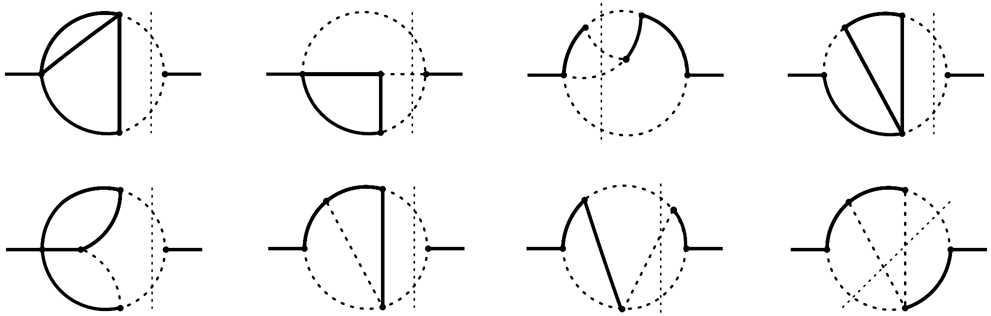
6. Solving the system $(*)$ numerically [A.C. Hindmarsch, `http://www.netlib.org/odepack`] along an ellipse in the complex z plane. Doing so along several different ellipses allows us to estimate the numerical error.

Sample three-loop propagator-type integrals that parameterize large-z expansions of the master integrals:



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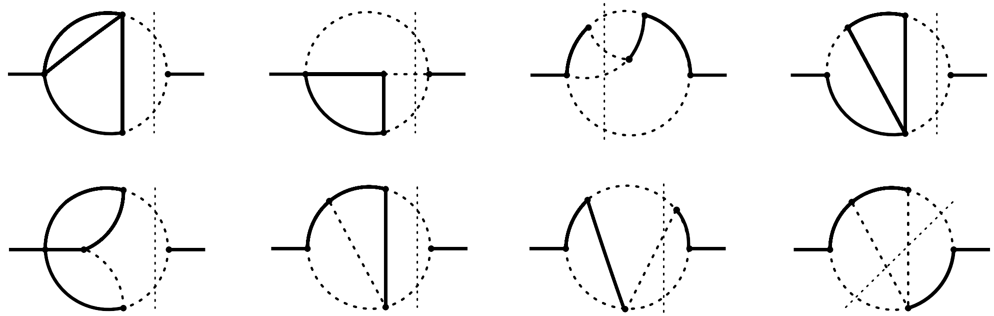
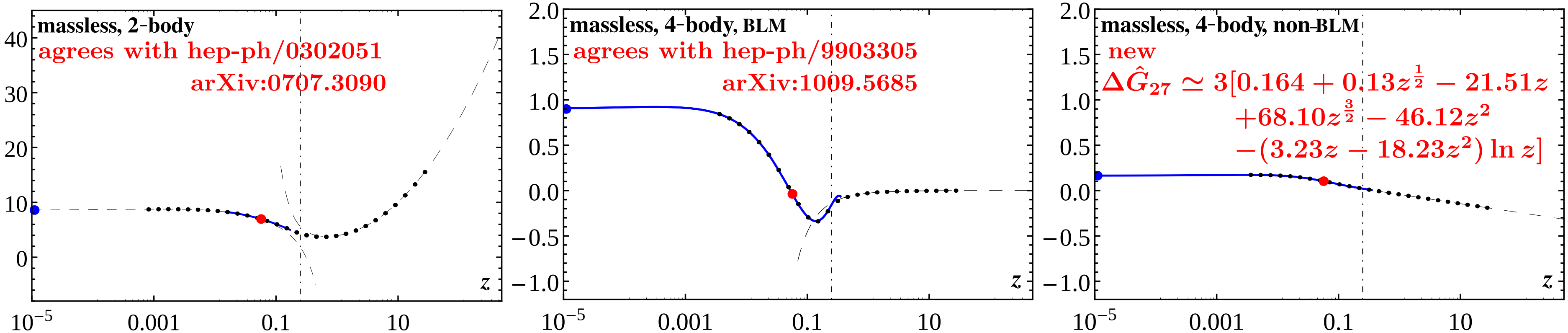
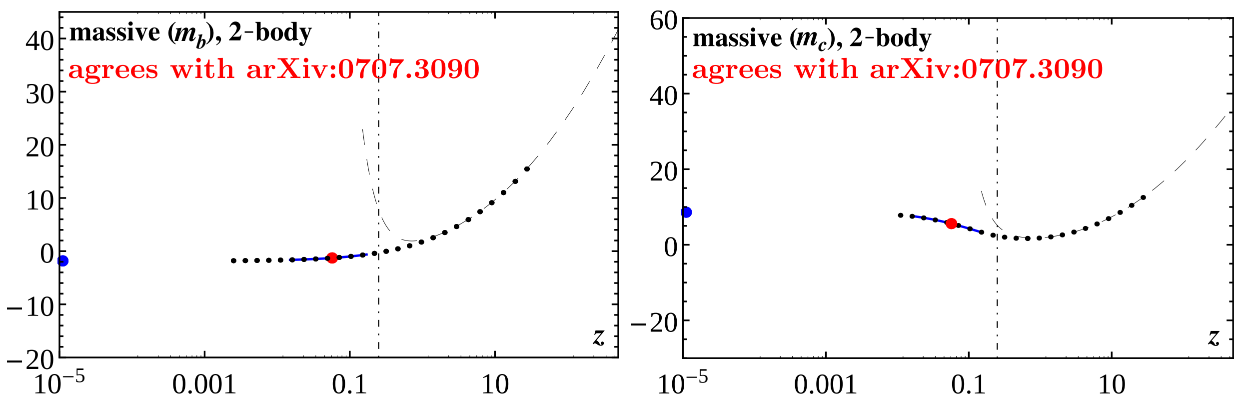
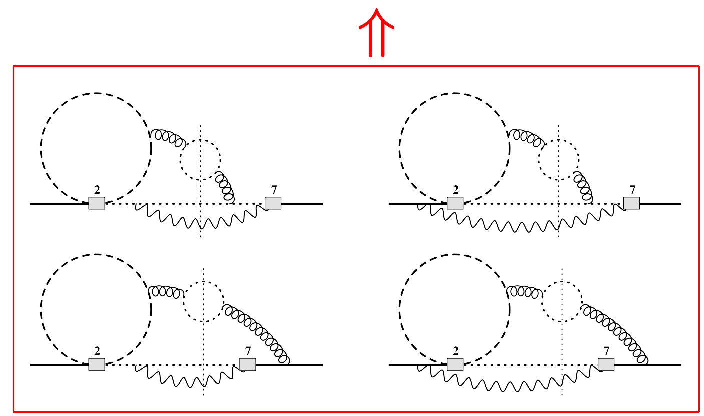


Contributions to $\hat{G}_{27}(E_0 = 0)$ from diagrams with closed loops of massless fermions



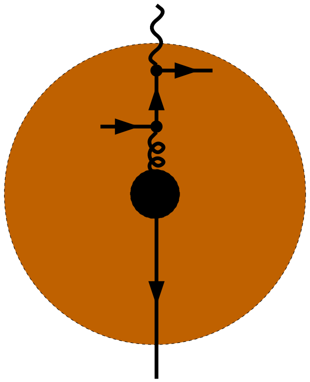
And from diagrams with closed loops of massive fermions





UV renormalization has been carried out using the results from arXiv:1702.07674.

Non-perturbative contribution from gluon-to-photon conversion in the QCD medium.



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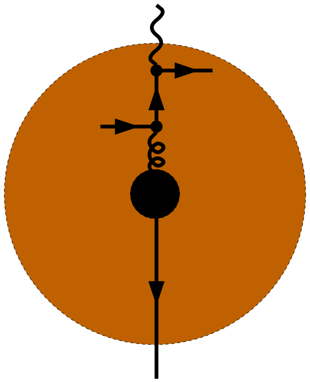

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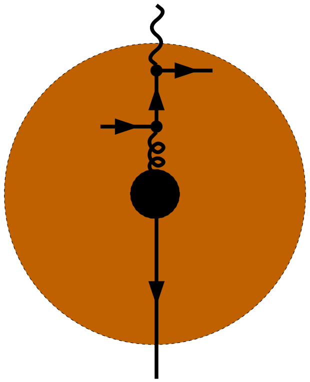

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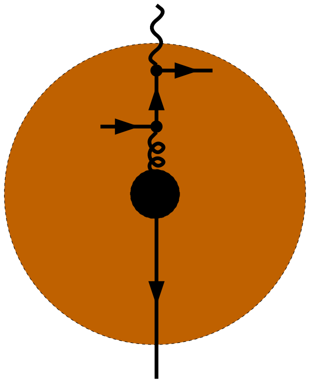

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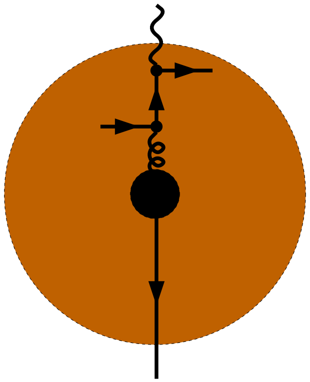

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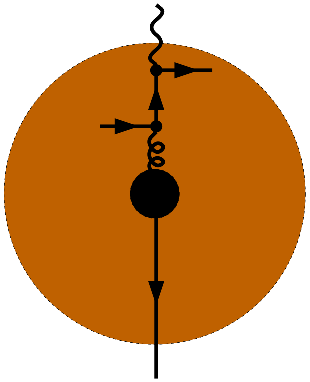

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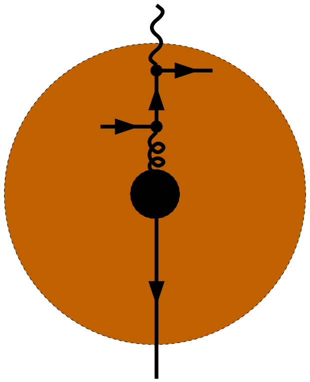

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$$\frac{\delta\Gamma_c}{\Gamma} \simeq -\tfrac{1}{3}\Delta_{0-}\left[1 + 2\,\tfrac{D-C}{C-B}\right] \;=\; -\tfrac{1}{3}\underbrace{(-0.48 \pm 1.49 \pm 0.97 \pm 1.15)}_{\text{Belle, arXiv:1807.04236, } E_0 = 1.9\,\text{GeV}}\% \times (1 \pm 0.3) \;=\; (0.16 \pm 0.74)\%$$

Recall: $(x \pm \sigma_x)(y \pm \sigma_y) = xy \pm \sqrt{(x\sigma_y)^2 + (y\sigma_x)^2 + (\sigma_x\sigma_y)^2}$

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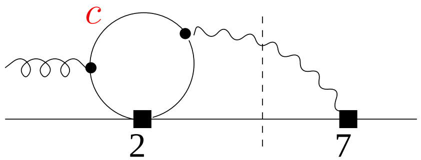
$\langle \bar{B} |$  $| \bar{B} \rangle$

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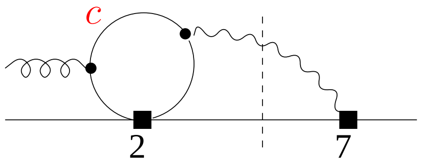
$\langle \bar{B} |$  $| \bar{B} \rangle$

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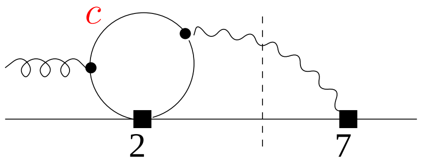
$\langle \bar{B}|$  $|\bar{B}\rangle$

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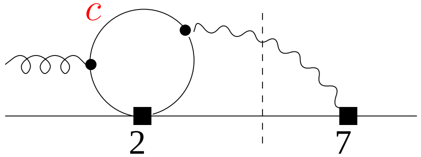
$\langle \bar{B} |$  $| \bar{B} \rangle$

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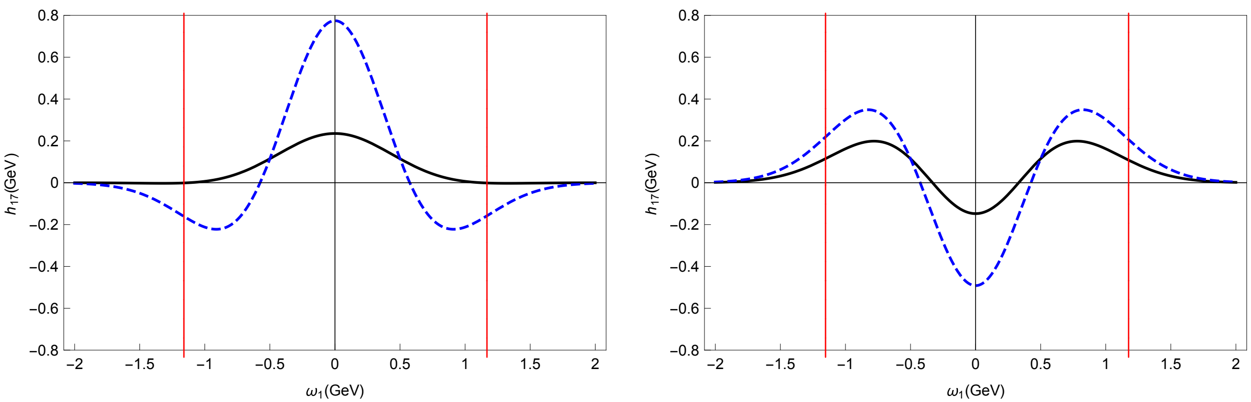

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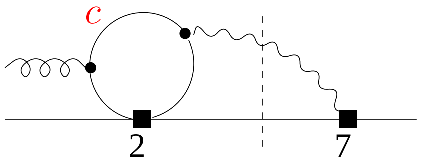
$\langle \bar{B} |$  $| \bar{B} \rangle$

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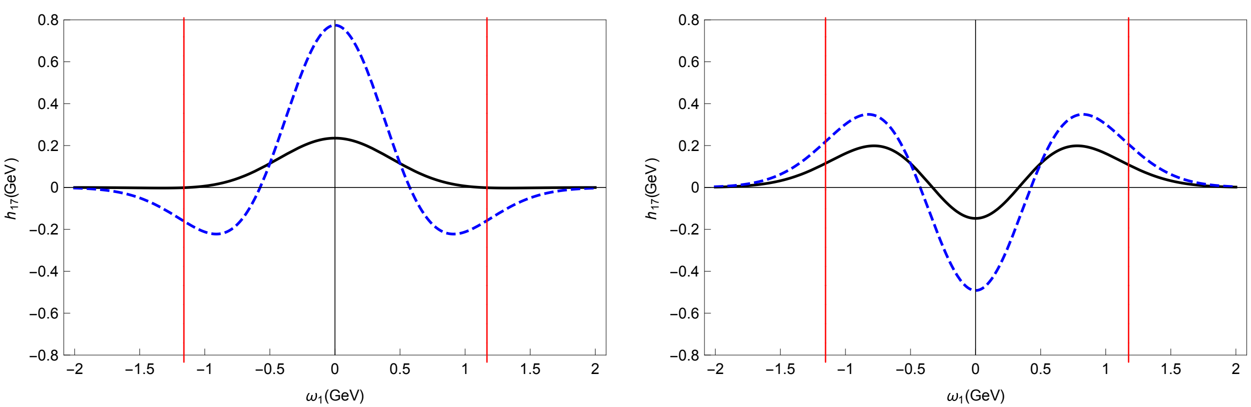
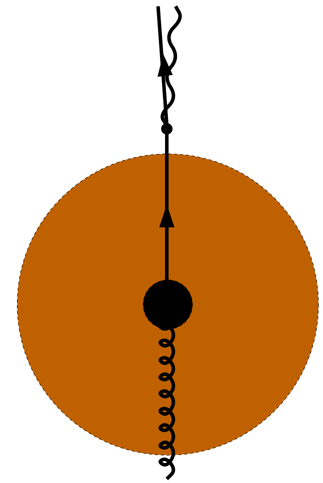


G+P numerically:
$\Lambda_{17} \in [-24, 5]$ MeV for $m_c = 1.17$ GeV.
Factor-of-3 improvement w.r.t. BLNP.

In our code: $\kappa_V = 1.2 \pm 0.3$.
Warning: scheme for m_c!

Non-perturbative contribution proportional to $|C_8|^2$



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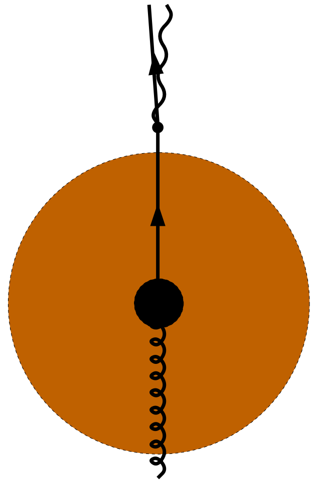

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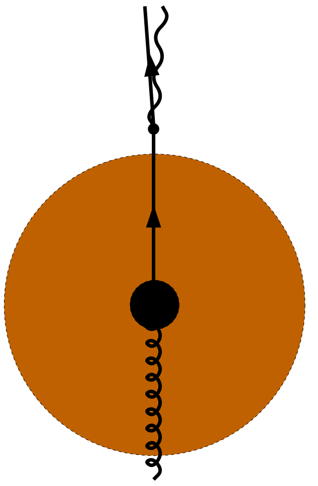

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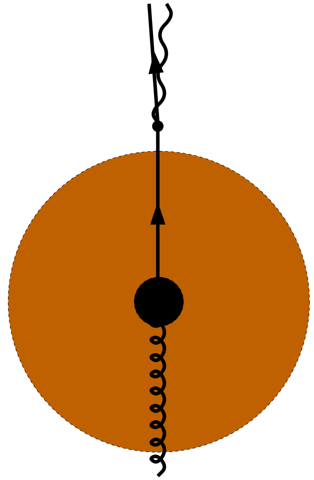

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Numerically, we can reproduce this range by performing a replacement

$$\ln \frac{m_b}{m_s} \to \kappa_{88} \ln 50 \qquad \text{with} \qquad \kappa_{88} = 1.7 \pm 1.1$$

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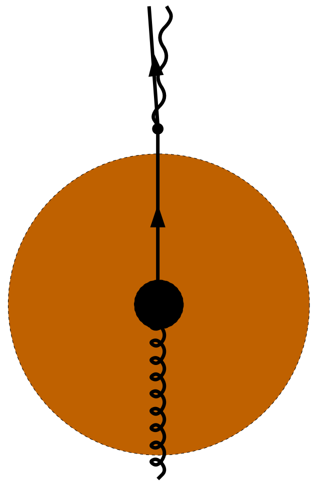

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$$\ln \frac{m_b}{m_s} \to \kappa_{88} \ln 50 \qquad \text{with} \qquad \kappa_{88} = 1.7 \pm 1.1$$

in all the perturbative contributions proportional to $|C_8|^2$.

The $[\ln 10, \ln 50]$ range remains used in other (small) terms where collinear logs arise.

Updated SM predictions for $\mathcal{B}_{s\gamma}$ and $R_\gamma \equiv \mathcal{B}_{(s+d)\gamma}/\mathcal{B}_{c\ell\bar{\nu}}$ (with $E_0 = 1.6\,\text{GeV}$):

$\mathcal{B}_{s\gamma} = (3.40 \underset{(\pm 5.0\%)}{\pm 0.17}) \times 10^{-4}$ compare to $(3.36 \underset{(\pm 6.9\%)}{\pm 0.23}) \times 10^{-4}$ in arXiv:1503.01789

$R_\gamma = (3.35 \underset{(\pm 4.8\%)}{\pm 0.16}) \times 10^{-3}$ compare to $(3.31 \underset{(\pm 6.7\%)}{\pm 0.22}) \times 10^{-3}$ in arXiv:1503.01789

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Shifts in uncertainties related to $\frac{\delta\Gamma_c}{\Gamma}$, κ_V and κ_{88}:

formerly: $1.25\% + 2.85\% + 1.10\% = 5.20\%$ (in quadrature: 3.30%)

at present: $0.74\% + 0.88\% + 0.92\% = 2.54\%$ (in quadrature: 1.48%) $\sqrt{1.48^2 + 2.01^2}\% = 2.49\% \simeq 2.5\%$

Summary for the radiative decay

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Determination of $|V_{cb}|$ from the inclusive $\bar{B} \to X_c \ell \nu$ rate and spectra

$$|V_{cb}| = (42.00 \pm \underbrace{0.64}_{1.5\%}) \times 10^{-3}$$

[P. Gambino, K. J. Healey and S. Turczyk, arXiv:1606.06174]

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$$\sqrt{\underbrace{(3.0\%)^2}_{|V_{cb}|^2} + \underbrace{(2.3\%)^2}_{\text{other}}} \simeq 3.8\%$$

[C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou and M. Steinhauser, arXiv:1311.0903],
[M. Beneke, C. Bobeth and R. Szafron, arXiv:1908.07011].

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Impact on the uncertainty in the SM prediction for ϵ_K:

$$\sqrt{\underbrace{(5.3\%)^2}_{|V_{cb}|^4} + \underbrace{(6.4\%)^2}_{\text{other}}} \simeq 8.3\% \quad \text{(roughly)}$$

using Eq. (17) of [J. Brod, M. Gorbahn and E. Stamou, arXiv:1911.06822].

Inclusive Decays

- Optical Theorem
- OPE – Heavy Quark Expansion (HQE): $p_b = m_b v_B + k$

Observables can be written as:

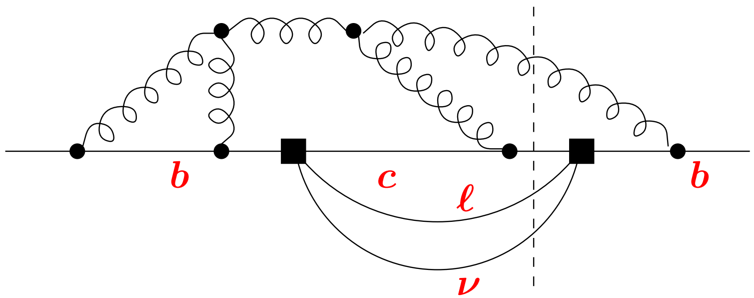
$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \ldots$$

- $d\Gamma_i$ are computed in **perturbative QCD**
- The non-perturbative dynamics is enclosed into the HQE parameters: $\mu_\pi, \mu_G, \rho_D, \rho_{LS} \sim \langle B | \, \bar{b}_v iD^\mu \ldots iD^\nu \Gamma_{\mu\ldots\nu} b_v \, | B \rangle$
- HQE parameters are extracted from data.

Reviews:
Benson, Bigi, Mannel, Uraltsev, Nucl.Phys. B665 (2003) 367;
Dingfelder, Mannel, Rev.Mod.Phys. 88 (2016) 035008.

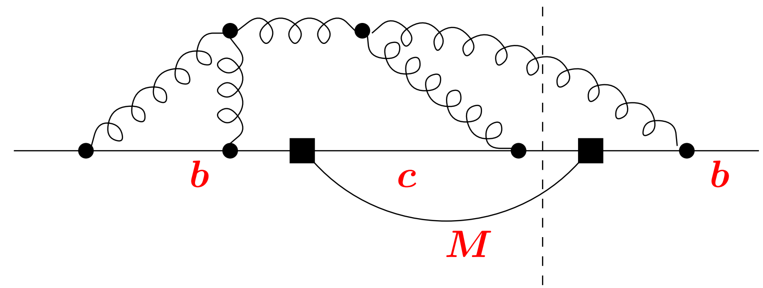
	tree	α_s	α_s^2	α_s^3	
1	✓	✓	✓	!	Jezabek, Kuhn, NPB 314 (1989) 1; Gambino et al., NPB 719 (2005) 77; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015.
μ_π	✓	✓	!		Becher, Boos, Lunghi, JHEP 0712 (2007) 062.
μ_G	✓	✓	!		Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PRD 92 (2015) 054025.
ρ_D	✓	✓			Mannel, Pivovarov, PRD100 (2019) 093001.
ρ_{LS}	✓	!			
$1/m_b^4$	✓				Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
$1/m_b^5$	✓				Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109
m_b^{kin}		✓	✓		Bigi, Shifman, Uraltsev, Vainshtein, PRD 56 (1997) 4017; Czarnecki, Melnikov, Uraltsev, PRL 80 (1998) 3189.

Feasibility of $b \to X_c \ell \bar{\nu}$ @ N^{3}LO



contribution to Γ

$\longrightarrow$



contribution to $d\Gamma/dq^2$ for $q^2 = M^2$

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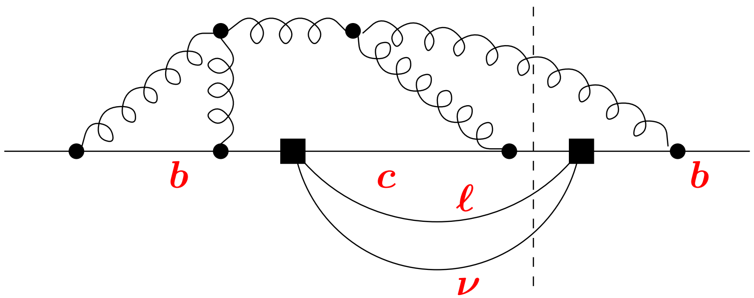

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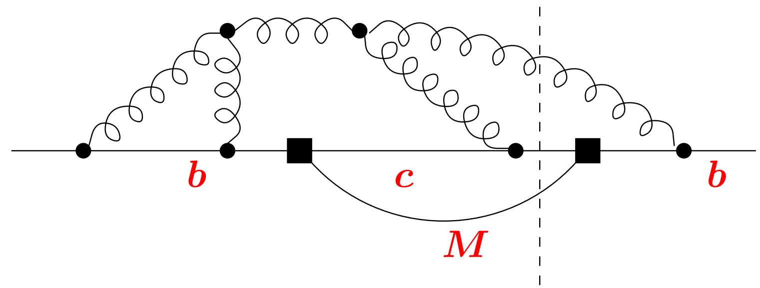
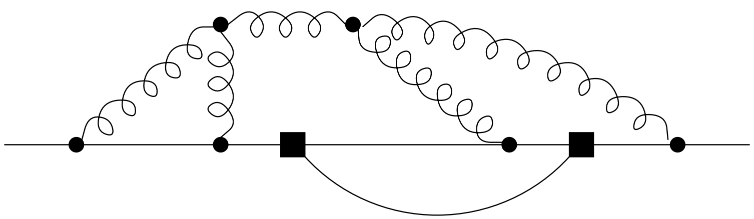
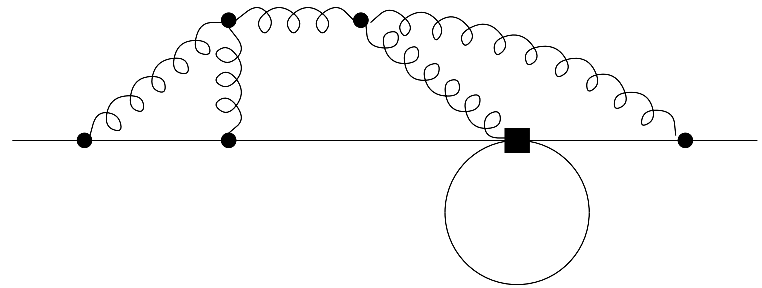


contribution to $d\Gamma/dq^2$ for $q^2 = M^2$

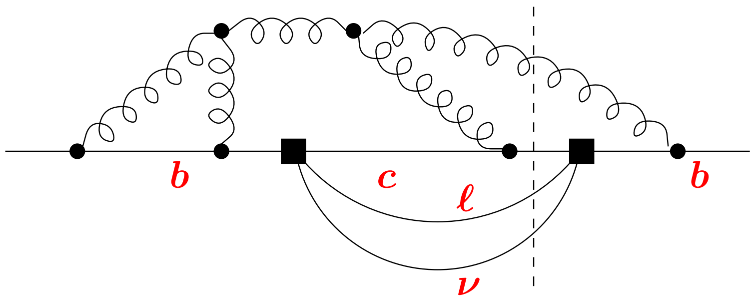
Let us consider $q^2 = m_c^2$:

Im $\Big($  $\Big)$ from



Real boundary condition for the differential equations at $m_c \gg m_b$

Feasibility of $b \to X_c \ell \bar{\nu}$ @ $\mathrm{N}^3\mathrm{LO}$



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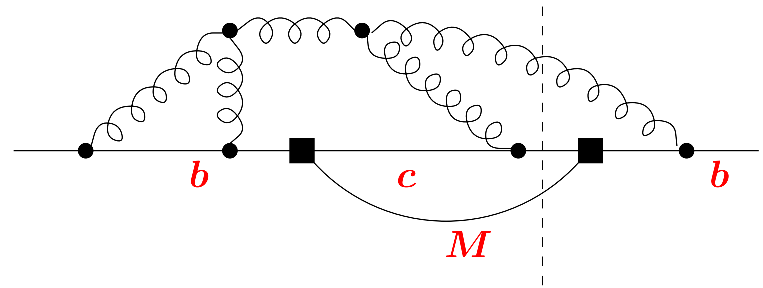

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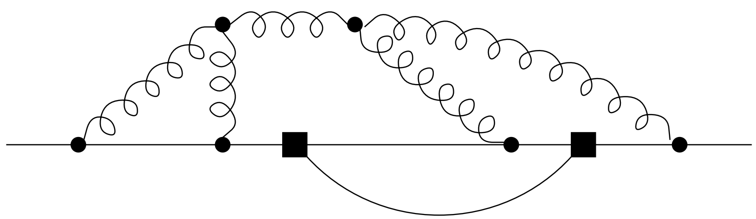
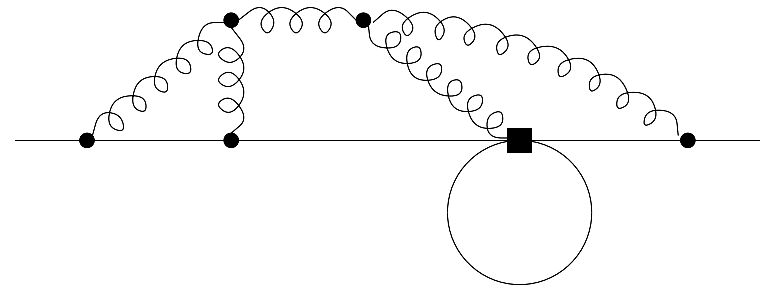
Im $\Big($  $\Big)$ from 

Real boundary condition for the differential equations at $m_c \gg m_b$

Possible IBP outsourcing: **Fraunhofer Institute for Industrial Mathematics**
[D. Bendle *et al.*, arXiv:1908.04301]

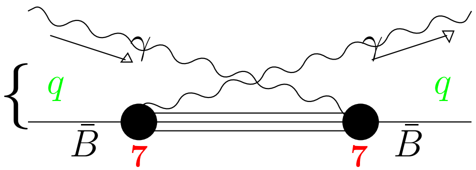
BACKUP SLIDES

The "hard" contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\sum_{X_s} \left| C_7(\mu_b) \langle X_s\gamma | O_7 | \bar{B}\rangle + C_2(\mu_b) \langle X_s\gamma | O_2 | \bar{B}\rangle + \ldots \right|^2$

The "77" term in this sum is "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

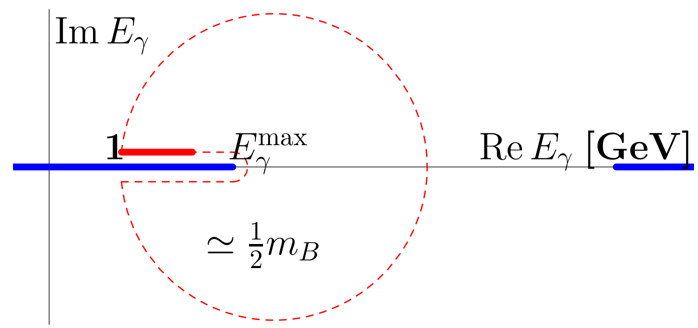
$\mathrm{Im}\{$  $\} \equiv \mathrm{Im}\, A$

When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**

However, the $\bar{B} \to X_s\gamma$ **photon spectrum is dominated by hard photons** $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ **is considered as a function of arbitrary complex** E_γ, $\mathrm{Im}A$ **turns out to be proportional to the discontinuity of** A **at the physical cut. Consequently,**

$$\int_{1\,\mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma\ \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma\ A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ **is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives**

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0) | Q^{(j)}_{\mathrm{local\ operator}} | \bar{B}(\vec{p}=0)\rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b.

At $(\Lambda/m_b)^0$: $\langle \bar{B}(\vec{p}) | \bar{b}\gamma^\mu b | \bar{B}(\vec{p})\rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to X_s^{\mathrm{parton}}\gamma) + \mathcal{O}(\Lambda/m_b)$.

At $(\Lambda/m_b)^1$: **Nothing! All the possible operators vanish by the equations of motion.**

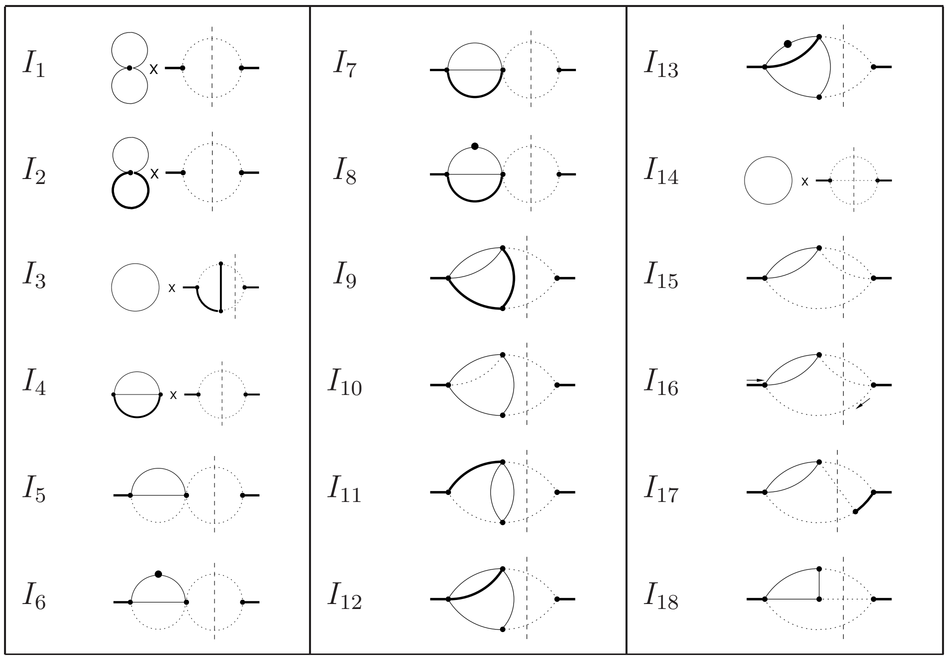
At $(\Lambda/m_b)^2$: $\langle \bar{B}(\vec{p}) | \bar{b}_v D^\mu D_\mu b_v | \bar{B}(\vec{p})\rangle \sim m_B\, \mu_\pi^2$,
$\langle \bar{B}(\vec{p}) | \bar{b}_v g_s G_{\mu\nu}\sigma^{\mu\nu} b_v | \bar{B}(\vec{p})\rangle \sim m_B\, \mu_G^2$,

The HQET heavy-quark field: $b_v(x) = \frac{1}{2}(1+\not{v})b(x)\exp(i m_b\, v\cdot x)$ **with** $v = p/m_B$.

The same method has been applied to the 3-loop counterterm diagrams

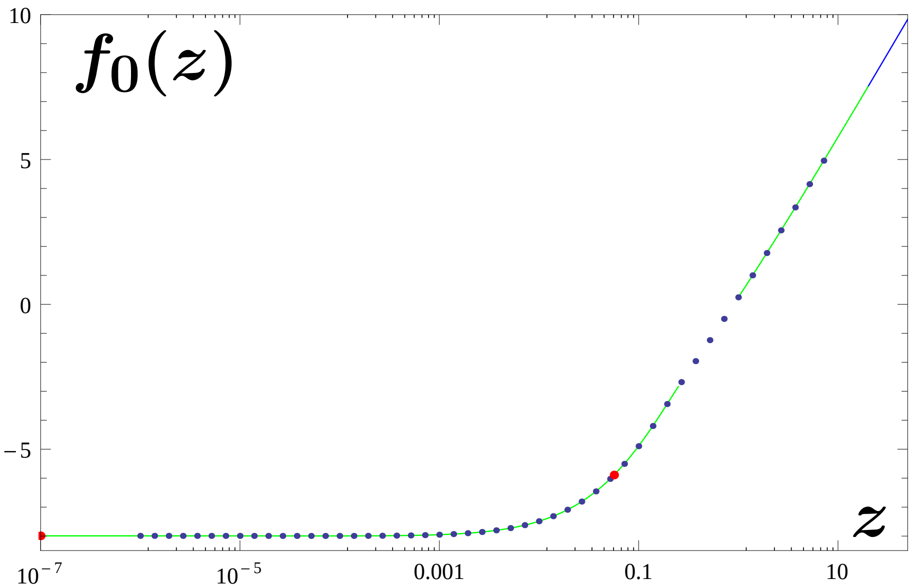
[MM, A. Rehman, M. Steinhauser, PLB 770 (2017) 431]

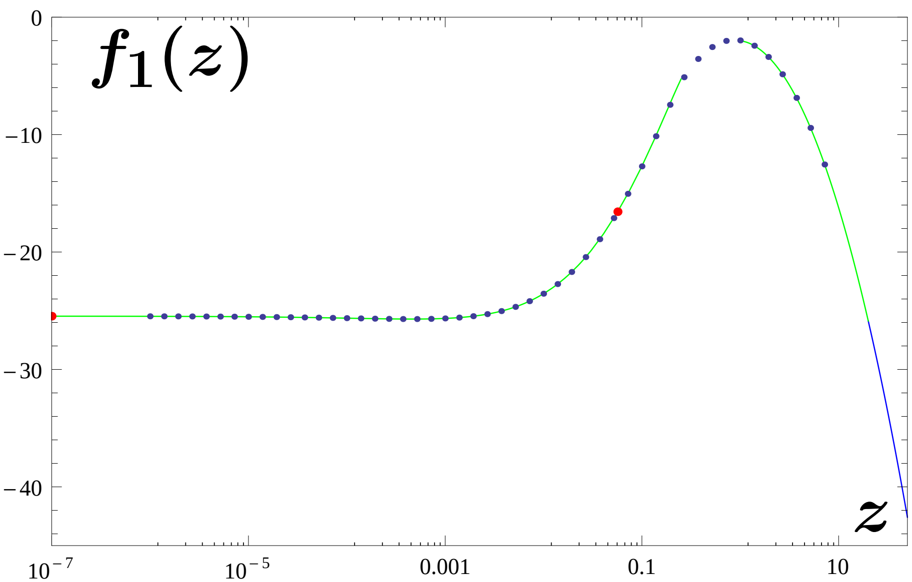
Master integrals:



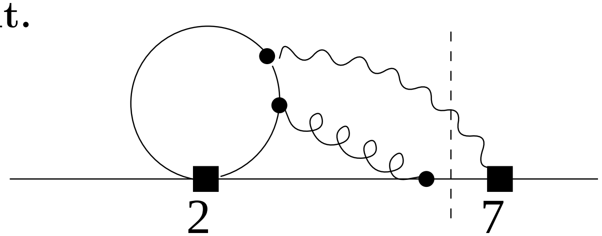
Results for the bare NLO contributions up to $\mathcal{O}(\epsilon)$:

$$\hat{G}_{27}^{(1)2P} = -\frac{92}{81\epsilon} + f_0(z) + \epsilon f_1(z) \quad \xrightarrow{z\to 0} \quad -\frac{92}{81\epsilon} - \frac{1942}{243} + \epsilon\left(-\frac{26231}{729} + \frac{259}{243}\pi^2\right)$$





Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.
Lines: large- and small-z asymptotic expansions



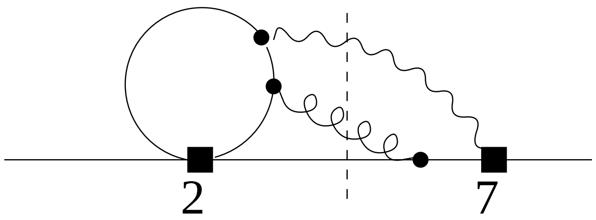
Small-z expansions of $\hat{G}_{27}^{(1)2P}$:

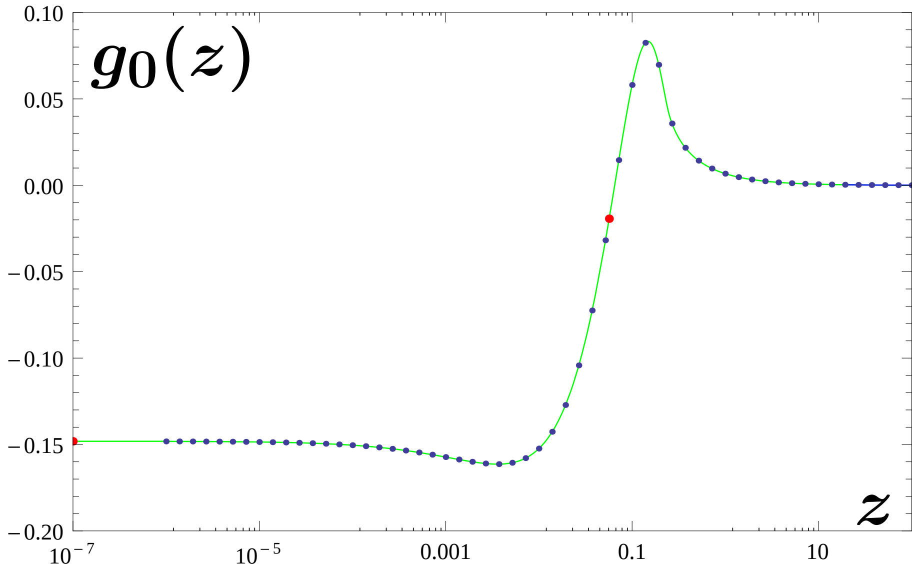
f_0 from **C. Greub, T. Hurth, D. Wyler, hep-ph/9602281, hep-ph/9603404,**
A. J. Buras, A. Czarnecki, MM, J. Urban, hep-ph/0105160,

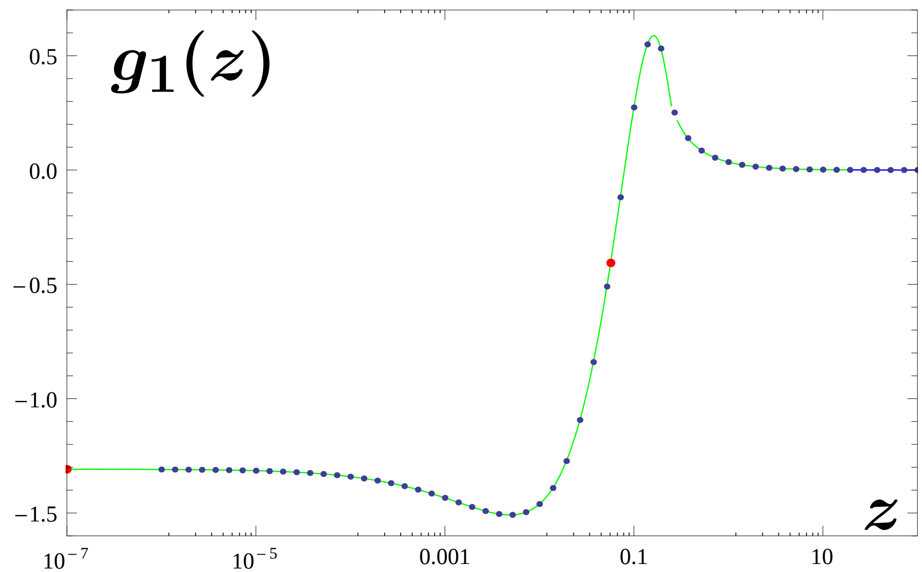
f_1 from **H.M. Asatrian, C. Greub, A. Hovhannisyan, T. Hurth and V. Poghosyan, hep-ph/0505068.**

Analogous results for the 3-body final state contributions ($\delta = 1$):

$$\hat{G}_{27}^{(1)3P} = g_0(z) + \epsilon g_1(z) \quad \xrightarrow{z \to 0} \quad -\frac{4}{27} - \frac{106}{81}\epsilon$$







Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.
Lines: exact result for g_0, as well as large- and small-z asymptotic expansions for g_1.

$$g_0(z) = \begin{cases} -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, s\, L + \frac{16}{9}z(6z^2-4z+1)\left(\frac{\pi^2}{4} - L^2\right), & \text{for } z \leq \frac{1}{4}, \\ -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, t\, A + \frac{16}{9}z(6z^2-4z+1)\, A^2, & \text{for } z > \frac{1}{4}, \end{cases}$$

where $s = \sqrt{1-4z}$, $L = \ln(1+s) - \frac{1}{2}\ln 4z$, $t = \sqrt{4z-1}$, and $A = \arctan(1/t)$.