



Flavor hierarchies and anomalies from a 5D perspective

Javier Fuentes-Martín
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A 5D model that...

- ★ Explains/Justifies the flavor hierarchies from a multi-scale origin
- ★ Reduces to 4321 (U_1 UV completion) at low energies
- ★ Stabilizes the Higgs hierarchy (Higgs as a pNGB)

[JFM, Isidori, Lizana, Selimović, Stefanek, [2203.01952](#)]

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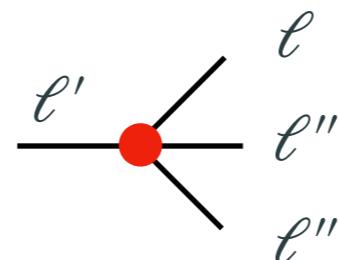
Multi-scale solution of the flavor problem/puzzle

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \sum_{i,d} \frac{1}{\Lambda_i^{d-4}} C_i \mathcal{O}_i^d$$

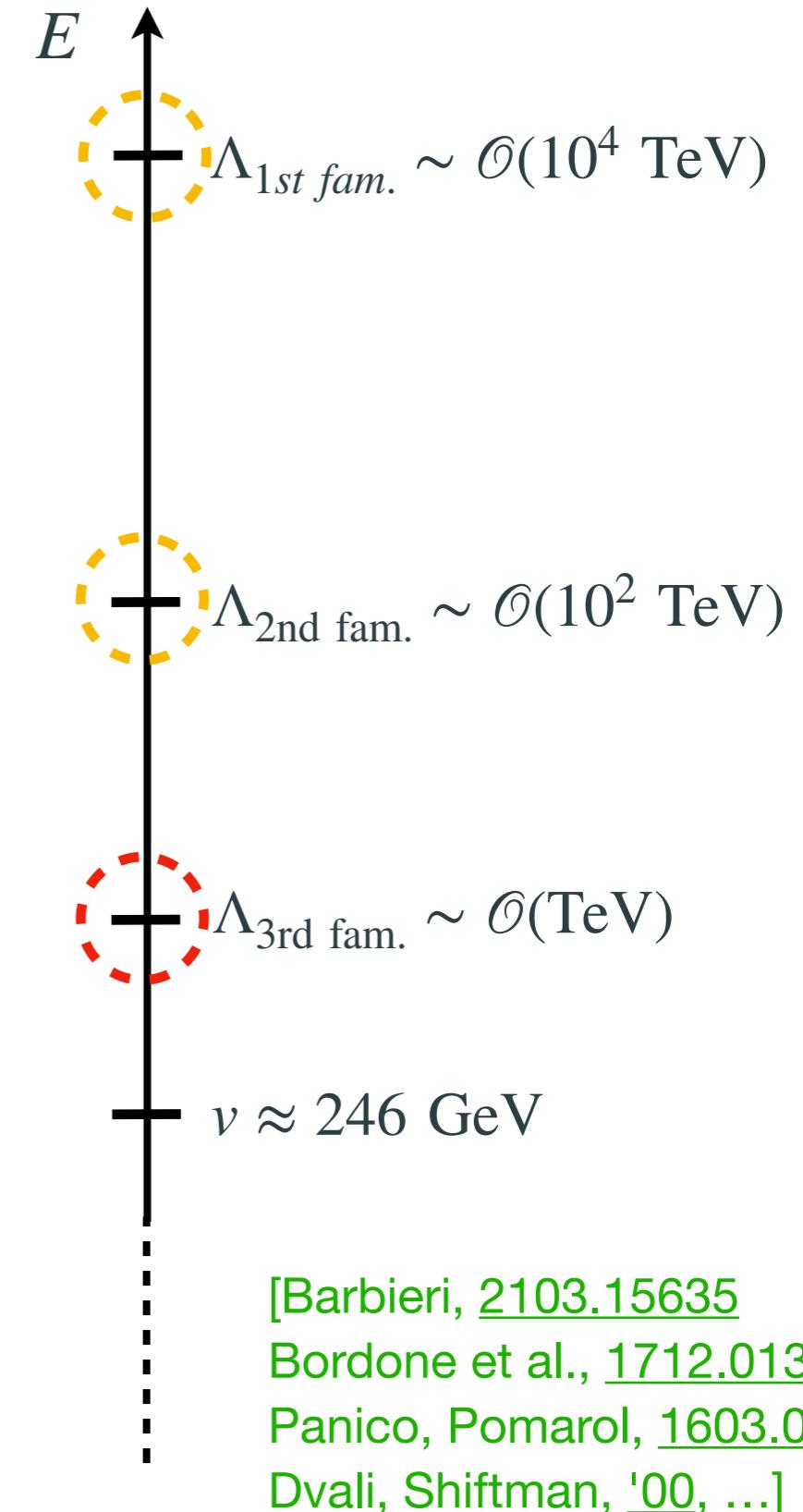
Non-trivial UV imprints

- ★ The SM Yukawas are very different because they originate at separate scales!
- ★ TeV-scale NP dominantly coupled to third and (to a lesser extent) second families
[protection from flavor constraints]

e.g. from $\frac{1}{\Lambda^2} (\psi_i \psi_j)^2$



- ★ Direct production of new states at the LHC is naturally more suppressed [NP scale can be lower]



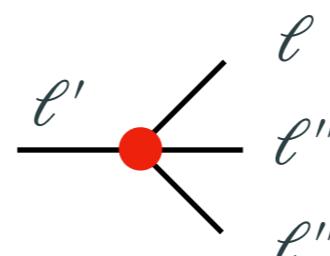
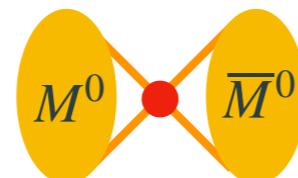
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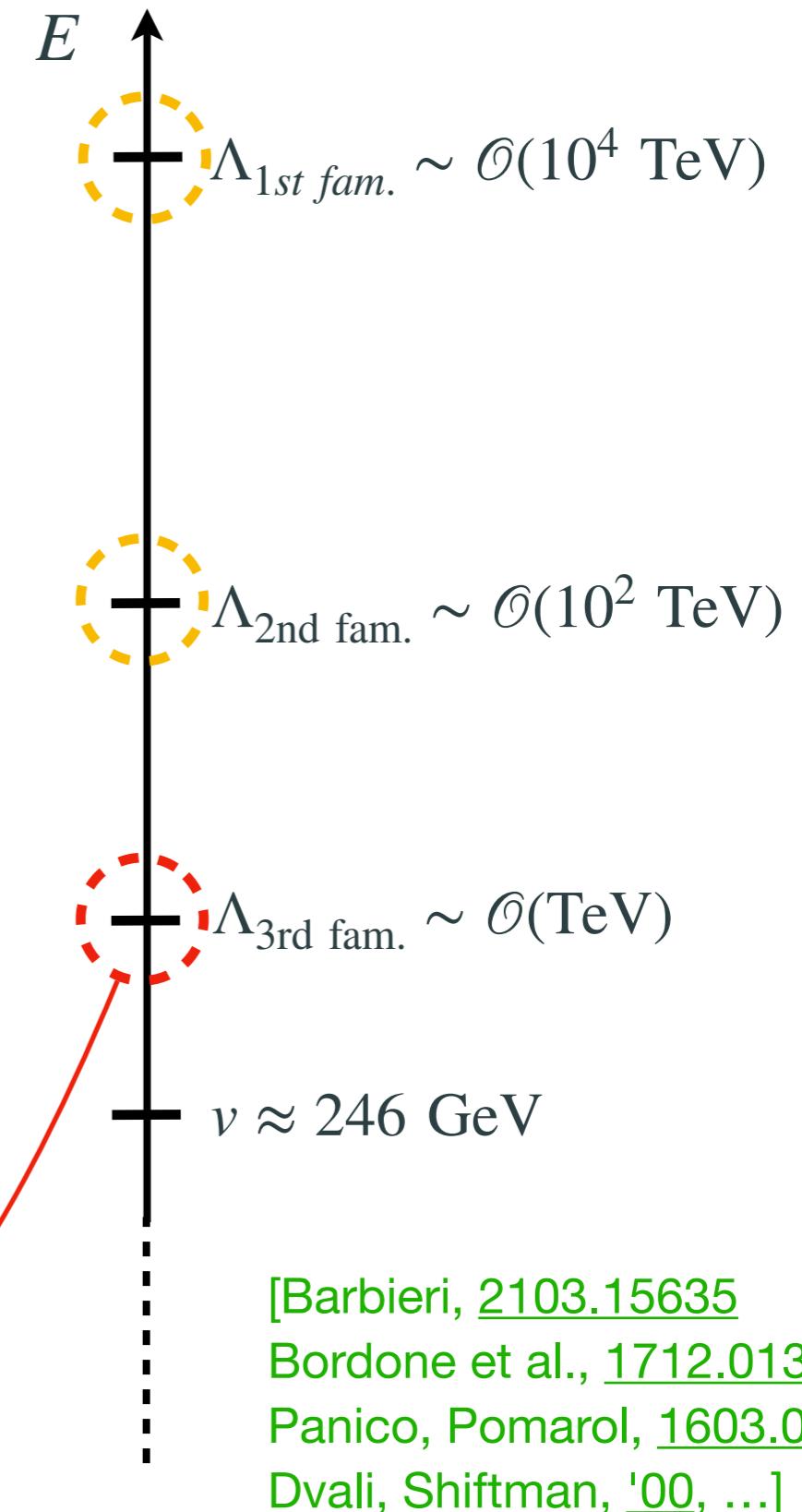
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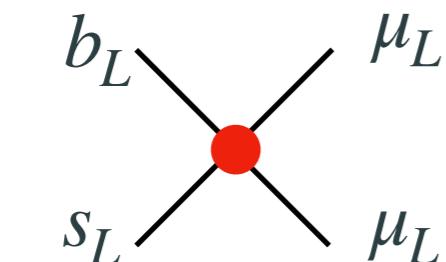
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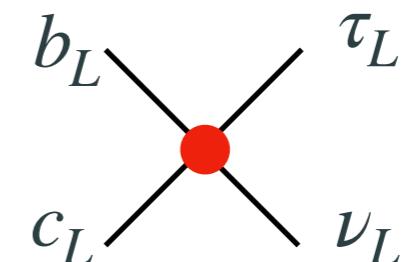
Perhaps we are already seeing hints in
B-meson decays?



Combined explanation of B anomalies



$$\sim \frac{1}{(40 \text{ TeV})^2}$$



$$\sim \frac{1}{(3 \text{ TeV})^2}$$

$$3_q \rightarrow 2_q 2_\ell 2_\ell$$

$$3_q \rightarrow 2_q 3_\ell 3_\ell$$

The only source of **Lepton Flavor Universality Violation** in the SM (Yukawas) follows a very similar trend: $y_e \ll y_\mu \ll y_\tau$

Combined explanation of B anomalies

$$\begin{array}{ccc} \begin{array}{c} b_L \\ s_L \end{array} & \begin{array}{c} \text{red circle} \\ \swarrow \searrow \end{array} & \begin{array}{c} \mu_L \\ \mu_L \end{array} \\ & \text{red double-headed arrow} & \\ \sim \frac{1}{(1 \text{ TeV})^2} |V_q| |V_\ell|^2 & & \sim \frac{1}{(1 \text{ TeV})^2} |V_q| \\ \\ \begin{array}{c} 3_q \rightarrow 2_q 2_\ell 2_\ell \end{array} & & \begin{array}{c} 3_q \rightarrow 2_q 3_\ell 3_\ell \end{array} \end{array}$$

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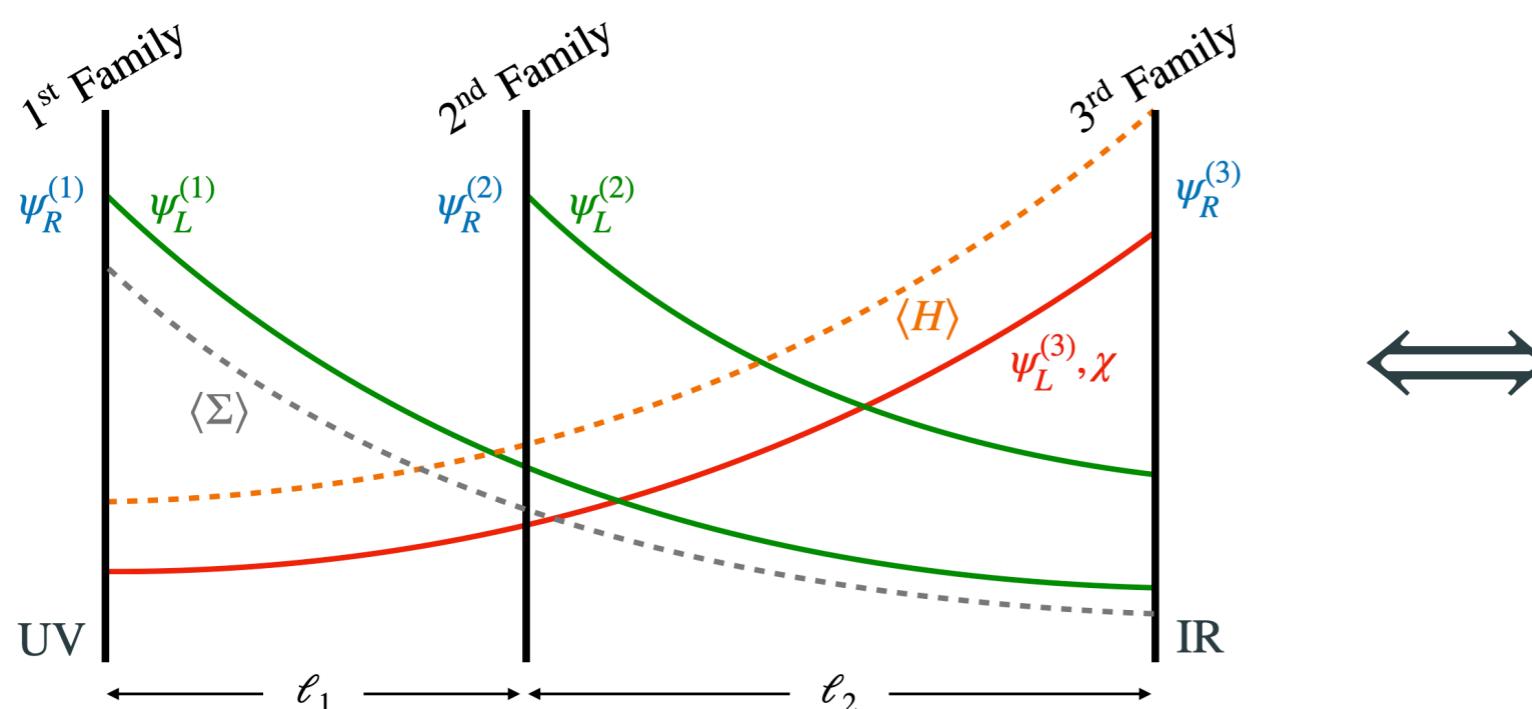
Data consistent with TeV-scale NP with a Yukawa-like scaling with $|V_q|, |V_\ell| \sim 0.1$
[roughly the size inferred from the SM Yukawa $|V_q| \sim V_{cb} \approx 0.04$]

[JF, Isidori, Pagès, Yamamoto, [1909.02519](#)]

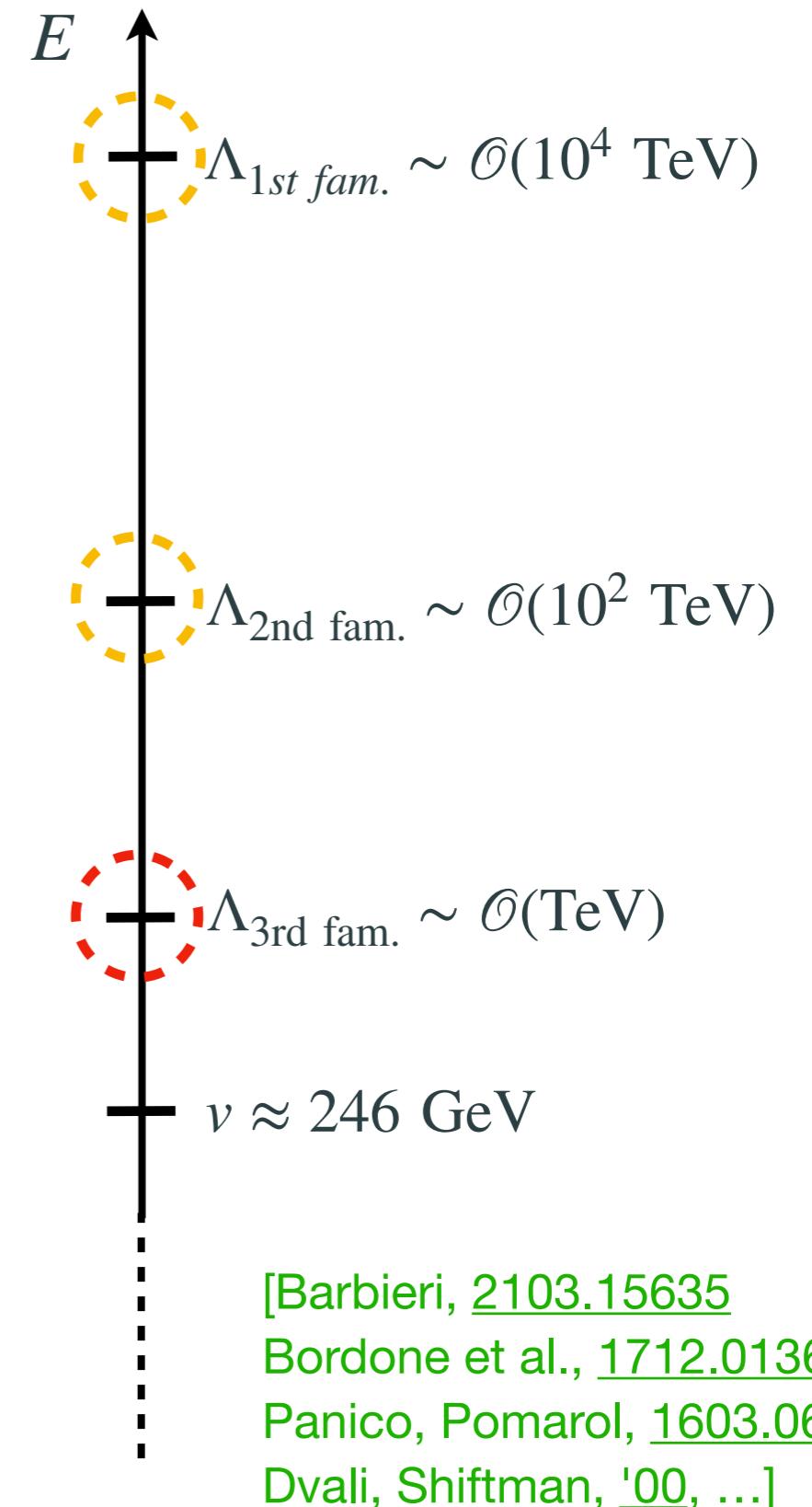
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Non-trivial UV imprints



Flavor \longleftrightarrow fermion (quasi-)localization
along an extra dimension

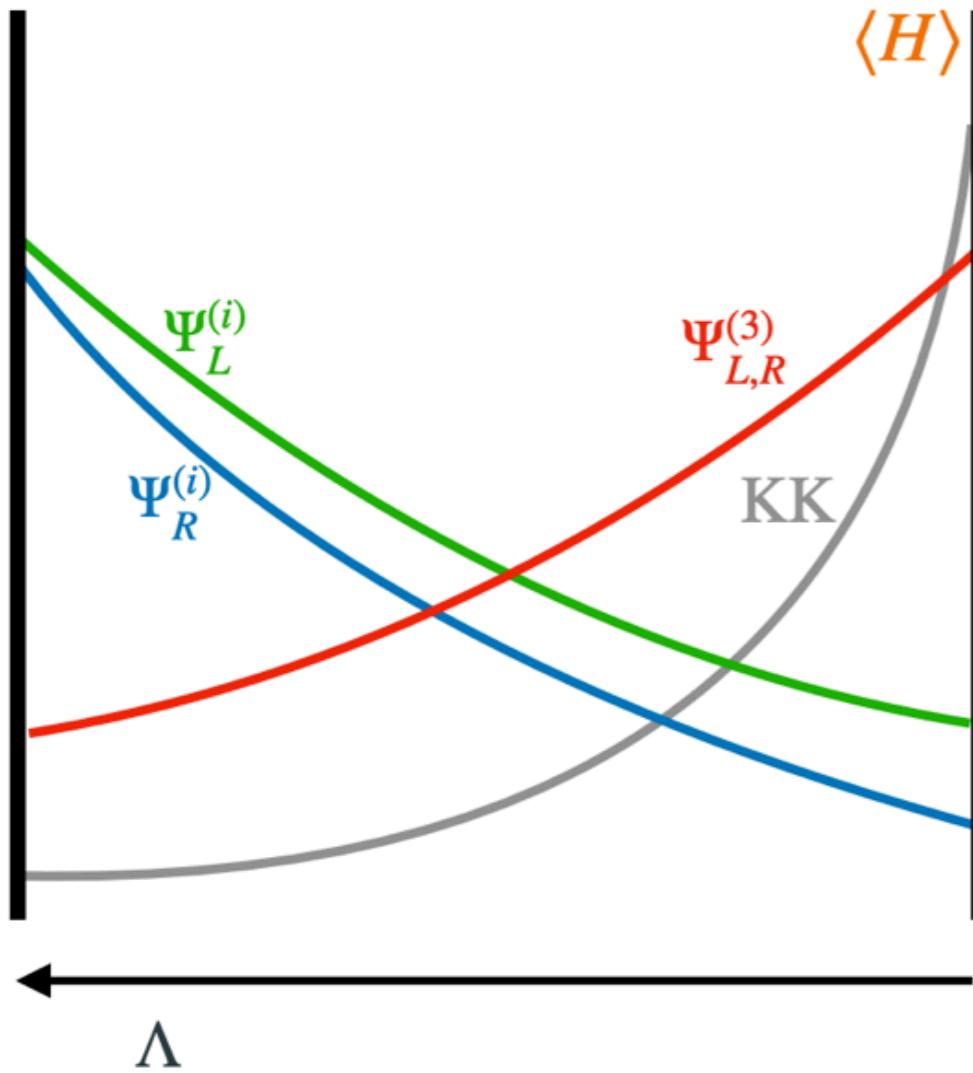


Flavor in Randall-Sundrum

Curvature of the AdS slice

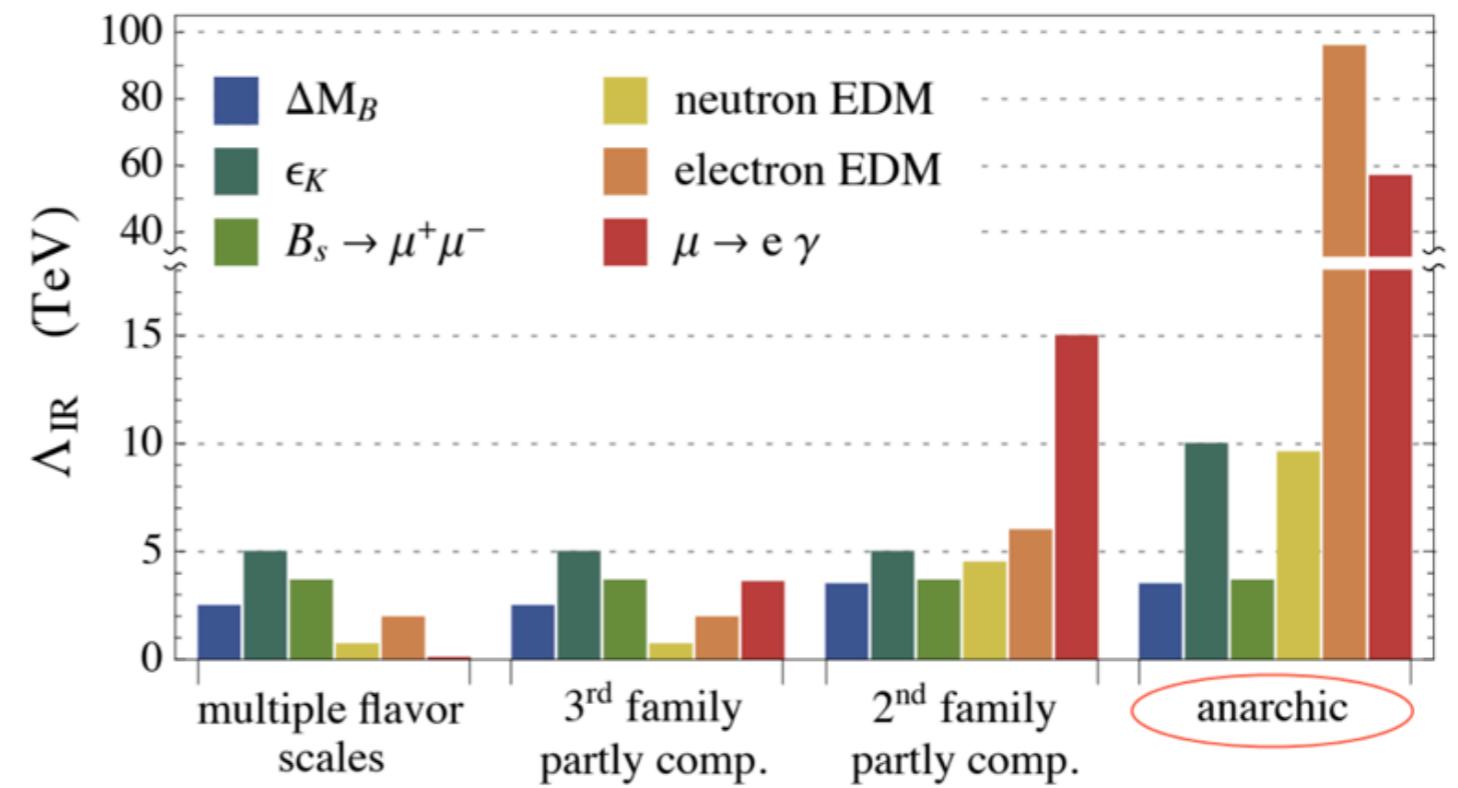
Warped 5D geometry (RS): $ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$

- ▶ Justification of the Yukawa hierarchies through exponentiation + flavor anarchy
- ▶ Analogous to partial compositeness in composite models



Dangerous dipoles (among others)
generated at the IR scale

$$\sim \frac{g_*^2}{16\pi^2} \frac{m_e}{\Lambda_{\text{IR}}^2} \bar{e}_L \sigma_{\mu\nu} e_R F^{\mu\nu}$$



[Panico, Pomarol, [1603.06609](#)]

A 5D model that...

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[JFM, Isidori, Lizana, Selimović, Stefanek, [2203.01952](#)]

Gauge UV completion for the U_1 leptoquark

$$U_1 \sim (3,1,2/3) \longrightarrow SU(4) \longrightarrow PS = SU(4) \times SU(2)_L \times SU(2)_R$$

$$SU(4) \sim \begin{pmatrix} G^a & U^\alpha \\ (U^\alpha)^* & Z' \end{pmatrix} \quad \psi_{L,R} = \begin{bmatrix} q_{L,R}^1 \\ q_{L,R}^2 \\ q_{L,R}^3 \\ l_{L,R} \end{bmatrix}$$

Leptons as the fourth “color”

[Pati, Salam, [Phys. Rev. D10 \(1974\) 275](#)
(only 7 years after the SM was proposed)]

- ✓ $SU(4)$ is the smallest group containing the $U_1 \sim (3, 1, 2/3)$
- ✓ No proton decay (protected by symmetry)
- ✗ Flavor-blind U_1 mediates $K_L \rightarrow \mu e \Rightarrow m_{U_1} \gtrsim 100 \text{ TeV}$
- ✗ Extra fermions can make the U_1 non-universal, not the Z'
- ✗ Strongly coupled, universal Z' would be excessively produced at the LHC

4321 model(s)

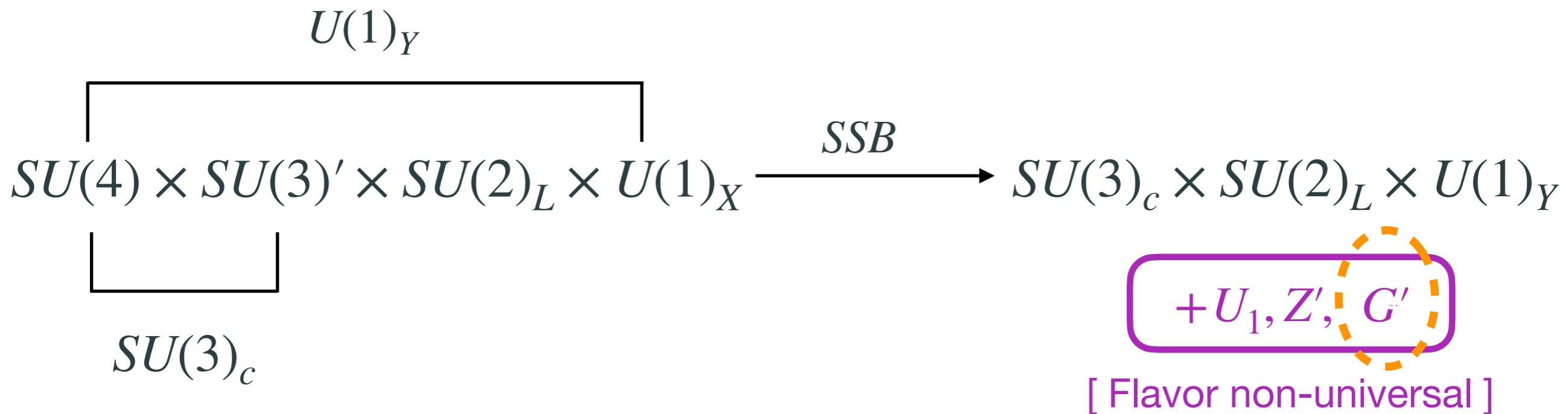
[Georgi and Y. Nakai, 1606.05865; Diaz, Schmaltz, Zhong, 1706.05033;
Di Luzio, Greljo, Nardecchia, 1708.08450.
See also Fornal, Gadam, Grinstein, 1812.01603]

We can “protect” the light families by de-correlating $SU(4)$ from the SM color group ($g_4 \gg g_3$)

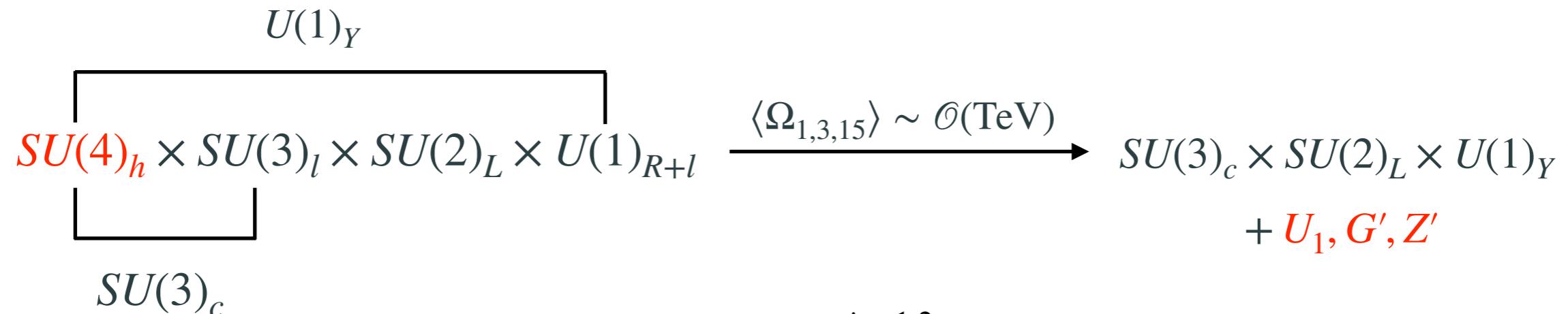
PS group: $\mathcal{G}_{\text{PS}} \supset \textcolor{red}{SU(4)} \times \textcolor{blue}{SU(2)_L} \times \textcolor{blue}{U(1)_R}$ [Flavor universal]



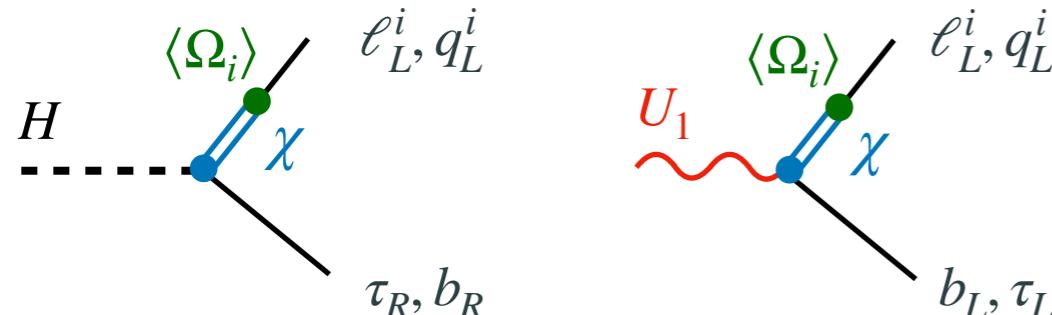
4321 group: $\mathcal{G}_{\text{4321}} \equiv \textcolor{blue}{SU(4)_h} \times \textcolor{red}{SU(3)_l} \times \textcolor{blue}{SU(2)_L} \times \textcolor{blue}{U(1)_{R+l}}$ [Flavor non-universal]



Third-family quark-lepton unification at the TeV scale



- ★ Direct new physics couplings to 3rd family only
- ★ CKM mixing and NP couplings to light families via (small) mixing with vectorlike fermions χ



$i = 1, 2$

Field	$SU(4)$	$SU(3)'$	$SU(2)_L$	$U(1)_X$
q_L^i	1	3	2	1/6
u_R^i	1	3	1	2/3
d_R^i	1	3	1	-1/3
ℓ_L^i	1	1	2	-1/2
e_R^i	1	1	1	-1
ψ_L	4	1	2	0
ψ_R^\pm	4	1	1	$\pm 1/2$
χ_L^i	4	1	2	0
χ_R^i	4	1	2	0
H	1	1	2	1/2
Ω_1	4	1	1	-1/2
Ω_3	4	3	1	1/6
Ω_{15}	15	1	1	0

1st & 2nd
families

3rd family
vectorlike
fermions

4321
breaking
scalars

- ★ Fully calculable loop contributions

[JFM, Isidori, König, Selimovic,
1910.13474, 2006.16250, 2009.11296]

[Bordone, Cornella, JFM, Isidori 1712.01368, 1805.09328;
Greljo, Stefanek, 1802.04274;
Cornella, JFM, Isidori 1903.11517]

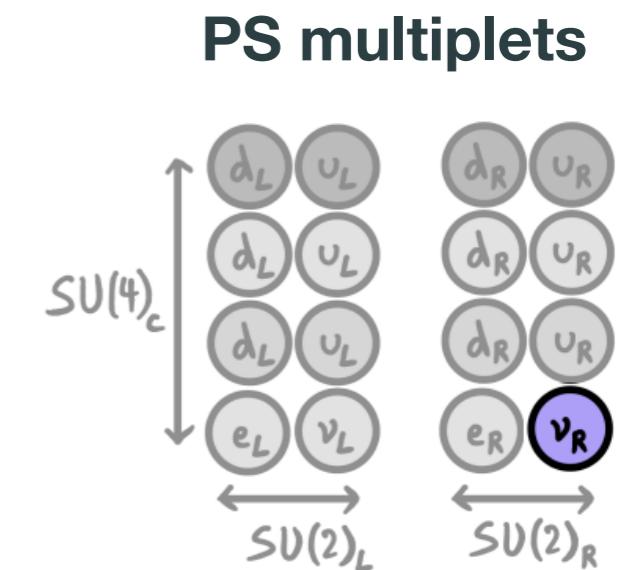
Third-family quark-lepton unification at the TeV scale

In first approximation, third-family quark-lepton unification implies

$$y_\tau = y_b \quad \checkmark$$

$$[y_\tau = 0.8 y_b \text{ at 2 TeV}]$$

$$y_{\nu_\tau} = y_t$$



TeV-scale unification limits Majorana mass for ν_R to $m_{\nu_R} \lesssim \text{TeV}$

Type-I see-saw: $m_\nu \approx \frac{m_D^2}{m_{\nu_R}} \sim 10 \text{ GeV} \quad \times$

$$m_D \equiv y_\nu v / \sqrt{2}$$

Solution: Inverse seesaw via new fermion singlets S_L^i with hierarchical Majorana masses μ^i

[Greljo, Stefanek, [1802.04274](#)
Fileviez, Wise, [1307.6213](#)]

$$m_\nu \approx m_D m_R^{-1} \mu (m_R^{-1})^\top m_D^\top$$



$$\mu^i \sim (10^7, 10^{-1}, 10^{-9}) \text{ GeV}$$

$$m_D^i \approx m_u^i \sim (10^{-2}, 1, 10^2) \text{ GeV}$$

Third-family quark-lepton unification at the TeV scale

Model prediction: mixing between active neutrino and pseudo-Dirac heavy neutral leptons yields

PMNS unitarity violation

with the expected pattern:

$$\eta \equiv |1 - NN^\dagger| \sim \left| \frac{m_D^3}{m_R^3} \right|^2 \begin{pmatrix} \epsilon_L^4 & \epsilon_L^3 & \epsilon_L^2 \\ \epsilon_L^3 & \epsilon_L^2 & \epsilon_L \\ \epsilon_L^2 & \epsilon_L & 1 \end{pmatrix} \quad \epsilon_L \approx 0.1$$

First sign of violation in 33 entry:

$$\eta_{33} \approx \left| \frac{m_D^3}{m_R^3} \right|^2 \sim \left| \frac{100 \text{ GeV}}{2 \text{ TeV}} \right|^2 = 2.5 \times 10^{-3}$$

$$\eta_{33}^{\text{exp}} < 5.3 \times 10^{-3} \quad (90\% \text{ C.L.})$$

[Antusch, Fischer, [1407.6607](#)]

A 5D model that...

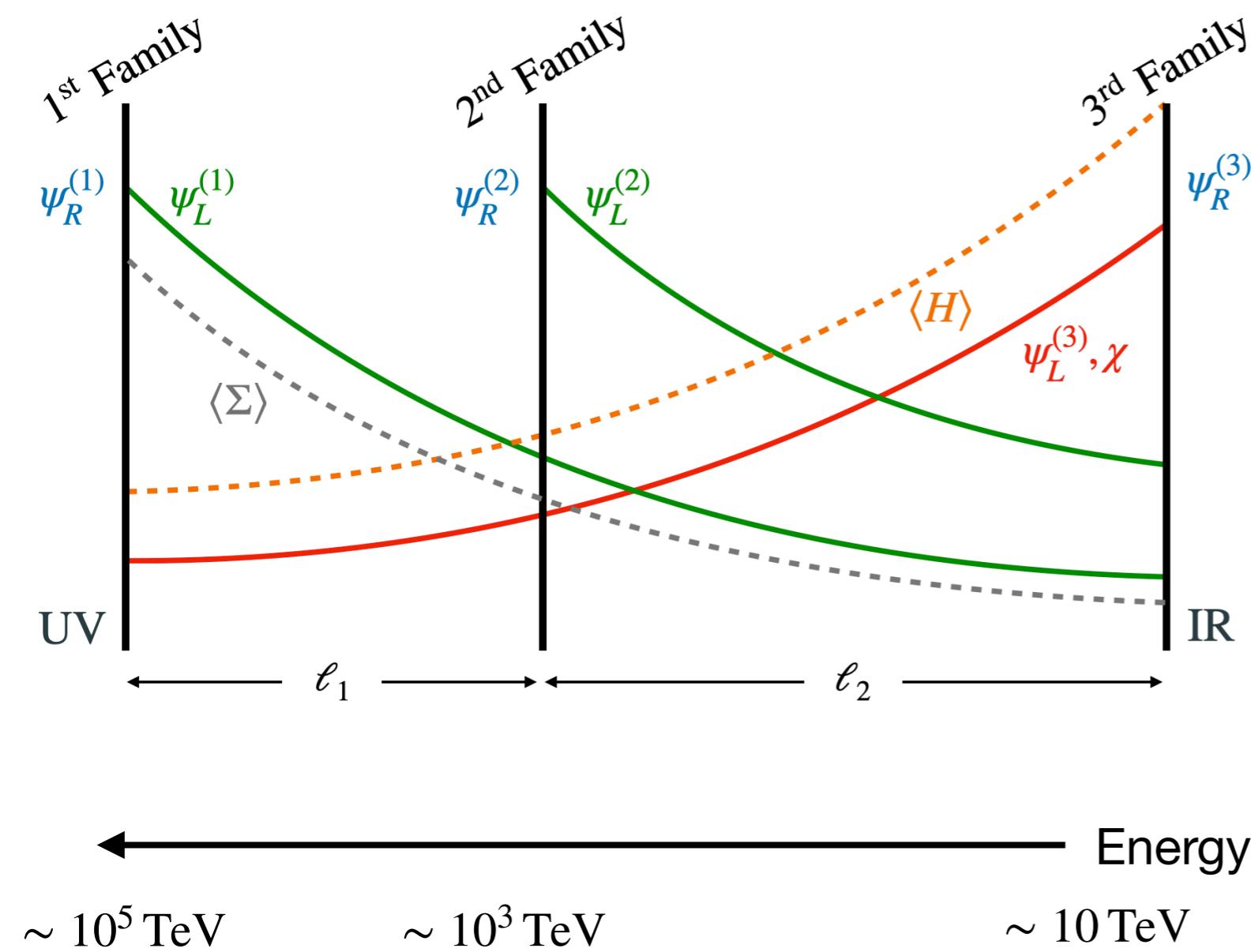
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[JFM, Isidori, Lizana, Selimović, Stefanek, [2203.01952](#)]

A 5D UV completion of 4321

[JF, Isidori, Pagès, Stefanek, [2012.10492](#)
 JF, Isidori, Lizana, Selimovic, Stefanek, [2203.01952](#)]

Attempt to construct a **full theory of flavor** by embedding the 4321 group in a compact warped extra dimension (**AdS₅**) with multiple four-dimensional branes



Flavor \longleftrightarrow fermion (quasi-)localization
in each of the branes

[Dvali, Shifman, '00; Panico, Pomarol, [1603.06609](#)]

$$y_{ij} \approx y_t e^{-k(L-\ell_j)} e^{-k(c_i - 1/2)(y_i - \ell_j)}$$

k : Curvature of the AdS slice

Same dynamics that breaks 4321
also generates a pNGB Higgs \longleftrightarrow
stabilization of the EW hierarchy with
an $\mathcal{O}(0.1\%)$ tuning (little hierarchy)

Anarchic neutrino masses via
inverse see-saw mechanism

Gauge sector

$$\mathcal{G}_{\text{bulk}}^{12} = SU(4)_h \times SU(4)_l \times SO(5)$$

$\downarrow \Lambda_2$ (6 broken)

$$\mathcal{G}_{\text{bulk}}^{23} = SU(4)_h \times SU(3)_l \times U(1)_l \times SO(5)$$

$\downarrow \Lambda_3 = \Lambda_{\text{IR}}$ (15 + 4 broken)

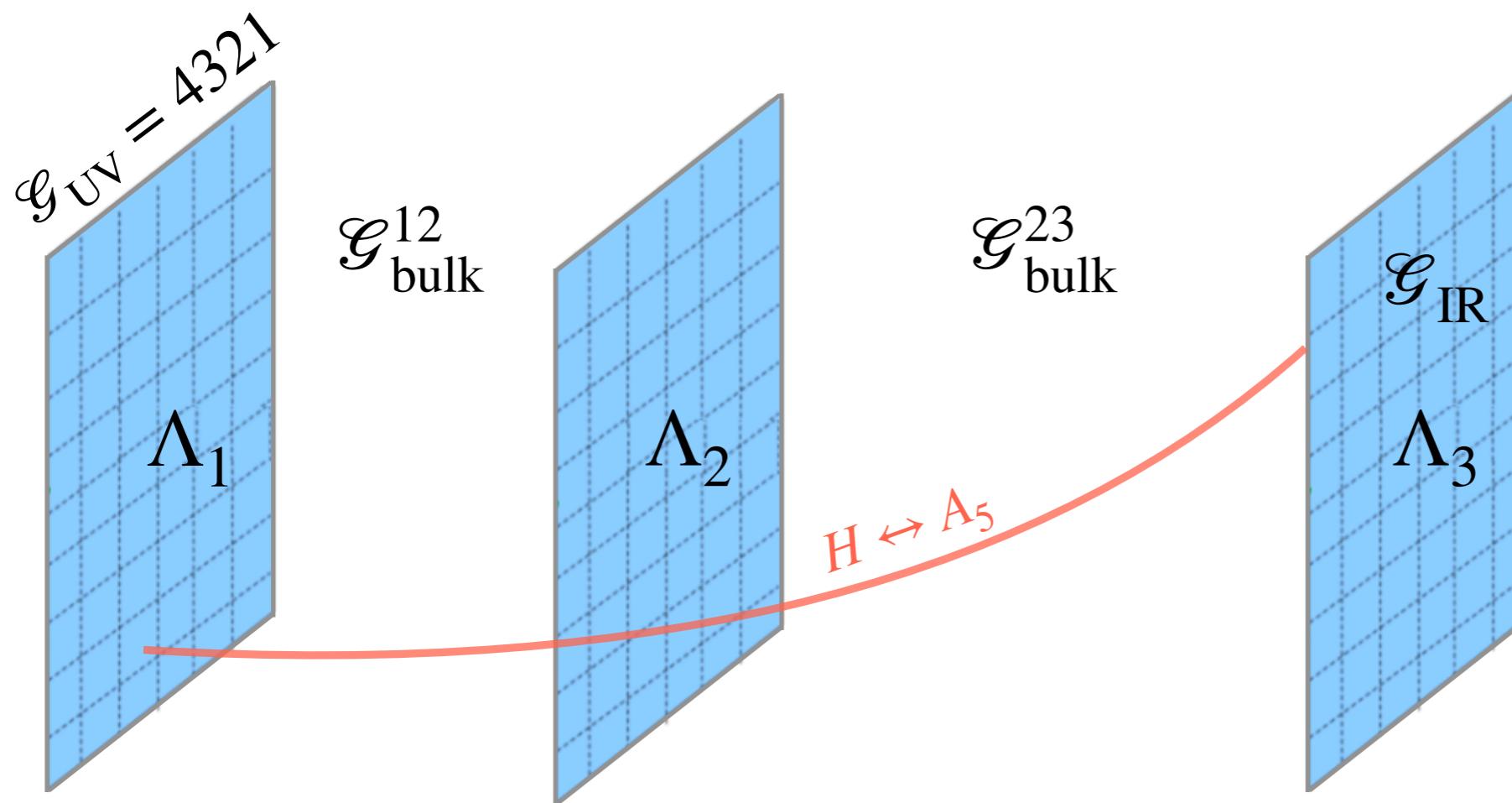
$$\mathcal{G}_{\text{IR}} = SU(3)_c \times U(1)_{B-L} \times SO(4)$$

Quark-lepton unification of light families

15 eaten by U_1, G', Z' , $M_{4321} \sim \frac{M_{\text{KK}}}{\sqrt{2kL}}$

4 as pNGB Higgs

$$SO(5) \rightarrow SO(4) = SU(2)_L \times SU(2)_R$$



Minimal composite
Higgs model

[Agashe, Contino, Pomarol, '04]

Fermion and scalar sector

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
$\Psi^3 \quad \Psi_d^3 \quad \chi^{(')}$	4	1	4
$\Psi^j \quad \Psi_{u,d}^j$	1	4	4
\mathcal{S}^i	1	1	1
Φ	1	1	1
Ω	1	4	4
Σ	1	1	5

SM fermions and VLF
 For neutrinos
 For light Yukawas

Fermions
 Scalars

[JFM, Isidori, Pages, Stefanek, 2012.10492]

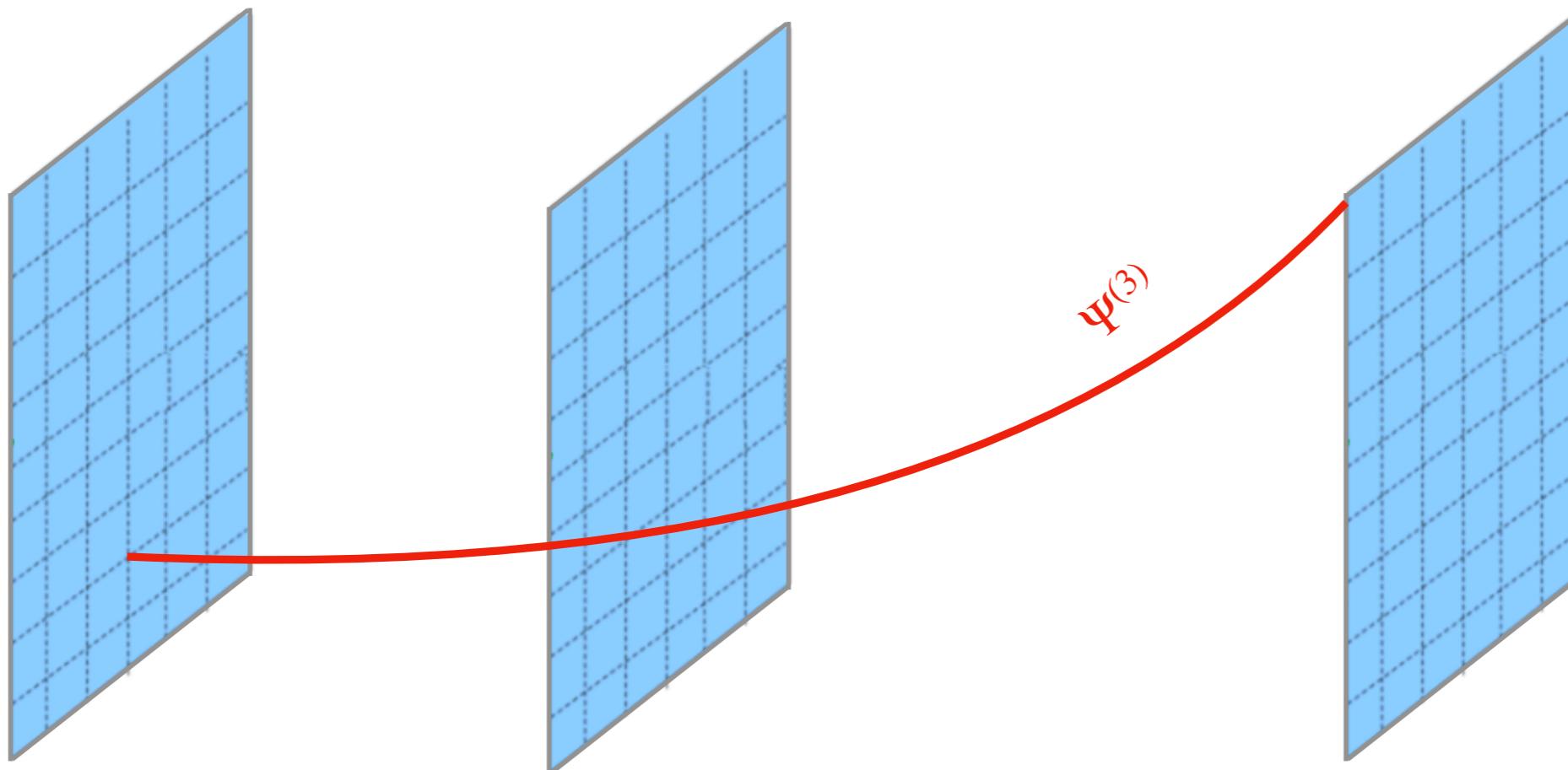
Top Yukawa

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
Ψ^3	4	1	4

$$\Psi^3 = \begin{bmatrix} q_L \\ t_R \\ \times \end{bmatrix} \left. \begin{array}{c} \textcolor{blue}{SU(2)_L} \\ \textcolor{red}{SU(2)_R} \end{array} \right\}$$

Top Yukawa from
 $\bar{\Psi}^3 A_5 \Psi^3$ coupling
in the bulk

$$y_t = \frac{g_*}{2\sqrt{2}} \textcolor{blue}{P}(M_{\Psi^3}) \quad (g_*^2 = g_5^2 k) \quad \text{For } y_t : g_* \geq 2.2$$

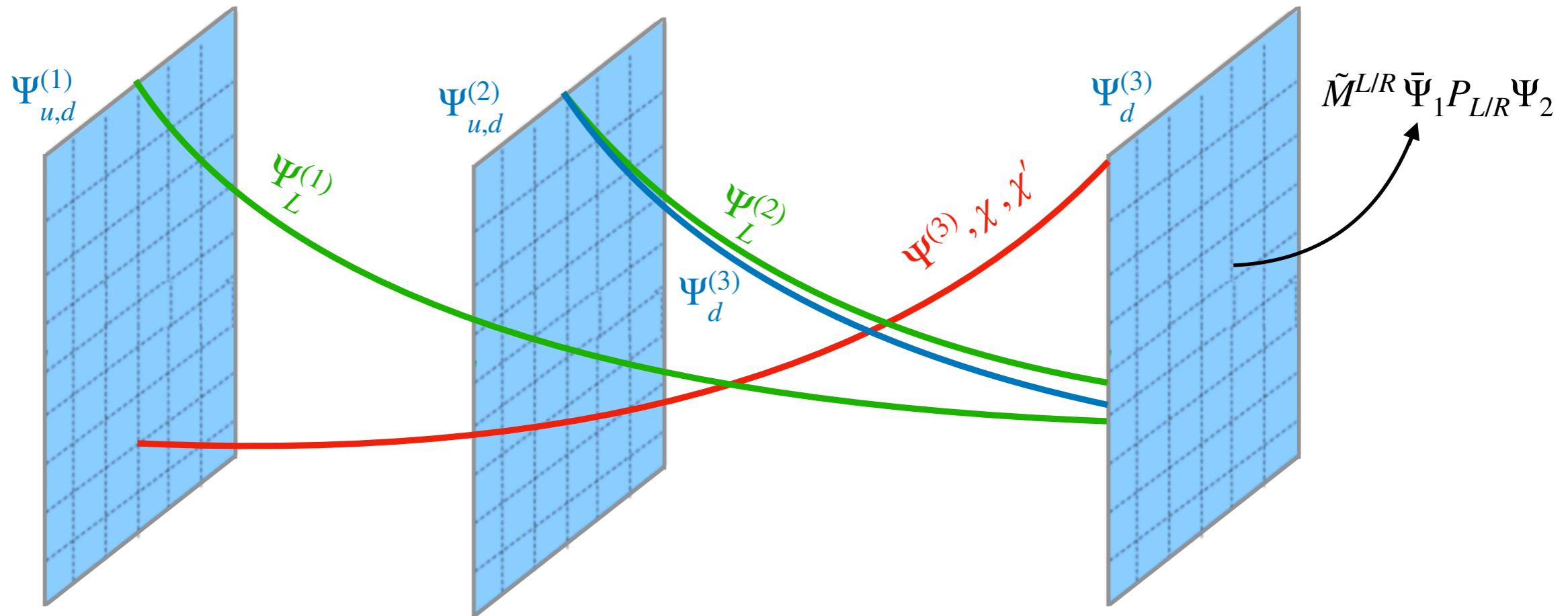


Light-heavy mixing

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
$\Psi^3, \Psi_d^3, \chi^{(')}$	4	1	4
$\Psi^j, \Psi_{u,d}^j$	1	4	4

Mass mixing of light families with VLF and light-3rd family Yukawas from masses in the IR brane

$$y_{f_1 f_2} = \frac{g_*}{2\sqrt{2}} (\tilde{M}^L - \tilde{M}^R) \times \text{(profile suppression)}$$

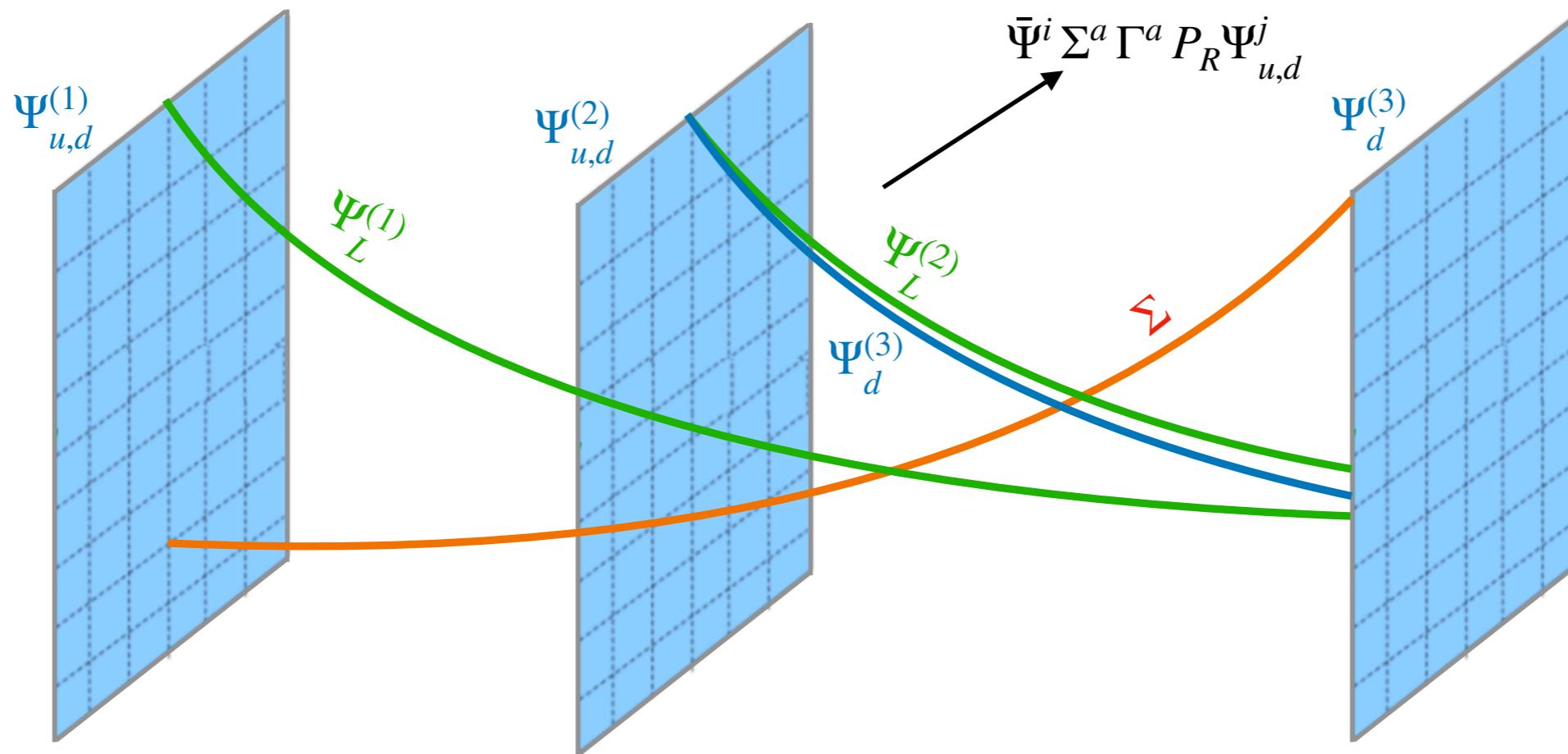


Light Yukawas

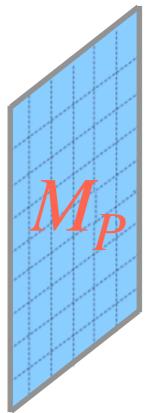
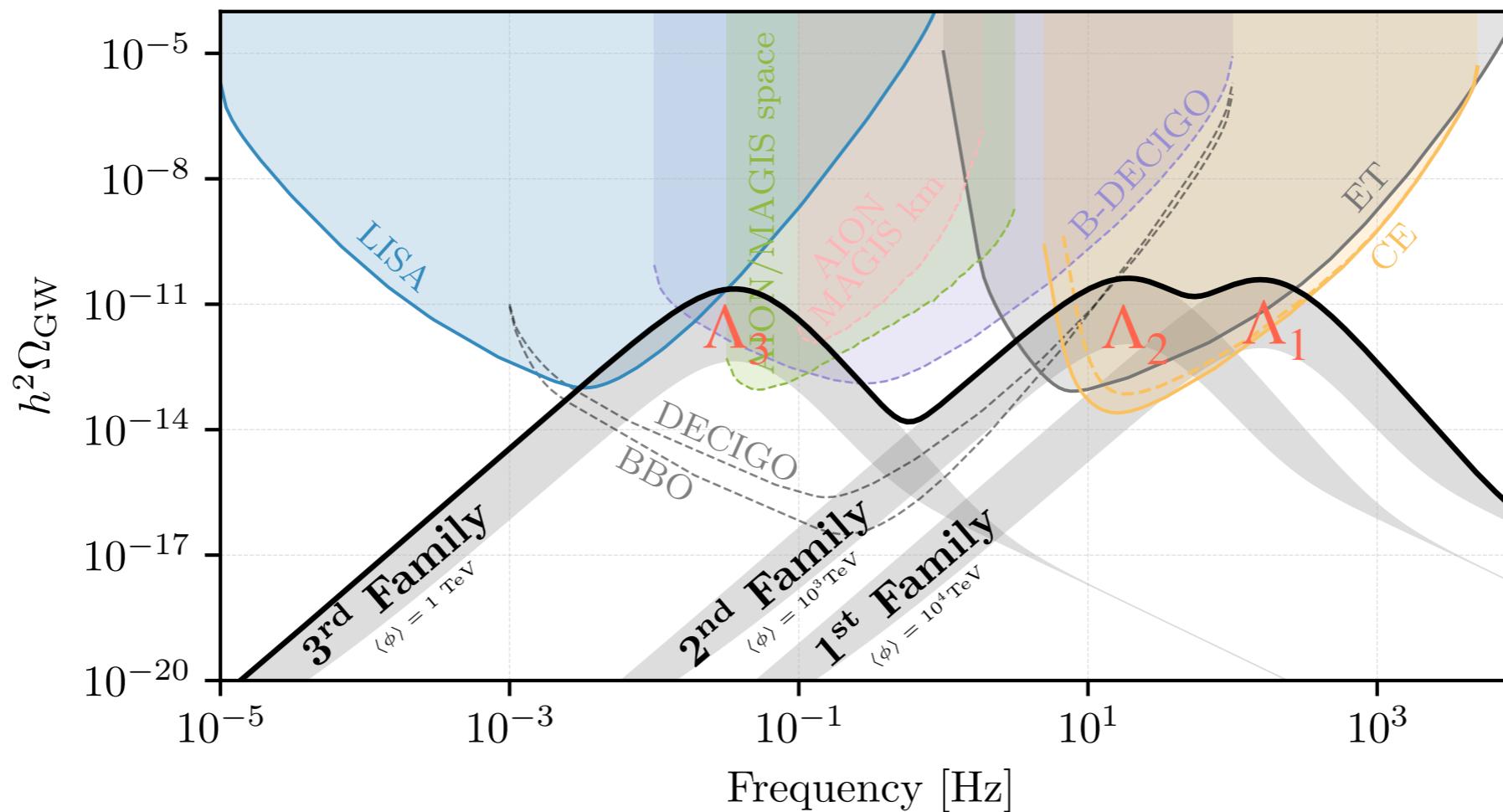
Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
$\Psi^j, \Psi_{u,d}^j$	1	4	4
Σ	1	1	5

$\Sigma^T \sim (H' \phi)$ takes a VEV along the singlet direction and propagates the breaking of $SO(5)$ into the bulk

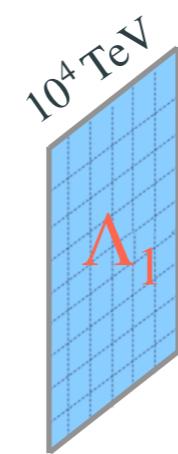
$$y_{u,d}^{ij} = \frac{g_*}{2\sqrt{2}} \tilde{Y}_{u,d}^{ij} \frac{\langle \Sigma_{\text{IR}} \rangle}{\Lambda_{\text{IR}}} \times (\text{profile suppression})$$



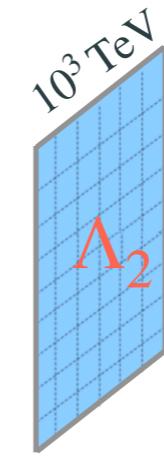
Extension to Planck and Cosmological Signatures



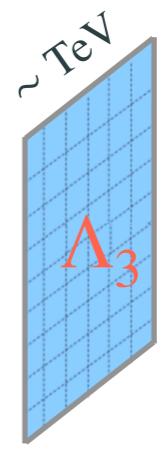
$$4_3 \times 4_2 \times 4_1$$



$$4_3 \times 4_2 \times 3_1$$



$$4_3 \times 3_{12}$$



$$\mathcal{G}_{\text{IR}}$$

[Greljo, Opferkuch, Stefanek, [1910.02014](#);
JFM, Isidori, Lizana, Selimovic, Stefanek, [2203.01952](#)]

N.B.: $N_i \equiv SU(N)_i$

Conclusions

The flavor puzzle and the hierarchy problem, when considered together, point to a **multi-scale picture with the first NP threshold around the TeV**

- The B -physics anomalies nicely fit with this picture and might be its first manifestation

A flavor non-universal 4321 gauge theory provides a consistent framework for **third-family quark-lepton unification at the TeV scale**. This model can explain the B-anomalies, while retaining a flavor structure consistent with the Yukawas and the multi-scale picture

This UV solution can be justified from a **multi-brane extra-dimensional construction** where the Higgs emerges as a pNGB from the same strong dynamics that breaks 4321

Thank you!

Backup

Inverse see-saw and third-family PS unification

Inverse Seesaw setup

$$M_\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^\top & 0 & m_R^\top \\ 0 & m_R & \mu \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ S_L^c \end{pmatrix}$$

Neutrino solution

$$m_\nu \approx m_D m_R^{-1} \mu (m_R^{-1})^\top m_D^\top$$

for $m_D^{(i)} \sim m_u^{(i)} \sim (10^{-2}, 1, 10^2) \text{ GeV}$
 $m_R^{(i)} \sim (10^7, 10^5, 10^3) \text{ GeV}$
 $\mu \sim (10^7, 10^{-1}, 10^{-9}) \text{ GeV}$
 $\Rightarrow 10^{-2} \text{ eV} \lesssim m_\nu^{(i)} \lesssim 10^{-1} \text{ eV}$

μ generated dynamically by singlet scalar Φ_i breaking spontaneously $U(1)_F \leftarrow$ fermion number

$$-\mathcal{L}_\nu \supset \text{Dirac mass } m_D \quad \bar{\Psi}_L^{(i)} H_i \Psi_R^{(i)} + \text{ } m_R \text{ matrix } S_L^{(i)} \Sigma_i^\dagger \Psi_R^{(i)} + \text{ } \text{Mixing } (\epsilon_S \bar{S}_L^{(2)} \Sigma_1^\dagger \Psi_R^{(1)} + \dots) + \text{ } \text{Majorana mass } \mu \bar{S}_L^{(i)} \Phi_i S_L^{(i)c}$$

$\langle H_1 \rangle = \epsilon_H \langle H_2 \rangle = \epsilon_H^2 \langle H_3 \rangle$

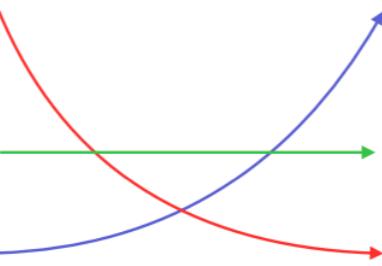
$\langle \Sigma_3 \rangle = \epsilon_\Sigma \langle \Sigma_2 \rangle = \epsilon_\Sigma^2 \langle \Sigma_1 \rangle$

$\langle \Phi_3 \rangle = \epsilon_\Phi \langle \Phi_2 \rangle = \epsilon_\Phi^2 \langle \Phi_1 \rangle$

Erase $U(2)^5$ in
the neutrino sector:

hierarchical ratio $\frac{m_D^{(i)}}{m_R^{(i)}}$

very hierarchical $\mu^{(i)} \sim \langle \Phi_i \rangle$



\Rightarrow anarchical $m_\nu^{(i)}$
(both masses and mixing angles)

Realised for:

$\epsilon_\Phi \sim \epsilon_H^2 \epsilon_\Sigma^2$
 $\epsilon_S \sim \epsilon_H \epsilon_\Sigma$

The 5D model

[JFM, Isidori, Lizana, Selimovic, Stefanek, [2203.01952](#)]

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
$\Psi^3, \Psi_d^3, \chi^{(\prime)}$	4	1	4
$\Psi^j, \Psi_{u,d}^j$	1	4	4
\mathcal{S}^i	1	1	1
Σ	1	1	5
Ω	1	4	4
Φ	1	1	1

$$\Psi^3 = \begin{bmatrix} \psi^3 (+,+) \\ \psi_u^3 (-,-) \\ \tilde{\psi}_d^3 (+,-) \end{bmatrix},$$

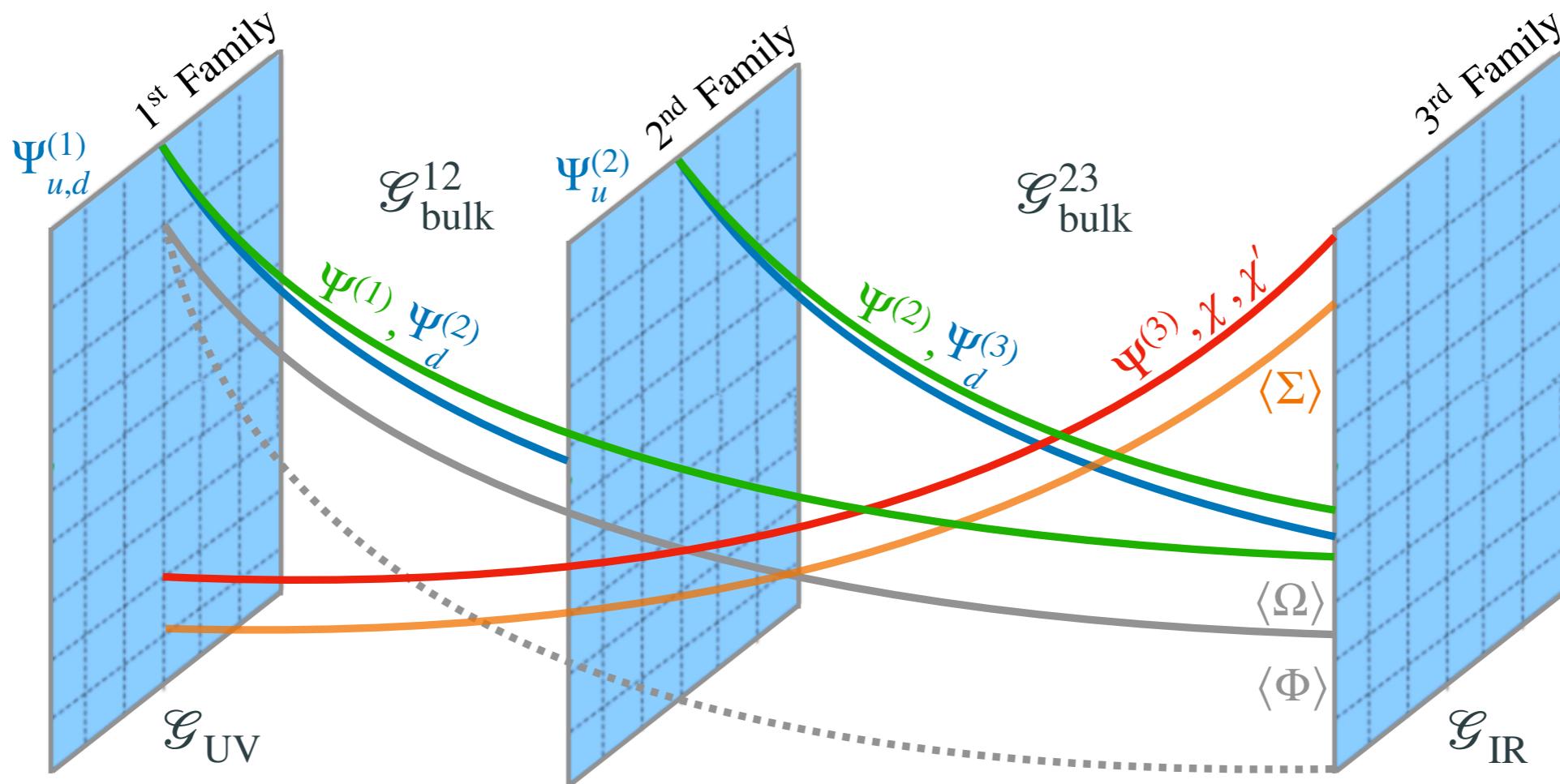
$$\chi^{(\prime)} = \begin{bmatrix} \chi^{(\prime)} (\pm,\pm) \\ \chi_u^{(\prime)} (\mp,\pm) \\ \chi_d^{(\prime)} (\mp,\pm) \end{bmatrix},$$

$$\Psi_u^j = \begin{bmatrix} \tilde{\psi}^j (+,-) \\ \psi_u^j (-,-) \\ \hat{\psi}_d^j (+,-) \end{bmatrix},$$

$$\Psi_d^3 = \begin{bmatrix} \tilde{\psi}^3 (+,-) \\ \tilde{\psi}_u^3 (+,-) \\ \psi_d^3 (-,-) \end{bmatrix},$$

$$\Psi^j = \begin{bmatrix} \psi^j (+,+) \\ \tilde{\psi}_u^j (-,+) \\ \tilde{\psi}_d^j (-,+) \end{bmatrix},$$

$$\Psi_d^j = \begin{bmatrix} \hat{\psi}^j (+,-) \\ \hat{\psi}_u^j (+,-) \\ \psi_d^j (-,-) \end{bmatrix},$$



Higgs Potential

$$V(h) = \sum_r \frac{N_r}{16\pi^2} \int_0^\infty dp p^3 \log [\rho_r(-p^2)]$$

Field	$SU(4)_h$	$SU(4)_l$	$SO(5)$
Ψ^3	4	1	4
Σ	1	1	5
Ω	1	4	4

$$V(h) \approx \alpha(h) \cos\left(\frac{h}{f}\right) - \beta(h) \sin^2\left(\frac{h}{f}\right)$$

VEV :

$$\alpha_\Omega \approx (\tilde{M}_\Omega^R - \tilde{M}_\Omega^L) \Lambda_{\text{IR}}^2 \langle \Omega_{\text{IR}} \rangle^2$$

$$\alpha_{\Psi^3}(h) \approx \frac{3N_c f^4}{32\pi^2} \zeta(3) y_t^2 g_*^2 - 2\beta_{\Psi^3}(h)$$

$$\cos(\langle h \rangle/f) = -\frac{\alpha}{2\beta}$$

Quartic :

$$\beta_\Sigma \approx \frac{1}{2} (\tilde{M}_{H'} - \tilde{M}_S) \frac{\Lambda_{\text{IR}}^2}{(kL)^2} \langle \Sigma_{\text{IR}} \rangle^2$$

$$\beta_{\Psi^3}(h) \approx \frac{N_c f^4}{16\pi^2} y_t^4 \left[\gamma + \log \frac{\Lambda_{\text{IR}}^2}{m_t^2(h)} \right]$$

$$\beta_{\text{EW}} \approx -\frac{9f^4}{512\pi^2} g_*^2 \zeta(3) (3g_L^2 + g_Y^2)$$

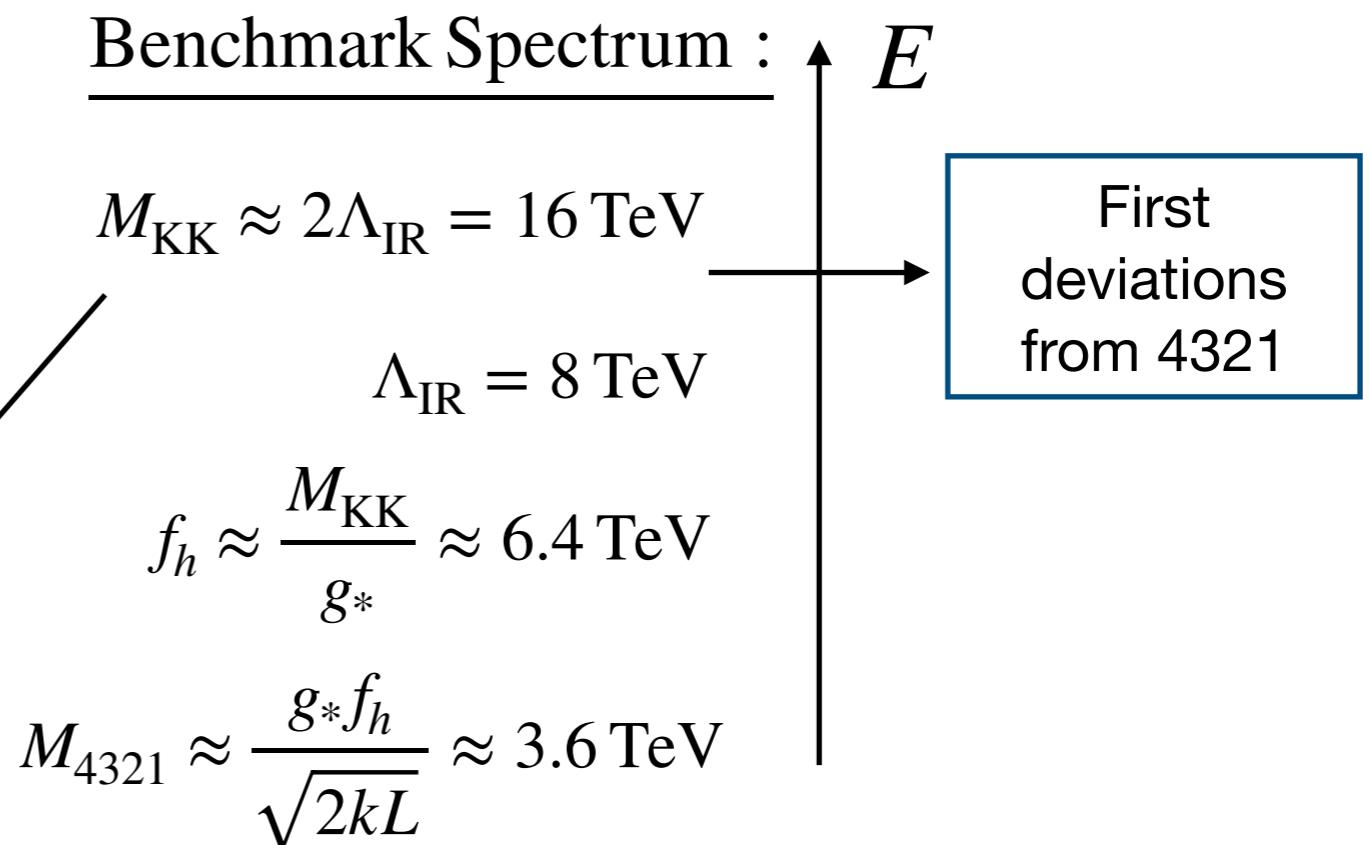
[JF, Isidori, Lizana, Selimovic, Stefanek, [2203.01952](#)]

Low-energy phenomenology

- Below KK scale, same phenomenology as 4321 (B-anomalies)
- Main experimental limit coming from coloron direct searches:

$$M_{4321} \gtrsim 3.5 \text{ TeV}$$

[Cornella et al., [2103.16558](#)]



$[f > 2.5 \text{ TeV } (M_{\text{KK}} > 6 \text{ TeV})]$

[JF, Isidori, Lizana, Selimovic, Stefanek, [2203.01952](#)]