Lattice QCD for quark flavour physics

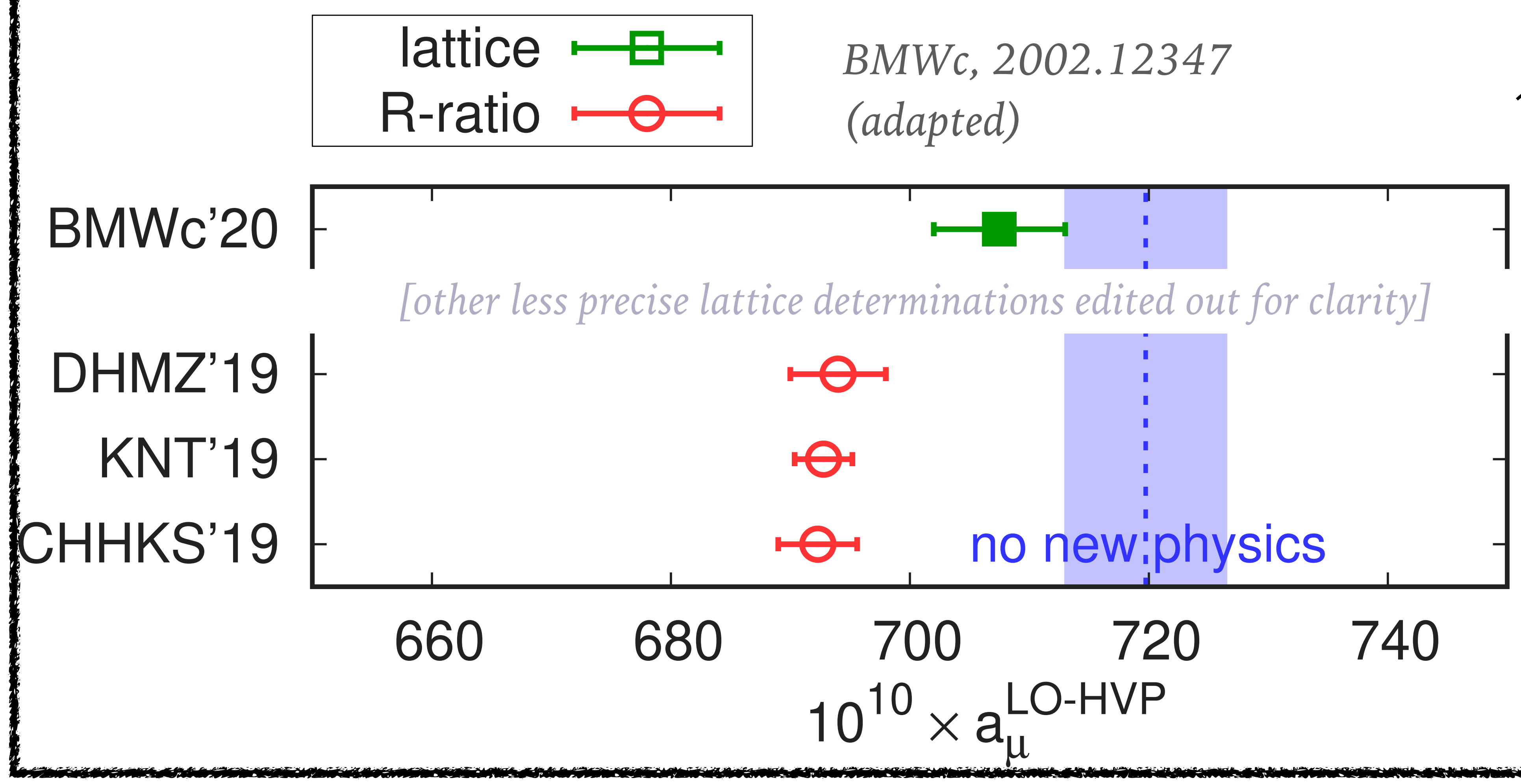
@ Rencontres du Vietnam Flavour Physics Conference 2022

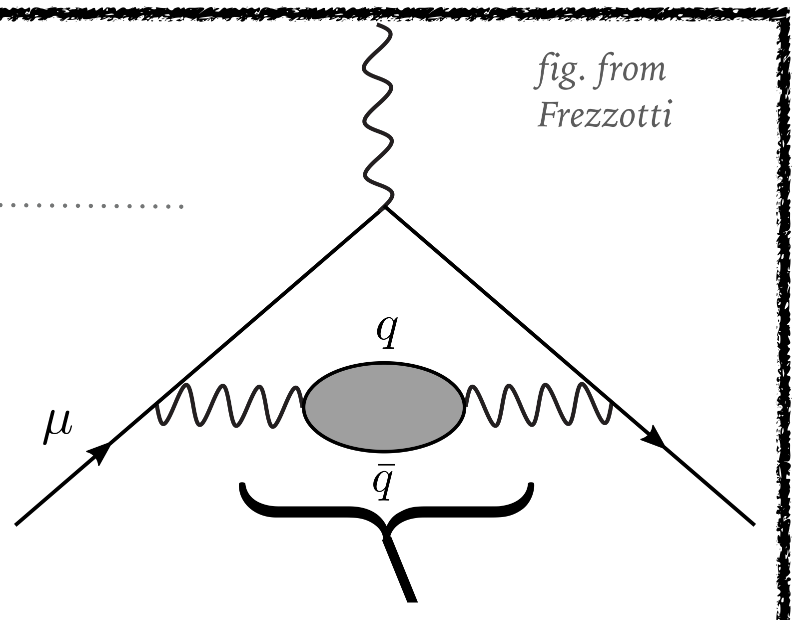
Shoji Hashimoto (KEK Theory Center/SOKENDAI)

Lattice seems like a competitive tool, now

Slides from G. Salam @ ICHEP2022

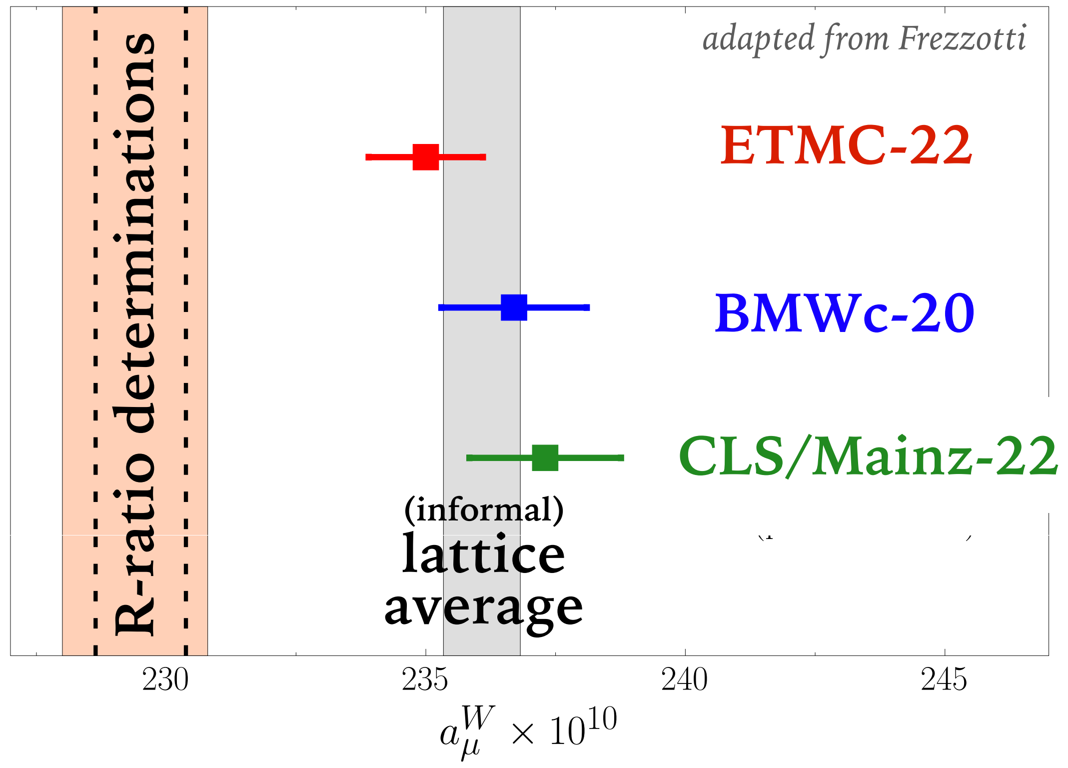
g_μ–2: largest theory uncertainty is "HVP"





Hadronic Vacuum Polarisation (HVP)

intermediate window: 2 new lattice results (ETMC-22 & CLS/Mainz-22)



- Highest precision lattice results mutually consistent (more to come soon)
- Difference in this window alone (~7) not enough to explain g_μ-2 (~25), but enhances credibility of full BMWc-20 result
- Tension between lattice & e^+e^- data clearly needs to be understood

- strong tension with a_μ^W (HVP-LO) results driven by experimental e^+e^- data :

at $\sim 4.2\sigma_{combined}$ if WP-proc.('22) (2205.12963, Colangelo et al.), see *light-red band*, is used

at $\sim 5.8\sigma_{combined}$ if KNT('19-'22) (1911.00367 + private comm.), see *dashed lines*, is used

Should we trust the lattice?
Any problem, or limitations?

Lattice QCD

We believe it's the right way to go, because …

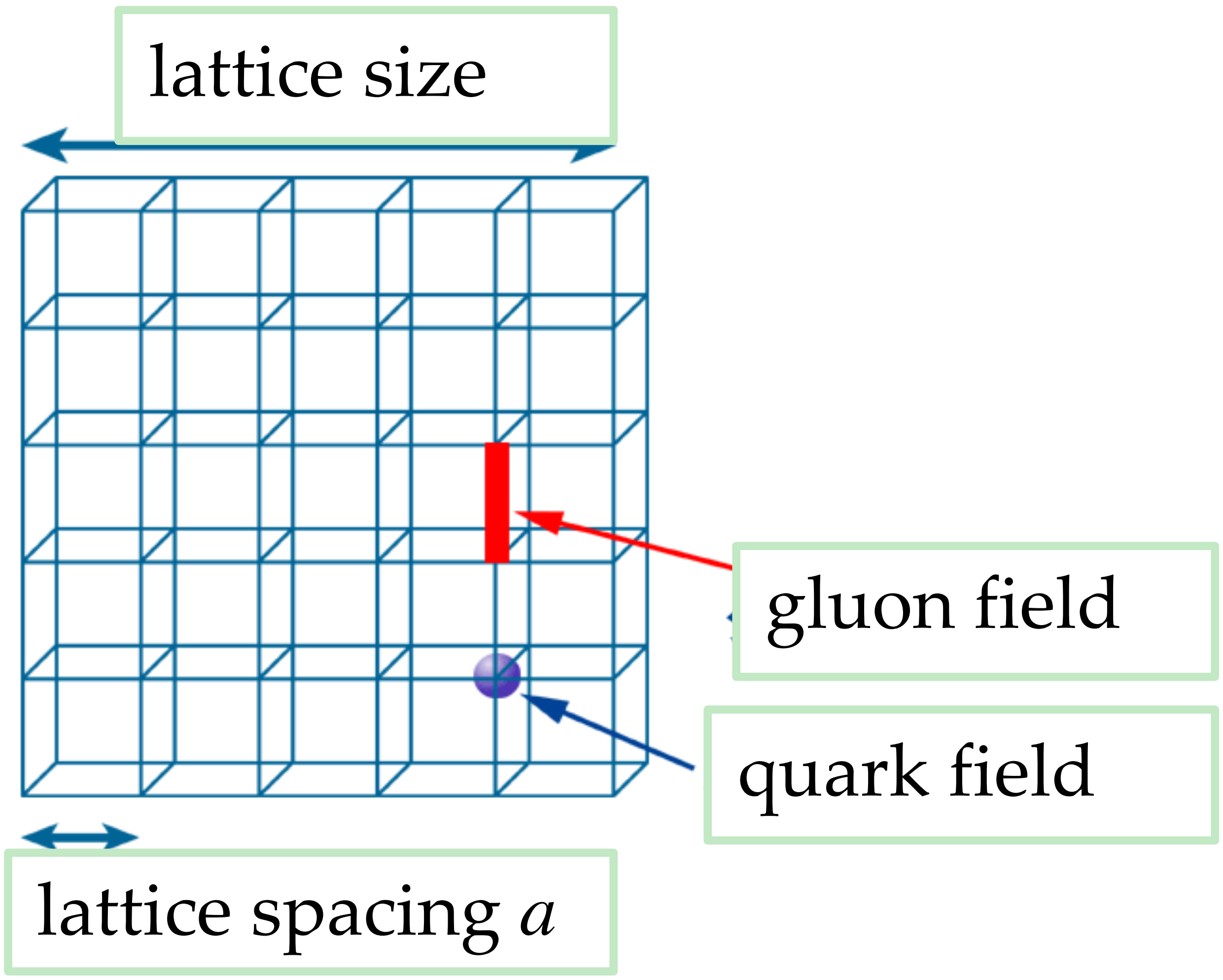
Firm theoretical framework = Quantum Field Theory

- Well-defined theory
  - QCD Lagrangian + path integral quantization

$$S = \int d^4x \left\{ \frac{1}{4} \mathrm{Tr}\, F_{\mu\nu}^2 + \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f \right\},$$

$$Z = \int [dA_\mu] \prod_f [d\psi][d\bar{\psi}] \exp\left[-S \right]$$

- Universality
  - Renormalizable, the unique continuum theory = QCD
  - Lattice is merely a regularization (irrelevant after taking $a \to 0$)



Lattice QCD

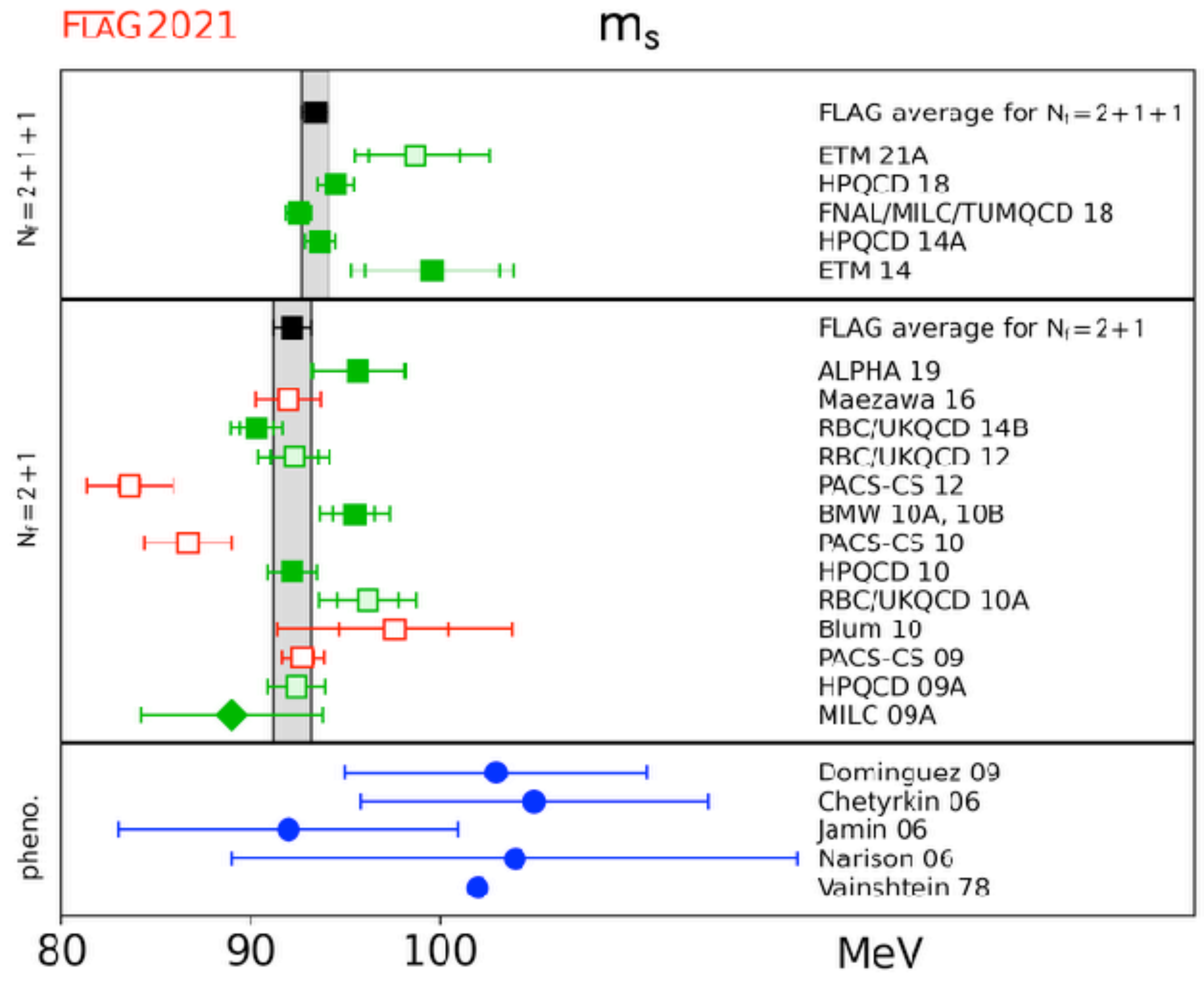
Limitations need to be understood

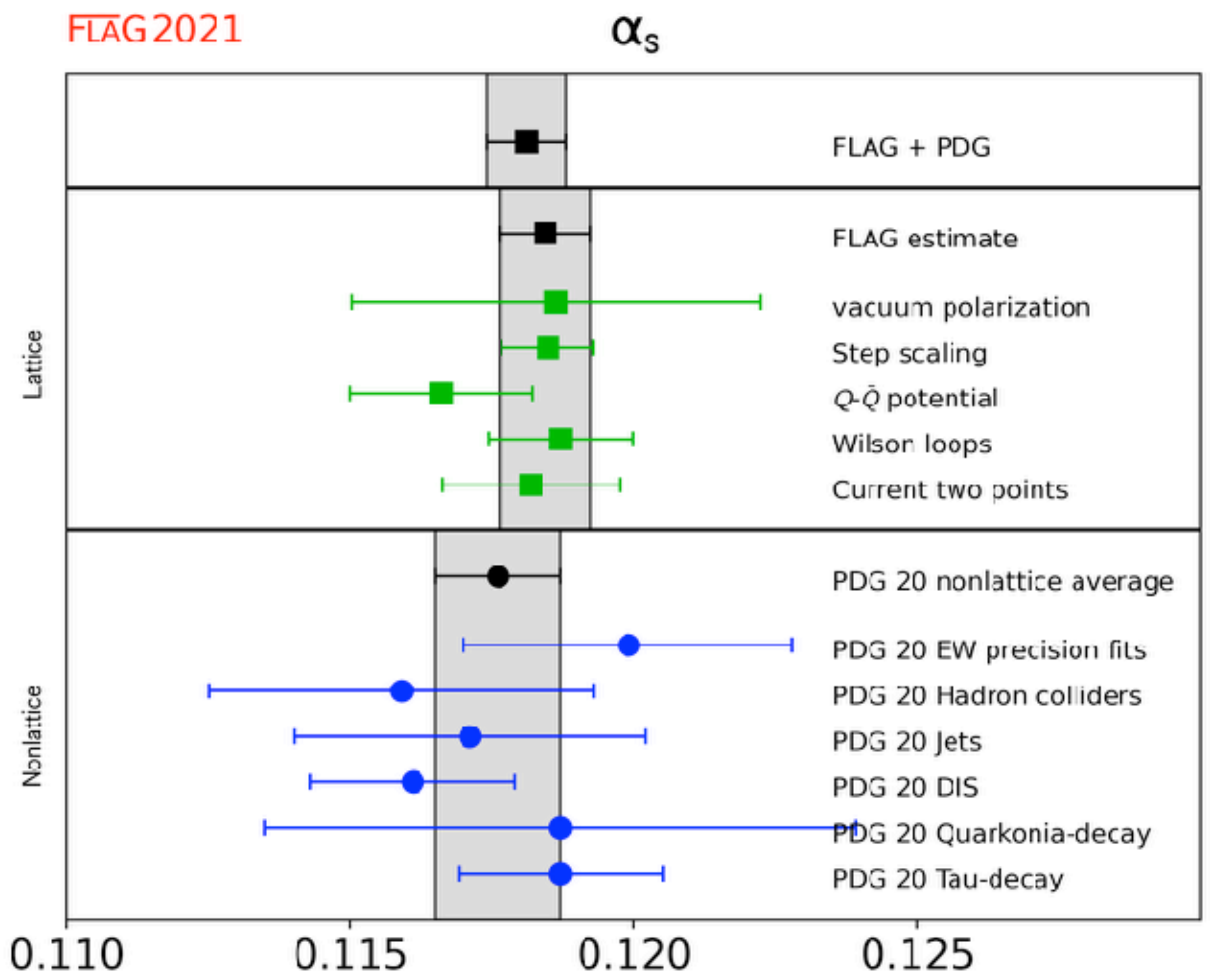
- Systematic errors
  - Discretization, finite volume, etc. : main subject of research for some *easy* quantities.
- Intrinsic limitations
  - (exponentially growing) Noise.
  - Euclidean lattice: no on-shell particles, nor energy conservation. Put severe limits on quantities that can be computed.

This talk

not a comprehensive review

- Resources:
  - PDG review: SH and S. Sharpe (2022 edition)
  - FLAG (Flavour Lattice Averaging Group): http://flag.unibe.ch/2021/
    - World averages (quark masses, decay constants, form factors, etc.)
    - Quality control with well-defined criteria
  - Lattice review talks
    - Kaneko's @ Lattice 2022





This talk

not a comprehensive review

- Snowmass reports:
  - "Lattice QCD and particle physics," arXiv:2207.07641
  - "A lattice QCD perspective on weak decays of b and c quarks," arXiv:2205.15373
  - "Hadron spectroscopy with lattice QCD," arXiv:2203.03230
  - "Lattice QCD calculations of parton physics," arXiv:2202.07193
  - "Lattice QCD and the computational frontier," arXiv:2204.00039

Submitted to the Proceedings of the US Community Study
on the Future of Particle Physics (Snowmass 2021)

Lattice QCD and Particle Physics

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(USQCD Collaboration)

This talk

not a comprehensive review

- Nice reviews, averages may be found as above.
  - Look for your favorite quantities, or ask me (but for limited knowledge)
- Rather, my talk focuses on the *limitations*
  - How do we understand the systematic errors?
  - What can be easily computed? What is (much) more challenging? And, why?

Systematic errors

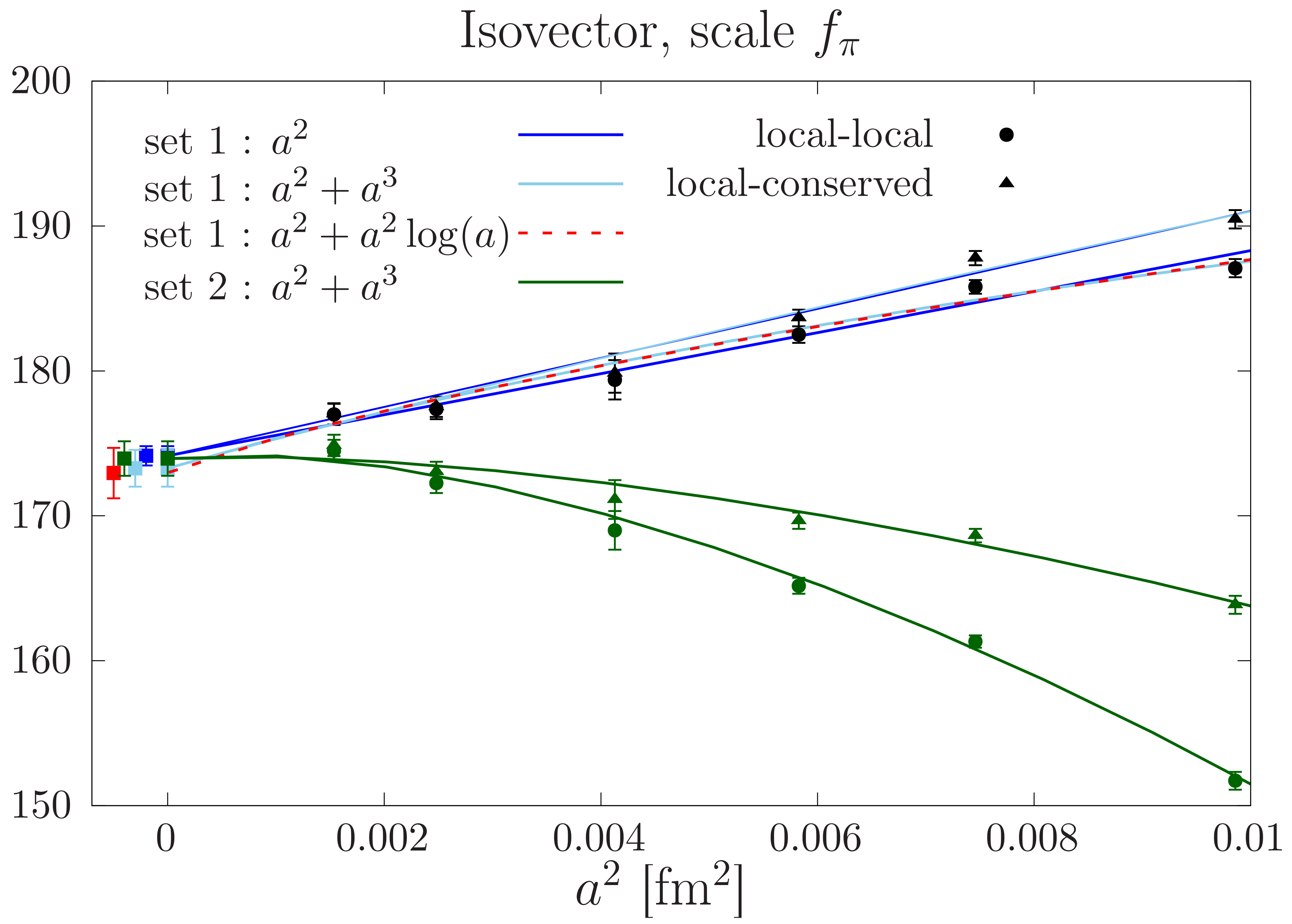
Discretization effects

Fine if taking the continuum limit ··· trivial?

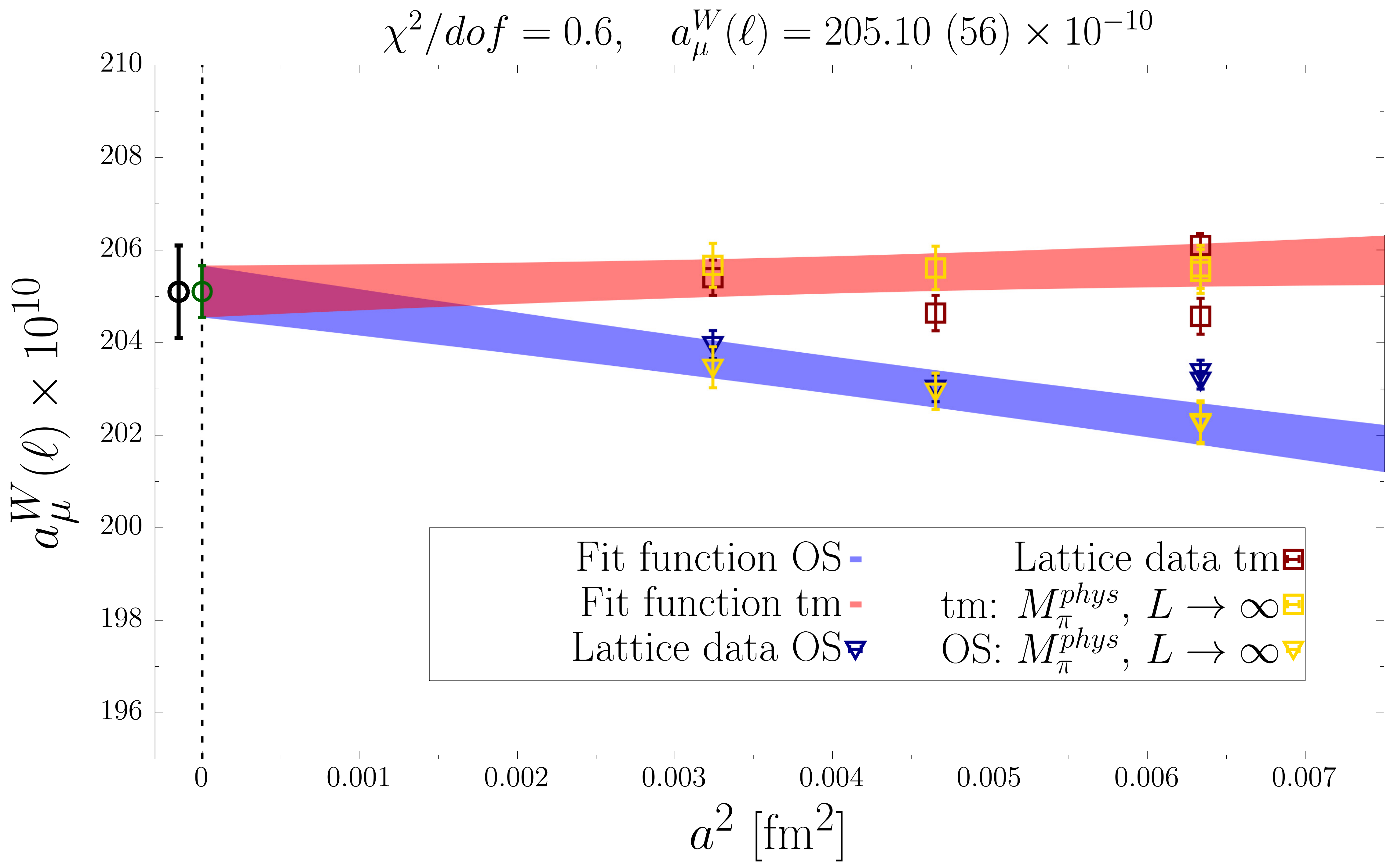
- Typical size of the discretization effect:
  - $(a\,\Lambda)^2$, with Λ the QCD scale
  - $(a\,\mathrm{M})^2$, when heavy quark of mass M is involved
  - $(a\,\mathrm{p})^2$, when external momenta are injected.
- Typical lattice spacing of the present simulations
  - $a^{-1} \sim 2 \sim 4$ GeV; so a rough idea of the error → extrapolation

Examples of the "continuum limit" : HVP (window) for muon g-2

Mainz group, 2206.06582

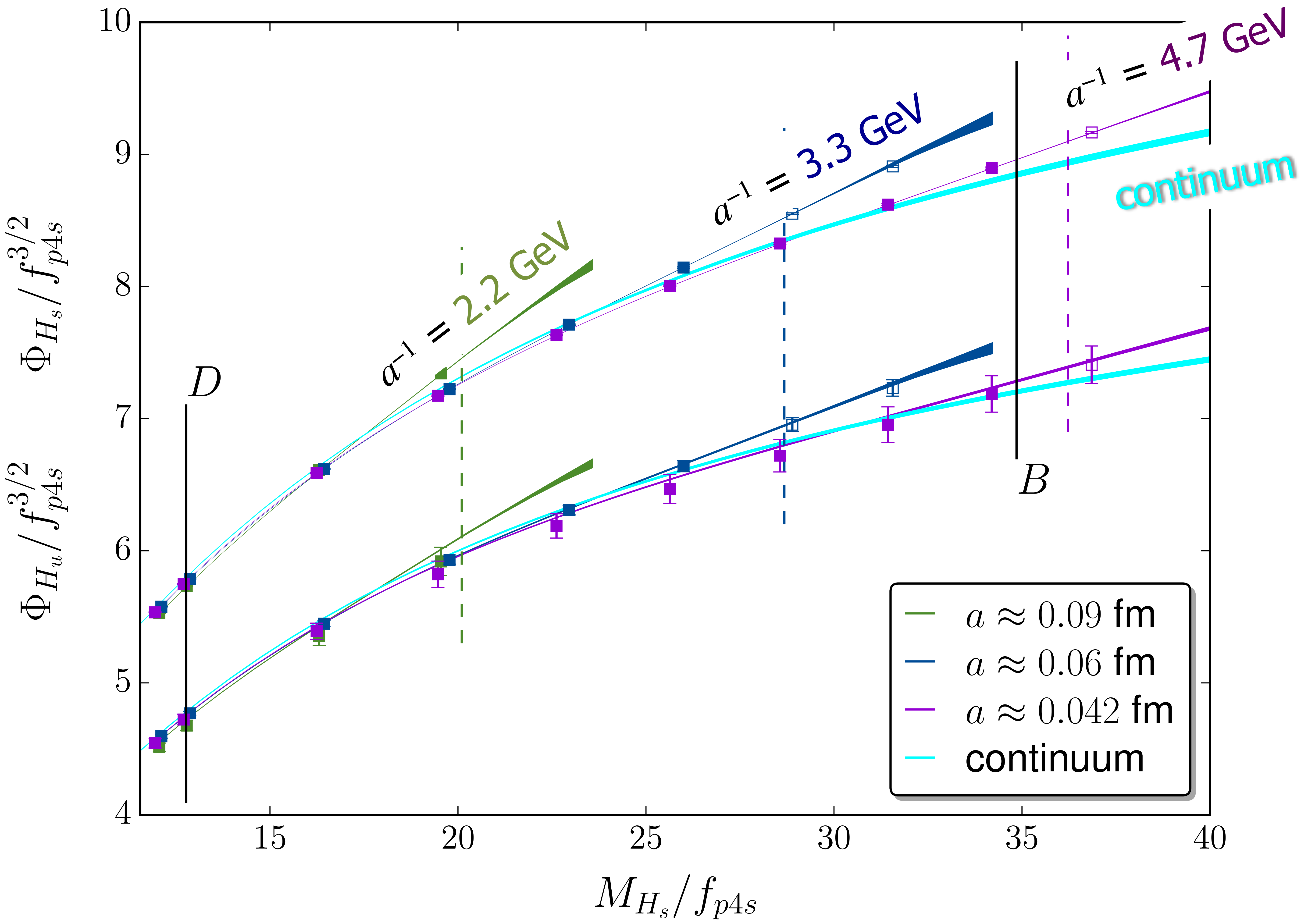


ETMC, 2206.15084



Lattice simulations at several values of a ; then extrapolate.

B meson decay constant:
Is the disc. effect with heavy quarks under control?



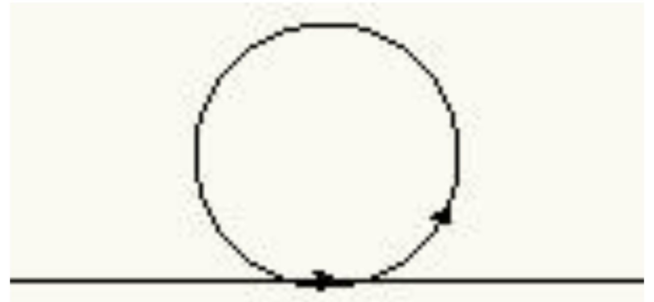
Fermilab/MILC, 1712.09262
the most precise f_B to date

Under good control with the help of heavy quark scaling.

Finite volume effects

Simple for one particle in a box

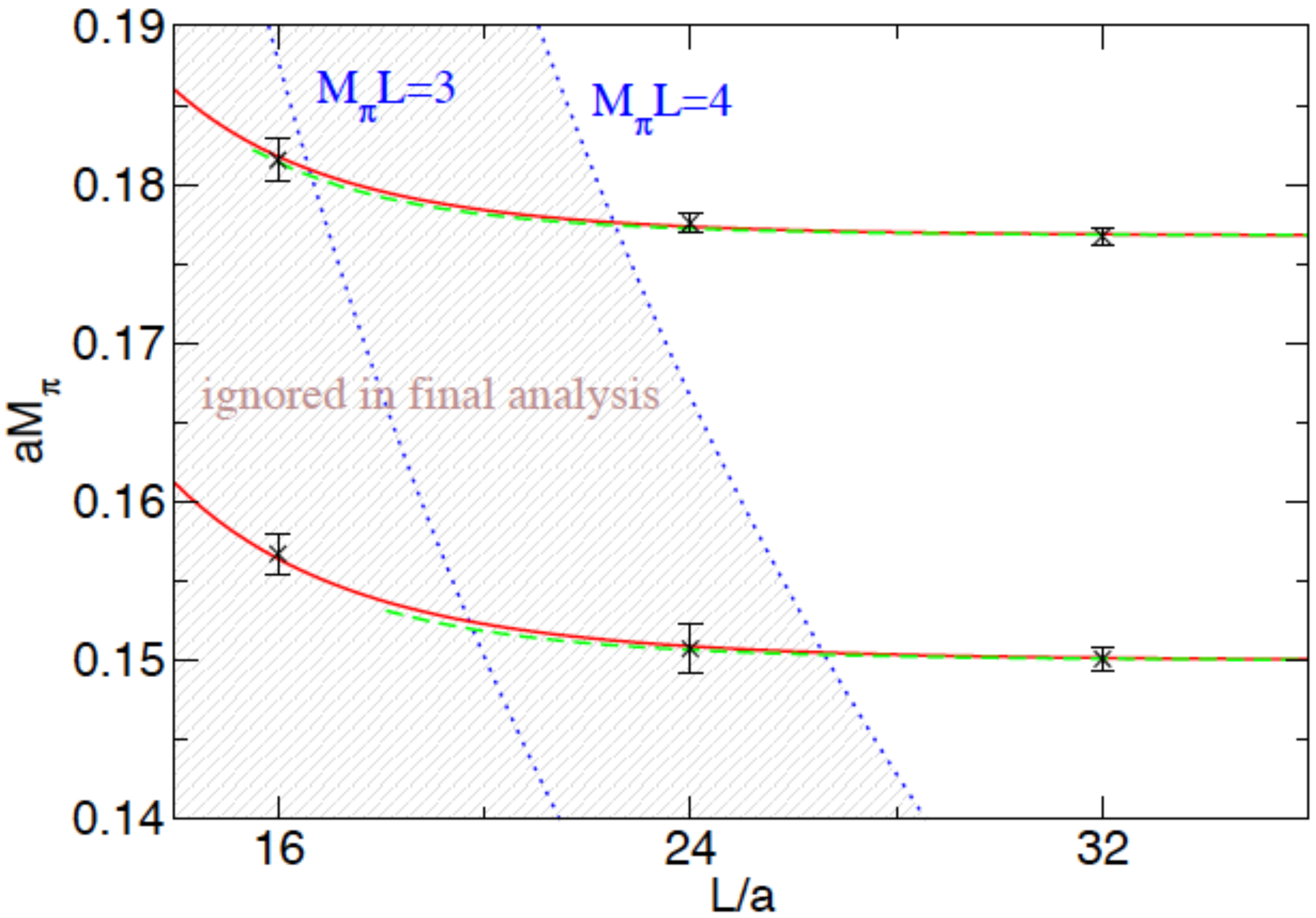
Dominant contribution from the lightest particle, i.e. pion. It's loop effect can be estimated using chiral effective theory.



$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+m^2} \to \sum_k \frac{1}{k^2+m^2}$$

Effects of $\sim \exp(-m_\pi L)$

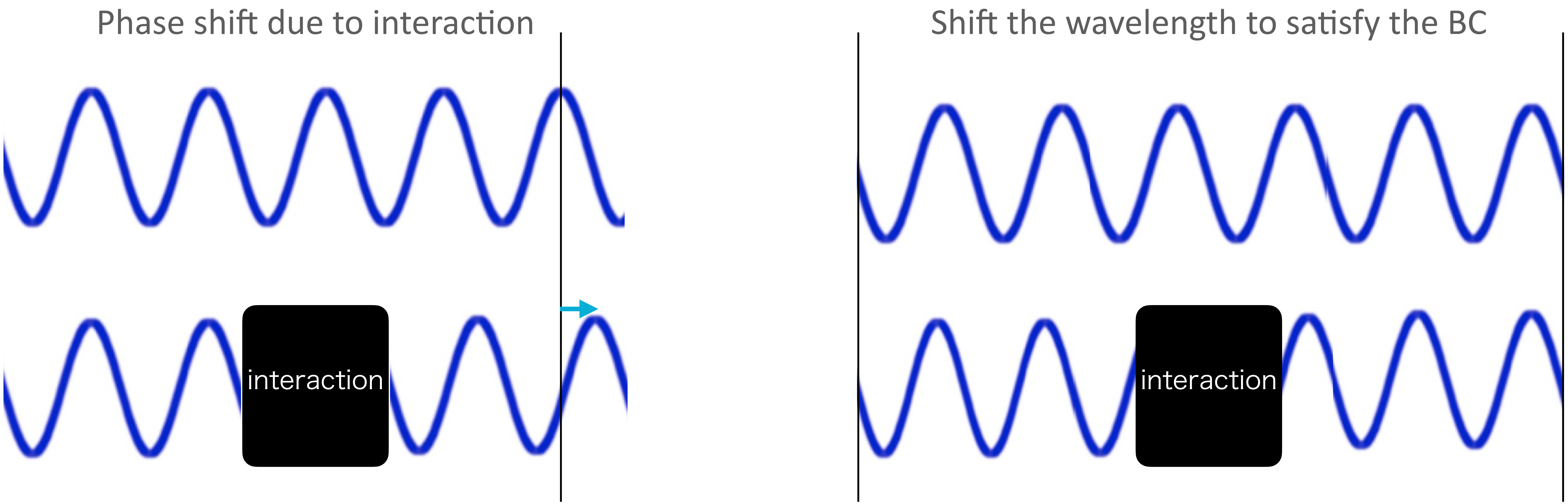
BMW (2011)



Finite volume effects

More involved with multiple particles

Lüscher's method: FVE behaves as $1/L^3$. Can be used to extract phase shift.

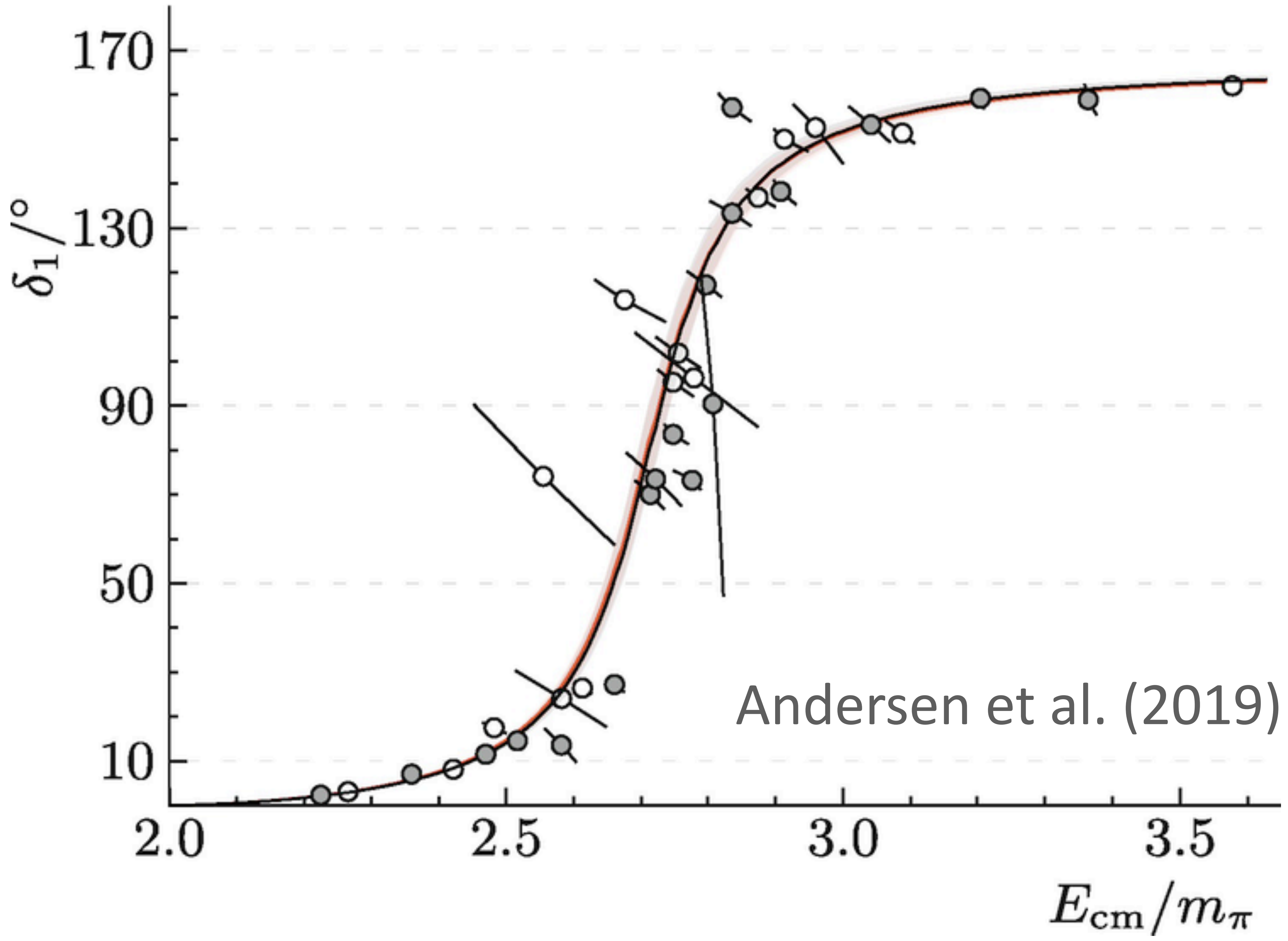


Phase shift converted to the energy shift: **a friend, rather than an enemy!**

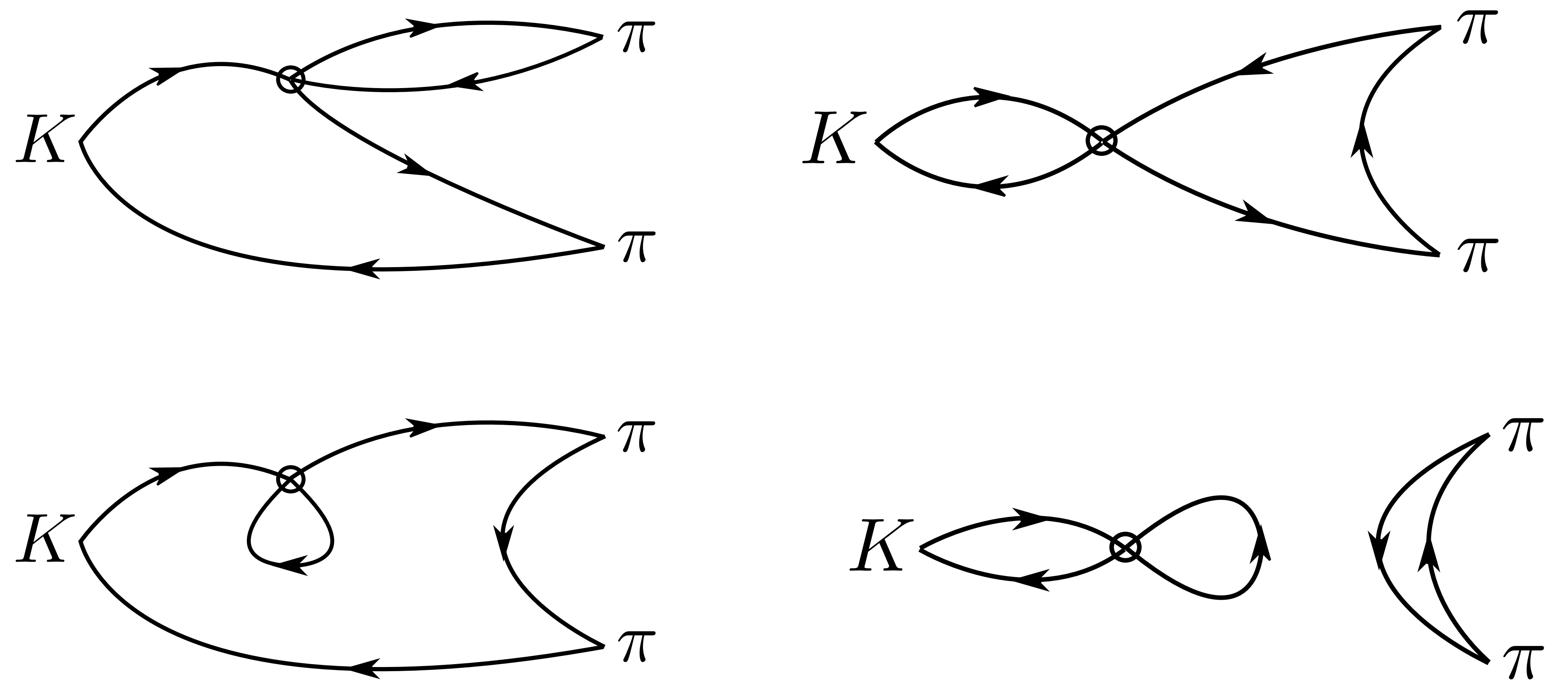
Lüscher's method

Successfully applied for many two-body scatterings.

I = 1, ππ phase shift



Extension to the (non-leptonic) decay, such as $K \to \pi\pi$ (Lellouch-Lüscher 2001)

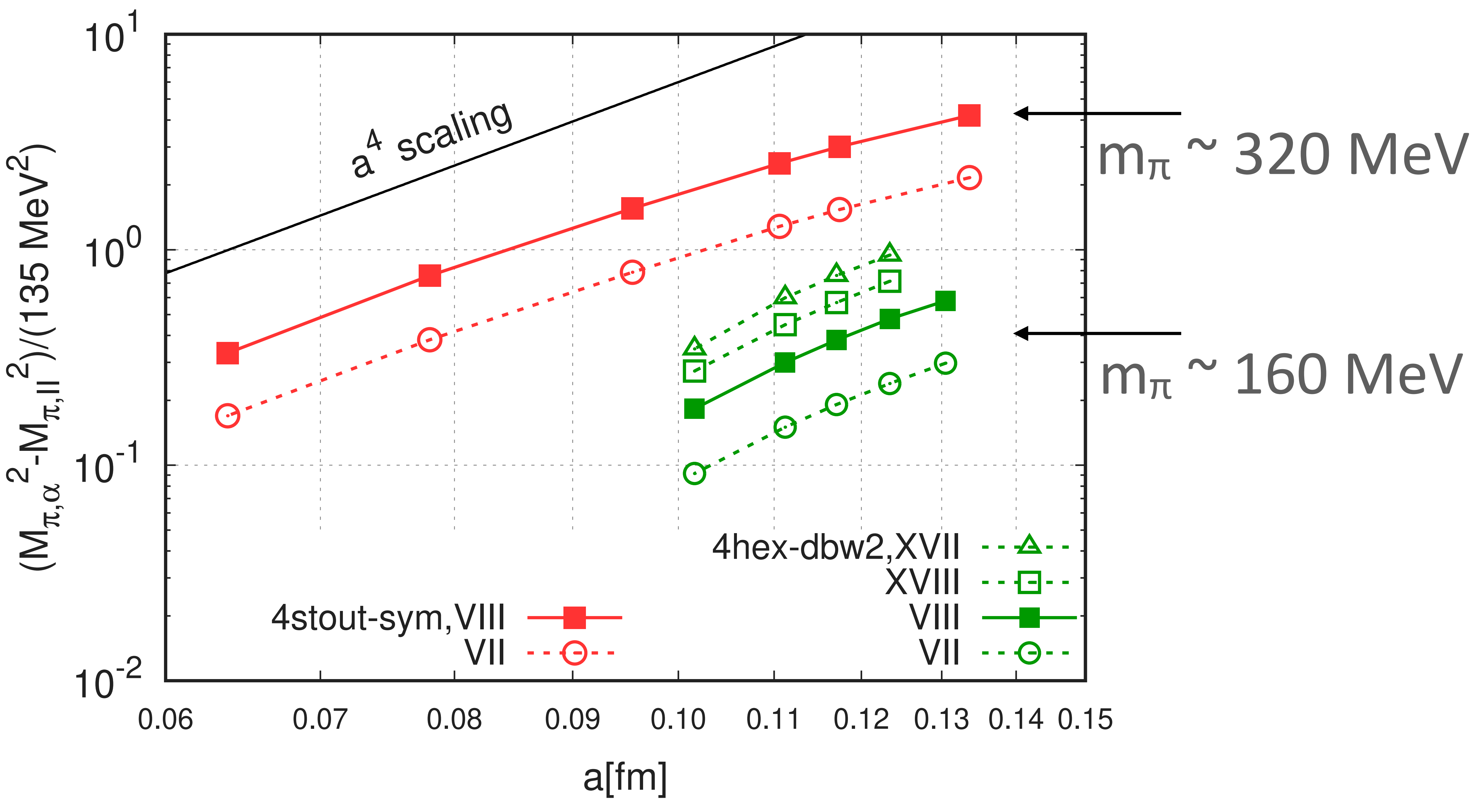


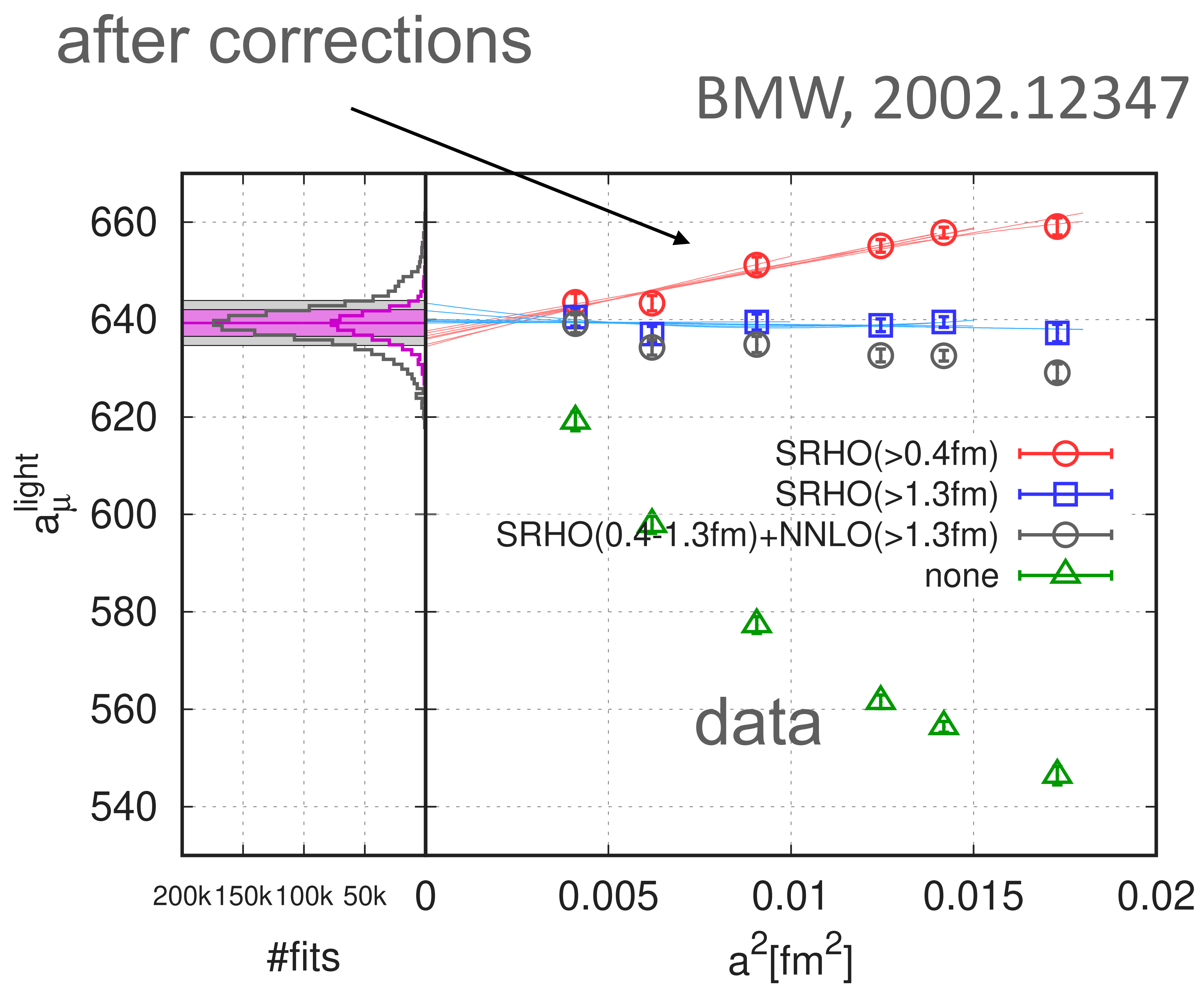
See, heroic contributions by RBC-UKQCD over years (come back to this later)

Combined effect?

So, sometimes tricky

With the staggeredferminon, disc effect distorts the pion spectrum, which may thus induce finite volume effects.

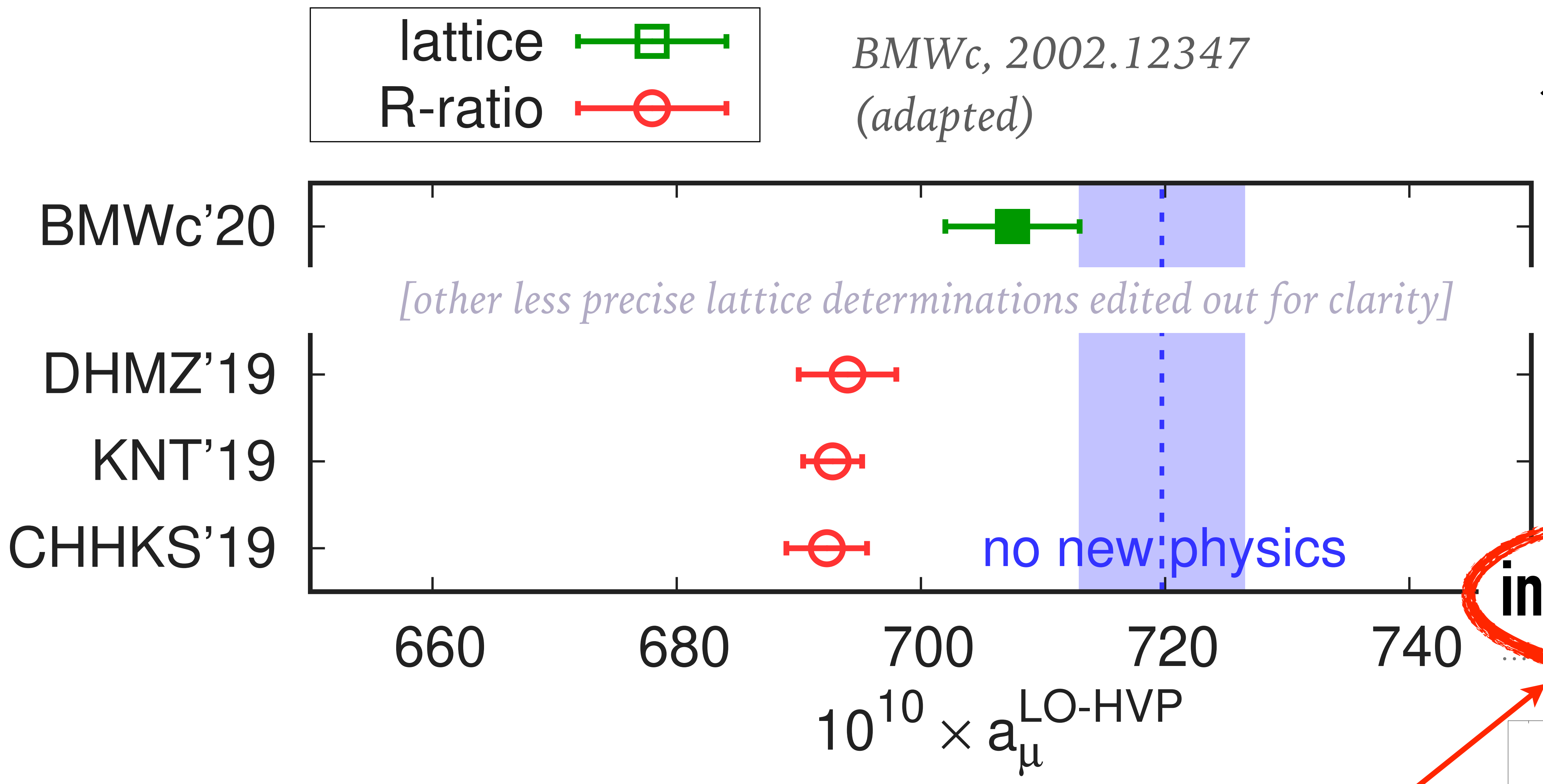


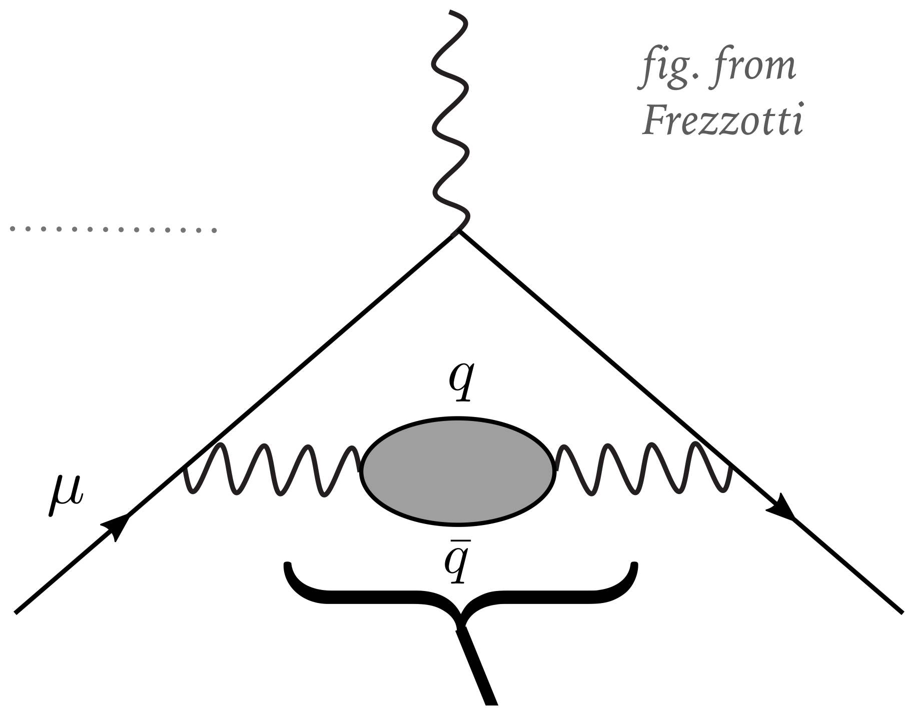


BMW, 2002.12347

g_μ-2: largest theory uncertainty is “HVP”

Slides from G. Salam @ ICHEP2022



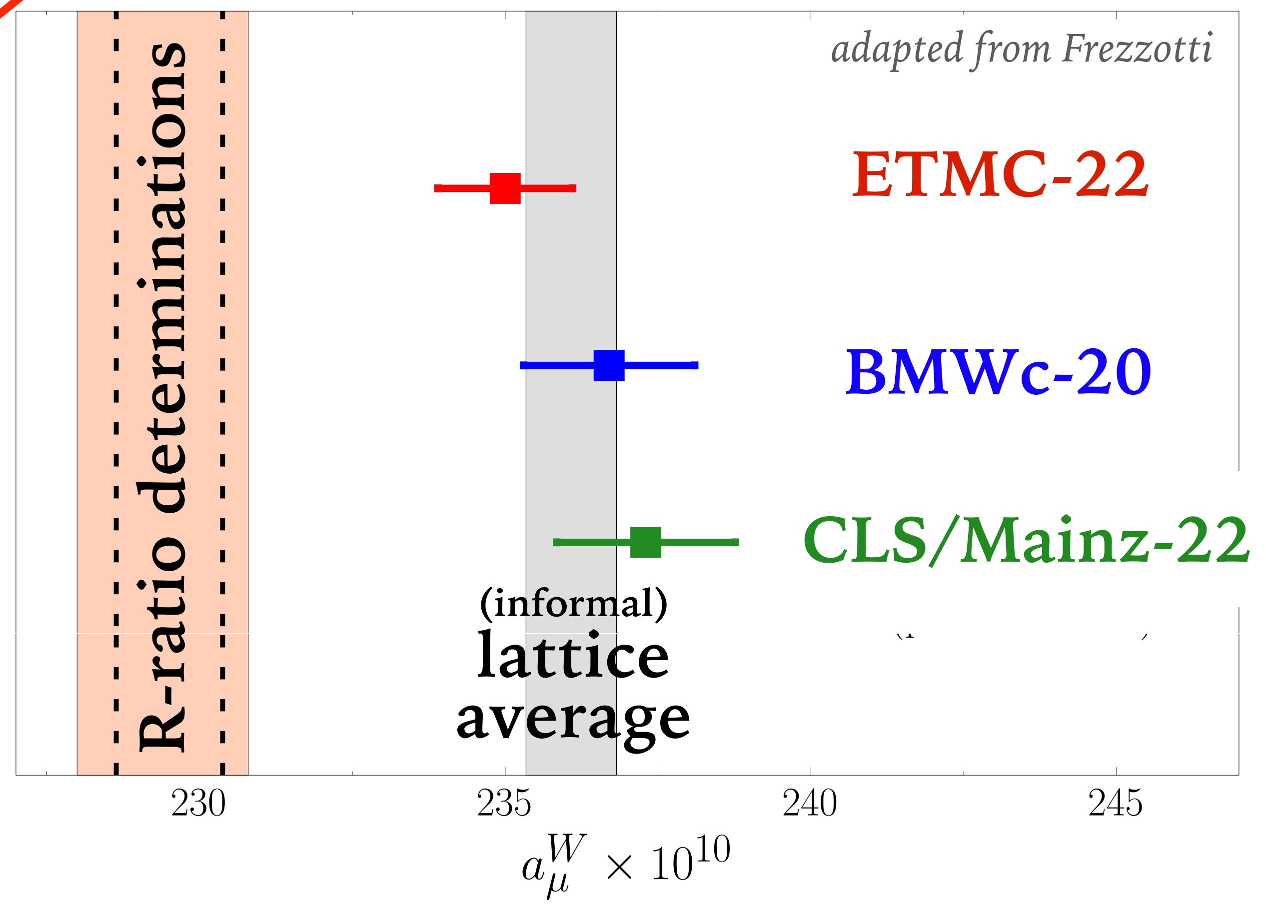


Hadronic Vacuum Polarisation (HVP)

Contrib. from mid-scale:

- less disc. effect
- less finite volume effect

intermediate window: 2 new lattice results (ETMC-22 & CLS/Mainz-22)



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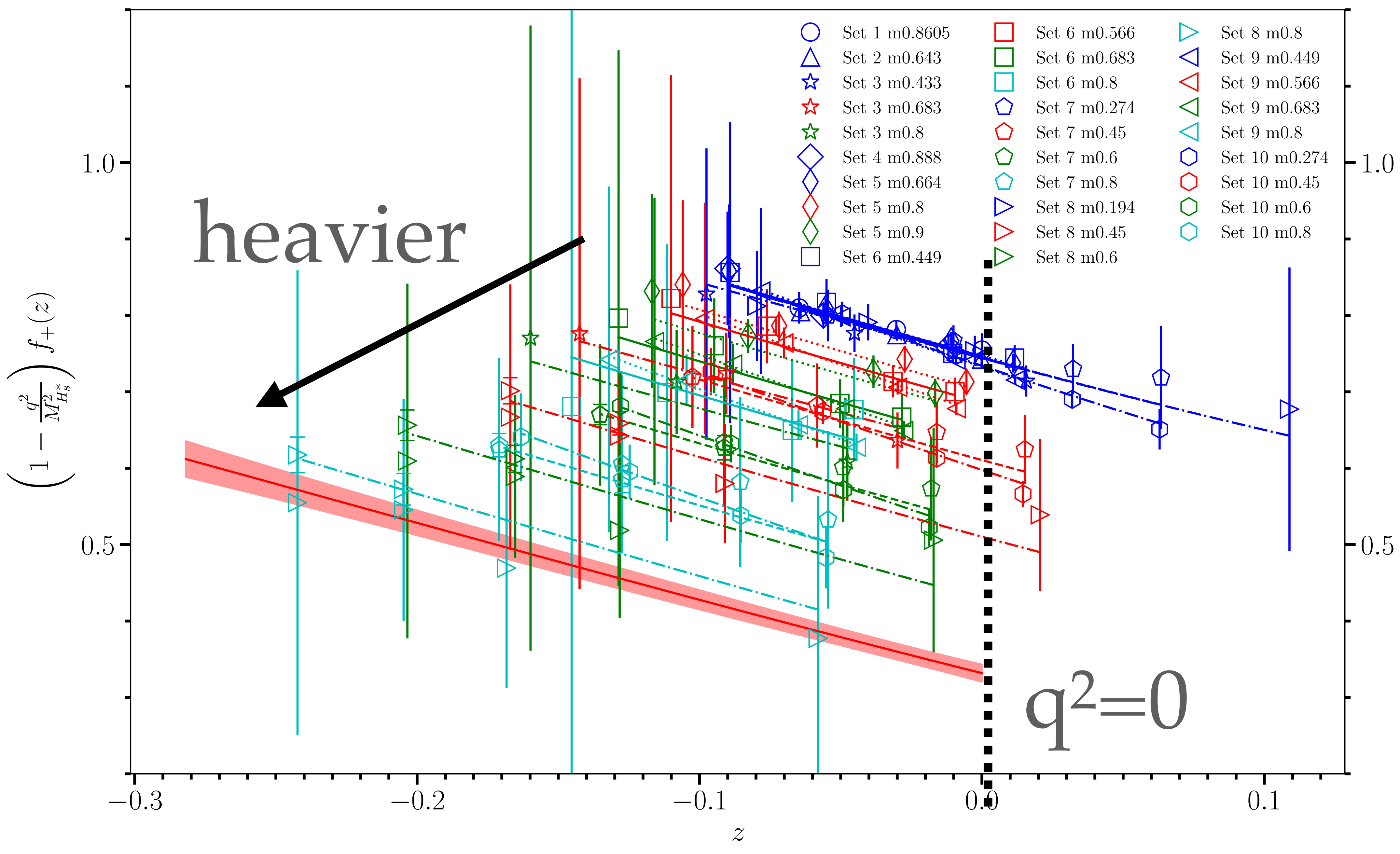
Another important quantity for NP search

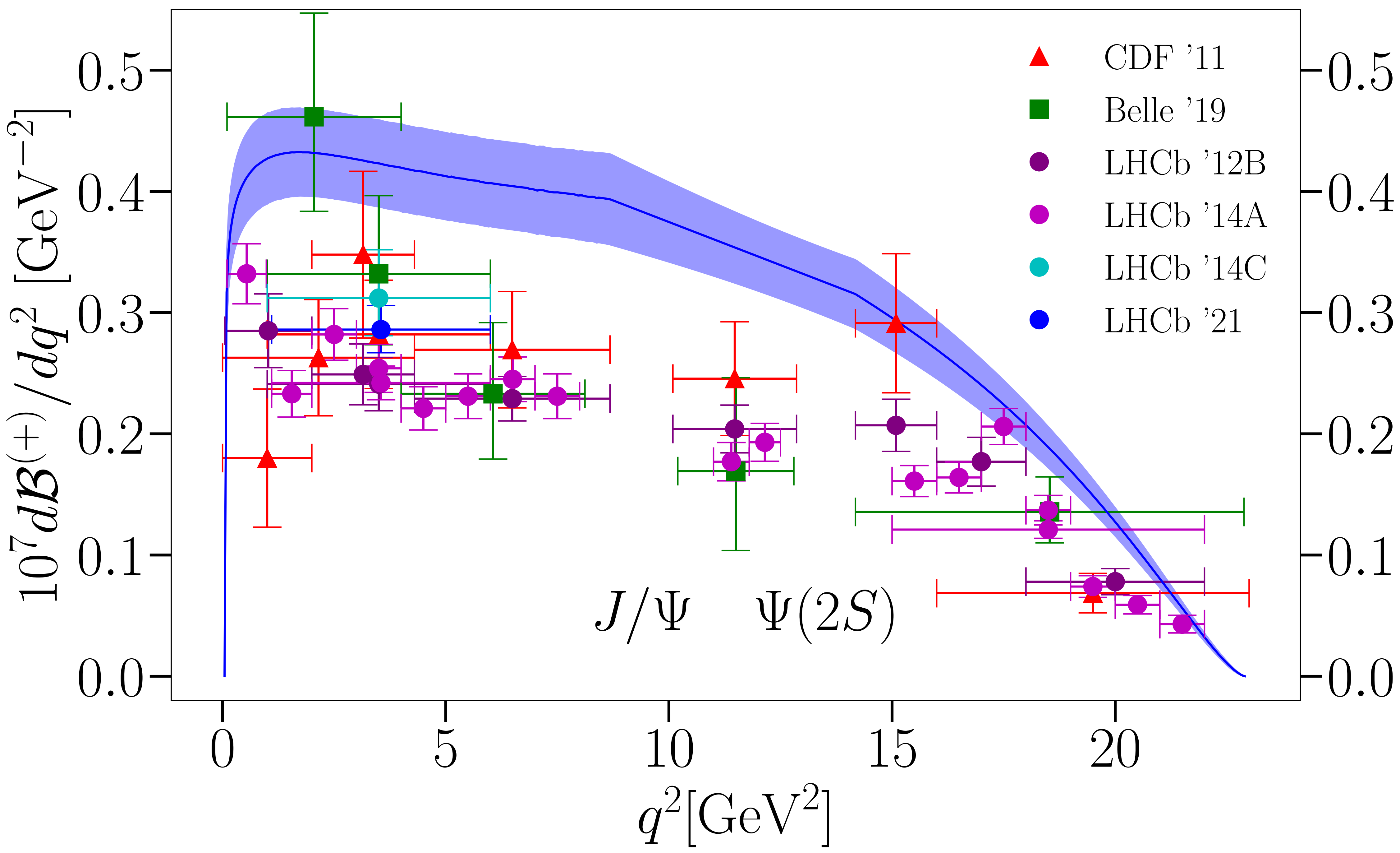
Decay rate involving FCNC (B→Kll)

HPQCD, 2207.12468



HPQCD, 2207.13371

B→Kll differential decay rate



Huge deviation. Clear sign of new physics?

More challenges ahead

Fundamental issue

No such thing as energy conservation, on the lattice

We are working on the **Euclidean** lattice; no real particles exist.

$$\frac{i}{p^2 - m^2 + i\epsilon} \to \frac{1}{p^2 + m^2}$$

Particle properties from the pole
(position, residue, …)

Use the exponential fall-off, instead

$$\sum_n A_n e^{-E_n t} \to A_0 e^{-E_0 t}$$

Only the lowest energy state can be extracted cleanly.

What to do then

Extraction of the non-ground states involves $A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + A_2 e^{-E_2 t} + \cdots$

- Use some elaborate method, such as machine learning??
  - May work for a few lowest states, with very precise data.
  - Often, nearby states are mixed up, making a *garbage can*.

- Signal-to-noise kills you.
  - Long-distance correlators are exponentially noisy.
  - Good signal only for the single pion. (Even a single nucleon is challenging.) Multi-particle states are noisy. Finite mom is noisy…

$$S_q \sim e^{-(m_\pi/2)|x|}$$

$$S_N \sim e^{-m_N|x|} \ll S_q^3$$

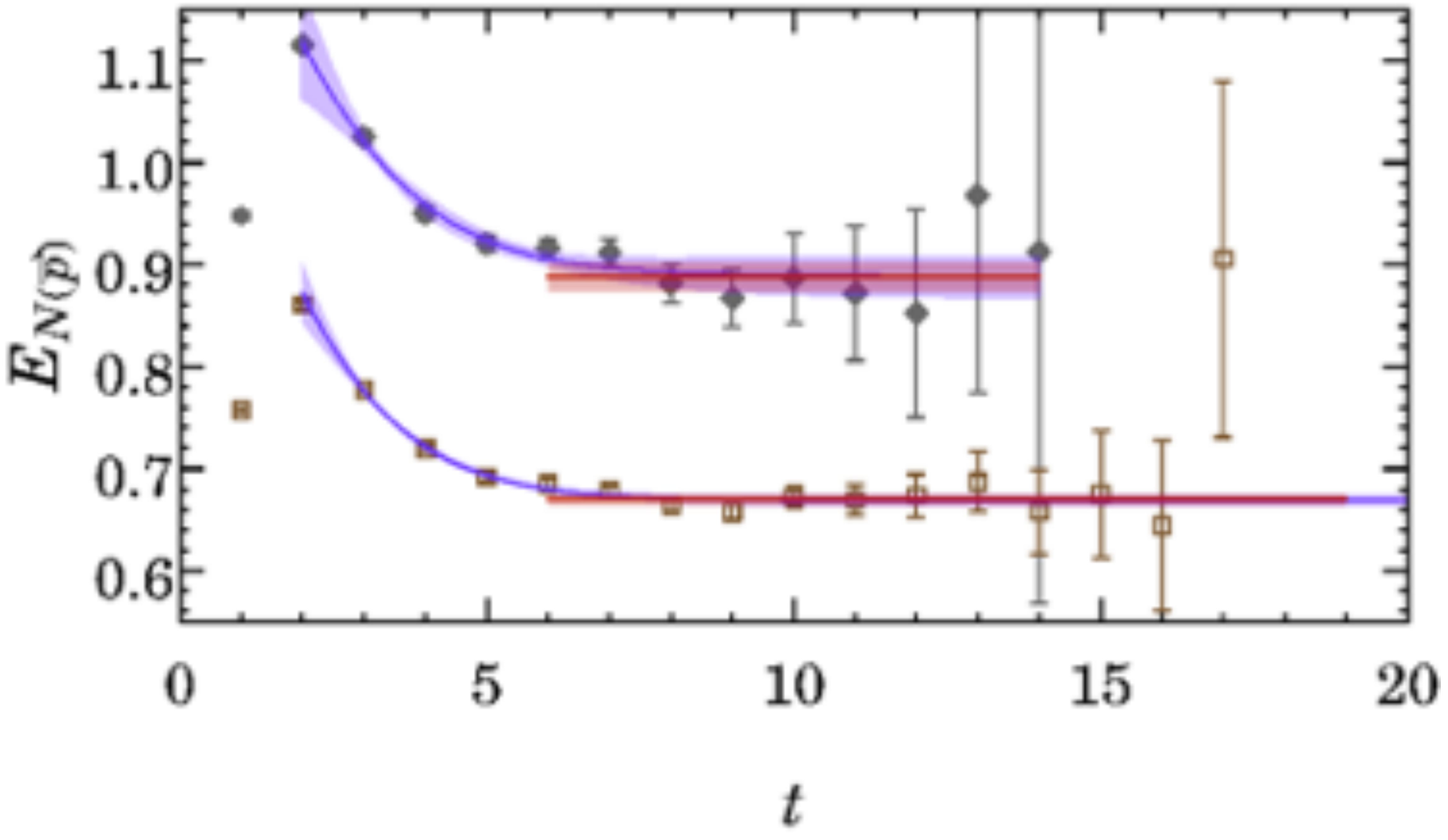
A lot of cancellations among various components.

$$A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + A_2 e^{-E_2 t} + \cdots$$

PNDME (2014)

"effective mass"

$$E(t) = \ln \frac{C(t)}{C(t+1)}$$



No-go theorem

Maiani-Testa (1984)

Why relevant?

Lattice data (imaginary time)

$$C(t_f, t_i) = \langle \mathcal{O}_{\pi\pi}(t_f) H_W(0) \mathcal{O}_K(t_i) \rangle$$

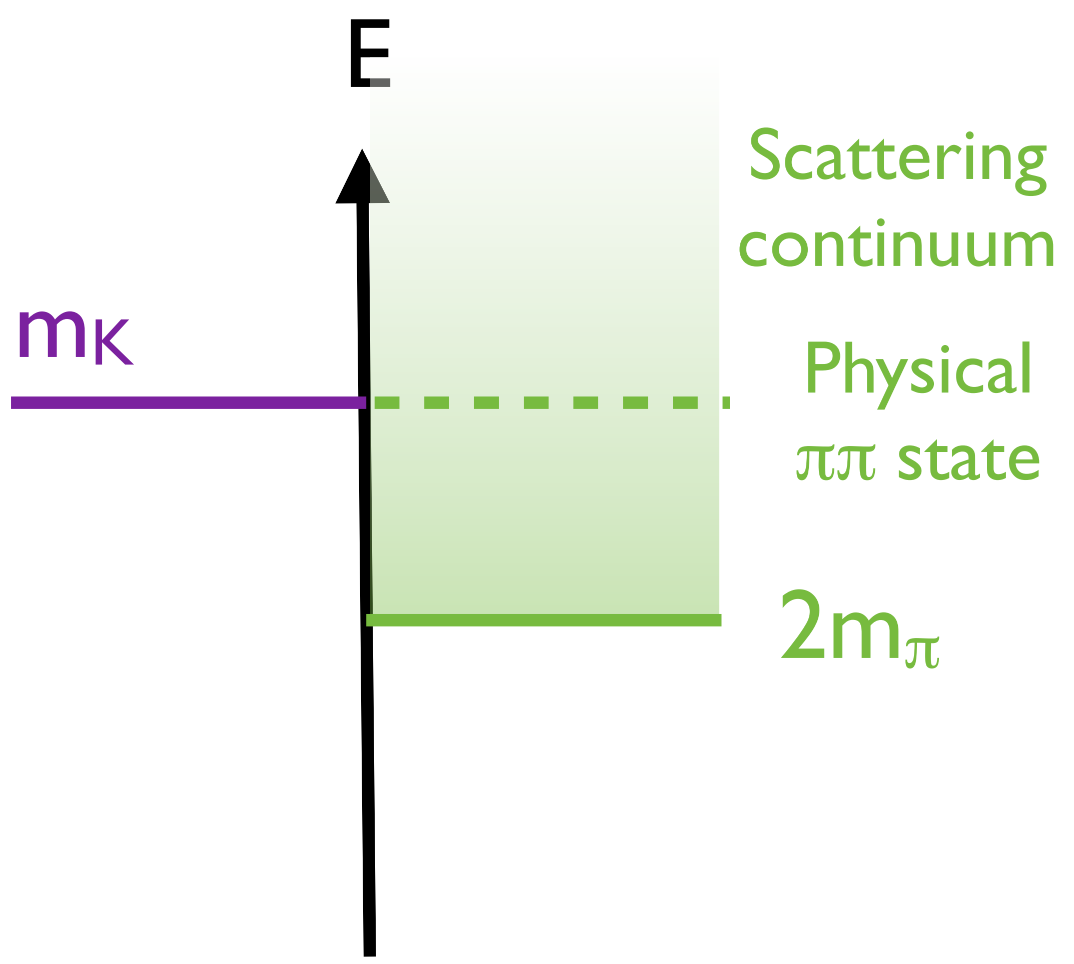
$$\Downarrow$$

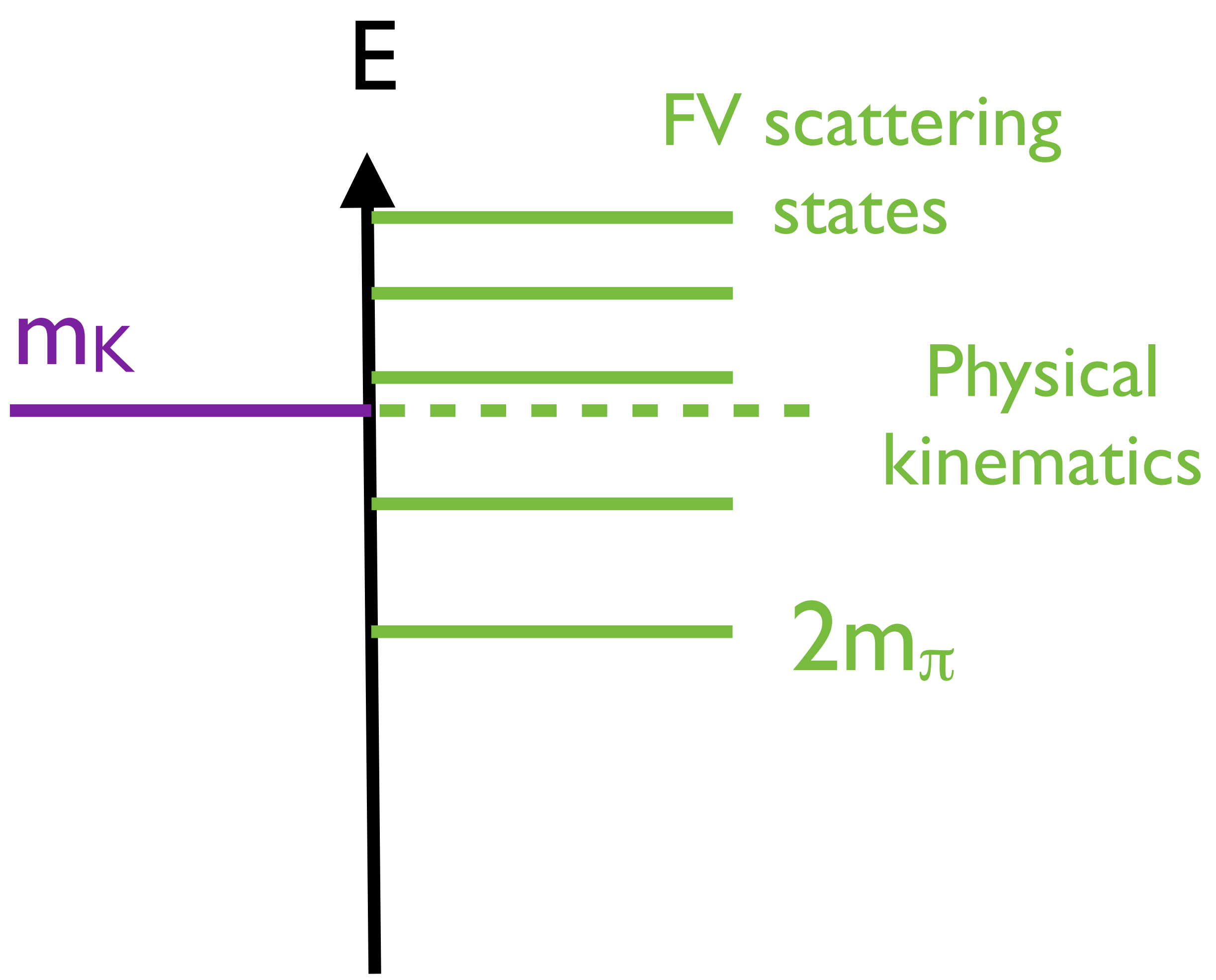
$$\langle \pi(\boldsymbol{p})\pi(-\boldsymbol{p}) | H_W | K(\boldsymbol{0}) \rangle$$

can’t be simply obtained. Rather, you get

$$\langle \pi(\boldsymbol{0})\pi(-\boldsymbol{0}) | H_W | K(\boldsymbol{0}) \rangle$$

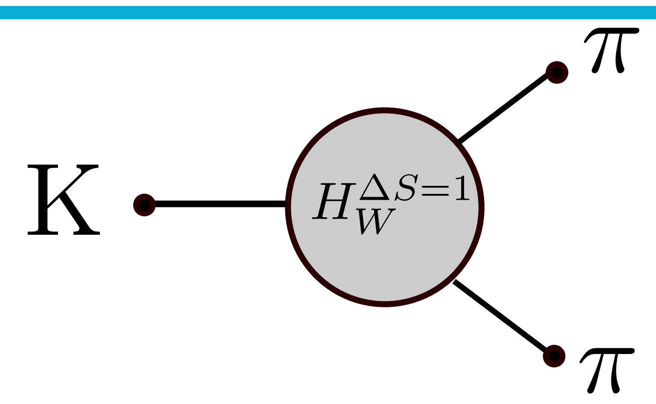
Have to tune the final state energy by
- tuning the lattice volume
- designing (special) boundary conditions, so that no 0-momentum state exists





$K \to \pi\pi$ RBC-UKQCD collab. (2004.09440)

Kelley @ Lattice 2021



LL finite-volume correction

renormalization matrix (mixing)
Use RI-SMOM
convert to MSbar perturbatively

$$A^I = F \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[\left(z_i(\mu) + \tau y_i(\mu) \right) Z_{ij}^{\mathrm{lat} \to \overline{\mathrm{MS}}} M_j^{I,\,\mathrm{lat}} \right]$$

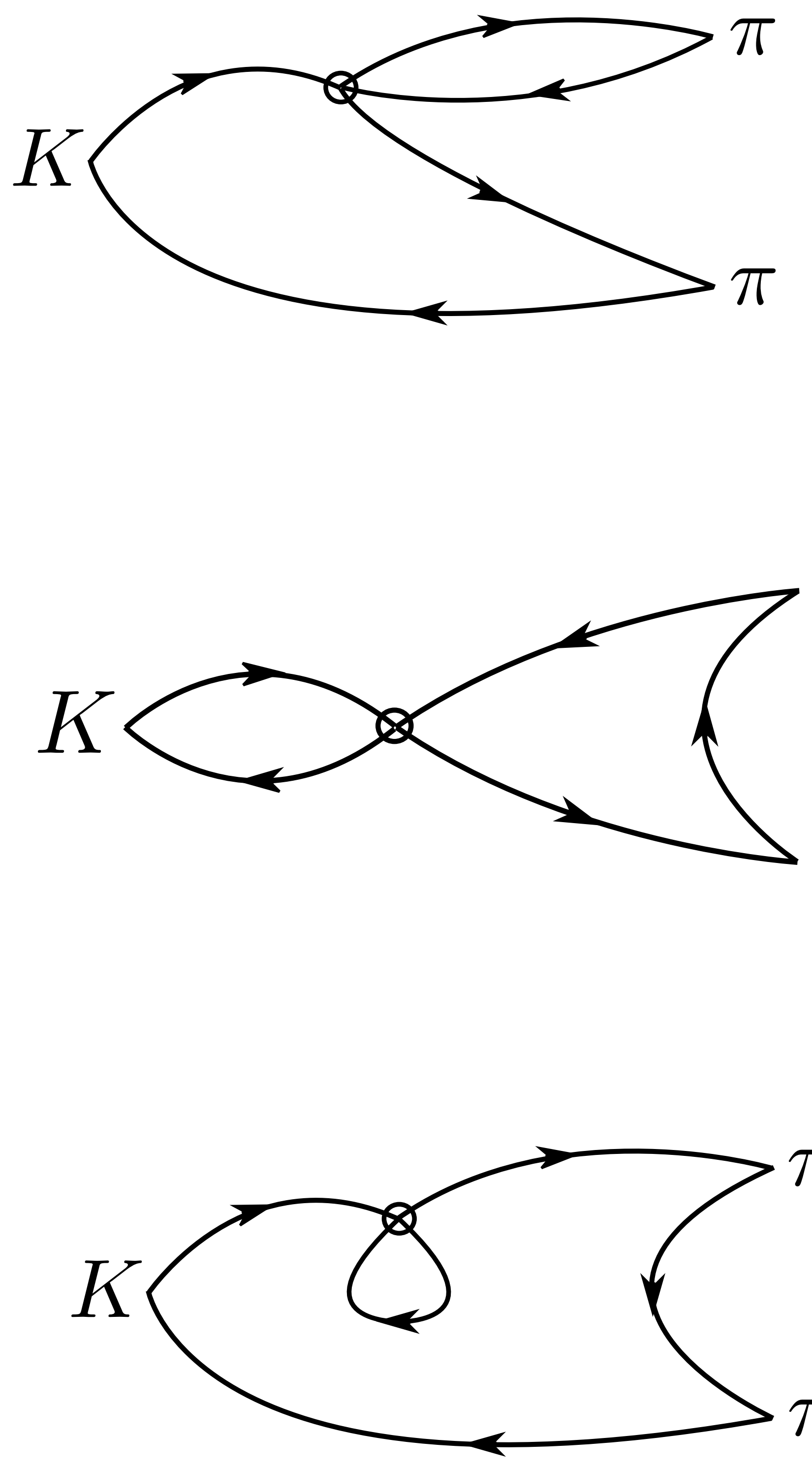
perturbative Wilson coeffs.

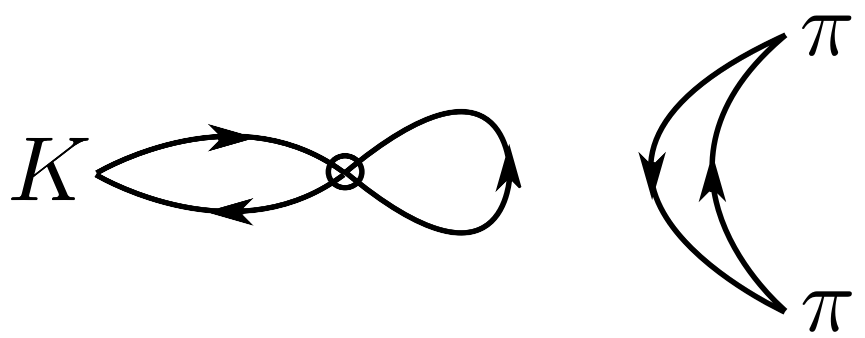
$M_j^{I,lat} = \langle (\pi\pi)_I | Q_j | K \rangle$ (lattice)

10 effective four-quark operators

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

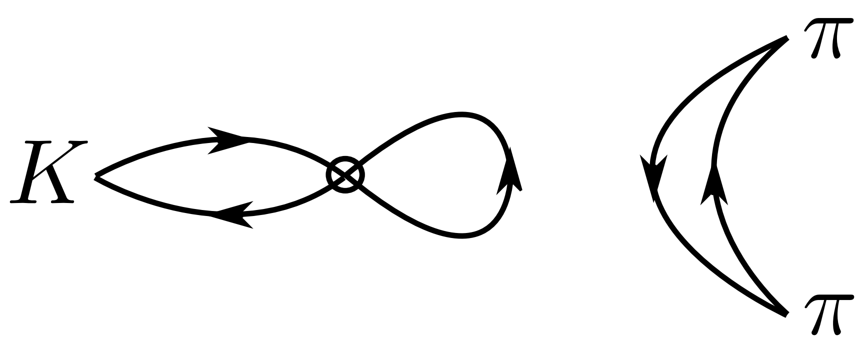
quark-line diagrams:

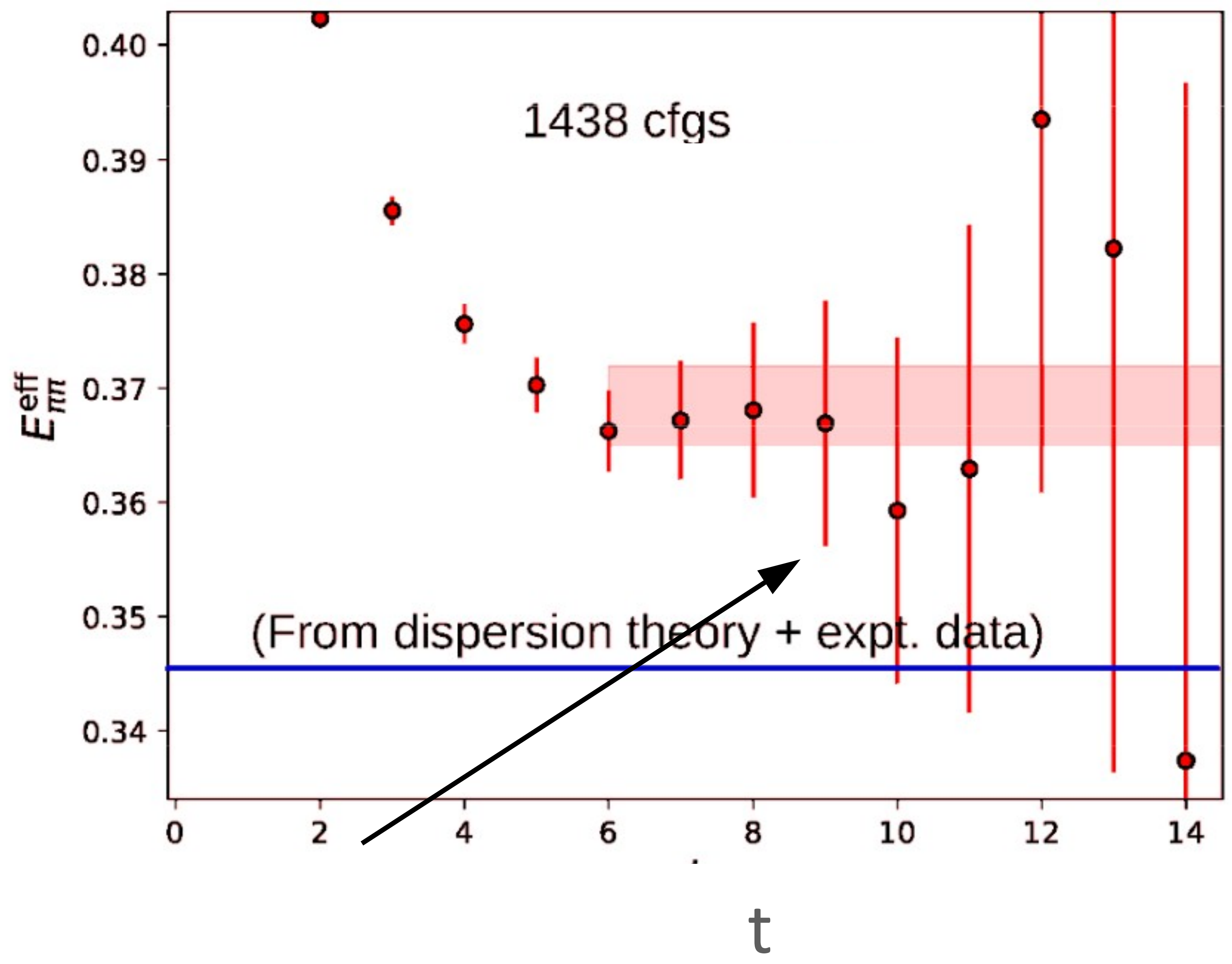




Only for I = 0

Large noise for I=0

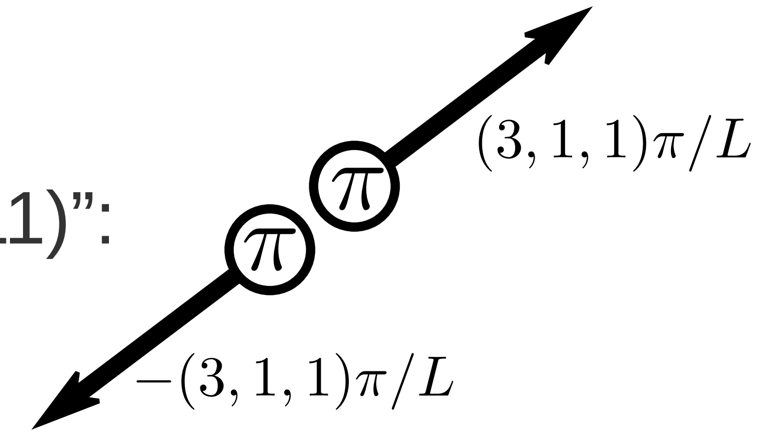




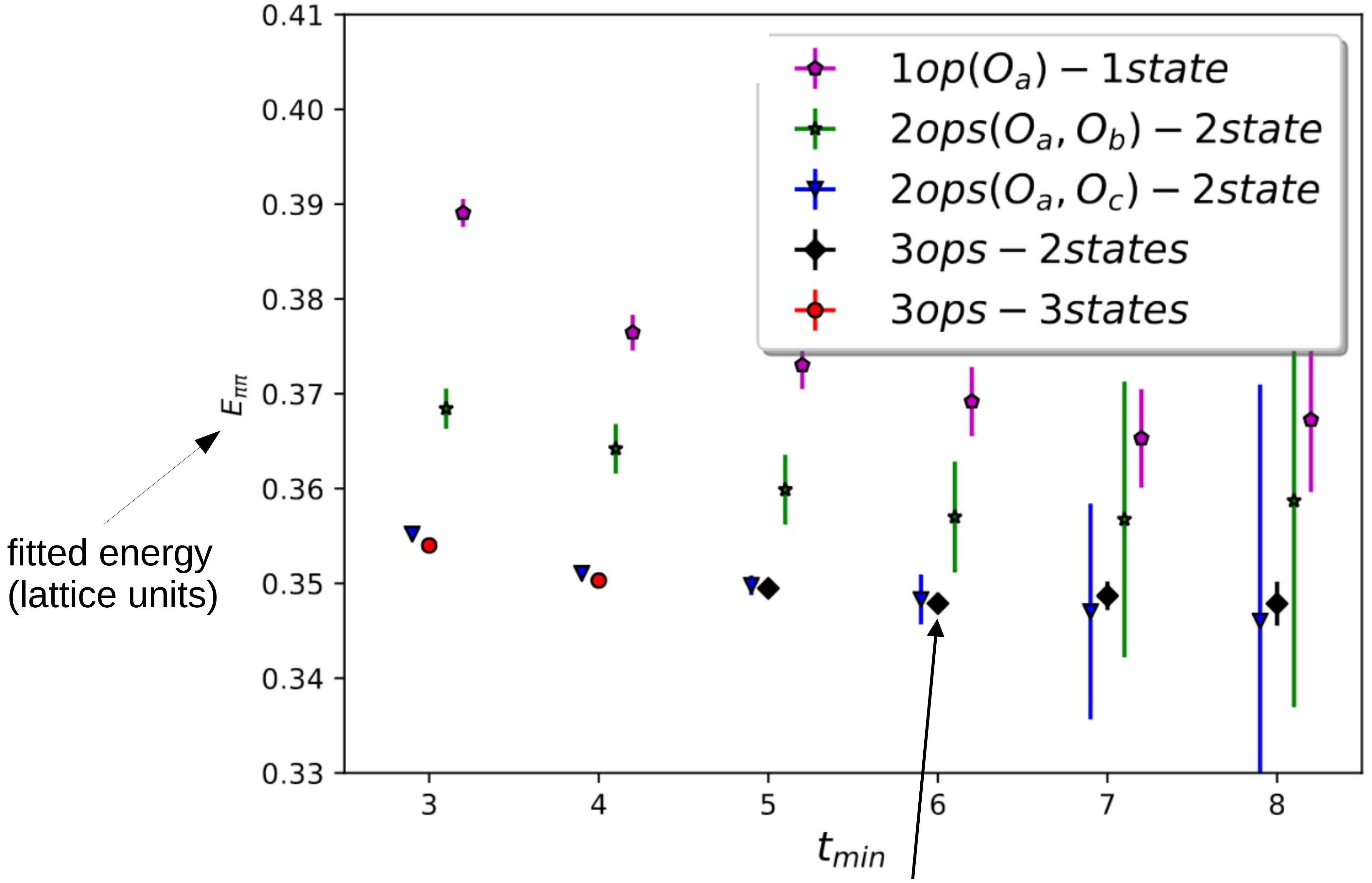
inconsistent with exp't of scattering phase shift

Better id of state by adding more Ops

"ππ(311)":



$$\sigma = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

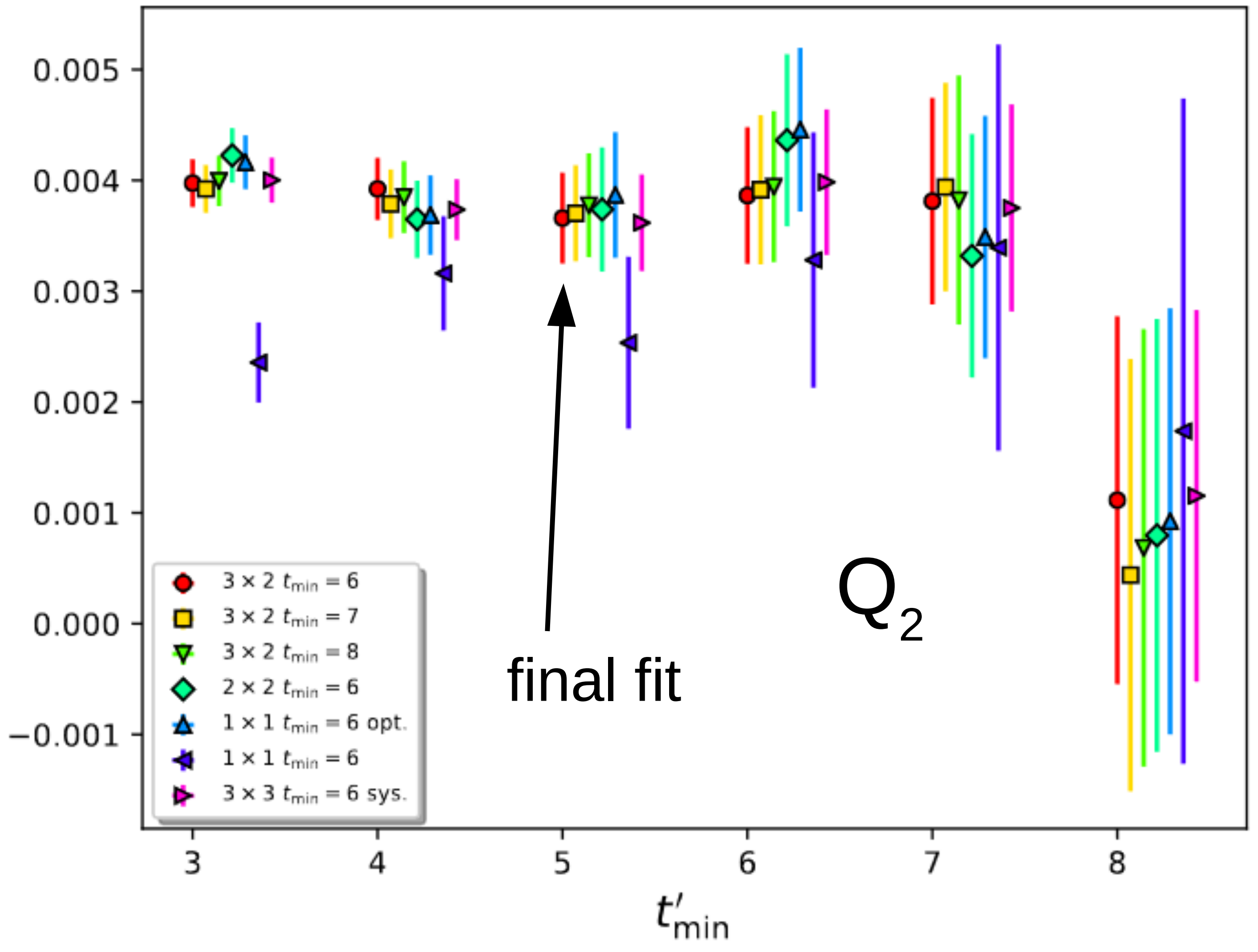


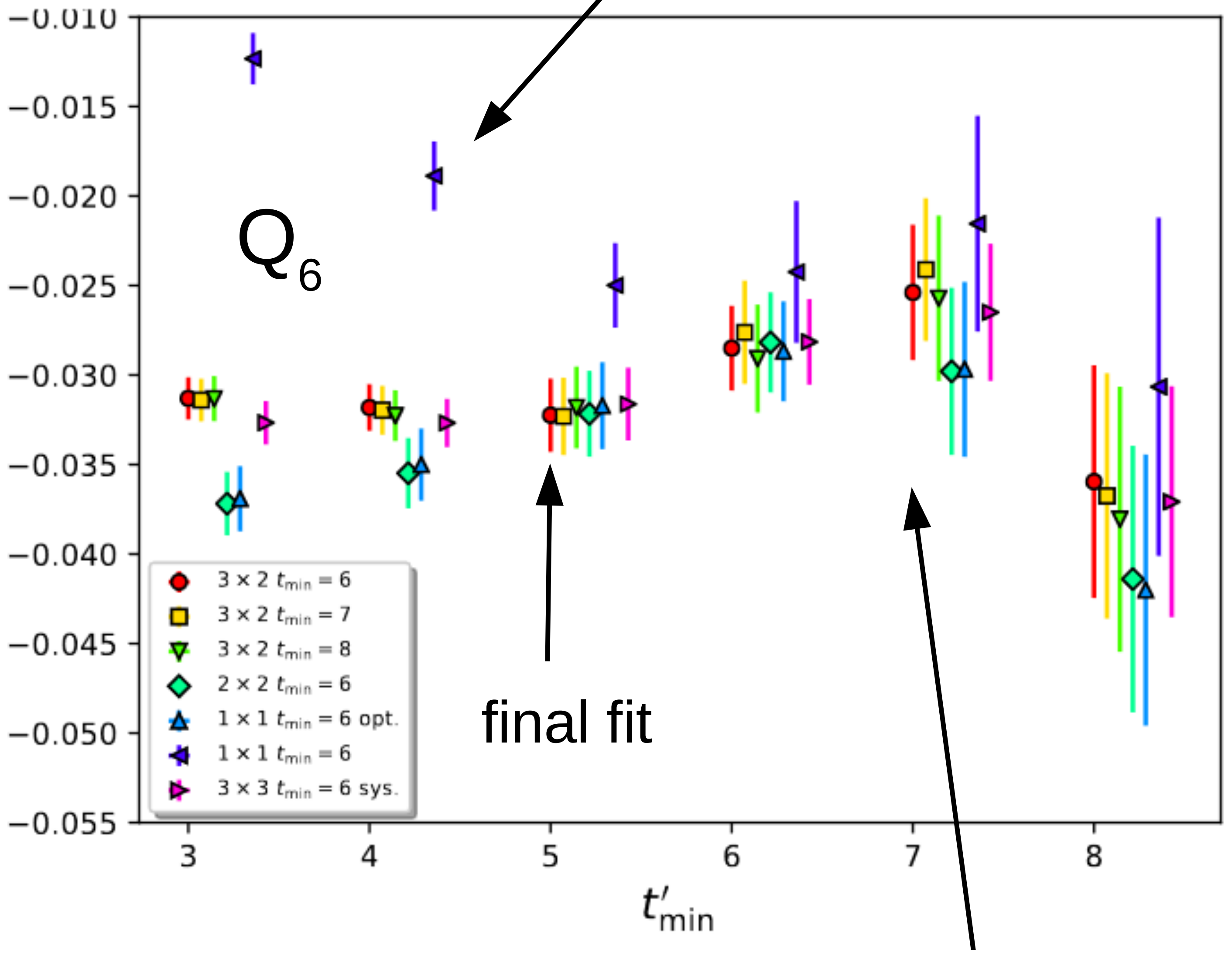
improved (2021)

Matrix elements $\langle \pi(\boldsymbol{p})\pi(-\boldsymbol{p})|H_W|K(\boldsymbol{0})\rangle$

little indication of exc. stat. cont. for Q_2

exc. state contam. with 1op
(this was t'_{min} for 2015 work)

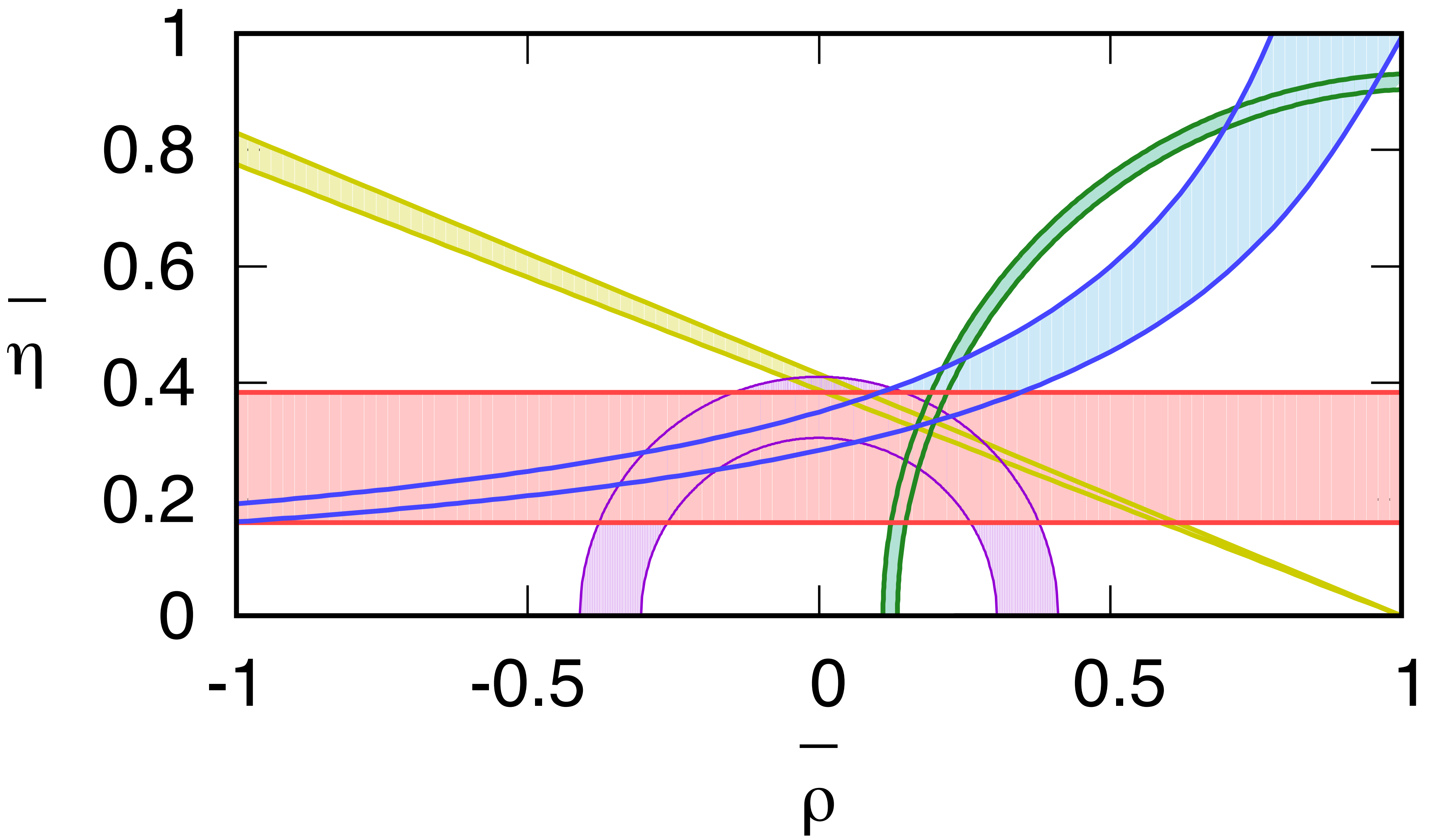




$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= 0.00217(26)(62)(50)$$

stat sys IB + EM

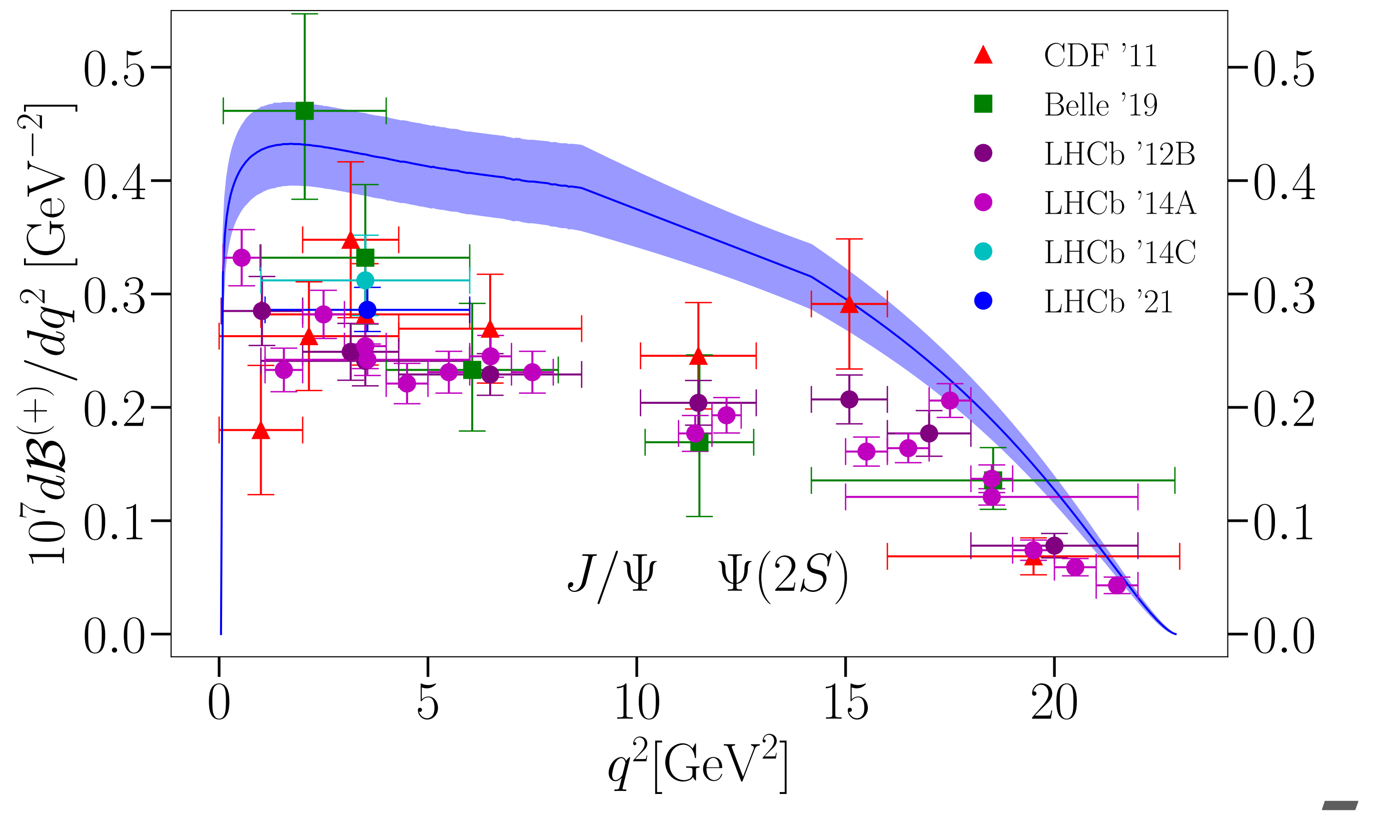


Consistent with experimental result:

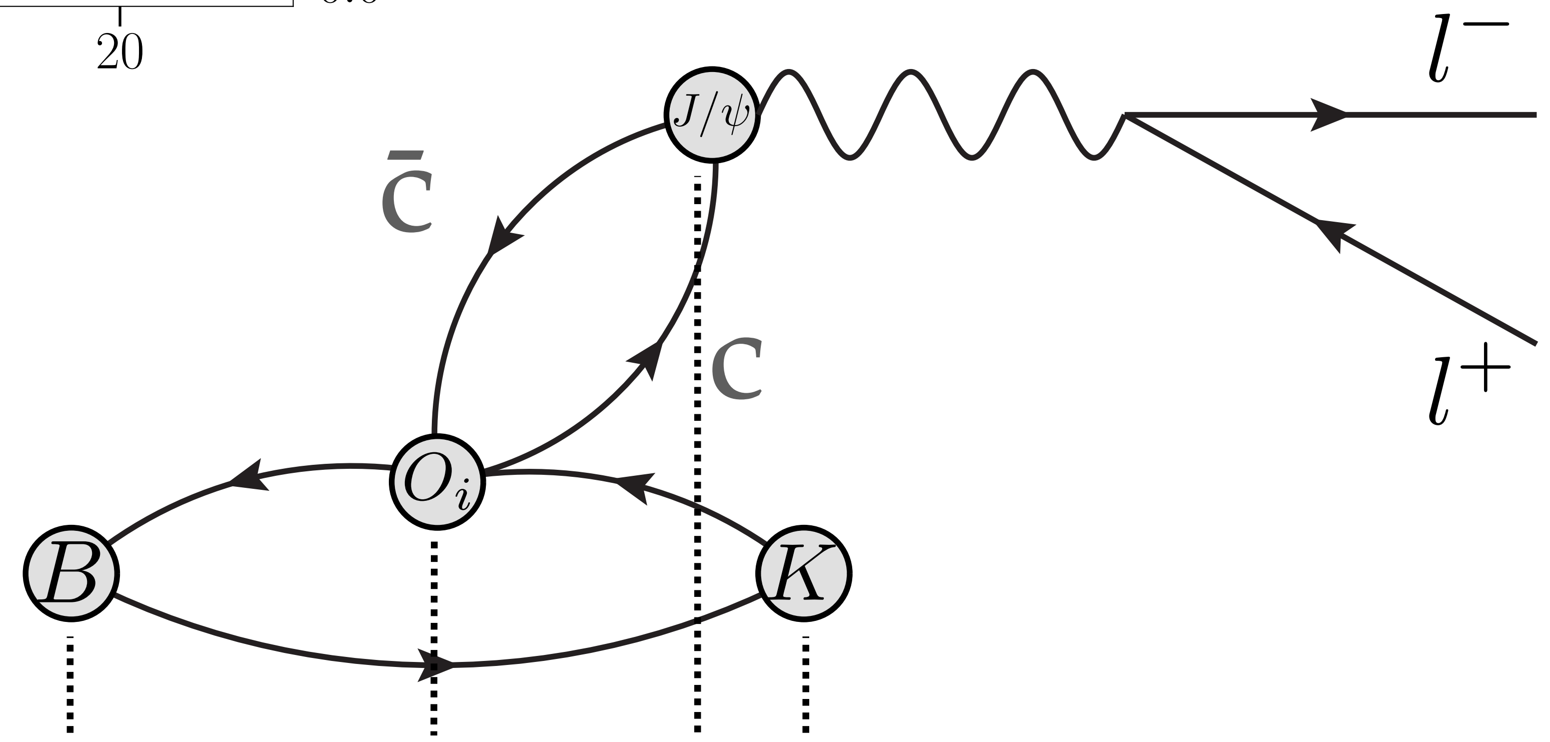
$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{expt}} = 0.00166(23)$$

Back to B→Kll

What is the problem?

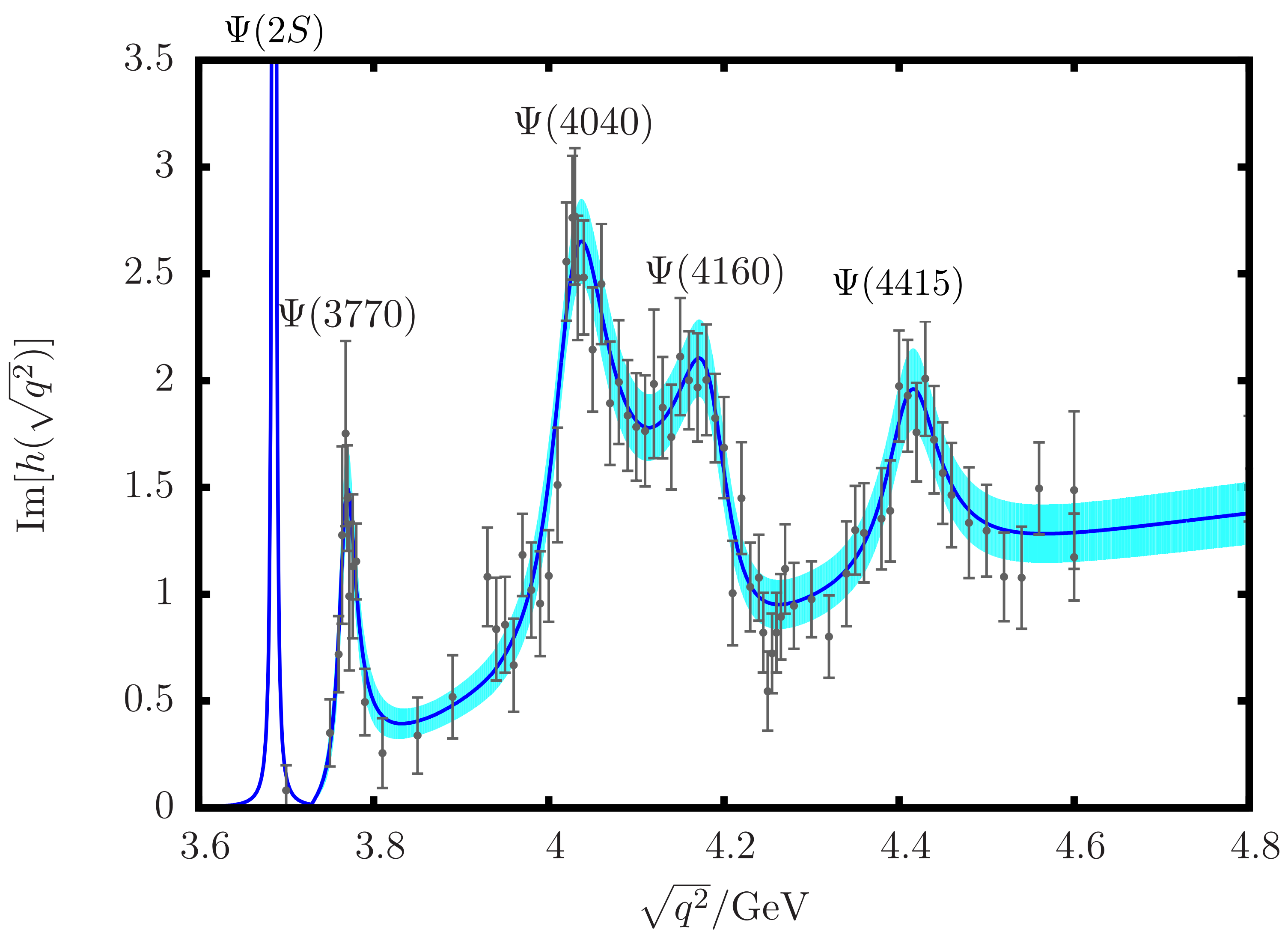


Other than potential systematic errors



- Charm loop is missing
  - Intrinsically (or virtually) multi-body hadronic state, even though the final state is not.
  - Need to select the state of a specified energy at J_{em}^{μ}
  - **NoGo**: Many (lower) energy states exist: B → ψ(~**0**) K(~**0**) → ll(-**p**) K(**p**)
- An example of larger class of problems that need to disentangle full spectrum of states

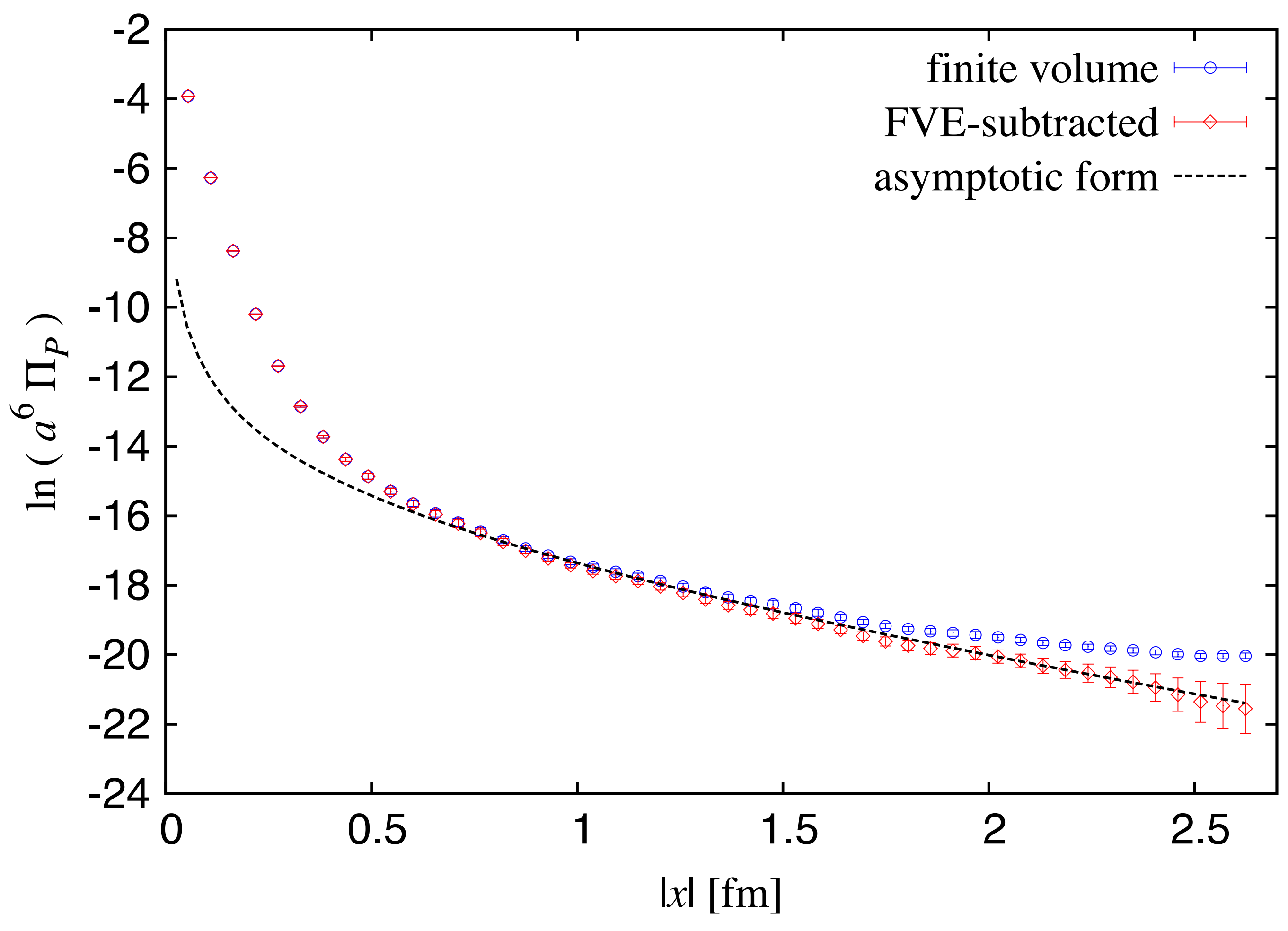
Lyon, Zwicky, arXiv:1406.0566

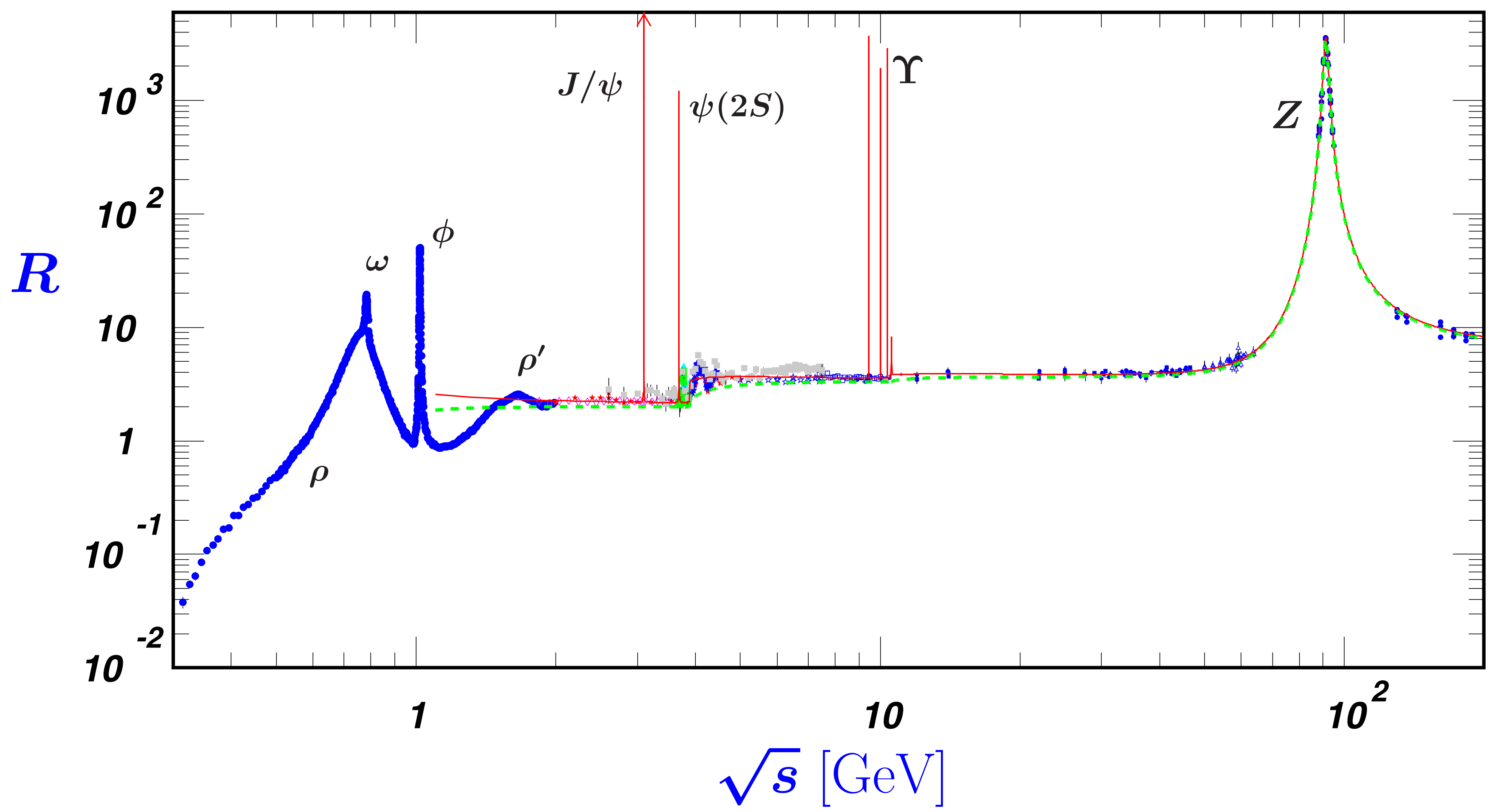


Ill-posed problem

We want spectral functions

$$A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + A_2 e^{-E_2 t} + \cdots \xrightarrow{\text{inf volume}} \langle \Omega | T J(t) J(0) | \Omega \rangle \sim \int_0^\infty d\omega \, \rho(\omega) e^{-\omega t}$$





No way to extract the rich structure, without infinitely precise data.

Make it more feasible

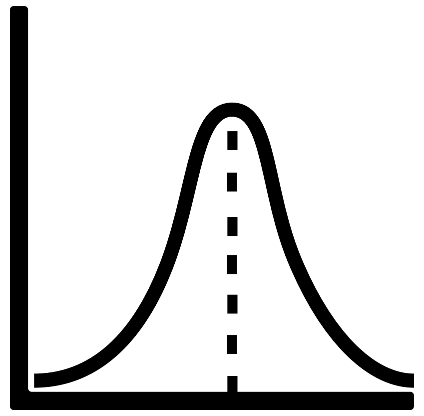
= smeared spectrum

Hansen, Lupo, Tantalo (2019); Bailas, SH, Ishikawa (2020)

Lattice correlator $C(t) = \int_0^\infty d\omega\, \rho(\omega) e^{-\omega t}$ $\sim \langle 0 | J\, e^{-\hat{H} t}\, J | 0 \rangle$

Spectral function $\rho(\omega) \propto \sum_X \delta(\omega - E_X) |\langle X | J | 0 \rangle|^2$ $\sim \langle 0 | J\, \delta(\omega - \hat{H})\, J | 0 \rangle$

Smeared spectrum $\bar{\rho}_\Delta(\omega) = \int_0^\infty d\omega'\, S_\Delta(\omega, \omega') \rho(\omega)$ $\sim \langle 0 | J\, S_\Delta(\omega, \hat{H})\, J | 0 \rangle$



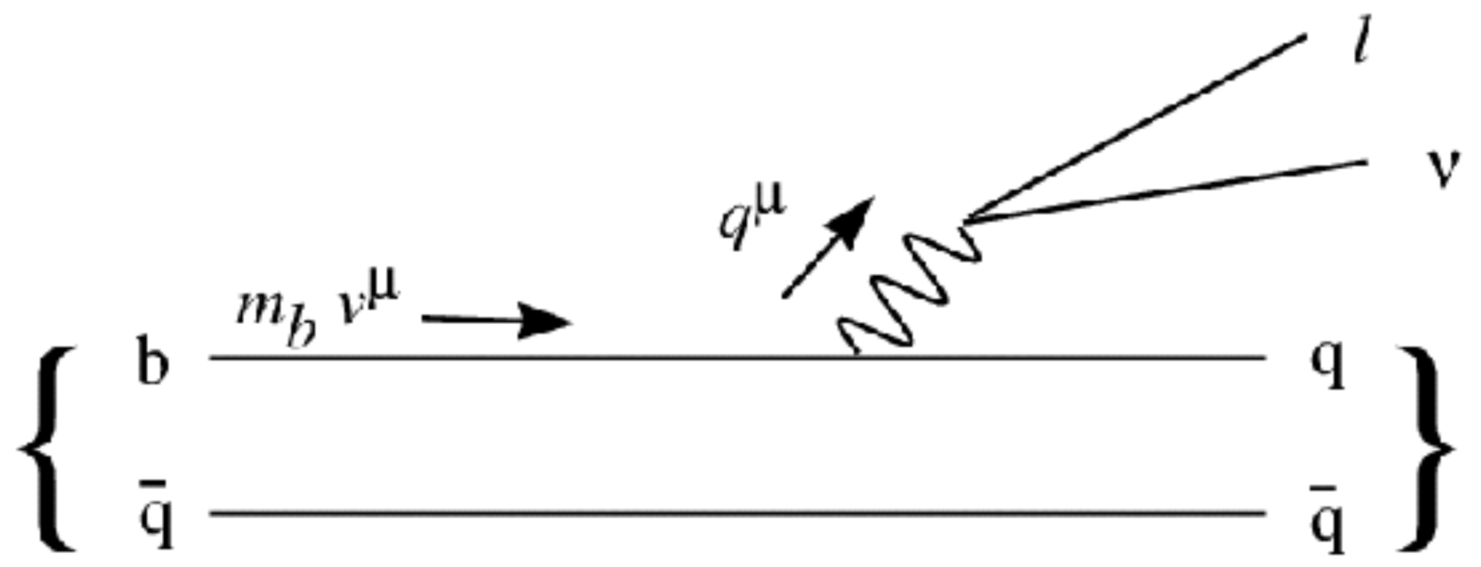
Approx: $S_\Delta(\omega, \hat{H})$ by $e^{-\hat{H} t}$?

Inclusive processes from lattice?

Gambino, SH (2020)

Differential decay rate:

$$d\Gamma \sim |V_{cb}|^2 l^{\mu\nu} W_{\mu\nu}$$



Structure function (or hadronic tensor):

$$W_{\mu\nu} = \sum_X (2\pi)^2 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

$$\rightarrow \langle B(\mathbf{0})|\tilde{J}_\mu^\dagger(-\boldsymbol{q};t)\ \delta(\omega - \hat{H})\ \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle$$

Total decay rate:

$$\Gamma \propto \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q} \int_{\sqrt{m_D^2+\boldsymbol{q}^2}}^{m_B-\sqrt{\boldsymbol{q}^2}} d\omega\, K(\omega;\boldsymbol{q}^2)\langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})\delta(\omega-\hat{H})\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

kinematical (phase-space) factor

Energy integral to be evaluated:

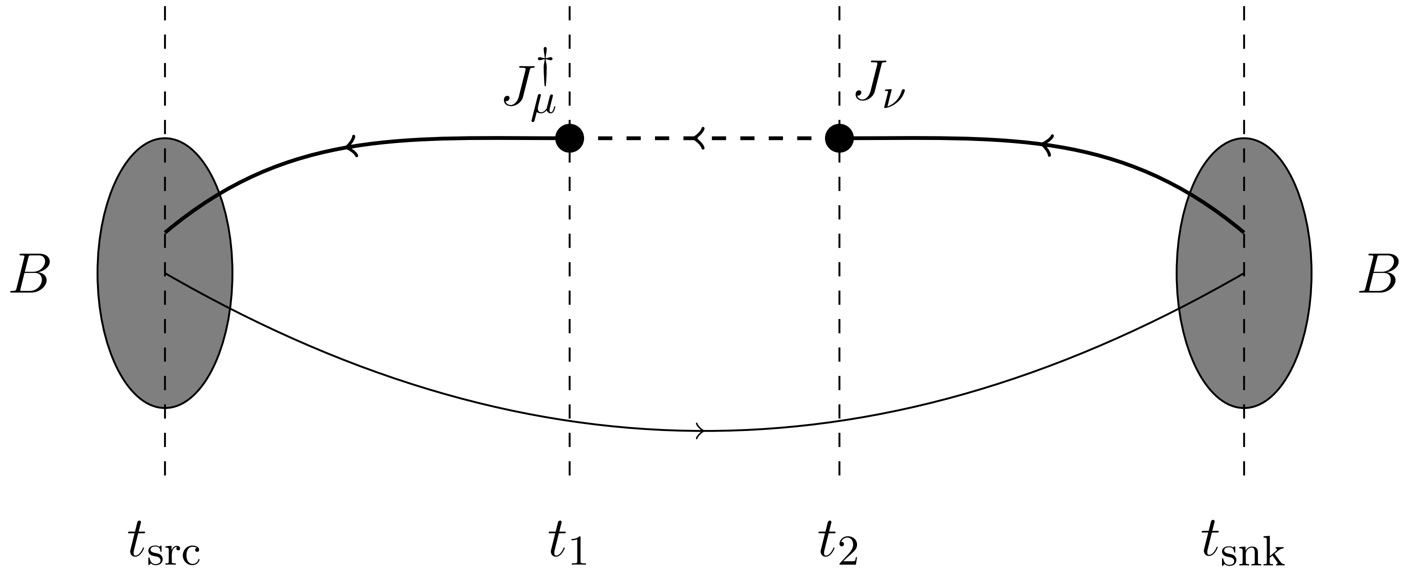
$$\Gamma \propto \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q} \int_{\sqrt{m_D^2+\boldsymbol{q}^2}}^{m_B-\sqrt{\boldsymbol{q}^2}} d\omega\, K(\omega;\boldsymbol{q}^2)\langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})\delta(\omega-\hat{H})\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

$$= \langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})K(\hat{H};\boldsymbol{q}^2)\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

smeared spectrum!

Compton amplitude obtained on the lattice:

$$\langle B(\mathbf{0})|\tilde{J}^\dagger_\mu(-\boldsymbol{q};t)\ \ \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle \longrightarrow \langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})e^{-\hat{H}t}\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$



Approx :

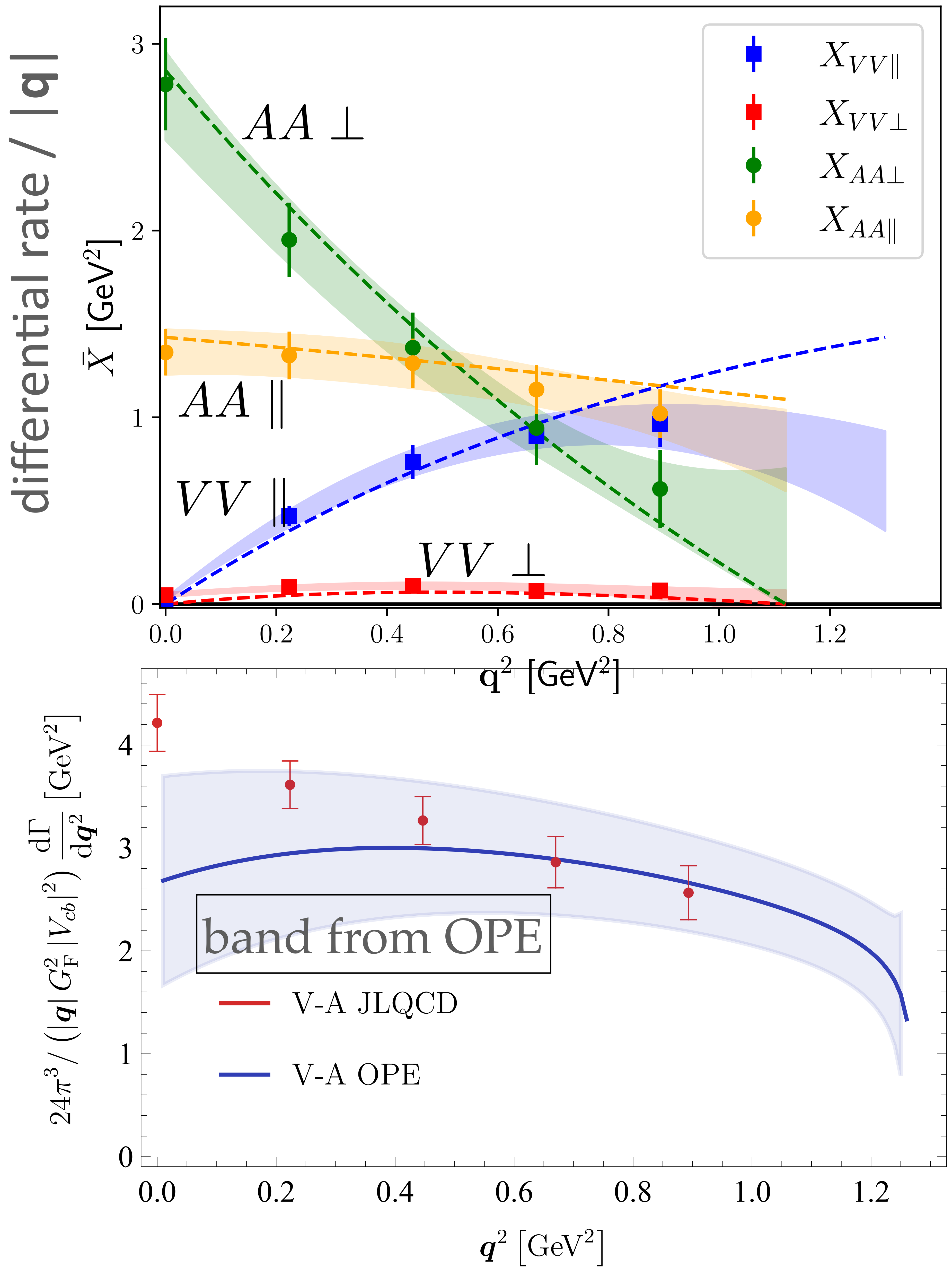
$$K(\hat{H}) = k_0 + k_1 e^{-\hat{H}} + k_2 e^{-2\hat{H}} + \cdots + k_N e^{-k_N \hat{H}}$$

Inclusive decay rate

Gambino et al., 2203.11762

- Prototype lattice calculation
  - $B_s \rightarrow X_c$
  - unphysical, i.e. the b quark is lighter than physical.

- Decay rate in each channel
  - VV and AA
  - parallel or perpendicular to the recoil momentum
  - compared to “exclusive”
    - $VV_{||}$ is dominated by B→D
    - Others are by B→D*
- Compared with OPE
  - With inputs from exp’t analysis



Challenges ahead

many interesting processes

- Only some "smeared" spectrum is accessible (like inclusive). The limit of zero smearing feasible? Detailed studies needed. (e.g. Bulava et al., 2111.12774)
- Relevant problems include
  - $K \to \pi\pi$ (without BC), … even (too much) harder for $B \to \pi\pi$
  - Set the energy of ll in $B \to Kll$
  - D^0-D^0bar mixing
  - νN inelastic (or inclusive) scattering cross section

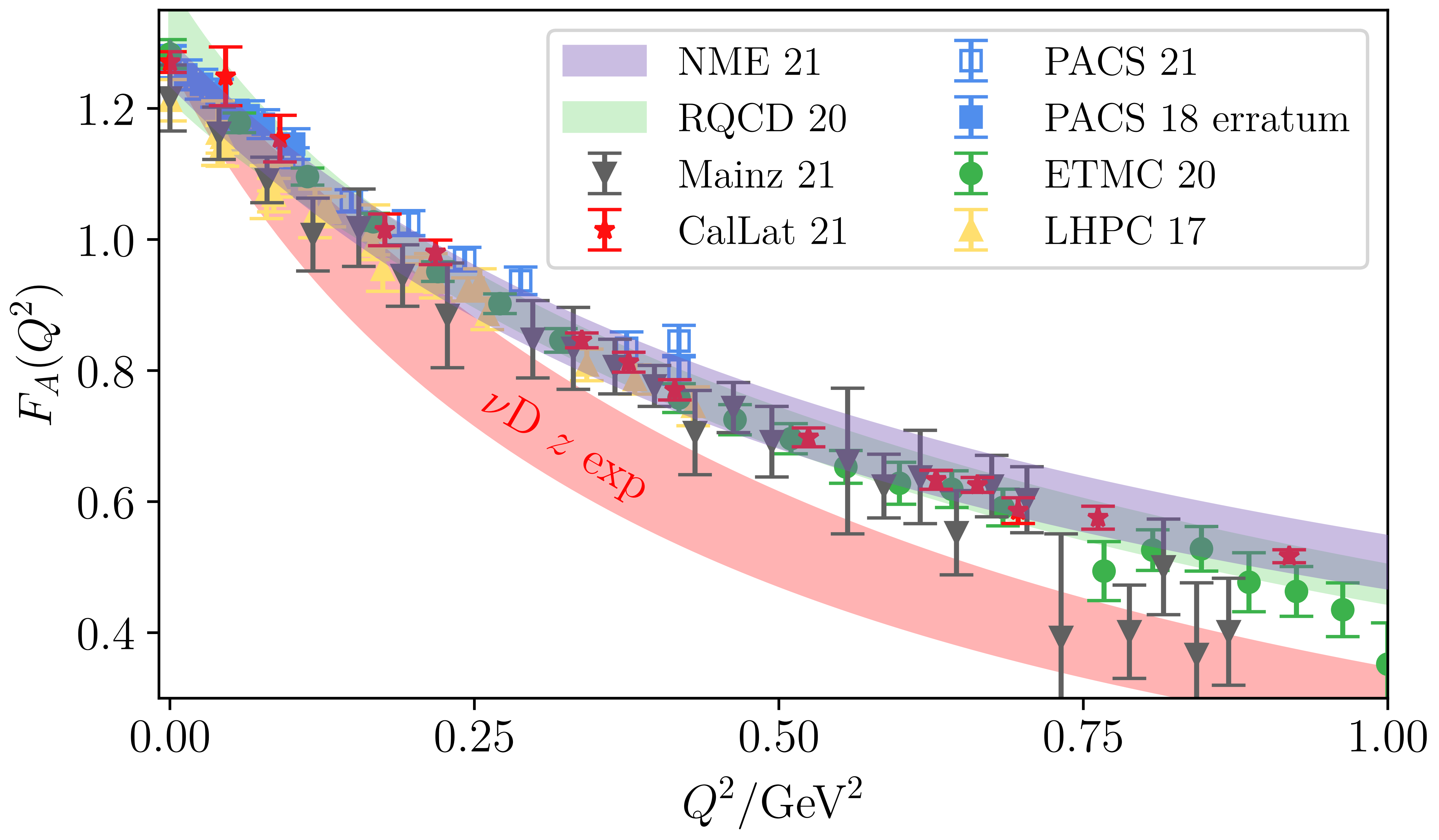
See, Fukaya et al., arXiv:2010.01253

The other flavour physics

Neutrino

Immediate contribution: QE nucleon axial form factor

From Meyer, Walker-Loud, Wilkinson, 2201.01839



Inelastic: more challenging:
- Resonance (Δ, etc) contrib.
- $N\pi$ final states
- inclusive

Nuclear effect:
- will need a new framework (in collaboration with NP)

To summarize …

Lattice QCD for flavour physics

matured and infant

- Precision frontier!
    - masses, decay constants, form factors, … [see FLAG] + g-2
- Under development:
    - Many other interesting quantities. Extracting (or avoiding) a given energy state would be a key.
- Not mentioned:
    - Quantities that need light-cone separation, e.g. PDF or light-cone distribution amplitude. A lot of recent works for PDF (Ji’s proposal to approach from space-like); very challenging to achieve precise results.