Structure-dependent QED effects using SCET

Philipp Böer

mainly for non-leptonic decays; based on: 2008.10615, 2107.03819, 2108.05589 and 2204.09091

with M. Beneke, G. Finauri, J. N. Toelstede and K. K. Vos

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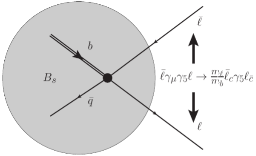
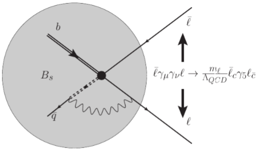
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Motivation

- Precision: Traditionally focus on hadronic uncertainties. Time to look at QED.
  - $\rightarrow$ QED effects can cause large logarithms $\ln m_\ell / m_B, \ln m_\pi / m_B, \ldots$
  - $\rightarrow$ structure-dependent log's could be of similar size as: $m_\pi \sim m_\mu \sim \Lambda_{\mathrm{QCD}}$
- Photons couple weakly to strongly interacting quarks
  - $\rightarrow$ probe of hadronic physics, requires (mostly unknown) factorization theorems
- Theoretically interesting: Photons have long-range interactions with charged mesons
  - $\rightarrow$ QED factorization is more complicated than QCDF
- Qualitatively new effects, e.g.
  - $\rightarrow$ violation of isospin symmetry ($Q_u \neq Q_d$)
  - $\rightarrow$ power-enhancement in $B_s \rightarrow \mu^+ \mu^-$
  - $\rightarrow$ ...



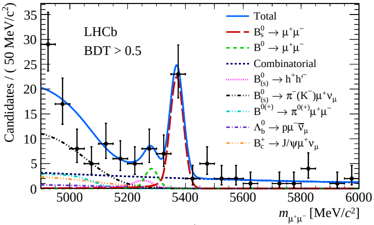


IR finite observable:

→ must include ultrasoft photon radiation

→ soft-photon inclusive width

$$\Gamma(\Delta E) \equiv \Gamma[\bar{B} \to M_1 M_2 + X_s]\big|_{E_{X_s} \leq \Delta E}$$



[LHCb, $B_s \to \mu^+\mu^-$, 1703.05747]

for $\Delta E \approx 60\text{MeV} \ll \Lambda_{\text{QCD}}$: factorizes in non-radiative amplitude and u-soft function (electrons ↯)

$$\Gamma(\Delta E) = |\mathcal{A}(\bar{B} \to M_1 M_2)|^2 \times \sum_{X_s} |\langle X_s|(\bar{S}_v^{(Q_B)} S_{v_1}^{\dagger(Q_1)} S_{v_2}^{\dagger(Q_2)})|0\rangle|^2\, \theta(\Delta E - E_{X_s})$$

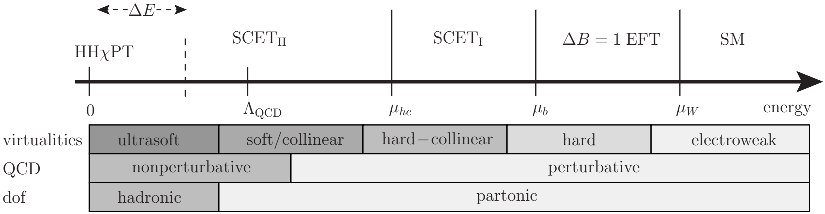
Simple classification:

- ultrasoft photons with energy $\ll \Lambda_{\text{QCD}}$ see pointlike mesons ("universal")
- (virtual) photons with energy $\gtrsim \Lambda_{\text{QCD}}$ probe the partonic structure of the mesons!
  → Non-universal, structure dependent corrections
- present treatment: pointlike coupling up to the scale m_B ↯
  (Pointlike coupling requires wavelenght $\gg$ typical size of the meson $\sim 1/\Lambda_{\text{QCD}}$)

Scales and EFTs

Hierarchy of energy scales:

$\Delta E \sim 60\,\text{MeV} \ll$ few times $\Lambda_{\text{QCD}} \ll m_b \sim 4.2\,\text{GeV} \ll M_W \sim 80\,\text{GeV}$



- ✓ short-distance QED at $\mu \gtrsim m_b \to$ Wilson coefficients of weak eff. Lagrangian
- ✓ Far IR (ultrasoft) region $\mu_{\text{us}} \ll \Lambda_{\text{QCD}}$ described by point-like hadrons

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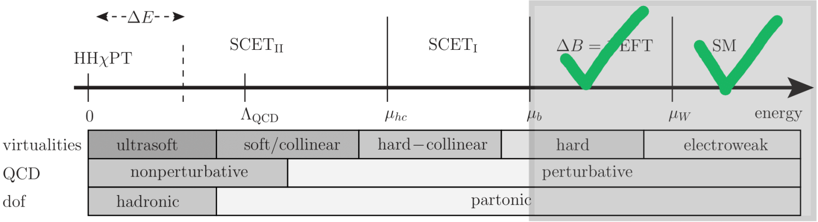

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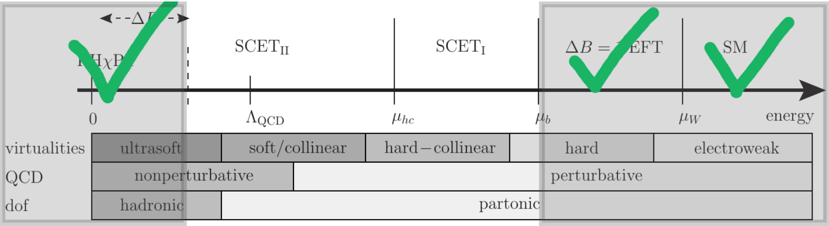

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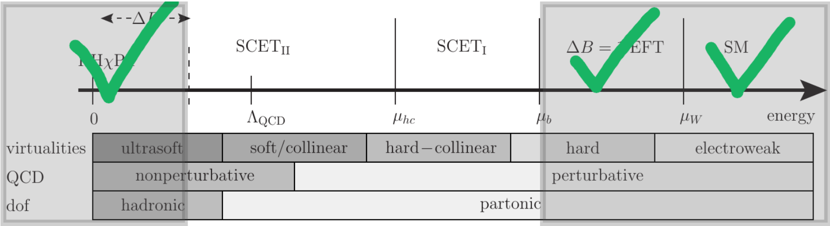

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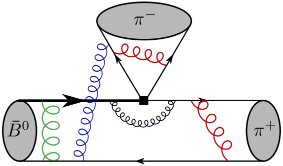
Goal: develop theory for QED between m_b and Λ_{QCD} ("structure dependent effects")

→ systematic $1/m_B$ expansion using Soft-collinear effective theory (SCET)

→ Factorization theorems hold to all orders in $(\alpha_s, \alpha_{\mathrm{em}})$ in the heavy-quark limit

→ universal soft $\ln \Delta E/m_B$ and collinear log's $\ln m/m_B$ from interplay of multiple scales

Factorization Theorem for Non-Leptonic Decays



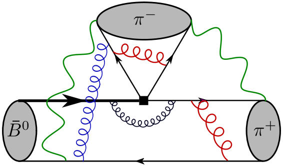
$$\langle M_1 M_2 | Q_i | \bar{B} \rangle = F^{B\to M_1}(q^2=0) \int_0^1 \mathrm{d}u\, \mathbf{T}_i^{\mathrm{I}}(u)\, f_{M_2}\phi_{M_2}(u)$$
$$+ \int_0^\infty \mathrm{d}\omega \int_0^1 \mathrm{d}u\, \mathrm{d}v\; \mathbf{T}_i^{\mathrm{II}}(u,v,\omega)\, f_{M_1}\phi_{M_1}(v)\, f_{M_2}\phi_{M_2}(u)\, f_B\phi_B(\omega) + \mathcal{O}(\Lambda/m_B)$$

- perturbative scattering kernels $\mathbf{T}_i^{\mathrm{I,II}}$
- convoluted with universal non-perturbative LCDAs for heavy and light mesons

State of the art:

- NNLO scattering kernels [e.g. Bell, Beneke, Huber, Li]
- 3-loop (2-loop) anomalous dimension for ϕ_M (ϕ_B) [Braun et al.]
- this work: QEDF [Beneke, PB, Finauri, Toelstede, Vos]

Factorization Theorem for Non-Leptonic Decays



$\otimes = (Q_{M_1}, Q_{M_2})$

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \big|_{\text{non-rad.}} = \mathcal{F}^{B\to M_1}_{Q_2}(q^2=0) \int_0^1 \mathrm{d}u\, \mathbf{T}^{\mathrm{I}}_{i,Q_2}(u)\, f_{M_2} \Phi_{M_2}(u)$$
$$+ \int \mathrm{d}\omega \int_0^1 \mathrm{d}u\, \mathrm{d}v\, \mathbf{T}^{\mathrm{II}}_{i,\otimes}(u,v,\omega)\, f_{M_1} \Phi_{M_1}(v)\, f_{M_2} \Phi_{M_2}(u)\, f_B \Phi_{B,\otimes}(\omega) + \mathcal{O}(\Lambda/m_B)$$

- looks like the QCDF formula, but non-perturbative objects need to be generalized
  $\rightarrow$ similar for $B \to DL$ transitions
- form factor $F^{B\to\pi} \to$ semi-leptonic amplitude $A^{B\to\pi\ell\bar{\nu}_\ell}$ for charged M_2
- **process-dependent soft functions** inherit soft Wilson lines from charged mesons

$$S^{(q)}_{n_+}(x) = \exp\left\{ -iQ_q e \int_0^\infty \mathrm{d}s\, n_+ \cdot A_s(x + s n_+) \right\}$$

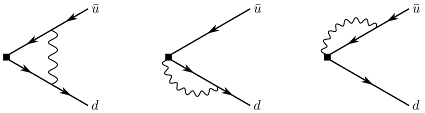
Light-Cone Distribution Amplitudes in QED

Appear in the factorization of the non-rad. amplitude for sufficiently small $\Delta E \ll \Lambda_{\mathrm{QCD}}$
$\to$ exclusive matrix elements, IR-divergent

Light-Meson LCDA

$$\langle \pi^-(p)|R_c(\bar{d}W^{(d)})(tn_+)\not{n}_+\gamma_5(W^{\dagger(u)}u)(0)|0\rangle = -2iEf_\pi \int_0^1 du\; e^{iu(n_+p)t}\Phi_{\pi^-}(u;\mu)$$

- infinite-length QED Wilson-lines lead to IR-divergent UV-scale evolution
  - $\to$ anom. dimension well-defined after soft rearrangement (→ more details in backup)



$$\gamma(u,v) = -\frac{\alpha_{\mathrm{em}}}{\pi}\delta(u-v)Q_M\left(Q_M \ln\frac{\mu}{2E} - Q_d \ln u + Q_u \ln(1-u) + \frac{3}{4}\right)$$
$$-\left(\frac{\alpha_s C_F}{\pi} + \frac{\alpha_{\mathrm{em}}}{\pi}Q_uQ_d\right)\left[\left(1+\frac{1}{v-u}\right)\frac{u}{v}\theta(v-u) + \left(1+\frac{1}{u-v}\right)\frac{1-u}{1-v}\theta(u-v)\right]_+$$

$\to$ double-logarithmic; asymmetric; no longer diagonalized by Gegenbauer polynomials

$\to$ numerically $\sim \mathcal{O}(1\%)$, can be larger than two-loop QCD evolution (→ more details in backup)

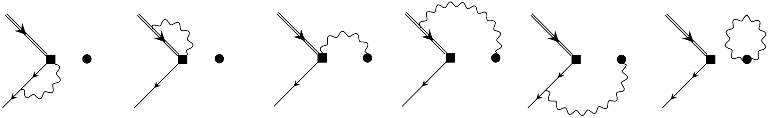
Light-Cone Distribution Amplitudes in QED

B-Meson LCDA/Soft Function

$$iF_{\text{stat}}(\mu)\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\Phi_{B,+-}(\omega;\mu) = \frac{1}{R_c R_{\bar{c}}}\langle 0|\,\bar{q}_s^{(d)}(tn_-)[tn_-,0]^{(d)}\not{n}_-\gamma_5 h_v(0)\,(S^\dagger_{n_+}S^\dagger_{n_-})(0)\,|\bar{B}_v\rangle$$

- different objects compared to standard QCD B-meson LCDA $\Rightarrow$ “Soft functions”
  - $\rightarrow$ process dependent (soft photons feel final-state charges + directions of flight)
  - $\rightarrow$ imag. parts; different support properties $\omega \in (-\infty,\infty)$ for charged anti-coll. meson

(→ more details in backup)



$$\gamma_{+-}(\omega,\omega') = \frac{\alpha_s C_F}{\pi}\left[\left(\ln\frac{\mu}{\omega - i0} - \frac{1}{2}\right)\delta(\omega-\omega') - H_+(\omega,\omega')\right]$$
$$+\frac{\alpha_{\text{em}}}{\pi}\left[\left(-\frac{5}{9}\ln\frac{\mu}{\omega - i0} - \frac{5}{36} - \frac{2\pi i}{3}\right)\delta(\omega-\omega') - \frac{1}{9}H_+(\omega,\omega') + \frac{1}{3}H_-(\omega,\omega')\right]$$

- numerically $\sim \mathcal{O}(1\%)$, but smaller than two-loop QCD evolution

(→ more details in backup)

Numerical Estimates for πK Final States

(DL final states in backup)

Ultrasoft photon radiation included:

→ universal part of Sudakov factors combines to $U(M_1 M_2)$; dresses non-rad. amplitude

→ resums large soft $\sim \ln \Delta E / m_B$ and collinear $\sim \ln m_i / m_B$ logarithms

$$U(M_1 M_2) = \left(\frac{2\Delta E}{m_B}\right)^{-\frac{\alpha_{\rm em}}{\pi}\left(Q_B^2 + Q_1^2\left[1+\ln\frac{m_1^2}{m_B^2}\right] + Q_2^2\left[1+\ln\frac{m_2^2}{m_B^2}\right]\right)}$$

For $\Delta E = 60$ MeV: ($\Delta E = \pi K$ invariant mass window around m_B)

$$U(\pi^+ K^-) = 0.914 \qquad U(\pi^0 K^-) = 0.976$$

$$U(\pi^- \bar{K}^0) = 0.954 \qquad U(\bar{K}^0 \pi^0) = 1$$

Virtual corrections in non-rad. amplitude partly included:

✓ Electroweak scale to m_B: QED corrections to Wilson coefficients

✓ m_B to $\Lambda_{\rm QCD}$: $\mathcal{O}(\alpha_{\rm em})$ corrections to short-distance kernels

↯ QED effects in LCDAs/form factors (log-enhanced effects now under control!)

Ratios and Isospin Sum Rules

1. Consider ratios where QCD uncertainties drop out:

$$R_L = \frac{2\mathrm{Br}(\pi^0 K^0) + 2\mathrm{Br}(\pi^0 K^-)}{\mathrm{Br}(\pi^- K^0) + \mathrm{Br}(\pi^+ K^-)} = R_L^{\mathrm{QCD}} + \cos\gamma \mathrm{Re}\,\delta_{\mathrm{E}} + \delta_U$$

[Beneke, Neubert '03]

$$R_L^{\mathrm{QCD}} - 1 \approx (1 \pm 2)\% \qquad \delta_E \approx 0.1\% \qquad \delta_U \approx 5.8\%$$

→ QED corrections larger than QCD and QCD uncertainty, but short-distance QED negligible

2. Isospin sumrule

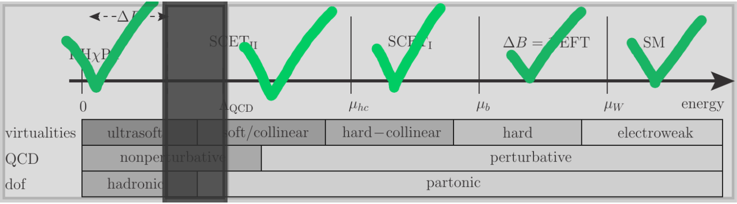
$$\begin{aligned}\Delta(\pi K) &\equiv A_{\mathrm{CP}}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 K^-) - \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\mathrm{CP}}(\pi^0 \bar{K}^0) \\ &\equiv \Delta(\pi K)^{\mathrm{QCD}} + \delta\Delta(\pi K)\end{aligned}$$

[Gronau, Rosner '06]

$$\Delta(\pi K)^{\mathrm{QCD}} = (0.5 \pm 1.1)\% \qquad \delta_\Delta(\pi K) \approx -0.4\%$$

→ Isospin sumrule robust against QED effects (QED of similar size but small)

QED in B Decays: Open Tasks



1.) This work: consistent treatment of QED effects between m_b and $\Lambda_{\rm QCD}$

→ matching of SCET$_{\rm II}$ onto theory of ultrasoft photons (and electrons) coupling to boosted ultrarelativistic point-like hadrons at the matching scale $\sim \Lambda_{\rm QCD}$ must be done non-perturbatively

2.) Non-perturbative determination of QED LCDAs

→ difficult on the lattice due to non-local light-cone operators that extend to ∞

3.) Electrons live at much lower scales (jet-like objects due to collinear radiation)

4.) Comparison between theory and experiment

→ requires precise statement about how QED effects are treated

→ Experiments use PHOTOS, which captures only a subset of QED effects

→ Dedicated Monte Carlo for QED compatible with EFT description above $\Lambda_{\rm QCD}$

Conclusion

Take-home messages:

1. QED factorization more complicated than QCD-alone due to charged external states. We now understand how to systematically include QED effects between $\Lambda_{\rm QCD}$ and m_B, but description requires new non-perturbative hadronic matrix elements.

2. Generalized heavy- and light-meson LCDAs exhibit novel properties (semi-universality, support properties, re-scattering phases, etc.). In particular soft physics is qualitatively different from standard hard-scattering picture.

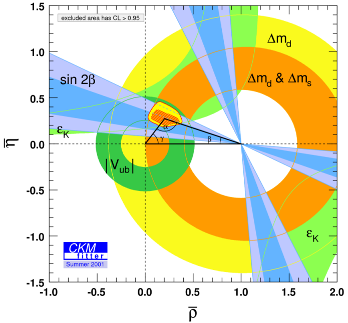
3. Structure dependent contributions turn out to be small, but can compete with QCD uncertainty.

4. Comparison between theory and experiment requires precise statement about how QED effects are treated in the exp. analysis

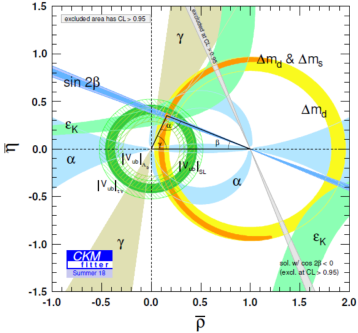
Backup-Slides

Testing the Standard Model

2001 (B factory turn-on)



2018 (Precision flavour physics)

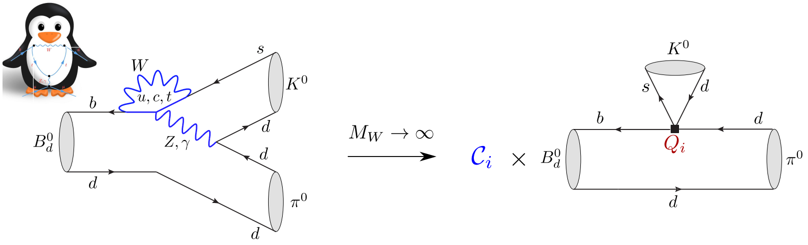


The quest for new physics

- Energy frontier: direct searches in high-energy collisions (limited by beam energy)
- Precision frontier: Flavour physics searches for new physics in small quantum fluctuations
  - $\rightarrow$ can probe much higher energy scales
  - $\rightarrow$ look for tiny deviations from the SM requires precise theoretical predictions
  - $\rightarrow$ currently observed "B-anomalies" (P_5', R_{D^*}, . . .)

Weak Effective Hamiltonian

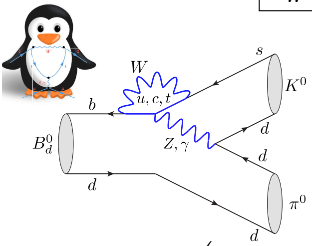
Non-leptonic decays: CP violation, determination of CKM angles, new physics searches



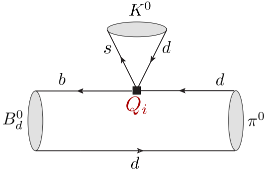
Weak Effective Hamiltonian

Non-leptonic decays: CP violation, determination of CKM angles, new physics searches

$$\boxed{M_W \sim 80\text{GeV} \gg m_B \sim 5.2\text{GeV}}$$



$\xrightarrow{M_W \to \infty}$ $\mathcal{C}_i \times$



see Buras, Buchalla, Lautenbacher [1995]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{j=u,c} V^*_{jq} V_{jb} \left(\sum_{i=1,2} C_i(\mu)\, Q_i^{jq}(\mu) + \sum_{i=3}^{10} C_i(\mu)\, Q_i^{q}(\mu) \right) \qquad \text{e.g. } Q_{1,2} = (\bar{q}\Gamma b)(\bar{d}\Gamma' q)$$

$C_i(\mu)$: short-distance Wilson coefficient $\qquad$ $\langle Q_i \rangle$: long-distance physics

- Decoupling of W,Z,t,H at $\mu_W \sim 80$ GeV (“matching”)
- Renormalization Group evolution from $\mu_W \to \mu_b \sim 5$GeV (“running”)
  $\rightarrow$ resummation of large logarithms $\ln m_b/M_W$

Soft Rearrangement

- effective SCET operator composend of soft and (anti-)collinear fields
  $\rightarrow$ has well-defined UV-scale evolution ✓
- But: Factorization requires renormalization of each individual mode

↯ Problem: UV-scale evolution of individual pieces IR-divergent! ("factorization anomaly")

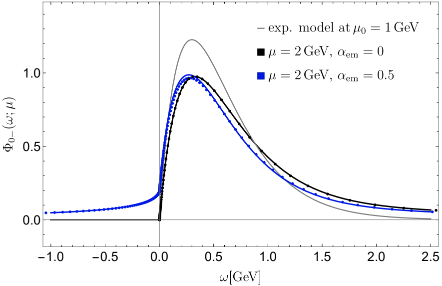
$\rightarrow$ Anomalous dimension depends on IR-regulator (off-shellness, quark masses, ...)

$\rightarrow$ can be cured by a "soft rearrangement" that removes the soft overlap

cf. [Beneke, Bobeth, Szafron]

$$\mathcal{O} = \mathcal{O}_{\bar{c}} \times \mathcal{O}_{s,C} \to (\mathcal{O}_{\bar{c}} R_{\bar{c}}) \times \left(\frac{\mathcal{O}_{s,C}}{R_{\bar{c}}} \right) \quad \text{with} \quad \left| \langle 0 | S_{n_+}^{\dagger (Q_2)} S_{n_-}^{(Q_2)} | 0 \rangle \right| \equiv R_{\bar{c}} R_c$$

B-Meson LCDA/Soft Function



$i0$ prescription from hard-collinear jet function generates imag. part in inverse moments:

$$\lambda_B^{-1}(\mu) = \int_{-\infty}^{\infty} \frac{d\omega}{\omega - i0}\, \Phi_B(\omega,\mu)\,, \qquad \sigma_n(\mu) = \lambda_B(\mu) \int_{-\infty}^{\infty} \frac{d\omega}{\omega - i0} \ln^n \frac{\tilde{\mu}}{\omega - i0}\, \Phi_B(\omega,\mu)$$

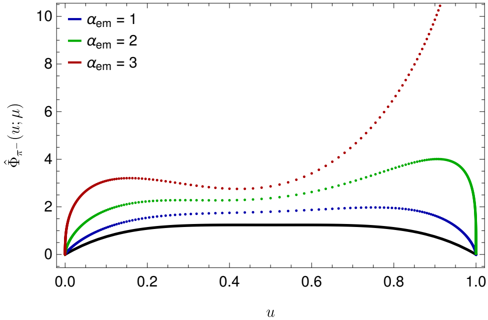
$\rightarrow$ λ_B^{-1} well-defined, but higher moments remain endpoint-divergent! (as in QCD)

QED correction at most 1.2% (for exp. model at $\mu_0 = 1$GeV with $\lambda_B^{-1}(\mu_0) = 3.33$):

	$\mu_0 = 1$ GeV	$\mu = 2$ GeV				
	initial	QCD	(0,0)	$(-,0)$	$(0,-)$	$(+,-)$
λ_B^{-1}	3.333	2.792	2.792	2.802	$2.790 + 0.010i$	$2.798 + 0.010i$
σ_1	0	-0.213	-0.213	-0.210	-0.214	-0.211

Light-Meson LCDA

Discretize integro-differential evolution equation:



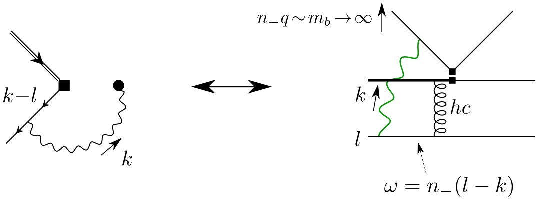
Numerical results for inverse moments: ($a_2^\pi = 0.116$ @ 2GeV)

$$\left\langle \bar{u}^{-1} \right\rangle_{\pi^-} (5.3\,\text{GeV}) = 0.9997\Big|^{\text{QED}}_{\text{point charge}} \left(3.285^{+0.05}_{-0.05}\Big|_{\text{LL}} - 0.020\Big|_{\text{NLL}} + 0.017\Big|^{\text{QED}}_{\text{partonic}}\right)$$

$$\left\langle \bar{u}^{-1} \right\rangle_{\pi^-} (80.4\,\text{GeV}) = \;\; 0.985\Big|^{\text{QED}}_{\text{point charge}} \left(3.197^{+0.03}_{-0.03}\Big|_{\text{LL}} - 0.022\Big|_{\text{NLL}} + 0.042\Big|^{\text{QED}}_{\text{partonic}}\right)$$

$\rightarrow$ QED as important as two-loop QCD evolution

On the Support of QED B LCDAs



- even for on-shell massive partons with $\Phi^{(0)}(\omega) = \delta(\omega - m)$ the one-loop soft photon exchange with the anti-coll. π^- generates a support for $\omega < 0$
- diagram has e.g. the following contribution

$$\int \mathrm{d}^d k \, \frac{\delta(\omega - n_-\ell + n_-k)}{(k^2 + i0)[(k-\ell)^2 - m^2 + i0]\,(n_+k - i0)} \quad \Big| \text{pick up residues in } (n_+k)$$

$$\sim \Gamma(\epsilon) \int_{n_-\ell}^{\infty} \mathrm{d}(n_-k)\,(n_-k)^{-1-\epsilon} \delta(\omega - m + n_-k) = \Gamma(\epsilon)\,(m-\omega)^{-1-\epsilon}\theta(-\omega)$$

- QED B LCDA no longer linear in ω as $\omega \to 0$ but rather const.

  $\to$ no endpoint singularity in first inverse moment

$$\int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega}{\omega - i0} \Phi_{B,+-}(\omega)$$

Ratios and Isospin Sum Rules

$$R_L^{(0),(*)}(\Delta E) \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+} L^-)(\Delta E)}{d\Gamma^{(0)}(\bar{B}_d \to D^{(*)+}\mu^-\bar{\nu}_\ell)/dq^2|_{q^2=m_L^2}}$$

$$R_L^{(*)}(\Delta E) \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+} L^-)(\Delta E)}{d\Gamma(\bar{B}_d \to D^{(*)+}\mu^-\bar{\nu}_\ell)(\Delta E)/dq^2|_{q^2=m_L^2}}$$

- short-distance QED effects $\approx -1\%$, ultrasoft up to $\approx -7\%$ for pions, depending on the semi-leptonic normalization.
- not large enough to explain the -15% amplitude deficit [Bordone et al., 2020], but highlights the importance of proper treatment of ultrasoft radiation effects.

$R_L^{(*)}$	LO	QCD NNLO	$+\delta_{\rm QED}$	$+\delta_{\rm U}$ ($\delta_{\rm U}^{(0)}$)
R_π	0.969 ± 0.021	$1.078^{+0.045}_{-0.042}$	$1.069^{+0.045}_{-0.041}$	$1.074^{+0.046}_{-0.043}$ ($1.003^{+0.042}_{-0.039}$)
R_π^*	0.962 ± 0.021	$1.069^{+0.045}_{-0.041}$	$1.059^{+0.045}_{-0.041}$	$1.065^{+0.047}_{-0.042}$ ($0.996^{+0.043}_{-0.039}$)
$R_K \cdot 10^2$	7.47 ± 0.07	$8.28^{+0.27}_{-0.26}$	$8.21^{+0.27}_{-0.26}$	$8.44^{+0.29}_{-0.28}$ ($7.88^{+0.26}_{-0.25}$)
$R_K^* \cdot 10^2$	6.81 ± 0.16	$7.54^{+0.31}_{-0.29}$	$7.47^{+0.30}_{-0.29}$	$7.68^{+0.32}_{-0.30}$ ($7.19^{+0.29}_{-0.28}$)

Table 3: Theoretical predictions for $R_L^{(*)}$ expressed in GeV^2 at LO, NNLO QCD and subsequently adding $\delta_{\rm QED}$ given in (82) and the ultrasoft effects $\delta_{\rm U}$ (or in brackets $\delta_{\rm U}^{(0)}$). The last column presents our final results.