QED in $B \to K\ell\ell$ and LFU

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Rencontre de Vietnam, Flavour Conference

14-20 August 2022 — Quy Nhon

yesterday

R_K-anomaly ..testing LFU

- **QCD** blind to lepton flavour, **hence hadronic** effects cancel in ratios:

$$R_H = \frac{\int \frac{d\Gamma(B\to H\mu^+\mu^-)}{dq^2}\, dq^2}{\int \frac{d\Gamma(B\to He^+e^-)}{dq^2}\, dq^2}$$

Hiller Kruger'03

$$R_K\left[1.1\mathrm{GeV}^2, 6\mathrm{GeV}^2\right] = 0.846^{+0.042+0.013}_{-0.039-0.012}$$

LHCb (2103.11769)
2-3σ

- $R_K = 1 + \Delta_{QED}$ as **QED** does not respect LFU

What could go wrong?

- **QED-effects** large due to (soft)-hard collinear logs $O(\alpha)\ \ln m_e/m_b$; when treated by point-like approximation (=scalar QED)

  - 1. **Structure-dependent** effects new hard-collinear logs
  - 2. **PHOTOS** (QED Monte-Carlo) not in harmony with point-like approx.*
  - 3. **Resonances** impact on [1.1,6]GeV^2-bin *

- To understand, need to learn/recap few things:
  - **IR logs** (and when they cancel and not) [relevant to 1. and 2.]
  - also depends on **kinematic variables** [relevant to 3.]

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* for 2,3 partial answers (as approximations) in Bordone, Pattori, Isidori'16

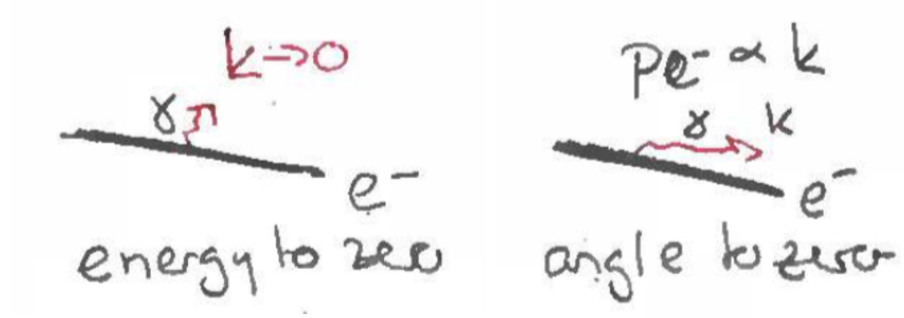
Overview

- I. **Recap** of basics what we know and what not about IR-logs
- II. **2 Theory Results** relevant to understanding R_K-safety

  Isidori, Nabeebaccus, RZ 2009.00929
- III. Comparison with PHOTOS & dangerous charmonium resonances

  Isidori, Lancierini, Nabeebaccus, RZ 2205.08635
- END. Summary & Conclusions

Recap on IR sensitive terms for Rates



- d=4 IR-divergences are **logarithmic**:
  - "soft" photon momentum $k \to 0$
  - "collinear" photon momentum $k \propto p_{ex}$

- **Kinoshita-Lee-Nauenberg theorem** (1962)
  Total (decay) rates all divergences (IR-sensitive terms) cancel

- **Loopholes**:
  i) not photon inclusive (next slide)
  ii) differential (sizeable collinear $\alpha \ln m_e/m_b$ can remain)

What is clear and what isn't.

Soft & soft-collinear logs: $\ln m_\gamma$ & $\ln m_\gamma \ln m_\ell / m_B$

- Soft and soft-coll. logs captured by point-like approximation

- Soft and soft-coll. logs cancel @ differential level
  (from YFS resummation 61', Weinberg'65, coherent states 60's)

- For $E_\gamma < \Delta E$, soft logs ``$\ln m_\gamma |_{real} \to -\ln \Delta E$" at leading log

Hard-collinear logs: $\ln m_\ell / m_B$

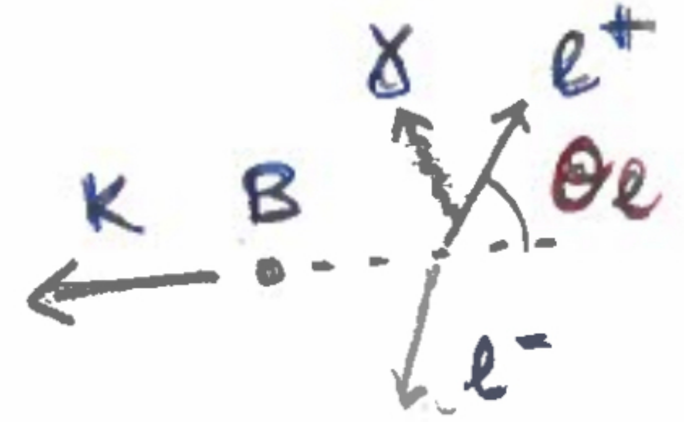
- Unclear whether they cancels at differential level
  ``cancellation miraculous [=unitarity by KLN] as topologies unrelated"

- Unclear whether captured by point-like approximation ``not soft"

Generalisation of decay Kinematics for radiative rate

- Also unclear how to generalise differential variables $c_\ell \equiv \cos\theta_\ell$

To: $\bar{B}(p_B) \to \bar{K}(p_K)\ell_1(\ell_1)\bar{\ell}_2(\ell_2)\gamma(k)$

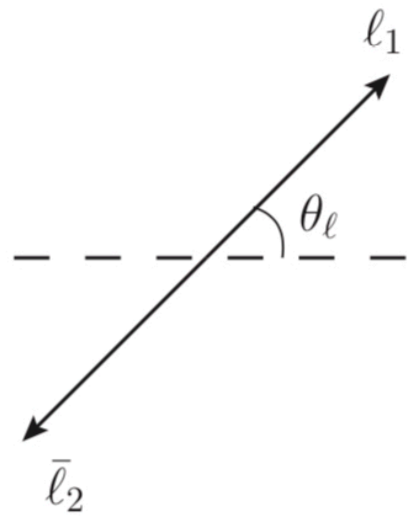


$$\frac{d^2\Gamma(B \to K\ell\ell(\gamma))}{dq^2 dc_\ell}$$

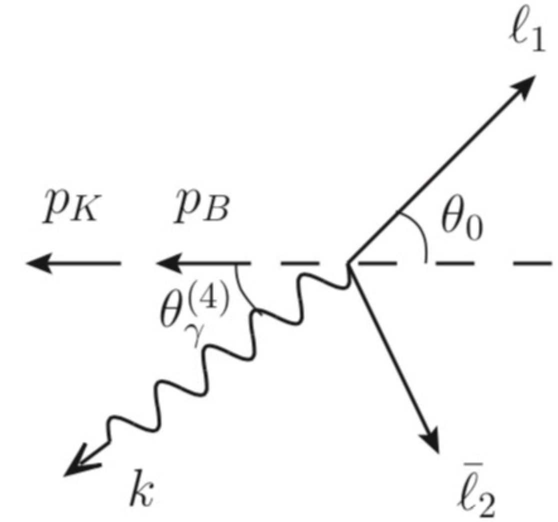
related to previous question?

- Natural choices from kinematics viewpoint

$$\{q_a^2, c_a\} = \begin{cases} q_\ell^2 = (\ell_1+\ell_2)^2, \quad c_\ell = -\left(\frac{\vec{\ell_1}\cdot\vec{p}_K}{|\vec{\ell_1}||\vec{p}_K|}\right)_{q-\mathrm{RF}} & [\text{“Hadron collider” variables}]\ , \\ q_0^2 = (p_B - p_K)^2\ ,\ c_0 = -\left(\frac{\vec{\ell_1}\cdot\vec{p}_K}{|\vec{\ell_1}||\vec{p}_K|}\right)_{q_0-\mathrm{RF}} & [\text{“B-factory” variables}]\ , \end{cases}$$



q-RF



q_0-RF

- **Theory Result 1**: in $\{q_0^2, c_0\}$-variables hc logs cancel differentially

  - understood by explicit computation first
  - now also by IR-safety notion from collider physics q_0^2 = jet variable
  - splitting function approach to collinear divergences
    (close to PHOTOS)

Theory result 2: are hc logs are universal

Or if B, K-meson resolved (structure-dependence), further collinear logs?

- Write in meson-EFT: $A^{(1)} = \hat{Q}_{\ell_1} \frac{a_{\ell_1}}{\ell_1 \cdot k} + \delta A^{(1)}$

1) $$\hat{Q}^2_{\ell_1} \int_\gamma \left| \frac{a_{\ell_1}}{\ell_1 \cdot k} \right|^2 = O(1)\, \hat{Q}^2_{\ell_1} \ln m_{\ell_1} + . \qquad \int_\gamma Rest \xrightarrow{m_{\ell_1} \to 0} finite^*.$$

collinear-log **IR-safe**

* by gauge invariance: collinear region: $A = \epsilon^\mu A_\mu \Rightarrow \ell_1^\mu A_\mu = \mathcal{O}(m_{\ell_1})$

2) Hence $\delta A \to \delta A + A^{B,K}_{structure}$, no new real collinear logs

3) Since real & virtual cancel (in q_0^2, c_0 variables),
no new virtual collinear logs either

Gauge invariance acts as **custodian** that sweeps away all the ``**dangerous" hc logs** beyond pt-like app. Point 1. clarified with positive answer

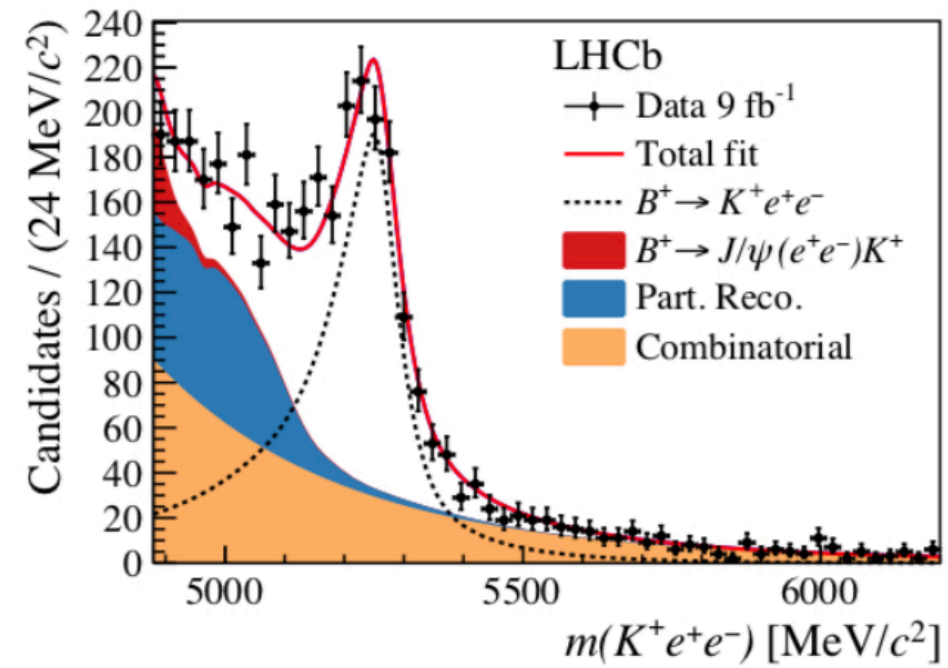
Agreement with PHOTOS?

- Experiment uses PHOTOS [PHOTONS++Sherpa] Monte Carlo event by event simulation [to produce 5-fold diff. distribution]

E.g. single differential plot
(at detector level from LHCb)

$$\frac{d}{dm_{K\ell\ell}} \text{ equivalent to } \frac{d}{dE_\gamma}$$

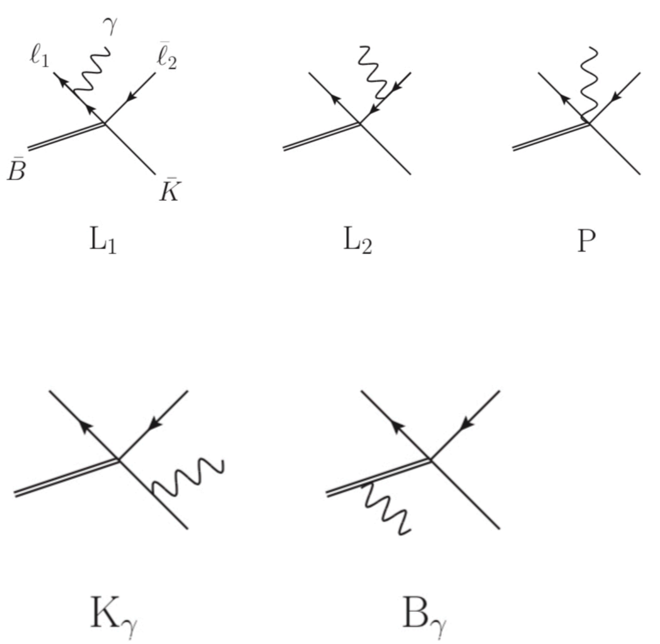
$$m_{K\ell\ell}^2 \equiv m_{Brec}^2 = m_B^2 - 2m_B E_\gamma = (1-\delta)m_B^2$$



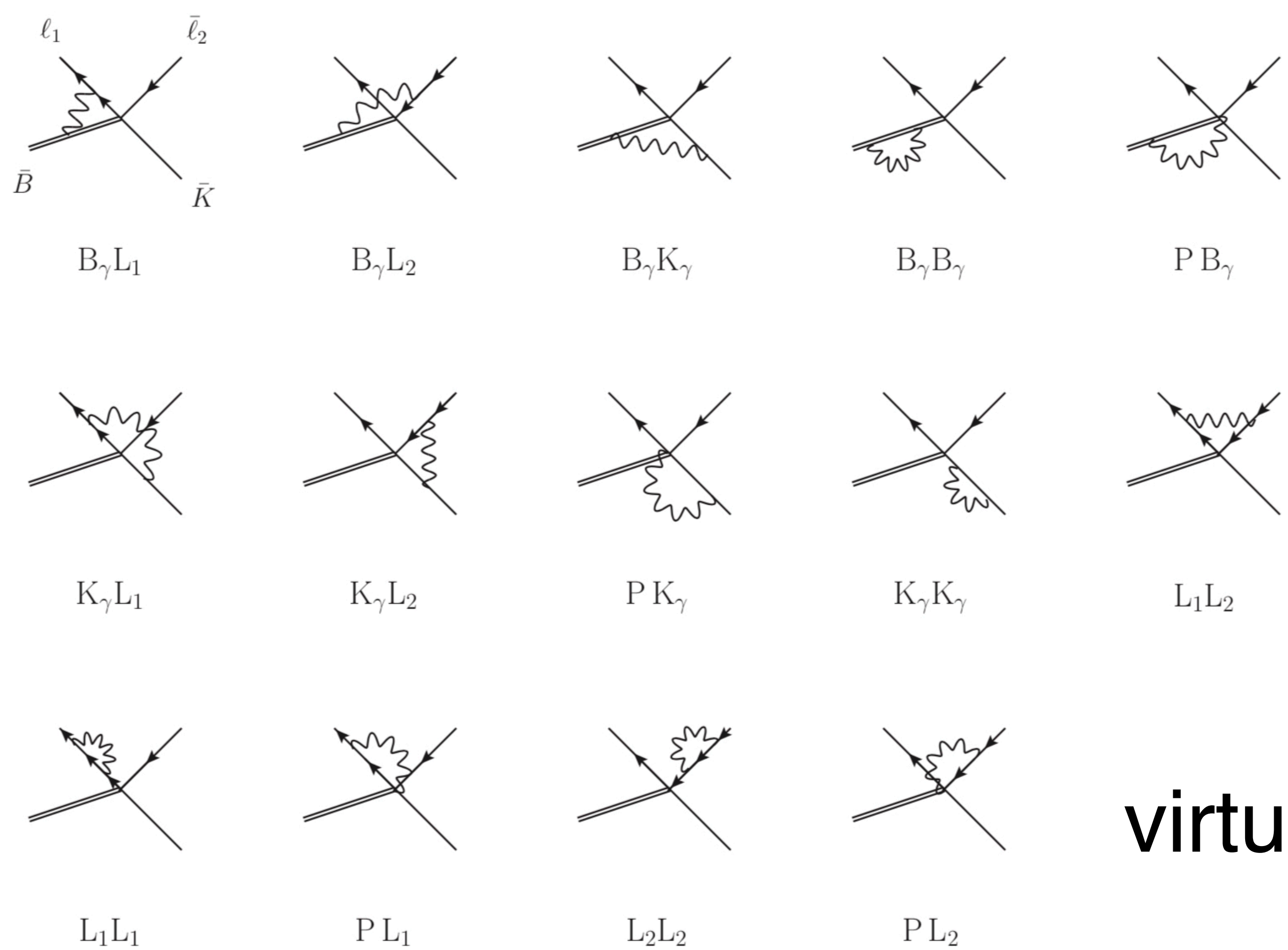
- Ought to test whether we agree with PHOTOS.
  We test for fixed $m_{Brec} > (4.88, 5.18) GeV$ for (electrons, muons) as this comes closest to what LHCb does.

What we do

- Take our point-like computation from Isidori, Nabeebaccus, RZ 2009.00929



real

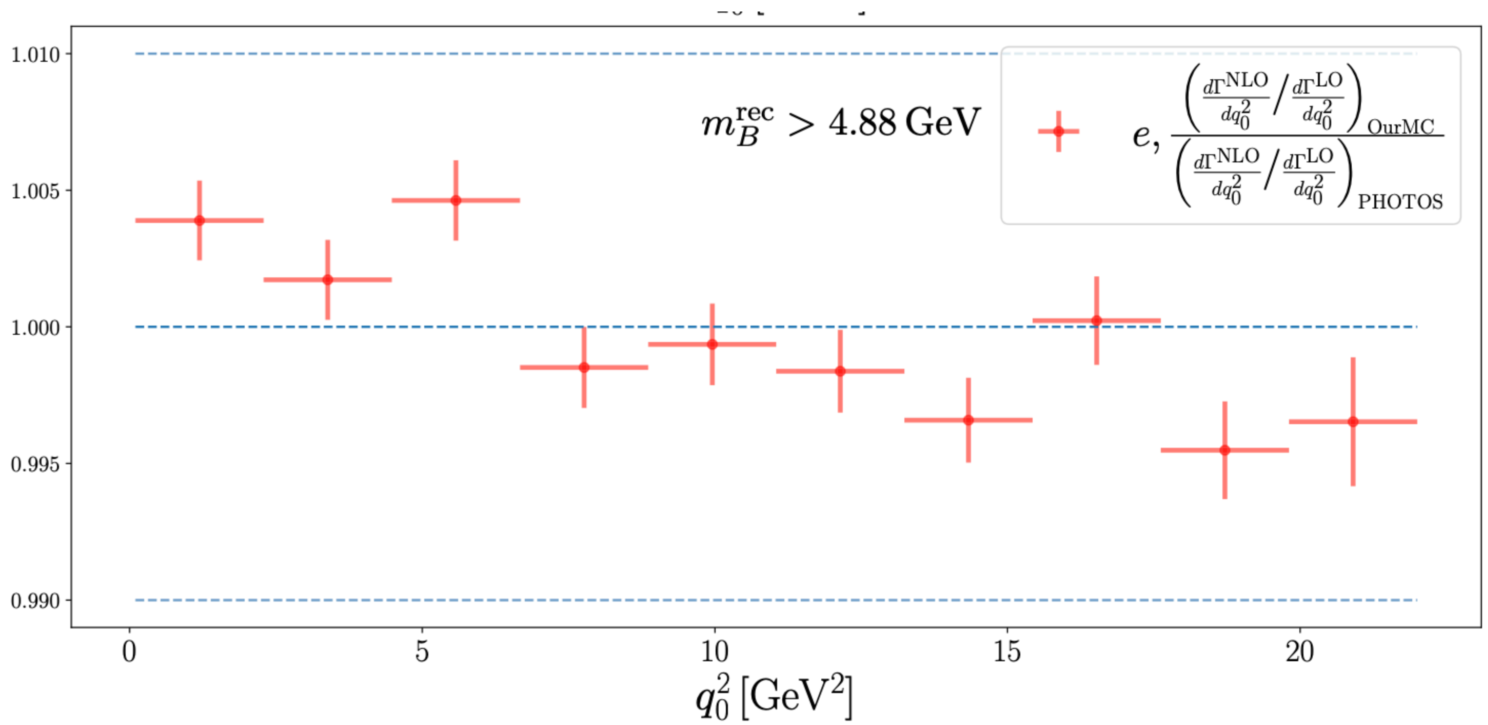


virtual

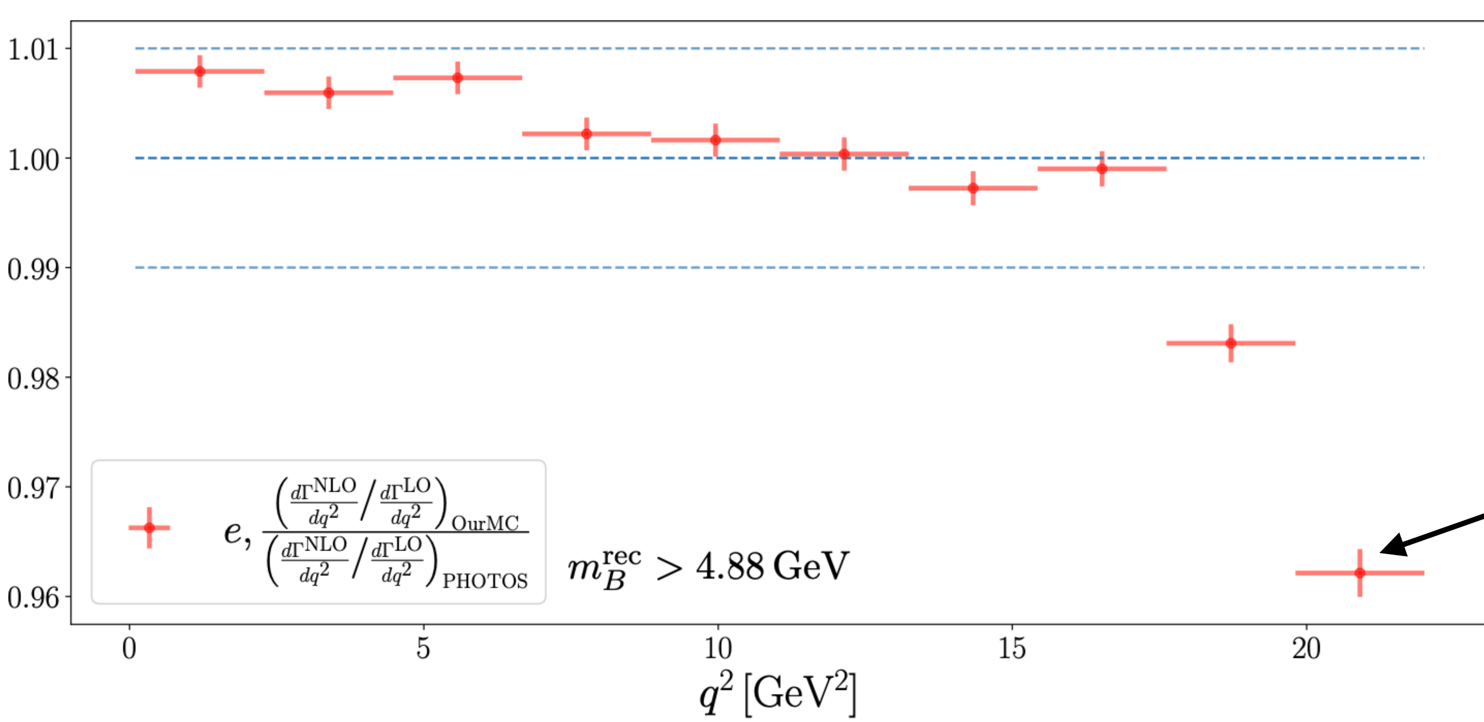
- Use it as basis for Monte Carlo Generator (hit or miss algorithm) and compare to PHOTOS using same events

  (many events = distribution)

Plots for electrons (muons in backup slides)



- Our Monte Carlo/PHOTOS in both q_0^2 and q^2 variable



- Agreement 1% level

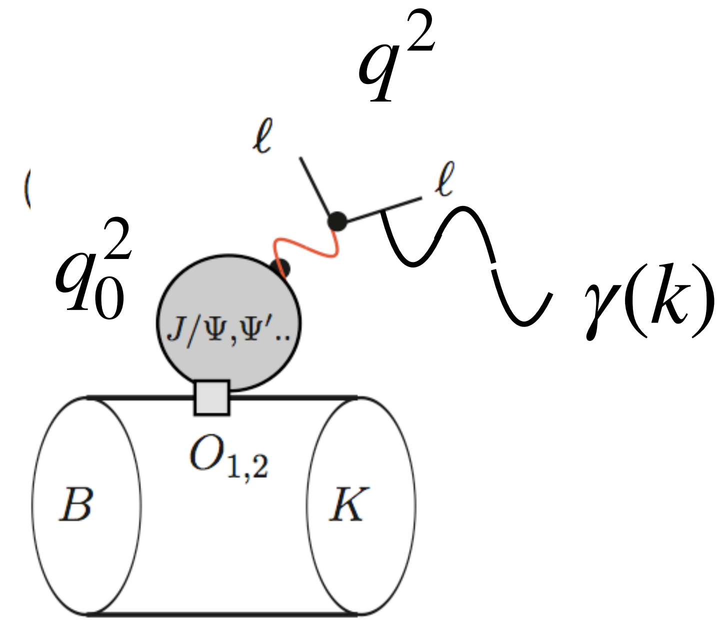
- Except endpt effects 20% NNLL needed (next slide)

Point 2. clarified, PHOTOS is a good program

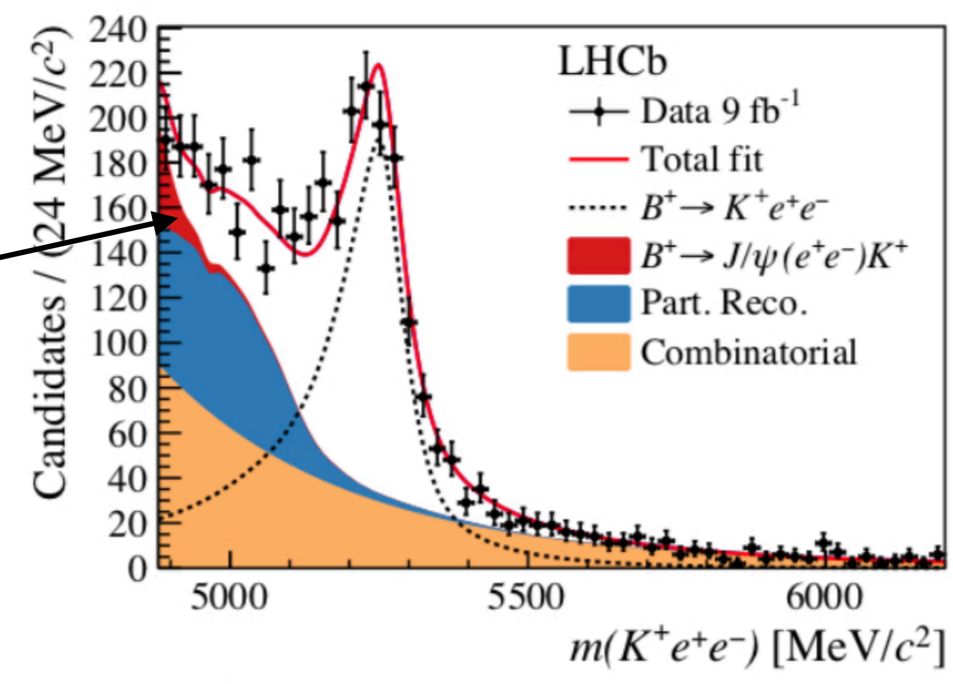
The charmonium (mainly J/Ψ) resonances

- LHCb uses q^2-variable, now since

ℓ	$m_B^{\rm rec}$[GeV]	$\delta_{\rm ex}$	$(q_0^2)_{\rm max}$
μ	5.18	0.0486	$q^2 + 1.36\ \mathrm{GeV}^2$
e	4.88	0.146	$q^2 + 4.07\ \mathrm{GeV}^2$



and $m_{J/\Psi}^2 \approx 9.6\,GeV^2$,
$q^2 = 6\,GeV^2$ ``sees”
the peak of the resonance $O(10^4)$-effect



- LHCb neglects interference of rare and resonant mode ….

Collinear logs from splitting function

- With **splitting function** one can reproduce all collinear logs (and numerically this dominates so can use it for assessment*)

splitting fct LO-rate

$$\Delta_{QED} \propto \ln \frac{\mu_{\rm hc}}{m_{\ell_1}} \left(\frac{1}{\Gamma^{\rm LO}} \int_{\max(\hat{q}^2, z^\delta_{\ell_1})}^1 dz P_{f\to f\gamma}(z) \frac{d^2\Gamma^{\rm LO}(\hat{q}_0^2, c_0)}{d\hat{q}_0^2 dc_0} \right) J_{\ell_1}(c_\ell, z)$$

$$P_{f\to f\gamma}(z) \;=\; \lim_{z^*\to 0}\left[\tfrac{1+z^2}{(1-z)}\theta((1-z^*)-z) + (\tfrac{3}{2} + 2\ln z^*)\delta(1-z)\right]$$

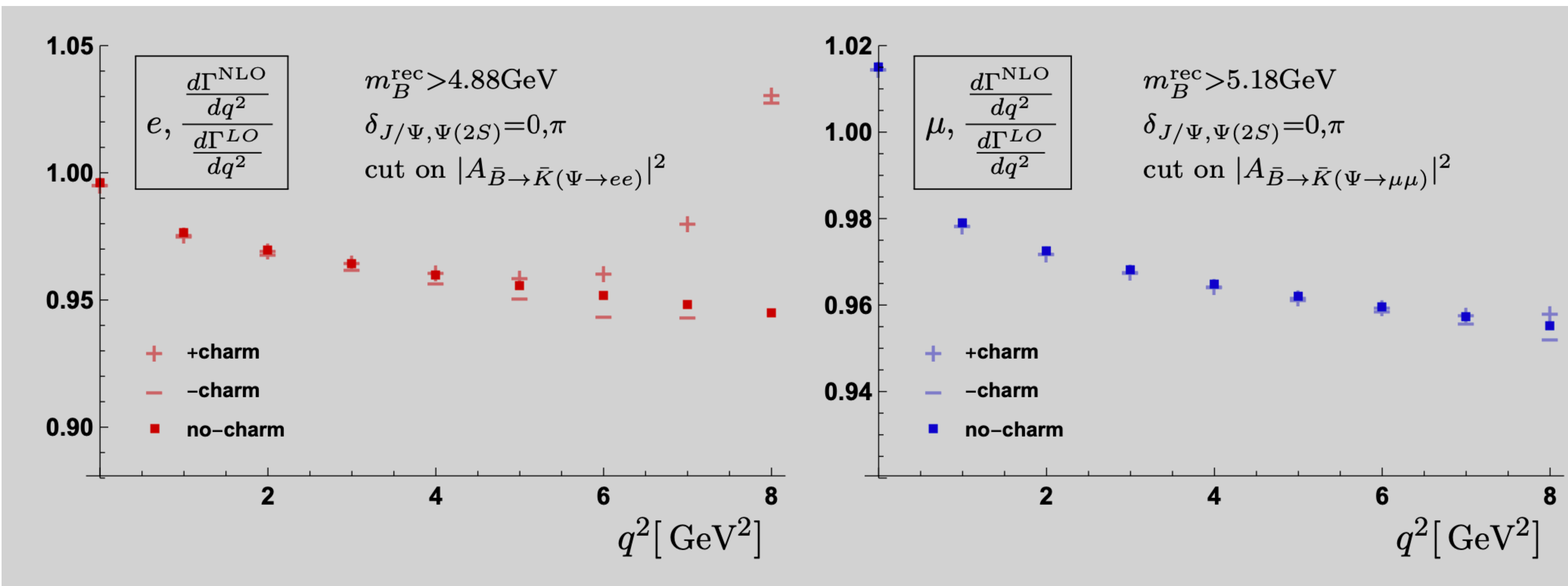
$$q^2 = z q_0^2\,, \quad c_0|_{m_K=0} = \frac{c_\ell(1+z)+\bar{z}}{c_\ell \bar{z} + 1 + z}\,, \quad J_{\ell_1}(c_\ell, z)|_{m_K=0} = \frac{4}{(c_\ell \bar{z} + 1 + z)^2}$$

$$z^\delta_{\ell_1}|_{m_K=0} = \frac{1+\hat{q}^2 - \delta + c_\ell(1-\hat{q}^2-\delta)}{1+\hat{q}^2 + \delta + c_\ell(1-\hat{q}^2-\delta)}$$

* With its resummed version (electron structure function), we can fix the difference with PHOTOS at high q^2.

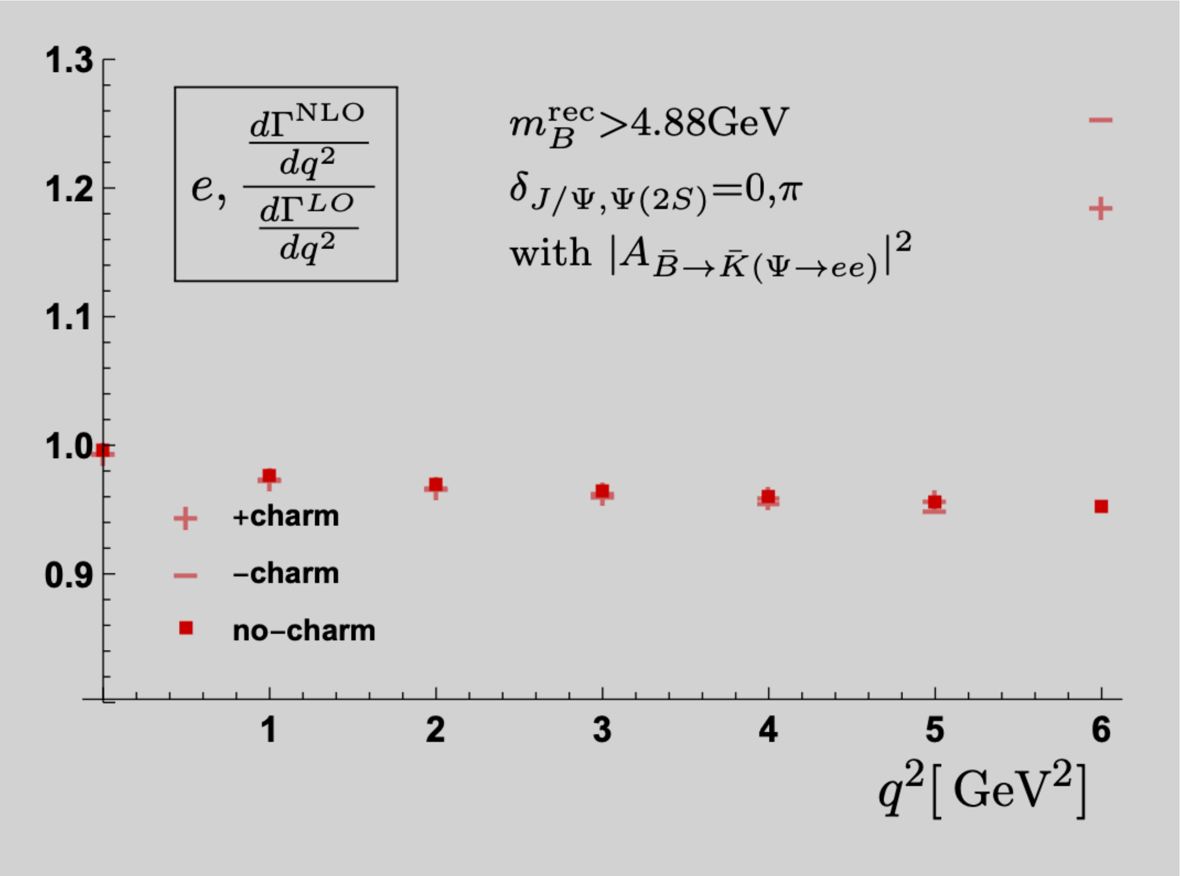
Assessing neglecting interference

- The **issue** is that we do **not know** the **LO-rate** precisely for the **charm**, e.g. strong phase of the J/Ψ
- To assess we minimise and maximise the interference effect cut out resonant mode (it's amplitude square to mimic LHCb)



- Effects only begin to be sizeable for $q^2 > 6\,GeV^2$

Assessment including full resonant mode



- Effects more sizeable at $q^2 = 6\,GeV^2$
- On $[1,1,6]\,GeV^2$ bin effect is not dramatic
  However, it would be unwise to extend as at $q^2 = 7{,}8\,GeV^2$ 600, 4000%

Point 3. clarified, J/Ψ does not hugely impact on $[1,1,6]\,GeV^2$-bin

- Hence, it would be wise and great to **provide** finely **binned** R_K

Conclusions & Outlook

- **Theory results:**
  1) **no** new **hard collinear logs** beyond pt-like approx.
  2) sensitivity of **hard collinear logs** depends on diff-variables
- **PHOTOS Monte Carlo**
  3) Good program and sufficient when pt-like approx. is sufficient
  4) by pt 2), PHOTOS is then sufficient!
- **J/Ψ-resonance by migration of radiation**
  5) is just ok for $[1,1,6]\, GeV^2$-bin
  6) please provide binned R_K
- **Going beyond pt-like approximation [in progress]**
  7) relevant for non-LFU e.g. $\ln m_{\pi,K}/m_B$ sizeable (backup plots)
  8) challenging and interesting (lattice,SCET,QCD sum rules)

The end as time is surely up!

Backup

Cancellation of logs (photon-inclusive)*

cancel?	$\frac{d^2\Gamma}{dq^2 dc_\ell}$	$\frac{d^2\Gamma}{dq_0^2 dc_0}$
soft	yes	yes
soft-collinear	yes	yes
collinear	no	yes

"main result 1"

- Note: once photon energy cut-off restored (all logs come back)

total rates agree

$$\Gamma(\Delta E) = \int_{\Delta E} \frac{d^2\Gamma}{dq^2 dc_\ell} dq^2 dc_\ell = \int_{\Delta E} \frac{d^2\Gamma}{dq_0^2 dc_0} dq_0^2 dc_0$$

* use photon energy cut-off - all done analytic
(technical aspect: soft energy and angular integral shown to be separately Lorentz-invariant!)

III) Plots

- parameterise relative QED-correction

$$d^2\Gamma_{\bar{B}\to\bar{K}\ell_1\bar{\ell}_2}(\delta_{\rm ex}) \;=\; d^2\Gamma^{\rm LO}\left[1+\Delta^{(a)}(q_a^2,c_a;\delta_{\rm ex})\right]\, dq_a^2 dc_a$$

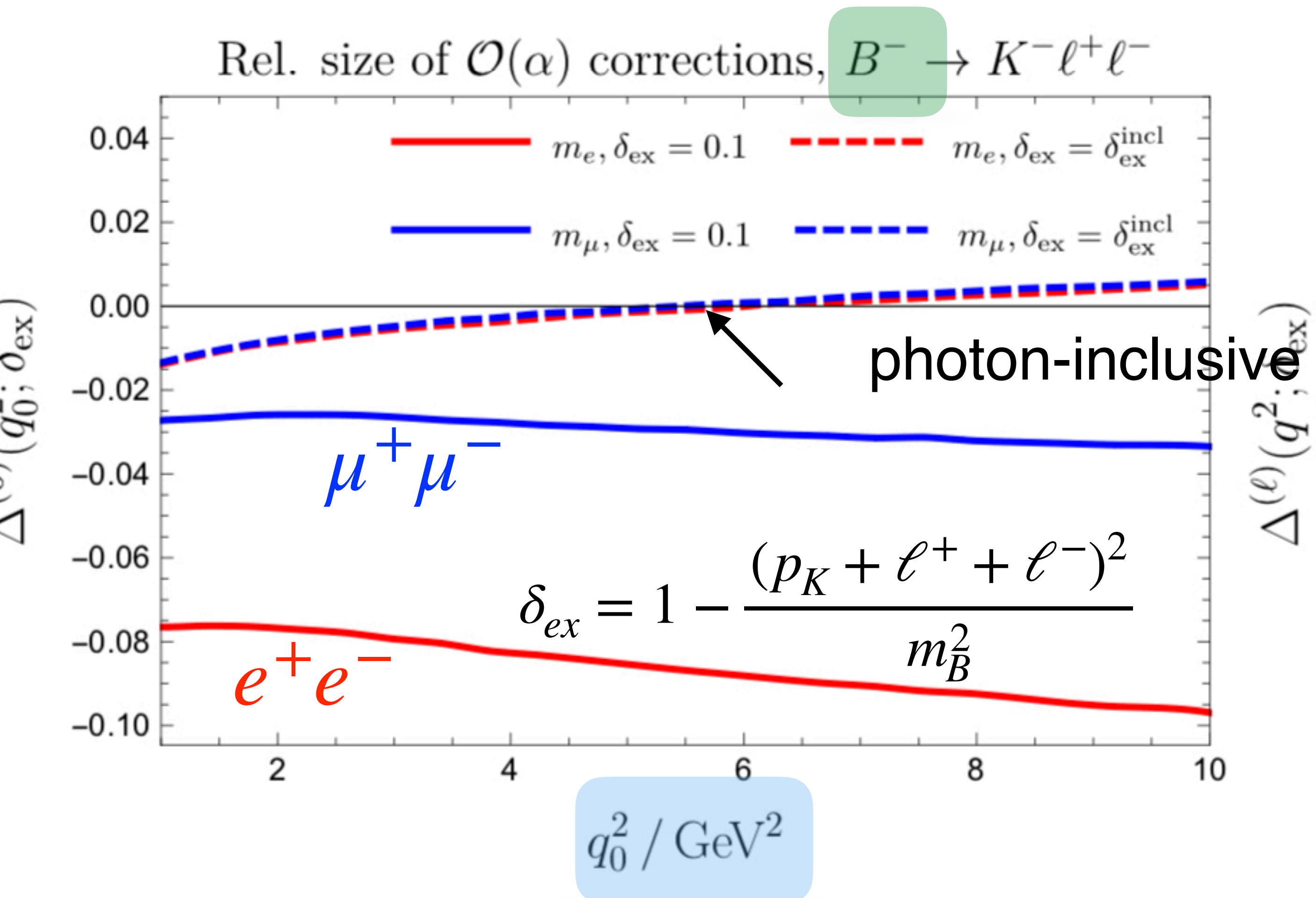
- manifest Lorentz-invariant photon "energy" cut off

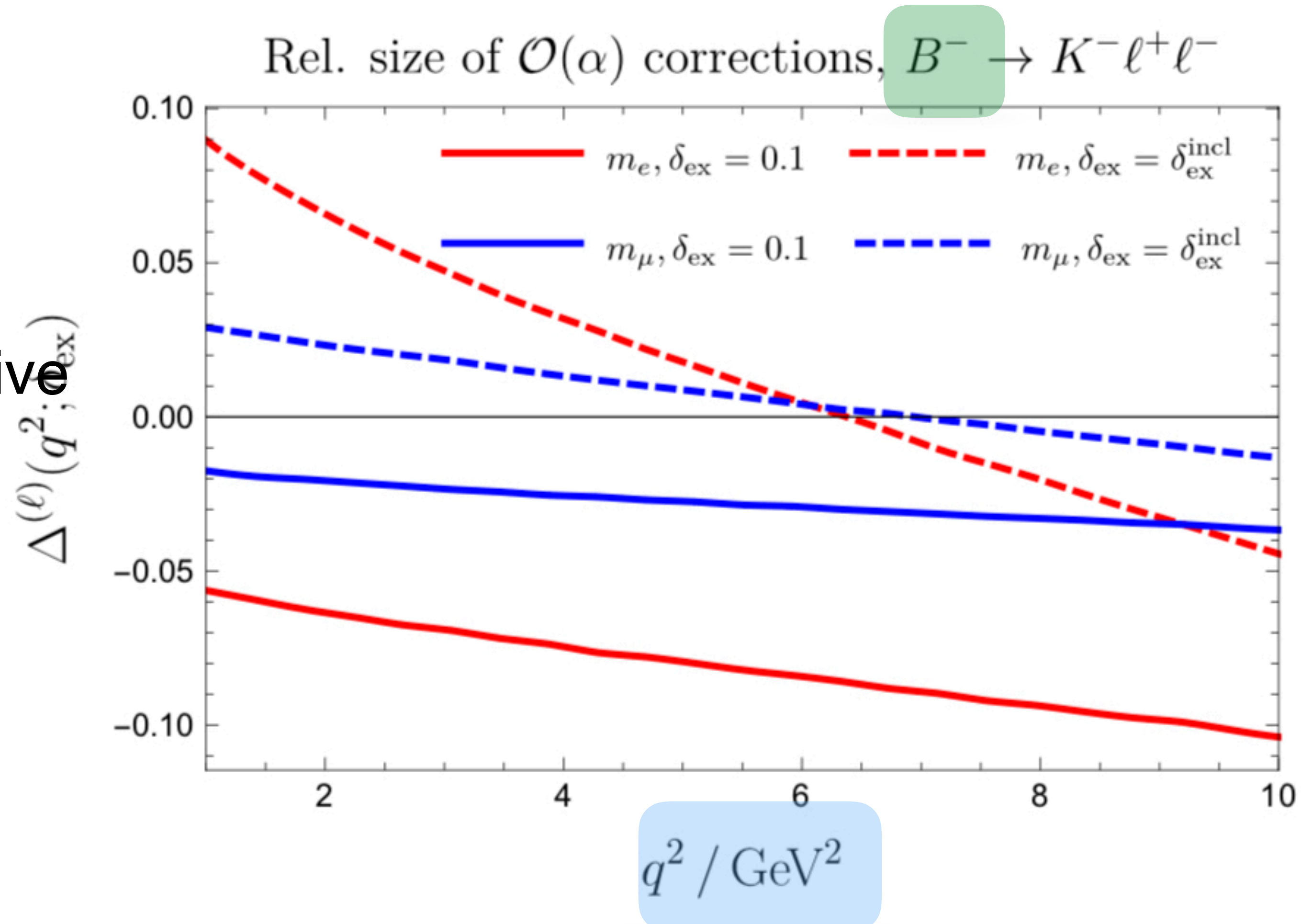
$$\delta_{ex} = 1 - \frac{(p_K + \ell^+ + \ell^-)^2}{m_B^2}$$

visible final states

corresponds to previous ΔE

Charged meson case





photon-inclusive (dashed)

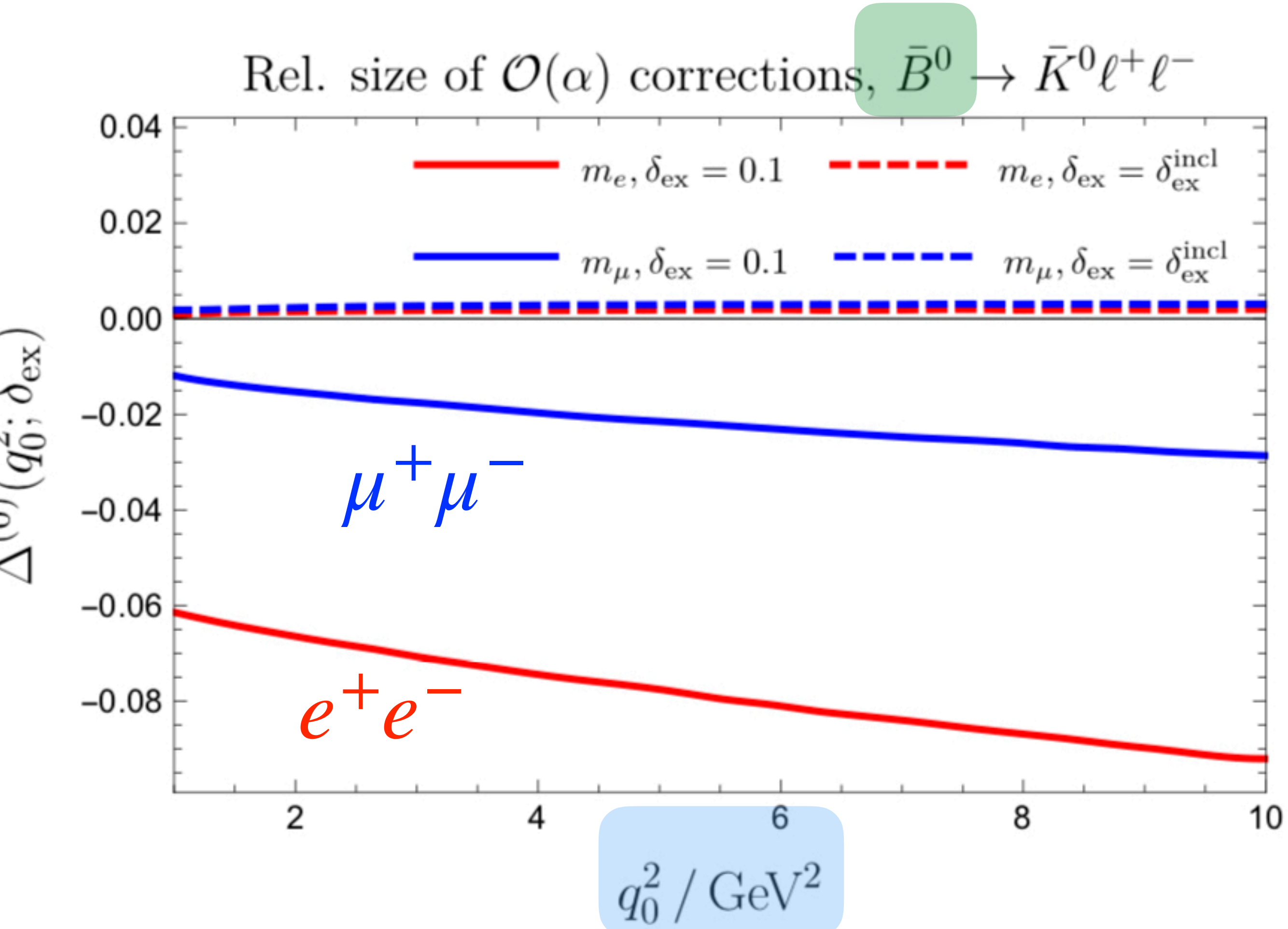
approx lepton-universality (LU) LU broken by collinear logs $ln\frac{m_\ell}{m_b}$

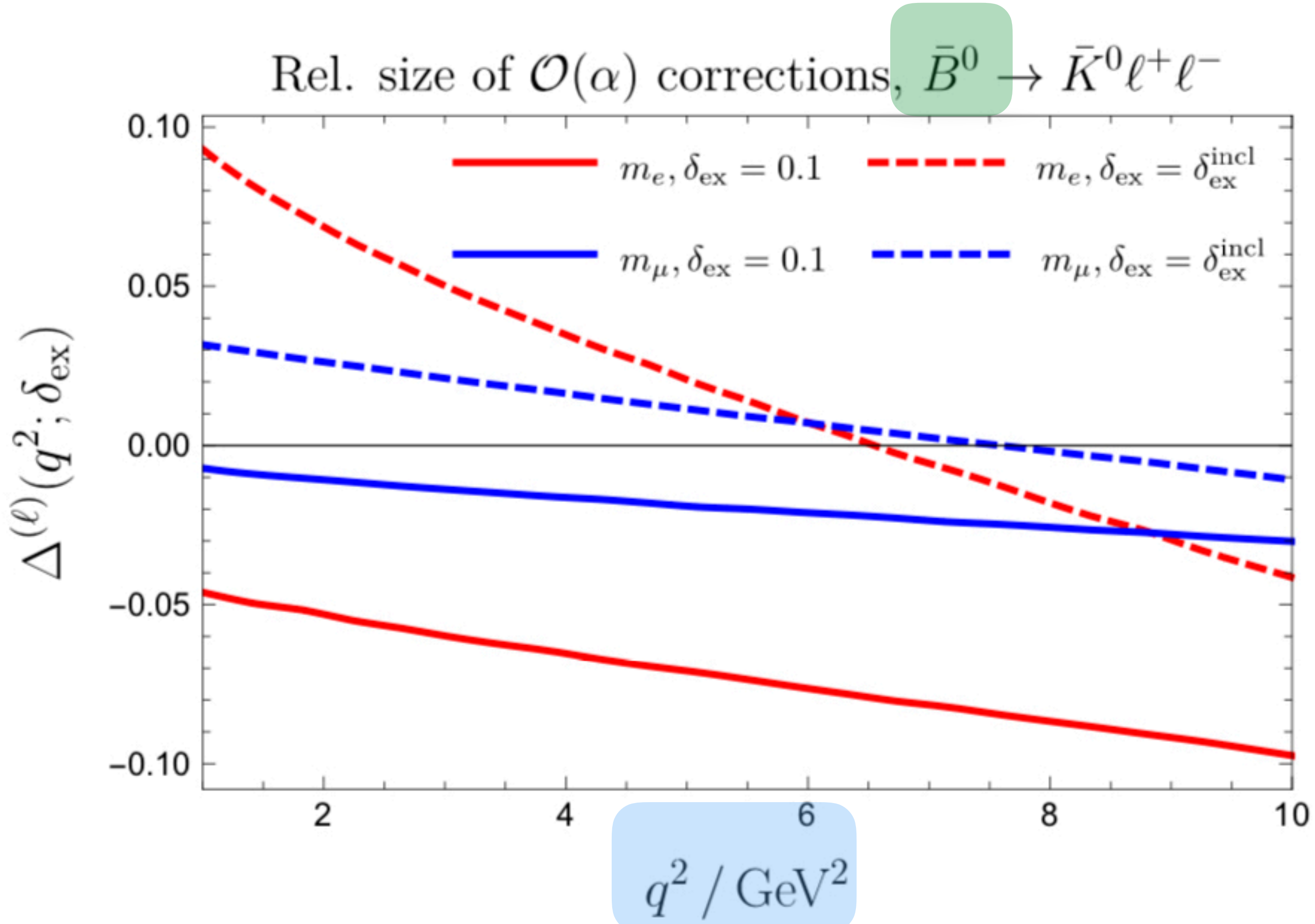
photon-energy cut (straight)

μ : $3-4\,\%$, e : $6-10\,\%$, effects (LU broken by soft-col. & col. logs)

Neutral meson case

skip in talk





- differences to charged case due to absence of collinear $\ln m_K/m_B$
- N.B. did not plot much beyond $10 GeV^2$ to avoid charmonium (for semileptonic case no problem)

LFU R_K etc

- SM: lepton universality broken by lepton masses. Small but not in QED.
  BSM: lot's of excitement …

$$R_K\Big|_{q_0^2\in[q_1^2,q_2^2]GeV^2}^{m_B^{rec}} = \frac{\Gamma[\bar{B}\to\bar{K}\mu^+\mu^-]}{\Gamma[\bar{B}\to\bar{K}e^+e^-]}\Bigg|_{q_0^2\in[q_1^2,q_2^2]GeV^2}^{m_B^{rec}} \approx 1 + \Delta_{\rm QED}^{m_B^{rec}} R_K$$

$$R_K\Big|_{q_0^2\in[1,6]GeV^2} = 0.846^{+0.060+0.016}_{-0.054-0.014}$$ LHCb (1903.09252)

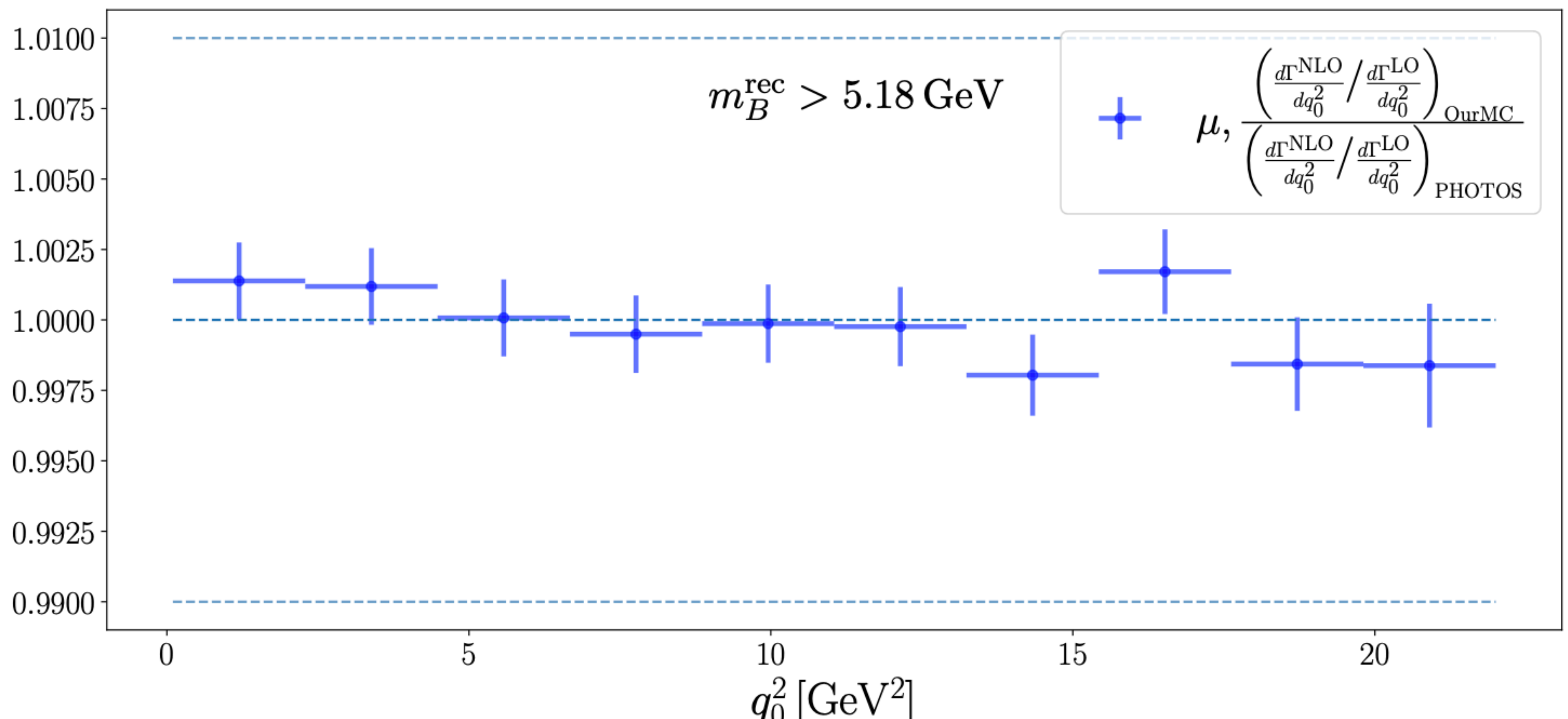
Q: do we miss terms col.-logs in $\Delta_{\rm QED}^{m_B^{rec}} R_K$?

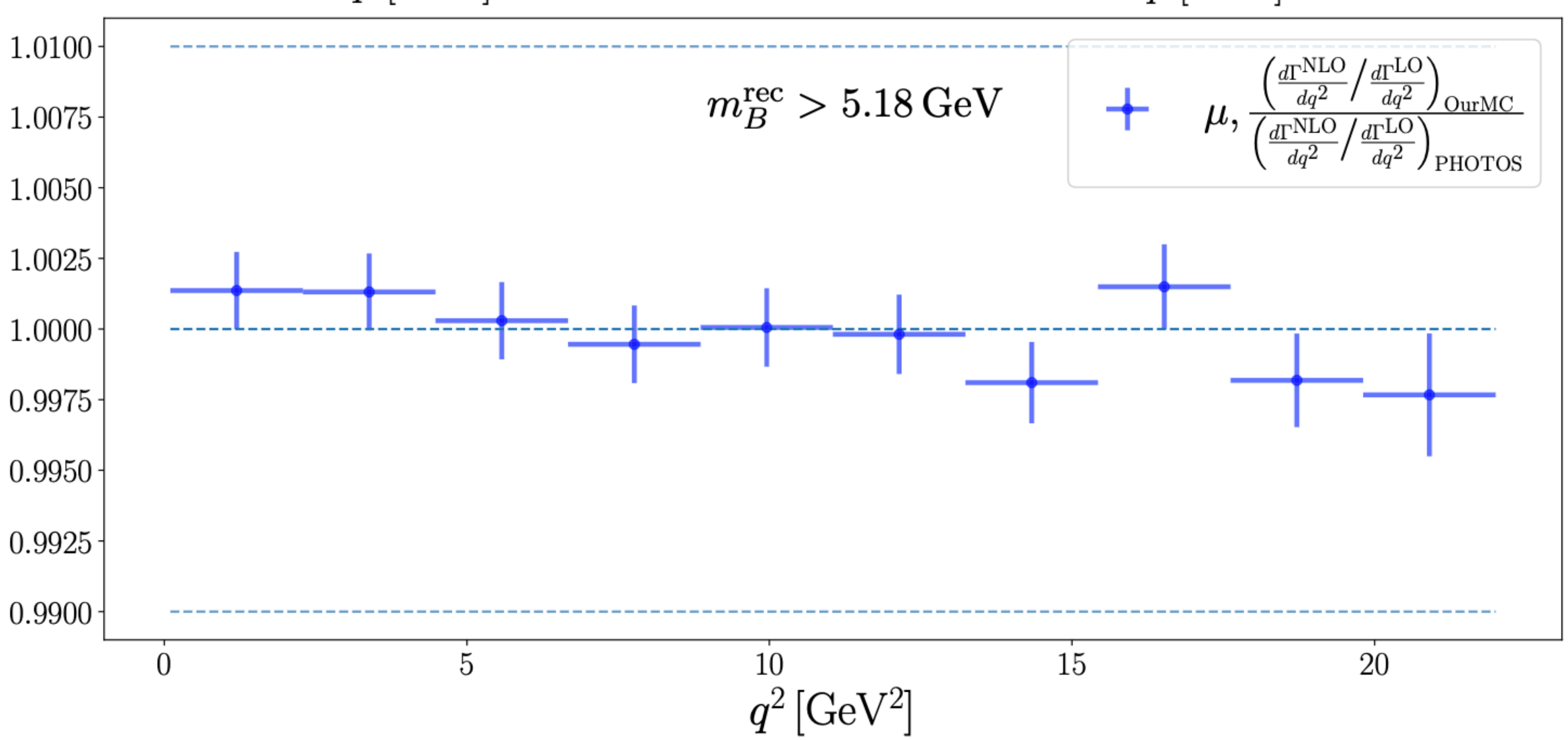
A: no by our main result 2 (by gauge invariance)

⇒ **LFU ratios** of $R_K, R_{K*}, R_D, R_{D*}, R_\phi$-type are, from the theory side, **under control w.r.t.** $\ln m_\ell/m_B$ **logs**

"main result 3"

Additional Plots for muons

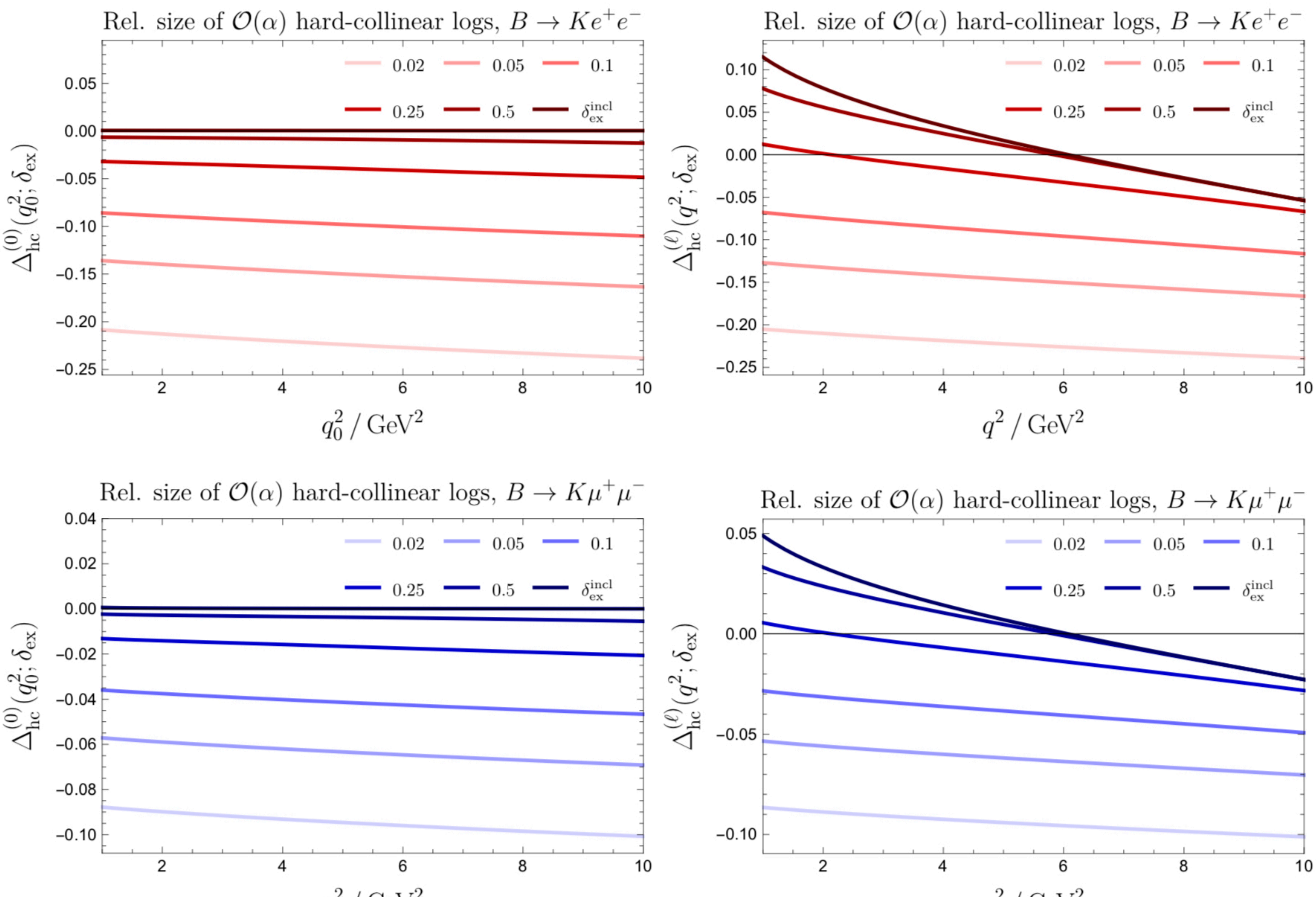




Additional Plots

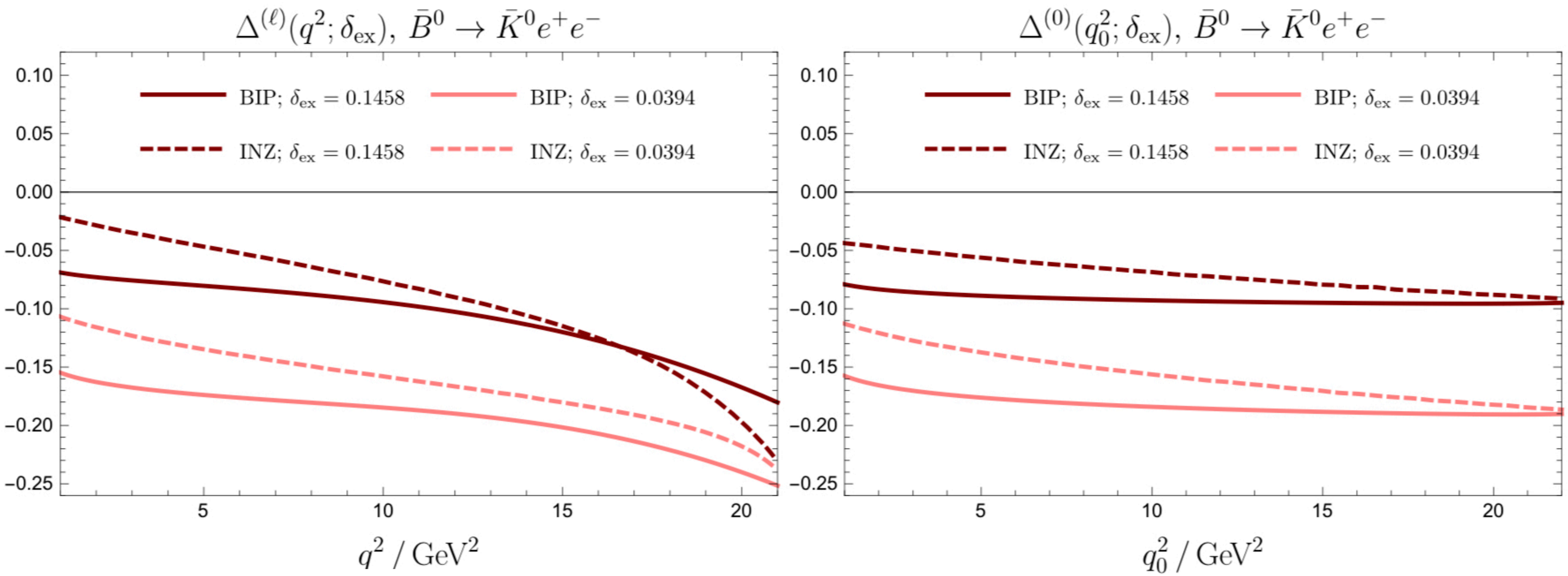
Dependance on photon “energy" cut-off

$$\delta_{ex} = 1 - \frac{(p_K + \ell^+ + \ell^-)^2}{m_B^2}$$



Comparison with BIP

Appendix A.2



$$\frac{d\Gamma}{dq_0^2} = \frac{\alpha}{\pi}\left[\frac{d\Gamma}{dq_0^2}\right]^{\rm LO}\left(A_0 \ln\delta_{\rm ex} + C_0\right)\ln m_\ell + \text{non-collinear}$$

differences partly due to tight angle cut in BIP

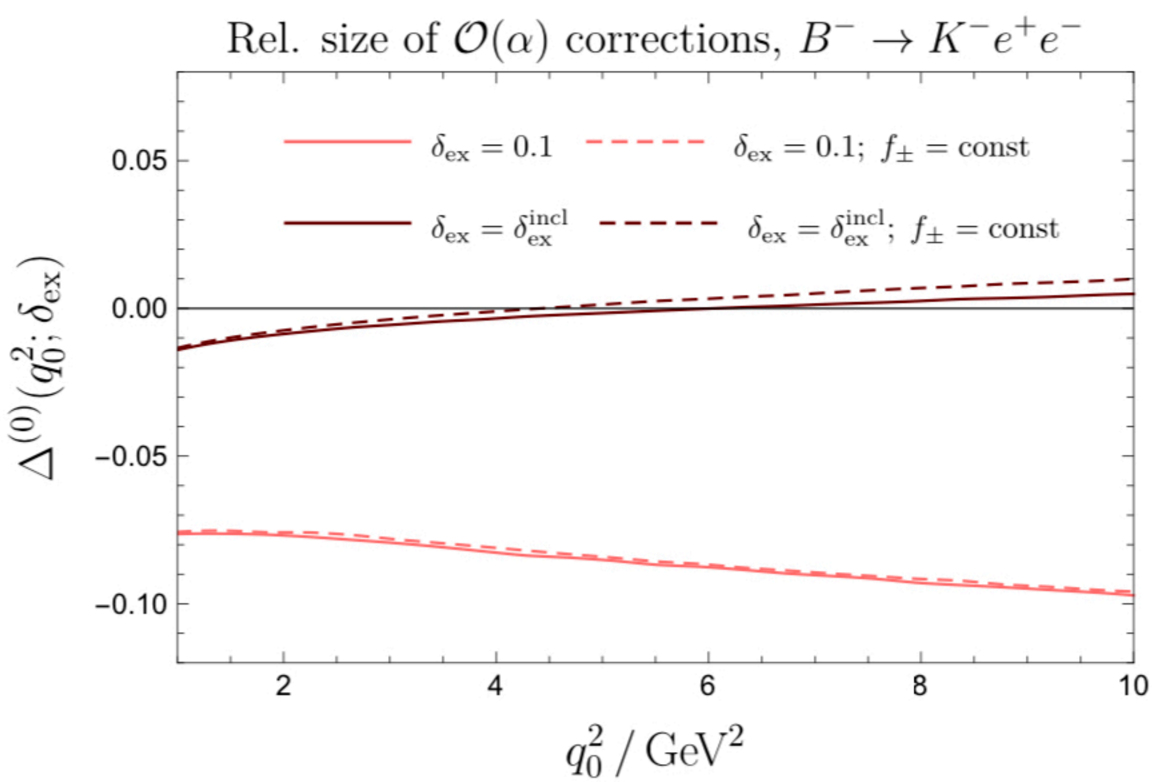
$$A_0 = A_0^{\rm INZ} = A_0^{\rm BIP} = -4$$

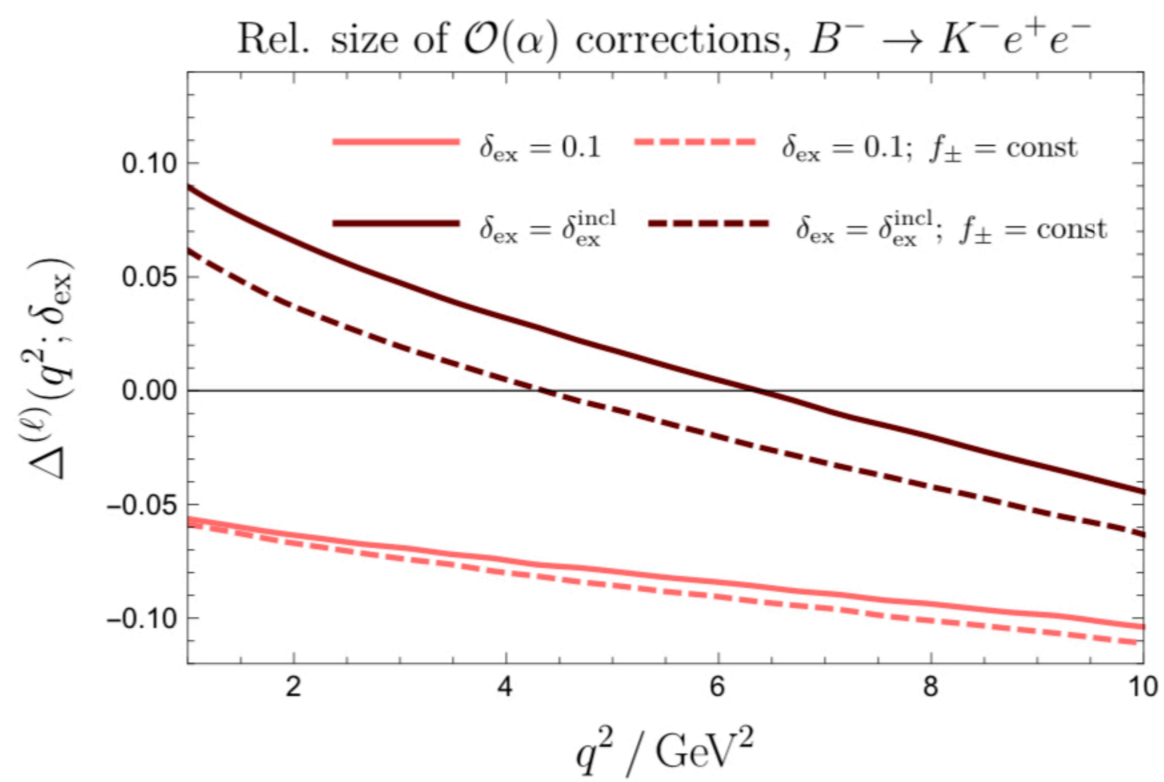
$m_K \to 0$ limit

$$C_0^{\rm INZ} = -\frac{19}{3} + 8\frac{\hat{q}_0^2}{(1-\hat{q}_0^2)^2} + 4\frac{(3-\hat{q}_0^2)}{(1-\hat{q}_0^2)^3}\hat{q}_0^4 \ln\hat{q}_0^2 + \mathcal{O}(\delta_{\rm ex}) \xrightarrow{\hat{q}_0^2 \to 1} -3 + \mathcal{O}(\delta_{\rm ex}) + \mathcal{O}(\hat{q}_0^2 - 1)$$

$$C_0^{\rm BIP} = -3 + \mathcal{O}(\delta_{\rm ex})\,. \qquad \text{(A.13)}$$

Constant versus non-constant form factor & distortion of q^2-spectrum





$$(q_0^2)_{\rm max} = q^2 + \delta_{\rm ex} m_B \left(E_{q_0}^{(1)} + |\vec{q}_0^{\,(1)}| \cos\theta_\gamma^{(1)}\right) .$$

$$(q_0^2)_{\rm max} = \begin{cases} q^2 + \delta_{\rm ex} q_0^2 & \cos\theta_\gamma^{(1)} = -1 \quad \text{tight-angle cut} \\ q^2 + \delta_{\rm ex} m_B^2 & \cos\theta_\gamma^{(1)} = +1 \quad \text{max-angle} \end{cases}$$

Additional Literature on QED corrections

2-body decays

- $B_s \to \mu\mu$ in SCET Beneke, Bobeth, Szafrom'17'19
  Power enhanced effects $\frac{m_\ell}{\Lambda} \to \frac{m_b}{\Lambda}$ chirality suppression in different disguise

  Factorisation in QED considerably more complicated than in QCD
  Process dependent distribution amplitudes

- $B \to K\pi$ in SCET Beneke, Boer, Toelstede, Vos '20
  Even more distribution amplitudes discussed

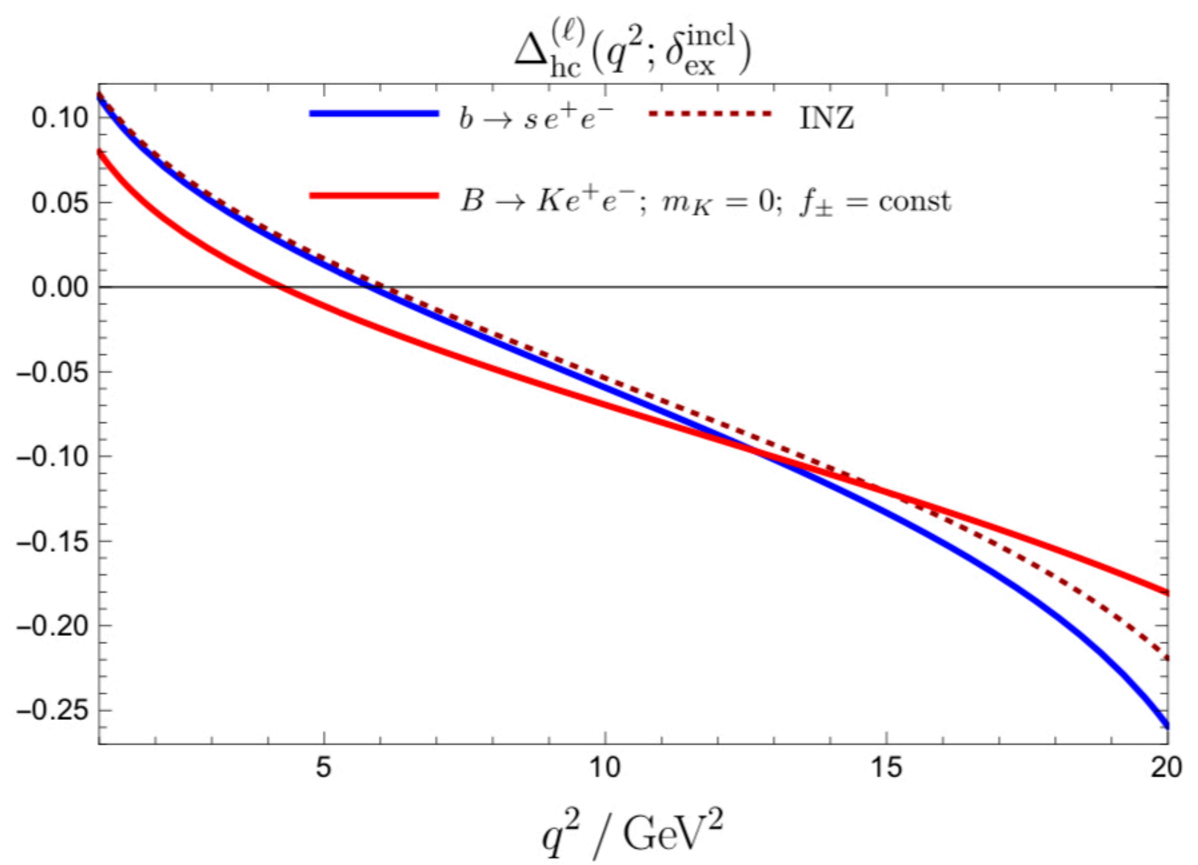
3-body decays

- $K \to \pi\ell\ell$ in "scalar QED" , Kubis, Schmidt '10
  Became aware only after publication

- $B \to D\ell\nu$ in "scalar QED" with constant form factor
  Ginsparg '67 (no approximation), de Boer, Kitahara, Nisandzic '18 (soft photon)

- $K \to \pi\ell\nu$ in ChiPT ,Cirigliano, Knecht, Neufeld, Rugensberger,Talavera '01
  structure-dependence, also derivative expansion but ChiPT as EFT (more mature)

- $b \to s\ell^+\ell^-$(inclusive) ,Huber, Lunghi, Misiak, Wyler'05
  Dedicated comparison in paper in appendix.
  Beautiful universality of coll-logs emerging depending on the spin encoded in splitting functions for the leptons



$$\tilde{\Delta}_{\rm hc}^{(\ell)}(\hat{q}^2) = \frac{1}{\Gamma^{\rm LO}}\left(\int_{\hat{q}^2}^{1}\frac{dz}{z}P_{f\to f\gamma}(z)\frac{d\Gamma^{\rm LO}(\hat{q}^2/z)}{d\hat{q}^2/z}\right)\ln\frac{\Lambda_b}{m_\ell}$$

$$P_{f\to f\gamma}(z) = \frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)$$

$$\frac{1}{\Gamma^{\rm LO}}\frac{d\Gamma^{\rm LO}(\hat{q}^2)}{d\hat{q}^2} = \begin{cases} 2(1-\hat{q}^2)^2(2\hat{q}^2+1) & b\to s\ell^+\ell^- \\ 4(1-\hat{q}^2)^3 & \bar{B}\to\bar{K}\ell^+\ell^- \end{cases}$$

- $b \to s\ell^+\ell^-$(inclusive) ,Huber, Hurth, Lunghi,'15
  Includes angular analysis (not looked at closely)

- K_{l2}, K_{l3} (lattice QCD & current insertion) Sachrajda, Martinelli, …'15'& later
  challenges with massless photon and exponentially growing states
  And other lattice groups: Portelli et al , Patella et al (C* boundary conditions)