Soft photon QED effects to the ratio of CKM elements

(Based on : A. Bansal, N. Mahajan, D.M., JHEP 03 (2022) 130)

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Introduction

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$$|V_{ub}|_{incl} = 4.40 \times 10^{-3}$$

$$\left|V_{ub}\right|_{excl} = 3.72 \times 10^{-3},$$

$\sim 3.5\sigma \ldots ?$

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[1] pdg.lbl.gov

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New Physics or Theoretical uncertainties ...?

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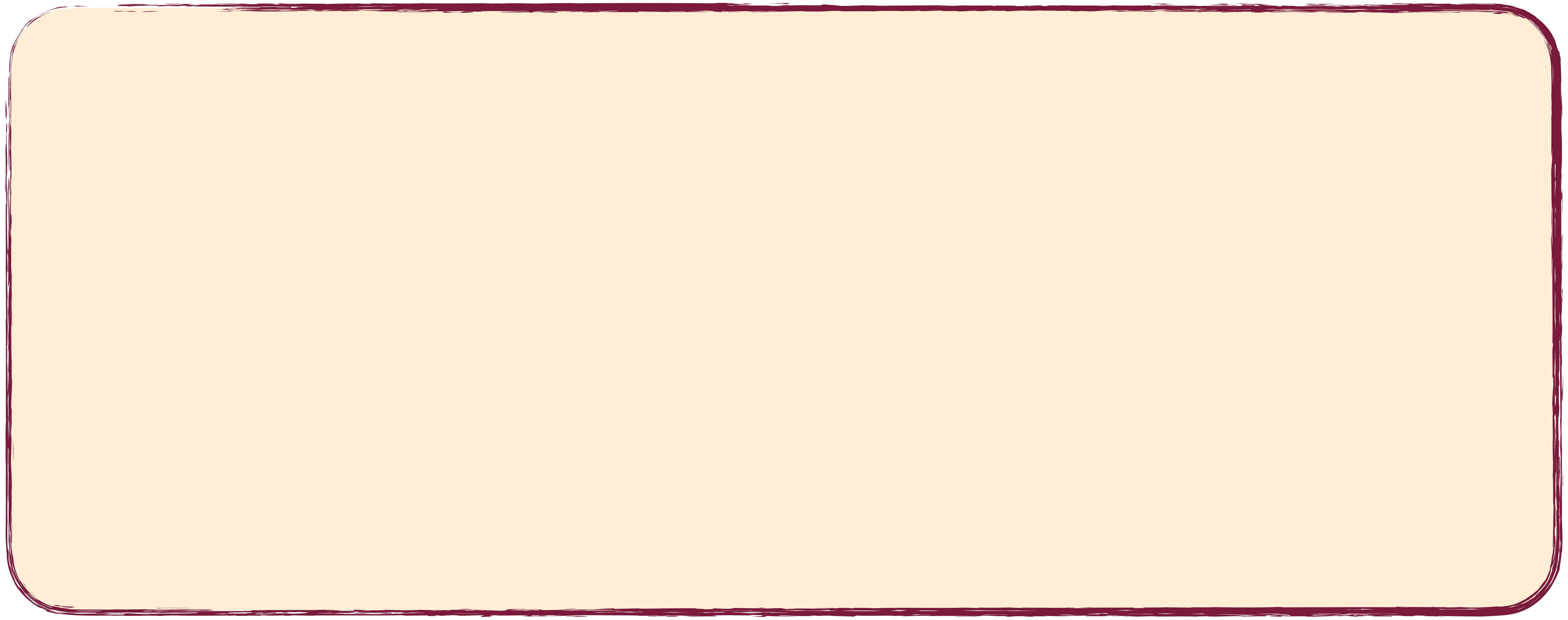

Origin of Theoretical uncertainties

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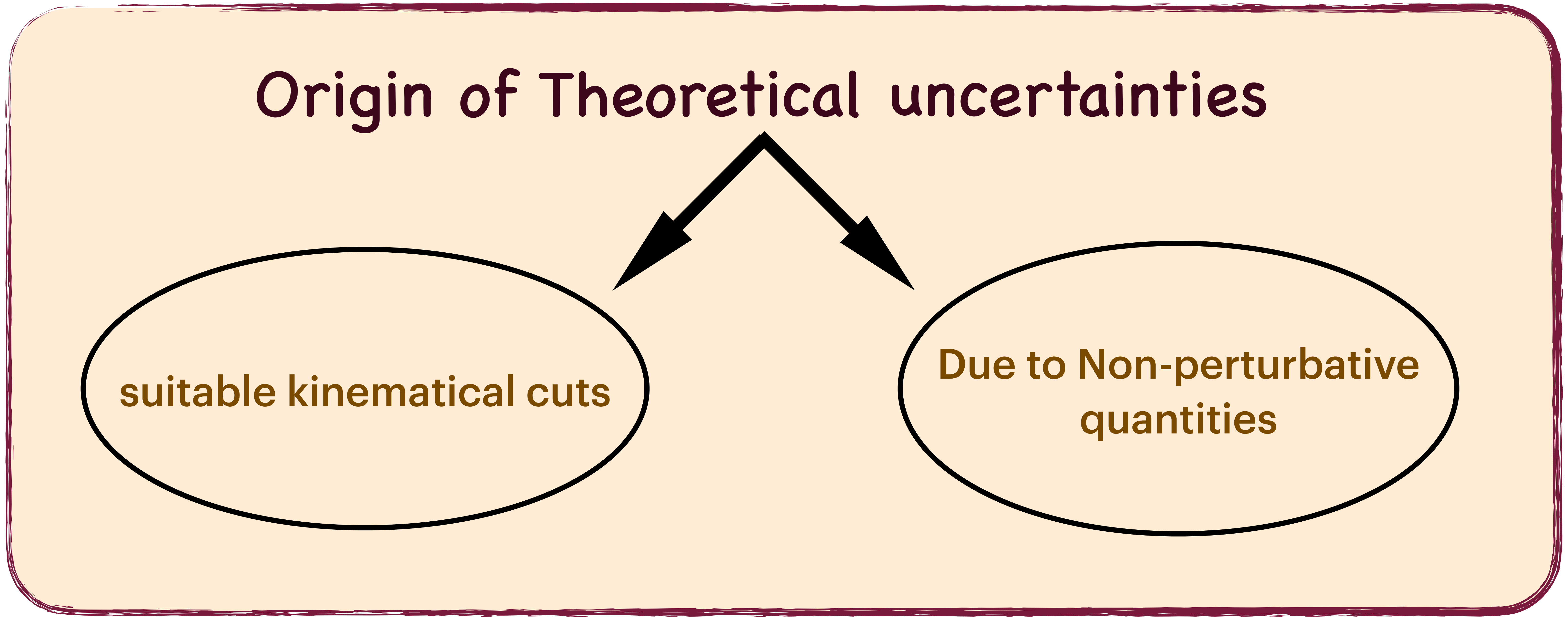
suitable kinematical cuts


Origin of Theoretical uncertainties
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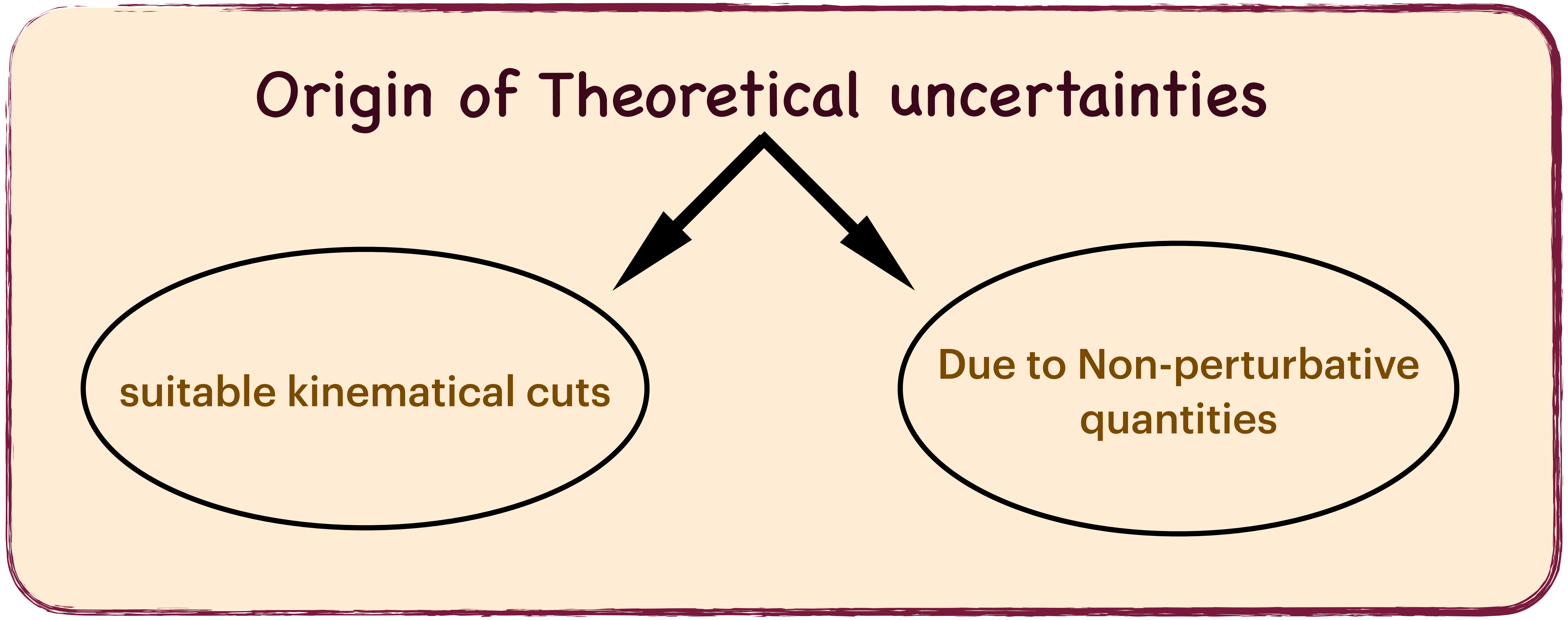
Due to Non-perturbative quantities



Possible Solution:
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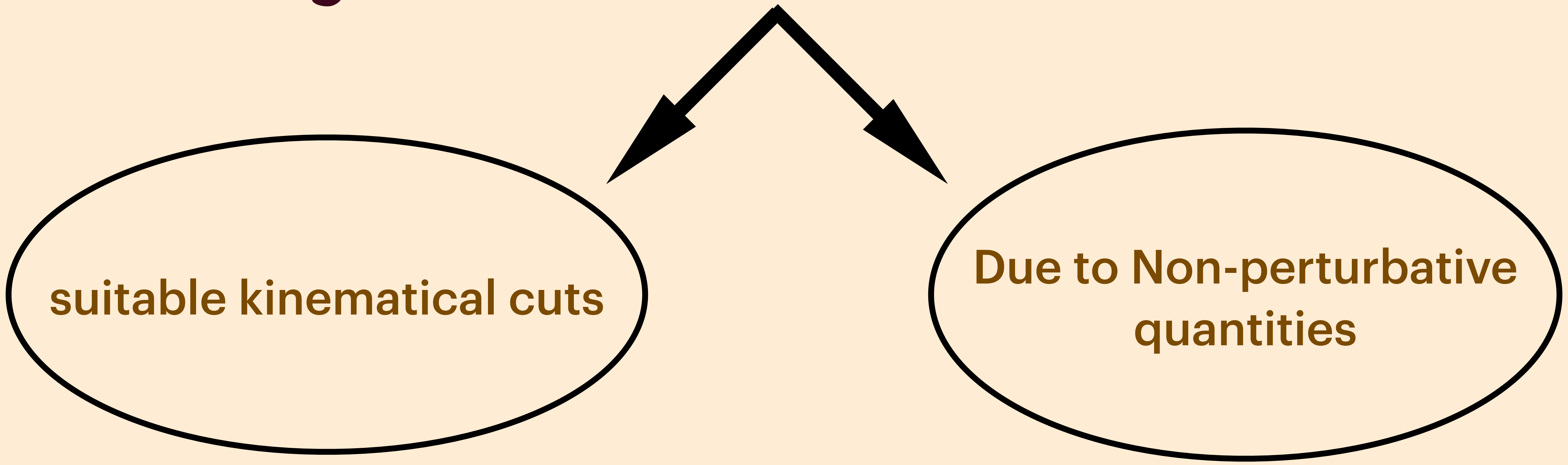

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Proposed LFU ratios R_K or R_D : less sensitive to hadronic uncertainties but Soft photon QED corrections[2,3]...?

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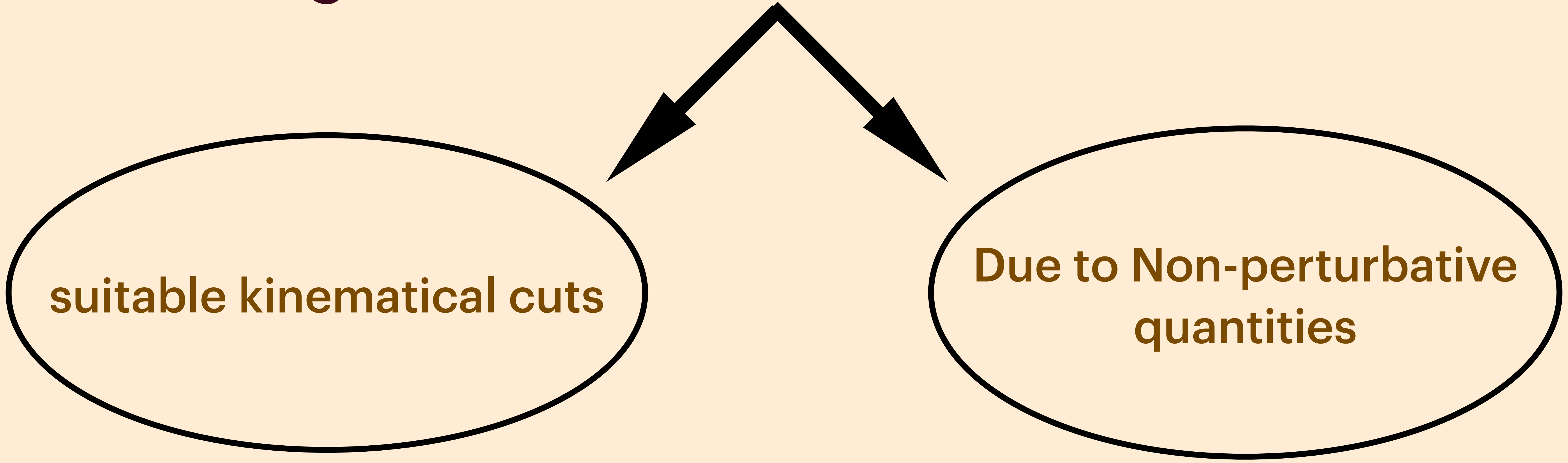

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$$R_K^{\mu e} \equiv \frac{\int dq^2 \frac{d\Gamma(B \to K \mu^+ \mu^-)}{dq^2}}{\int dq^2 \frac{d\Gamma(B \to K e^+ e^-)}{dq^2}}$$

$$R_D \equiv \frac{\int dq^2 \frac{d\Gamma(B \to D \tau \nu_\tau)}{dq^2}}{\int dq^2 \frac{d\Gamma(B \to D \mu \nu_\mu)}{dq^2}}$$

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constructed

$$R_V = \frac{|V_{ub}|}{|V_{cb}|}$$

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- Experimental (LHCb) results :

1. $\Lambda_b^0 \to p\mu^-\bar{\nu}_\mu$ and $\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu}_\mu$ modes : (Aaij et.al., 2015)

$$R_V\,(\text{high } q^2) = 0.083 \pm 0.004$$

2. $B_s^0 \to K^-\mu^+\nu_\mu$ and $B_s^0 \to D_s^-\mu^+\nu_\mu$ modes: (Aaij et.al., 2021)

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- R_V **as constructed using measurements of** $|V_{ub}|$ **and** $|V_{cb}|$ **obtained from PDG :**

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$\Longrightarrow$ **Motivation to study R_V**

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- Matrix element : $\mathscr{M}_0(B \to P\ell\bar{\nu}_\ell) = \frac{G_F}{\sqrt{2}} V_{qb} \mathscr{H}_\mu(p_P, p_B) \mathscr{L}^\mu$

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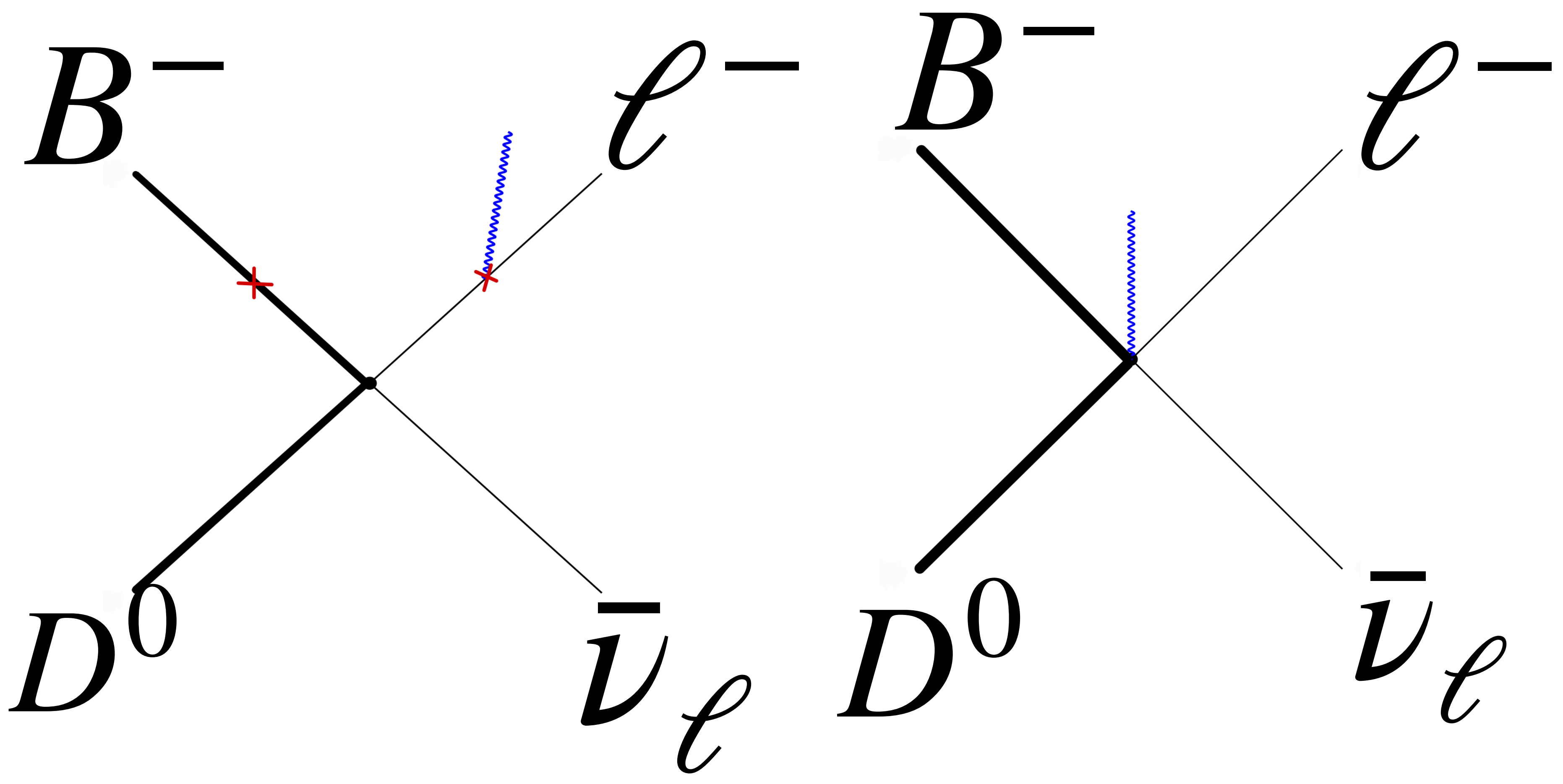

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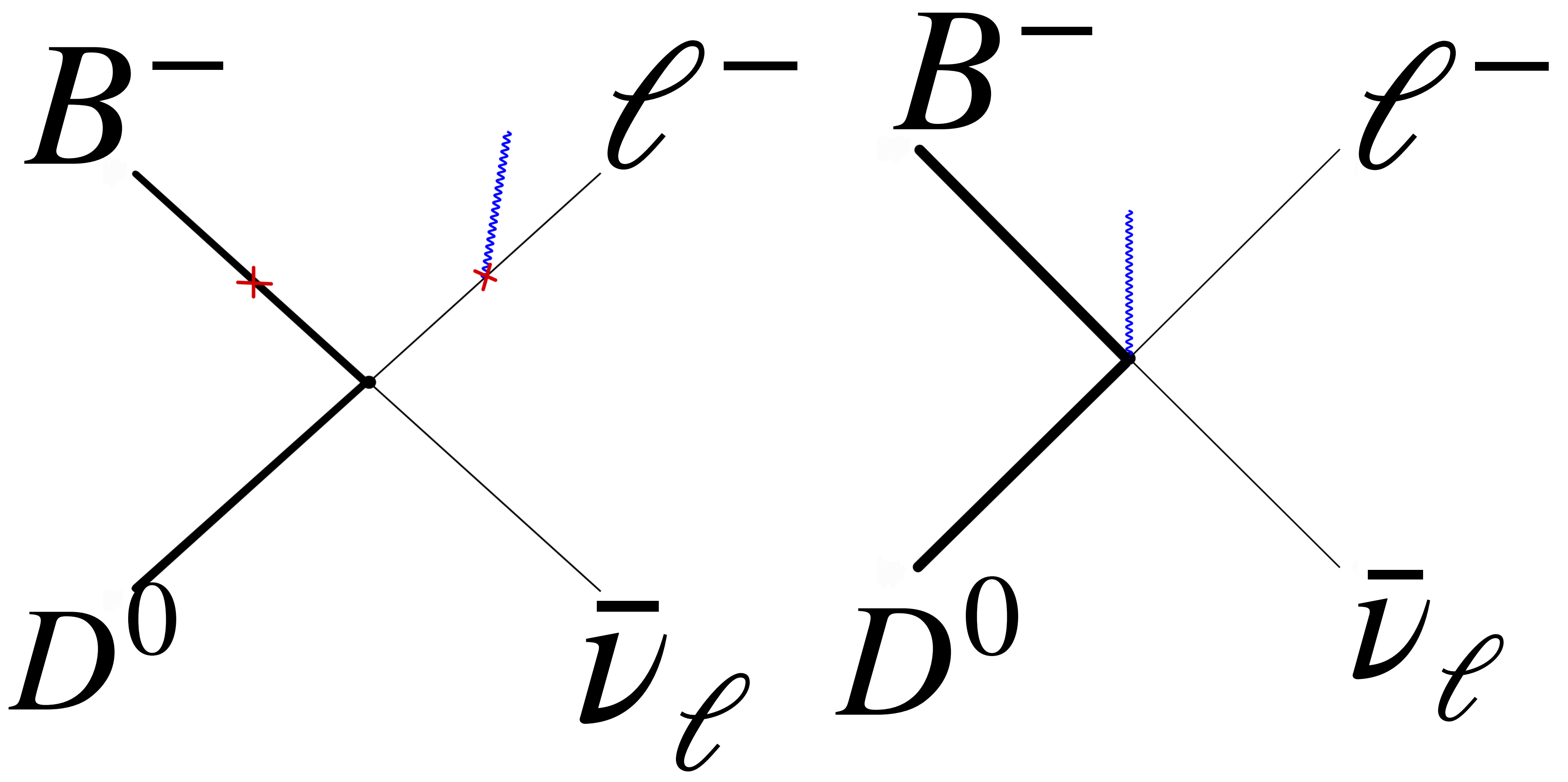
Real photon emission :

- Gauge invariant matrix element :

$$\mathscr{M} = e\epsilon_\alpha(k)\Big[\mathscr{M}_0\left(-\frac{p_B^\alpha}{p_B\,.\,k} + \frac{p_\ell^\alpha}{2p_\ell\,.\,k}\right) + \bar{u}(p_\ell)\frac{\gamma^\alpha\gamma_\mu k^\mu}{2p_B\,.\,k}\Gamma_\mu v(p_\nu)\mathscr{H}^\mu$$
$$-(f_+ - f_-)\bar{u}(p_\ell)\left(\frac{p_B^\alpha}{p_B\,.\,k}\gamma_\mu k^\mu - \gamma^\alpha\right)(1-\gamma^5)v(p_\nu)\Big]$$



- General decay width form for $B \to P\ell\bar{\nu}_\ell\gamma$:

$$\Gamma|_{B\to P\ell\nu_\ell\gamma} = \frac{1}{2m_B}\int\frac{d^3p_P}{(2\pi)^3 2E_P}\int\frac{d^3p_l}{(2\pi)^3 2E_l}\int\frac{d^3p_\nu}{(2\pi)^3 2E_\nu}\int\frac{d^3k}{(2\pi)^3 2E_k}(2\pi)^4\delta^4\left(Q-p_\nu-k\right)\left|\mathscr{M}\right|^2_{B\to P\ell\nu_\ell\gamma}$$

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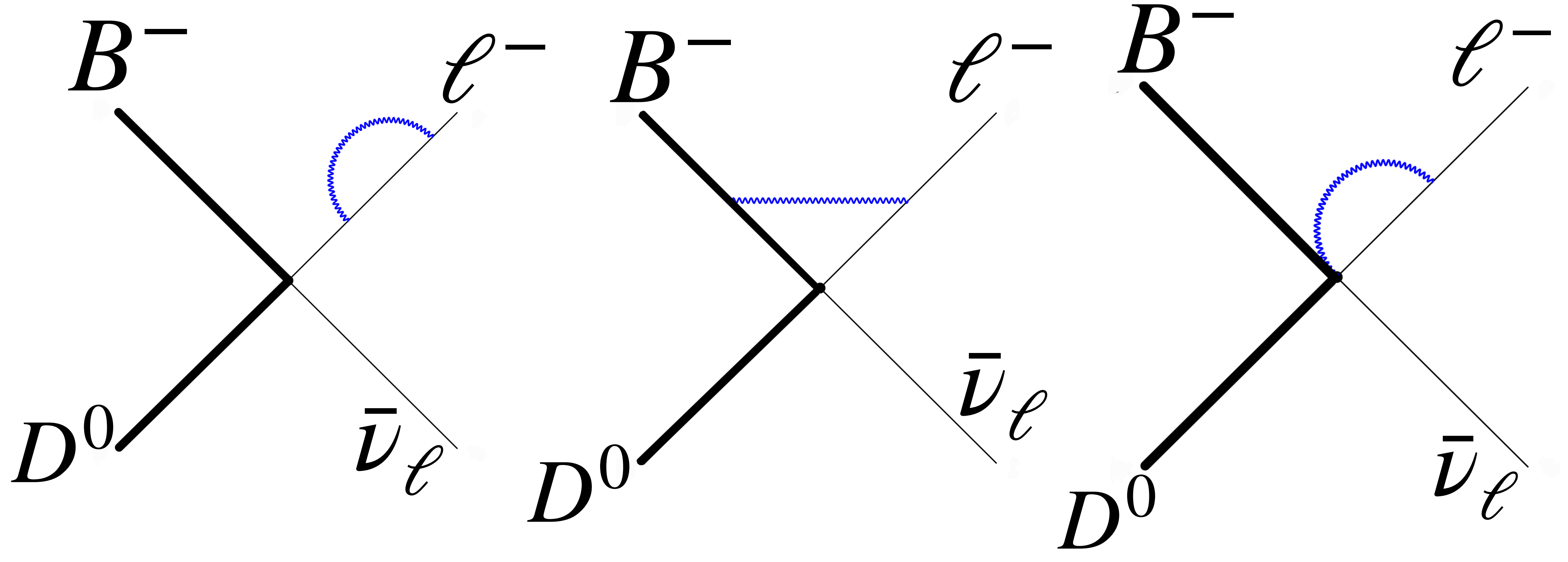
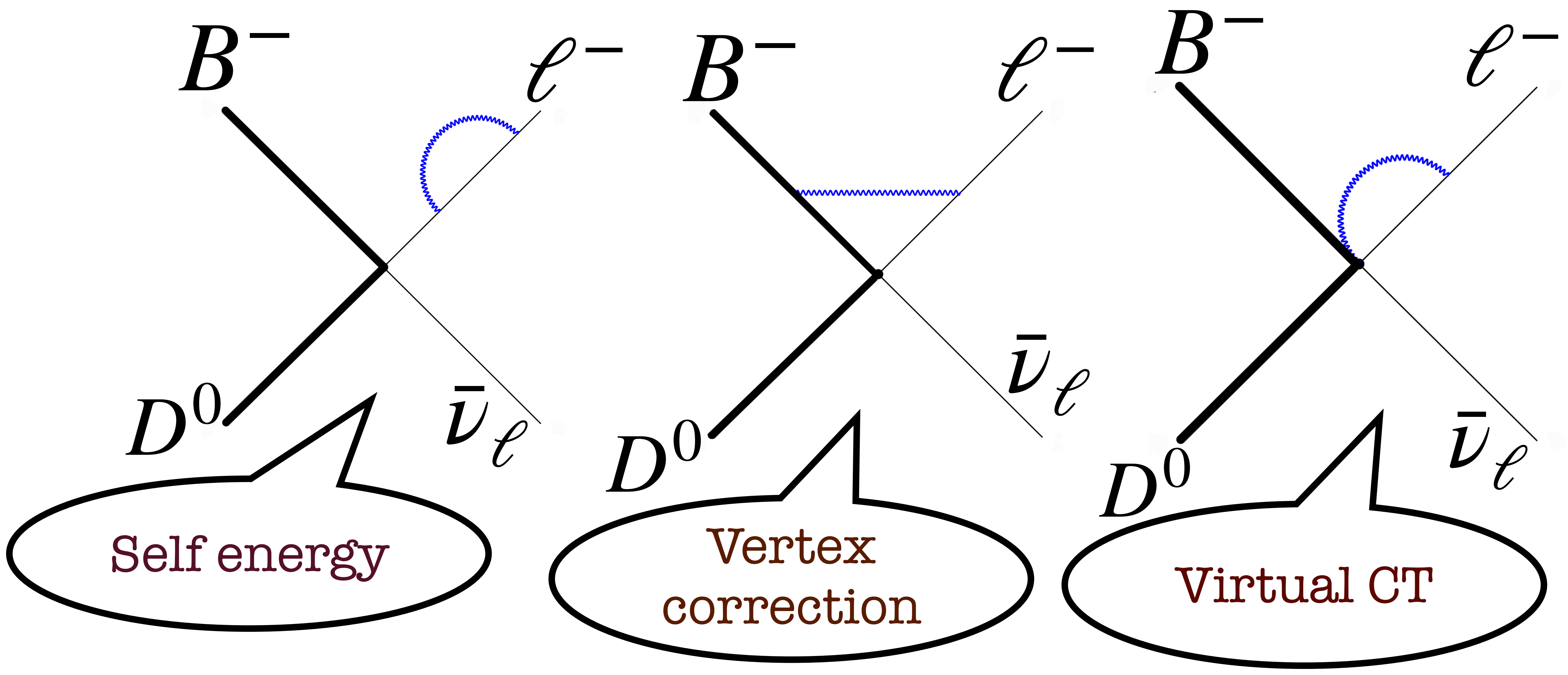
- Choosing the kinematical cut, total decay width can get rid of Collinear divergences


B^-
ℓ^-
D^0
$\bar{\nu}_\ell$
B^-
ℓ^-
$\bar{\nu}_\ell$
D^0
B^-
ℓ^-
D^0
$\bar{\nu}_\ell$

Virtual Correction:



Virtual Correction:

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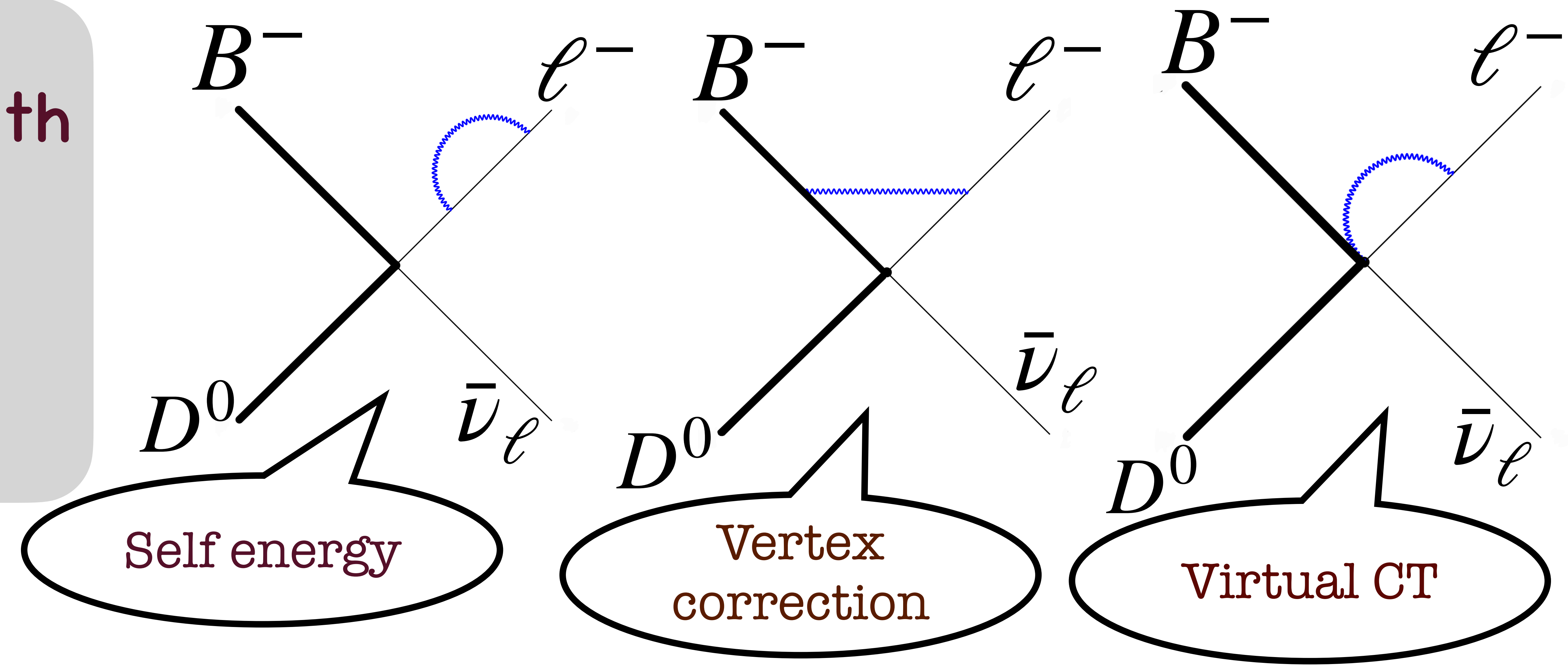
$$\frac{d^2\Gamma_{\text{vir}}}{dydz} = \frac{d^2\Gamma_0}{dydz}(2\alpha B) + \frac{d^2\Gamma'_{\text{vir}}}{dydz}$$



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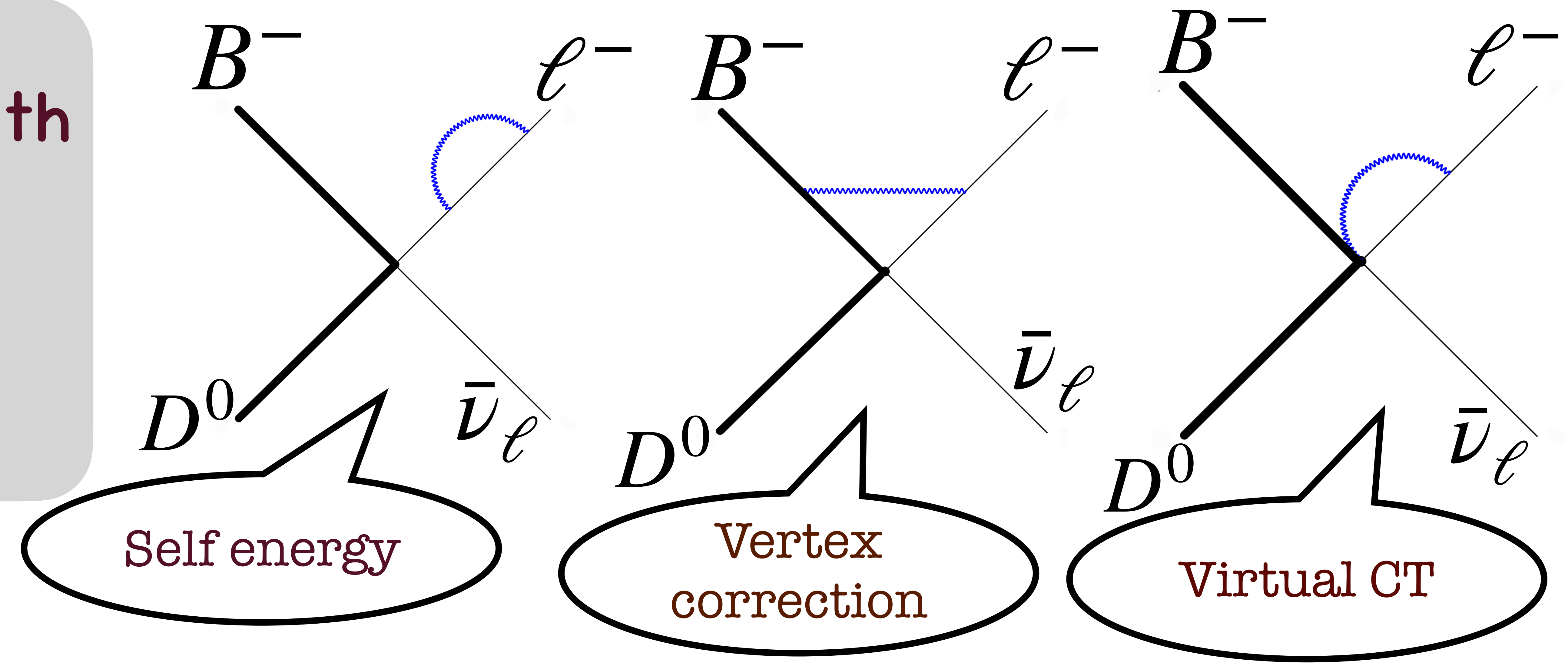

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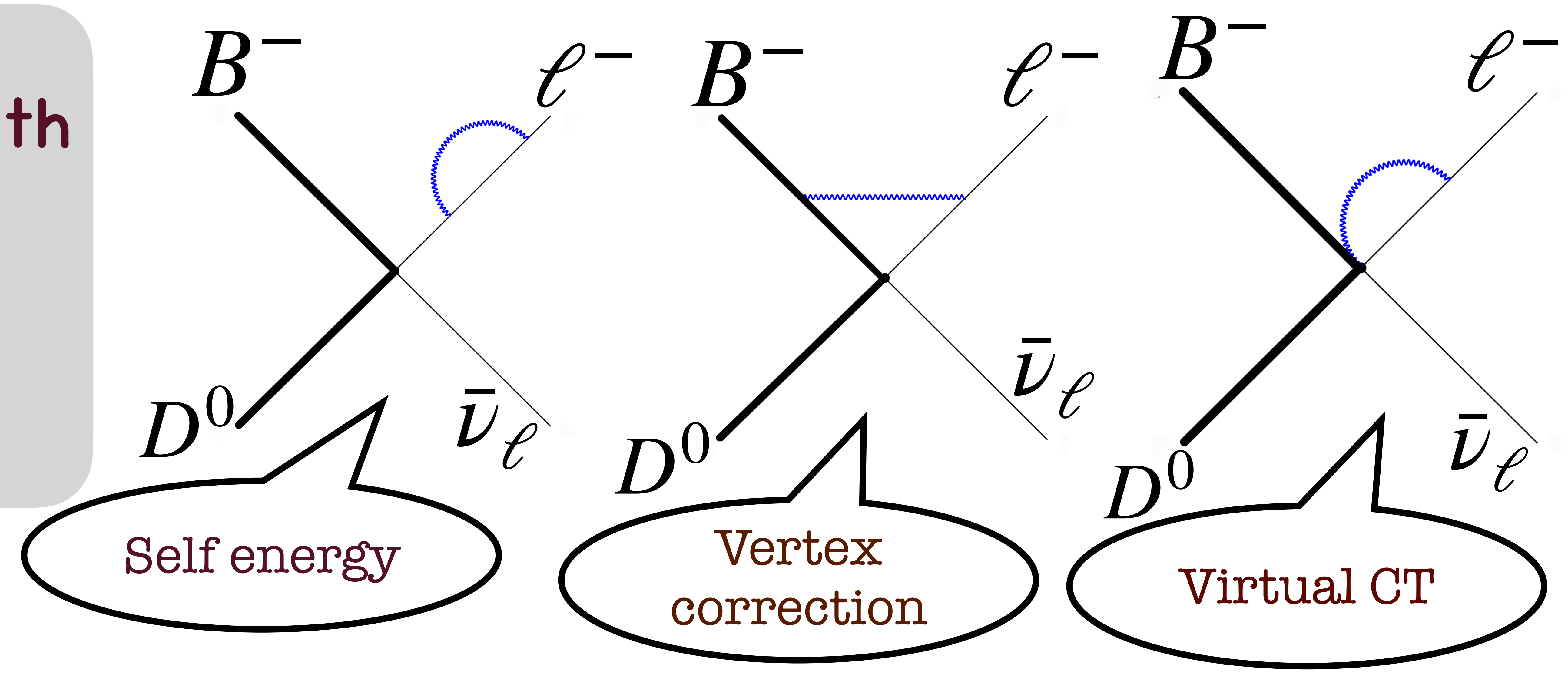

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2-point PV functions

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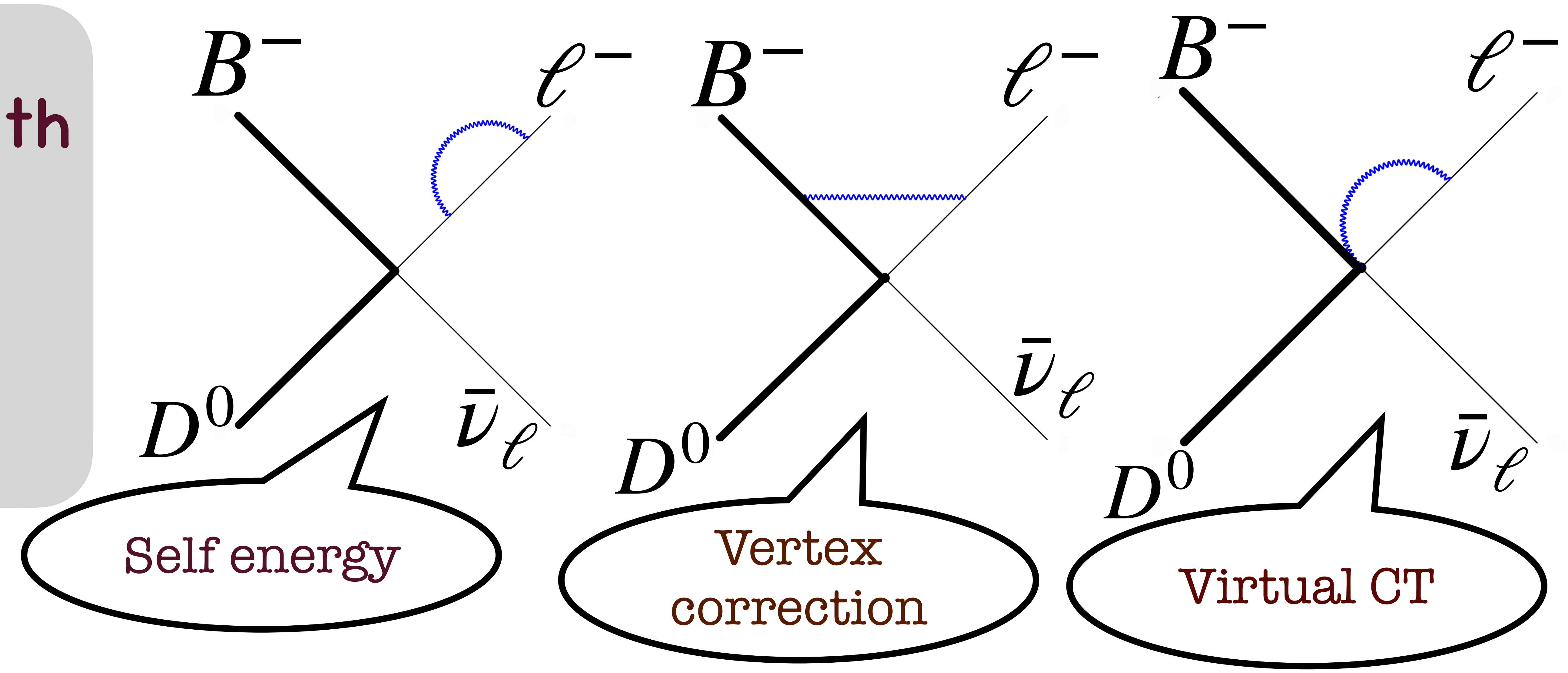

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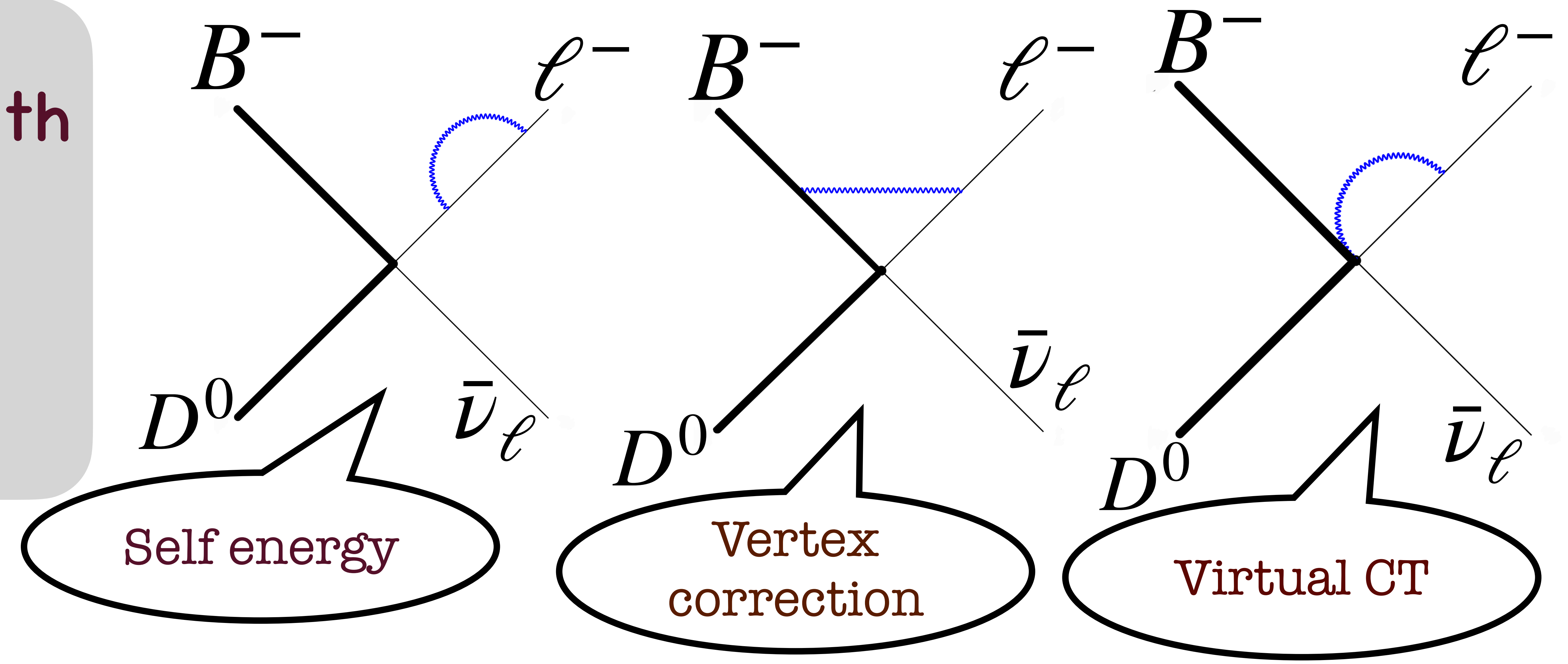
3-point PV functions

Virtual Correction:

- The second order differential decay width

$$\frac{d^2\Gamma_{\text{vir}}}{dydz} = \frac{d^2\Gamma_0}{dydz}(2\alpha B) + \frac{d^2\Gamma'_{\text{vir}}}{dydz}$$



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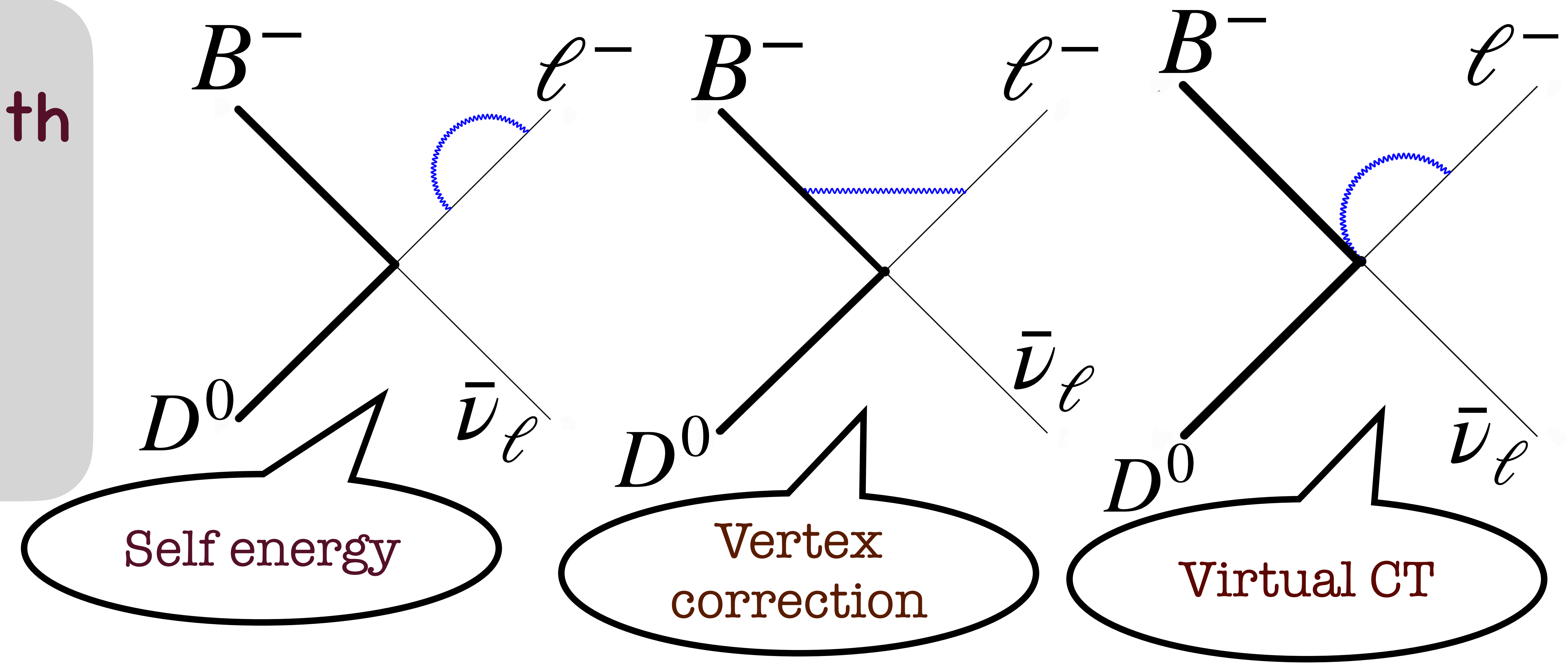

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QED correction factor

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Note: All quantities with '0' as superscript or subscript are non-radiative while without any superscript or subscript are $\mathcal{O}(\alpha)$ QED corrected quantity.

Results

$B^0 \to P^+(=D,\pi)\mu^-\bar{\nu}_\mu$

$B^- \to P^0(=D,\pi)\mu^-\bar{\nu}_\mu$

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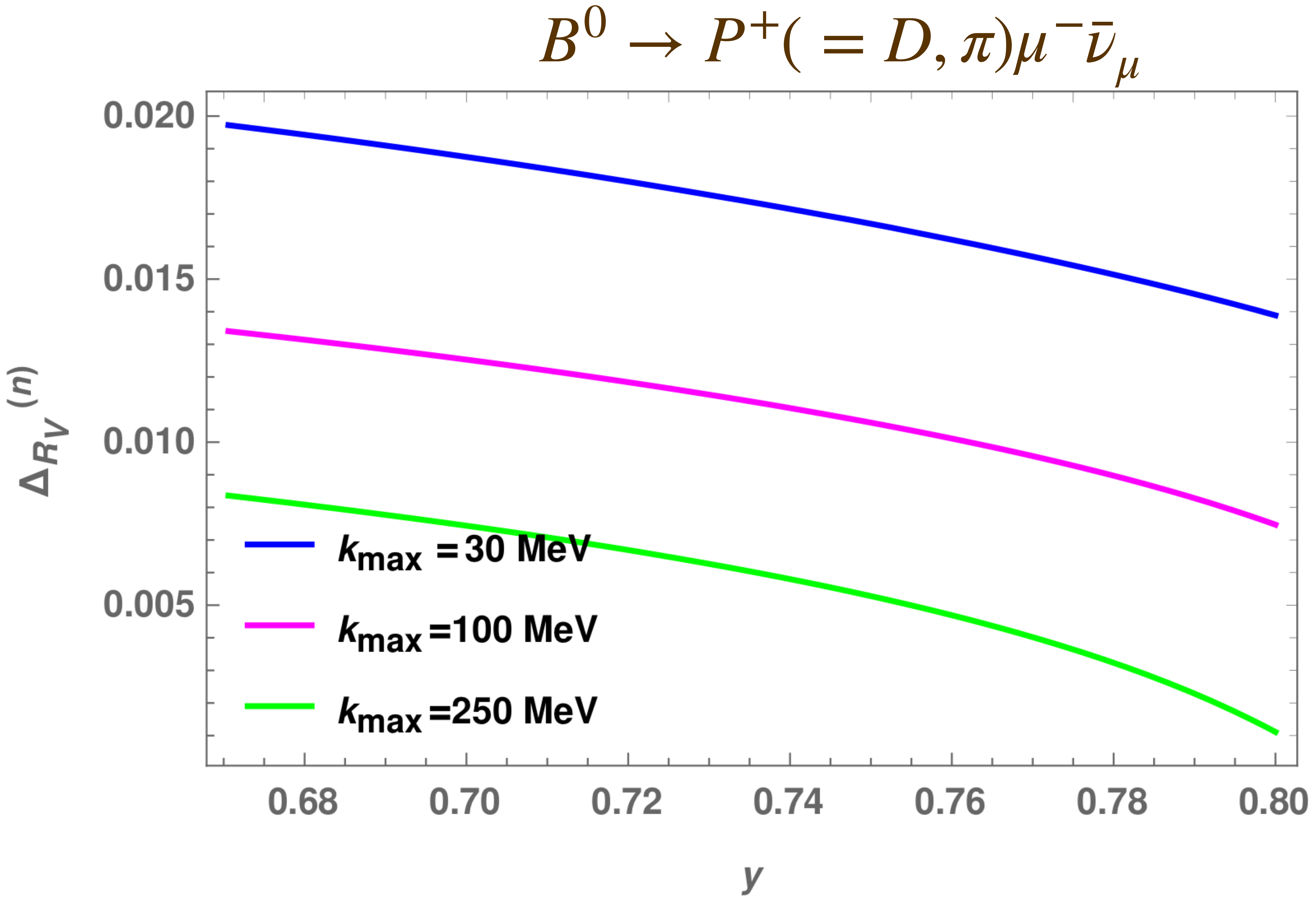

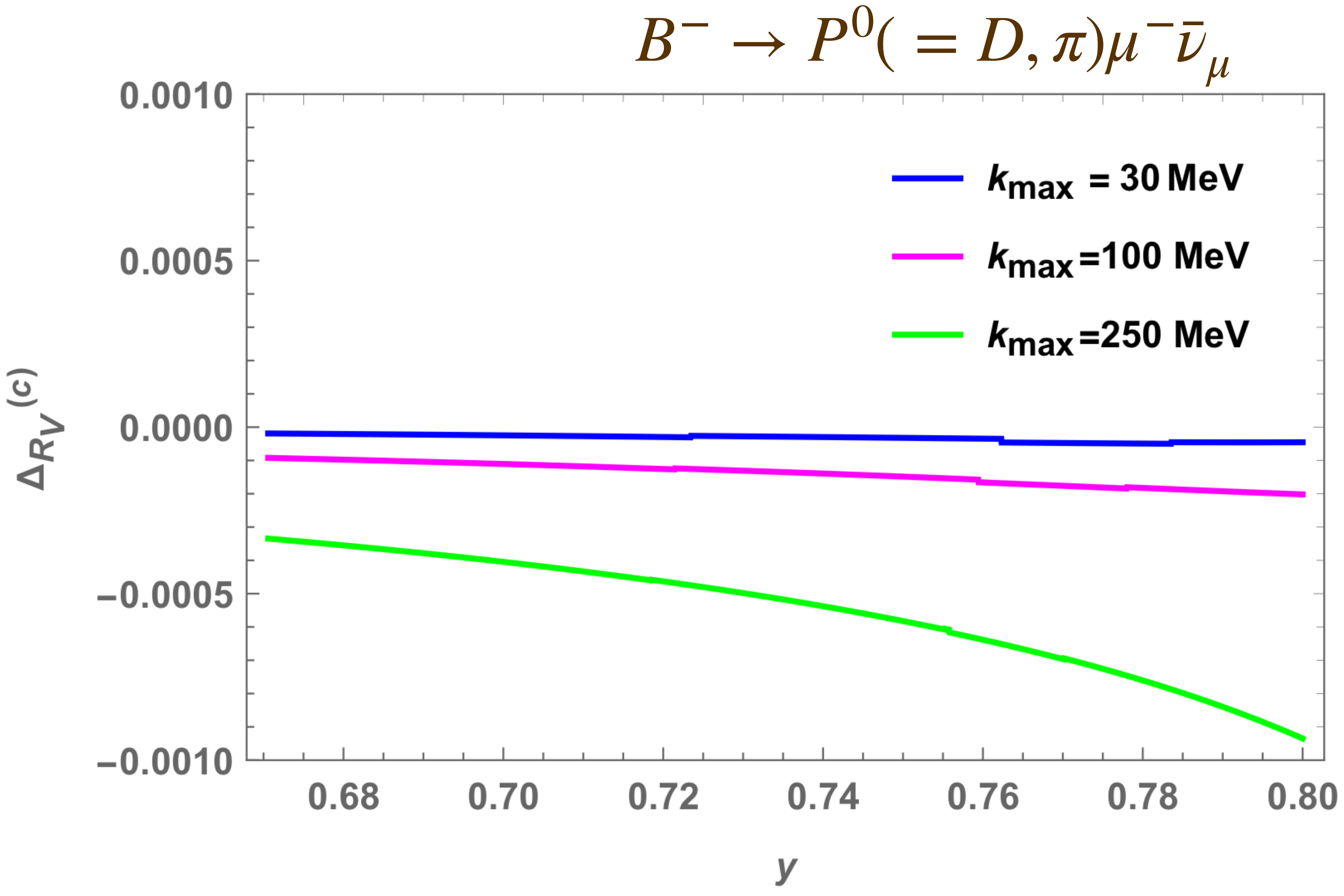

Radiative corrections to R_V for different thresholds on photon energy, k_{max} for (a) $B^0 \to P^+(=D,\pi)\mu^-\bar{\nu}_\mu$ and (b) $B^- \to P^0(=D,\pi)\mu^-\bar{\nu}_\mu$

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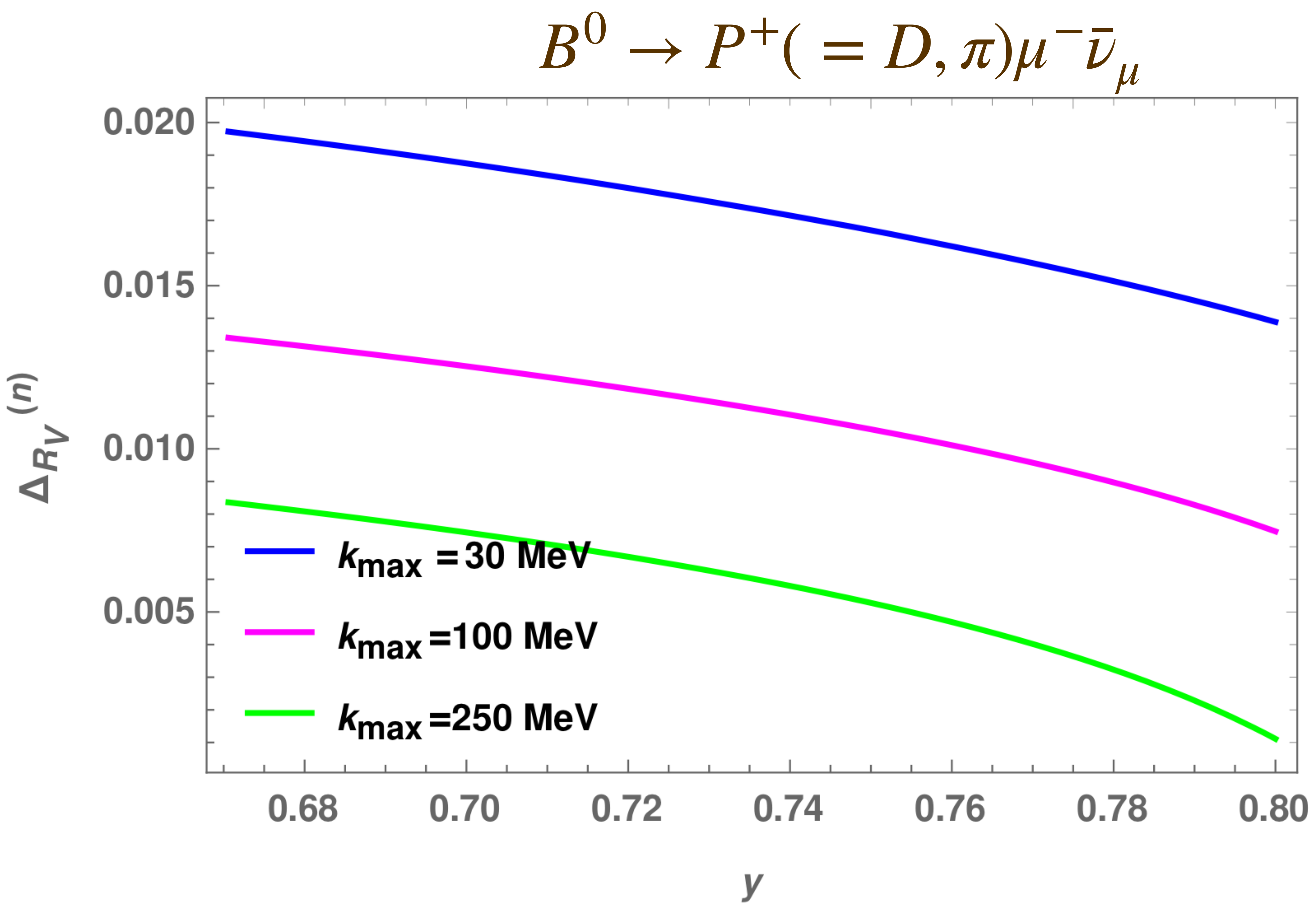

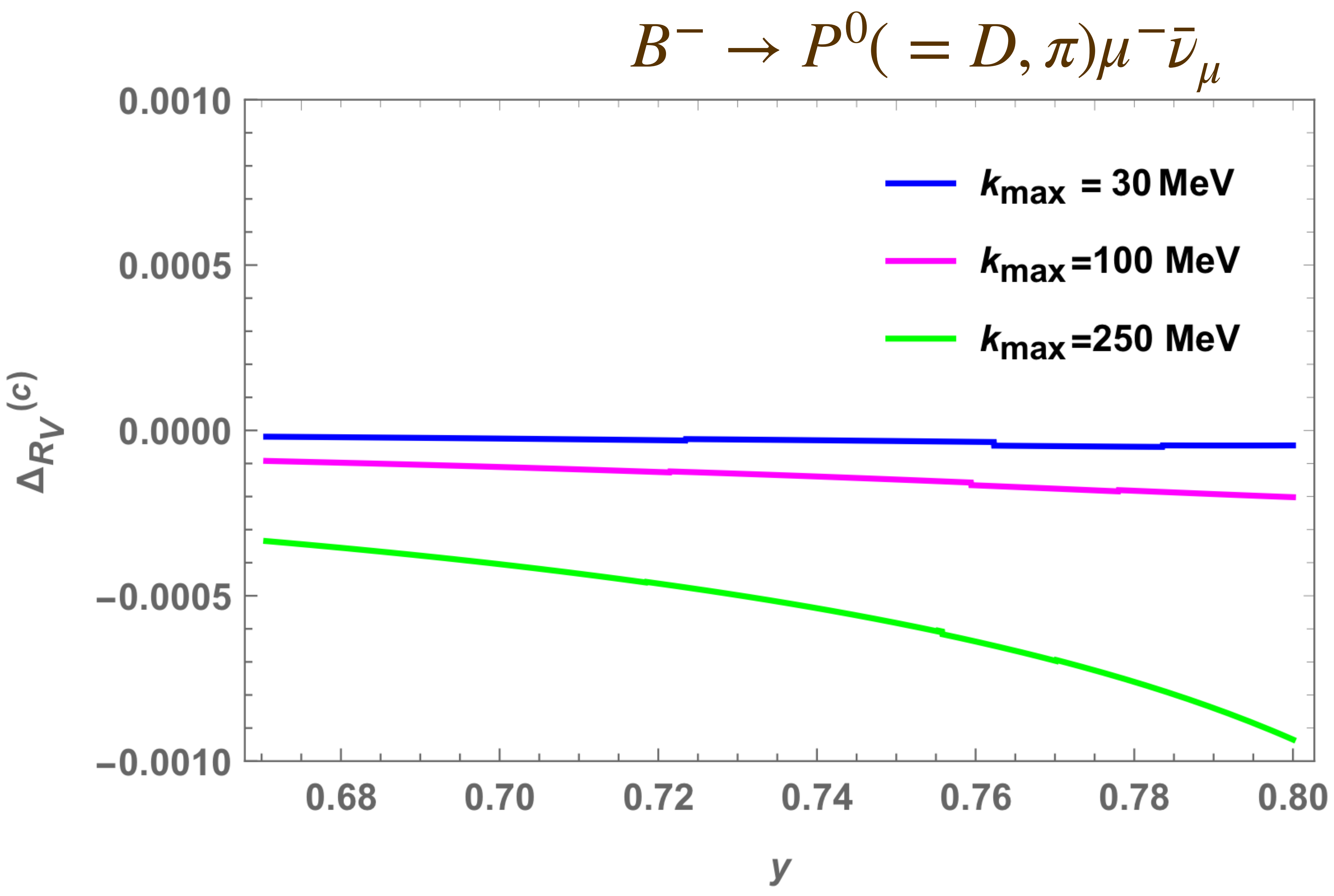

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- Neutral mode : very minute ($\sim \mathcal{O}(10^-3)$)

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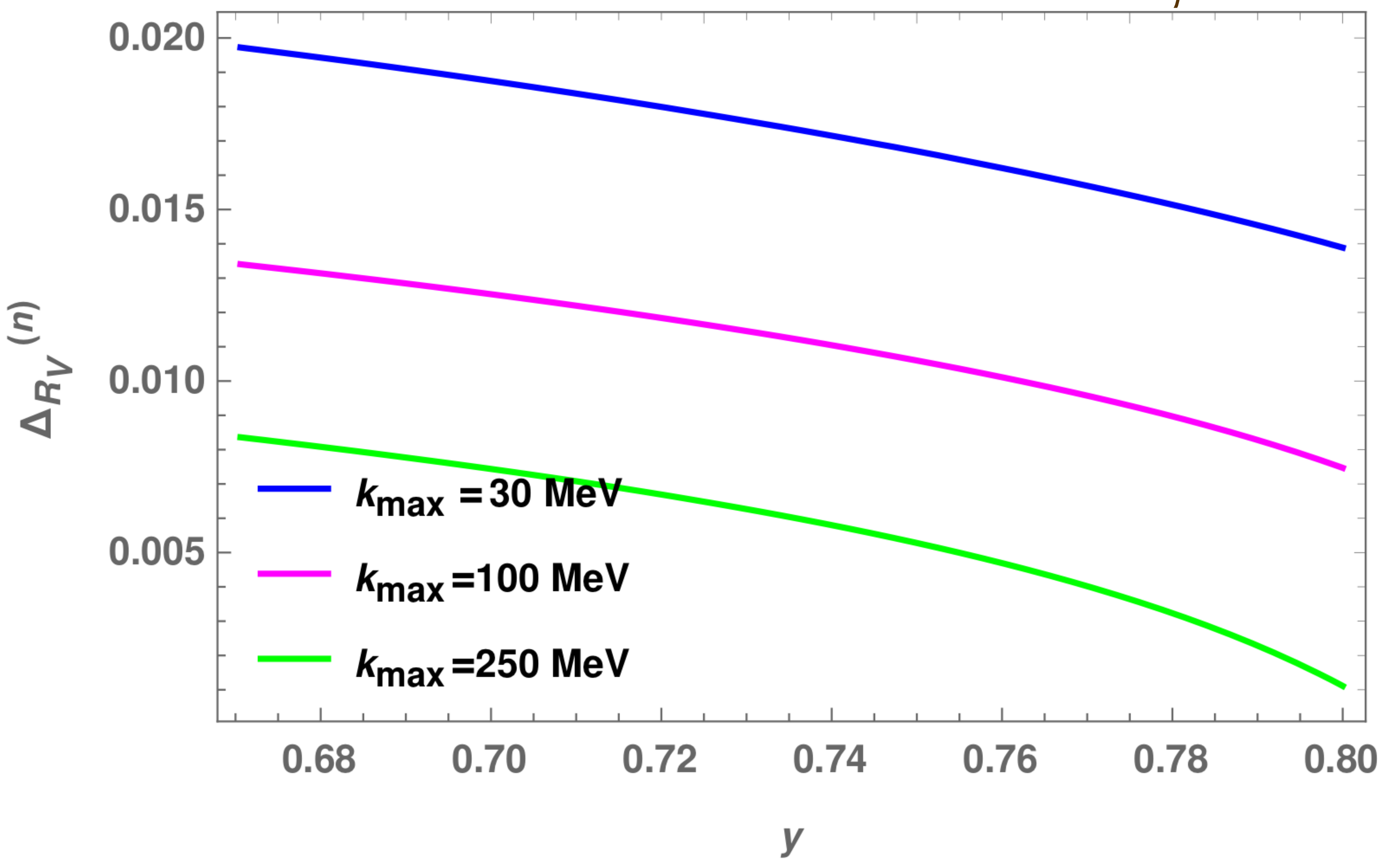

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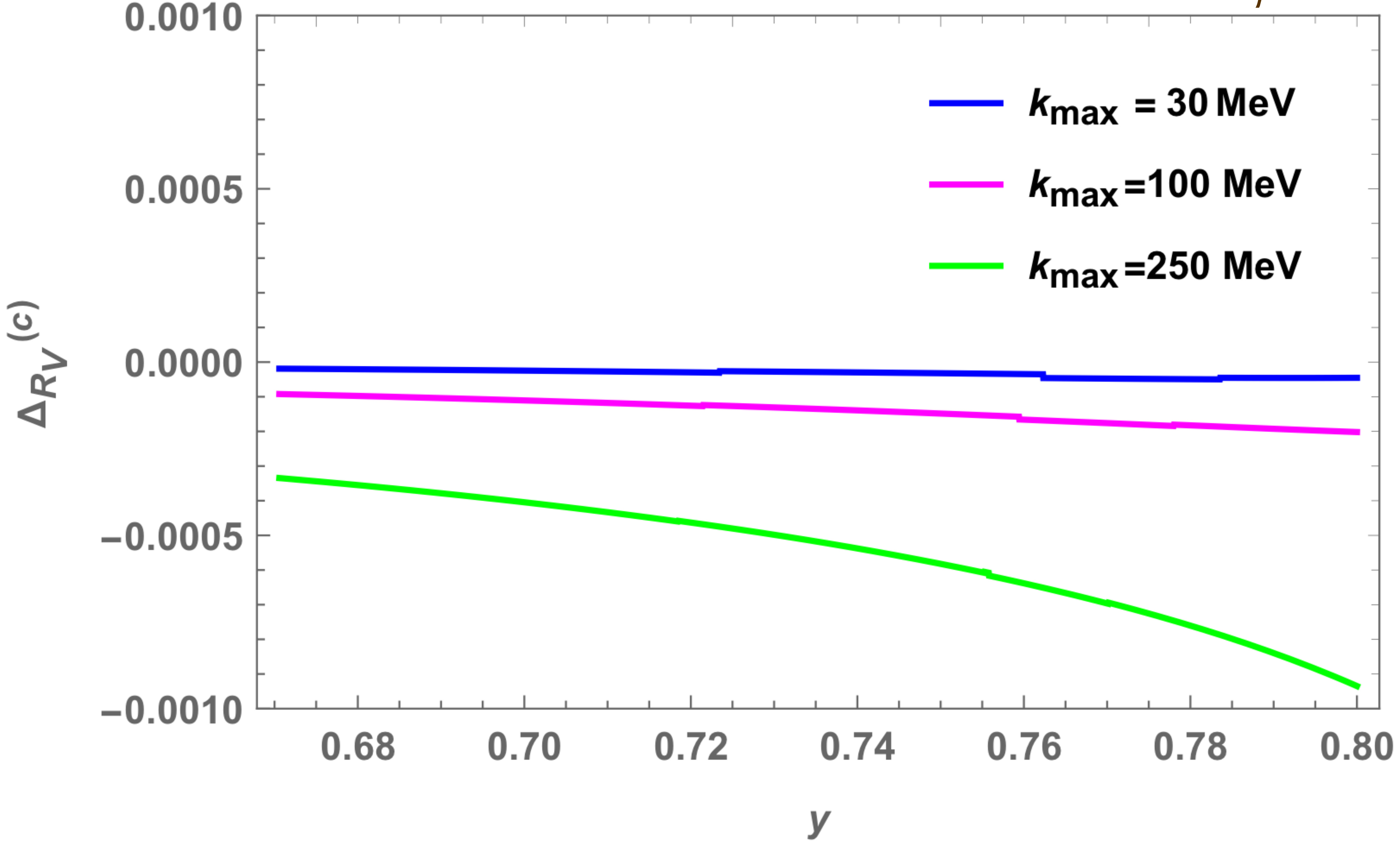

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Consequence of photon emission from D vs π mesons

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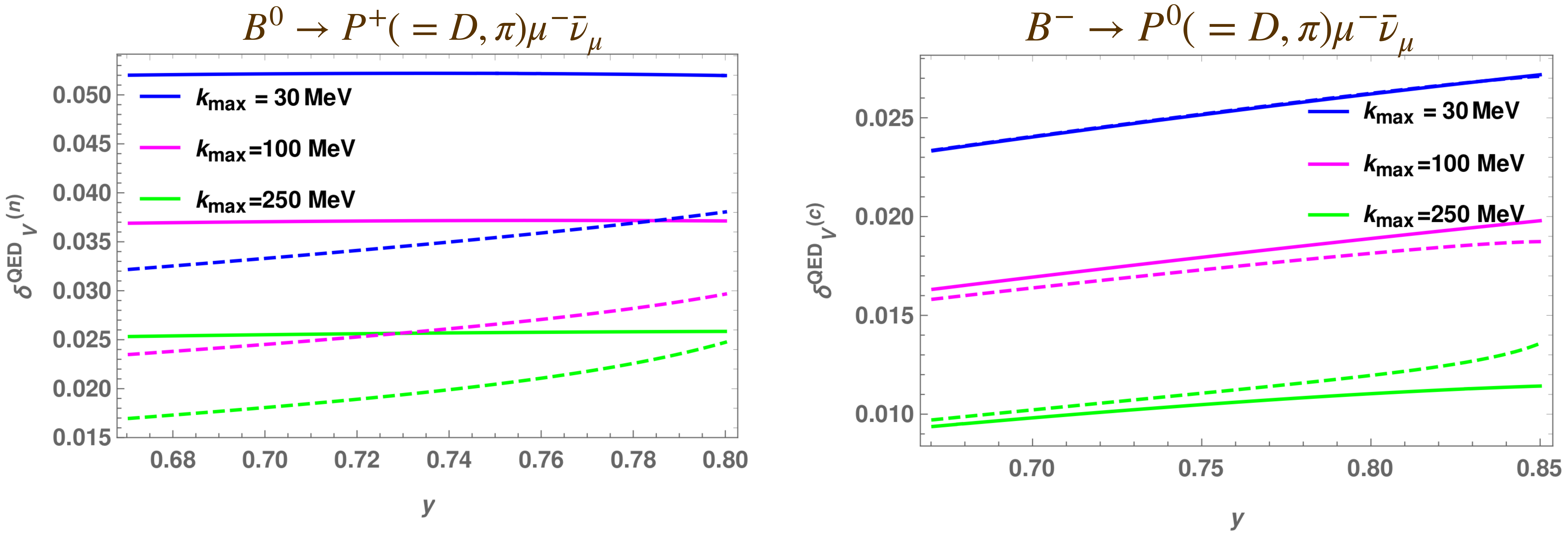
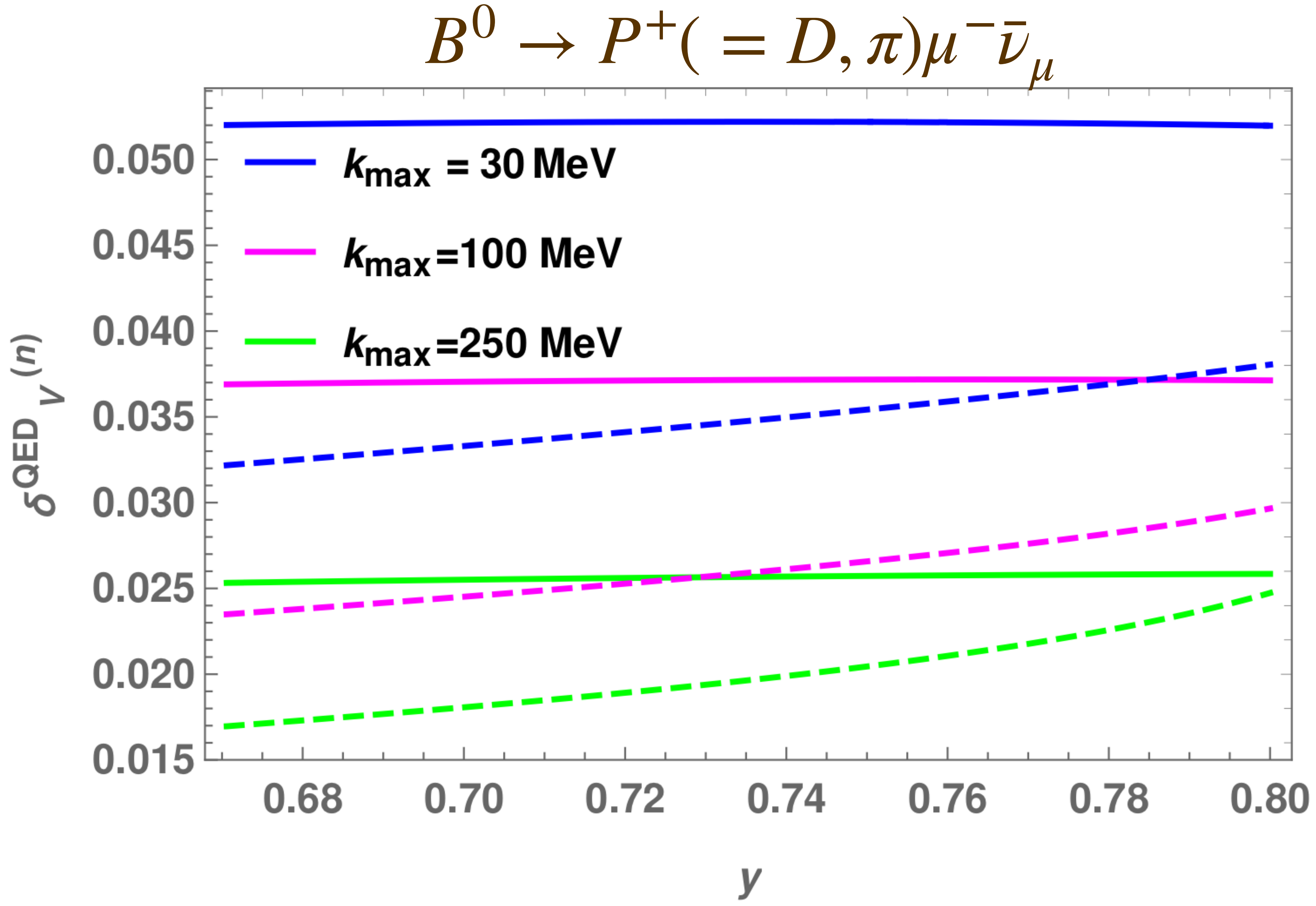
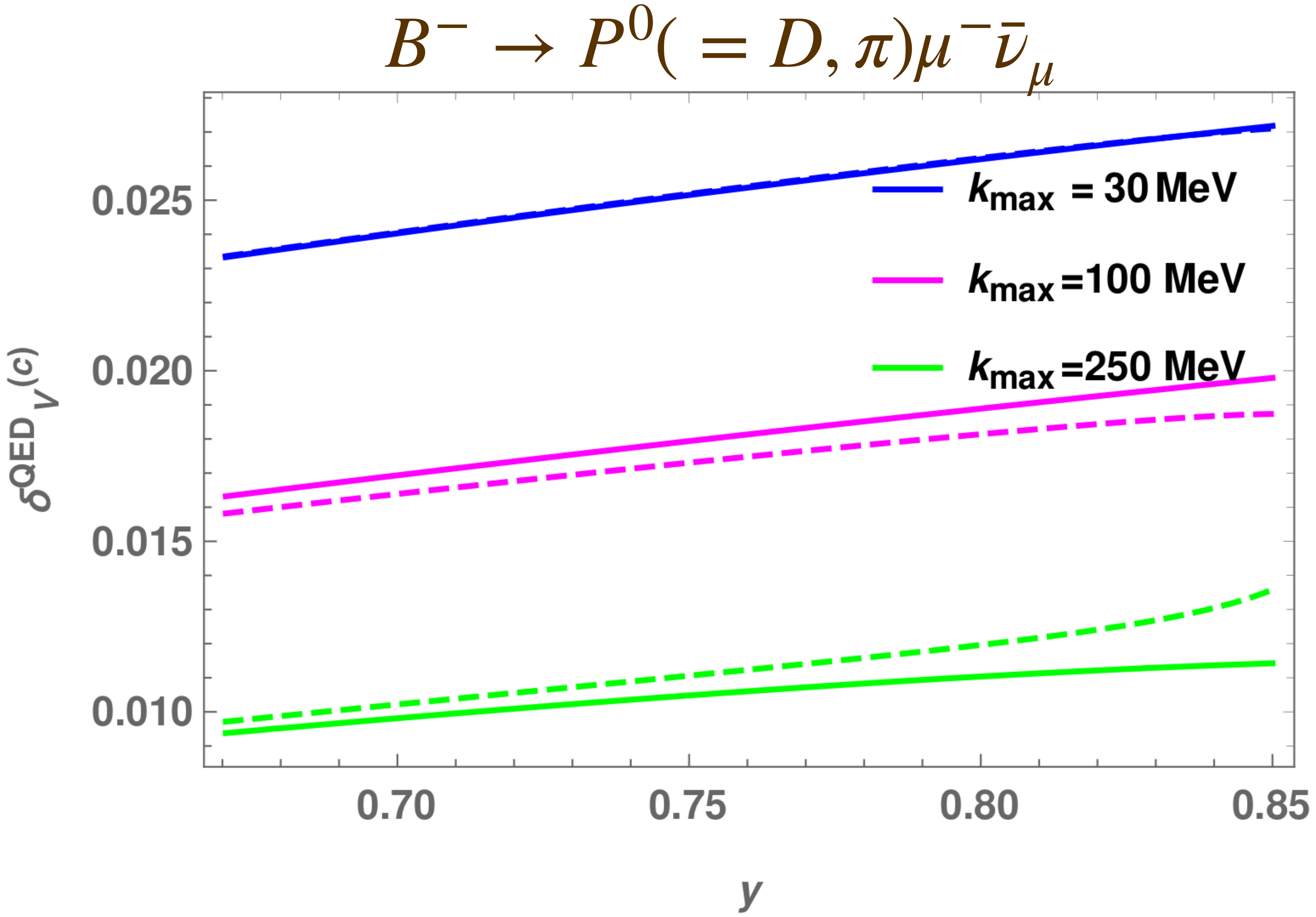


Radiative corrections to the CKM elements $|V_{cb}|$ (solid) and $|V_{ub}|$ (dashed) for different thresholds on photon energy, k_{max} for (a) $B^0 \to P^+(=D,\pi)\mu^-\bar{\nu}_\mu$ and (b) $B^- \to P^0(=D,\pi)\mu^-\bar{\nu}_\mu$





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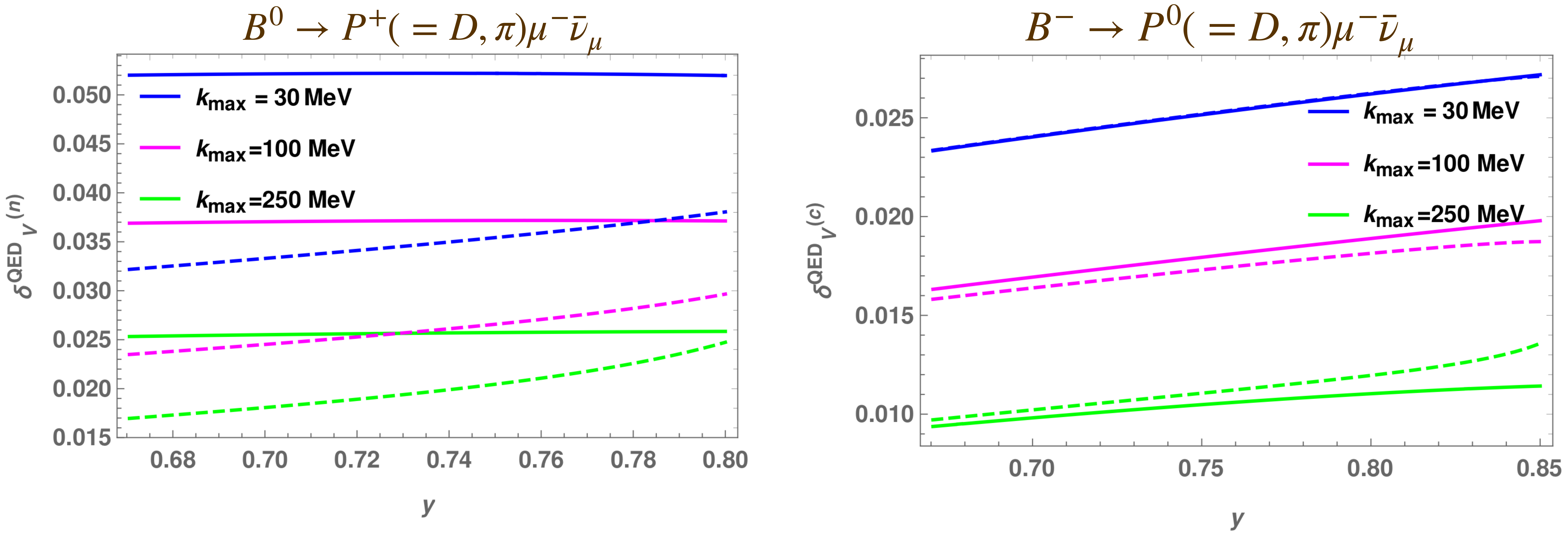

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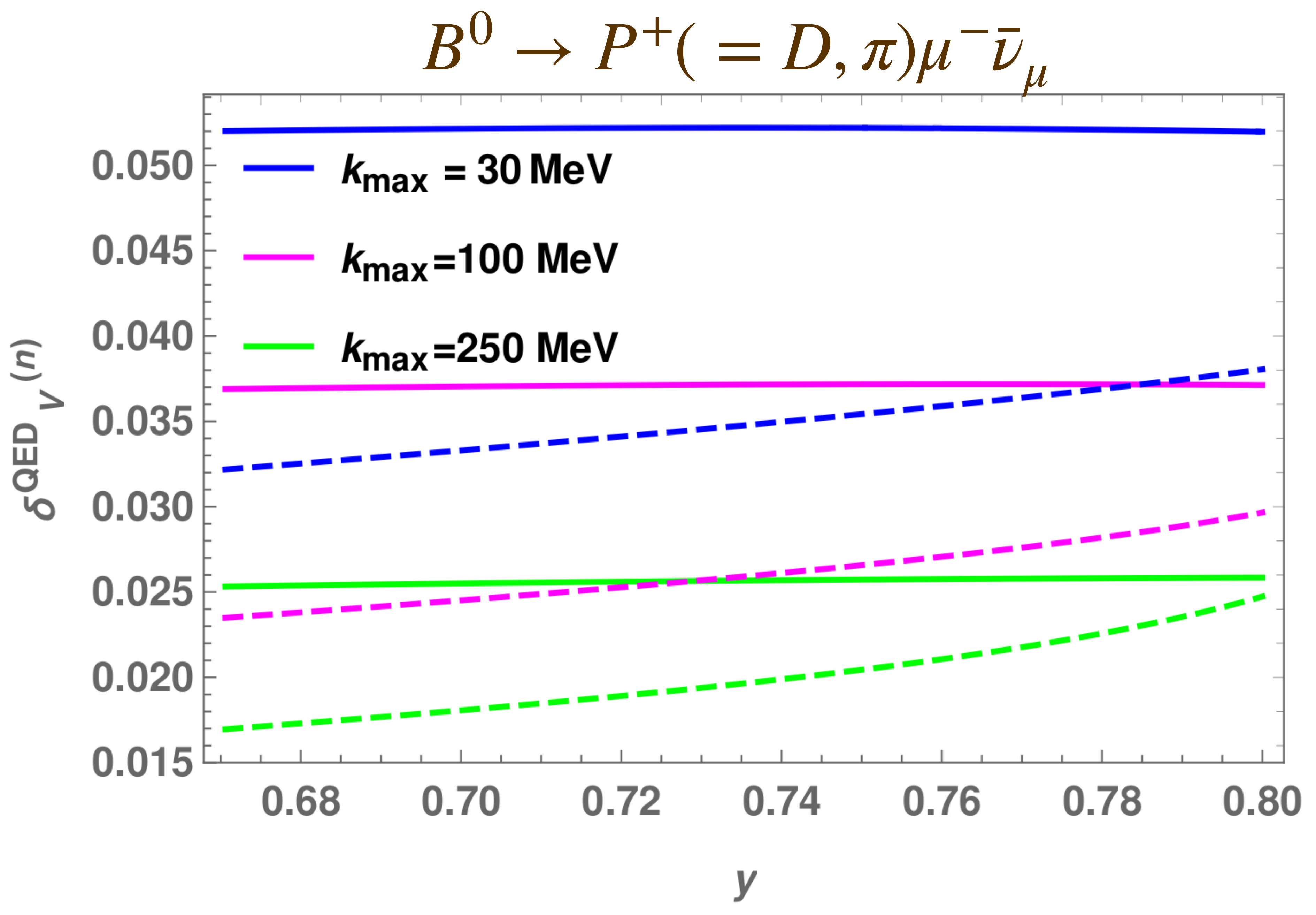

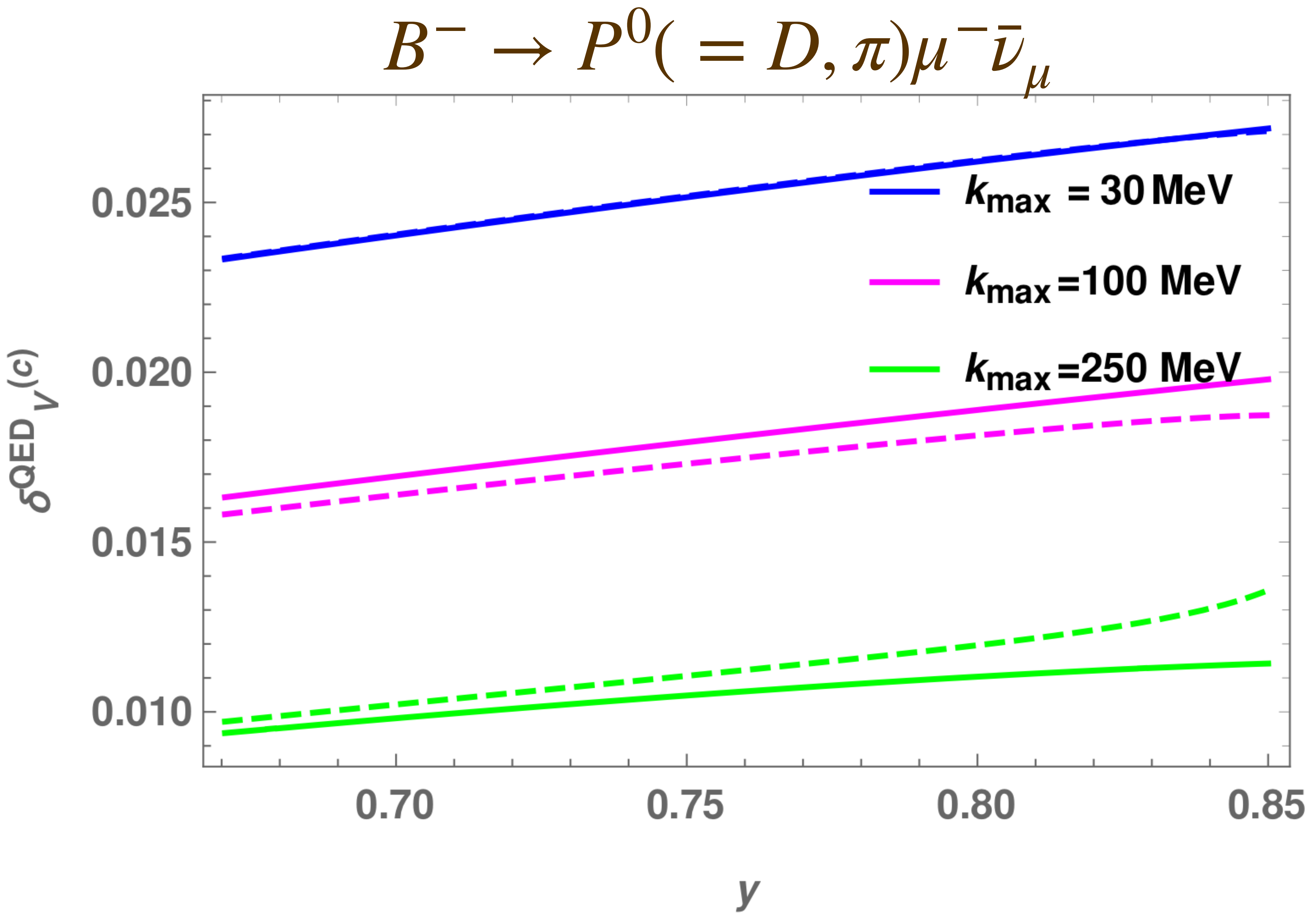

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Due to mass difference in π and D

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Note: we limit ourselves in large q^2 region

	$(f^{(I)}_{B\to\pi}; f^{(I)}_{B\to D})$	$(f^{(II)}_{B\to\pi}; f^{(I)}_{B\to D})$	$(f^{(I)}_{B\to\pi}; f^{(II)}_{B\to D})$	$(f^{(II)}_{B\to\pi}; f^{(II)}_{B\to D})$
R_V	0.091	0.093	0.091	0.093

The ratio of RV determined with the choice $f^{(A)}_{B\to\pi}$ and $f^{(A)}_{B\to D}$ for the corresponding form factors.

	$(f^{(I)}_{B\to\pi}; f^{(I)}_{B\to D})$	$(f^{(II)}_{B\to\pi}; f^{(I)}_{B\to D})$	$(f^{(I)}_{B\to\pi}; f^{(II)}_{B\to D})$	$(f^{(II)}_{B\to\pi}; f^{(II)}_{B\to D})$
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R_V turns out to be robust against soft photon corrections as well as choice of form factors

Phenomenological impact (an example)

- Consider new physics (NP) in the form of right handed currents in quarks :

$$H_{\mathrm{NP}} = \frac{4G_F}{\sqrt{2}} V_{qb} c_R^q (\bar{\ell}\gamma_\mu P_L \nu)\left(\bar{q}\gamma_\mu P_R b\right),$$

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NP impact on $|V_{qb}|$

	Modes	V_{qb}^{NP}
Exclusive Decays	$B \to D\ell\nu_\ell$	$V_{cb}^{NP} = \frac{V_{cb}^{(\mathrm{SM})}}{1+c_R^c}$
	$B \to D^*\ell\nu_\ell$	$V_{cb}^{NP} = \frac{V_{cb}^{(\mathrm{SM})}}{1-c_R^c}$
	$B \to \pi\ell\nu_\ell$	$V_{ub}^{NP} = \frac{V_{ub}^{(\mathrm{SM})}}{1+c_R^u}$
	$B \to \rho\ell\nu_\ell$	$V_{ub}^{NP} = \frac{V_{ub}^{(\mathrm{SM})}}{1-c_R^u}$
Inclusive Decay	$B \to X_c\ell\nu_\ell$	$V_{cb} = \frac{V_{cb}(\mathrm{SM})}{1-0.34c_R^c}$
	$B \to X_u\ell\nu_\ell$	$V_{ub} = V_{ub}^{(\mathrm{SM})} \quad (\text{for } \; m_u \sim 0)$

V_{qb}^{NP} is the corresponding CKM elements in the presence of NP

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- NP impact on the ratio of R_V^{NP} to R_V^{SM}

	$\frac{B\to X_u}{B\to X_c}$	$\frac{B\to\pi}{B\to D}$	$\frac{B\to\pi}{B\to D^*}$	$\frac{B\to\rho}{B\to D}$	$\frac{B\to\rho}{B\to D^*}$
$\left(\frac{\|V_{ub}\|}{\|V_{cb}\|}\right)^{\mathrm{NP}} / \left(\frac{\|V_{ub}\|}{\|V_{cb}\|}\right)_{\mathrm{SM}}$	$1-0.34c_R^c$	$1+c_R^c-c_R^u$	$1-c_R^c-c_R^u$	$1+c_R^c+c_R^u$	$1-c_R^c+c_R^u$

- NP impact on $|V_{qb}|$

	Modes	V_{qb}^{NP}
Exclusive Decays	$B \to D\ell\nu_\ell$	$V_{cb}^{NP} = \frac{V_{cb}^{(\mathrm{SM})}}{1+c_R^c}$
	$B \to D^*\ell\nu_\ell$	$V_{cb}^{NP} = \frac{V_{cb}^{(\mathrm{SM})}}{1-c_R^c}$
	$B \to \pi\ell\nu_\ell$	$V_{ub}^{NP} = \frac{V_{ub}^{(\mathrm{SM})}}{1+c_R^u}$
	$B \to \rho\ell\nu_\ell$	$V_{ub}^{NP} = \frac{V_{ub}^{(\mathrm{SM})}}{1-c_R^u}$
Inclusive Decay	$B \to X_c\ell\nu_\ell$	$V_{cb} = \frac{V_{cb}(\mathrm{SM})}{1-0.34c_R^c}$
	$B \to X_u\ell\nu_\ell$	$V_{ub} = V_{ub}^{(\mathrm{SM})}$ (for $m_u \sim 0$)

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Note: V_{qb}^{SM} is the corresponding CKM elements in the absence of NP

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- We get constraint on c_R^u : $c_R^u \in [-1.34, 1.34]c_R^c$ (actual power of R_V)

Application:

- Attempt to find the constraint on $\mathscr{BR}(B_c \to \tau\nu_\tau)$ using $\mathscr{BR}(B \to \tau\nu_\tau)$

$$\mathscr{BR}(B_{u(c)} \to \tau\nu_\tau) = (1 - 2c_R^{u(c)})\mathscr{BR}(B_{u(c)} \to \tau\nu_\tau)|_{\text{SM}}$$

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With $f_{B(B_c)} = 185(434)\mathrm{MeV}$ And $\mathcal{BR}(B \to \tau\nu_\tau)|_{exp} = 1.09 \times 10^{-4}$

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- This examples $\Longrightarrow$ V_{cb} puzzle and V_{ub} puzzle are not independent

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Thank You for your attention

BACKUP

Differential decay width for inclusive modes :

$$\frac{d\Gamma}{d\hat{q}^2}\bigg|_{SM} =\Big(1+\frac{\lambda_1}{2m_b^2}\Big)\lambda(1,\hat{q}^2,\rho^2)\Big\{\big[(1-\rho)^2+\hat{q}^2(1+\rho)-2(\hat{q}^2)^2\big]$$
$$+\frac{\hat{m}_\tau^2}{\hat{q}^2}\big[2(1-\rho)^2-\hat{q}^2(1+\rho)-(\hat{q}^2)^2\big]\Big\}+\frac{3\lambda_2}{2m_b^2}\Big\{\big[(1-\rho)^3(1-5\rho)-\hat{q}^2(1-\rho)^2(1+5\rho)$$
$$-3(\hat{q}^2)^2(5+6\rho+5\rho^2)+25(\hat{q}^2)^3(1+\rho)-10(\hat{q}^2)^4\big]$$
$$+\frac{\hat{m}_\tau^2}{\hat{q}^2}\big[2(1-\rho)^3(1-5\rho)-\hat{q}^2(5-9\rho-21\rho^2+25\rho^3)$$
$$+3(\hat{q}^2)^2(1+2\rho+5\rho^2)+5(\hat{q}^2)^3(1+\rho)-5(\hat{q}^2)^4\big]\Big\},$$

$$\frac{d\Gamma}{dq^2}\bigg|_{LR} = -12\sqrt{\rho}\hat{q}^2\Big(1+\frac{\lambda_1}{2m_b^2}\Big)\lambda(1,\hat{q}^2,\rho^2)+4\sqrt{\rho}\frac{3\lambda_2}{2m_b^2}\Big\{\big[2(1-\rho)^3-3\hat{q}^2(1-\rho)^2$$
$$+12(\hat{q}^2)^2(1+\rho)-7(\hat{q}^2)^3\big]+\frac{4\hat{m}_\tau^2}{\hat{q}^2}\big[(1-\rho)^3-3\hat{q}^2\rho(1-\rho)-3\rho(\hat{q}^2)^2+(\hat{q}^2)^3\big]\Big\},$$