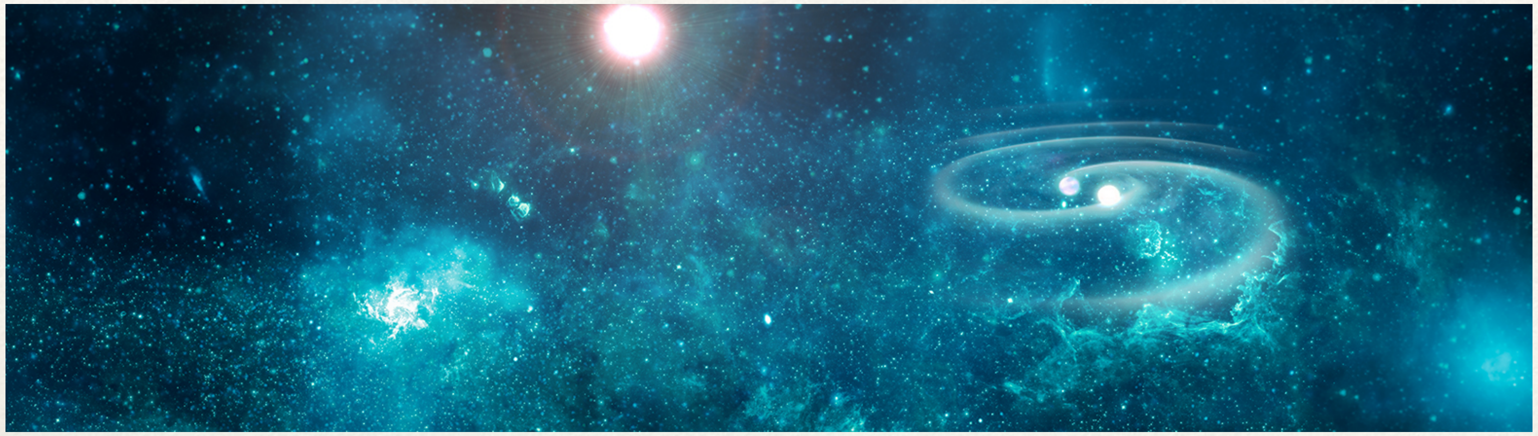
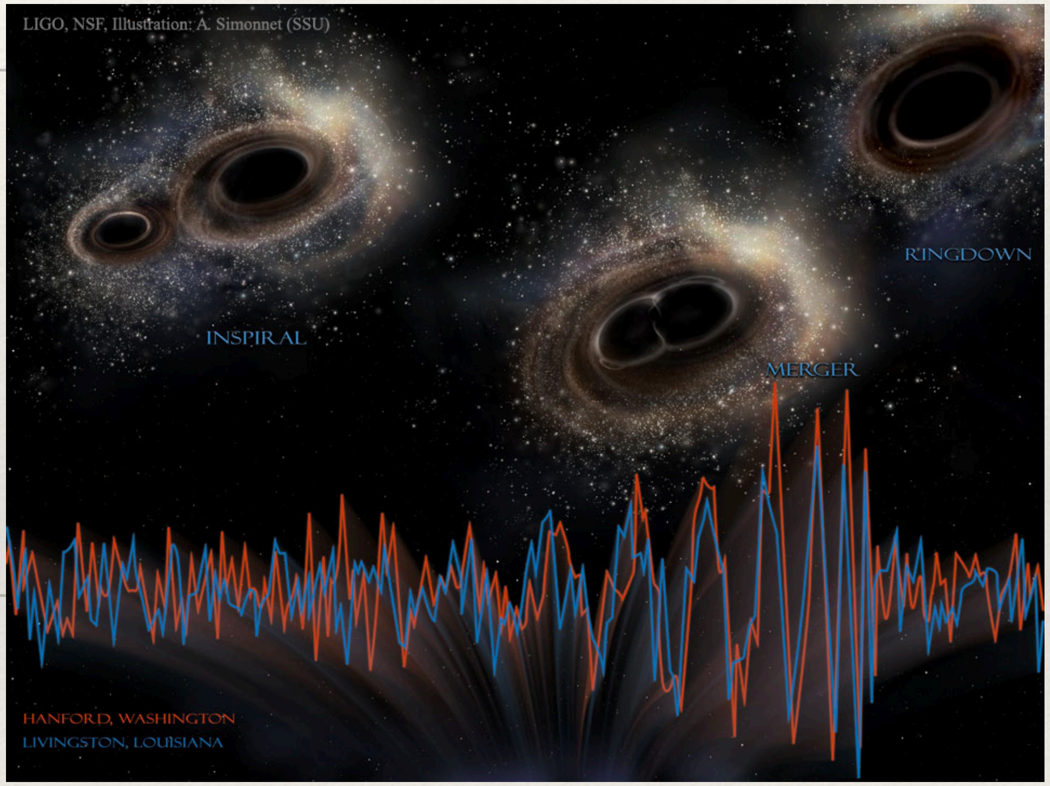
Stanislav (Stas) Babak.
AstroParticule et Cosmologie, CNRS (Paris)

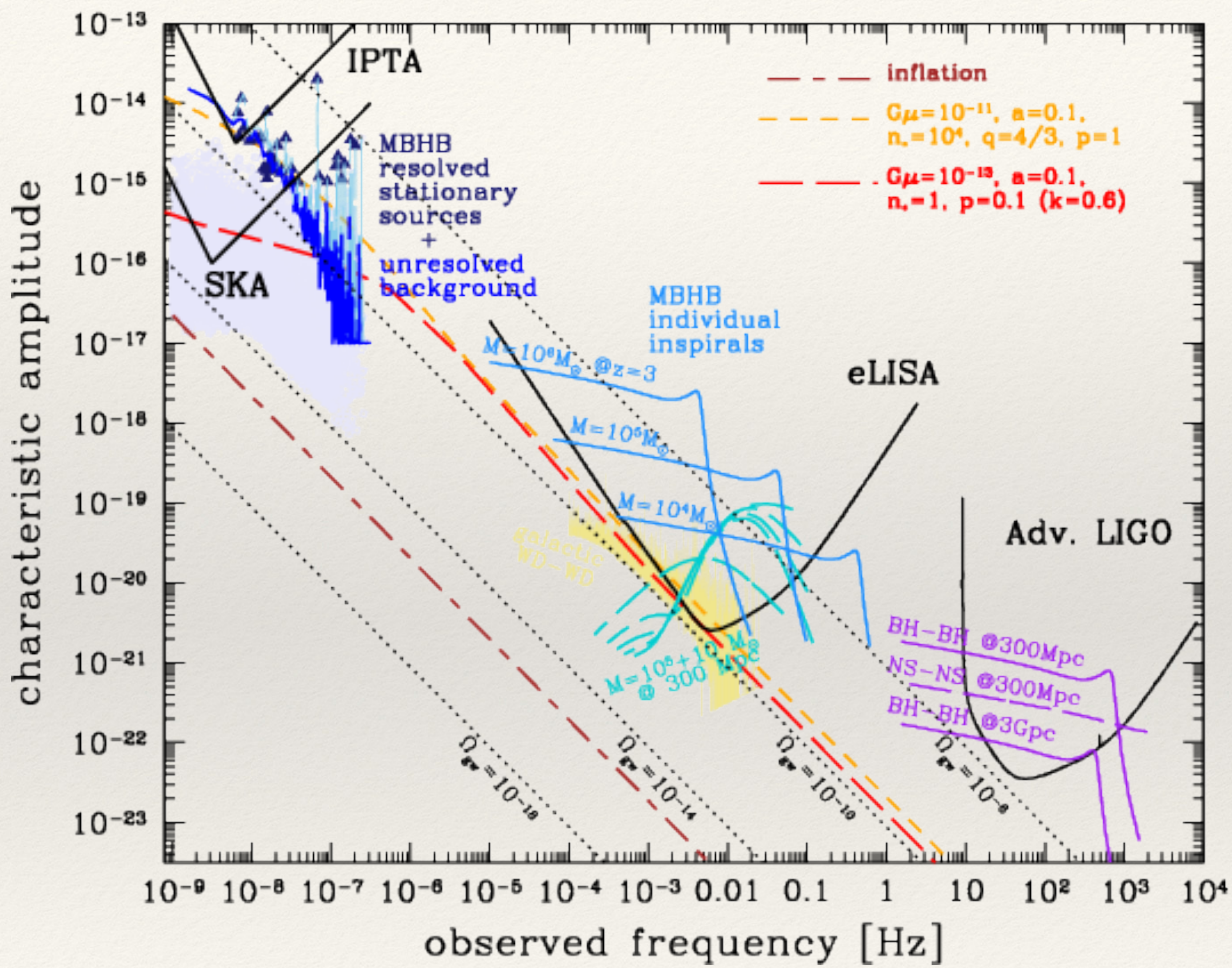


Pulsar Timing Array



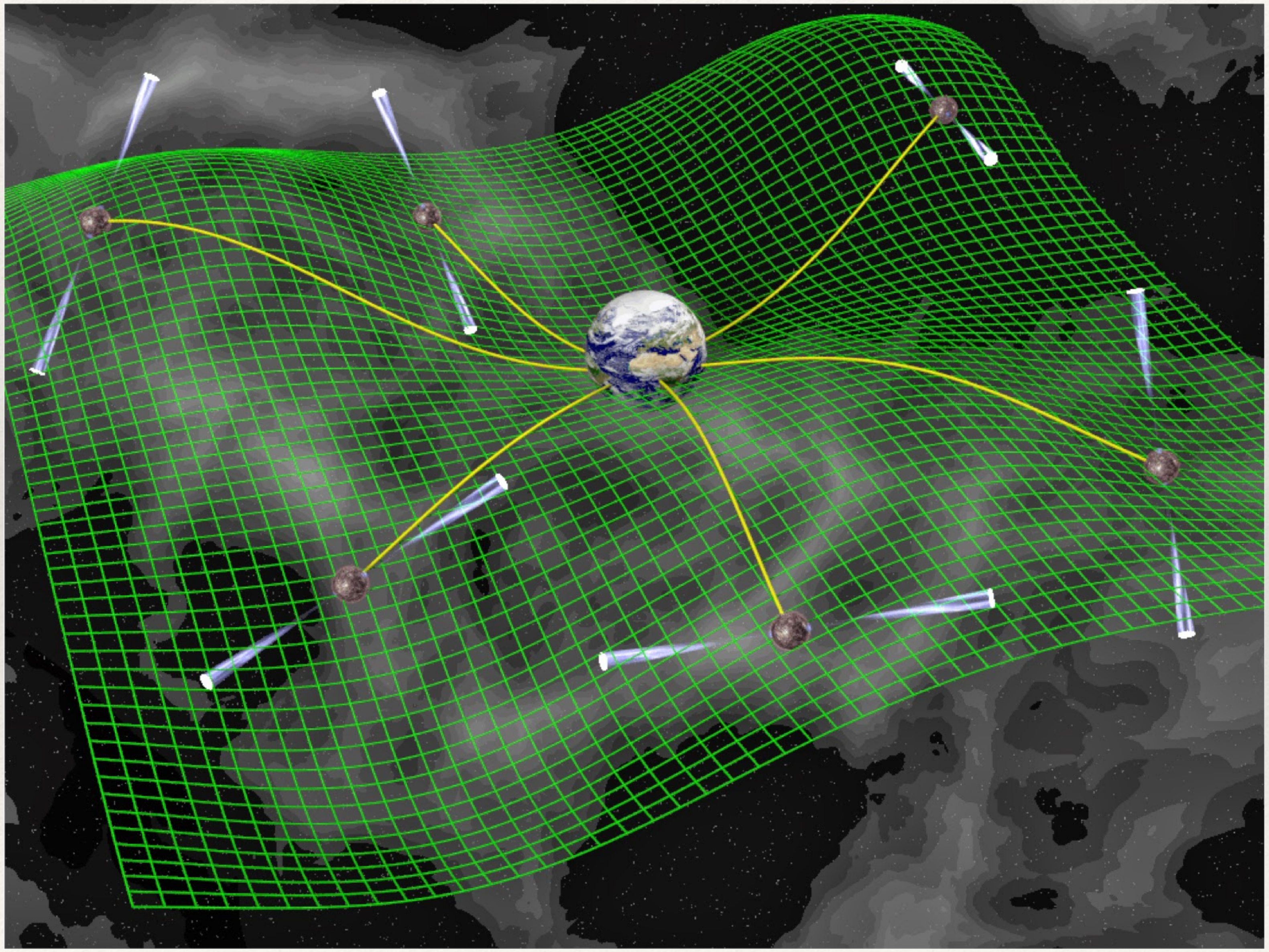
ISAPP, 7-17 June 2021

GW landscape



Pulsar Timing Array

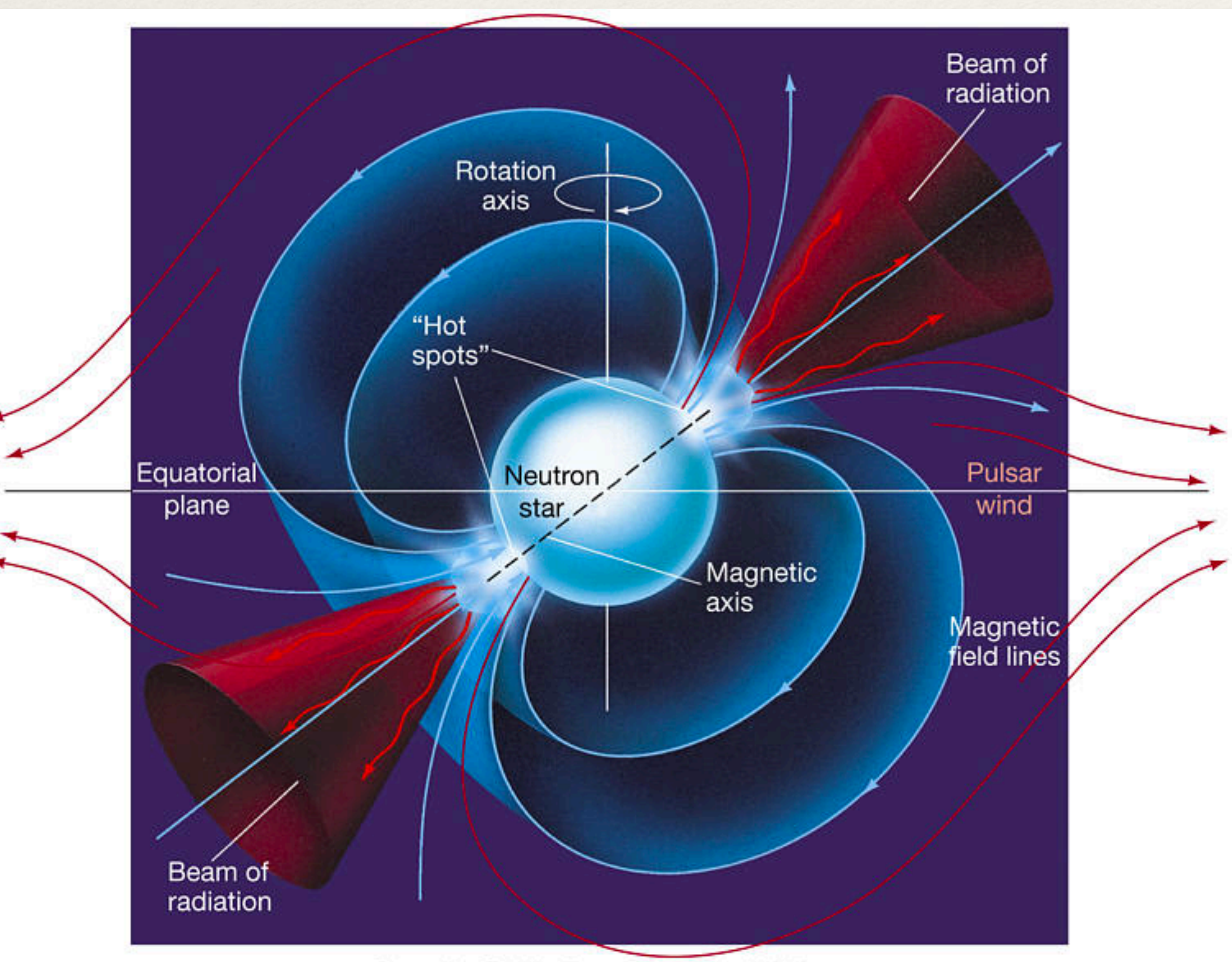
The main idea behind pulsar timing array (PTA) is to useultra-stable millisecond pulsars as beacons (clocks sending signals) for detecting GW inthe nano-Hz range (10^{-9} - 10^{-7} Hz).

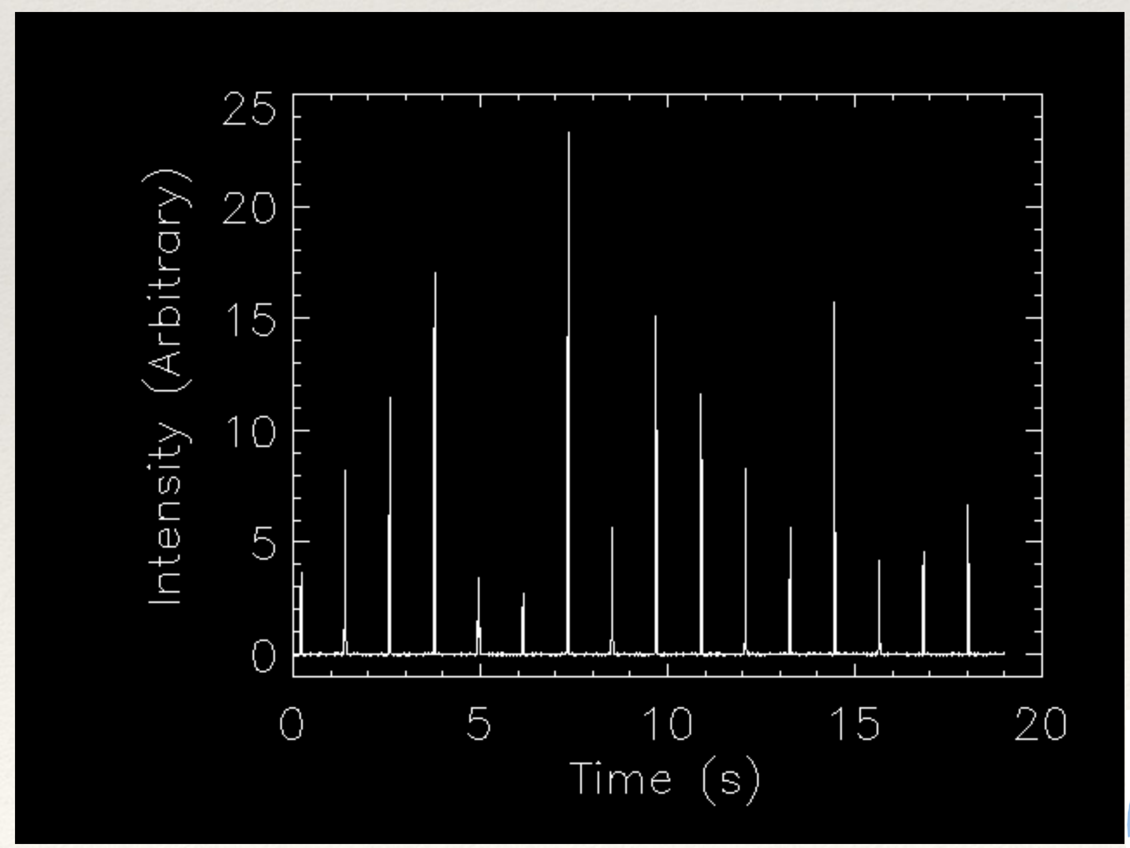


[Credits: D. Champion]

Millisecond pulsars

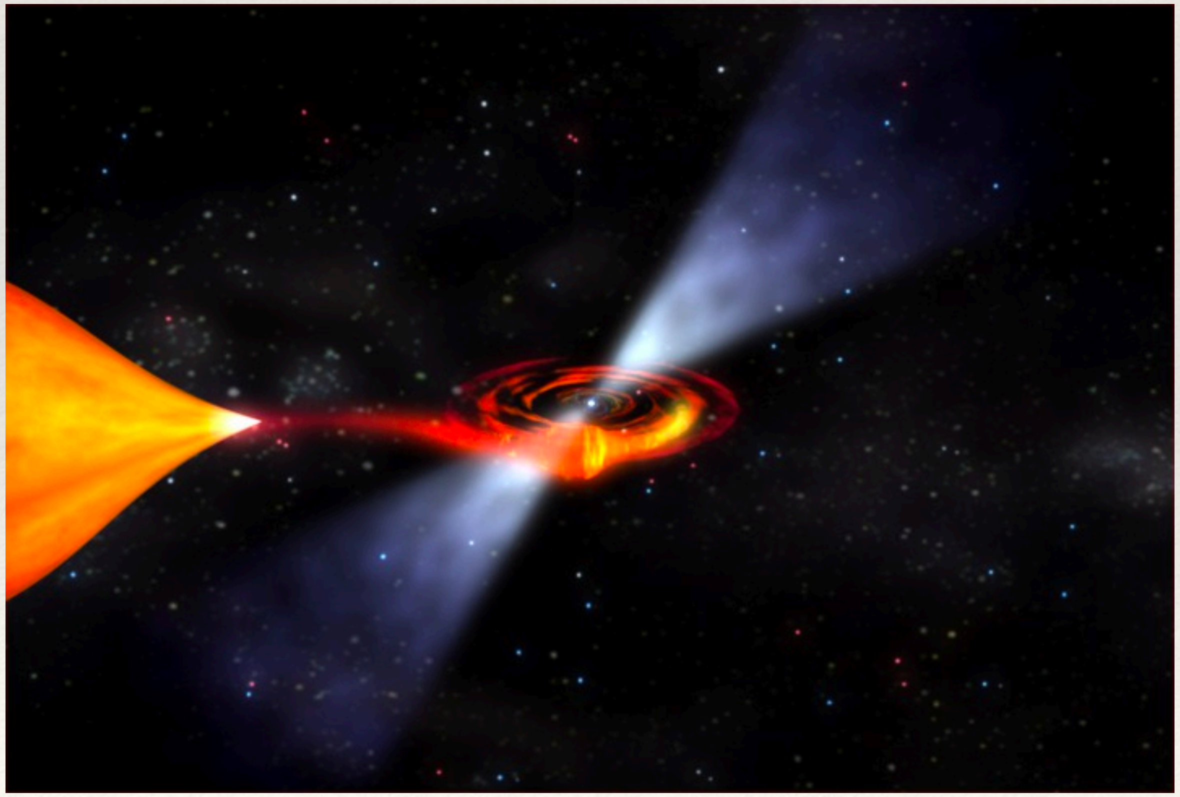
- Pulsars - neutron stars (end product of evolution of stars with the mass > 7 solar) with rapid rotation and strong magnetic field
- Emit beamed e/m radiation from the magnetic poles. Powered by rotation: spinning down.
- Beamed radio emission swaps across the line of sight — seen as pulses in observations (similar to the lighthouse)



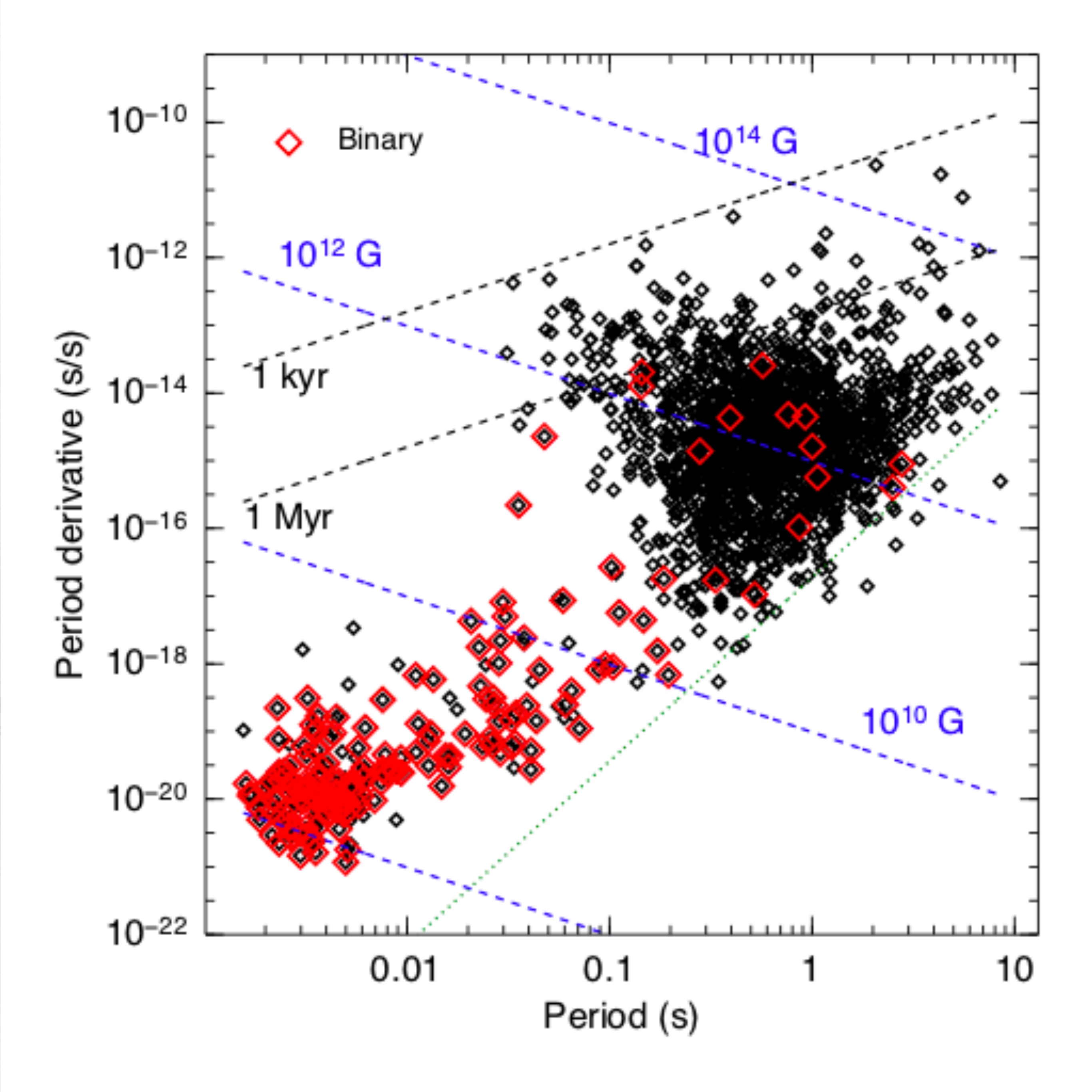


Millisecond pulsars

- Millisecond pulsars: period of rotation ~ millisec
- Often in binaries
- Very old NSs, very stable rotation
- The most accurate clock on the long time scale (decades)

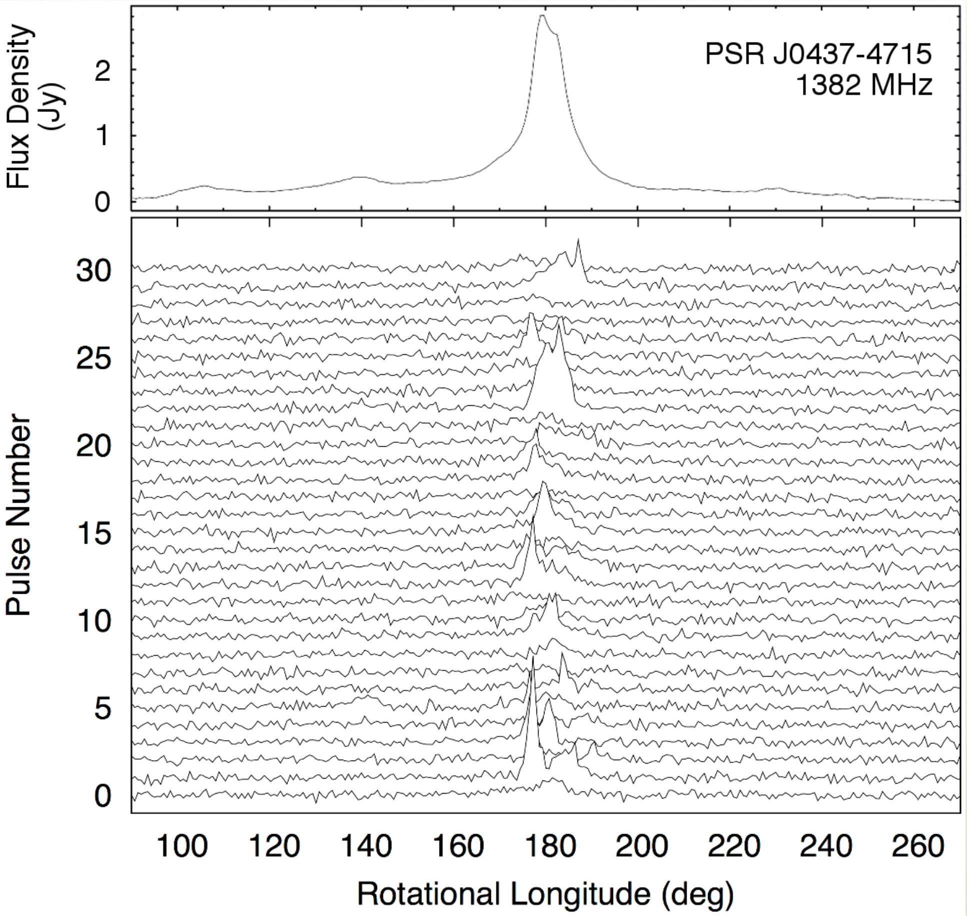


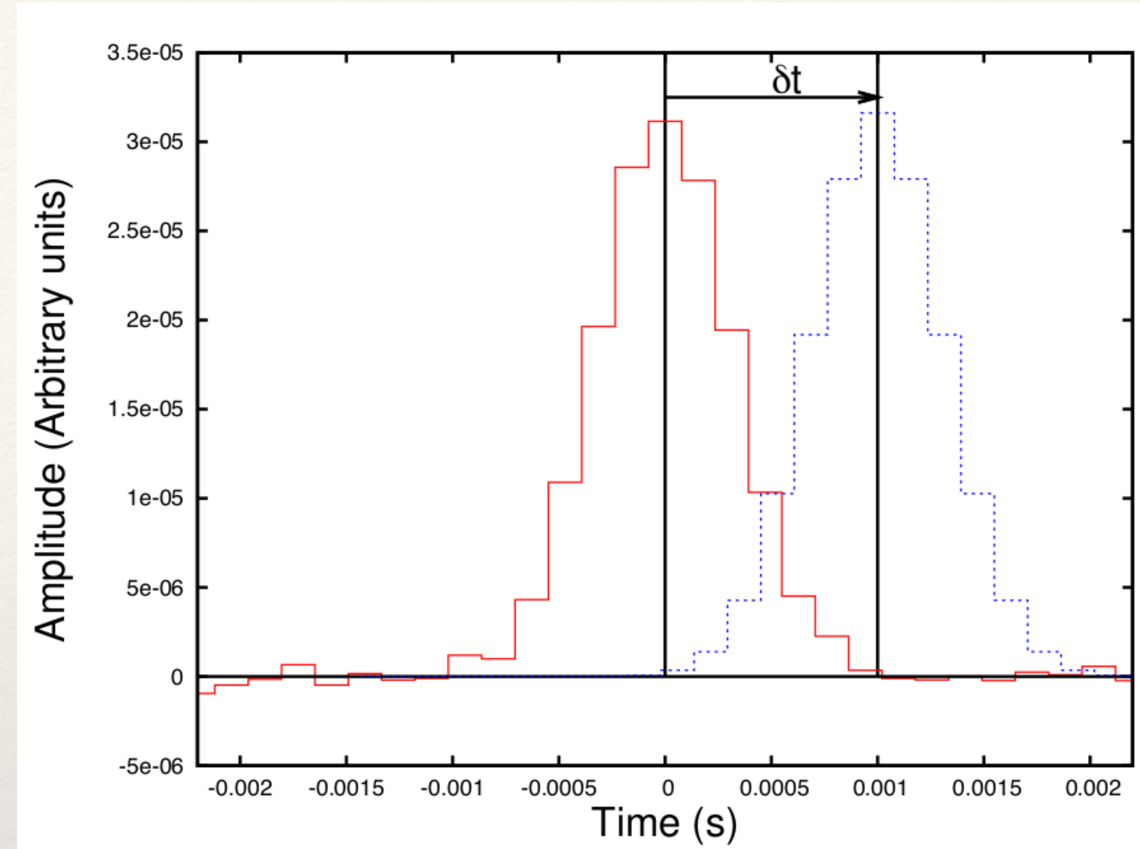
[Credits: NASA]



[Wikimedia]

Pulsar timing

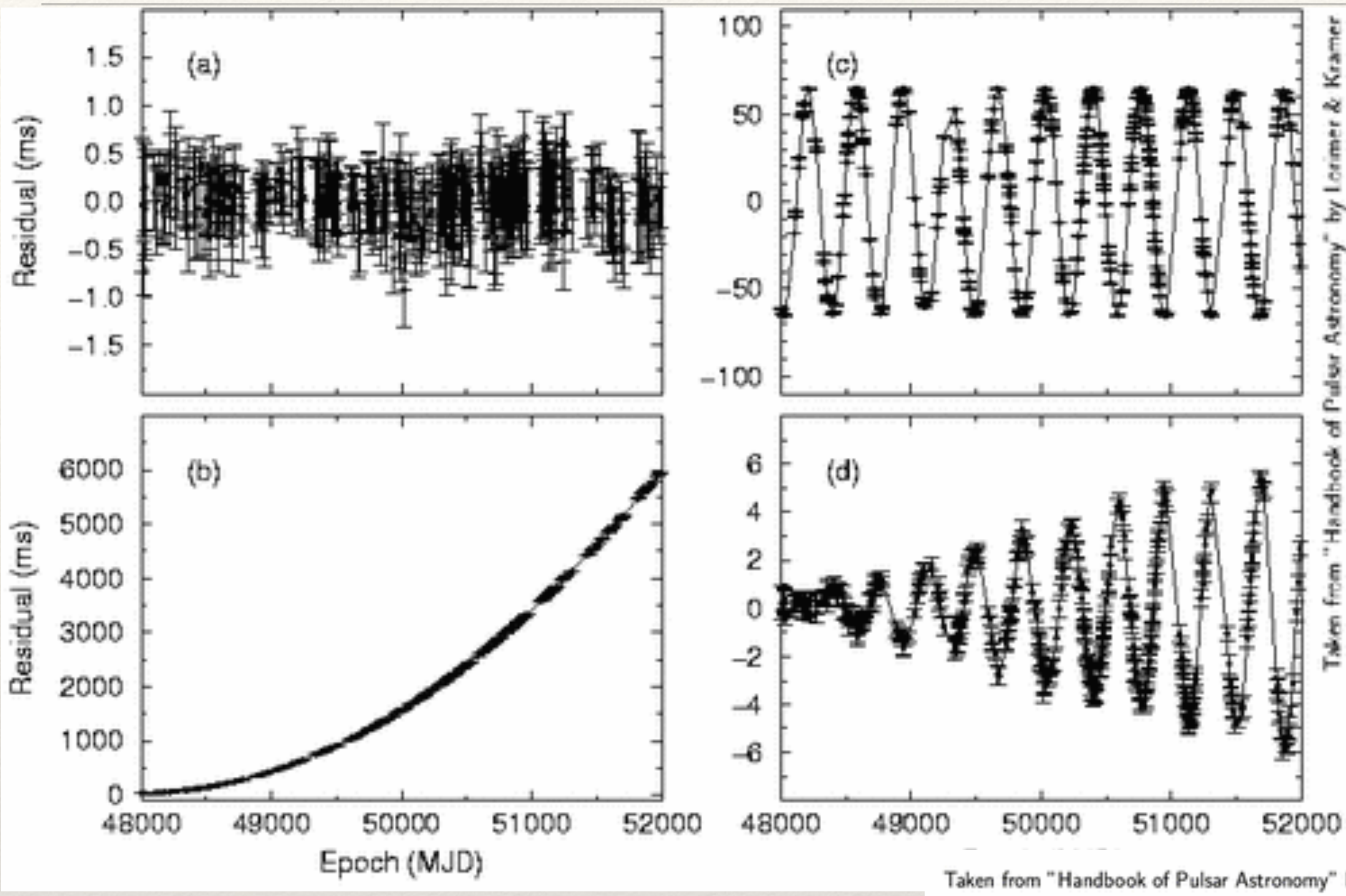




[Figs: credits
S. Burke-Spolar & L. Lentati]

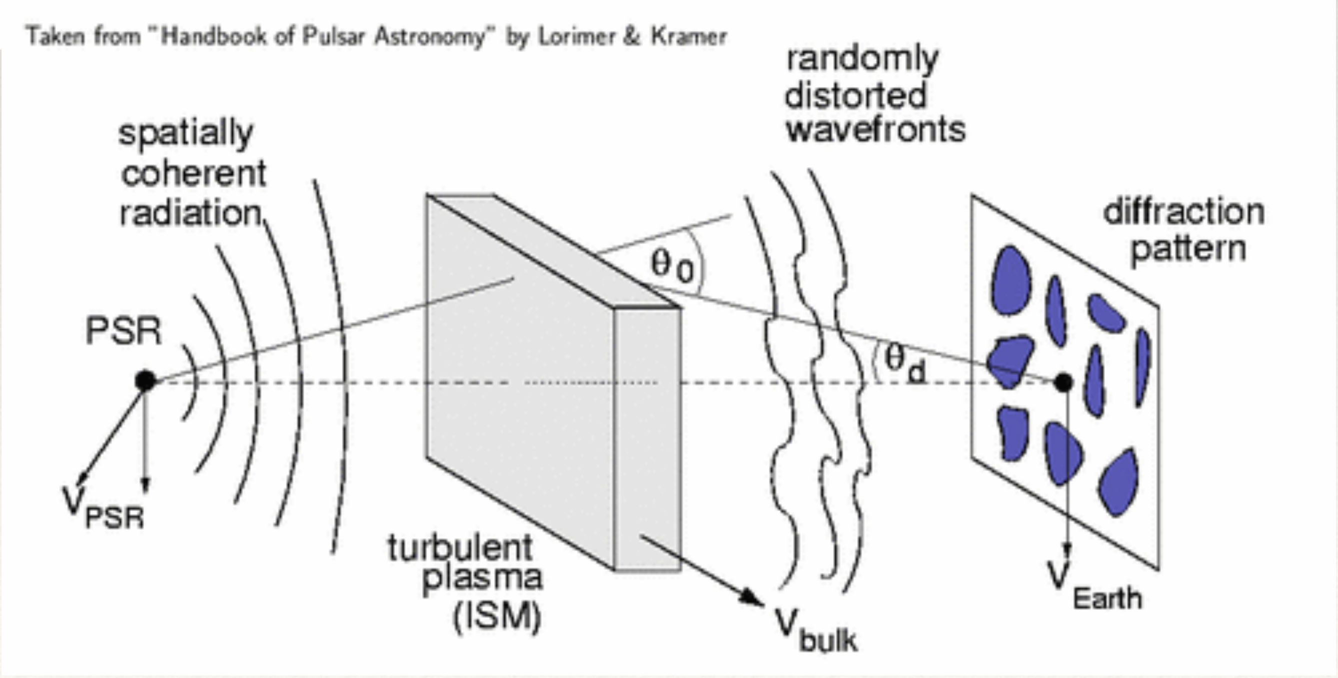
- Each observed radio pulse profile has a lot micro-structure. If we average over ~hour the (average) profile is very stable
- We can use the average pulse profile to estimate the time-of-arrival (TOA) of the pulses.
- The idea is to measure the TOA, and compare to the expected TOA. We know the spin of the pulsars, so we can predict the TOA. The difference between measure and expected TOA: ***residuals***

Timing pulsars



- We need to build a timing model to make accurate prediction for TOAs - take into account various physical effects
- Dispersion of e/m wave and its time dependence
- Rate of change of rotation (b)
- Sky position of the pulsar (c)
- Proper motion of a pulsar (d)

- Timing model could be quite complex if pulsar is in the binary



Residuals

- Building the timing model: depends on many parameters

$$t_{toa} = t_{toa}(P, \dot{P}, \ddot{P}, \Delta_{clock}, \Delta_{DM}(L), \Delta_{\odot-\oplus}, \Delta_E, \Delta_S)$$

$P, \dot{P}, \ddot{P}$ period of pulsar' rotation and its derivatives: spin-down

Δ_{clock} difference in the local clock and terrestrial standrad

$\Delta_{DM}(L)$ delays caused by propagation in the interstellar medium

$\Delta_{\odot-\oplus}$ Transformation from the local frame to the solar system barycentre

Δ_E Accounts for relative motion (Doppler) + gravitational redshift caused by the Sun, plantes or binary companion.

Δ_S Extra time required to trave in the curved spacetime containng Sun/companion (if in binary)

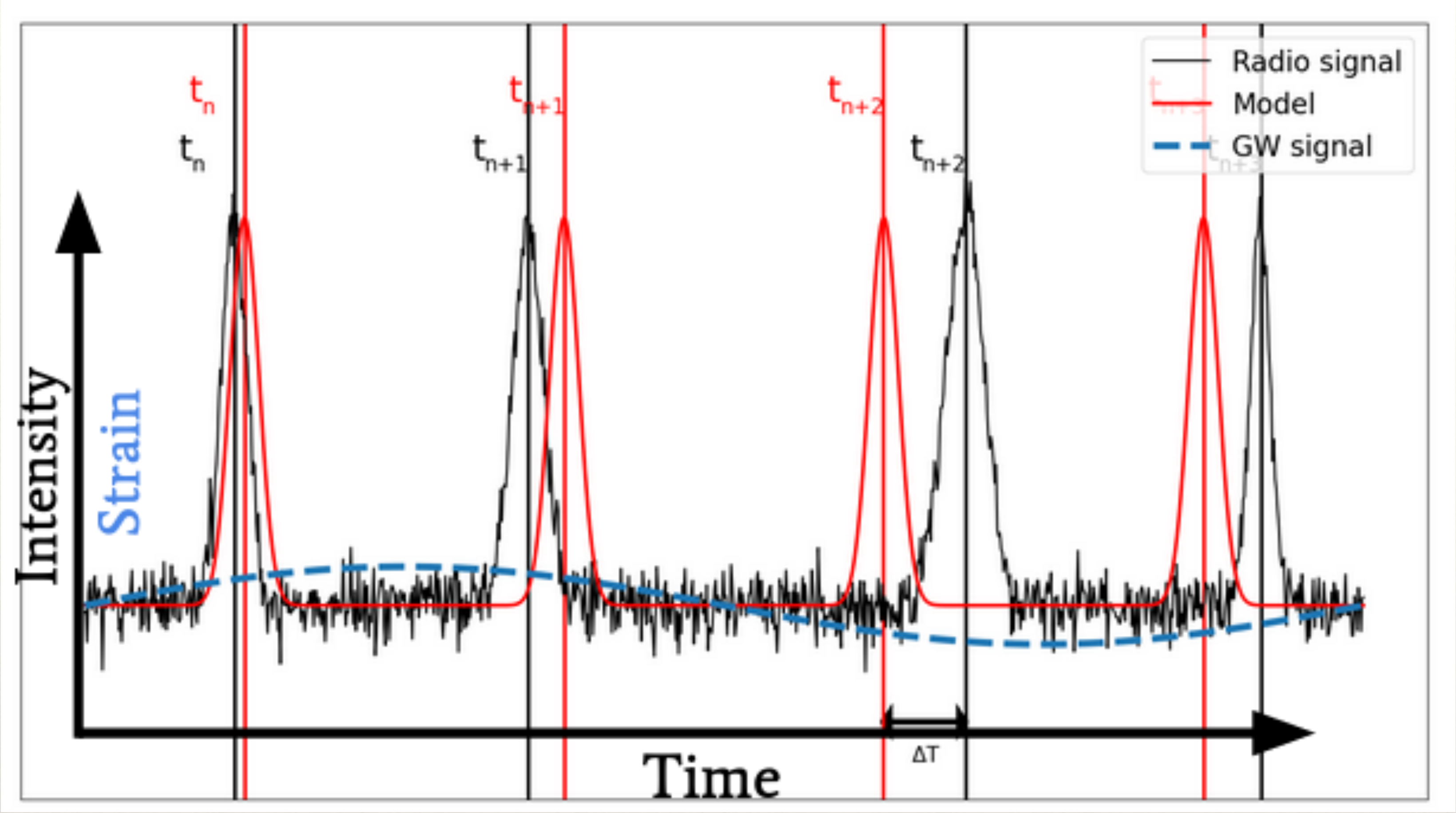
$$dt = t^p_{toa} - t^o_{toa} = dt_{errors} + \delta\tau_{GW} + noise$$

Errors in fitting the model

due to GWs

Timing Residuals

credits: Mikel Falxa

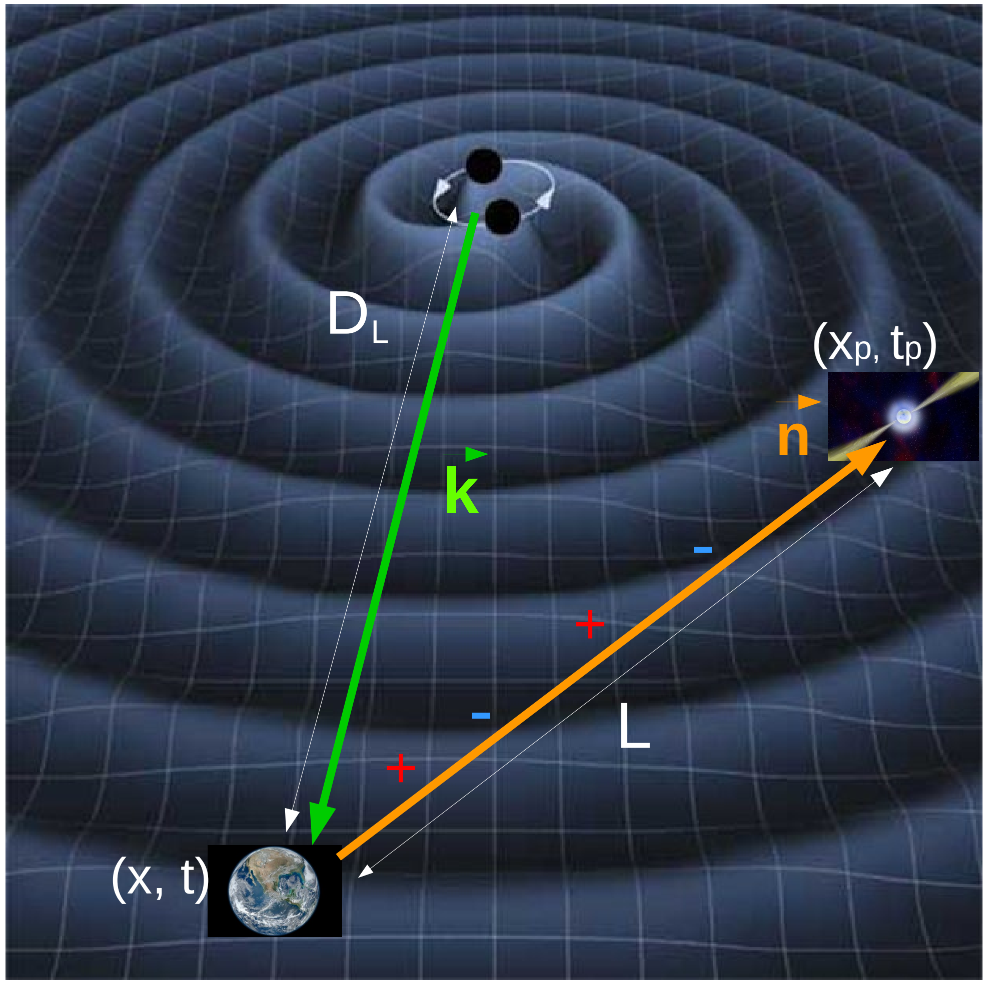


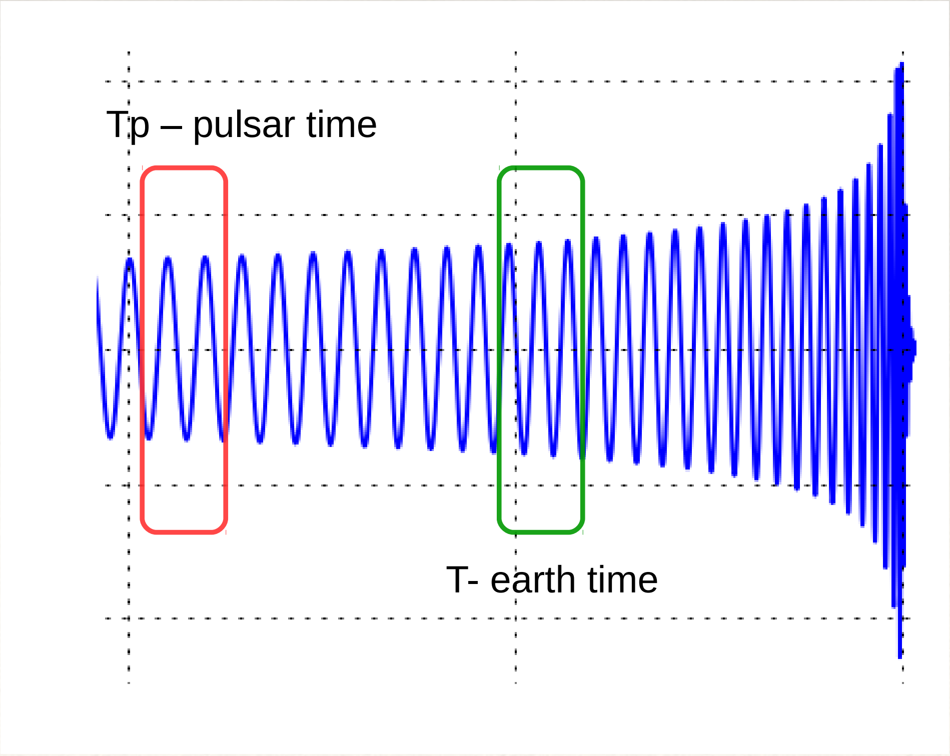
$$dt = t^p_{toa} - t^o_{toa} = dt_{errors} + \delta\tau_{GW} + noise$$

Errors in fitting the model

due to GWs

Response to GW signal





- PTA can be seen as a multi-arm detector where e/m signal travels only in onedirection (from a pulsar to the Earth). Pulsar plays role of an accurate clock, andwe measure change in phase (frequency) of arriving pulses (similar to thefrequency (phase) of the laser light)

- Important quantity which characterizes the response of any GW observatory is $\epsilon = (2\pi f_* L/c)$

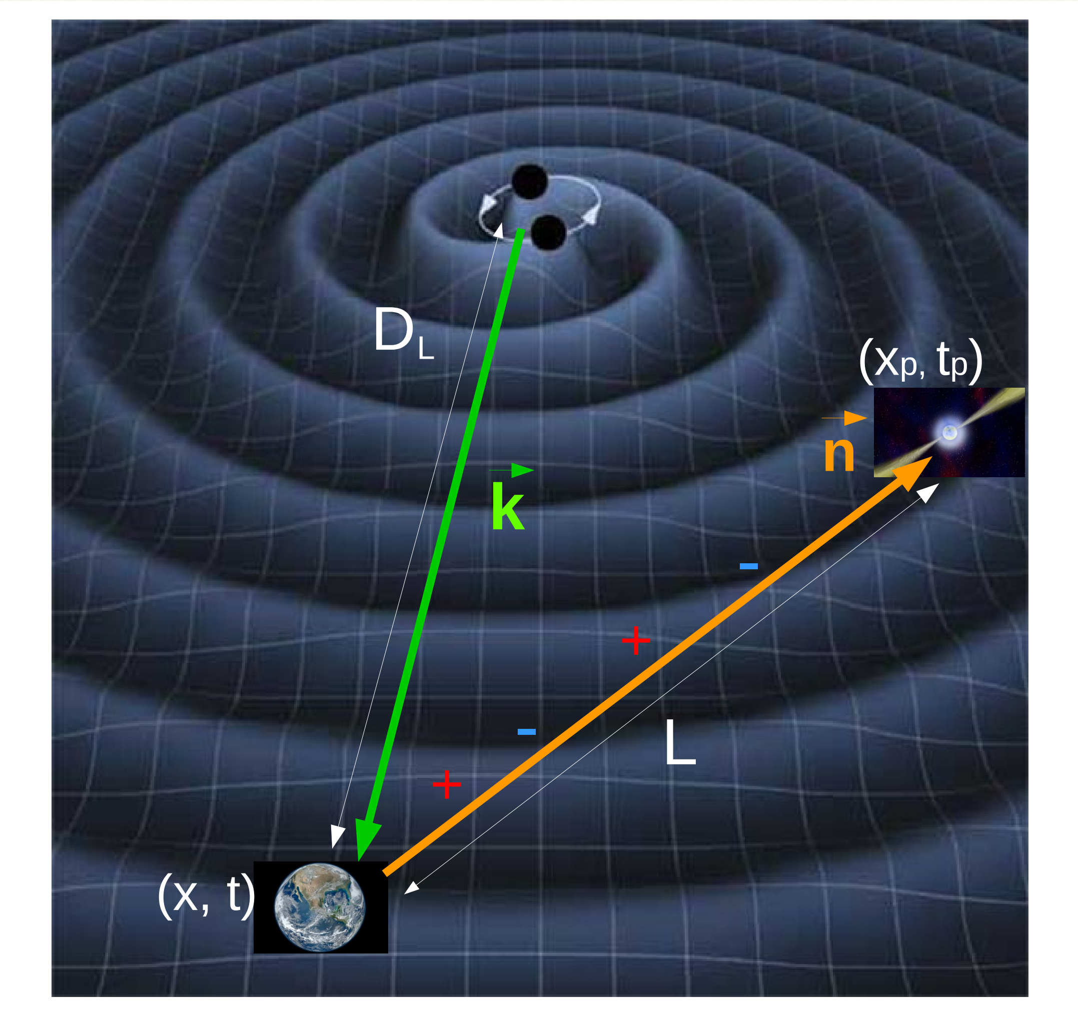
  size of GW detector

$\epsilon \ll 1 \rightarrow \; R \propto h_{ij} n^i n^j$ long wavelength approximation: LIGO/Virgo

$\epsilon = 1 \quad \rightarrow$ LIGO: f* ~12 kHz, LISA: f*~ 0.05 Hz, PTA: f*~0.002 nHz

PTA: $\epsilon \gg 1$

Response to GW signal

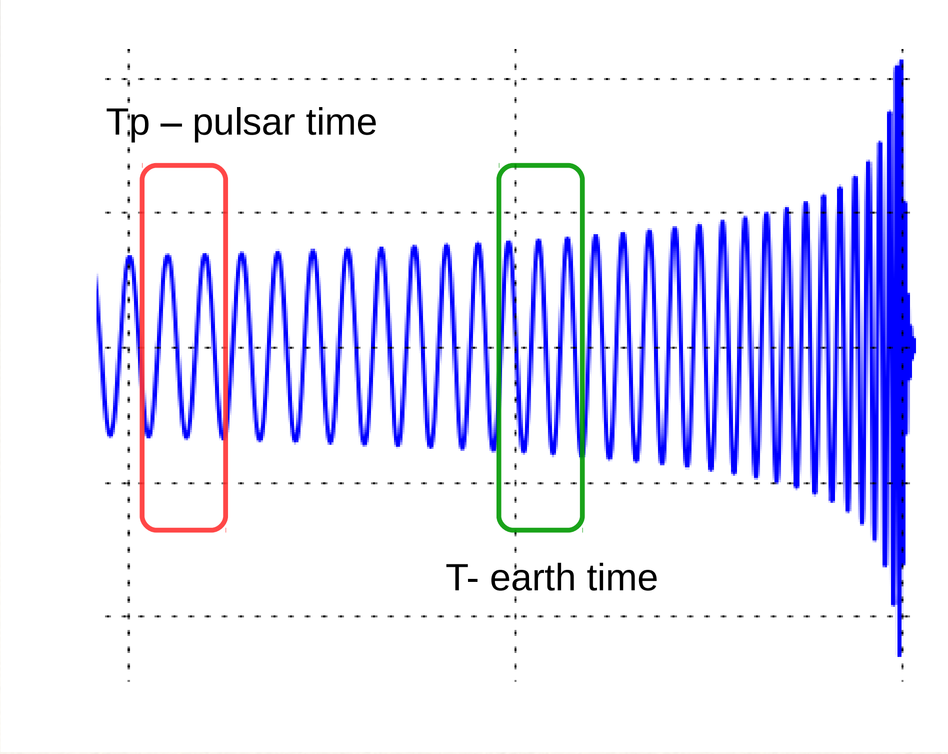


$$dt = t^p_{toa} - t^o_{toa} = dt_{errors} + \delta\tau_{GW} + noise$$

$$\delta\tau_{GW} = r(t) = \int_0^t \frac{\delta\nu}{\nu_0}(t')dt'; \quad \frac{\delta\nu}{\nu_0} = \frac{1}{2}\frac{\hat{n}^i\hat{n}^j\Delta h_{ij}}{1+\hat{n}.\hat{k}}$$

Familiar from LISA

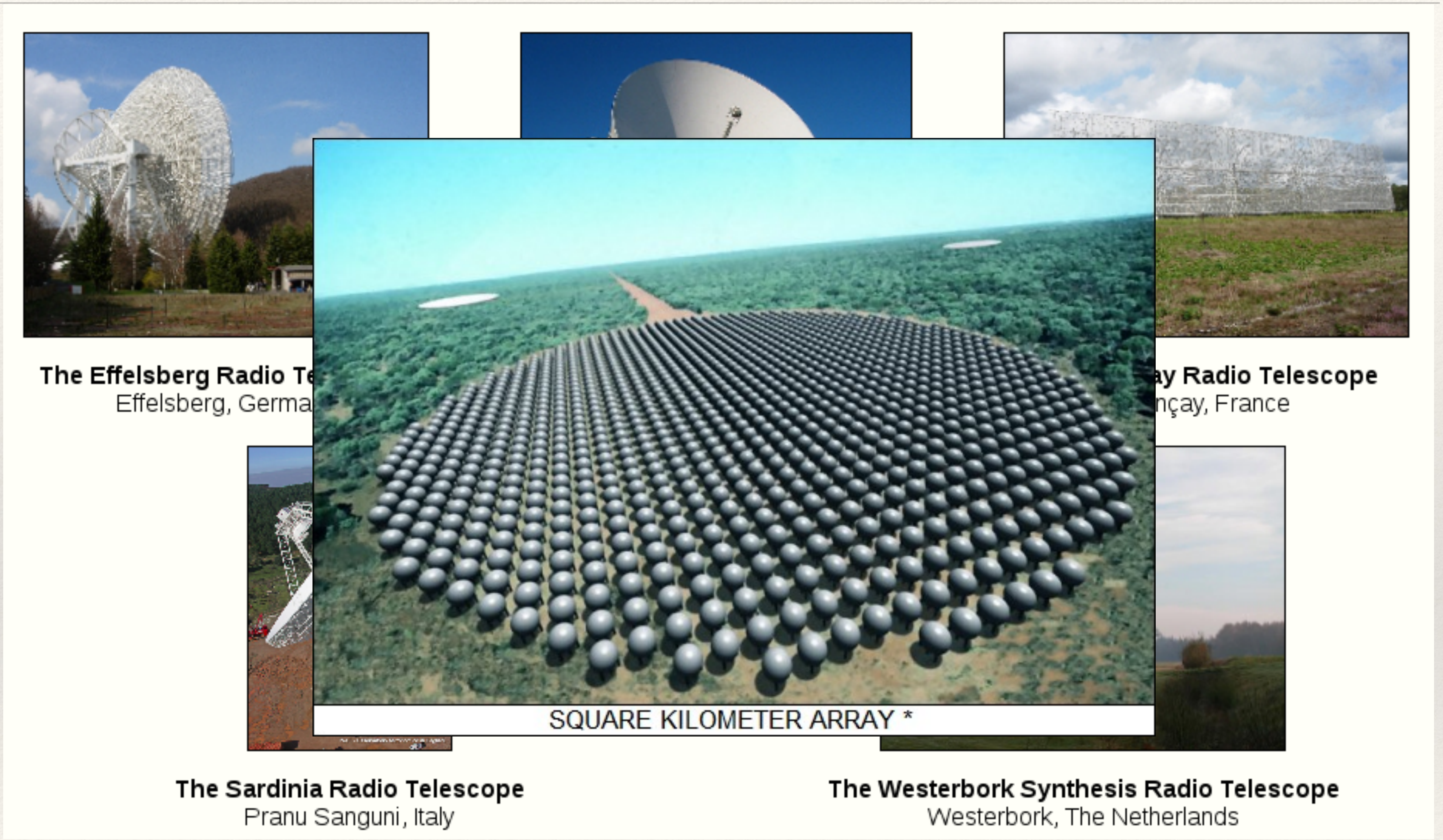
$$\Delta h_{ij} = h_{ij}(t_p = t - L(1+\hat{n}.\hat{k})) - h_{ij}(t)$$



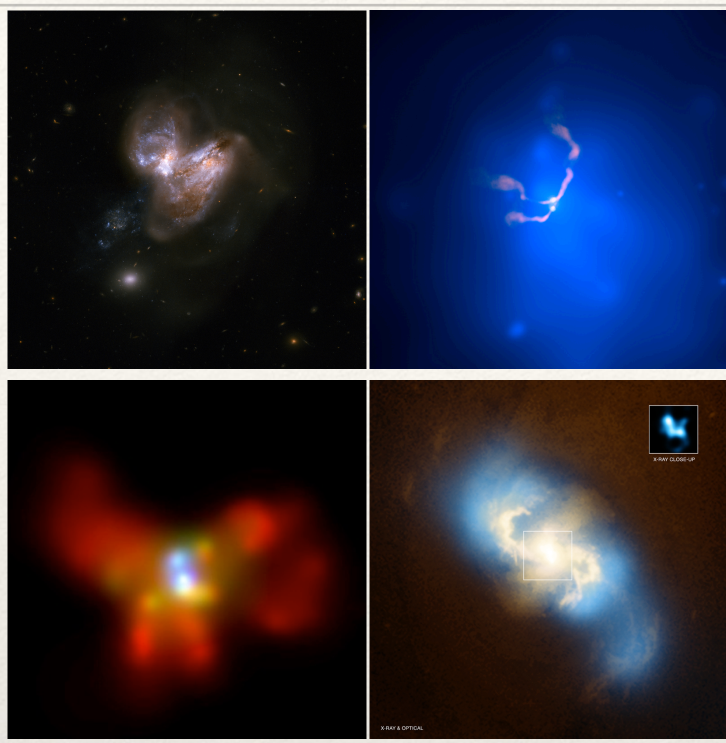
t_p — pulsar time, ~ time of emission of the radio pulse:

- depends on the relative position of a pulsar and GW source
- depends on the distance to the pulsar L
- L ~ few kpc ~10 000 years — “pulsar” term $h(t_p)$ contains info about the system 10^5 years in the past as compared to the “earth” term
- pulsar term depends on the pulsar.

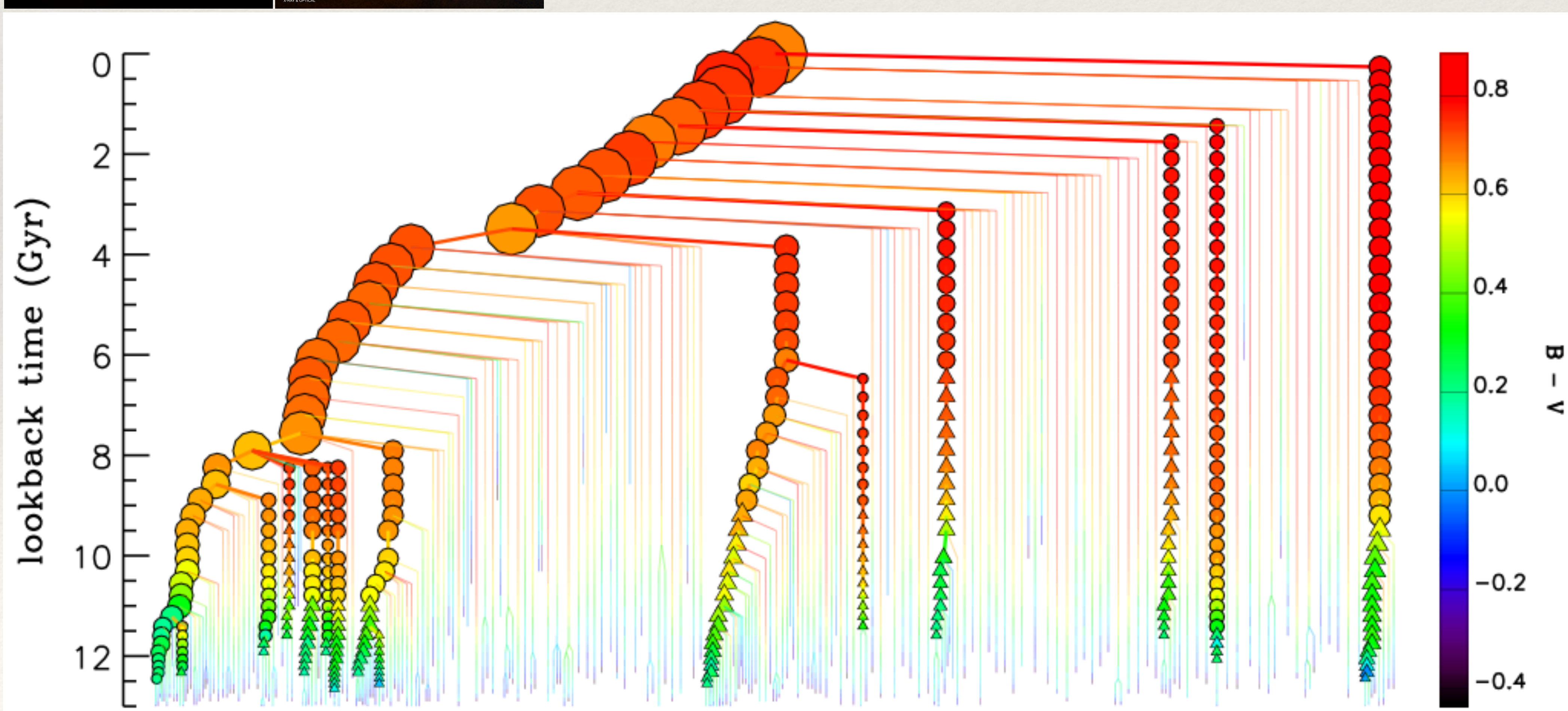
Radiotelescops: EPTA



Super-massive black holes (SMBHs)



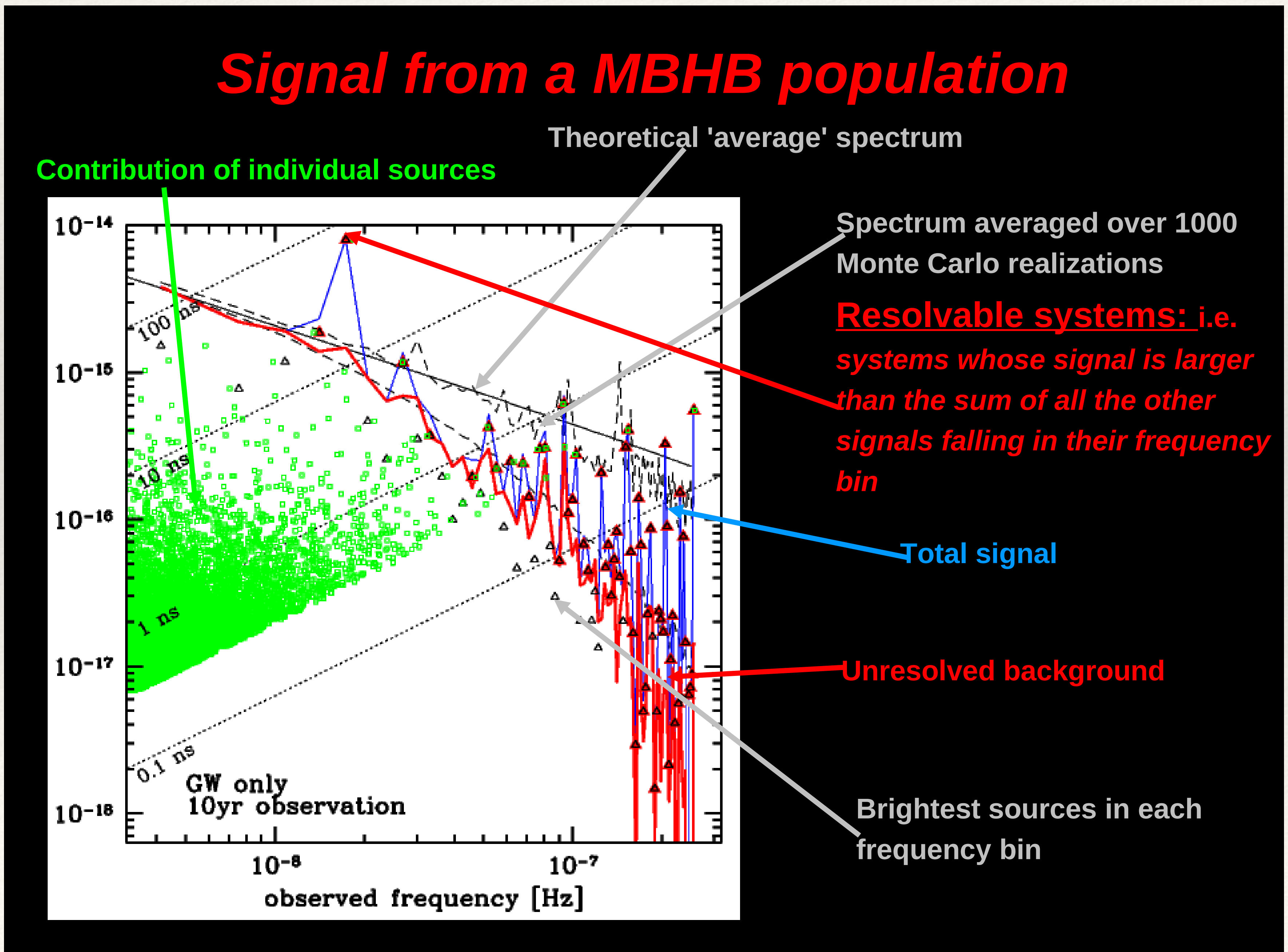
Massive black holes should reside in the nuclei of (we hope) every galaxy. (S)MBH are formed from relatively small seeds (remnants of popIII stars, direct collapse of giant protocloud) and acquire mass through accretion and major mergers (result of galactic encounter)



credits: G. De Lucia

Supermassive black hole binaries

- Main sources are supermassive black hole binaries (mass 10^7 — 10^{10} solar) on very broad orbit (period ~ year(s))
- The orbital evolution due to GW emission is very slow: $\frac{dE}{dt} \propto \eta (M/r)^5$ signal is (almost) monochromatic over period of observations



GW signal from the population of SMBH binaries: forms a stochastic signal at low freqs. (similar to Galactic binaries in LISA

[Credits: A. Sesana]

GW signal

Consider non-spinning SMBH binary in circular orbit

- pulsar and earth terms: each is monochromatic signal
- frequency. of pulsar term might or might not coincise with the erath term: $t_p = t - L(1 + \hat{n} \, . \, \hat{k})$
- amplitude of the pulsar term is larger: $\sim \omega^{-1/3}$

$$s_\alpha = F_\alpha^{+}(\hat{k}, \hat{n}_\alpha) \left[\frac{h_+(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha} - \frac{h_+(t, \omega)}{2\pi f} \right] +$$

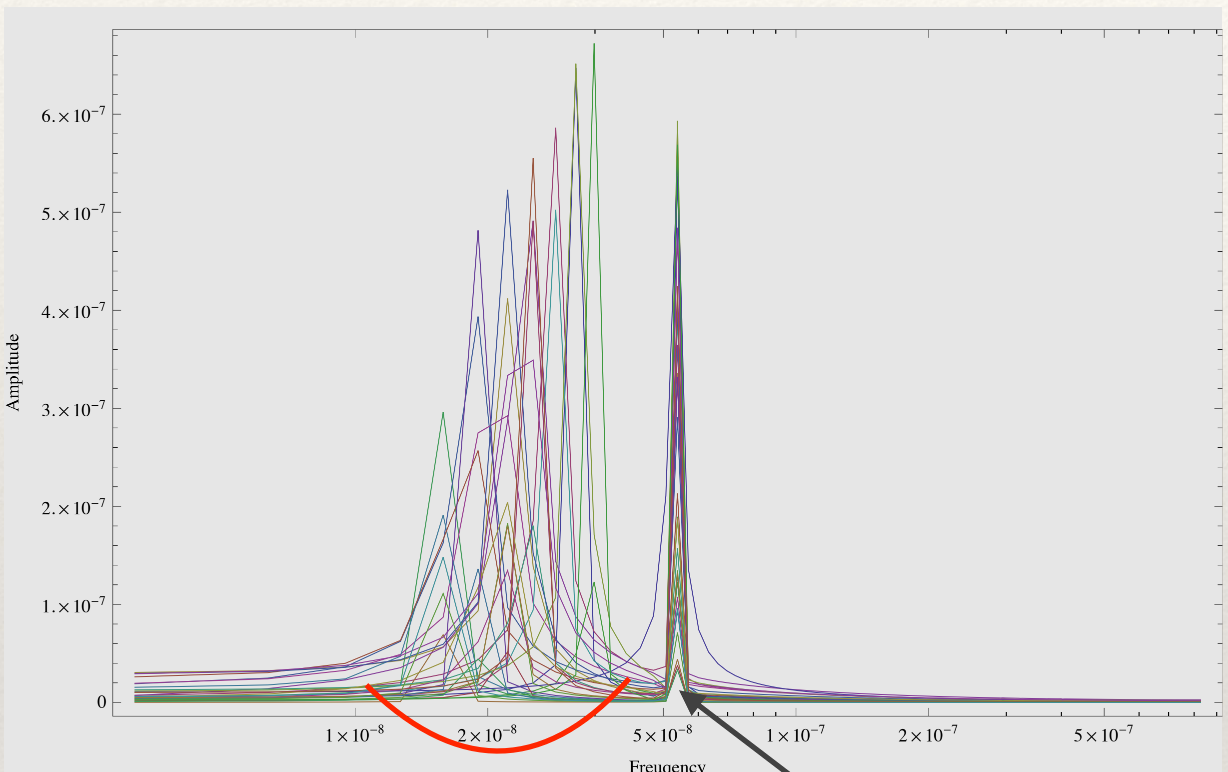
$$\underbrace{F_\alpha^{\times}(\hat{k}, \hat{n}_\alpha)}_{\text{relative position pulsar and GW source}} \left[\underbrace{\frac{h_\times(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha}}_{\text{Pulsar term } \omega_\alpha = \omega(t - L_\alpha(1 + \hat{n}_\alpha . \hat{k}))} - \underbrace{\frac{h_\times(t, \omega)}{2\pi f}}_{\text{Earth term coherent across pulsars}} \right]$$

α - pulsar index

GW signal in PTA

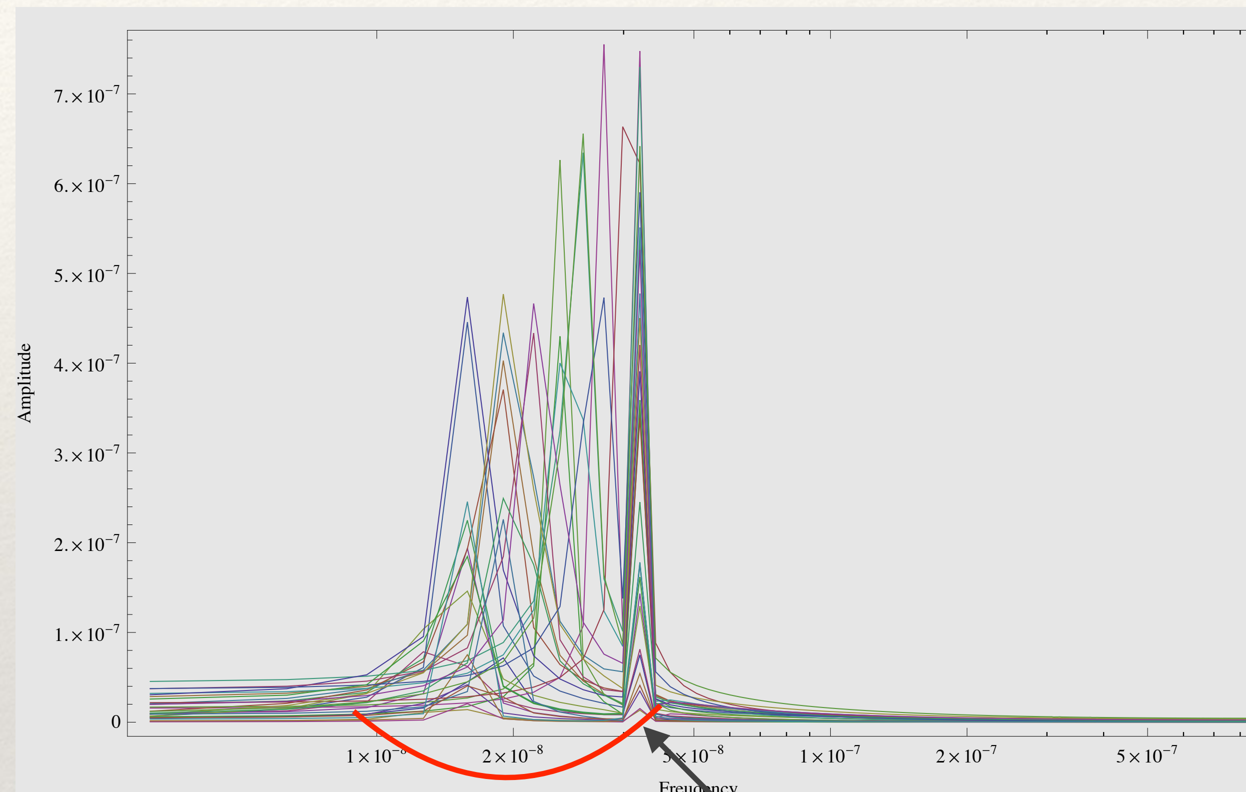
Response to GW signal of PTA in freq. domain

credits: A. Petiteau



Pulsar term contribution

or



Pulsar term contribution

Earth term contribution

Earth term contribution

Detection statistic and search algorithm

- We assume that noise is Gaussian: he likelihood function (likelihood of the signal with given parameters is

$$P(\vec{\delta t}, \vec{\theta}) = \frac{1}{\sqrt{(2\pi)^n det(C)}} \exp\left(-\frac{1}{2}(\vec{\delta t} - \vec{s})^T C^{-1} (\vec{\delta t} - \vec{s})\right),$$

- $\overrightarrow{\delta t}$ - concatenated residuals from all pulsars in the array: total size n
- $\vec{s}$ - is a model of deterministic signals (for example GW signals from individually resolvable SMBHBs
- C is the noise variance-covariance matrix (size $n \times n$)

$$C_{\alpha i,\beta j} = \underbrace{C^{wn}\delta_{\alpha\beta}\delta_{ij}}_{\text{white measurement noise}} + \underbrace{C^{rn}_{ij}\delta_{\alpha\beta}}_{\text{red noise spin noise}} + \underbrace{C^{dm}_{ij}\delta_{\alpha\beta}}_{\text{dispersion variation noise}} + \underbrace{C^{GW}_{\alpha i,\beta j}}_{\text{stochastic GW signal}} + \ldots$$

Noise modelling in PTA

- White noise — not very interesting. two parameters per backend per pulsar: unaccounted noise.
- Red noise: very generic noise description in freq. domain

$$S(f) = A_{rn}^2 f^{-\gamma}$$

common, uncorrelated red noise

$$S_\alpha(f) = A_{rn,\alpha}^2 f^{-\gamma_\alpha}$$

red noise in each. pulsar

- DM (dispersion measurement variation) noise: depends on the radio-frequency of observation

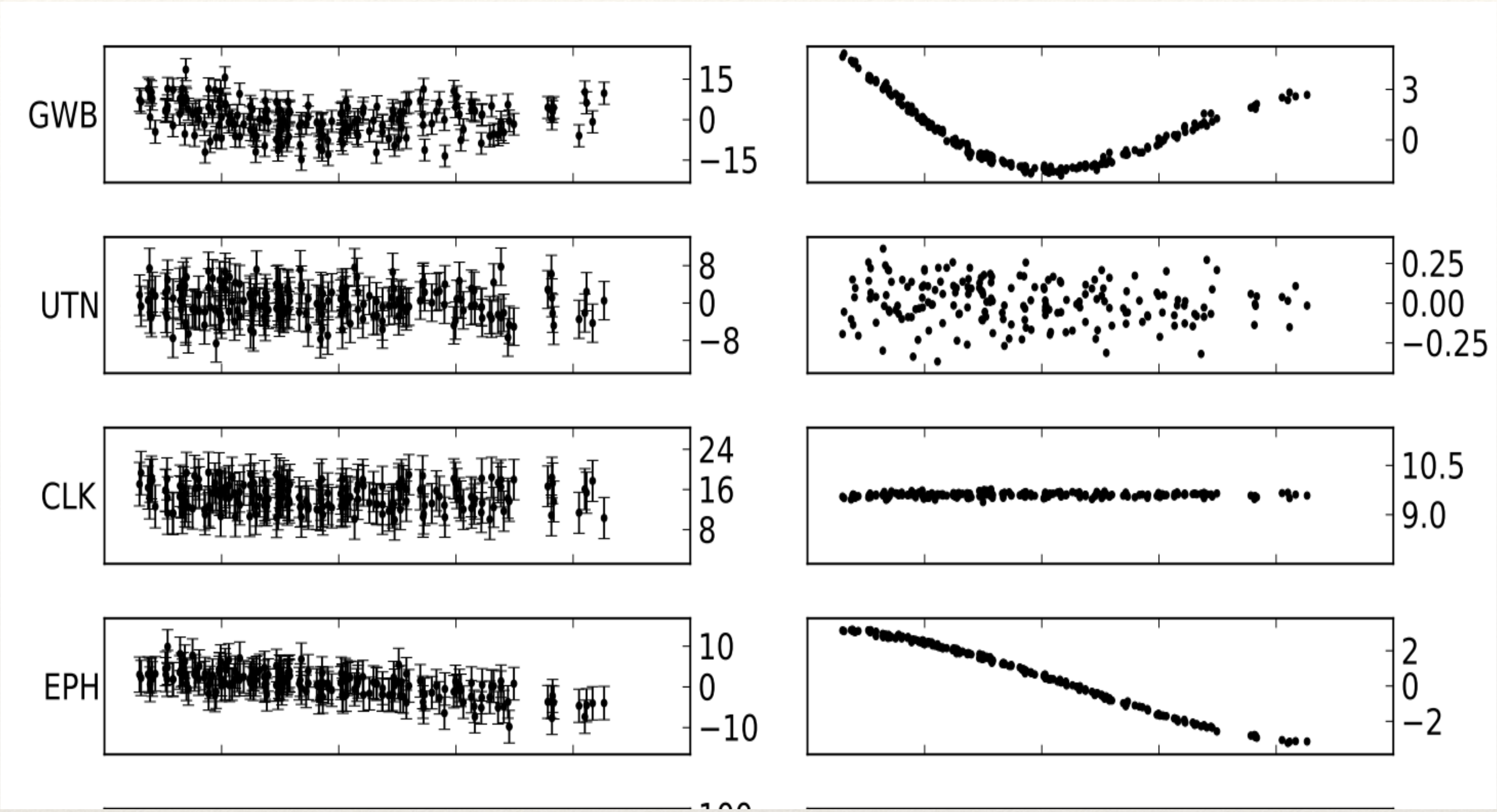
$$S_{DM}(f) \propto \frac{A_{dm}^2}{\nu^2} f^{-\gamma_{dm}}$$

- Correlated red noise processes

$$S_{\alpha\beta} = \Gamma_{\alpha\beta} A_{cor}^2 f^{-\gamma_{cor}}$$

— includes also cross spectrum between each pair of pulsars: $\Gamma_{\alpha\beta}$ - spacial correlation coefficients

Correlated noise



GWB (Quadrupole)

Uncorrelated RN

Clock error (monopole)

Ephemeris (dipole)

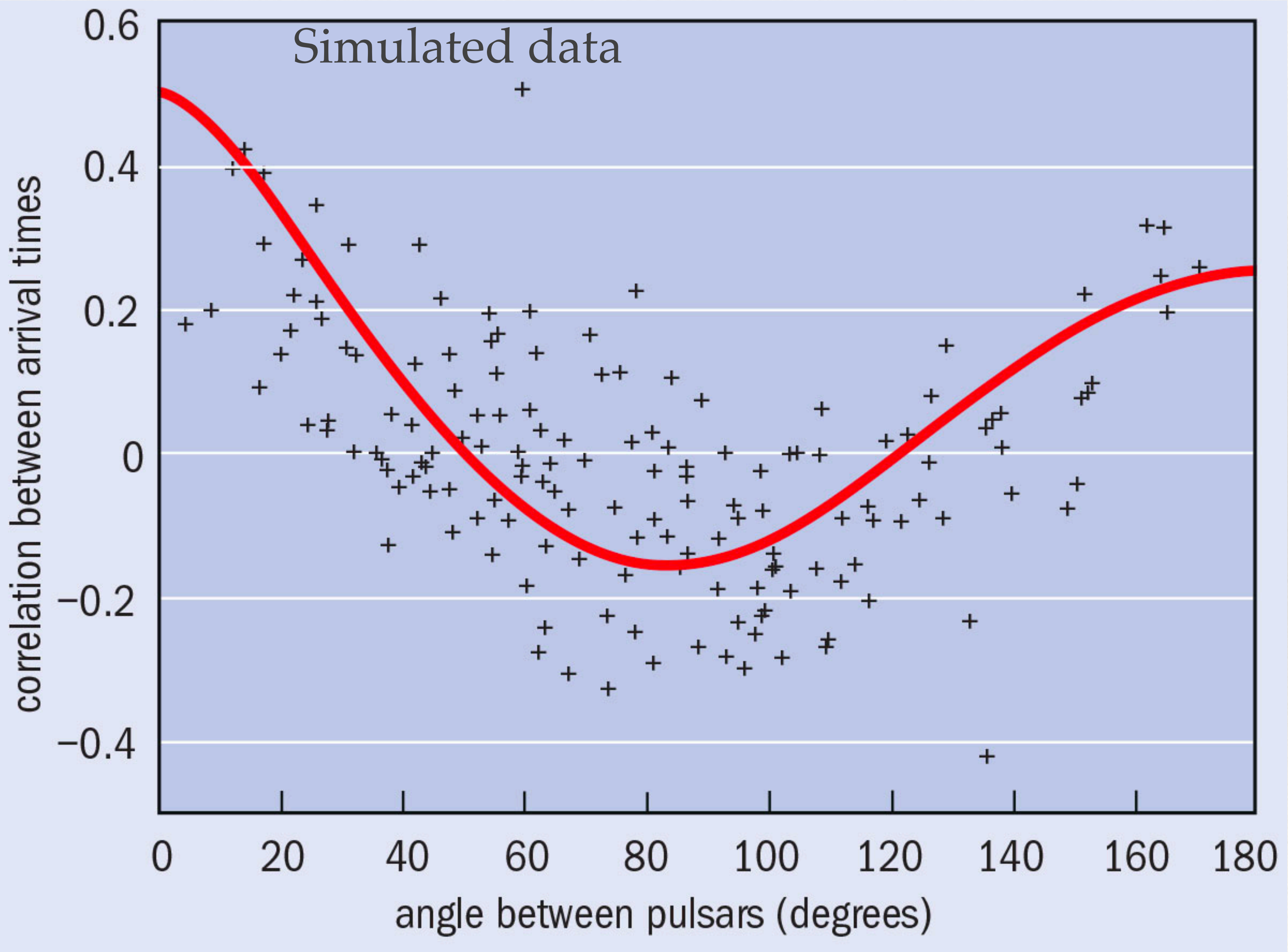
TOAs, time

Correlation, angular separation

stochastic GW from population of SMBHBs: $S_{\alpha\beta}^{SMBHB} = \Gamma_{\alpha\beta}^{H-D} A_{GW}^2 f^{-13/3}$

Hellings-Downs curve: stochastic GW signal

- Stochastic GW signal — noise like signal which is correlated in observation of all pulsars. The correlation due to GW is very specific: Hellings-Downs curve.
- Correlation for the isotropic stochastic GW signal depends only on the angular separation between the pairs of pulsars.



[Fig. from IOP, Physics World]

Gaussian-process approach to PTA: falling into a rabbit hole

Short intro into a Gaussian Process (GP)

- GP generalize the notion of Gaussian randiom variables to the case of infinite number of degrees of freedom
- GP can be specified in 2 equivalent ways:
    - as a sum of deterministic basis functions: $\sum_i \phi_i(x) w_i$ - where w_i are weights - Gaussian random variables $\mathcal{N}(w_i^0, \Sigma_{ij})$. weight-space view
    - as a continuous f-n $f(x)$ such that the ensemble average $\mathbb{E}[f(x)] = m(x)$ and the covariance: $\mathbb{E}[(f(x) - m(x))(f(x') - m(x'))] = k(x, x')$. function-space view
- Those two approaches are connected by

$$k(x, x') = \sum_{i,j} \phi_i(x) \Sigma_{ij} \phi_j(x')$$

Gaussian-process approach to PTA: falling into a rabbit hole

- Applying GP to the PTA likelihood function:

Gaussian prior on weights

GP

$$p(\delta t|w_i, GP) = \frac{e^{-\frac{1}{2}\cdot\sum_{ij}(\delta t_i - \sum_a \phi_a(t_i)w_a)(C^w_{ij})^{-1}(\delta t_j - \sum_a \phi_a(t_j)w_a)}}{\sqrt{(2\pi)^n \det(C^w)}} \times \frac{e^{-\frac{1}{2}\sum_{a,b} w_a(\Sigma_{ab})^{-1}w_b}}{\sqrt{(2\pi)^m \det(\Sigma)}}$$

weight-space approach

white noise

$$p(\delta t|w_i, GP) = \frac{e^{-\frac{1}{2}\cdot\sum_{ij}\delta t_i(C^w_{ij}+C^{rn}_{ij})^{-1}\delta t_j}}{\sqrt{(2\pi)^n \det(C^w + C^{rn})}} \quad \text{with} \quad C^{rn}_{ij} = k(t_i, t_j) = \sum_{a,b}\phi_a(t_i)\Sigma_{ab}\phi_b(t_j)$$

red noise variance-covariance matrix

In time domain, uncorrelated red noise:

$$C^{rn}_{ij} = A^2(f_L/\mathrm{yr}^{-1})\left\{\Gamma(1-\gamma)\sin\left(\frac{\pi\gamma}{2}\right)(f_L\tau_{ij})^{\gamma-1} - \sum_n \frac{(-1)^n(f_L\tau_{ij})^{2n}}{(2n)!(2n+1-\gamma)}\right\}$$

where $\tau_{ij} = |t_i - t_j|$ and f_L is low freq. cut-off

Gaussian-process approach to PTA: falling into a rabbit hole

- Alternatively we can use basis functions: based on the decomposition of residuals in the Fourier modes:

$$\delta t(t_i) \approx \sum_k a_k \sin 2\pi f t_i + b_k \cos 2\pi f t_i$$

weights

basis functions $\phi^F(f_a, t_i) = \phi^F_a(t_i)$

We use non-complete set of Fourier modes: covariance matrix can be approximated as

$$C_{ij}^{rn} \approx \sum_{a,b} \phi_a^F(t_i) \Sigma_{ab}^F \phi_b^F(t_j)$$ where

$$\Sigma_{ab}^F \propto \left(A_{rn}^2 f_a^{-\gamma}\right) \delta_{ab}/T$$ — red noise PSD

and for stochastic GW signal: $C^{GW}_{i\alpha,j\beta} = \sum_{i\alpha,j\beta} \Gamma_{\alpha\beta} \phi^F_a(t_{i\alpha}) \Sigma^{F,GW}_{ab} \phi^F_b(t_{j\beta})$, where

$$\Sigma^{F,GW}_{ab} = (A^2_{GW} f_a^{-\gamma_{gw}}) \delta_{ab}/T$$

Gaussian-process approach to PTA: falling into a rabbit hole

Advantage of this description: again likelihood

$$p(\delta t|w_i, GP) = \frac{e^{-\frac{1}{2}\cdot\sum_{ij}\delta t_i (C^w_{ij}+C^{rn}_{ij})^{-1}\delta t_j}}{\sqrt{(2\pi)^n \det(C^w + C^{rn})}}$$

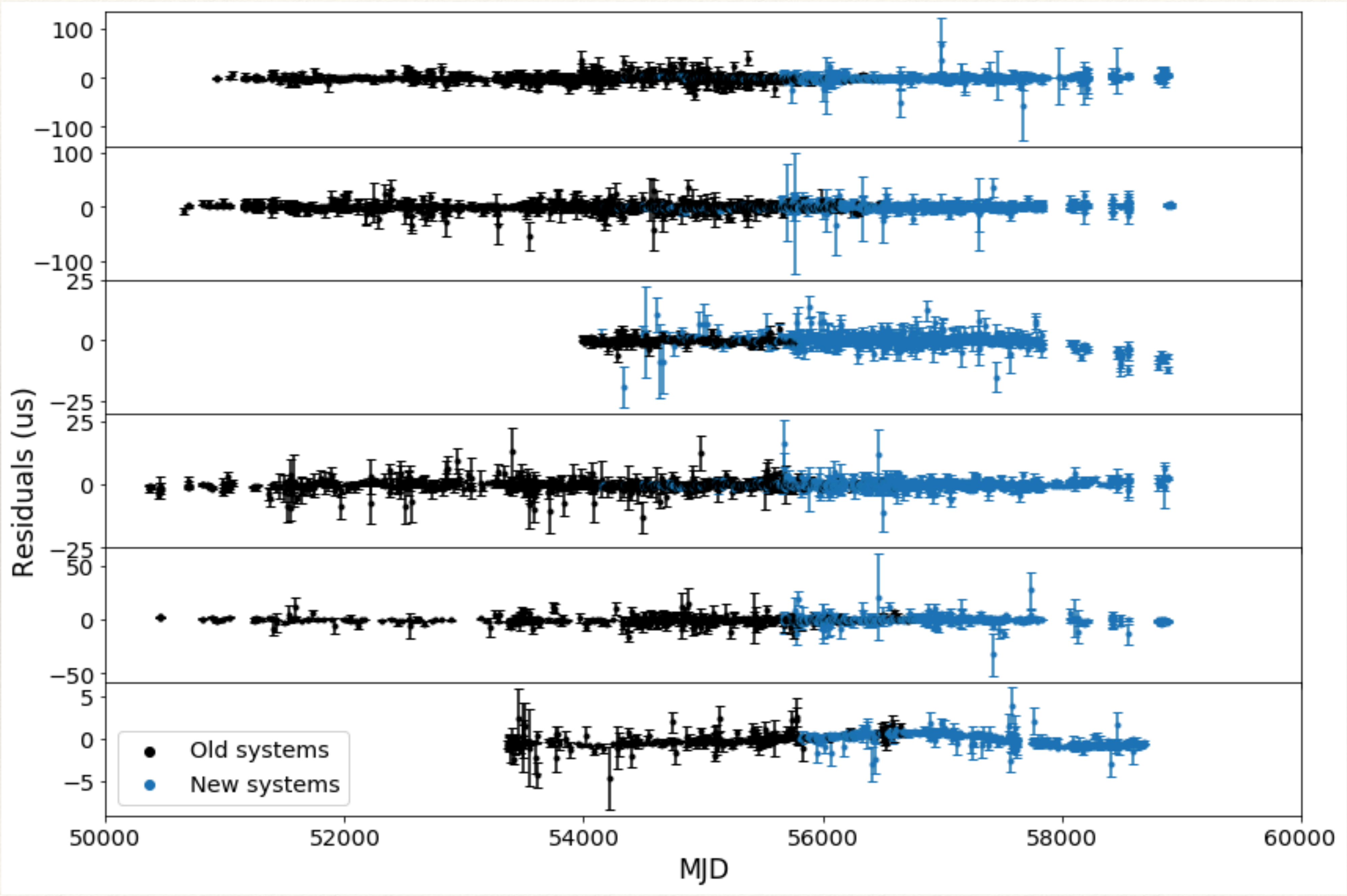
Data size: n - large, need to invert very large (covariance) matrices - $n \times n$
Can use Woodbury f-la

$$(C_w + C_{rn})^{-1} = (C_w + \Phi\Sigma\Phi^T)^{-1} = C_w^{-1} - C_w^{-1}\Phi \underbrace{\left(\Sigma^{-1} + \Phi^T C_w^{-1}\Phi\right)^{-1}}_{\text{inversion of } m \times m \text{ matrix}} \Phi^T C_w^{-1}$$

Number of modes: $m \ll n$ much faster and easier to invert, C_w is diagonal matrix

Residuals of 6 best EPTA pulsars

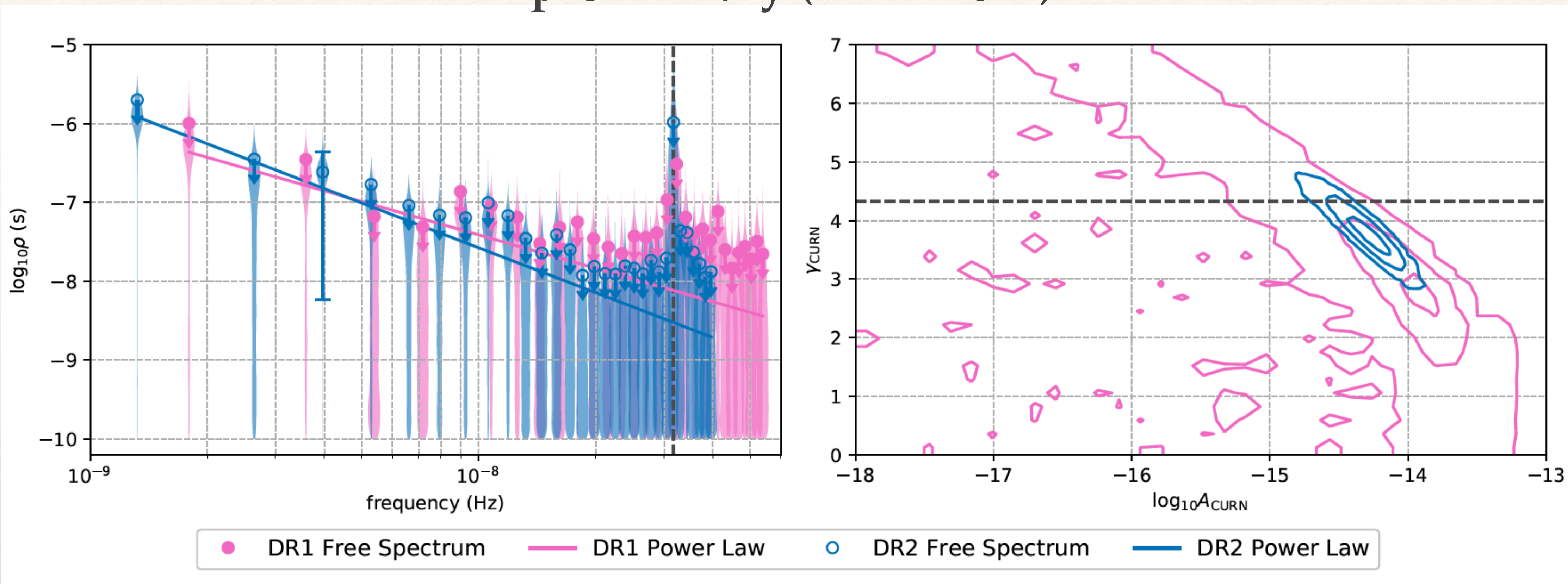
From top to bottom these are PSRs: J0613-0200, J1012+5307, J1600-3053,J1713+0747, J1744-1134, and J1909-3744

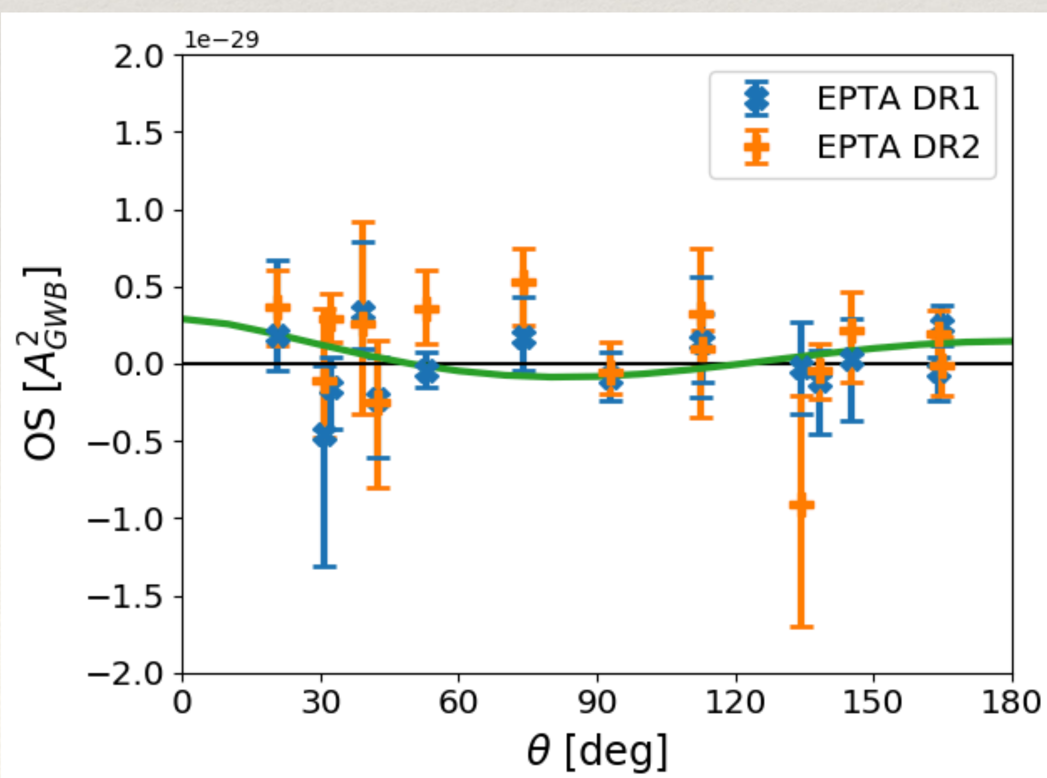


Up to 25 years of monitoring: black - DR1, blue - DR2 (data release)

Common red noise in EPTA data

preliminary (EPTA 2021)





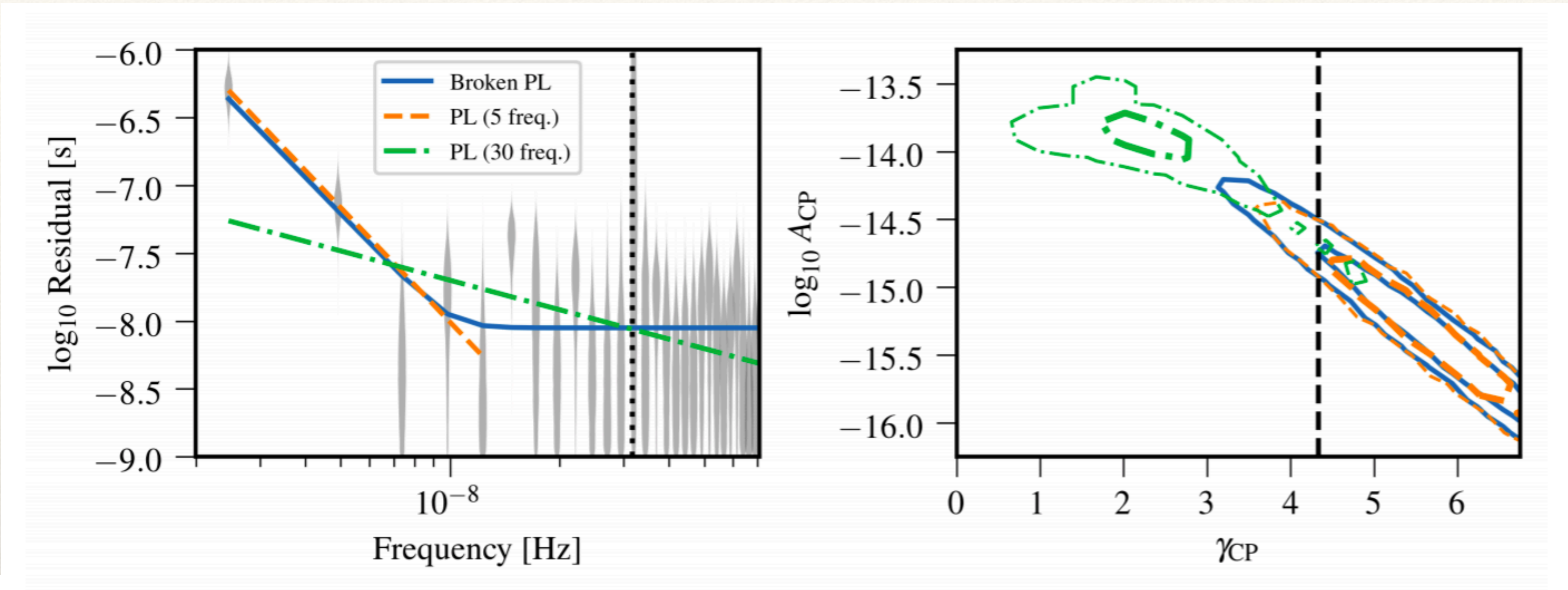
There is a strong statistical support for presence of common red noise

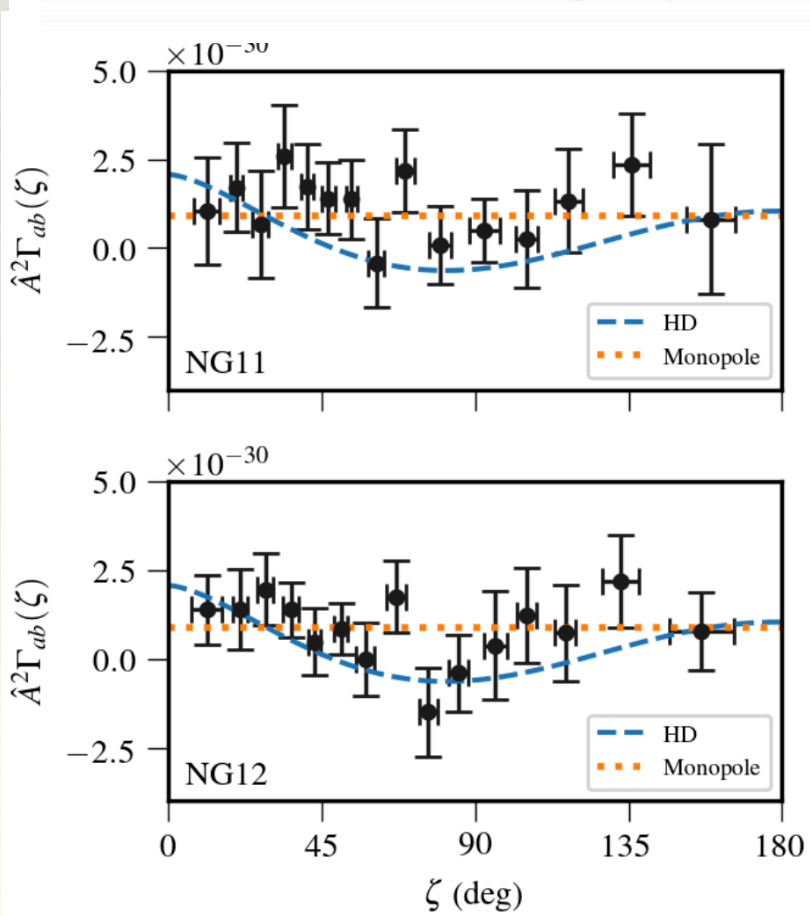
$$S(f) = A_{rn} f^{-\gamma}$$

common, uncorrelated red noise

NanoGrav 12.5 yrs data result

[Arzoumanian+ 2020]





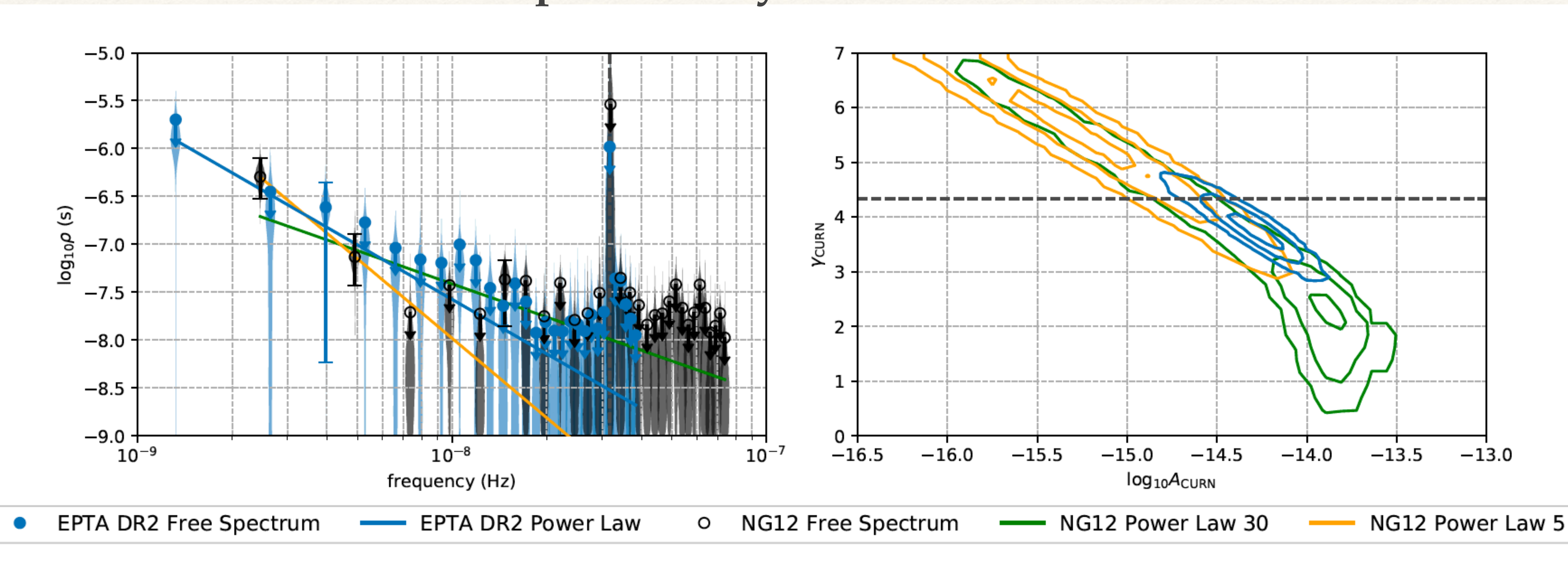
There is a strong statistical support for presence of common red noise

$$S(f) = A_{rn} f^{-\gamma}$$

common, uncorrelated red noise

Common uncorrelated red noise

NanoGrav , EPTA and PPTA support presence of a common red noise
preliminary (EPTA 2021)



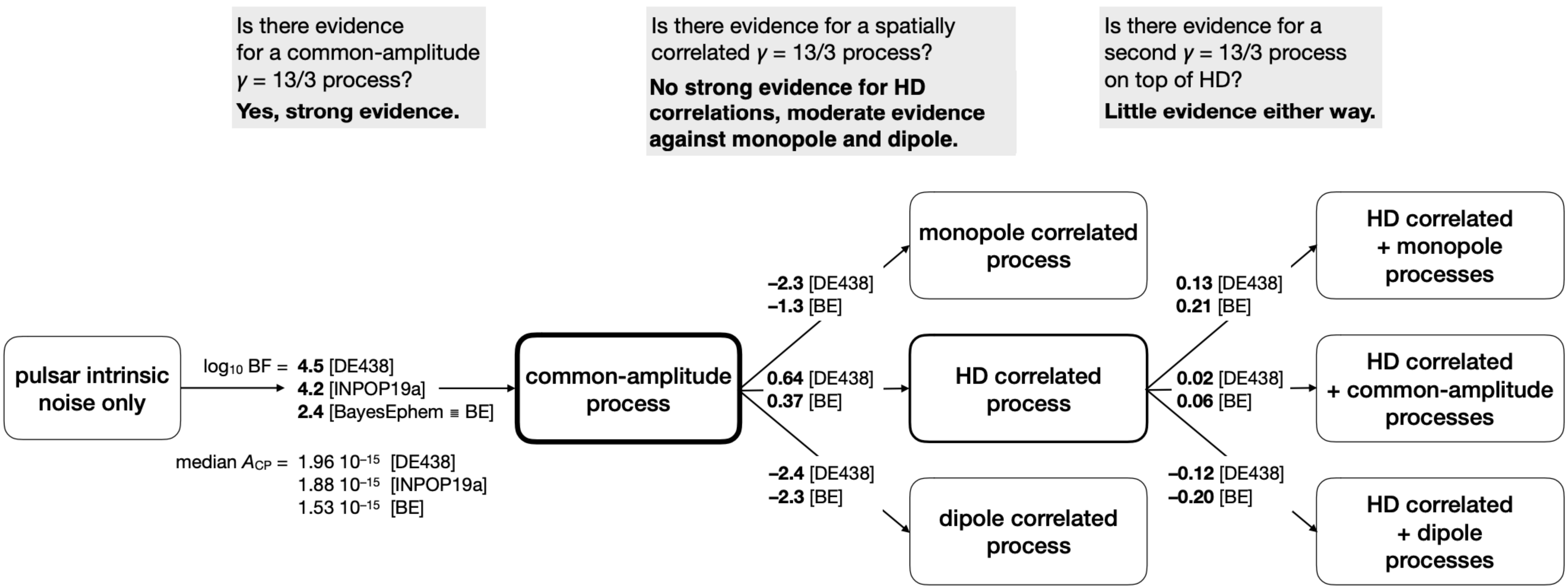
Bayesian analysis: model selection (hypothesis testing)

Odd ratio: $$O(M_1, M_2) = \frac{p(M_1\,|\,d)}{p(M_2\,|\,d)} = \underbrace{\boxed{\frac{p(d\,|\,M_1)}{p(d\,|\,M_2)}}}_{\text{Bayes factor}}\frac{\pi(M_1)}{\pi(M_2)}$$

Stochastic GW signal?

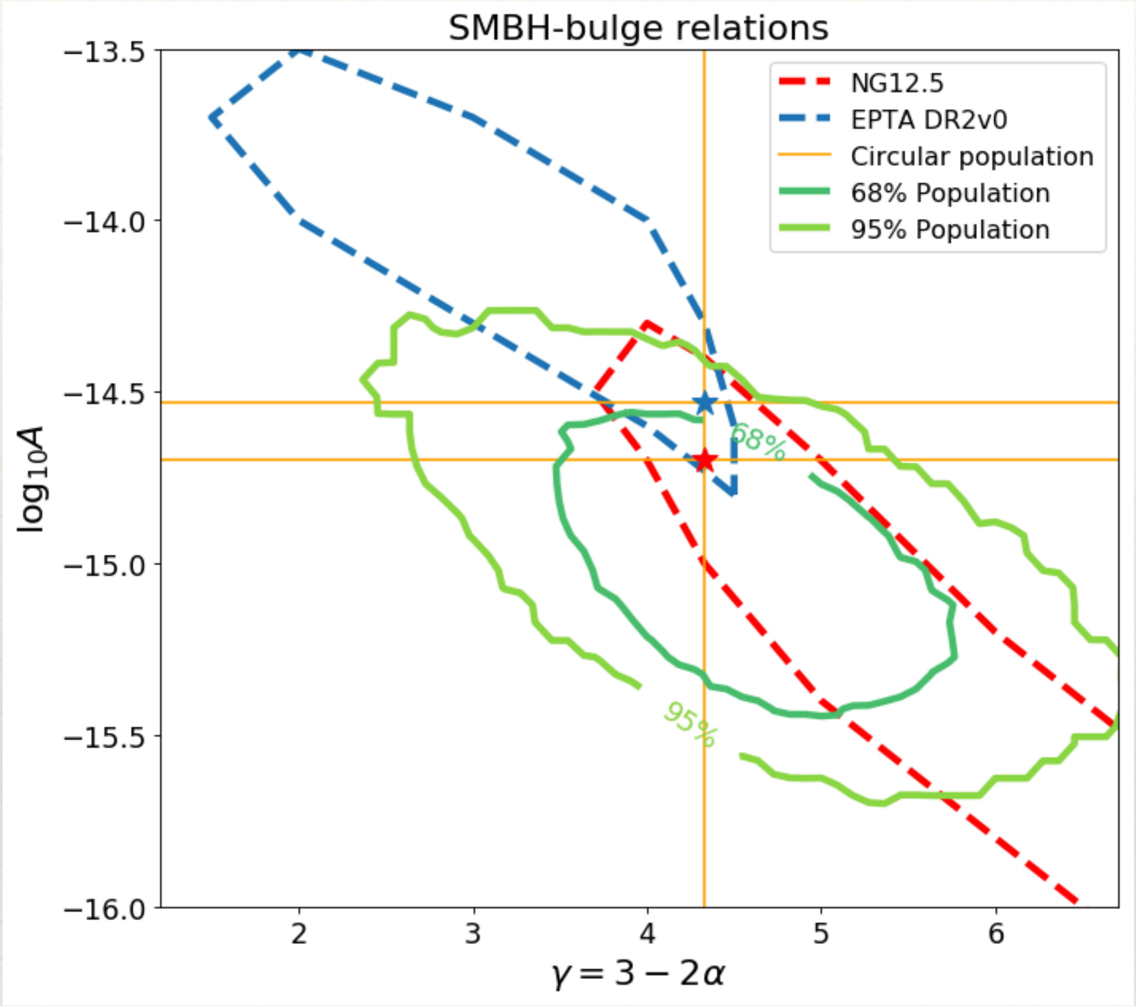
[Arzoumanian+ 2020]

THE NANOGRAV COLLABORATION



- No statistical significance to support Hellings-Downs spacial correlation:
  - Need more pulsars to compute more pair-wise correlations (EPTA->20)
  - Need longer data set to uncover more of the red signal (Nanograv-> 15 yrs)

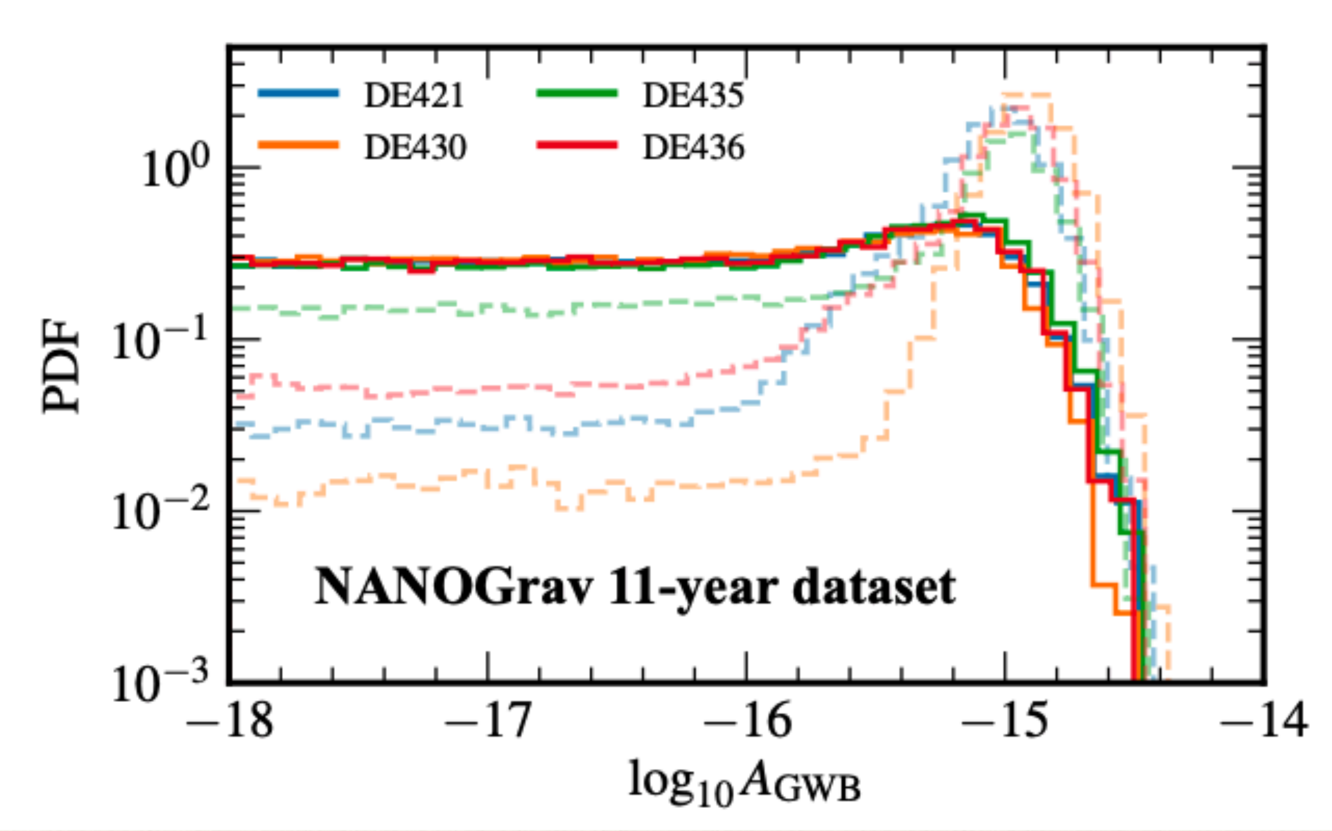
Can it be GW from SMBHs?



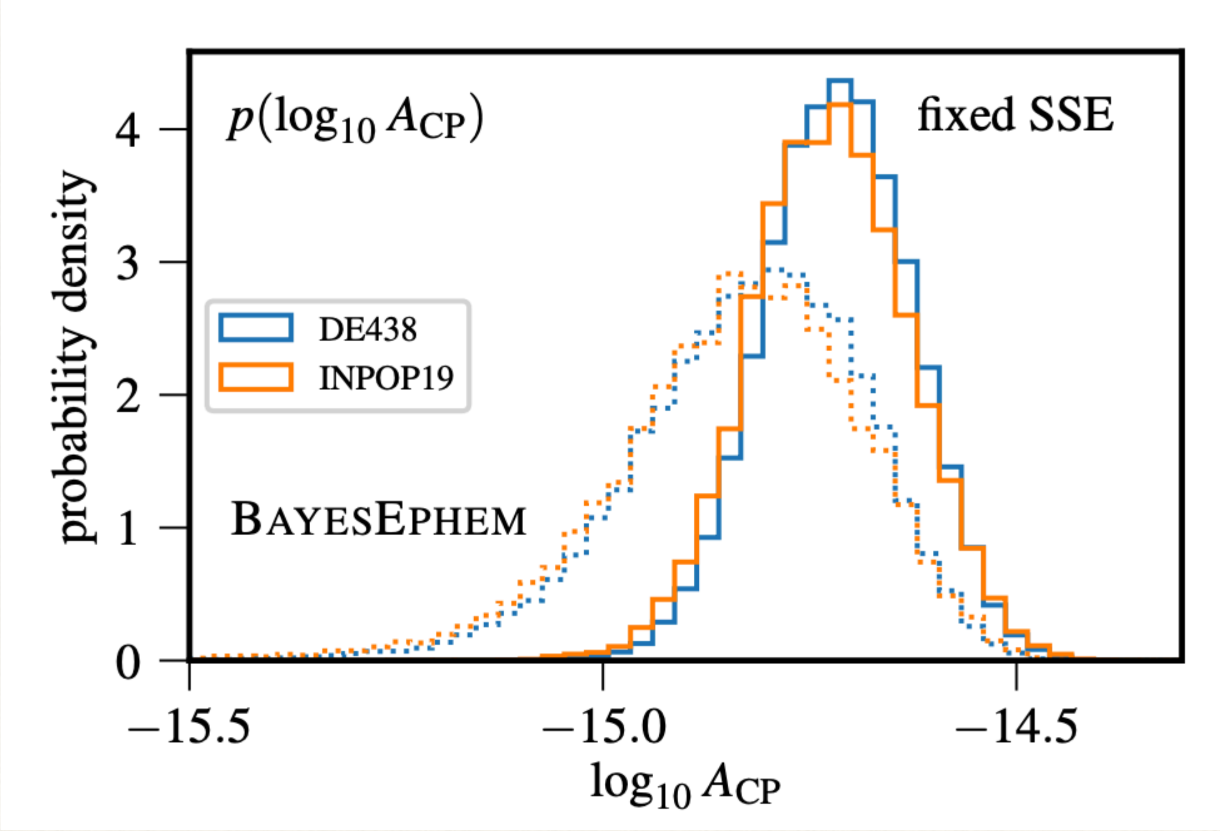
- Analytic prediction: spectral index
- Simulation of SMBHB populations is shown as green contours: wide range spectral indices
- Results of NanoGrav and EPTA are consistent with spectral index from the population of SMBHBs

Solar system ephemeris

- We use Solar system barycenter (SSB) as a reference system to reduce all observations
- The systematic error in SSB (from ephemeris) could create residual (dipolar cos-like spacial correlation) common signal with red-noise like spectrum
- Poorly determined position of SSB
- Use phenomenological model (vary orbital elements of Jupyter and Saturn) to mimick possible systematics (BayesEphem)



[Arzoumanian+ 2018]

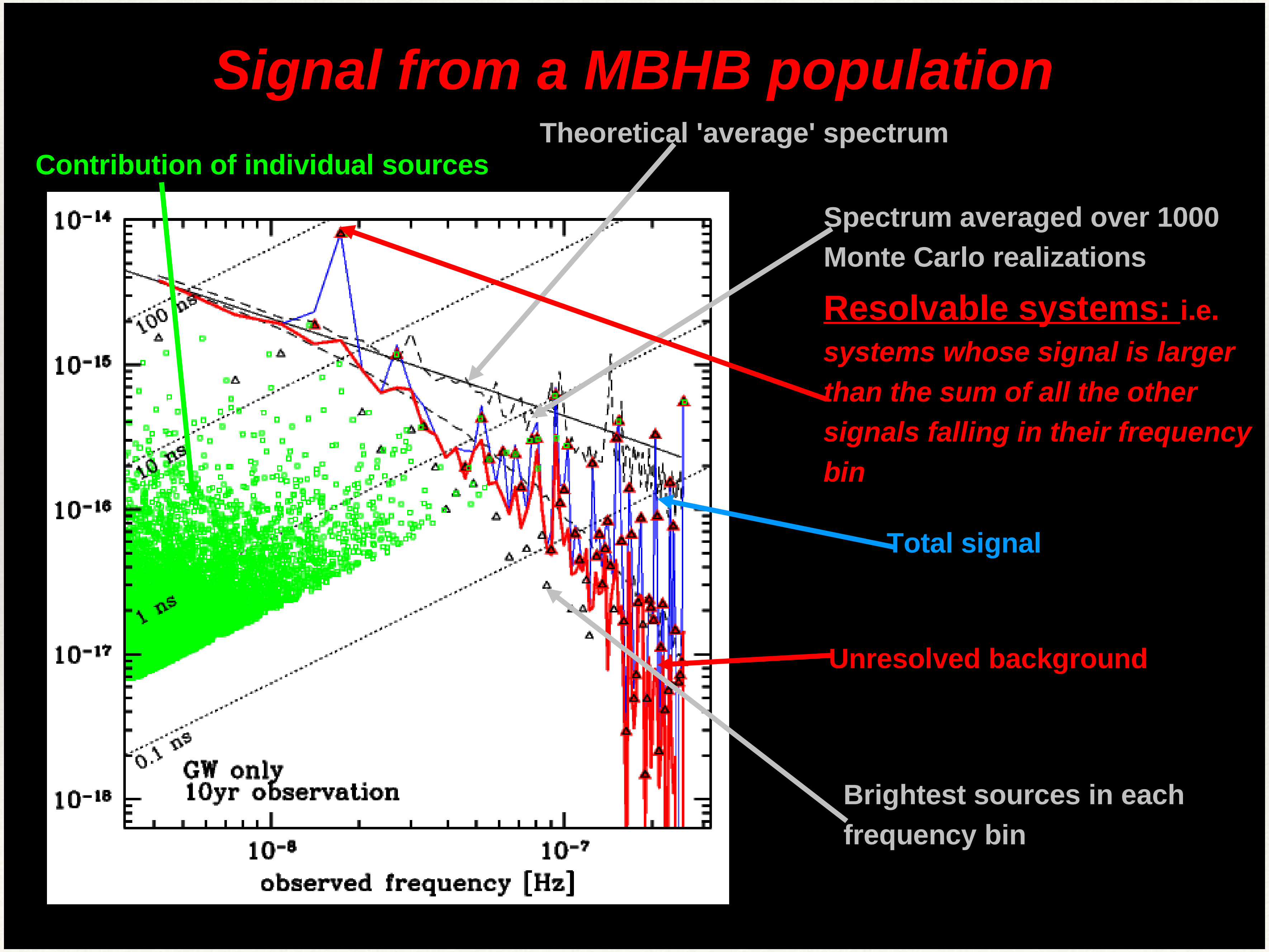


[Arzoumanian+ 2020]

Search for indivdual SMBHBs

Reminder: GW signal(s) from a population of SMBHBs:

- We are now after "loud" individual systems (hot spots) sticking above the stochastic component

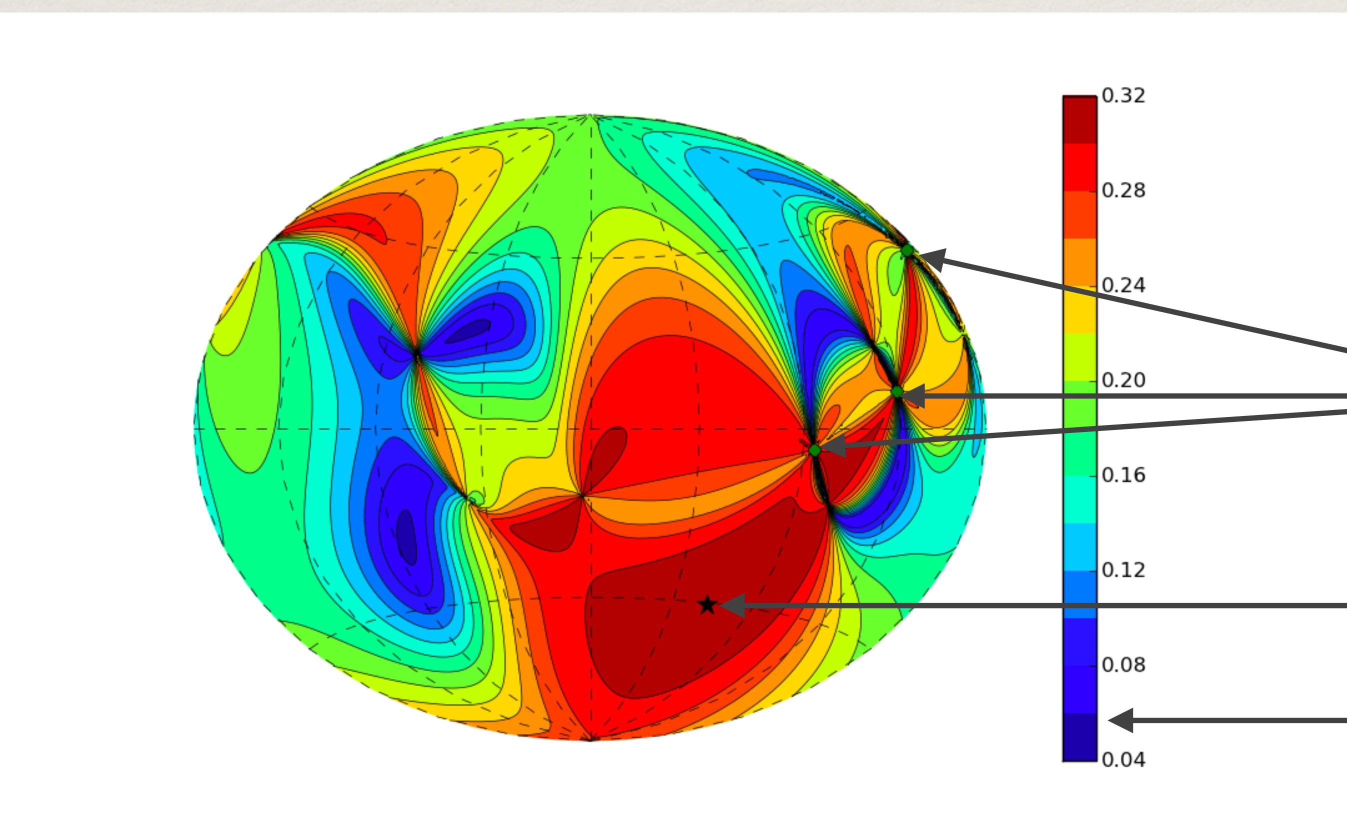


Continuous GW signal

- Each GW signal from SMBHB is characterized by:
  - Earth term: $A, \iota, \psi, \phi_0, f, \theta_{sky}, \phi_{sky}$
  - Pulsar term: L_α, M_c — distance to the pulsar (poorly known), chirp mass
  - In total $8 + N_p$ parameters
- Each pulsar gives 2 measurements: (real and imaginary at each freq.)
- Earth term depends on 6 params (for a given freq.)
  - We need at least 3 pulsars per GW source for parameter estimation

SIMULATED DATA

Sky map: 3 pulsars and 1 GW source



Pulsars

Source

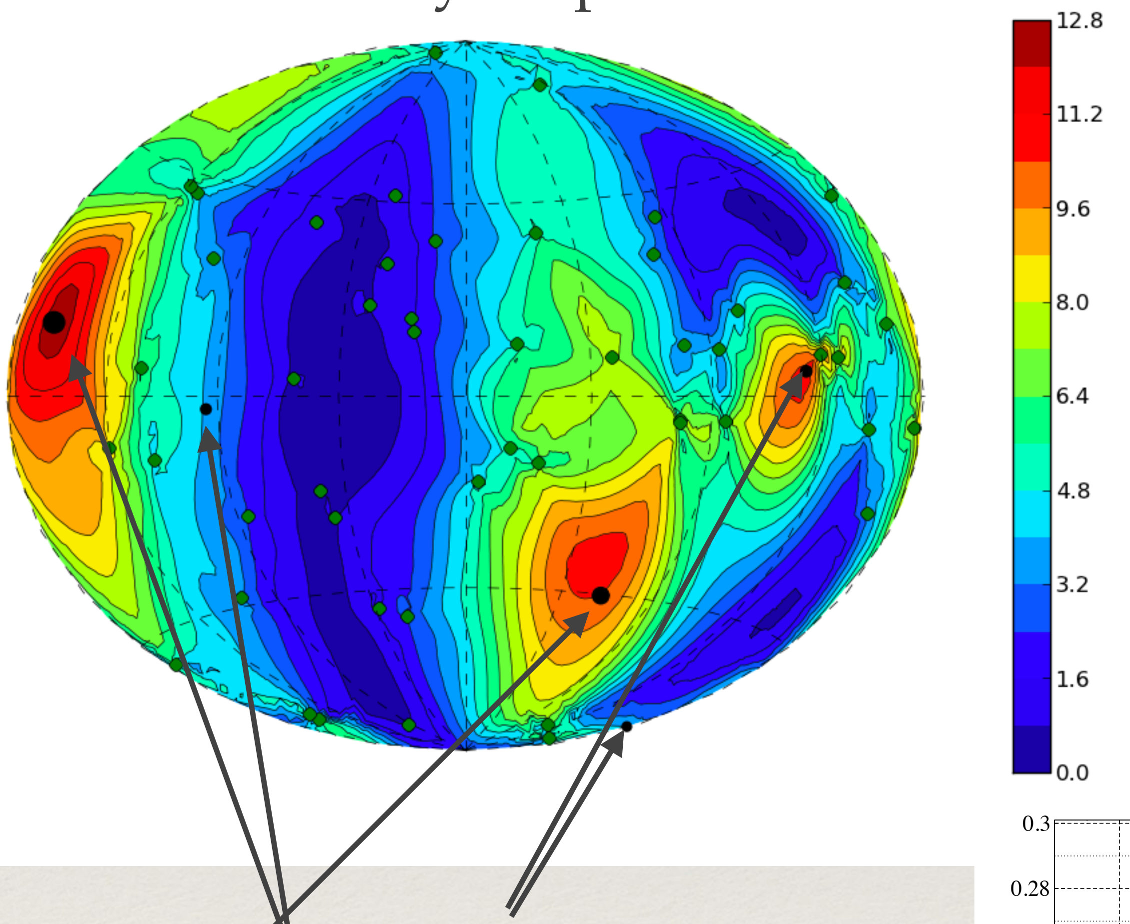
scaled likelihood

Continuous GW signal

Another example: 5 GW sources, and 50 pulsars.
Assume that there is only 1 GW source.

SIMULATED DATA

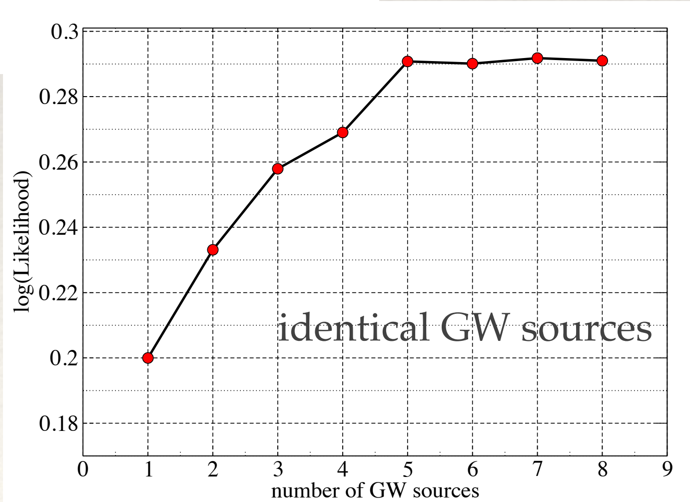
The likelihood sky map

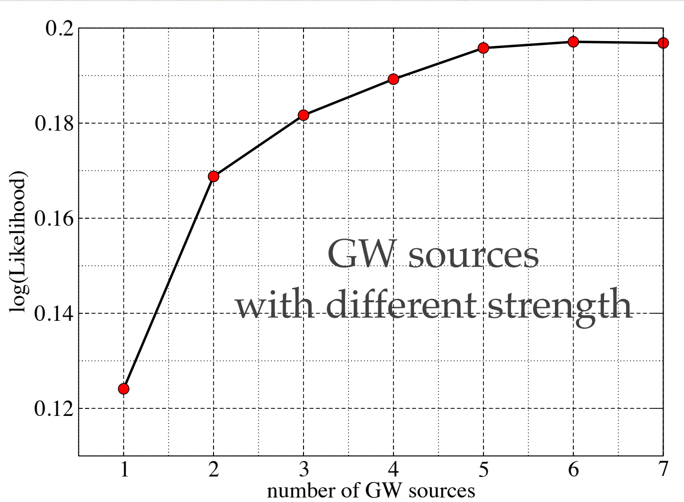


GW sources

With 1-source model we resolve three strongest sources: size of black circle is proportional to GW strain

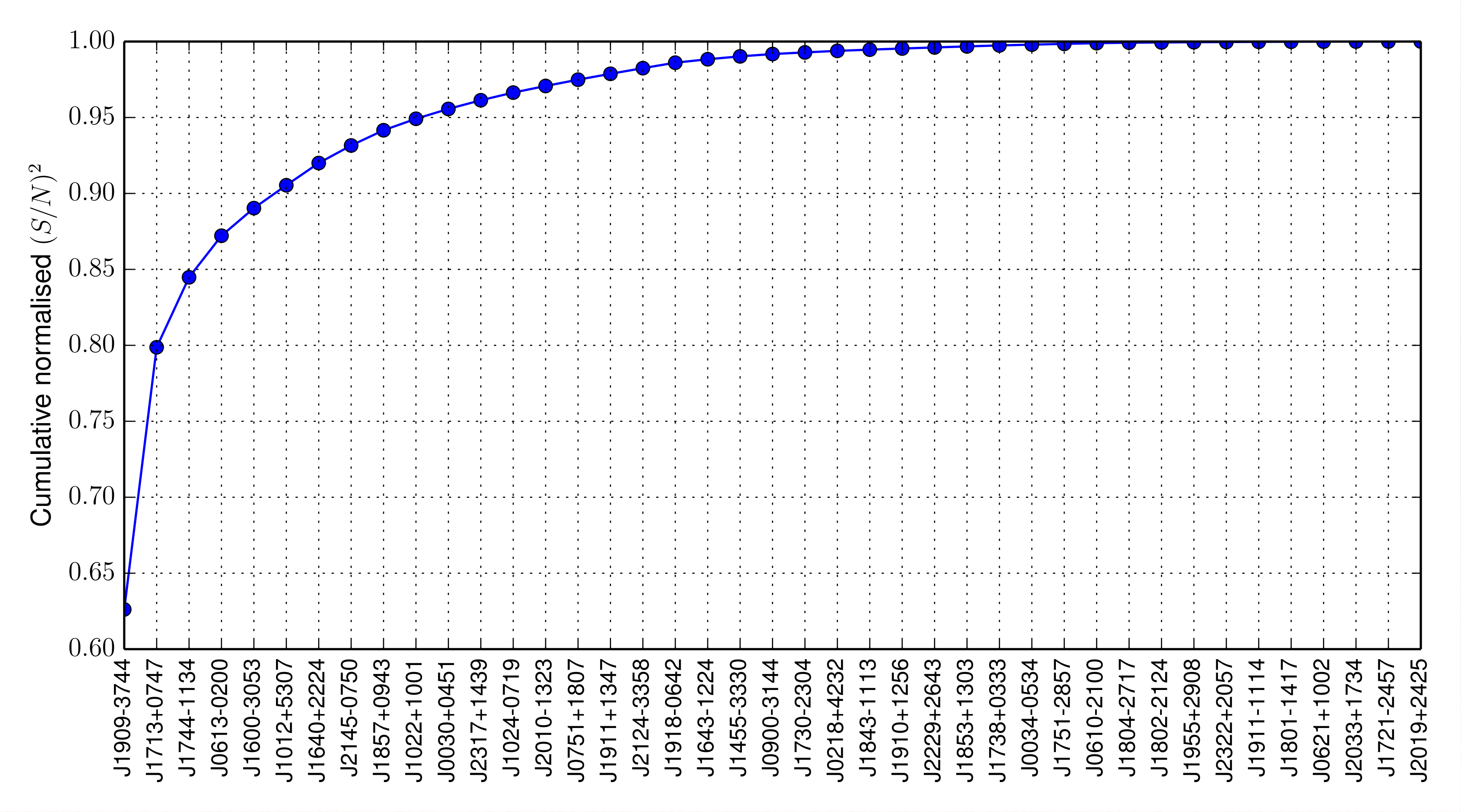
Likelihood for 2,3,4,5,6,7-source model





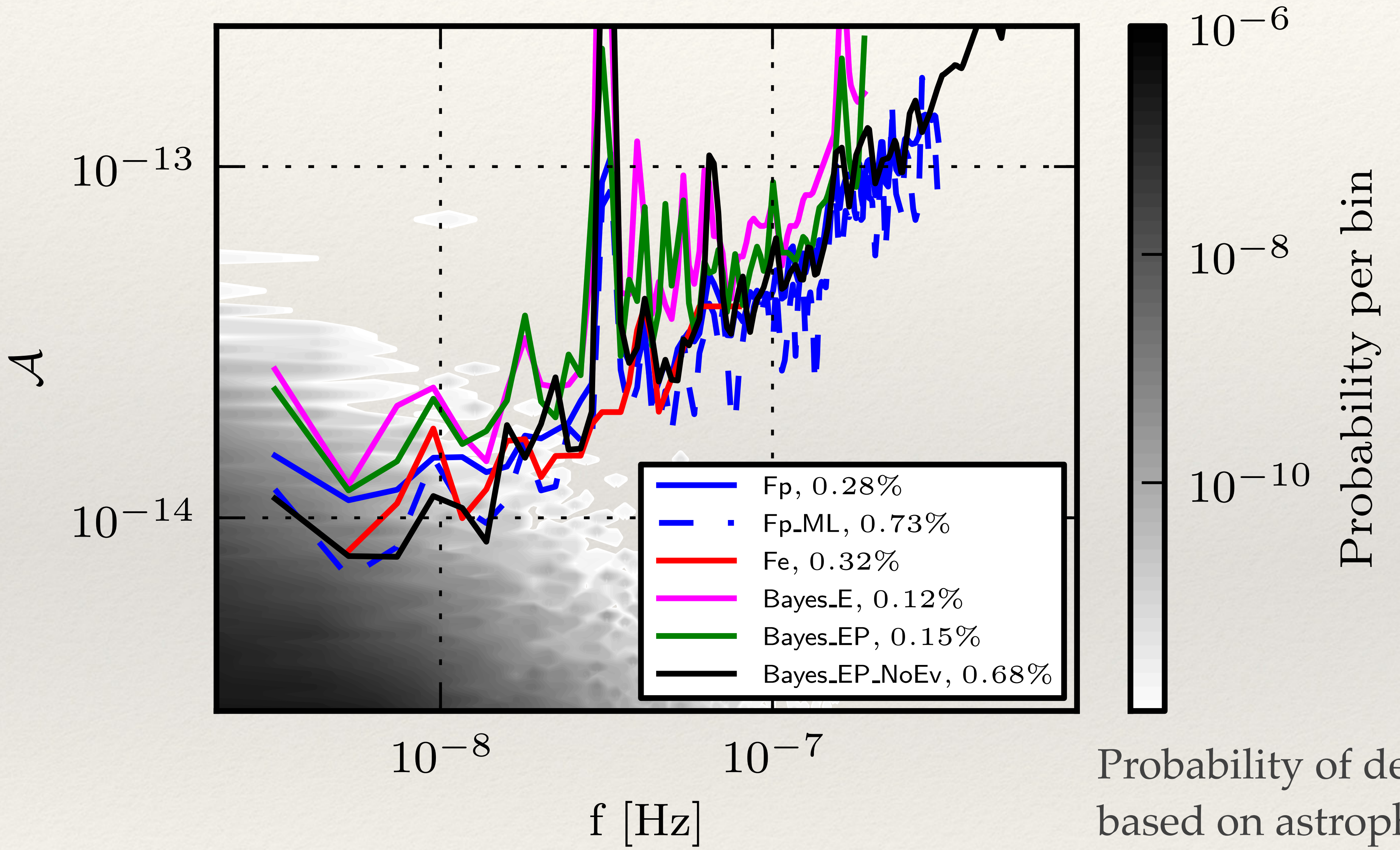
Continuous GW signal (EPTA)

- Search for continuous GW signal using frequentist and Bayesian techniques
  - analytic maximization (marginalization) over some parameters
- Search for continuous GW signal using earth-term only (coherent) or using earth+pulsar term (more expensive)
- Pulsar ranking: 41 pulsar in EPTA data, search is expensive - rank pulsars by “goodness” - how much they contribute to the total signal-to-noise ratio. Monte-Carlo simulation



Upper limit on continuous GW signal in EPTA data

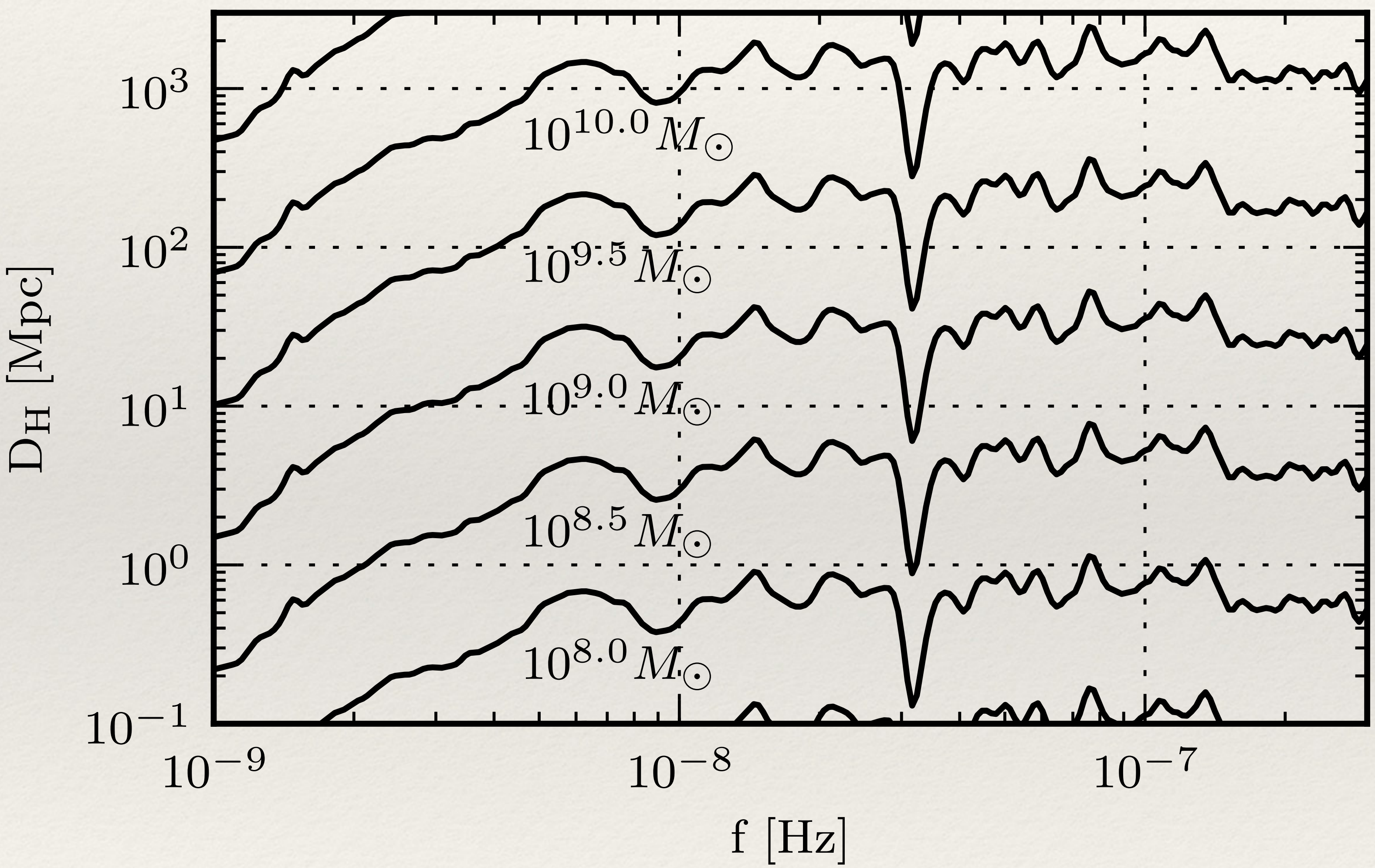
upper limit of GW strain using different statistics, methods, frameworks



Probability of detecting GW source based on astrophysical simulation of SMBHB population

Upper limit on continuous GW signal in EPTA data

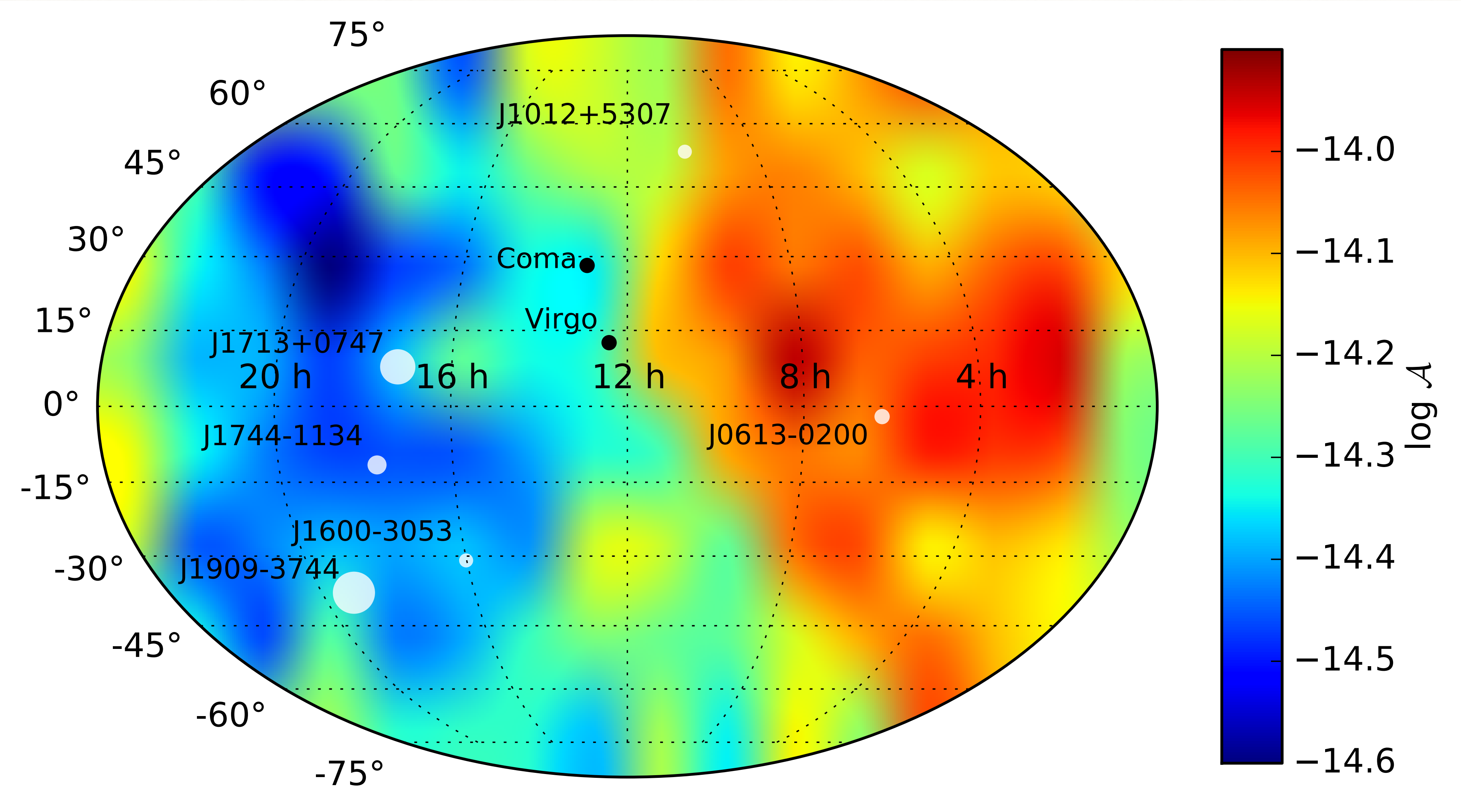
Upper limit distance: distance up to which we have not detected a circular binary of a particular mas



Upper limit on continuous GW signal in EPTA data

Directional upper limit (sky map) at 7nHz (best EPTA DR1 frequency)

- white circles: pulsars used to set upper limit, size proportional to “goodness”
- two nearest supeclusters: Coma and Virgo



[Babak+ 2015, EPTA]

What is next?

- NanoGrav: require longer observations (combining 15 yrs of data)
- EPTA: (i) need to finish analysis of 6 best pulsars, (ii) need to include more pulsars (20) to confirm H-D correlations
- PPTA: have very long observations and few very good MSP (south): another confirmation of common red process
- IPTA: combine all data together to see if significance grows as expected.
- Need to confirm GW (if it is GW signal) using methods [Cornish+ 2016, Taylor+ 2017] to destroy correlations and test statistical significance of our findings (preserving the noise properties)
- Wait longer:
  - new high quality data SKA (MeerKAT), Fast, …
  - Check SNR as a function of time

$< SNR > \propto T^{\gamma} \to \propto T^{1/2}, \gamma > 1$ — RN spectral index (e.g. $13/3$ for stochastic GW signal from SMBHBs)