Cosmology with gravitational waves

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A review of FRWL kinematic

- On large (Gpc) scales, the Universe is homogeneous and isotropic
- Friedmann-Robertson-Walker-LeMaitre (FRWL) metric

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right]$$

- $a(t)$ is the scale factor
- $k = -1,0,1$ for an open, flat, closed universe
- (t, r, θ, ϕ) are the *comoving* coordinates

A review of FRWL kinematic

- Source at co-moving distance r emitting at t_{emis}
- Signal detected at t_{obs}. From $ds^2 = 0$: $\int_{t_{emis}}^{t_{obs}} \frac{cdt}{a(t)} = \int_0^r \frac{dr}{\sqrt{1-kr^2}}$
- Considering a second pulse emitted at $t_{emis} + \Delta t_{emis}$, leads to $\Delta t_{obs} = \frac{a(t_{obs})}{a(t_{emis})} \Delta t_{emis}$
- The cosmological redshift z is defined as $1 + z = \frac{a(t_{obs})}{a(t_{emis})}$

Cosmological time dilation

- accounts for the time dilation measured by the observer compared to the time of the source

$$dt_{obs} = (1 + z)dt_{emis}$$

- So, one would measure a frequency

$$f_{obs} = \frac{f_{emis}}{1 + z}$$

Luminosity distance

- Consider
  - flux F (energy per unit time per unit area) at the observer
  - absolute luminosity $L = \frac{dE_s}{dt_s}$ in the source proper frame
- The luminosity distance d_L is defined via

$$F = \frac{L}{4\pi d_L^2}$$

Luminosity distance

- From $\frac{dE_o}{dt_o} = \frac{1}{(1+z)^2}\frac{dE_s}{dt_s}$ and the fact that in a FRWL universe the area of a sphere $A = 4\pi a(t)^2 r^2$

$$F = \frac{L}{4\pi a^2(t_o) r^2 (1+z)^2}$$

- Hence $d_L = (1+z)a(t_o)r$

- Taylor expanding $\frac{a(t)}{a(t_o)} \simeq 1 + H_0(t - t_o) + \dots$ one obtains Hubble's Law

$$H_0 d_L \simeq cz \qquad H_0 = \frac{\dot{a}(t_o)}{a(t_o)}$$

Luminosity distance

- In general

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}$$

- The relation between luminosity distance and redshift encodes the whole expansion history of the Universe

Hubble diagram

- Distance vs redshift
- The inverse of the slope is the Hubble constant
  - Rate of expansion of the Universe

$$D \simeq \frac{v}{H_0} = \frac{cz}{H_0} \quad z << 1$$

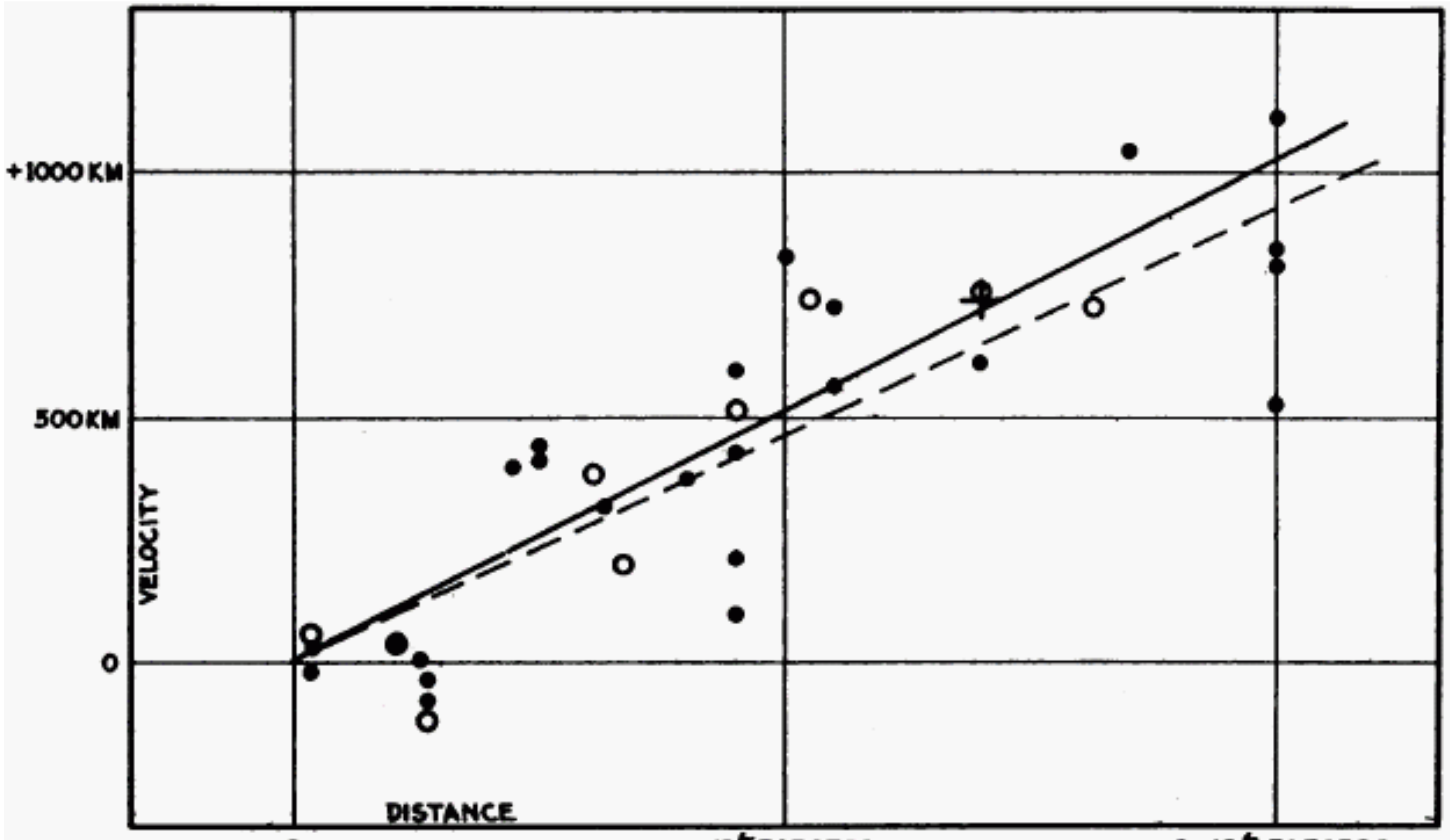


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Hubble, 1929

Propagation of GW in a FRWL universe

- Propagation of GWs in a FRWL universe is particularly interesting
- BBH systems have a unique time scale associated with them

$$\tau_s = \frac{G\mathscr{M}_s}{c^3}$$

- From the cosmological time dilation $\tau_o = (1+z)\tau_s$, so we would detect a system with a chirp mass $\mathscr{M}_o = (1+z)\mathscr{M}$
- Moreover, the amplitude of the GW will scale as d_L^{-1} (see Maggiore, Vol. 1, Sec. 4.1.4 for the full derivation)

Cosmography with GW

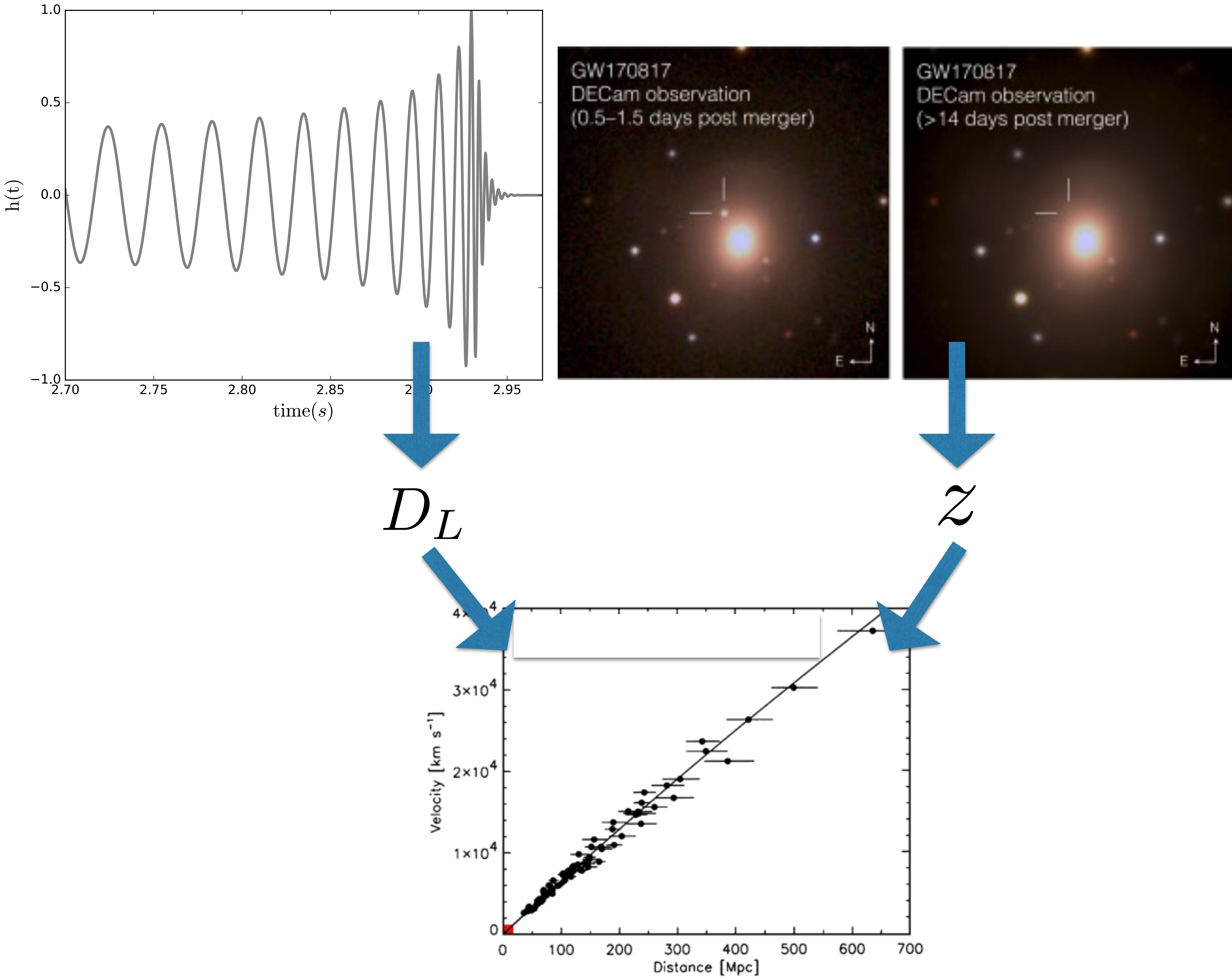
- GW are “self-calibrating” sources (Schutz 1986)

$$h \sim D_L^{-1}$$

  - Direct measurement of luminosity distance
  - “Standard sirens”

- In general, no redshift from GWs (Krolak & Schutz 1987)

$$m_{obs} = m_{src}(1+z)$$

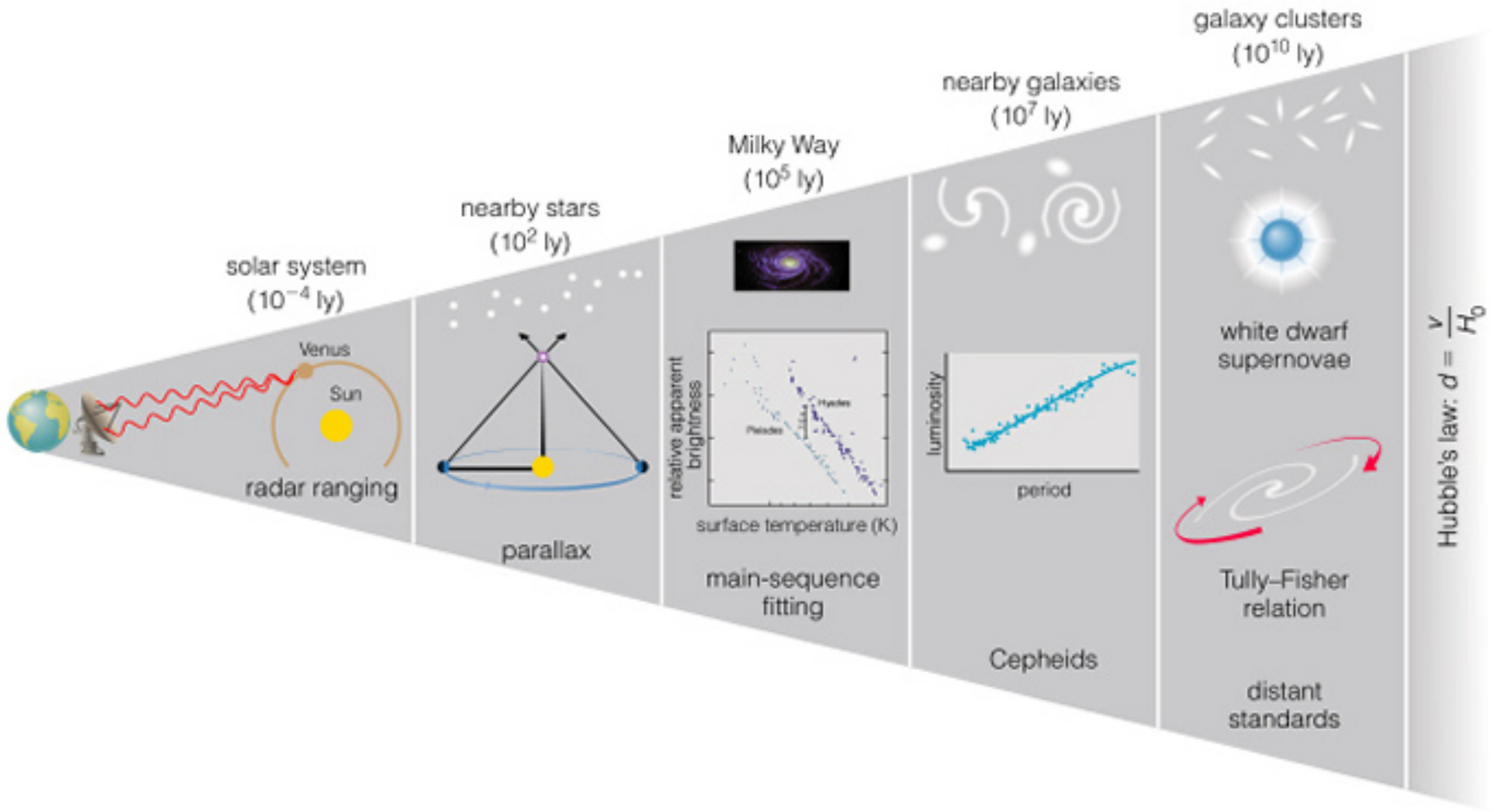


Classical determination of Hubble constant

- Spectroscopic redshift

- Distance requires:
    - identification of “standard candles”
    - cross-calibration of various candles
    - “Iterate and hope it converges” — S. N. Shore

- The “cosmic distance scale ladder”



The redshift problem

- GW cosmology is enabled by measuring the redshift:
    - with EM counterparts (Dalal et al. 2006, Sathyaprakash et al. 2010, Nissanke et al. 2010, Zhao et al. 2011, Del Pozzo 2012, Nissanke et al. 2013, Del Pozzo et al. 2018)
    - without EM counterparts (Chernoff & Finn 1993, Taylor et al. 2012, Taylor & Gair 2013, Messenger & Read 2012, Del Pozzo et al. 2017)

Redshift from a counterpart

- In this case one searches for an electromagnetic counterpart to get the redshift:
    - host galaxy identification (e.g. NGC 4993 & GW170817)
- The redshift information is (almost) certain
    - peculiar velocities
    - weak lensing

Redshift without a counterpart

- Knowledge of some intrinsic property of the system
  - equation of state (EOS) of neutron stars (Messenger & Read 2012)
  - mass function (e.g. Taylor et al. 2012)
- The redshift information is probabilistic
  - posterior distribution for the value of the redshift for each source

Inference of Ω

- Cosmological model H with cosmological parameters Ω, observation D

$$p(\Omega\,|\,D\,H\,I) = p(\Omega\,|\,H\,I)\frac{p(D\,|\,\Omega\,H\,I)}{p(D\,|\,H\,I)}$$

$$p(D\,|\,H\,I) = \int dx\,p(x\,|\,\Omega\,H\,I)p(D\,|\,x\,\Omega\,H\,I)$$

- x is the set of parameters characterising the GW signal

$$x \equiv (m_1, m_2, d_L, z, \iota, \alpha, \delta, \vec{s}_1, \vec{s}_2, \dots)$$

- In the simplest case, only d_L, z are relevant

Distance-redshift relation

- Given H, d_L, z are related via the distance-redshift relation
- FRWL:

$$d_L(\Omega, z) = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{E(z')}$$

$$E(z') = \sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda\, g(z', w_0, w_a)}$$

$$g(z', w_0, w_a) = (1+z')^{3(1+w_0+w_a)} e^{-3\frac{w_a z'}{1+z'}}$$

- Likelihood

$$p(D \,|\, \Omega\, H\, I) = \int dz\, dd_L\, p(d_L \,|\, z\, \Omega\, H\, I) p(z \,|\, \Omega\, H\, I) p(D \,|\, d_L\, z\, \Omega\, H\, I)$$

- $d_L(\Omega, z) \implies p(d_L \,|\, \Omega\, z\, I) = \delta(d_L - d_L(\Omega, z))$

$$p(D \,|\, \Omega\, H\, I) = \int dz\, p(z \,|\, \Omega\, H\, I) \boxed{p(D \,|\, d_L(\Omega, z)\, z\, \Omega\, H\, I)} \longleftarrow$$

From the GW measurement

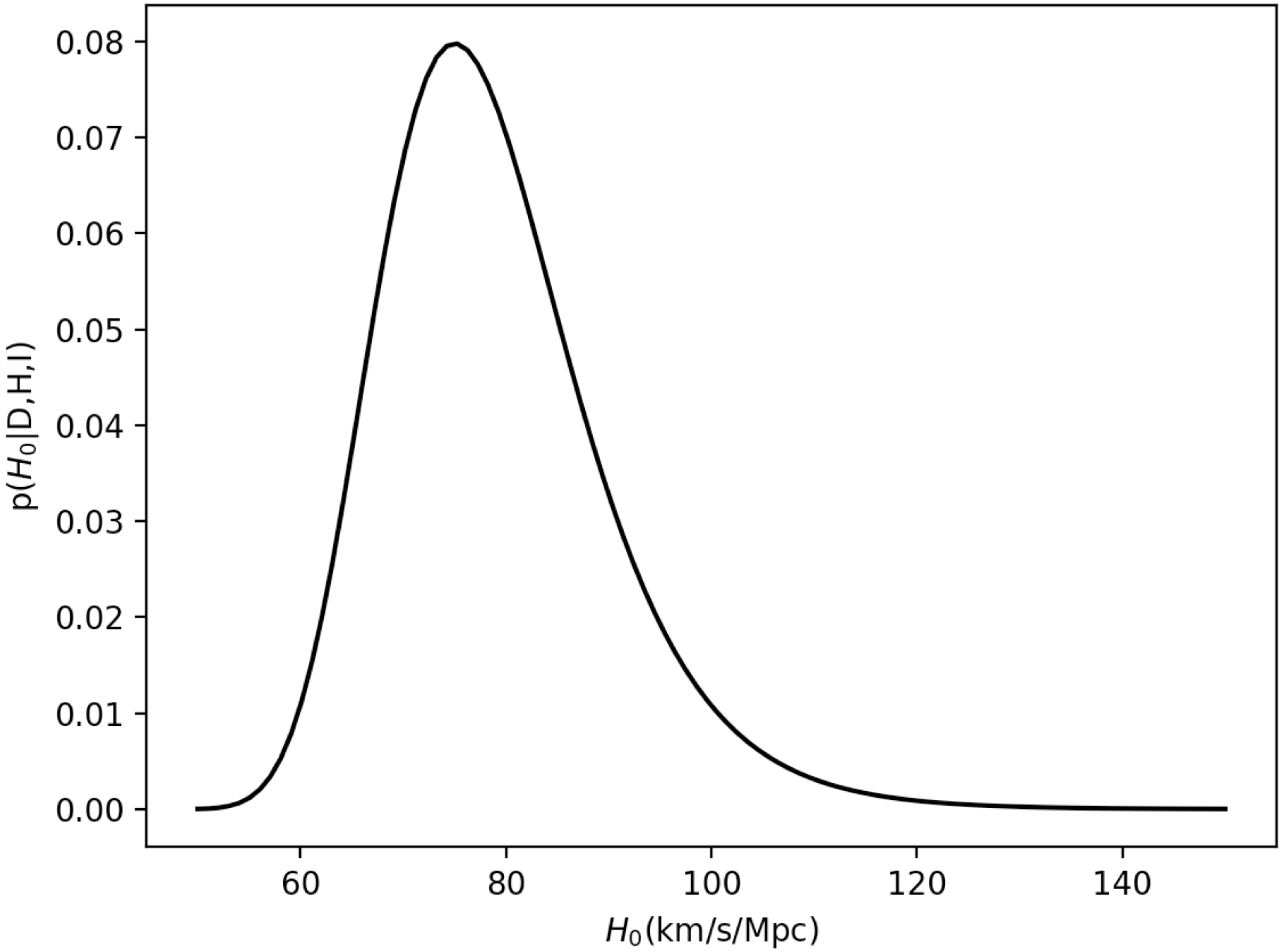
- The problem is reduced to determining $p(z \,|\, \Omega\, H\, I)$

Ideal case

- If we have a unique counterpart with a perfect redshift determination

$$p(z \,|\, \Omega\, H\, I) = \delta(z - \hat{z})$$

- For a Gaussian likelihood, $z << 1$, and no selection effects

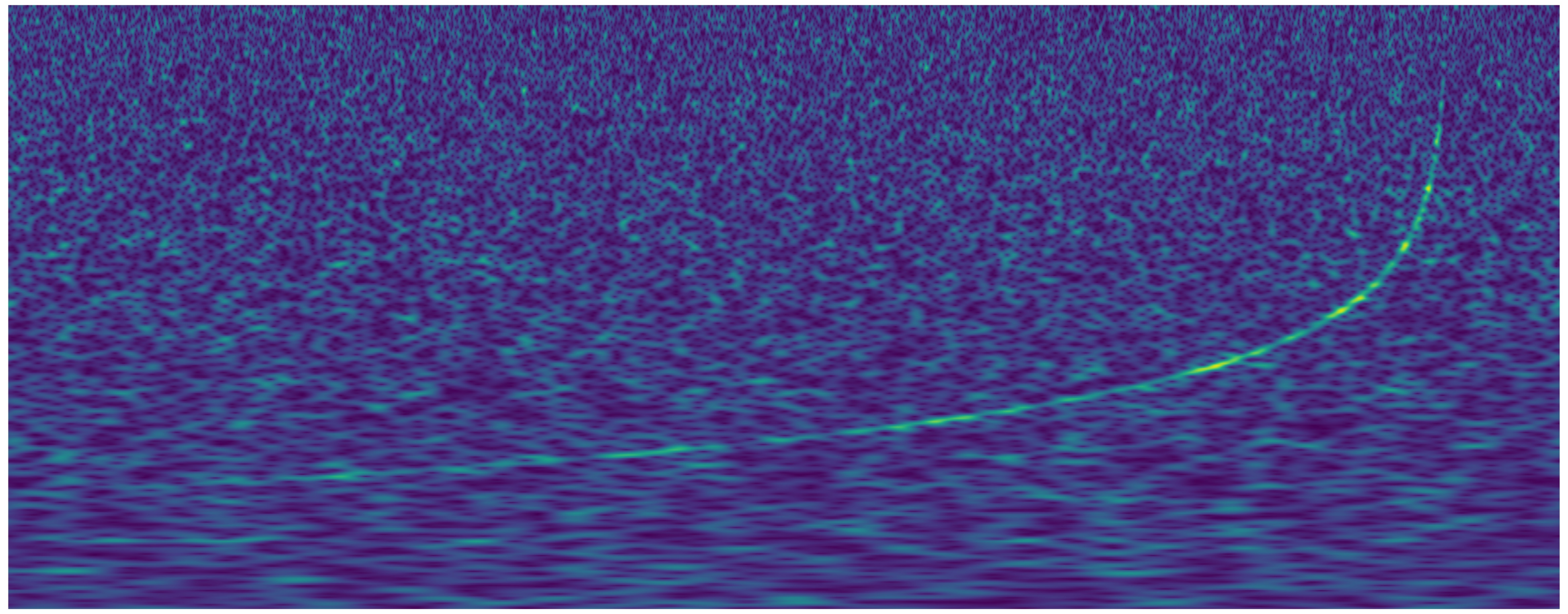


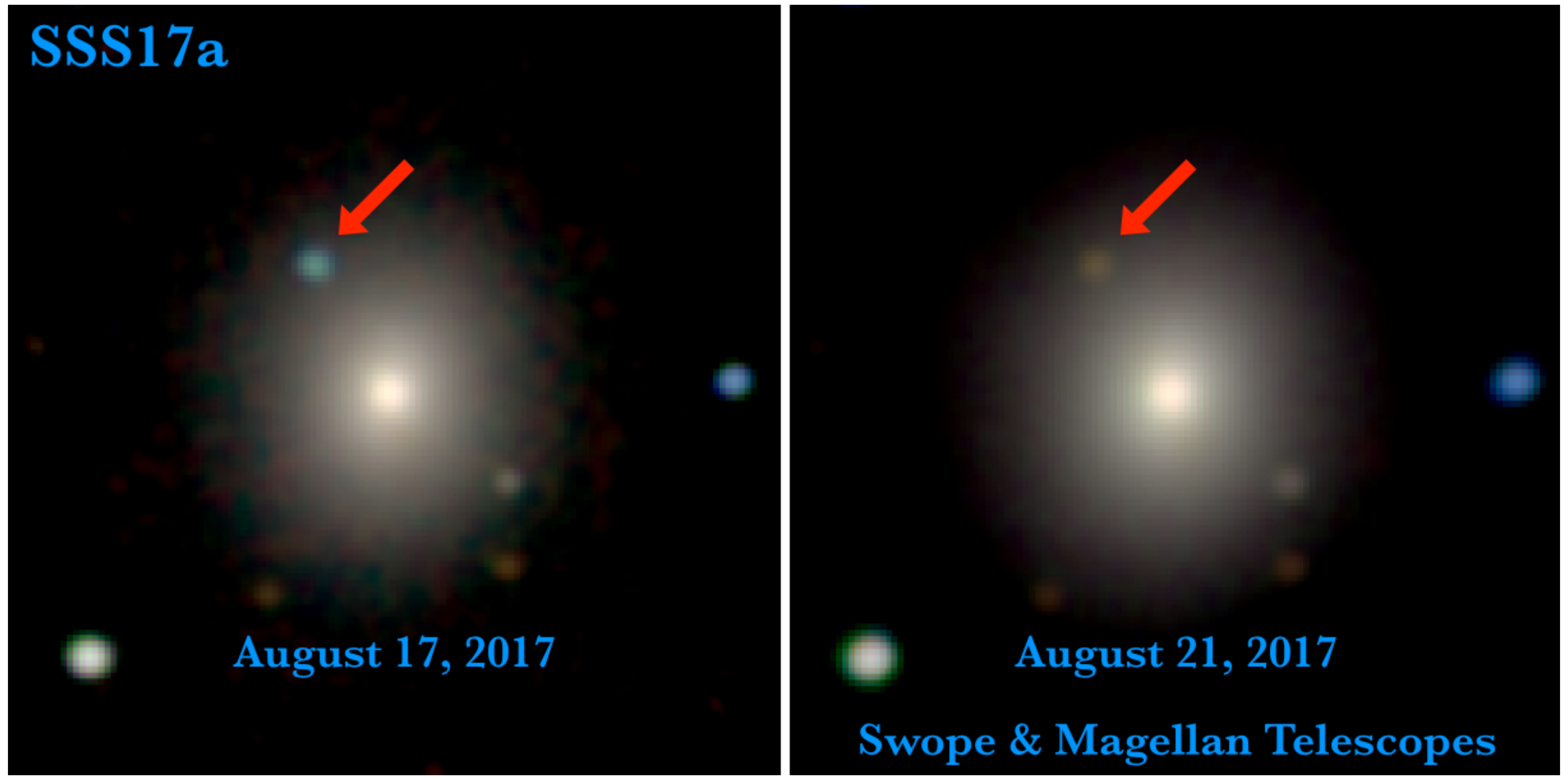
GW170817

- GW170817:
  - EM transient observed in coincidence with the GW event
    - host galaxy identification: NGC 4993
  - Correction for NGC 4993 peculiar velocity pv wrt its group centroids

$$p(z \,|\, \Omega H I) \sim \mathcal{N}(\hat{z}; pv)$$

  - Correction for GW sensitivity, function of H_0 and $\cos\iota$





GW170817

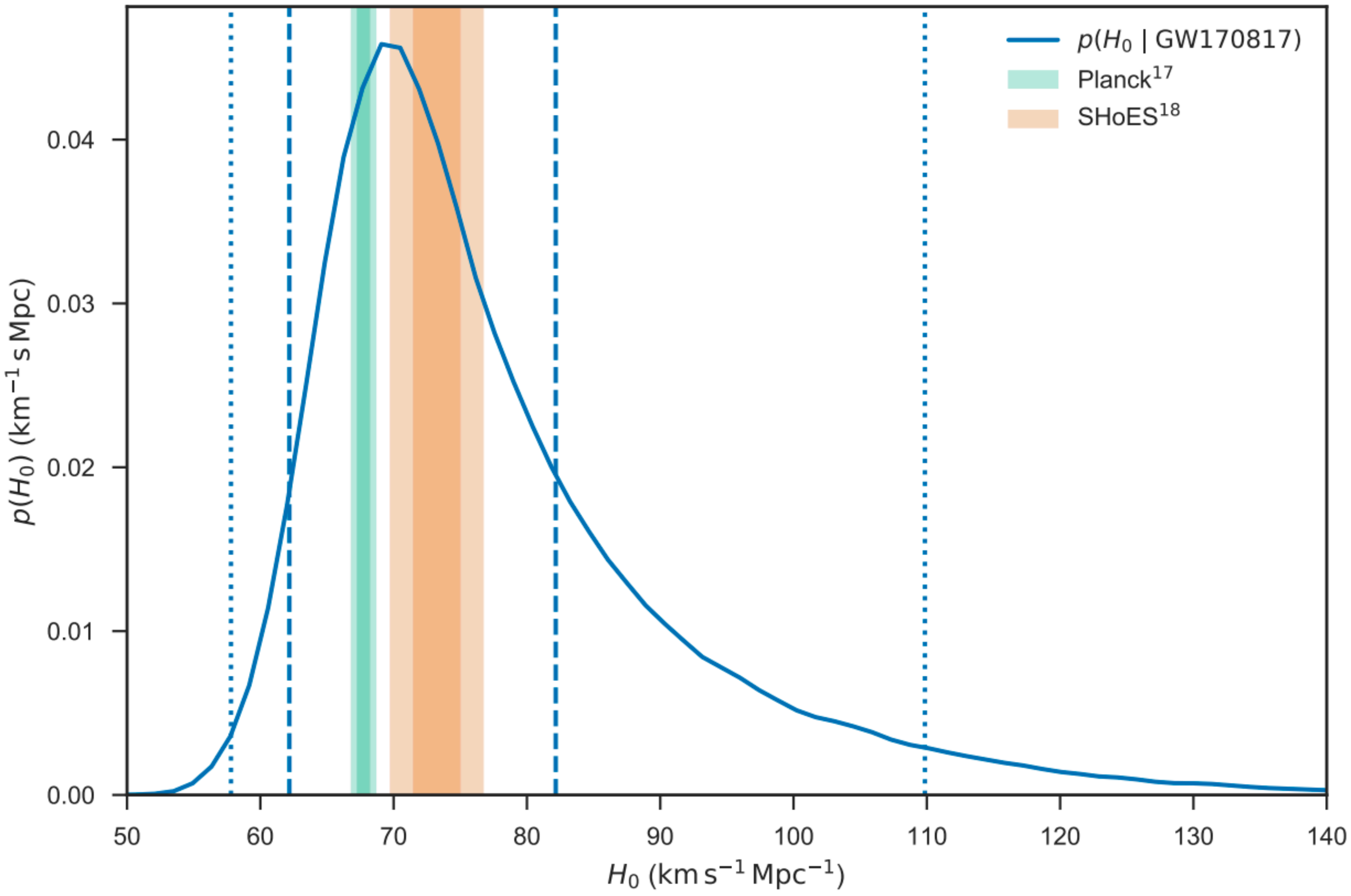
- From GW alone

$$D_L = 44^{+3}_{-7}\,\mathrm{Mpc}$$

- NGC 4993

$$z = 0.0098$$

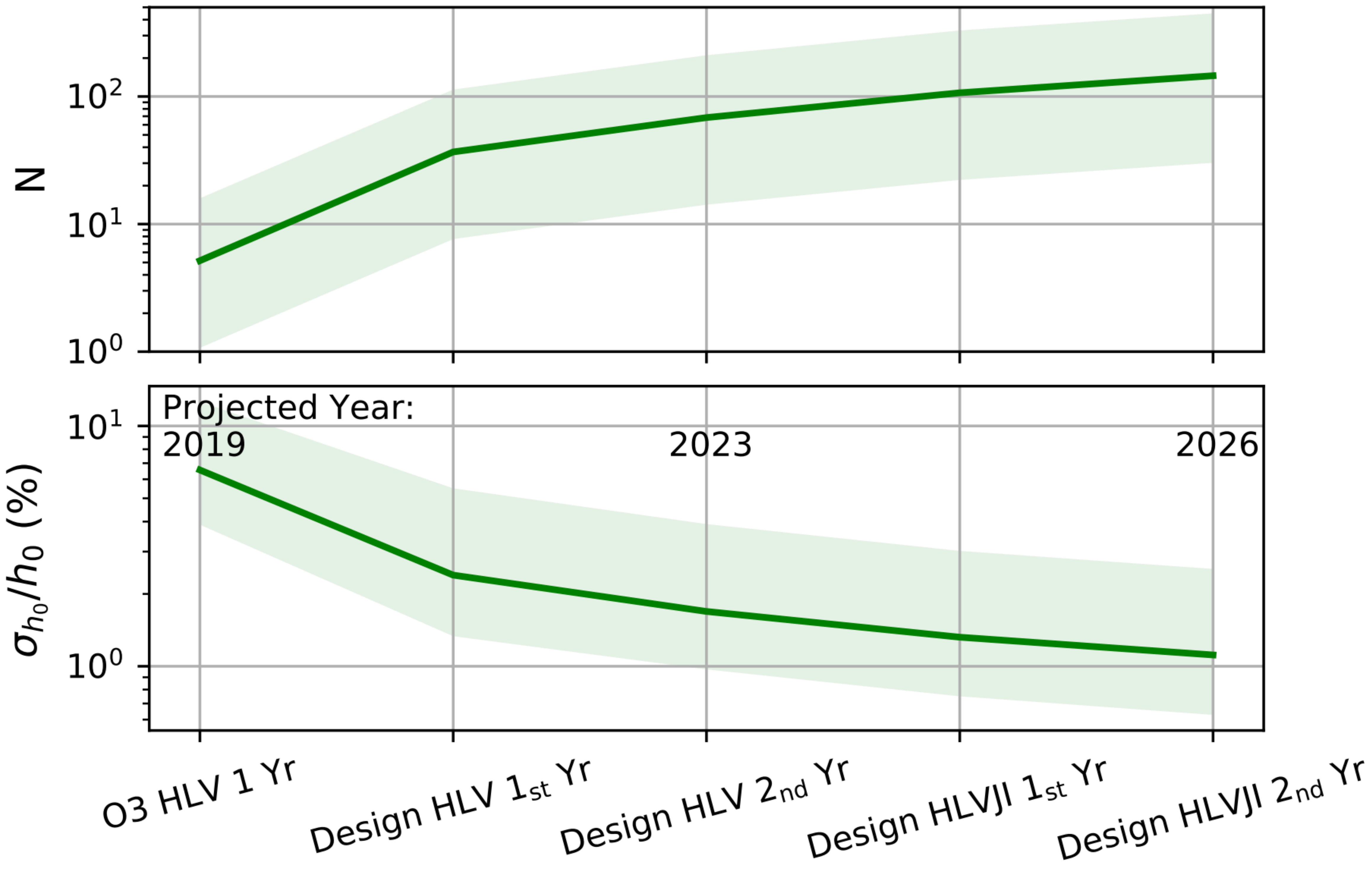
$$H_0 = 70^{+12}_{-8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$



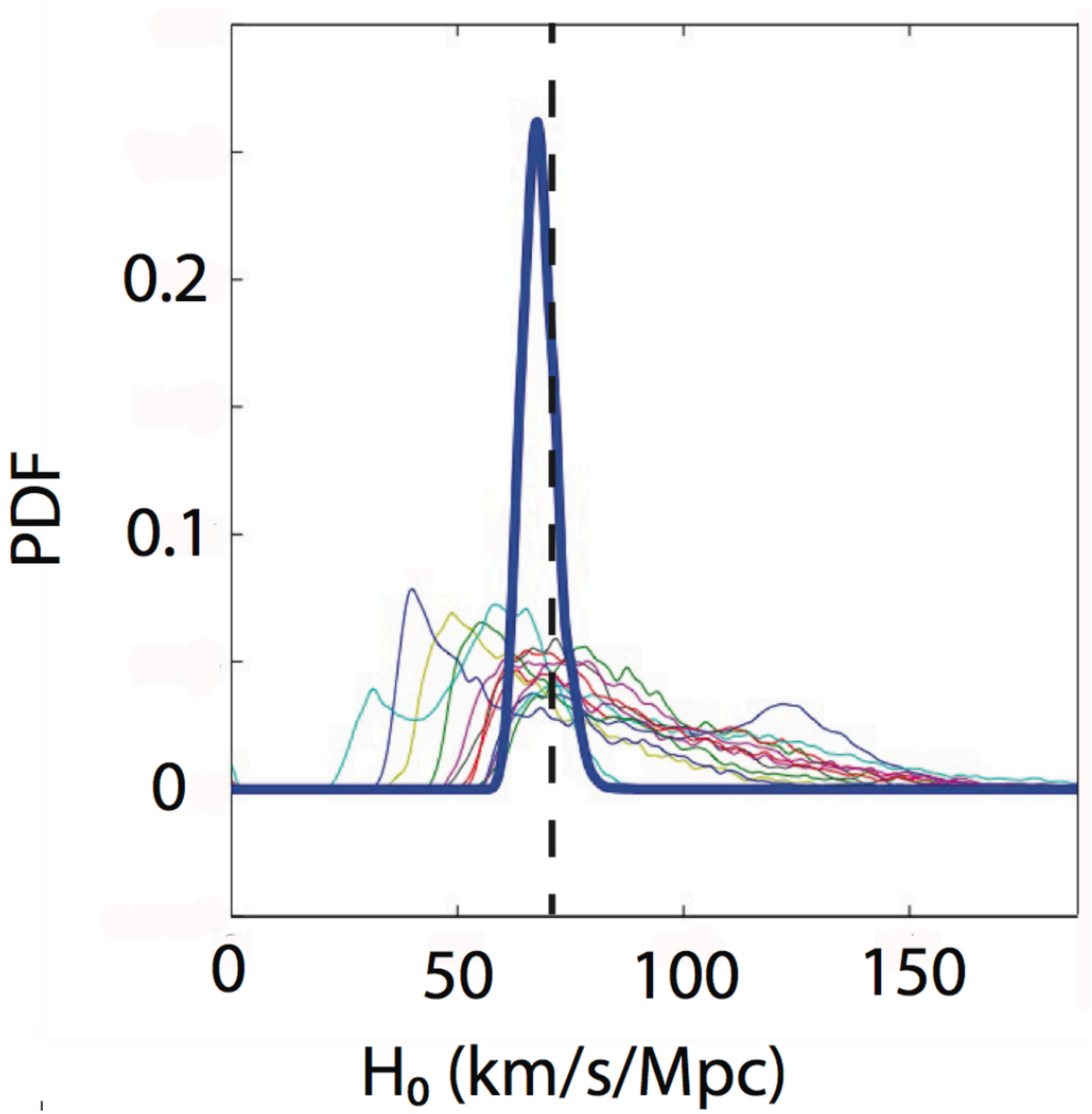
LVC et al, arXiv:1710.05835

Perspectives

- Combing O(100) BNS with transients, one can obtain ~ 5% accuracy on H0



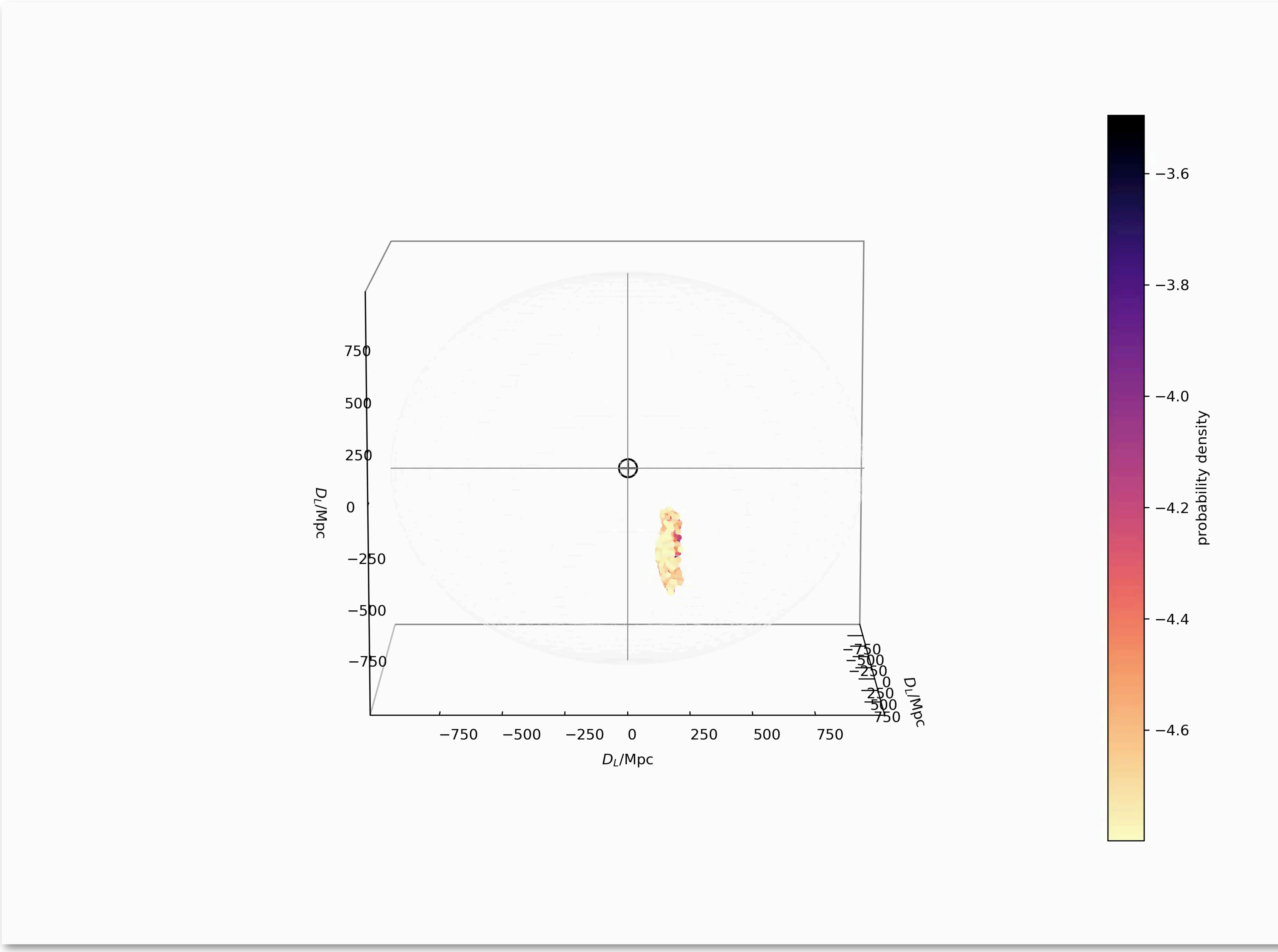
Chen et al, arXiv:1712.06531



Nissanke et al, arXiv:1307.2638

No transient counterpart

- Statistical association (e.g. Schutz 1986)
    - GW localised within some (unknown) galaxy
- Cross correlate with a galaxy catalog
    - Catalog completeness important for z > 0.3
- Not limited to BNS (or NSBH) source classes



Del Pozzo et al, arXiv:1801.08009

Statistical association

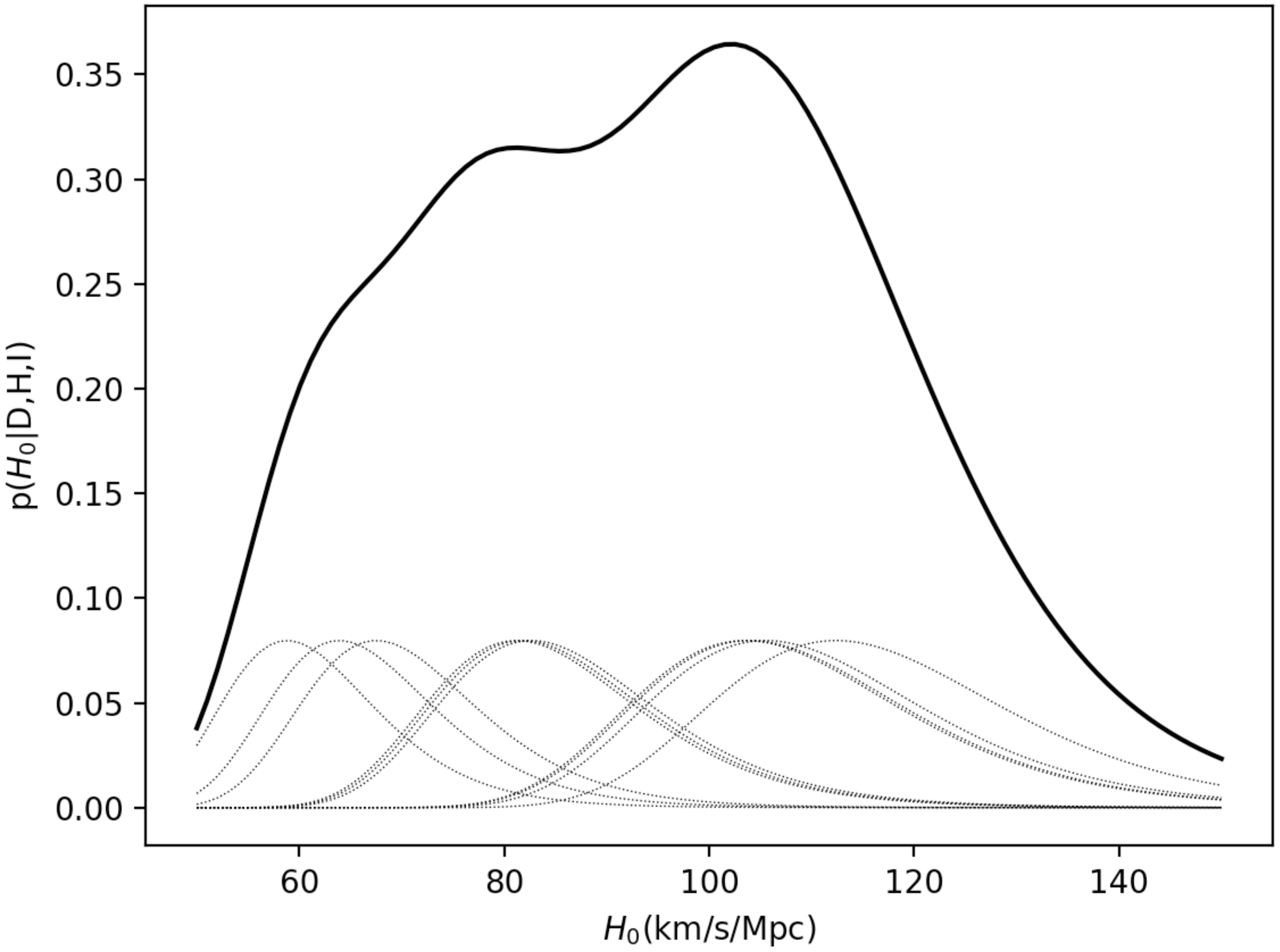
- If a GW is assumed to be located in a galaxy

$$p(z \,|\, \Omega H I) \propto \sum_i w_i \delta(z - \hat{z}_i)$$

- Including peculiar velocities

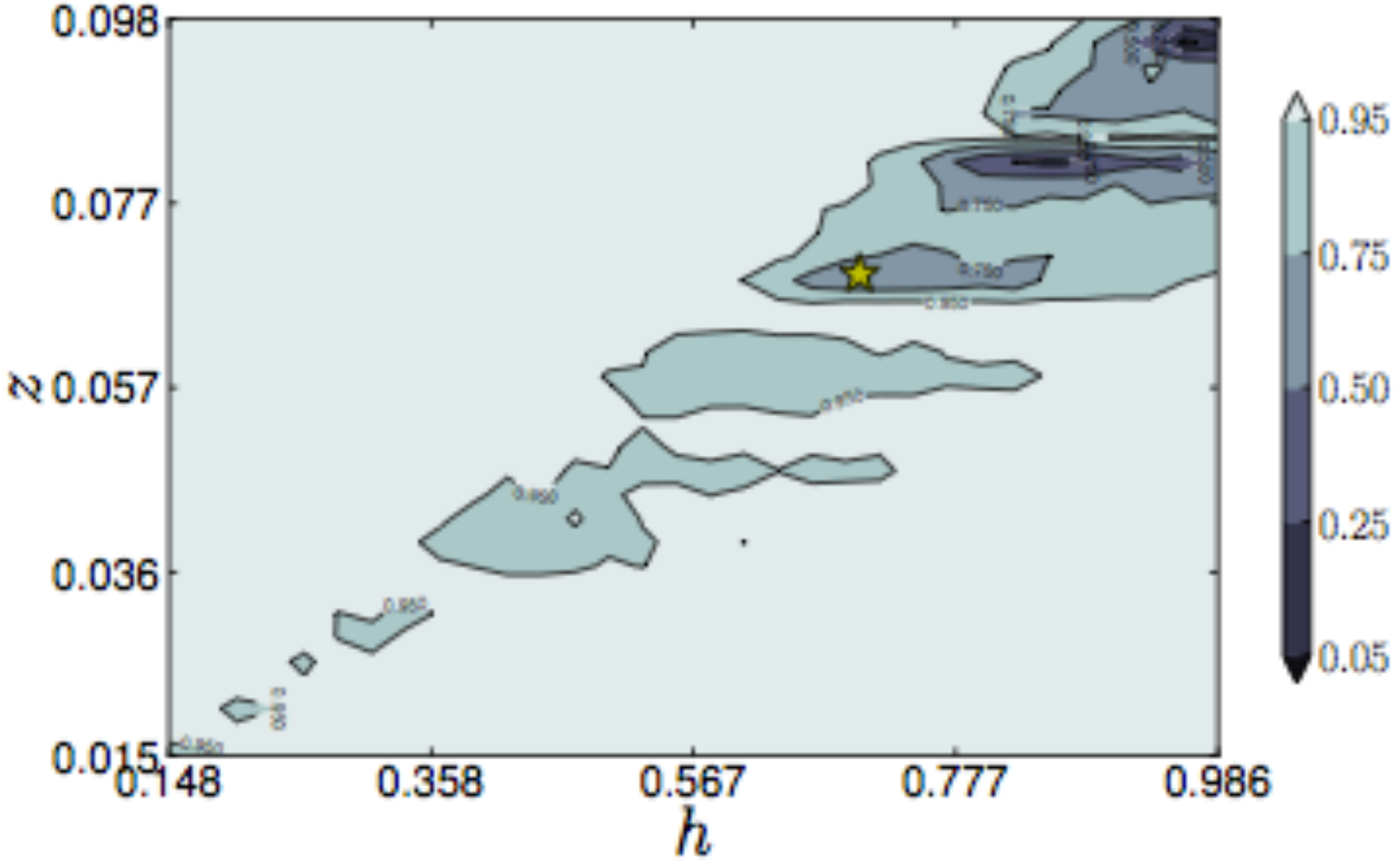
$$p(z \,|\, \Omega H I) \propto \sum_i w_i \mathcal{N}(\hat{z}_i; pv)$$

- For a Gaussian likelihood, $z < < 1$, and no selection effects



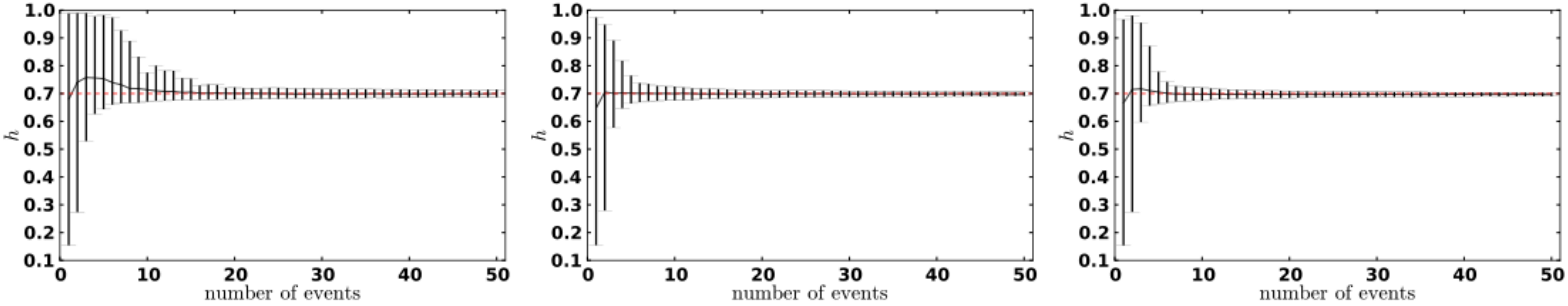
Single source posteriors

- Each event gives posteriors that track the large scale distribution of galaxies
- Idea exploited to use galaxy clustering as additional information in Mukherjee et al, arXiv:2007.02943



Del Pozzo, arXiv:1108.1317

Projections



	HLV			HLVJ			HLVJI		
# events	$\langle h_{2.5\%}\rangle$	$\langle\overline{h}\rangle$	$\langle h_{97.5\%}\rangle$	$\langle h_{2.5\%}\rangle$	$\langle\overline{h}\rangle$	$\langle h_{97.5\%}\rangle$	$\langle h_{2.5\%}\rangle$	$\langle\overline{h}\rangle$	$\langle h_{97.5\%}\rangle$
5	0.644	0.753	0.982	0.664	0.701	0.765	0.663	0.705	0.779
10	0.671	0.714	0.775	0.675	0.699	0.725	0.674	0.698	0.721
15	0.676	0.705	0.754	0.681	0.699	0.716	0.682	0.697	0.712
20	0.679	0.701	0.722	0.684	0.698	0.711	0.684	0.697	0.709
30	0.681	0.698	0.717	0.688	0.699	0.708	0.687	0.697	0.707
40	0.686	0.700	0.714	0.687	0.699	0.707	0.689	0.697	0.704
50	0.686	0.700	0.714	0.687	0.700	0.706	0.689	0.700	0.703

Del Pozzo, arXiv:1108.1317

Results from O2

- LV Collaboration, arXiv:1908.06060
  - 10 BBH+1BNS
  - Galaxy catalog completeness critical, $d_L(BBH) \sim O(Gpc)$

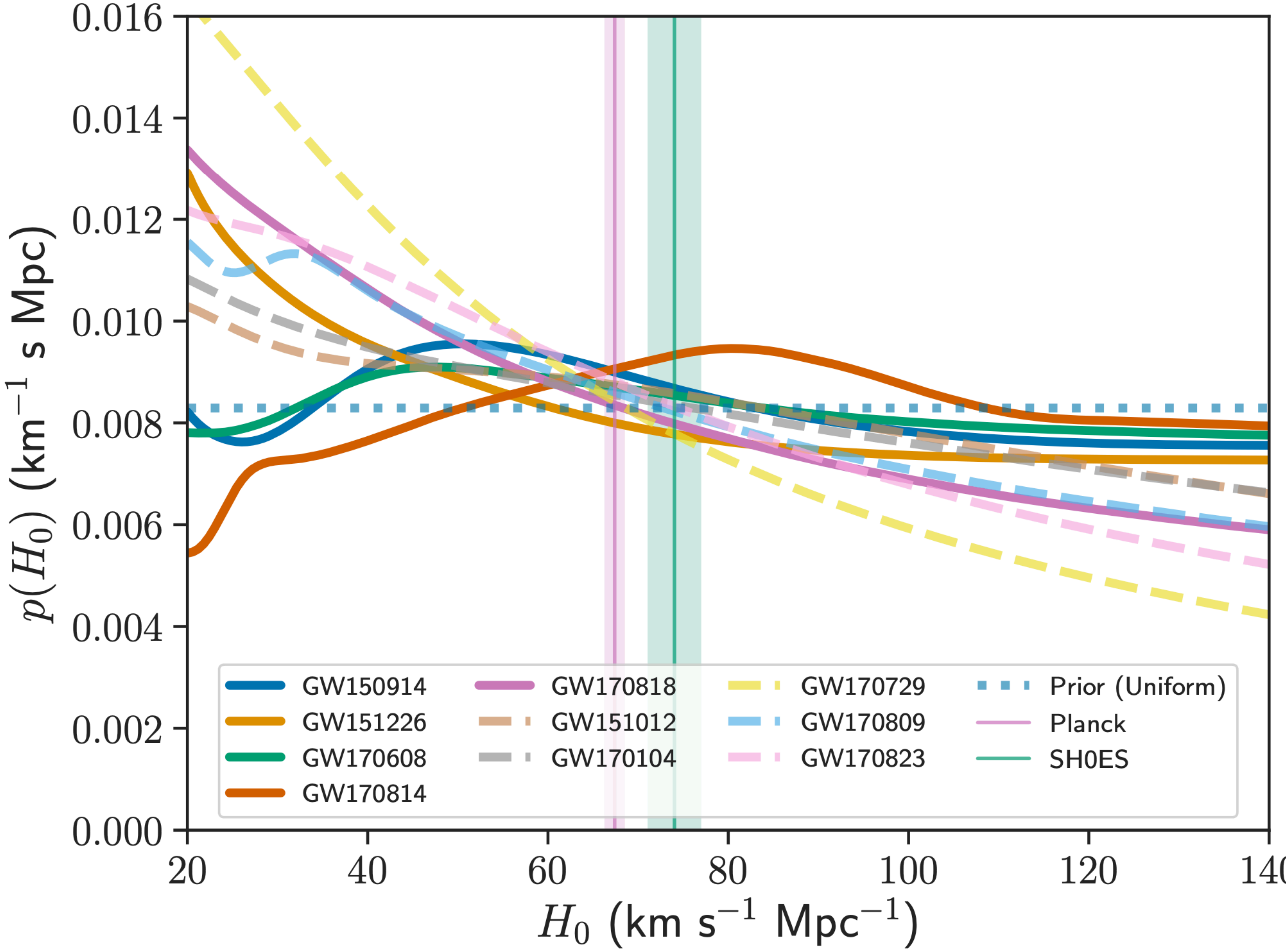
$$p(z\,|\,\Omega\,H\,I) = p(z\,|\,\Omega\,H\,G\,I)p(G\,|\,\Omega\,H\,I) + p(z\,|\,\Omega\,H\,\overline{G}\,I)p(\overline{G}\,|\,\Omega\,H\,I)$$

in catalog   not in catalog

  - Corrections depend on the cosmology, the survey completeness and galaxy population

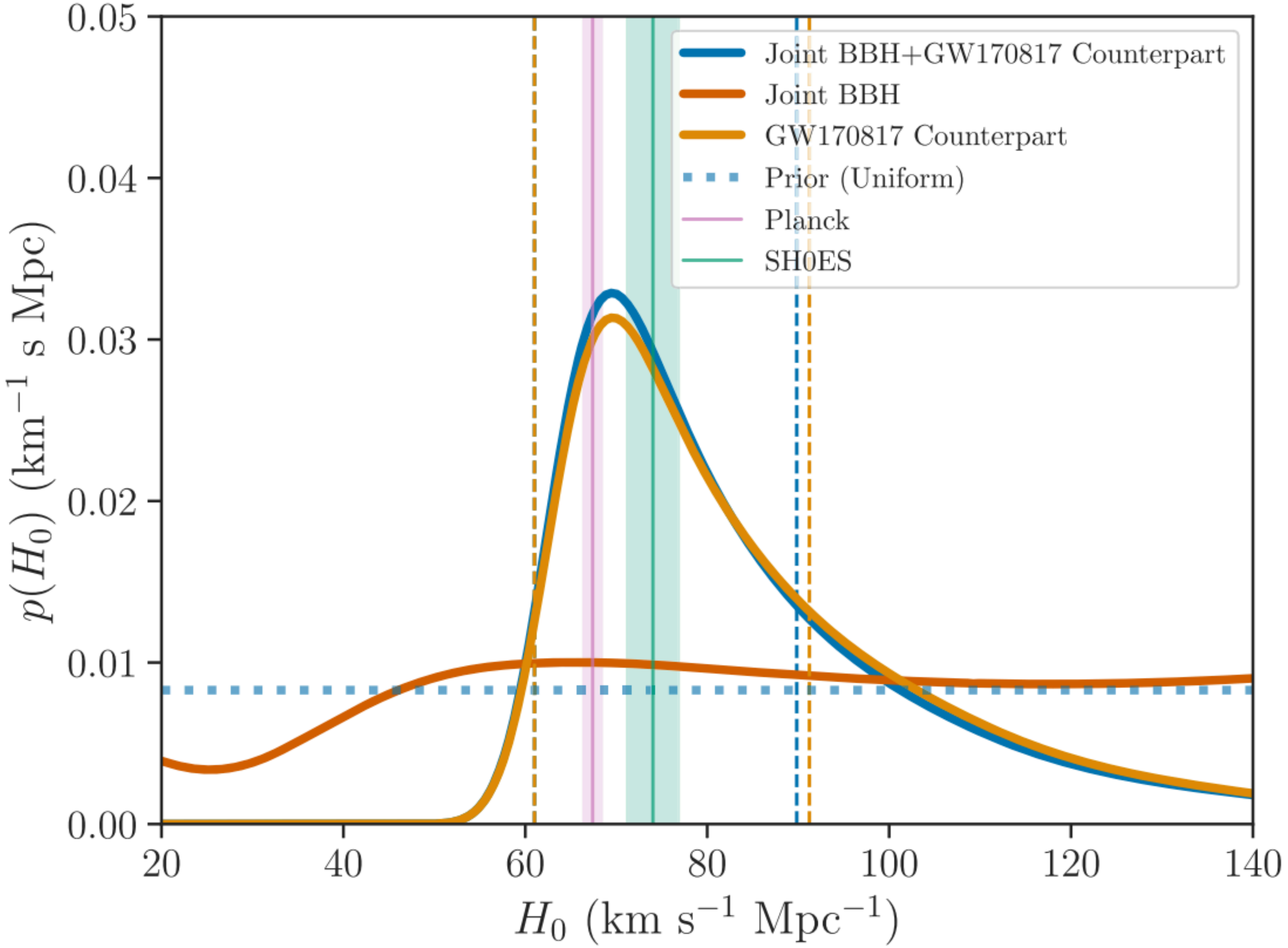
Results from O2 - individual BBHs

- Measurement not competitive yet
    - Individual posteriors are largely uninformative
- Catalog incompleteness is dominant



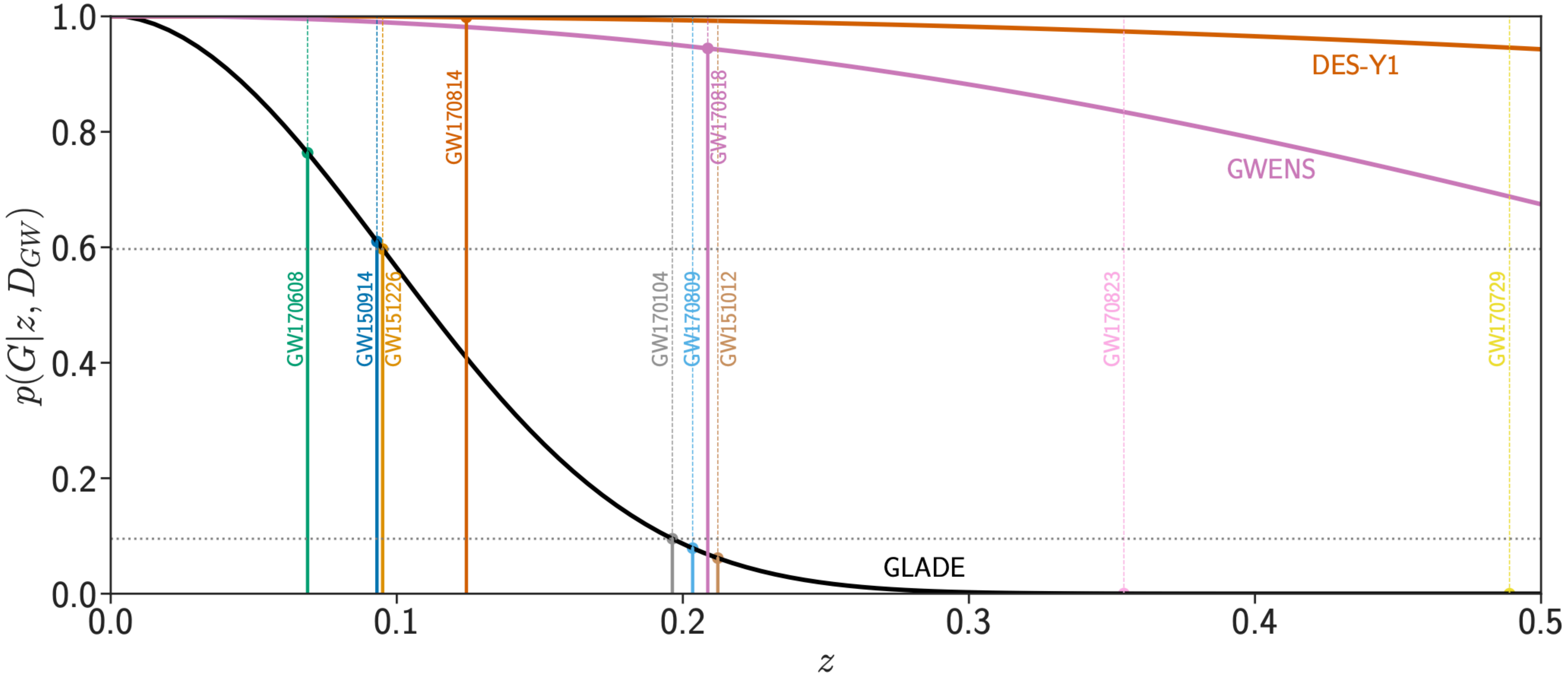
Joint results from O2

- Even a joint posterior is not competitive yet
  - GW170817 dominates the inference



Probability of host in catalog

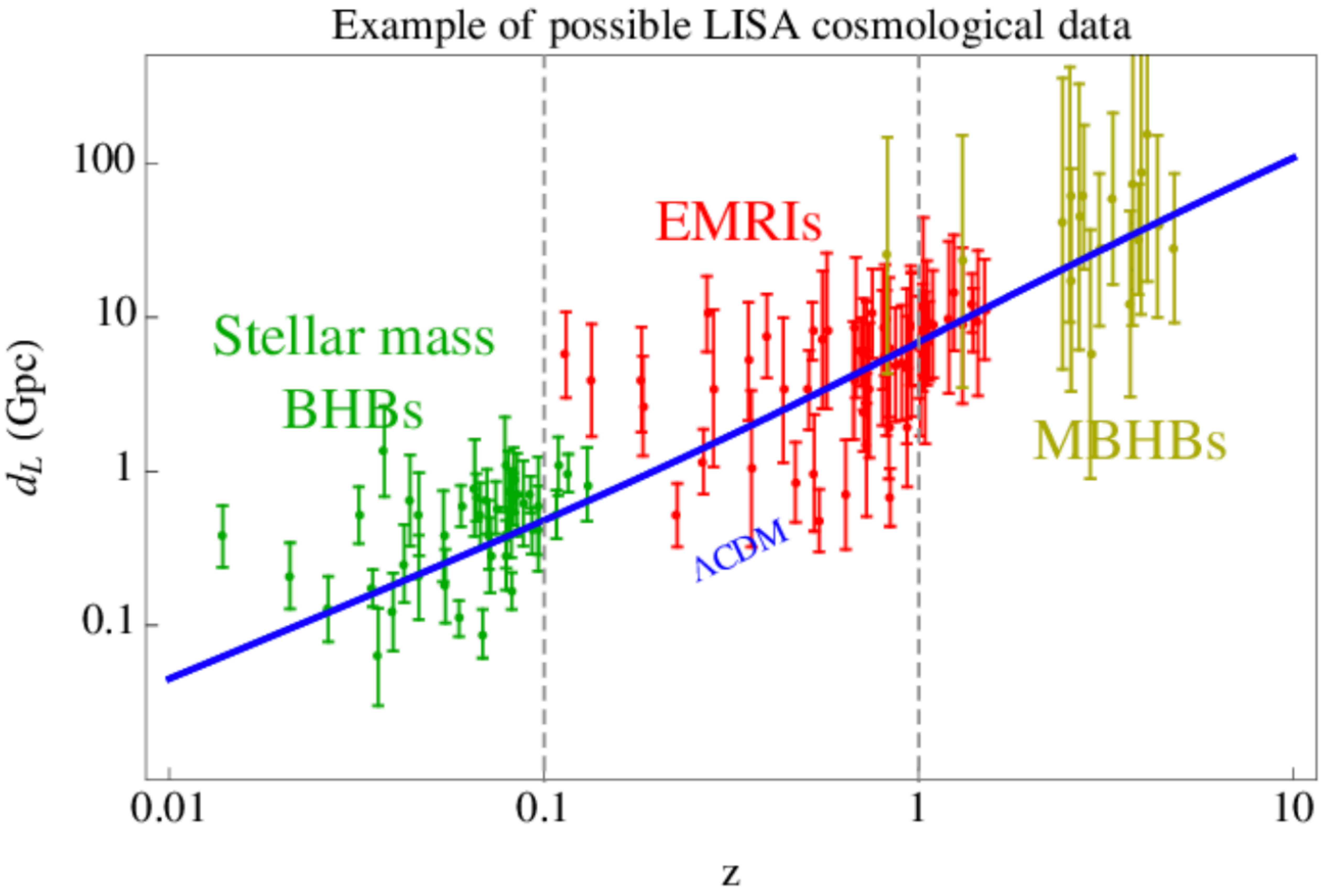
- Statistical association made assuming a combination of GLADE (Dalya+2018), DES Y1 (Drlica-Wagner+2018) and GWENS (Rahman+2019)



LISA as a cosmological probe

- LISA will observe sources across the Universe

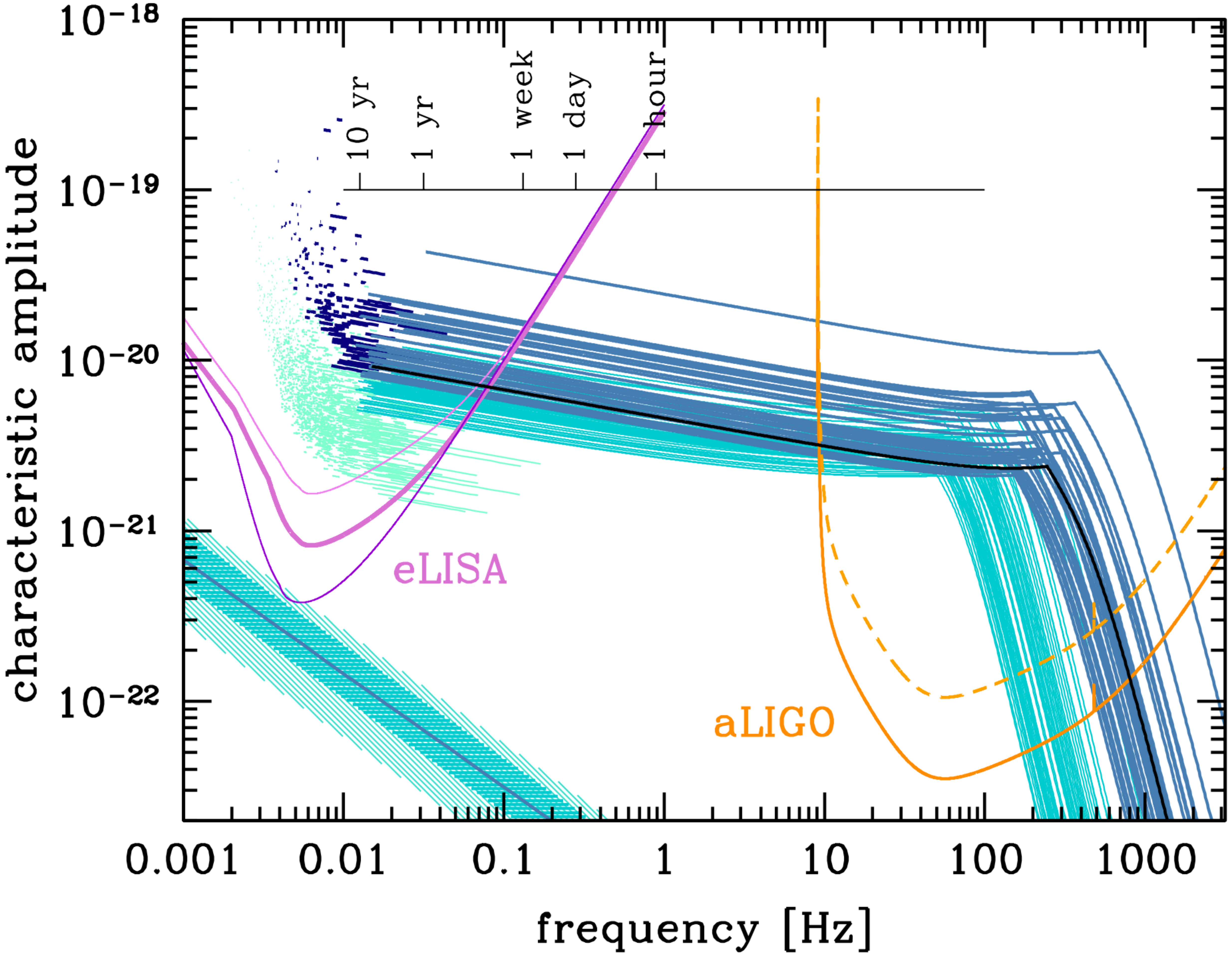
- Different sources will probe different redshift ranges
  - sBH: z < 0.1
  - EMRIs: z < 1
  - SMBHs: z < 10



Tamanini et al, arXiv:1601.07112

LISA observations of sBH

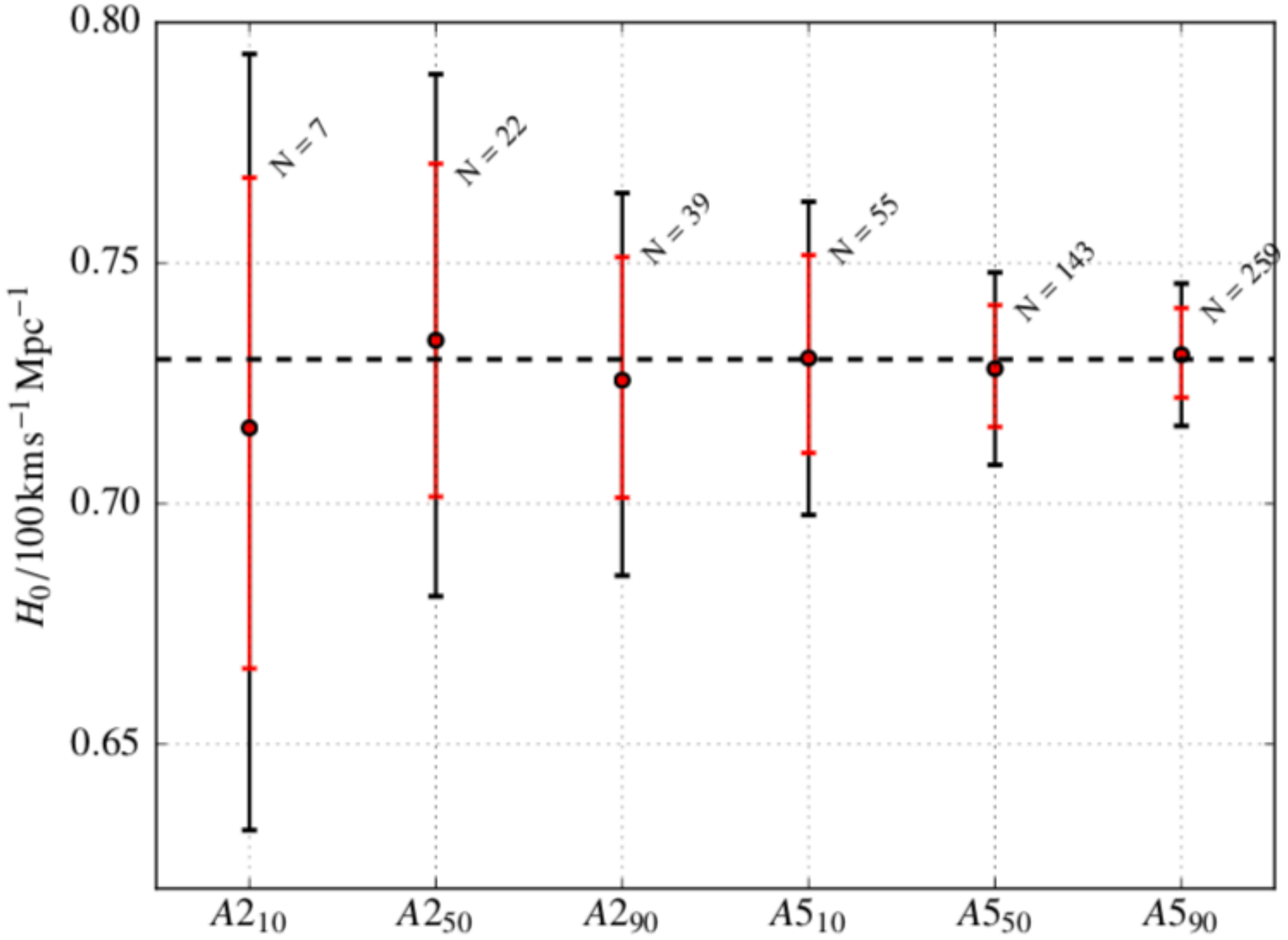
- Following GW150914, Sesana realised that many similar systems would be observed by LISA
- Low redshift (<0.1)
- Cross-correlation with Millennium simulation

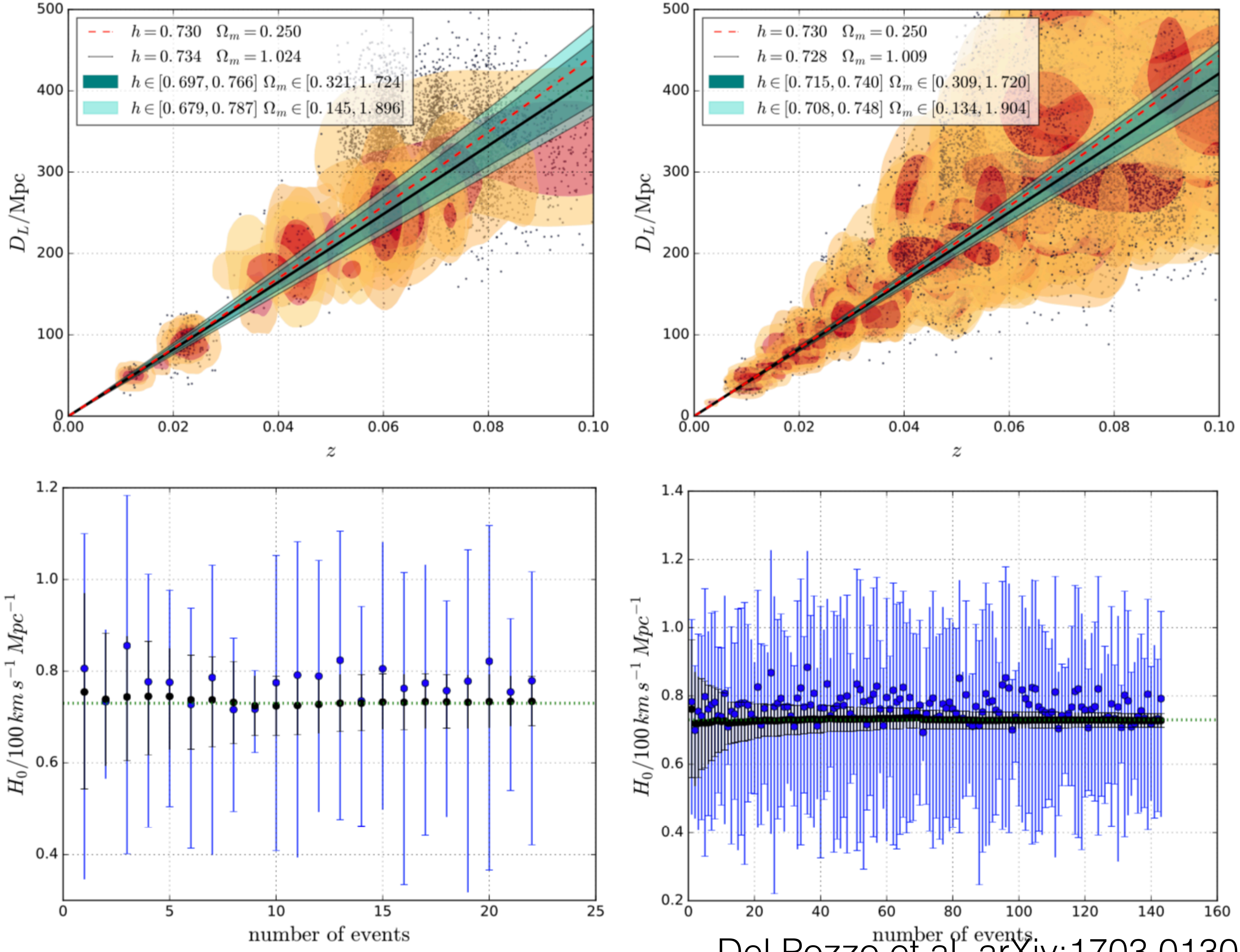


Sesana, arXiv:1602.06951

H0 from sBH

run	LISA design	$\mathcal{R}$ [yr^{-1}Gpc^{-3}]	$N_{\rm BHB}$	h(68%)
A2_{10}	N2A2M5L6	12	7	$0.716^{+0.052}_{-0.050}$
A2_{50}	N2A2M5L6	34	22	$0.734^{+0.037}_{-0.033}$
A2_{90}	N2A2M5L6	70	39	$0.726^{+0.026}_{-0.024}$
A5_{10}	N2A5M5L6	12	55	$0.730^{+0.021}_{-0.020}$
A5_{50}	N2A5M5L6	34	143	$0.728^{+0.013}_{-0.012}$
A5_{90}	N2A5M5L6	70	259	$0.731^{+0.010}_{-0.009}$

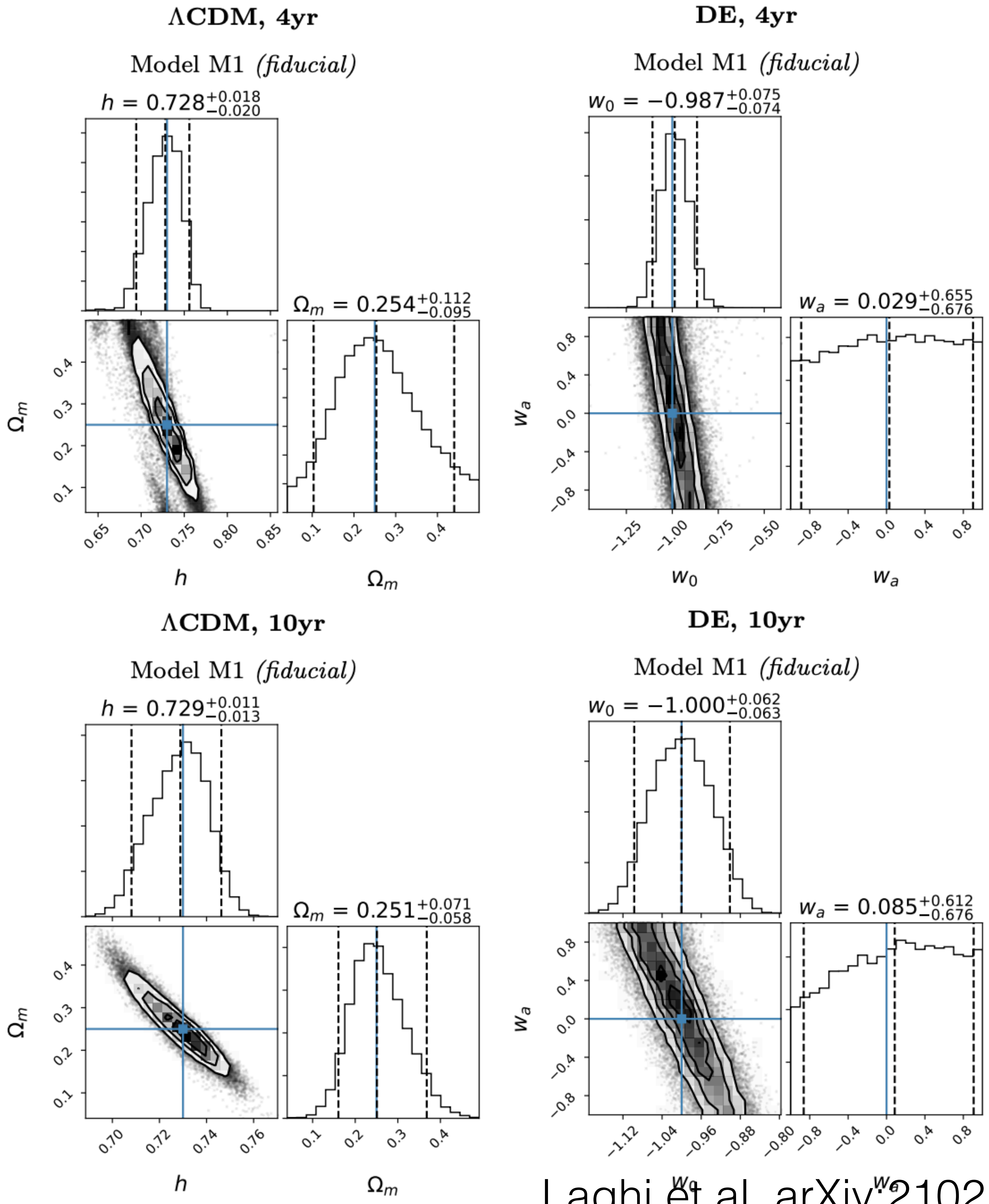




Del Pozzo et al, arXiv:1703.01300

Cosmography with EMRIs

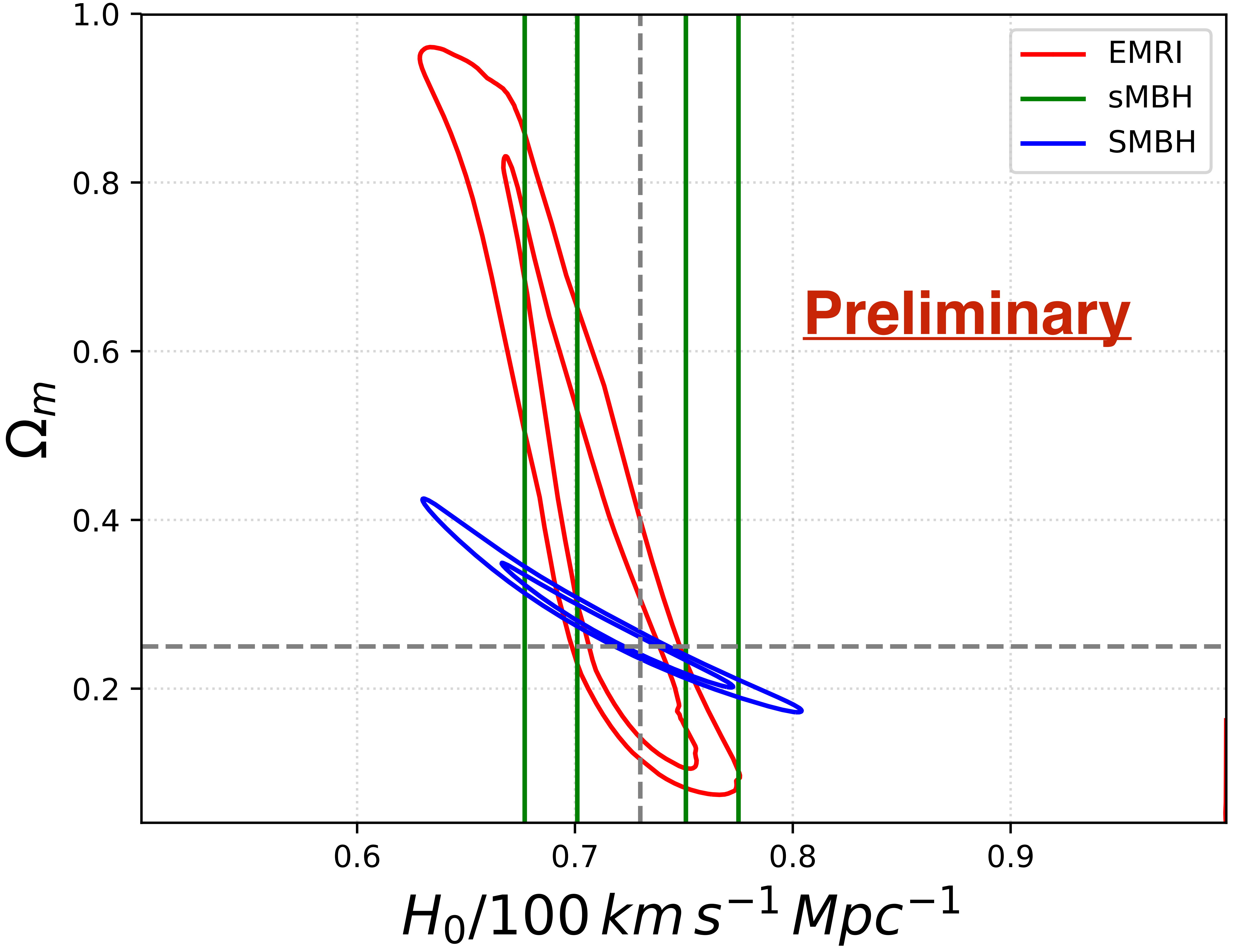
- EMRIs should be observable up to z < 1
- Detection rates depend on (largely unconstrained) population models
- Cross-correlation with Millennium simulation
- Constraints on DE parameters possible



Laghi et al, arXiv:2102.01708

LISA perspectives

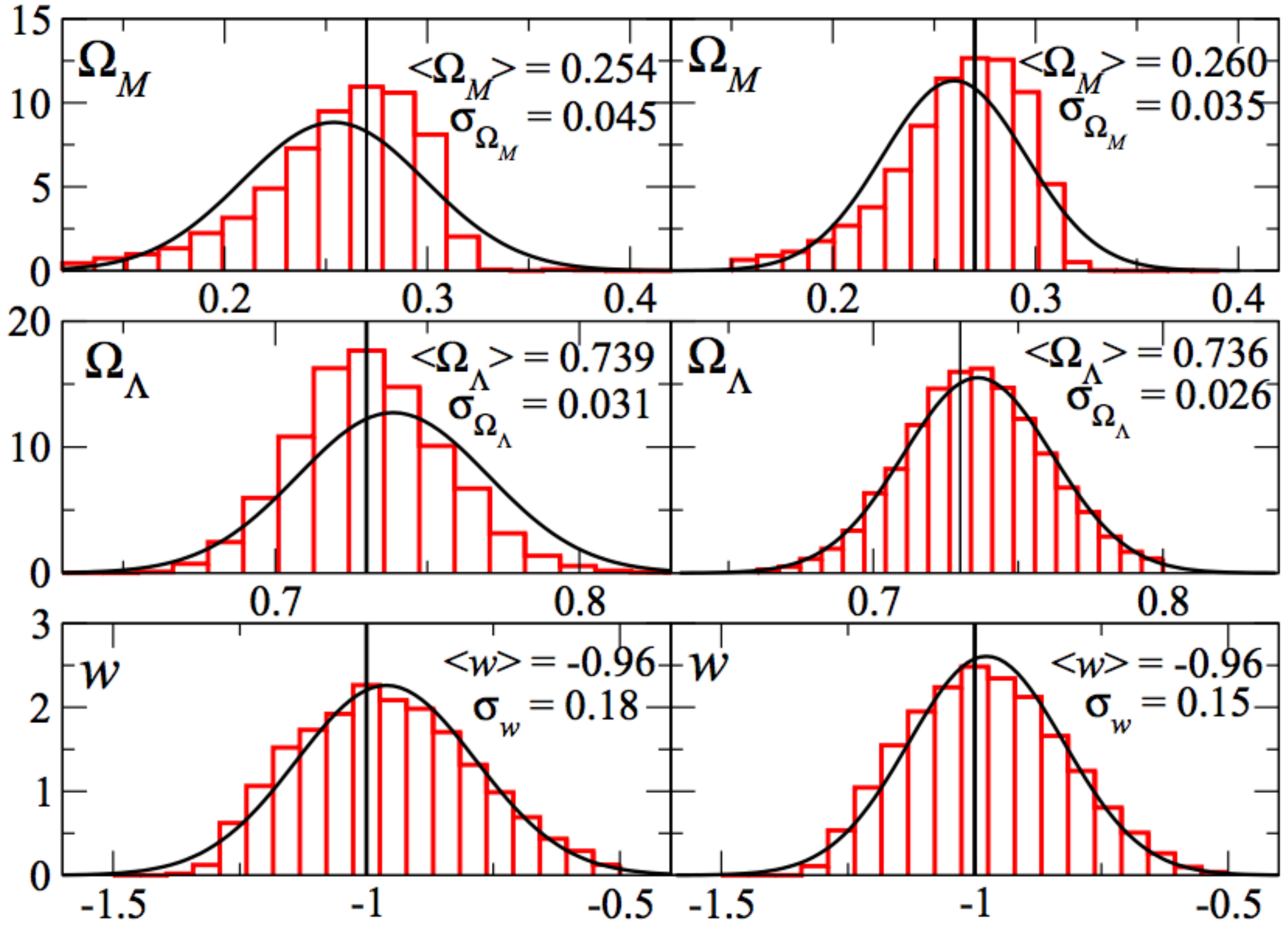
- O(10) SMBH up to z ~ 6
- O(20) EMRI up to z ~ 1
- O(40) sBH up to z ~ 0.1

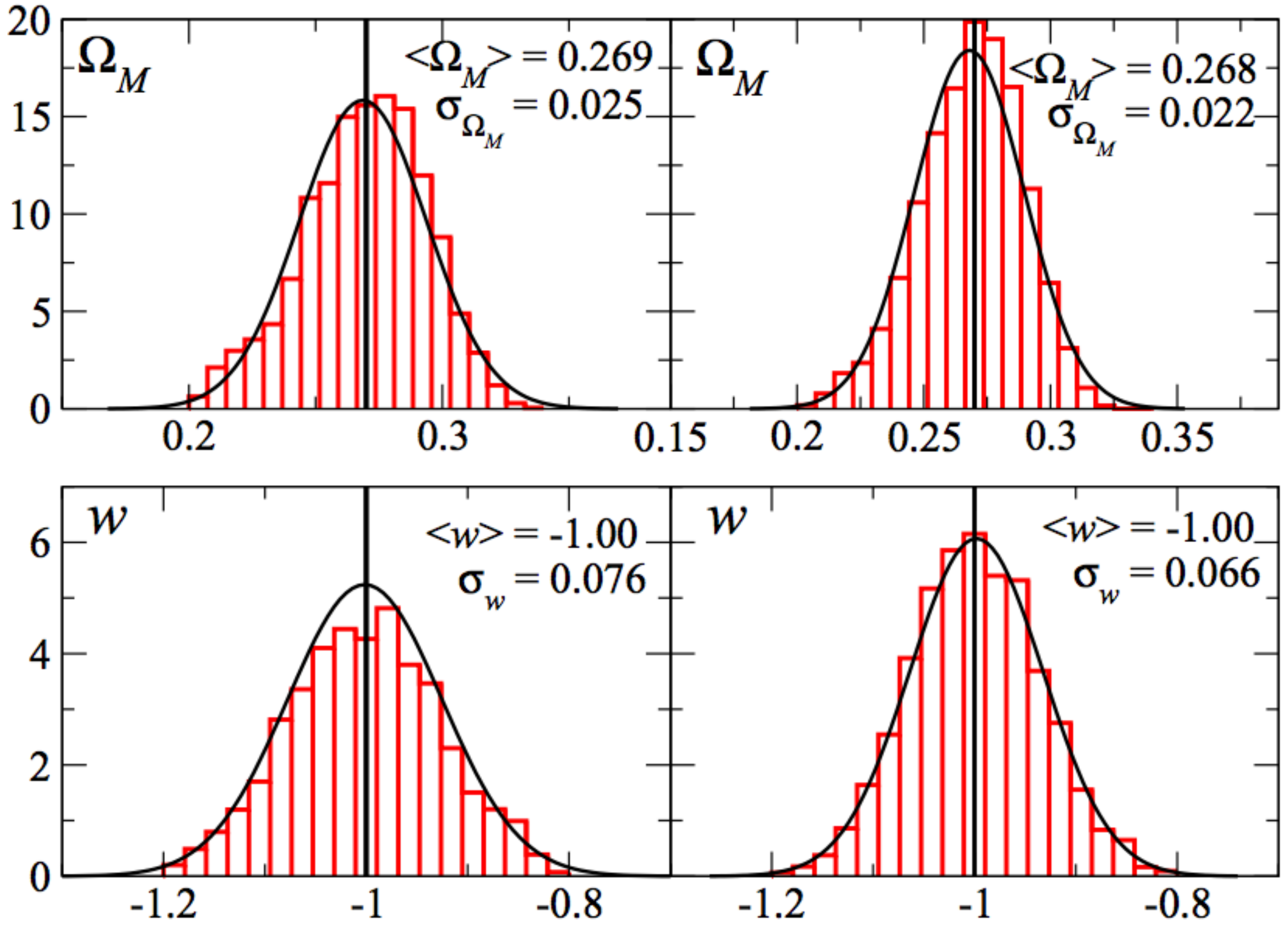


Tamanini et al, in preparation

Third generation instruments: ET

- BNS events with EM transient (z < 2)
- Energy density (and DE) parameters constrained to ~ % level
- Lensing become important

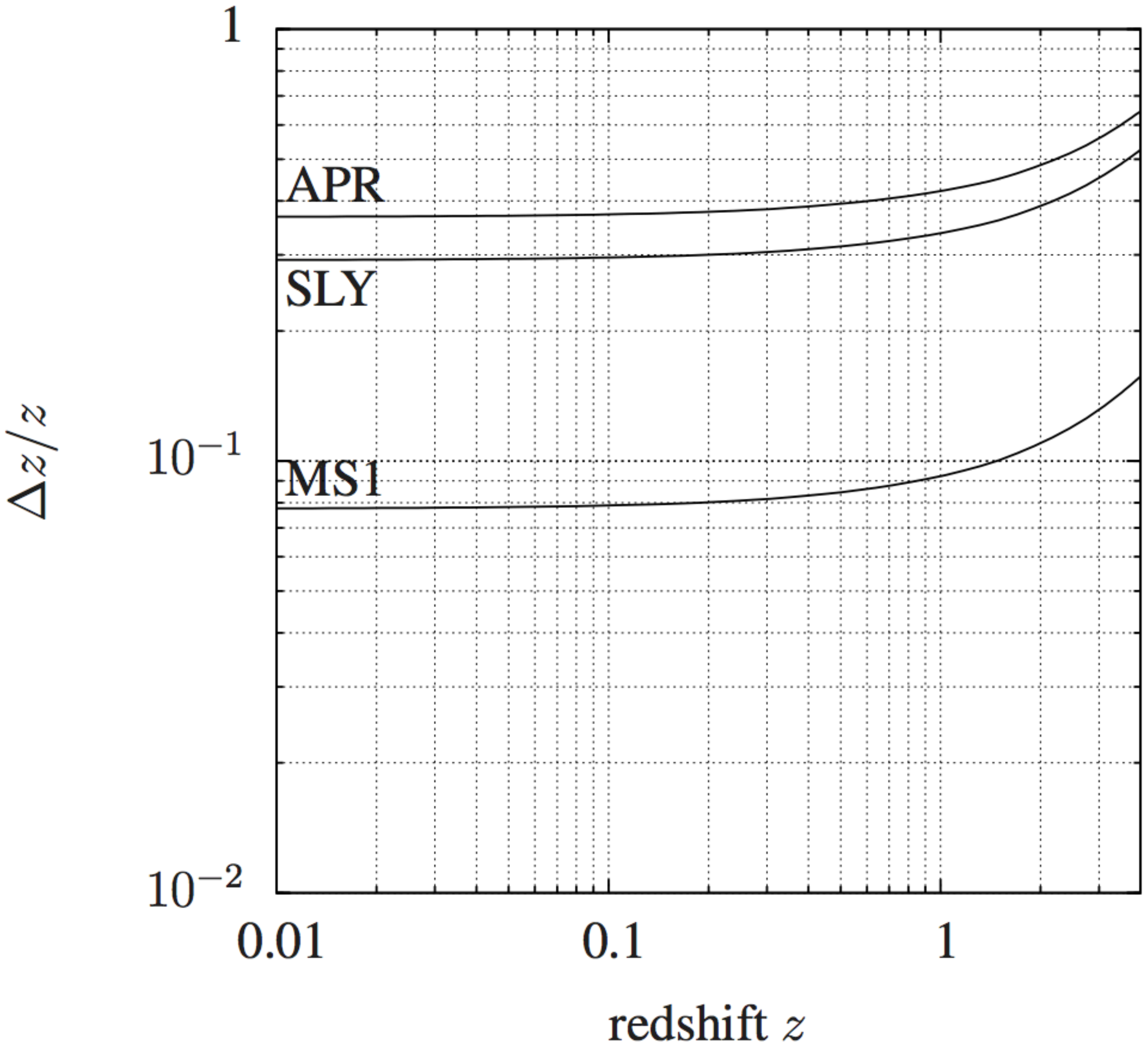




Sathyaprakash et al., arXiv:0906.4151

Cosmology from GW alone

- Messenger & Read 2012 realised that for BNS, knowledge of the equation of state allows to infer the redshift from GW alone
- Third generation IFOs such as ET



Messenger & Read, arXiv:1107.5725

Tidal effects in BNS

- Tidal effects enter through the tidal deformability

$$Q_{ij} = -\lambda(\mathrm{EOS}; m)\tau_{ij}$$

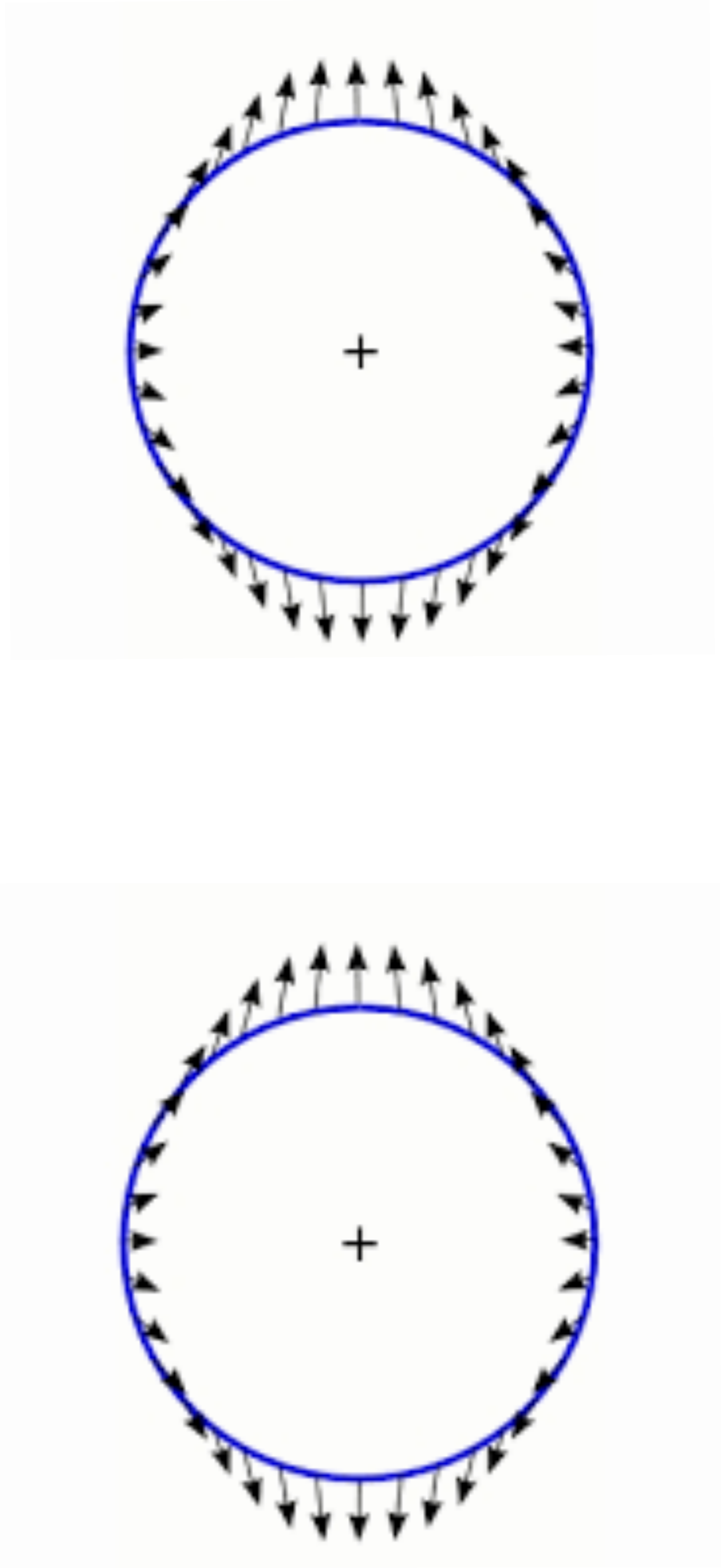
quadrupole moment

tidal field of companion star

$$\lambda(m) = \frac{2}{3}k_2 R^5(m)$$

second Love number

NS radius



- The tidal deformability depends on the EOS and on the rest-frame masses

- We get a “clock” in rest frame of the system => infer masses, distance and redshift simultaneously

Perspectives from ET (BNS alone)

Δh

N / Model	10^3	10^4	10^5	10^6	10^7
Flat FRW	0.5×10^{-1}	1.6×10^{-2}	0.5×10^{-2}	1.6×10^{-3}	0.5×10^{-3}
General FRW	4.6×10^{-2}	1.5×10^{-2}	4.6×10^{-3}	1.5×10^{-3}	4.6×10^{-4}
General FRW+DE	0.8×10^{-1}	2.5×10^{-2}	0.8×10^{-2}	2.5×10^{-3}	0.8×10^{-3}

$\Delta\Omega_m$

N / Model	10^3	10^4	10^5	10^6	10^7
Flat FRW	1.3×10^{-1}	4.0×10^{-2}	1.3×10^{-2}	4.0×10^{-3}	1.3×10^{-3}
General FRW	1.3×10^{-1}	4.2×10^{-2}	1.3×10^{-2}	4.2×10^{-3}	1.3×10^{-3}
General FRW+DE	1.9×10^{-1}	0.6×10^{-1}	1.9×10^{-2}	0.6×10^{-2}	1.9×10^{-3}

$\Delta\Omega_\Lambda$

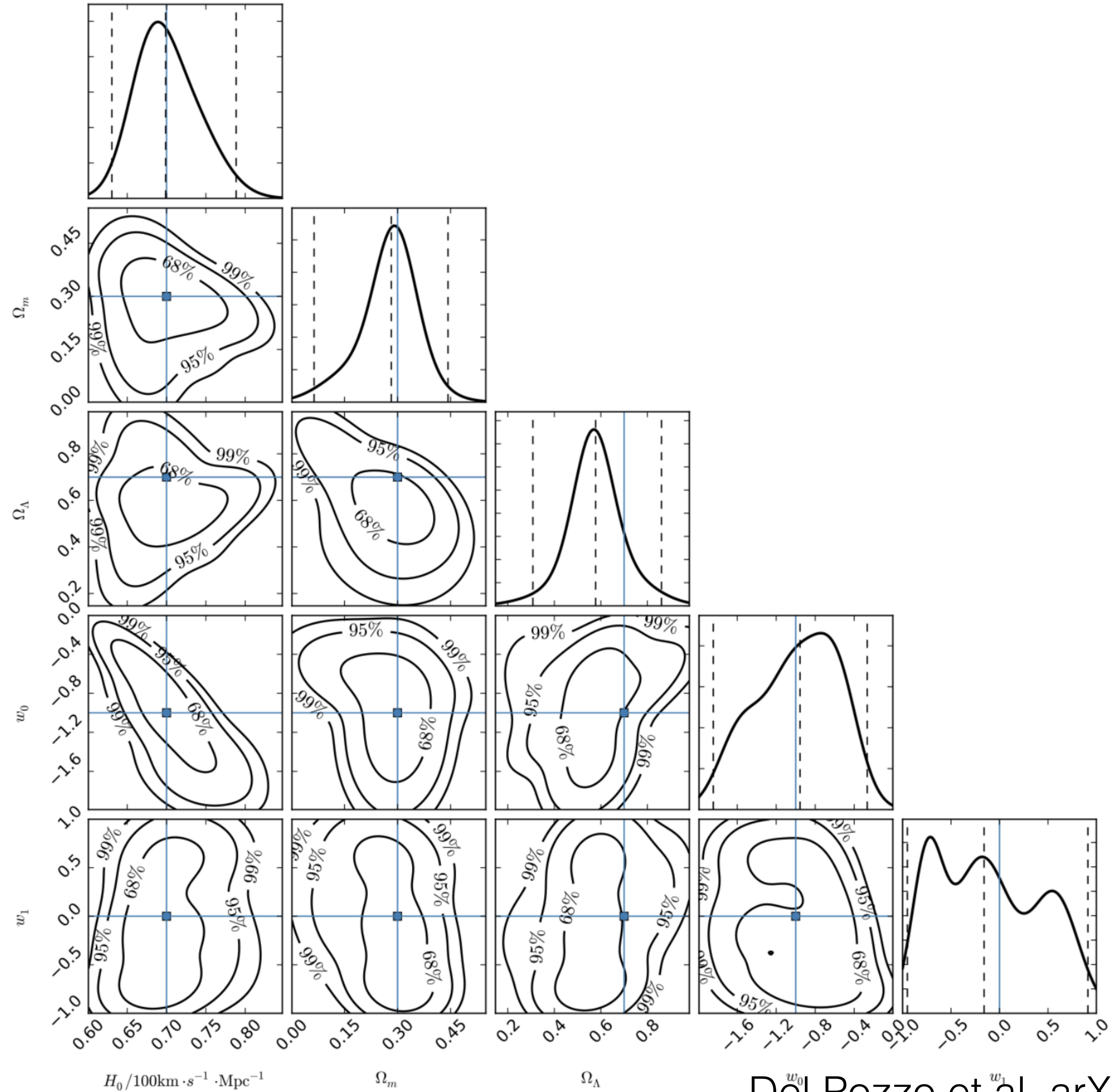
N / Model	10^3	10^4	10^5	10^6	10^7
General FRW	2.3×10^{-1}	0.7×10^{-1}	2.3×10^{-2}	0.7×10^{-2}	2.3×10^{-3}
General FRW+DE	2.8×10^{-1}	0.9×10^{-1}	2.8×10^{-2}	0.9×10^{-2}	2.8×10^{-3}

Δw_0

N / Model	10^3	10^4	10^5	10^6	10^7
General FRW+DE	0.8×10^{0}	2.5×10^{-1}	0.8×10^{-1}	2.5×10^{-2}	0.8×10^{-2}

Δw_1

N / Model	10^3	10^4	10^5	10^6	10^7
General FRW+DE	0.9×10^{0}	2.9×10^{-1}	0.9×10^{-1}	2.9×10^{-2}	0.9×10^{-2}



Del Pozzo et al, arXiv:1506.06590

Selection and population effects

- GW and EM have detection thresholds
    - e.g. for GW, $\rho(x, \Omega) > \rho_T$
- Consider a population of indistinguishable GW events

$$p(N \mid T, \Omega, \lambda, HI) = \frac{(R(\Omega, \lambda)T)^N e^{-R(\Omega,\lambda)T}}{N!}$$

- λ are population parameters (e.g. mass function)
- $R(\Omega, \lambda)$ is the integrated merger rate

Selection and population effects

- However, each GW event D_i is characterised by its own set of parameters x_i, so we need to consider

$$p((\overrightarrow{D}, \overrightarrow{x}) \,|\, T, \Omega, \lambda, H\, I) = \prod_{i=1}^{N} \underset{\substack{\uparrow \\ \text{individual likelihood}}}{p(D_i \,|\, x_i\, \Omega\, \lambda\, H\, T\, I)}\, \underset{\substack{\uparrow \\ \text{population based prior } \propto \frac{dR(\Omega, \lambda)}{dx_i}}}{p(x_i \,|\, \Omega\, \lambda\, H\, T\, I)}\, e^{-R(\Omega,\lambda)T}$$

- Define $p(D_i \,|\, x_i\, \Omega\, \lambda\, H\, T\, I) \equiv \mathscr{L}(D_i; x_i)$ and $p(x_i \,|\, \Omega\, \lambda\, H\, T\, I) \equiv f(x_i)$

- If $N = N_o + N_m$

$$p((\overrightarrow{D}, \overrightarrow{x}) \,|\, T, \Omega, \lambda, H\, I) = \prod_{i=1}^{N_o} \mathscr{L}(D_i; x_i) f(x_i) \prod_{j=1}^{N_m} \mathscr{L}(D_j; x_j) f(x_j) e^{-R(\Omega,\lambda)T}$$

Marginalisation over N_m

- We do not know how many events we missed, so we marginalise over them (e.g. Mandel et al, arXiv:1809.02063)

$$p((\overrightarrow{D}, \overrightarrow{x})_o \,|\, T, \Omega, \lambda, H\,I) = \prod_{i=1}^{N_o} \mathscr{L}(D_i; x_i) f(x_i) e^{-R_o(\Omega,\lambda)T}$$

$$R_o(\Omega, \lambda) = \int_{D|detection} dDdx \mathscr{L}(D; x) f(x)$$

- Which is the integral on all possible datasets and all possible population parameters that would be detected

Population effects

- Mis-modeling the population might lead to significant biases in Ω (e.g. Mastrogiovanni et al, arXiv:2103.14663)
- Simultaneous inference of λ and Ω
  - Larger number of GW events needed than “naive” simulations suggest
  - Significant computational challenge

EM selection effects

- The situation is further complicated for the statistical method

$$p(z\,|\,\Omega\,H\,I) = p(z\,|\,\Omega\,H\,G\,I)p(G\,|\,\Omega\,H\,I) + p(z\,|\,\Omega\,H\,\overline{G}\,I)p(\overline{G}\,|\,\Omega\,H\,I)$$

$$p(G\,|\,\Omega\,H\,I) \equiv p(\overrightarrow{L}_o\,\vec{z}_o\,|\,\Omega\,H\,I)$$

- $p(\overrightarrow{L}_o\,\vec{z}_o\,|\,\Omega\,H\,I)$ is obtained by marginalisation over the un-detected galaxy population (see Kelly et al, arXiv:0805.2946, LVC arXiv:1908.06060)

Selection effects: summary

- Selection effects are extremely important
  - both the GW and the EM selection functions depend on Ω
- Full, general treatment not yet demonstrated
  - Simplifications possible for local ($z<0.1$) sources
  - High redshifts will require full treatment

Conclusions

- GW observation allow cosmological measurements that are independent of the cosmic distance scale ladder
  - independent tests of current cosmological paradigm
- Second generation instruments will constrain Hubble constant
  - GW170817 stupendous glimpse in the future
  - Statistical methods based on cross correlations with catalogs are possible and should lead to ~few % H0 determination - but remember selection effects
- LISA ultimate probe for GW cosmology