STOCHASTIC BACKGROUND THEORY AND RESULTS

Tania Regimbau

ISAPP, June 14th 2021

Recommended bibliography

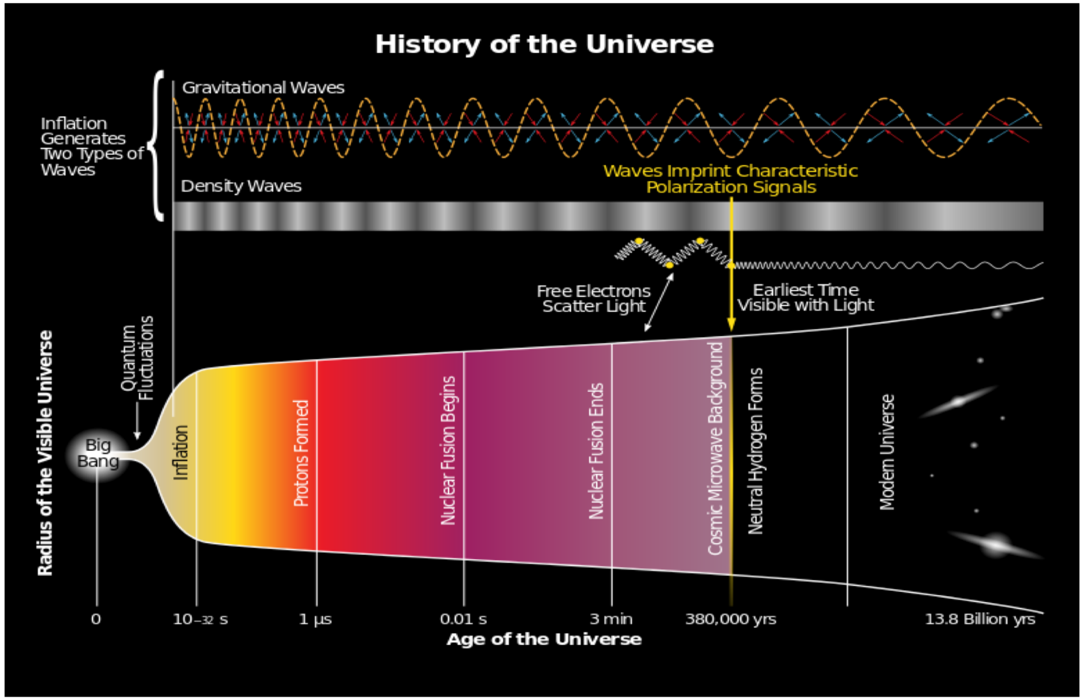
- **Gravitational Wave Experiments and Early Universe Cosmology** , M. Maggiore, gr-qc/9909001
- **Detecting a stochastic background of gravitational radiation: Signal processing strategies and sensitivities**, Allen B. and Romano J., gr-qc/9710117
- **Detection methods for stochastic gravitational-wave backgrounds: a unified treatment,** Romano J. and Cornish, arXiv:1608.06889
- **Cosmological Backgrounds of Gravitational Waves,** Caprini C. and Figueroa D., arXiv:1801.04268
- **The astrophysical gravitational wave stochastic background**, Regimbau T., arXiv:1101.2762
- **Gravitational wave background from binary systems,** Rosado P.

Stochastic GW background (SGWB)

A stochastic background of gravitational waves has resulted from the superposition of a large number of independent unresolved sources.

- cosmological (signature of the early Universe)

- Astrophysical (since the beginning of stellar activity)



SGWB = noise

Family dinner

Cocktail party

SGWB = symphony of the Universe

Orchestra

SGWB = symphony of the Universe

whether there is someone directing is out of the scope of the lecture

?

Orchestra

Decoupling

The condition for a particle to decouple from the primordial plasma is:

$$\Gamma \ll H$$

Interaction rate

Hubble time (expansion)

gravitons: 10^{-43} s

neutrinos: ~1 s

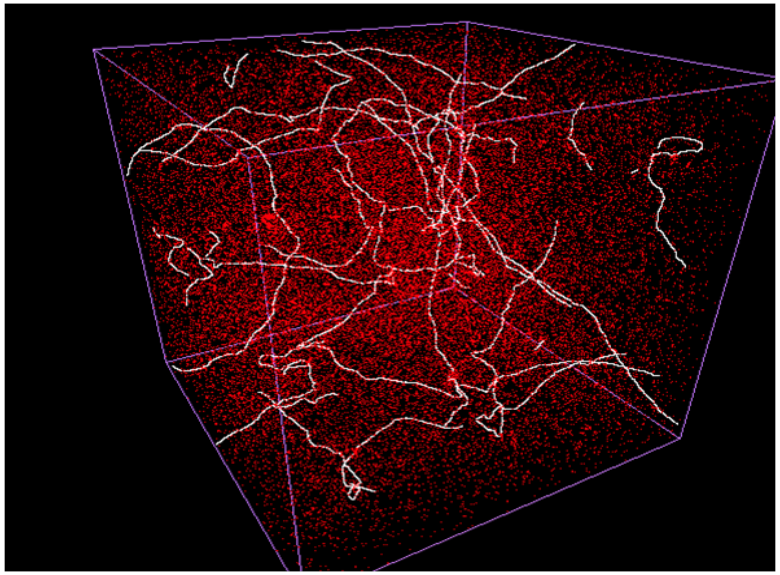
photons: ~380,000 yr

Cosmological background

- Unique window on the very early stages of the Universe and on the physical laws that apply at the highest energy scales (potentially up to the Grand Unified Theory (GUT) scale 10^{16} GeV).
- Results from the amplification of vacuum metric fluctuations during inflation (see arXiv:1610.06481)
- Active sources could have enhanced GW production at the end of inflation (particle production, reheating, spectator fields, primordial black holes)
- Other models include cosmic phase transitions, topological defects (cosmic (super)strings)

Cosmic strings (superstrings)

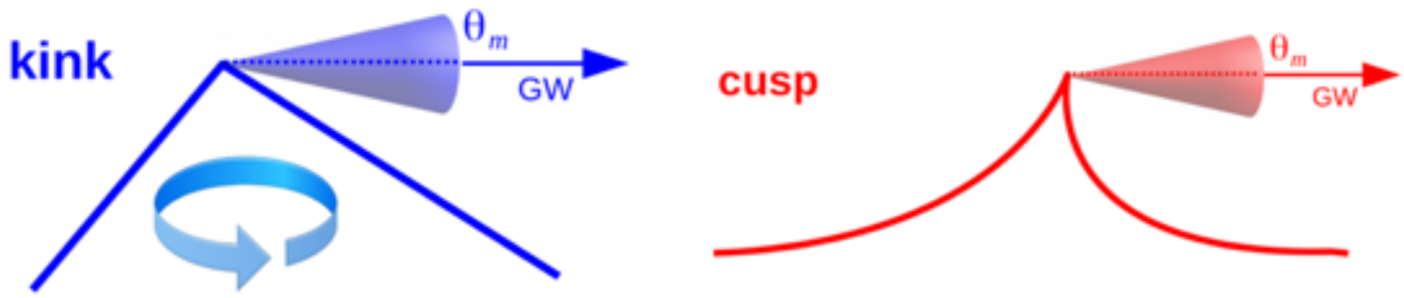
1D topological defects which can be formed in GUT-scale phase transitions in the early Universe. Strings are charactarized by 2 parameters: tension and intercommutation probability *p.*



Cosmic strings (superstrings)

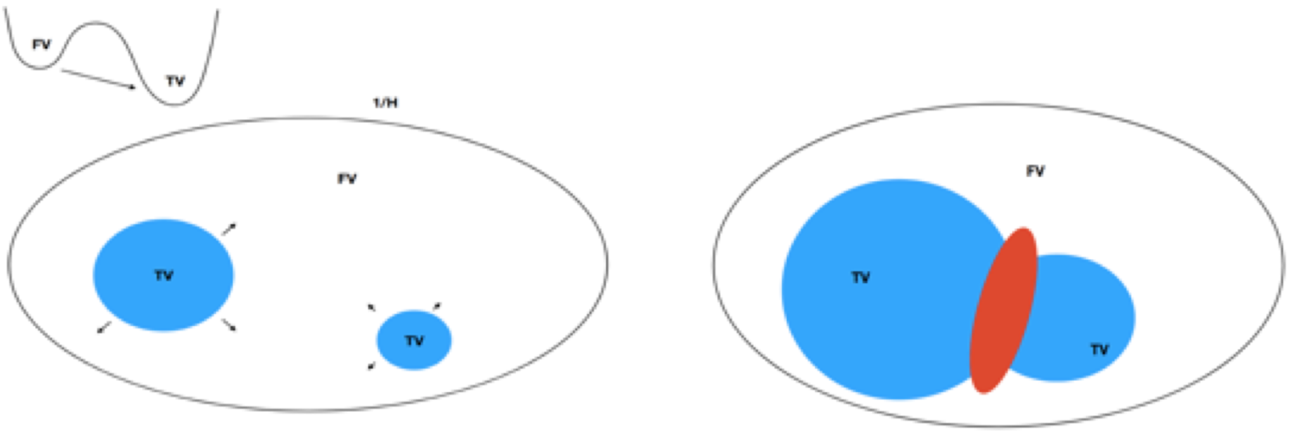
- 1D topological defects which can be formed in GUT-scale phase transitions in the early Universe. Strings are charactarized by 2 parameters: tension and intercommutation probability *p.*

- They can produce large amount of GWs through the production of loops (cusps and kinks).



Phase transitions

First order phase transitions in the early Universe can produce a large amount of GWs through bubble collisions, sound waves and magnetohydrodynamic turbulence.

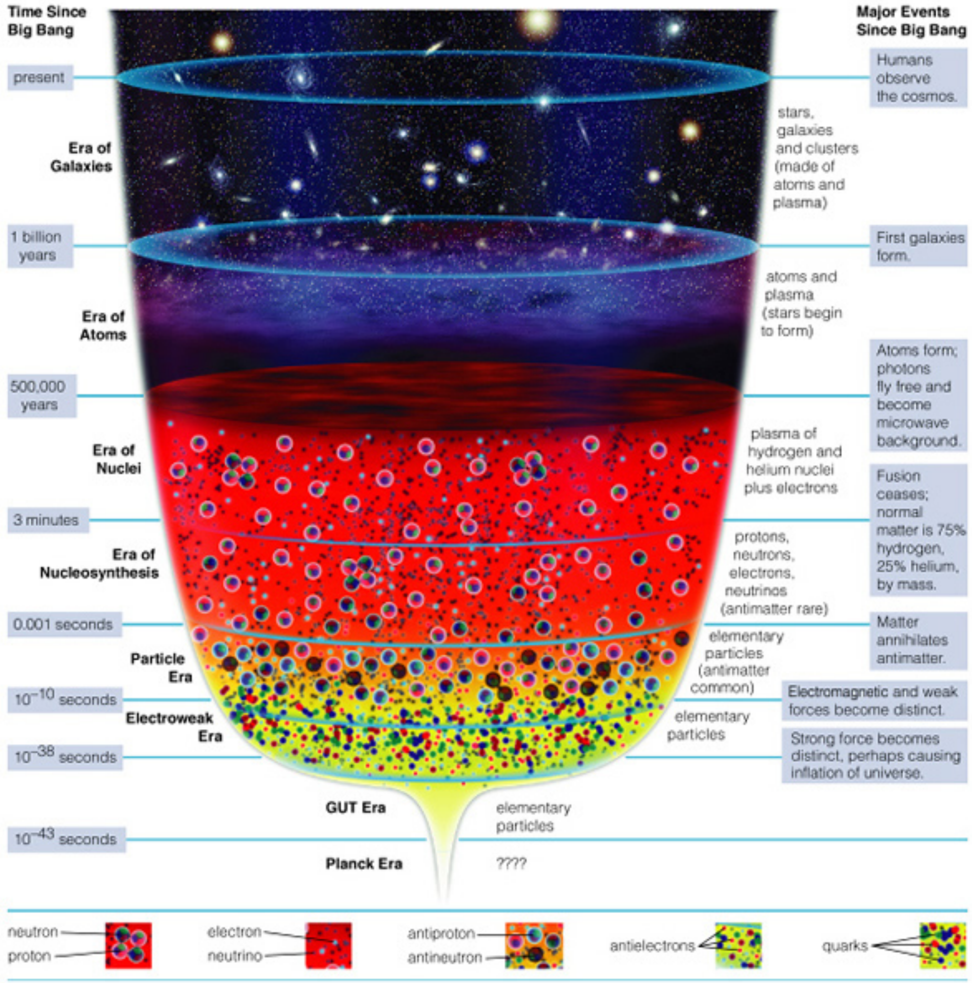


Phase transitions

- GUT (10^{15}-10^{16} GeV): Pulsar Timing Array

- Electroweak (130 GeV) : LISA

- In LIGO/Virgo PT occuring at 10^{7}-10^{10} GeV not accessible by LHC (see arXiv: 2102.01714)



Astrophysical Backgrounds

- All the sources that cannot be resolved individually (overlapping or below threshold)
- Complementary to individual detections (probe the high redshift population)
- Carry lots of information about the star formation history, the metallicity evolution, the average source parameters.
- May have different statistical properties: non continuous, non-Gaussian, non isotropic
- But can be a noise for the cosmological background

Astrophysical Backgrounds

- Formed by sources that cannot be resolved individually
- Compact binary coalescences, pulsars, core- collapse supernovae, BH ringdown, initial instabilities in neutron stars
- Complementary to individual detections (probe the high redshift population)
- Carry lots of information about the star formation history, the metallicity evolution, the average source parameters.
- But can be a noise for the cosmological background
- May have different statistical properties: non continuous, non-Gaussian, non isotropic

Characterizing the SGWB

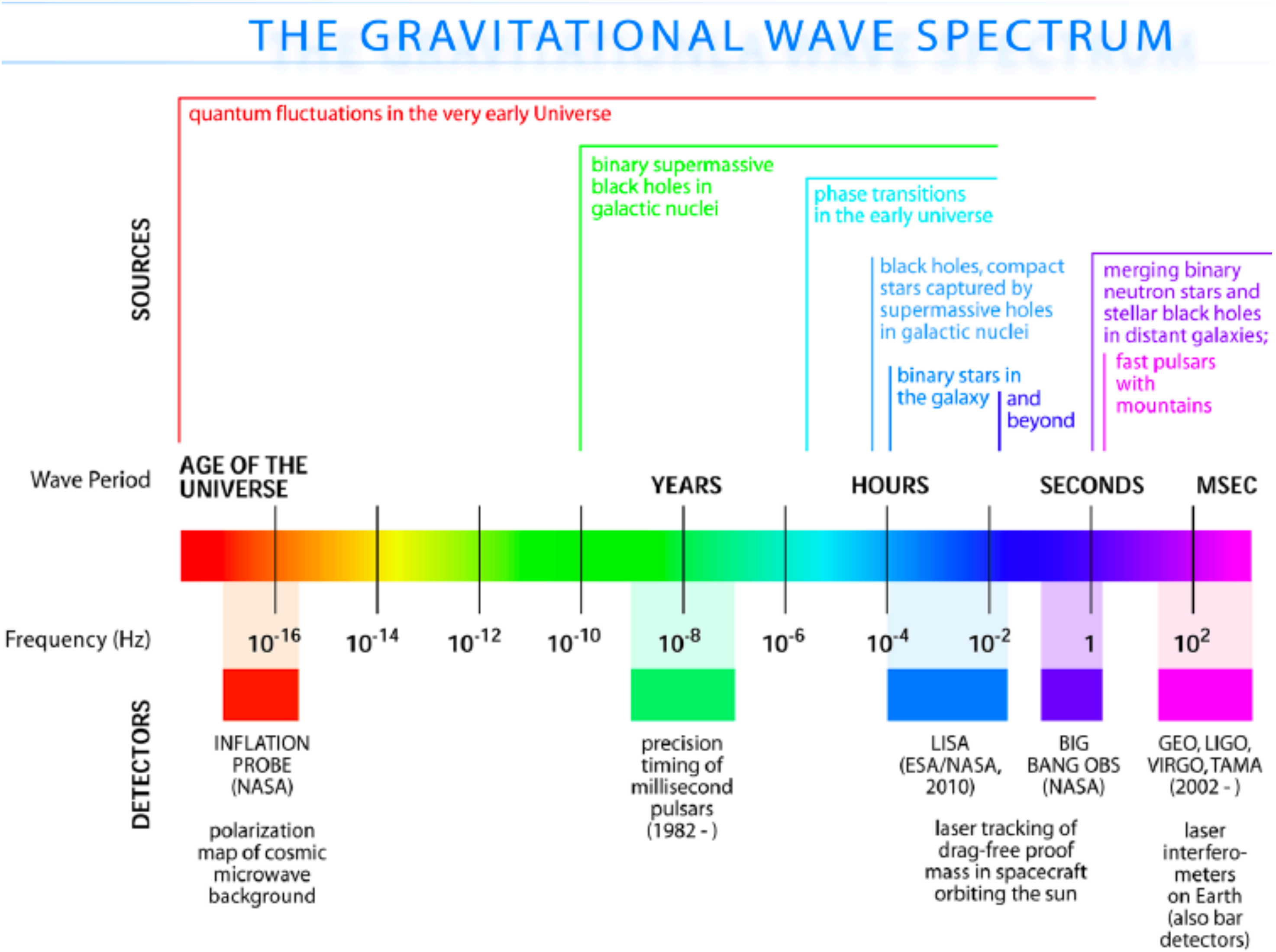
Assuming the background is Gaussian, stationary, isotropic and unpolarized (by analogy with the CMB), it can be completely characterized by the dimensionless spectral energy density parameter :

energy density in GWs

$$\Omega_{gw}(f) = \frac{f}{\rho_c} \frac{d\rho_{gw}}{df}$$

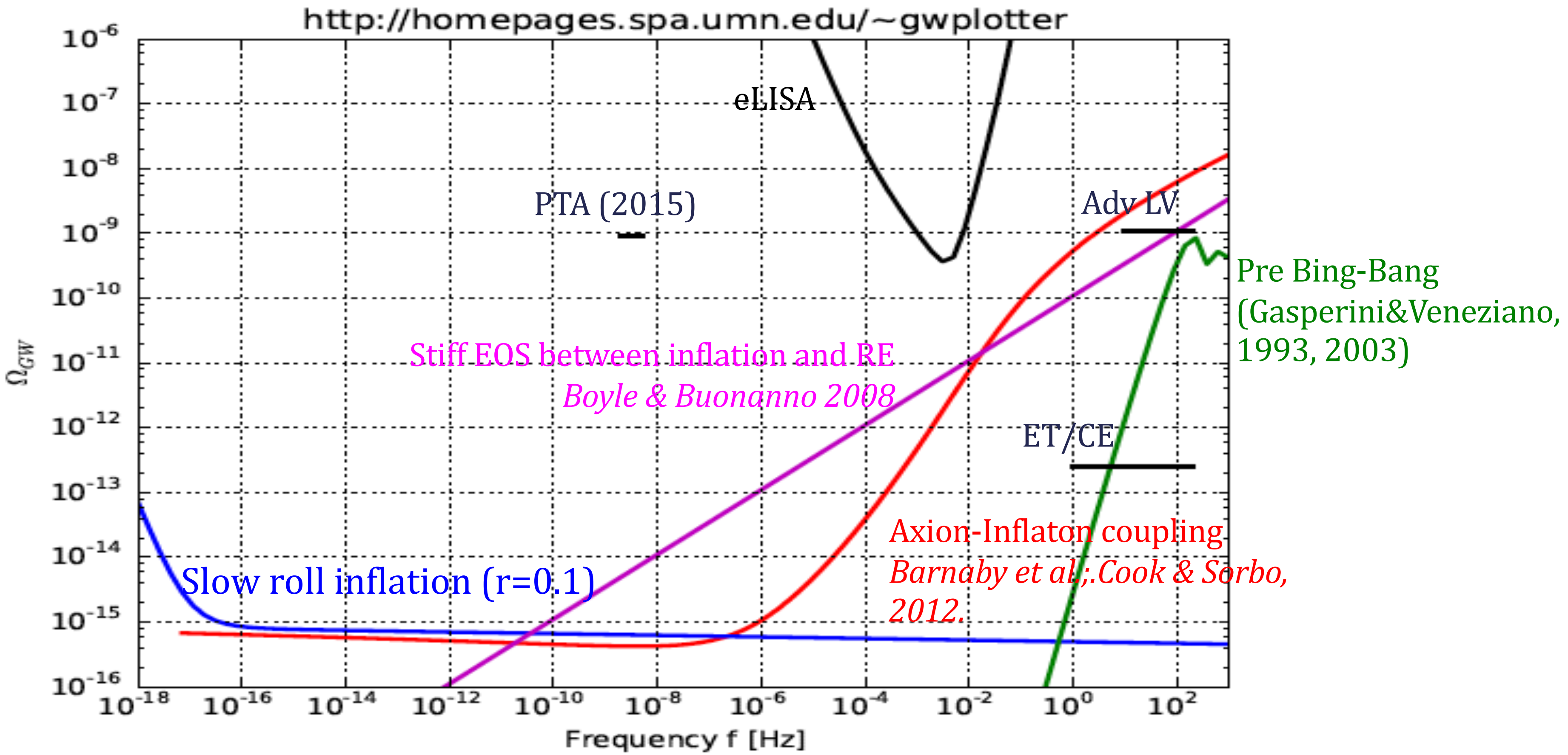
closure energy density

with $\rho_c = \dfrac{3H_0^2 c^2}{8\pi G}$



THE GRAVITATIONAL WAVE SPECTRUM
SOURCES
quantum fluctuations in the very early Universe
binary supermassive black holes in galactic nuclei
phase transitions in the early universe
black holes, compact stars captured by supermassive holes in galactic nuclei
binary stars in the galaxy
and beyond
merging binary neutron stars and stellar black holes in distant galaxies;
fast pulsars with mountains
Wave Period
AGE OF THE UNIVERSE
YEARS
HOURS
SECONDS
MSEC
Frequency (Hz)
10-16
10-14
10-12
10-10
10-8
10-6
10-4
10-2
1
102
DETECTORS
INFLATION PROBE (NASA)
polarization map of cosmic microwave background
precision timing of millisecond pulsars (1982 -)
LISA (ESA/NASA, 2010)
BIG BANG OBS (NASA)
laser tracking of drag-free proof mass in spacecraft orbiting the sun
GEO, LIGO, VIRGO, TAMA (2002 -)
laser interfero-meters on Earth (also bar detectors)

Background from inflation



Constraints from CMB

- Energy density in GWs :

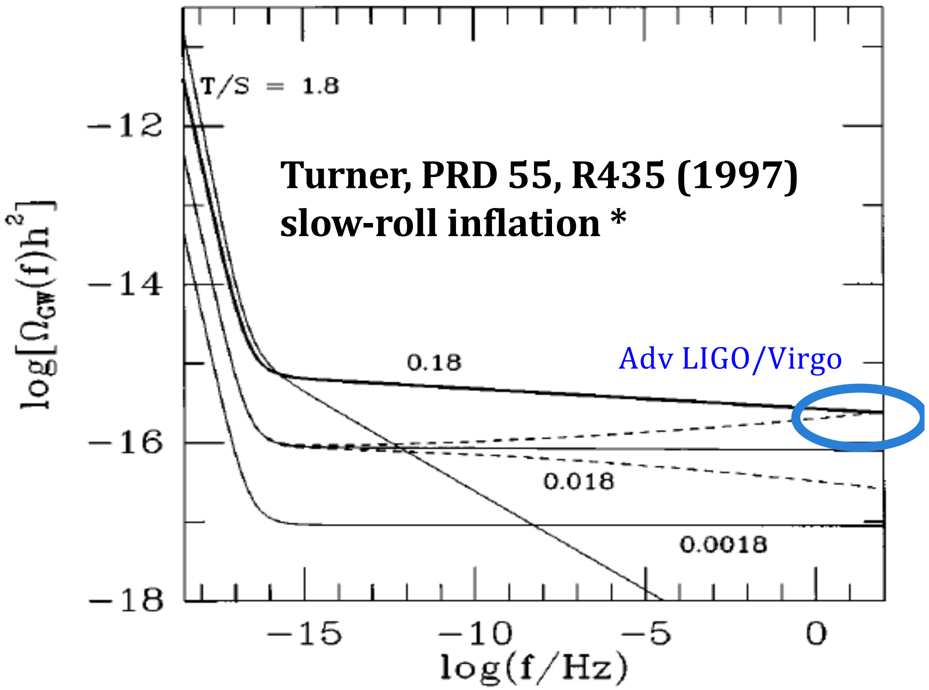
$$\Omega_{gw}(f) = \frac{f}{\rho_c}\frac{d\rho_{gw}}{df}$$

- Amplitude scales with *r=T/S*

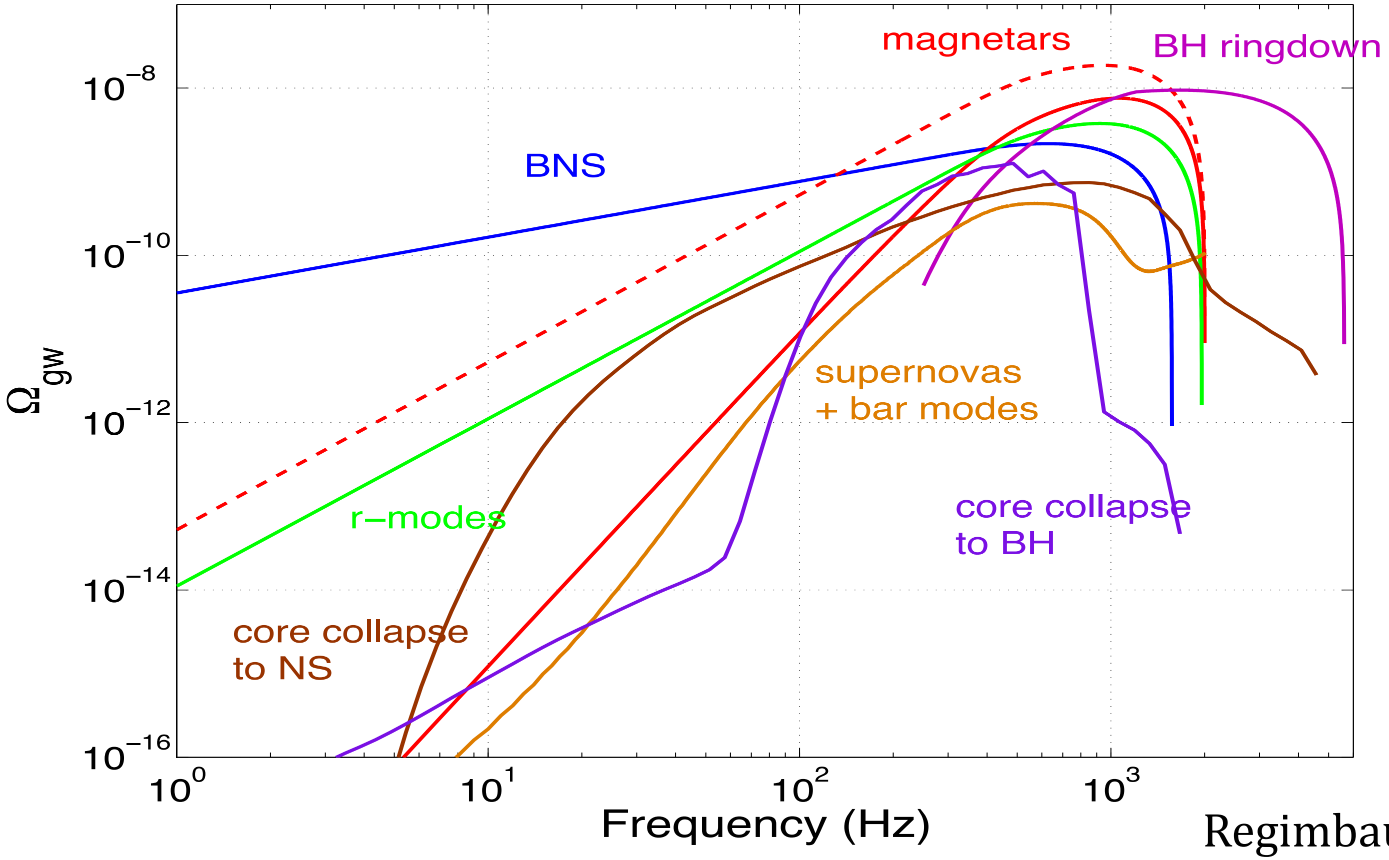
- Spectral shape depends on *r:*

$$\Omega_{gw}(f) \approx f^{n_T} \text{ with } n_T = -r/8$$

- Bicep2/Keck/Planck gives r<0.1 at 95% confidence (arXiv:1510.09217)



Astrophysical Backgrounds



Regimbau, arXiv:1101.2762

Implications of the LIGO/Virgo detections

- The events we detect now are loud individual sources at close distances

- Many more individual sources at larger distances that contribute to create a stochastic background, which could be the next milestone for LIGO/Virgo.

- Using mass distributions and local rates derived from the first observations, we were able to revise previous predictions of the GW background from BBHs and BNSs.

Stochastic background from BNSs and BBHs

Energy density in GWs given by:

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c c} f F(f)$$

with $$F(f) = \int_0^{z_{\max}} \frac{\mathrm{d}z}{4\pi D_{\rm L}^2(z)} \frac{dE_{gw}}{df}(f, \bar{\mathcal{M}}(1+z)) \frac{dR}{dz}(z)$$

Rate

Spectral properties
of individual sources

Stochastic background from BNSs and BBHs

The rate per redshift interval in the observer frame: $$\frac{dR}{dz}(z) = \frac{R_m(z)}{(1+z)} \frac{dV}{dz}$$

with $$\frac{dV}{dz}(z) = 4\pi \frac{c}{H_0} \frac{r(z)^2}{Ez(z)} \quad , \quad r(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{Ez(z')}$$

and $$R_m(z) = \int SFR(z) P(t_d) dt_d$$

Probability distribution of the delay between formation of the massive progenitors and merger

$$P(t_d) \sim \frac{1}{t_d} \text{ in } [t_{min} - t_{max}]$$

Stochastic background from BNSs and BBHs

The energy density, in the inspiral phase and up to the last stable orbit is:

$$\frac{dE_{gw}^{N,C}}{df_s}(f_s) = \frac{5(G\pi)^{2/3}\mathcal{M}_c^{5/3}F_\iota}{12} f_s^{-1/3}$$

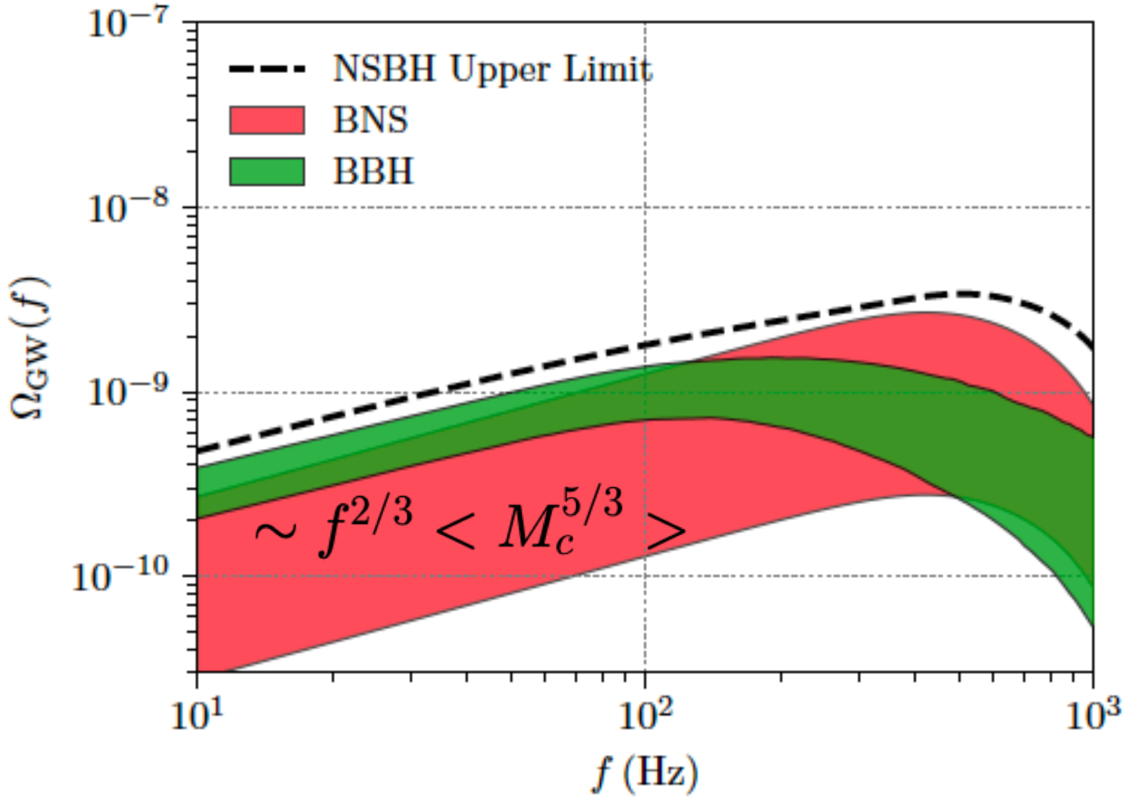
where $F_\iota = (1+\cos^2\iota)^2/4 + \cos^2\iota$

Adding post newtonian corrections and including merger and ringdown phase for BBHs:

$$\frac{dE_{gw}^{P,C}}{df_s}(f_s) = \frac{dE_{gw}^{N,C}}{df_s}(f_s) \begin{cases} (1+\sum_{i=2}^{3}\alpha_i\nu^i)^2 & \text{if } f_s < f_{merg} \\ f_s w_m (1+\sum_{i=1}^{2}\epsilon_i\nu^i)^2 & \text{if } f_{merg} \leq f_s < f_{ring} \\ f_s^{1/3} w_r \mathcal{L}^2(f_s, f_{ring}, \sigma) & \text{if } f_{ring} \leq f_s < f_{cut} \end{cases}$$

arXiv:0909.2867

Estimate from Detected Sources



$$\Omega_{\rm BNS}(25\,{\rm Hz}) = 2.1^{+2.9}_{-1.6} \times 10^{-10}$$

$$\Omega_{\rm BBH}(25\,{\rm Hz}) = 5.0^{+1.7}_{-1.4} \times 10^{-10}$$

arXiv:2101.12130

Time domain behavior

Depending average number of events overlapping (duty cycle):

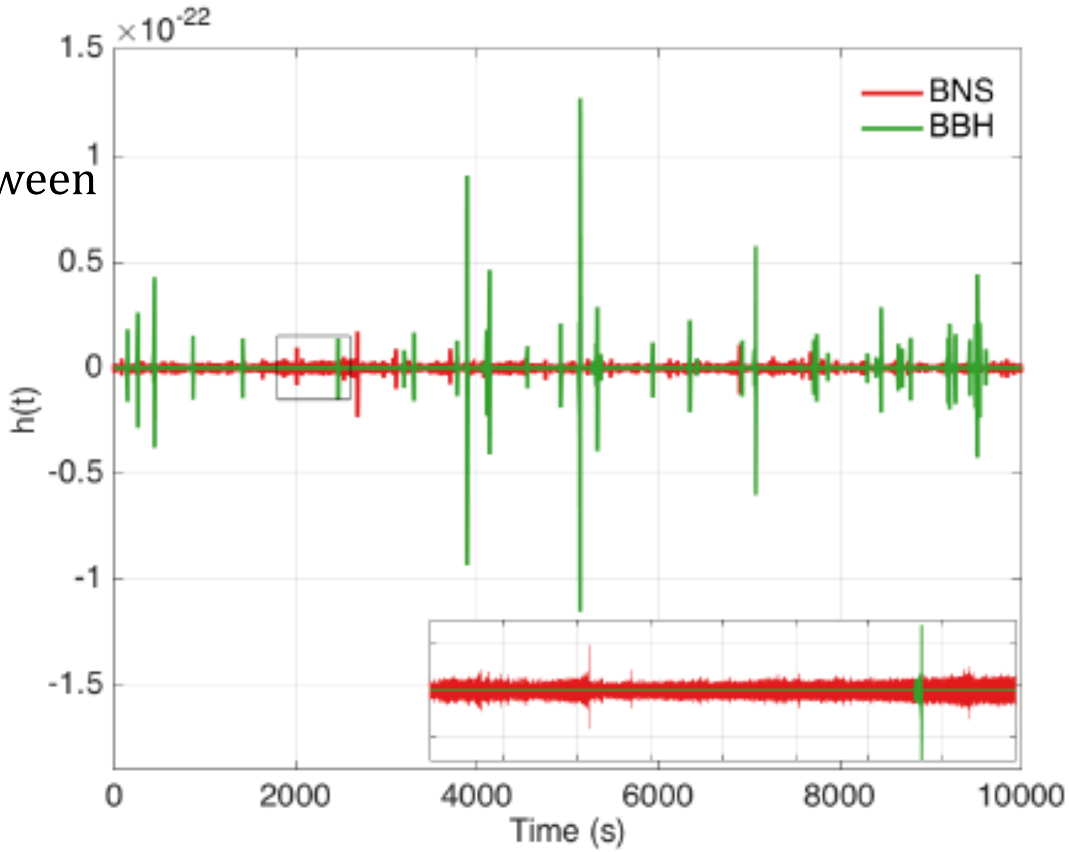
(time interval between events)$^{-1}$

waveform duration

$$\Lambda(z) = \int_0^z \tau^o(z) \frac{dR^o}{dz}(z) dz$$

the GW signal may result in a continuous (Λ>1) or popcorn (Λ<1) background.

BNS = continuous and BBH = popcorn



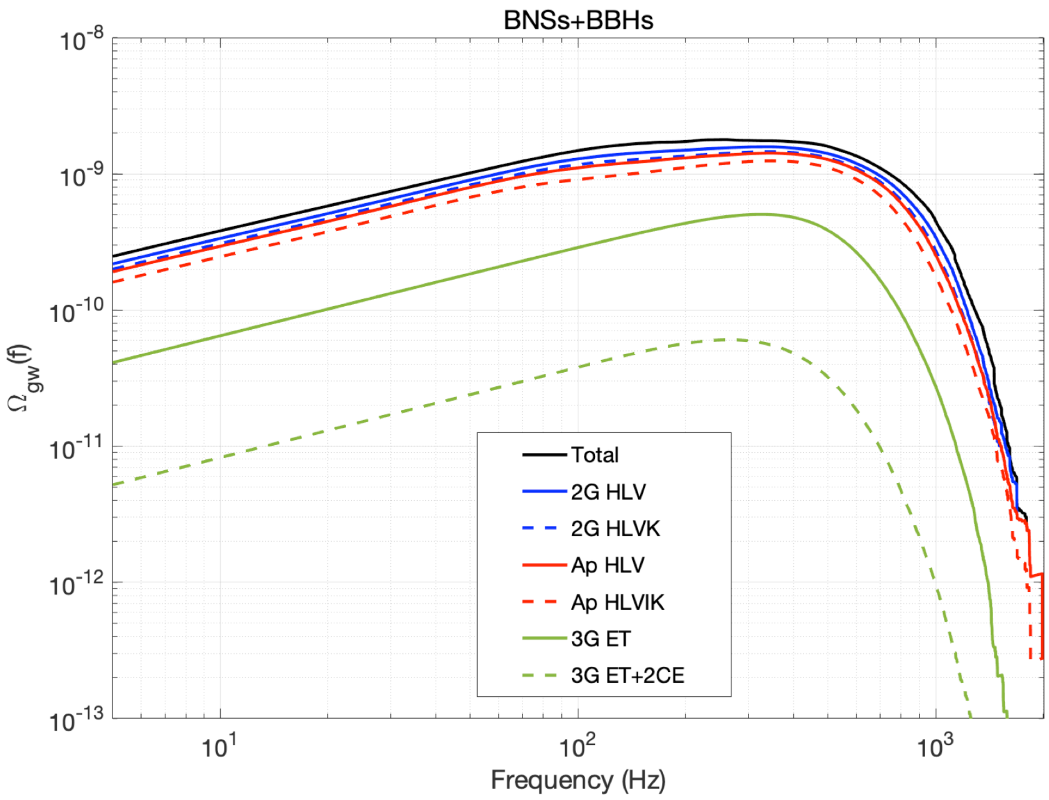
Residual background

“A stochastic background of gravitational waves has resulted from the superposition of a large number of independent unresolved sources.”

- Unresolved may mean that sources overlap (cosmological) or that the sources are too faint to be detected (example CBCs, see arXiv:2002.05365)
- The residual is the background after detected individual sources have been removed

Residual background

For CBCs the background decreases as the sensitivit of the detectors increases



Data Analysis Principle

Search for excess of coherence in the cross correlated data streams from multiple detectors with minimal assumptions on the morphology of the signal.

- Assume stationary, unpolarized, isotropic and Gaussian stochastic background.
- Cross correlate the output of detector pairs to eliminate the noise:

$$s_i = h_i + n_i$$

$$< s_1 s_2 > = < h_1 h_2 > + \underbrace{\cancel{< n_1 n_2 >}}_{0} + \underbrace{\cancel{< h_1 n_2 >}}_{0} + \underbrace{\cancel{< n_1 h_2 >}}_{0}$$

Cross Correlation Statistics

- Standard CC statistics (Allen & Romano, 1999, PRD, 59, 102001)

- Frequency domain cross product: $Y = \int \tilde{s}_1^*(f)\tilde{Q}(f)\tilde{s}_2(f)df$

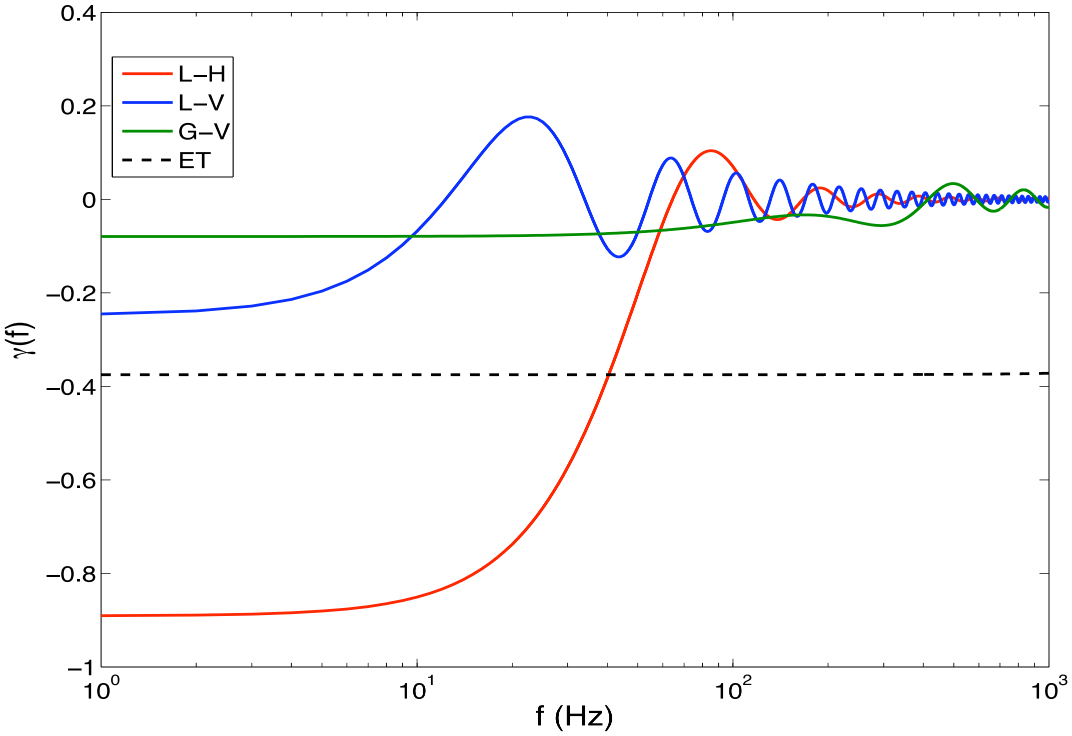
- optimal filter: $\tilde{Q}(f) \propto \dfrac{\gamma(f)\Omega_{gw}(f)}{f^3 P_1(f) P_2(f)}$ with $\Omega_{gw}(f) \equiv \Omega_0 f^{\alpha}$

- in the limit noise >> GW signal

$$\mathrm{Mean}(Y) = \Omega_0 T,\ \mathrm{Var}(Y) \equiv \sigma^2 \propto T,\ \mathrm{SNR} \propto \sqrt{T}$$

Overlap Reduction Function

Loss of sensitivity due to the separation and the relative orientation of the detectors.



Time delay

$$\gamma(f) = \frac{5}{8\pi} \sum_{A=\{+,\times\}} \int e^{2\pi i f \hat{\Omega}\Delta\vec{x}/c} F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega}) d\Omega$$

Detector response

Detectability

- The signal-to-noise ratio for multiple pairs of detectors:

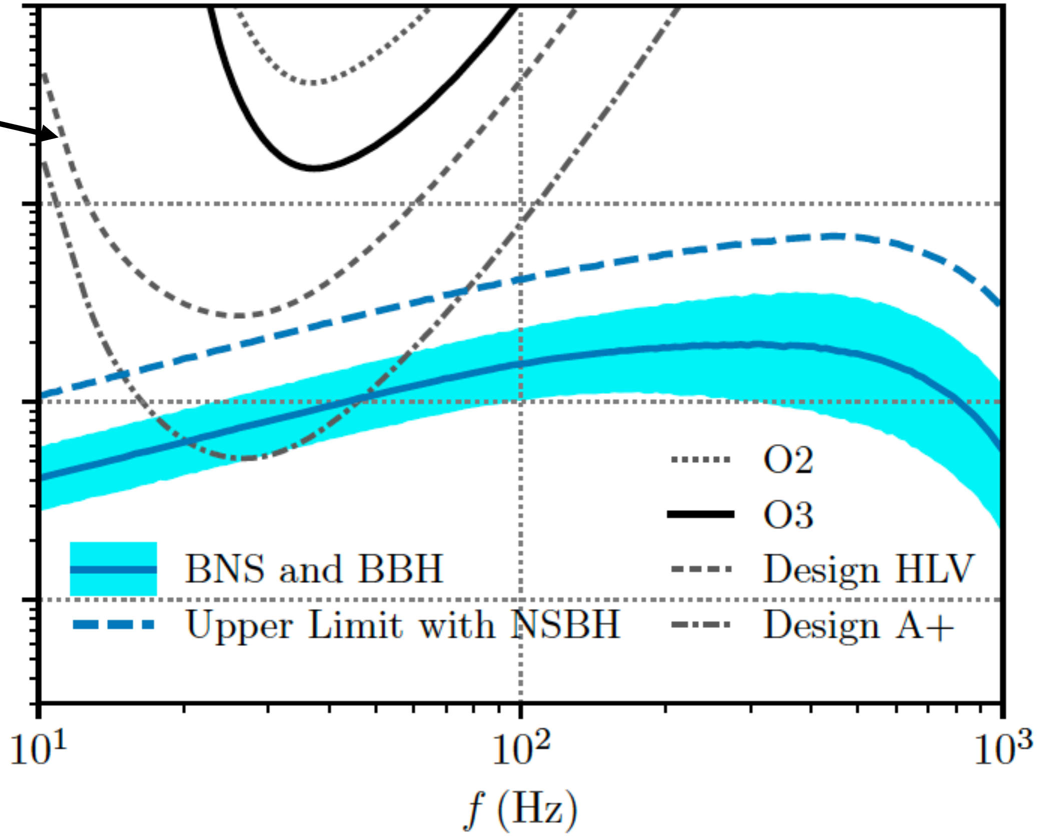
$$\mathrm{SNR} = \frac{3H_0^2}{10\pi^2}\sqrt{2T}\left[\int_0^\infty df \sum_{i=1}^{n}\sum_{j>i}\frac{\gamma_{ij}^2(f)\Omega_{gw}^2(f)}{f^6 P_i(f)P_j(f)}\right]^{1/2}$$

- The power integrated curves give a graphical representation of detector sensitivity for SGWB, taking into account the integration over time and frequency. See Thrane & Romano, arXiv:1310.5300.

- Can apply to LVK, LISA, PTA

Detectability

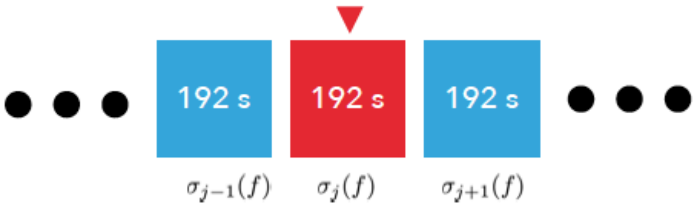
Power integrated curve:
if a GW spectrum crosses or is tangent to the curve it can be detected at 2-sigmas after 2 years (assuming 50% duty cycle)



Pre-analysis: data cut

- data split into half-overlapping 192s segments, downsampled to 1024 Hz, Hann windowed, HPF, Fourier transformed and coarse grained to 0. 03125 Hz.

- remove time segments where the noise is non stationary



$$\boxed{\left|\sigma_j - \frac{\sigma_{j+1} + \sigma_{j-1}}{2}\right| \leq 0.2\sigma_j}$$

- remove frequency bins which display coherence with auxiliary chanels (power mains, GPS timing, Schuman resonances).

- assume ~5% calibration uncertainty.

Constraints on the GW energy density from LVK

- No evidence for a stochastic background (cosmological or astrophysical).

- But set upper limits on the total energy density:

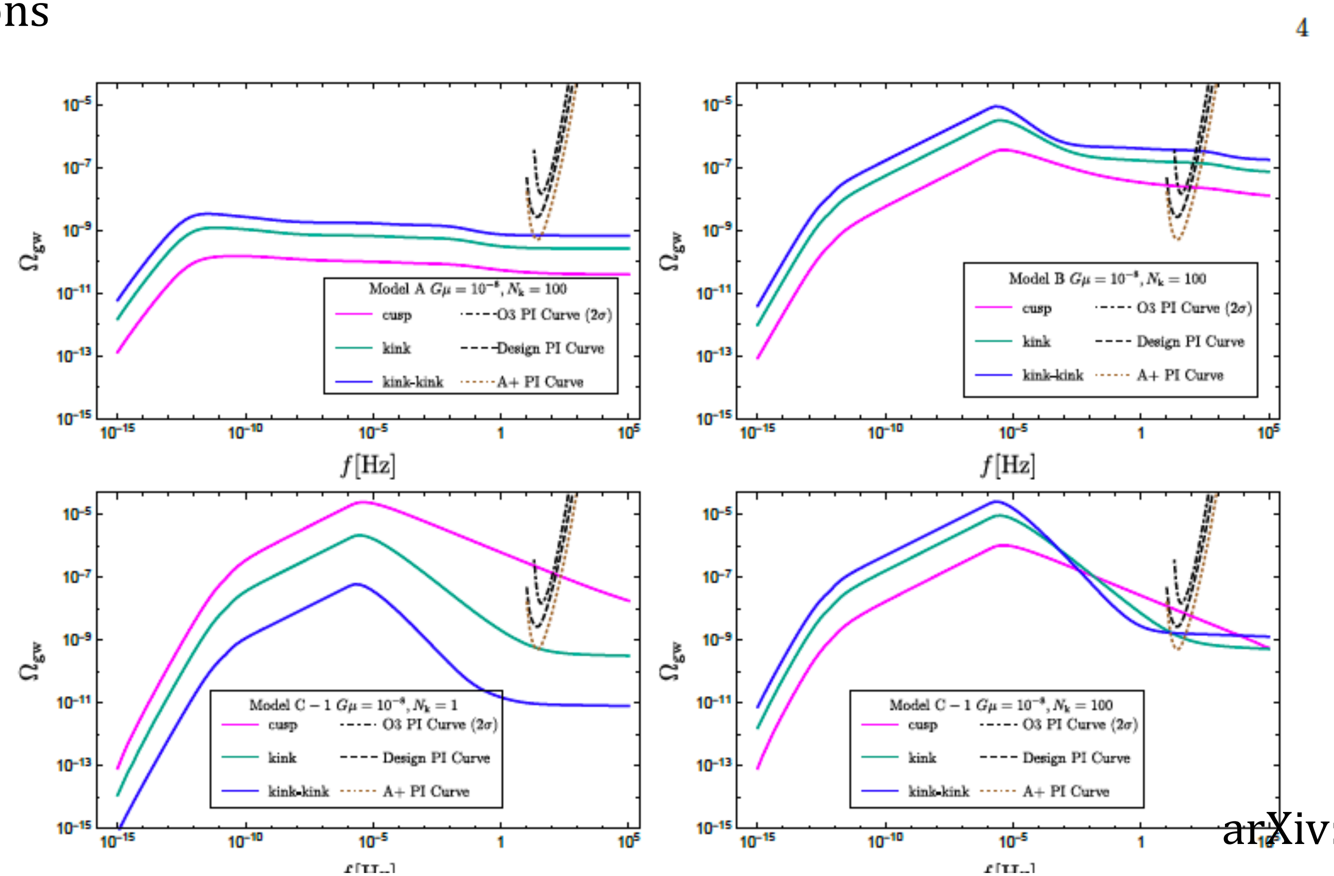
from CBCs

	Uniform prior			Log-uniform prior		
α	O3	O2 [43]	Improvement	O3	O2 [43]	Improvement
0	1.7×10^{-8}	6.0×10^{-8}	3.6	5.8×10^{-9}	3.5×10^{-8}	6.0
2/3	1.2×10^{-8}	4.8×10^{-8}	4.0	3.4×10^{-9}	3.0×10^{-8}	8.8
3	1.3×10^{-9}	7.9×10^{-9}	5.9	3.9×10^{-10}	5.1×10^{-9}	13.1
Marg.	2.7×10^{-8}	1.1×10^{-7}	4.1	6.6×10^{-9}	3.4×10^{-8}	5.1

arXiv:2101.12130

Constraints on cosmic strings models

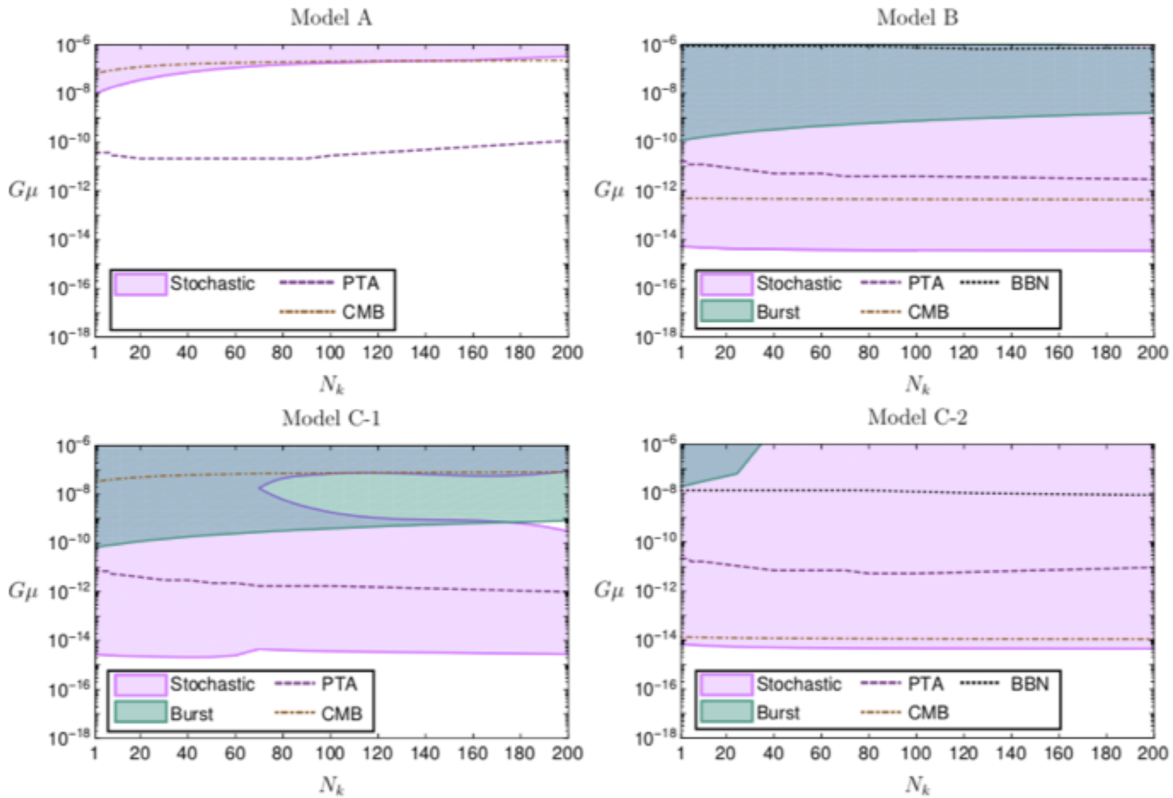
3 models based on Nambu-Goto numerical simulations (A, B and C) with cusps, kinks and kink-kink collisions



arXiv:2101.12248

Constraints on cosmic strings models (p=1)

The stochastic analysis of O1+O2+O3a gives the best constraints for B and C (PTA for A)



A: $G\mu \gtrsim (9.6 \times 10^{-9} - 10^{-6})$

B: $G\mu \gtrsim (4.0 - 6.3) \times 10^{-15}$

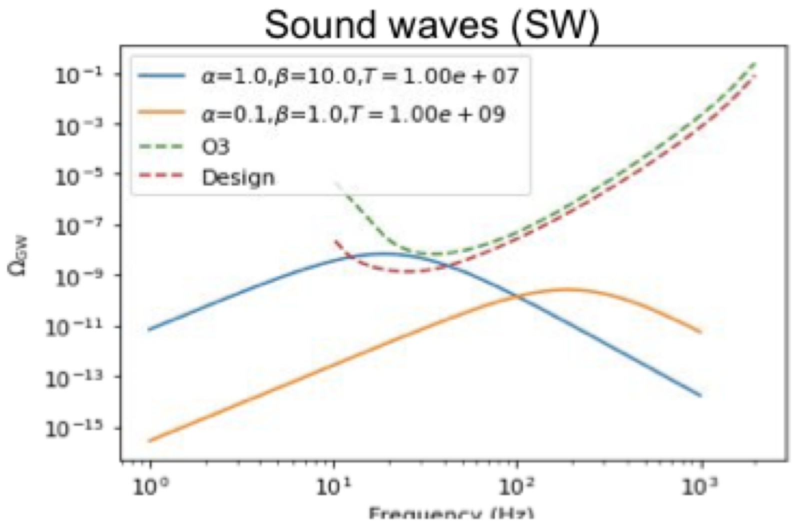
C1: $G\mu \gtrsim (2.1 - 4.5) \times 10^{-15}$

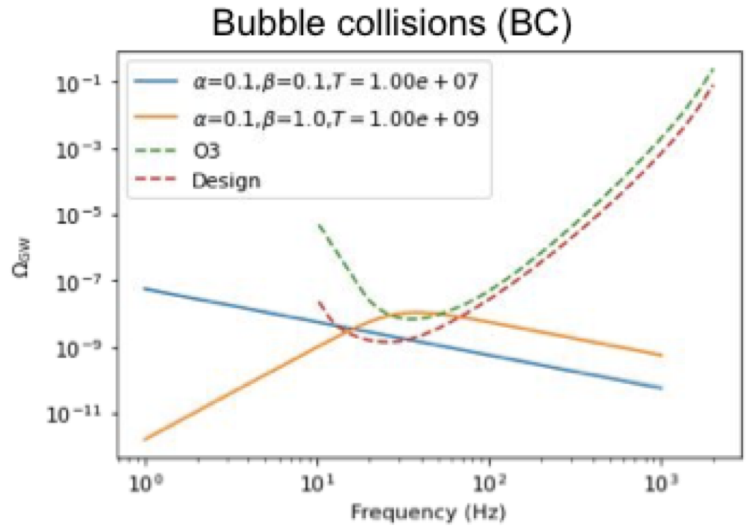
C2: $G\mu \gtrsim (4.2 - 7.0) \times 10^{-15}$

arXiv:2101.12248

Constraints on phase transitions fromLVK

Spectrum modeled by a broken power law whose peak frequency depends on the temperature of the PT



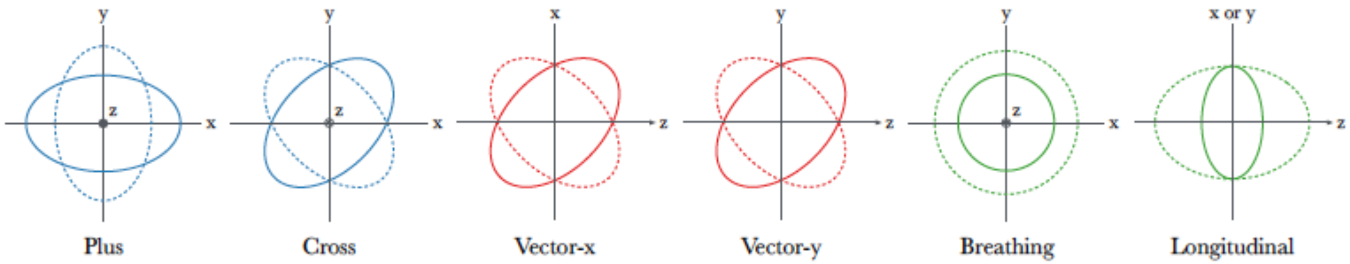


GW background due to bubble collisions is limited to $\Omega_{\rm pt} < 5.0 \times 10^{-9}$ and due to sound waves is limited to $\Omega_{\rm pt} < 5.8 \times 10^{-9}$ at 95% confidence level for temperatures above 10^8 GeV

arXiv:2102.01714

Search for extra polarization

- Most alternative theories of gravity have extra scalar and vector polarization modes and give additional contributions to the energy density of the SGWB.

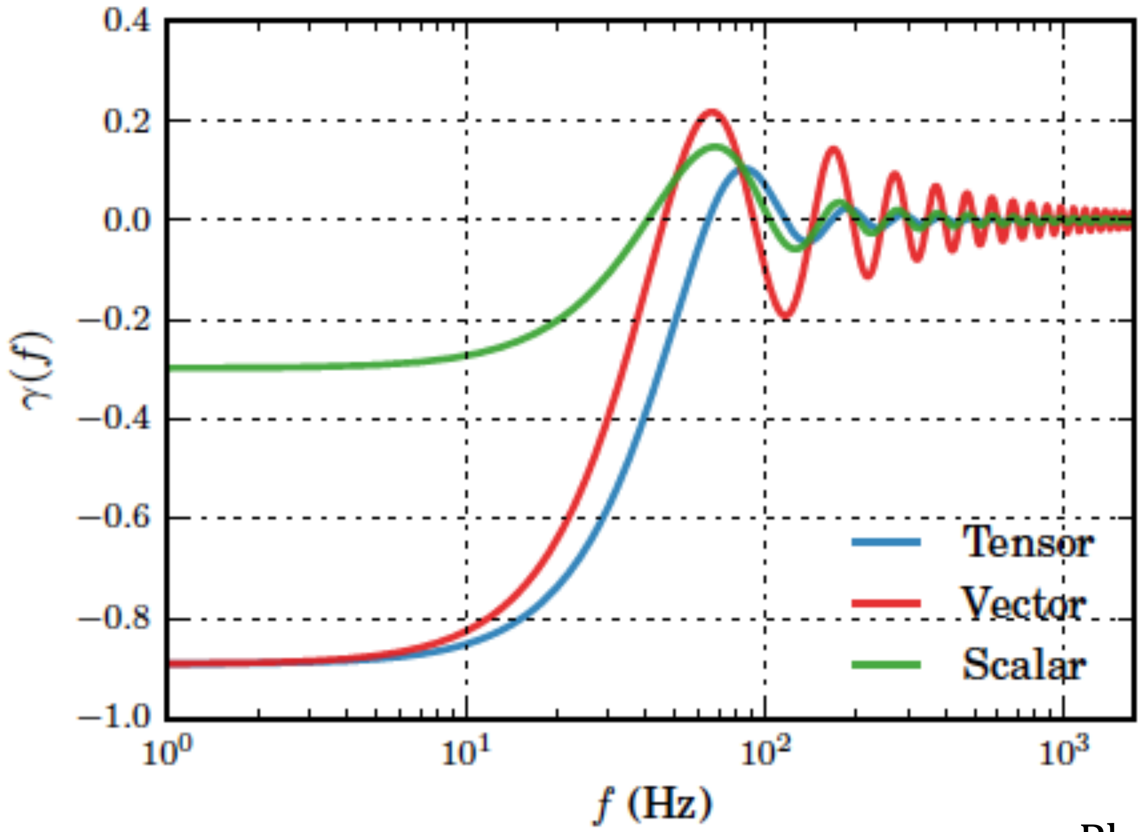


- We assume that the background is Gaussian, isotropic and stationary, uncorrelated between polarization modes and that the tensor and scalar contributions are individually unpolarized.

Overlap reduction functions

$$\langle \tilde{s}_1(f)\tilde{s}_2^*(f')\rangle = \delta(f-f')\sum_A \gamma_A(f) H^A(f)$$

Search for extra polarization



PhysRevX.7.041058

Constraints on extra polarization from LVK

Polarization	O3	O2 [43]	Improvement
Tensor	6.4×10^{-9}	3.2×10^{-8}	5.0
Vector	7.9×10^{-9}	2.9×10^{-8}	3.7
Scalar	2.1×10^{-8}	6.1×10^{-8}	2.9

arXiv:2101.12130

Directional searches

- relax assumption of isotropy and generalize to arbitrary angular distribution.

$$\Omega_{\rm GW}(f) \equiv \frac{f}{\rho_c}\frac{d\rho_{\rm GW}}{df} = \frac{2\pi^2}{3H_0^2} f^3 H(f) \int_{S^2} d\hat{\Omega}\, \mathcal{P}(\hat{\Omega})$$

$$\mathcal{P}(\hat{\Omega}) = \mathcal{P}_\alpha \mathbf{e}_\alpha(\hat{\Omega})$$

- by applying appropriate time varying delays between detectors it is possible to map the angular power distribution in a pixel or spherical harmonic basis

radiometer analysis for point-like sources: $\mathcal{P}(\hat{\Omega}) \equiv \eta(\hat{\Omega}_0)\delta^2(\hat{\Omega}, \hat{\Omega}_0)$

spherical harmonic decomposition : $\mathcal{P}(\hat{\Omega}) \equiv \sum_{lm} \mathcal{P}_{lm} Y_{lm}(\hat{\Omega})$

Models of Anisotropies

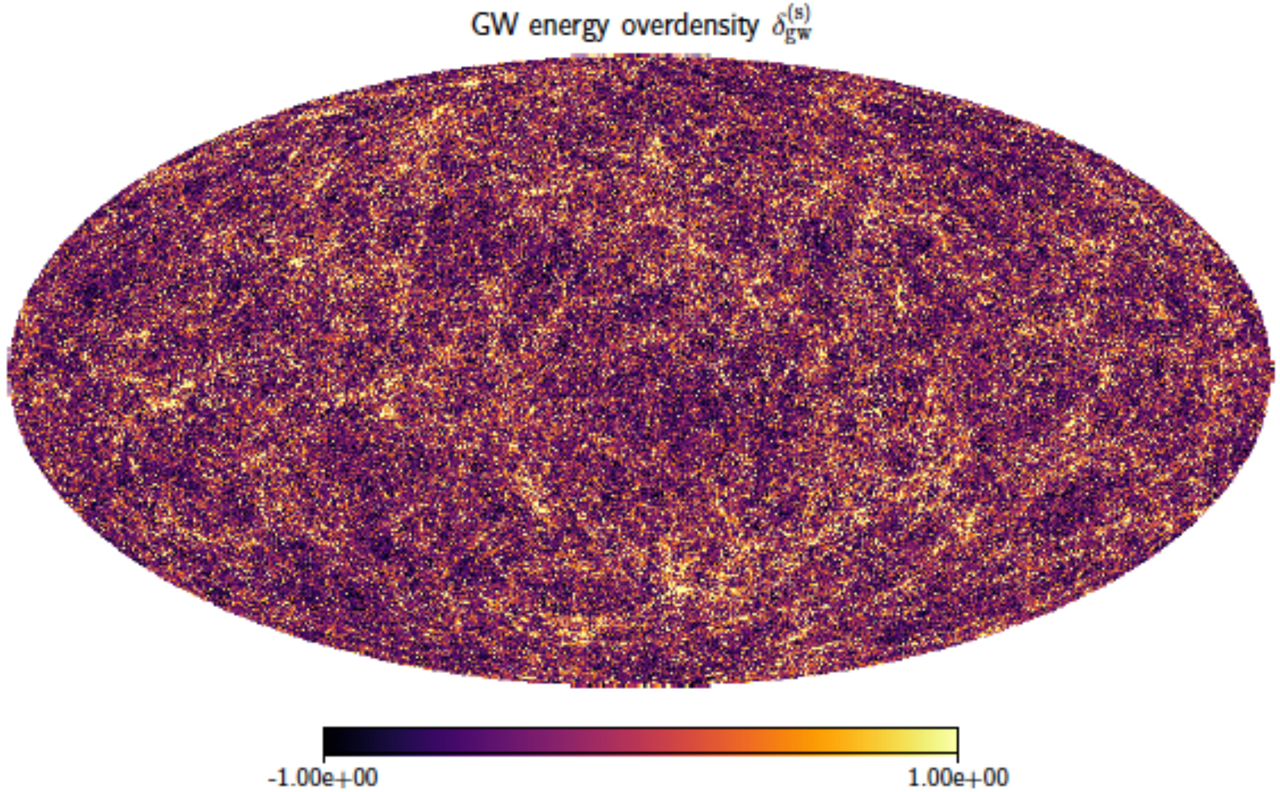
- We expect anisotropy due to the finitness of the number of sources, the nature of spacetime along the line of sight, and for astrophysical models the local distribution of matter.

- Recent efforts in modeling the anisotropy of the SGWB

Cusin et al., Phys.Rev.D96.103119, arXiv:1711.11345 (formalism), arXiv:1803.03236 (compact binary mergers), Jenkins & Sakellariadou arXiv:1802.06046 (formalism and cosmic strings), Jenkins, Sakellariadou, Regimbau & Slezac arXiv:1806.01718 (compact binary mergers, analytical + galaxy catalog).

- Can be extended to any type of SGWB from cosmological or astrophysical origin.

Anisotropies from Compact Binary Mergers



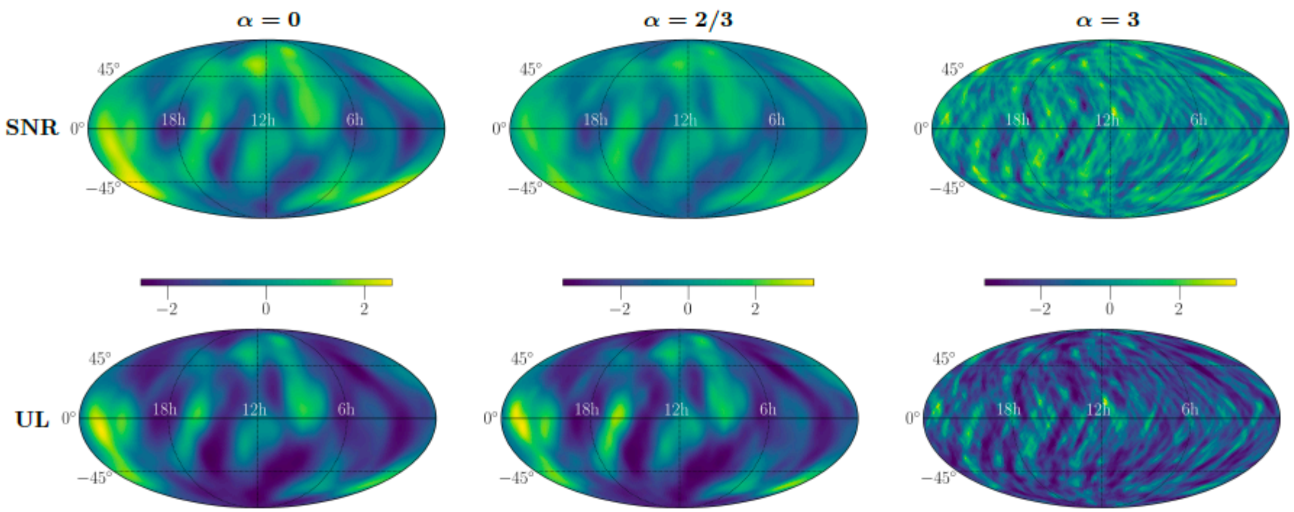
arXiv:1802.06046

Radiometer: LVK results

			Max SNR (% p-value)				Upper limit ranges (10^{-8})	
α	$\Omega_{\rm GW}$	$H(f)$	HL(O3)	HV(O3)	LV(O3)	O1+O2+O3 (HLV)	O1+O2+O3 (HLV)	O1 + O2 (HL)
0	constant	$\propto f^{-3}$	2.3 (66)	3.4 (24)	3.1 (51)	2.6 (23)	1.7 – 7.6	4.5 – 21
2/3	$\propto f^{2/3}$	$\propto f^{-7/3}$	2.5 (59)	3.7 (14)	3.1 (62)	2.7 (24)	0.85 – 4.1	2.3 – 12
3	$\propto f^{3}$	constant	3.7 (32)	3.6 (47)	4.1 (12)	3.6 (20)	0.013 – 0.11	0.047 – 0.32

SNR

90% UL



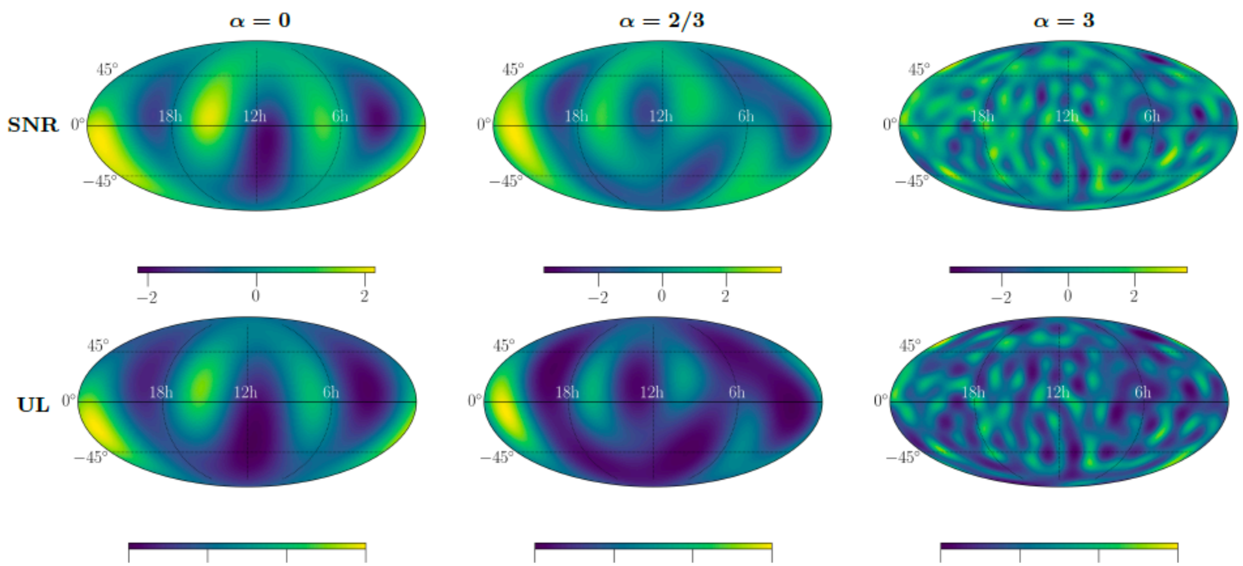
arXiv:2103.08520

Radiometer: LVK results

			Max SNR (% p-value)				Upper limit range (10^{-9})	
α	Ω_{GW}	$H(f)$	HL(O3)	HV(O3)	LV(O3)	O1+O2+O3 (HLV)	O1+O2+O3 (HLV)	O1 + O2 (HL)
0	constant	$\propto f^{-3}$	1.6 (78)	2.1 (40)	1.5 (83)	2.2 (43)	3.2–9.3	7.8–29
2/3	$\propto f^{2/3}$	$\propto f^{-7/3}$	3.0 (13)	3.9 (0.98)	1.9 (82)	3.7 (1.7)	1.9–9.7	6.5–25
3	$\propto f^{3}$	constant	3.9 (12)	4.0 (10)	3.9 (11)	3.2 (60)	0.56–3.4	1.9–11

SNR

90% UL



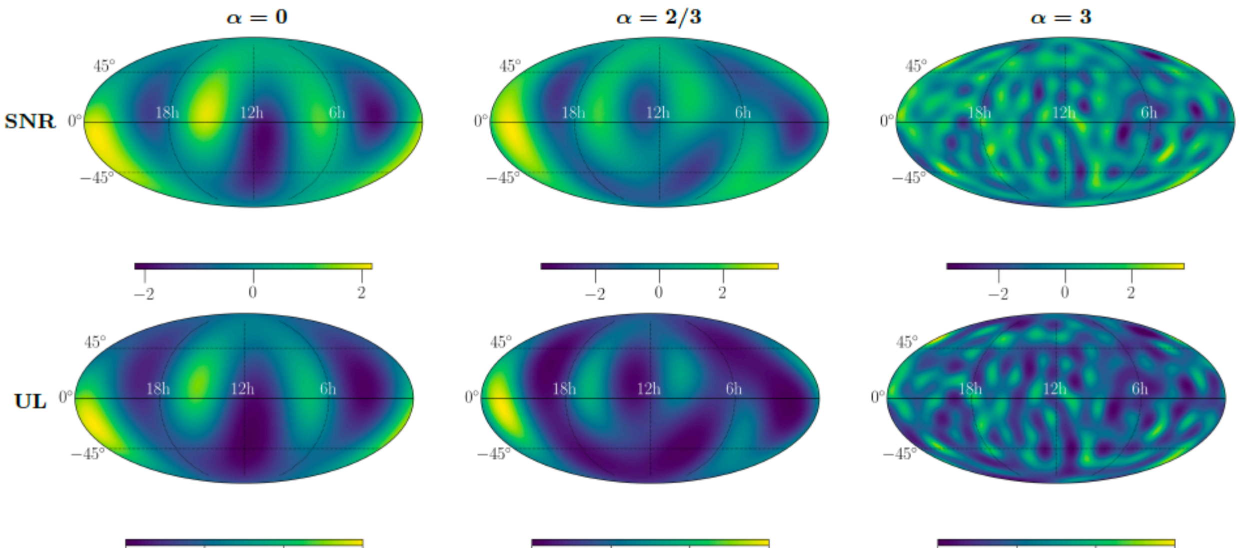
arXiv:2103.08520

SHD: LVK results

			Max SNR (% p-value)				Upper limit range (10^{-9})	
α	Ω_{GW}	$H(f)$	HL(O3)	HV(O3)	LV(O3)	O1+O2+O3 (HLV)	O1+O2+O3 (HLV)	O1 + O2 (HL)
0	constant	$\propto f^{-3}$	1.6 (78)	2.1 (40)	1.5 (83)	2.2 (43)	3.2–9.3	7.8–29
2/3	$\propto f^{2/3}$	$\propto f^{-7/3}$	3.0 (13)	3.9 (0.98)	1.9 (82)	3.7 (1.7)	1.9–9.7	6.5–25
3	$\propto f^{3}$	constant	3.9 (12)	4.0 (10)	3.9 (11)	3.2 (60)	0.56–3.4	1.9–11

SNR

90% UL



Conclusion

- The preliminary goal is to measure the isotropic SGWB.
- At the end of 2020, the NANOGrav collaboration gathered evidence of fluctuations in the timing data of 45 pulsars, which could be compatible with a stochastic gravitational wave background (SGWB) signal at nanohertz frequencies.
- The background from CBCs have a good chance to be detected in the next few years with LVK. CS and PT are also very promising candidates.
- With 3G the goal will be to subtact the background form CBCs, to recover the cosmological background below (Regimbau et al., PhysRevLett.118.151105; Sachdev et al. PhRvD.102b4051S).
- Many other searches in LVK can lead to very interesting results (non-isotropic, non standard polarization, non Gaussian?). These searches could be extended to LISA.