

LIGO/Virgo results and outlook Compact objects

S. Mastrogiovanni

Outline:

- Compact binaries and GWs before the LVC: Hulse and Taylor.
- Data analysis rehearsal
- GW150914 and its properties
- GW170814 the first triple detection
- GW170817 the first multimessanger observation
- GW190412 and GW190814, hearing at the GW higher modes
- The biggest one: GW190521
- Looking the population of CBC together
 - Basics of population analyses
 - Results for the rate and masses distributions and astro implications







For a binary system in quasi-circular orbits GR predicts the emission of GWs.

The Luminosity (energy radiated) in GWs is

 $L_{\rm GW} \equiv \frac{{
m d}E}{{
m d}t} = \frac{1}{5} \frac{G}{c^5} . \langle \widetilde{I}_{jk} \widetilde{I}_{jk} \rangle$ Quadrupole formula

Where the I are the third derivatives w.r.t time of the quadrupole moment of the system.

For a system of point mass particles

$$I_{jk} = \sum_{A} m_A \left[x_j^A x_k^A - \frac{1}{3} \delta_{jk} \left(x^A \right)^2 \right]$$



During the inspiral GWs are emitted and the system loses energy. The two objects rapidly approach each other.





Gravitational energy Gravitational energy
$$M = M_1 + M_2$$

We can derive the energy w.r.t to time to calculate the luminosity. By assuming that all the energy is emitted in GWs, we have

$$-\dot{E} = L_{\rm GW} = \frac{1}{2} \frac{G\mu M}{a^2} \dot{a} = -E \cdot \frac{\dot{a}}{a}$$



$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$



Implicit observation of a GW from a BNS: How to calculate L_gw



XX- component

$$M_{xx} = (M_1 a_1^2 + M_2 a_2^2) \cos^2 \phi + \text{constant terms}$$

= $\frac{1}{2} \mu a^2 \cos 2\phi + \text{constant terms},$

YY component

$$f_{yy} = -\frac{1}{2}\mu a^2 \cos 2\phi + \text{constant terms}$$

YX-XY component

$$I_{xy} = I_{yx} = \frac{1}{2}\mu a^2 \sin 2\phi + \text{constant terms}$$







Adding all the components together, with more and more steps...

$$\begin{split} L_{\rm GW} &= \frac{1}{5} \frac{G}{c^5} \langle \ddot{I}_{jk} \ddot{I}_{kj} \rangle \\ &= \frac{1}{5} \frac{G}{c^5} \cdot (2\Omega)^6 \cdot \left(\frac{1}{2} \mu a^2\right)^2 (\sin^2 2\Omega t + \sin^2 2\Omega t + 2\cos^2 2\Omega t) \\ &= \frac{32}{5} \frac{G}{c^5} \frac{(GM)^3}{a^9} (\mu a^2)^2 \\ &= \frac{32G^4}{5c^5} \frac{M^3 \mu^2}{a^5}. \end{split}$$



If the dynamic of the orbit is described by the Kepler law (radius of the orbit >> radius of the two bodies), then we can relate the shrinking of the orbit to the variation of the orbital period

But the shrinking of the orbit can also be related to the GW luminosity

Therefore we can relate the orbital period variation to the GW luminosity

$$\frac{\dot{P}}{P} = \frac{3}{2}\frac{\dot{a}}{a} = +\frac{3}{2}\frac{\dot{E}}{E} = \frac{3}{2} \cdot \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^5} \cdot \frac{(-2a)}{G\mu M} = -\frac{96}{5}\frac{G^3M^2\mu}{c^5a^4}$$

Implicit observation of a GW from a BNS: How to calculate L_gw





Implicit observation of a GW from a BNS: How to calculate L_gw





PSRB1913+16 time delay with respect to the orbital phase.

The time delay is due to the periastron precession.



Theory (taking into account ellipticity)

$$\frac{dP}{dt} = -2.4 \cdot 10^{-12}$$

Measured value

$$\frac{dP}{dt} = -\left(2.4184 \ \pm 0.0009\right) \cdot 10^{-12}$$

Good agreement between theory and observations.

In the following years this kind of measure has been repeated for other binary pulsars (PSR J0737-3039A)

A Data analysis rehearse



- We want to look for a signal matching templates with data: "Matched filtering techniques".
- Matching a template with data consists in convolution process. Your convolution will spike when there is a perfect overlap between the two.

$$C(\tau) = g(t) \star f(t) = \int_t f(t-\tau)g(t)dt$$



A Data analysis rehearse



- If the template does not describe well the signal, then your overlap will be smaller.
- If we mismatch the template we then lose some power from the recovered signal!



A Data analysis rehearse: Convolution and SNR



Let's assume that our data is composed by a superposition of signal and noise. If we apply the cross-correlation with a template we obtain

$$d(t) = n(t) + h(t)$$

$$c(t) = d(t) \star f(t) = n(t) \star f(t) + h(t) \star f(t)$$

The SNR is defined in the following way, where between the two steps we have sued the **Convolution theorem**

 $SNR^{2} = \frac{E[|h \star f|^{2}]}{E[|n \star f|^{2}]} = \frac{E[|h \star f|^{2}]}{f^{*} \star E[n \star n^{*}] \star f} \implies SNR^{2} = \frac{|h|^{2}}{S(f)}$

A Data analysis rehearse: Convolution and SNR

If the signal is evolving in frequency, then the SNR is defined as an integral

$$SNR^2 \equiv < h|h> = \int_{f_l}^{f_u} \frac{|h(f)|^2}{S_n(f)} df$$

The SNR is basically the ratio between the signal Fourier transform and the PSD







- 1. We take calculate a template for the waveform, corresponding to some physical parameters, e.g. masses etc.
- 2. We slide it on the data and look for excess in the cross-correlation/SNR.







- 1. When the template is matching the noise, we generate *noise background* realizations, that can be used to assess the significance of our candidate.
- 2. When we match the signal, we obtain a very strong outlier, which is not compatible with the background distribution of the noise.
- 3. A preliminary significance can be calculated using the p-value (or False alarm probability).





In a real search, we take all of our data and we slide pre-built templates for each interferometer. Then we match the interferometer results taking into account the travel-time between them. **Livingston data**





How do we generate noise backgrounds? In a real search we never know when the signal is present or not...





To generate noise backgrounds we perform the same search but matching the data between interferomenters ``*wrongly*``

Non physical time-shift Hanford data Contamin Contamination Noise Noise ation

Livingston data

The first GW observation GW150914

- GW150914 was detected on September 14, 2015 at 09:50:45 UTC.
- The signal was observed between 35 and 250. The strain peak was around 1e-21.
- The signal had a SNR of 24.





The first GW observation GW150914



- The significance of this event was 1 event per 203 000 years, equivalent to a significance greater than 5.1σ.
- The event was so loud that contaminated the noise backgrounds when time-sliding data.



Understanding the physical properties of GW150914

 $(M_2, \vec{\chi}_2)$

 $(M_1, \vec{\chi}_1)$

E



23

The parameters:

- Intrinsic: Spins, Masses, tidal deformability, ellipticity
- Extrinsic: Time, reference phase, sky position, luminosity distance, orbital orientation



Understanding the physical properties of GW150914





We can estimate the source-frame mass of the system by remembering that the chirp mass is related to the frequency evolution.

$$f_{\rm GW}^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5} \left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} (t_c - t)$$

The chirp mass can be estimated from the previous plot doing a linear fit. If you want to try it:

- Check for the <u>paper</u> describing this activity.
- The frequency of the GW is saved in this <u>file</u>.

Understanding the physical properties of GW150914



Once you have fit for the chirp mass, by assuming that the two masses are equal you will see that

$$m_1 = m_2 = 2^{1/5} \mathcal{M} = 35 \,\mathrm{M}_\odot$$
 $M = m_1 + m_2 = 70 \,\mathrm{M}_\odot$

We can then convert the total mass to the radius of the orbit using the Kepler's law. You will see that the radius of the orbit is of the same order of the Schwartchild radius of the two objects.

$$R = \left(\frac{GM}{\omega_{\text{Kep}}^2|_{\text{max}}}\right)^{1/3} = 350 \,\text{km} \qquad r_{\text{Schwarz}}(m) = \frac{2Gm}{c^2} = 2.95 \,\left(\frac{m}{M_{\odot}}\right) \,\text{km}$$

The BBHs from O1: a snapshot





Vanilla Black holes

- The source-frame masses does not excess what we expected for Stellar-mass origin black holes.
- The two BHs have similar masses.
- No sign of precession.
- We extensively tested the waveform systematics, deviations from waveform prescriptions etc.

The BBHs from O1: a snapshot



- The Sky localization for these events is huge 230 deg2, 850deg2 and 1600deg2 for GW150914, GW151226 and GW151012.
- There was no electromagnetic counterpart observed for these events.



GW170814: The first triple detection



- The event was detected with a False Alarm rate of 1/27000 years.
- The network SNR for this signal was 18
- The signal is barely visible in Virgo, where it has a small SNR.



GW170814: The first triple detection

- GW170814 was another "vanilla" BBH.
- The inferred masses were about 30 and 25 solar masses.
- The estimation of the spins were highly influenced by the priors: We cannot measure the spins very well.







GW170814: The first triple detection



- credible area on the sky is 1160 deg2 and shrinks to 100 deg2 when including Virgo data
- When including Virgo, the 90% localization volume goes from 71 × 10⁶ Mpc³, to 2.1 × 10⁶ Mpc³
- This is very important for cosmology (see Walter's talk).



GW170817: The first multimessenger detection

At 12:41:04 UTC a GW from the merger of two Neutron star is detected

- +2 seconds later Integral and Fermi detect a GRB.
- ~10 hrs later a kilonova emission from NGC4993 is observed.

With GW170817 we have been provided:

- Luminosity distance from GW170817.
- Redshift identification of the host galaxy from NGC4993.
- Peculiar motion of NGC4993.





GW170817: The first multimessenger detection



- The sky localization using the two LIGOs was 190 deg² and 31 deg² if we implement Virgo.
- High-spins or Low-spins for the NS?
 - When you run PE with priors on high spins you find a better luminosity distance.
 - This is due to the fact that, since you don't observe precession you know that the binary is not edge-on.
 - Spin-induced precession is more visible from edge-on binaries.



GW170817: The speed of Gravity



From GW170817 we measured the distance = 26 Mpc, and the time of arrival of the GRB (The GRB arrived 1.74s) later. Can we measure the speed of gravity?

$$d = v_{\rm gw}(t_{\rm det,gw} - t_{0,gw}) \qquad d = v_{\gamma}(t_{\rm det,\gamma} - t_{0,\gamma})$$

By equating the two and expanding to the first order...

$$\frac{v_{\gamma}}{v_{\text{gw}}}(t_{\text{det},\gamma} - t_{0,\gamma}) = (t_{\text{det},\text{GW}} - t_{0,\text{GW}})$$
$$\frac{v_{\text{GW}}}{v_{\gamma}} = \frac{v_{\gamma}}{D}(\Delta t_{0,GW-\gamma} - \Delta t_{det,GW-\gamma})$$

If we feed the numbers and consider that the EM counterpart can be emitted upt to +/- 10 s...

$$-3 \times 10^{-15} \leqslant \frac{\Delta v}{v_{\rm EM}} \leqslant +7 \times 10^{-16}$$





GW190814

35

GW190412 and GW190814: Asymmetric mass events

These are the first events that shows a mass ratio which is clearly not one. These are asymmetric mass events.





GW190412 and GW190814: Why are these events interesting?

Strong mass-asymmetric binaries could allow us to detect GWs higher modes (beyond the quadrupole formula.)





38

GW190412 and GW190814: Why are these events interesting?

- Detecting higher modes breaks the degeneracy between the luminosity distance.
- The presence of higher modes improves the luminosity distance estimation of the events.





GW190521: A very massive events



GW190521, with a three-detector network signal-to-noise ratio of 14.7, and an estimated false-alarm rate of 1 in 4900 yr



GW190521: A very massive events



- The primary mass has only a 0.32% probability of being below 65. This challenge our knowledge of BH formation [Pair (and pulsational) instability Supernova process].
- There was a small evidence for precession.



GW190521: A very massive events

We calculate the mass of the remnant to be 142, which can be considered an intermediate mass black hole (IMBH)







GW190521: What is the origin of this system?

Second generation of mergers?

- BBH formed in a dense astrophysical environment such as globular clusters
- BBH formed from two merged stars
- BBH formed in an AGN dense environment

Alternative models

- Possible, but really unlikely. You need either a triple star system (5% prob of producing excetring BBHs) or very dense environments (1% prob)
- Strong lensing: The lensing needed to transform BBH merger of 50-65 solar masses in this kind of signal is too high.
- Primordial black holes?: We do not know given the large uncertainties on the models. There are some studies.
- Cosmic string? The signal is very inconsistent with the template.









What do we have at the end of O3a?



46

- We select all the events from GWTC-1 and GWTC-2 with a False Alarm Rate (FAR) < 1 yr⁻¹.
 We use a total of: Key quantities for population inference
 - 2 BNS events
 - BH+lighter object (GW190814)
 - **44 confident BBH events** (the focus of this presentation)



What are the properties of the population of BBHs?





- Are there BBH systems with component masses higher than 45 Msun?
- What is the minimum mass of BH?
- Is there a preference to form nearly equal mass binaries?
- Does the merger rate evolve with redshift?

How do we infer the population properties?



We want to describe the population with some parameters \Lambda

 $p(\Lambda|\{x\}, N_{\rm obs}) \propto p(\{x\}, N_{\rm obs}|\Lambda) p(\Lambda) \implies p(\{x\}, N_{\rm obs}|\Lambda) p(\Lambda) = p(N_{\rm obs}|\Lambda) \prod_{i=1}^{N_{\rm obs}} p(x_i|N_i, \Lambda)$

Using the bayes theorem...

$$p(x_i|N_i,\Lambda) = \frac{p(N_i|x_i,\Lambda)p(x_i|\Lambda)}{p(N_i|\Lambda)}$$

- Probability of detecting N_obs events given some population parameters: A Poisson distribution
- Probability of detecting the event, given the current data set and the population parameters (one since we detected the event)
- Probability of detecting an event considering all the possible data sets (realization of the noise)
- Calculated from GW-likelihood

How to calculate how many events we expect



That is useful, for instance, when we want to infer the merger rate

$$p(N_{\rm obs}|\Lambda) = \frac{[N_{\rm exp}(\Lambda)]^{N_{\rm obs}} e^{N_{\rm exp}(\Lambda)}}{N_{\rm obs}!}$$

The expected number of events should take into account the 'detection probability'.

$$N_{\rm exp}(\Lambda) = \int p_{\rm det}(\cdot) \frac{dN}{dz d\vec{s} d\vec{m}_s dt_d} dz d\vec{s} d\vec{m}_s dt_d$$

Fraction of events produced per redshift, spin, mass and detector time

The fraction of events per parameter can be written as

$$\frac{dN}{d\theta dz dt_{\rm obs}}(\Lambda) = \frac{dN}{d\theta dV_c dt_{\rm s}} \frac{dV_c}{dz} \frac{dt_{\rm s}}{dt_{\rm obs}} = R_0 \frac{dV_c}{dz} \frac{1}{1+z} f(z)\pi(\theta|\Lambda)$$

Doing an hierarchical inference



$$N_{\rm exp}(\Lambda) = T_{\rm obs} R_0 \int p_{\rm det}(\cdot) \pi(\vec{s}|\Lambda) \pi(\vec{m}_s|\Lambda) \frac{dV_c}{dz} f(z) dz d\vec{s} d\vec{m}_s.$$

$$\mathcal{L}(x|\Lambda, N) = e^{-N_{\exp}(\Lambda)} N_{\exp}^{N_{obs}} \prod_{i=1}^{N_{obs}} \frac{\int \mathcal{L}(x_i|\theta, \Lambda) \pi(\theta|\Lambda) d\theta}{\frac{N_{\exp}}{N}}$$

- Doing an hierarchical inference consists in studying the parameters that govern the distributions of masses, redshift and spins of a population of BHs.
- This is potentially informative on the astrophysical processes that produces BBHs.
- You need to be careful to correct for ``selection biases``, i.e. how easy is to detect events w.r.t to a given choice of your population parameters.

A small example on the importance of selection biases



Example: We have a gaussian random generator with some **mean** and **std deviation**. By looking at the random numbers generated, we want to infer the mean and the std deviation.

However, we are able to register only random numbers above -1 (selection threshold)

How do you calculate correctly the mean?



A small example on the importance of selection biases



We can use a Bayesian framework, and be very careful about normalizations

The likelihood of obtaining a random sample x is absence of selection biases is

$$\mathcal{L}(x|\mu) = \frac{\exp[-(x-\mu)^2/(2\sigma^2)]}{\int_{-\infty}^{\infty} \exp[-(x-\mu)^2/(2\sigma^2)] dx} = \frac{1}{\sqrt{2\pi\sigma}} \exp[-(x-\mu)^2/(2\sigma^2)]$$

However, when we include a selection threshold, the normalization factor must be modified and it is also a part of your inference (note it changes with the mean)

$$\mathcal{L}(x|\mu) = \frac{\exp[-(x-\mu)^2/(2\sigma^2)]}{\int_{x_{thr}}^{\infty} \exp[-(x-\mu)^2/(2\sigma^2)] dx} = \frac{\exp[-(x-\mu)^2/(2\sigma^2)]}{I(\mu, x_{thr})}$$
Selection bias

A small example on the importance of selection biases



Result when you do not account selection bias for selection bias 0.035 Experiment nº1 Experiment nº1 Experiment nº2 Experiment nº2 0.05 0.030 Experiment nº3 Experiment nº3 Experiment nº4 Experiment nº4 0.025 0.04 Experiment n°5 Experiment n°5 PDF p(µ|{x}) PDF p(µ|{x}) All combined All combined 0.020 0.03 0.015 0.02 0.010 0.01 0.005 0.000 0.00 -0.50 -0.25 0.00 0.25 -1.00 -0.75 0.50 0.4 0.0 0.2 0.6 0.8 1.0 μ μ

Result when you account for the

53

0.75

1.00

The mass models that we fit on O3a



Analysis with 44 confident BBH. Primary mass distribution

 10^{1}

- The preferred model is a powerlaw + gaussian peak around 40 solar masses.
- No hard cut-off around 40 solar masses.
- This model is insensitive to the inclusion of GW190521 (GW190521 is not as trong outlier for this kind of population).

The mass models that we fit on O3a





Without GW190521

The mass models that we fit on O3a

- We rule out BHs masses below 2 solar masses and we rule out also sharp cut offs.
- GW190814 is in tension with this result. The secondary mass of GW190814 is too small to fit this model.
- The mystery of the secondary mass of GW190814 is even more thrilling.



The rate evolution

- The rate evolution today, at redshift 0, for BBHs is between [10, 35] Gpc⁻³ yr⁻¹
- The BBHs rate seems to evolve with respect to the redshift.
- When the Universe was younger, the merger rate was higher (it slightly follows the star formation rate**)

**We will confirm this better in the future.





Thank you for your attention! Questions?

59

REFERENCES

Hulse and Taylor <u>GW150914</u> <u>O1 BBHs</u> <u>GW170814</u> <u>GW170817</u> <u>GW higher order modes</u> <u>GW190814</u> <u>GW190412</u> <u>GW190521</u> <u>The population properties of the BBHs from O1+O2+O3</u> <u>A quide for population analysis with selection effects</u>

