GW data analysis – A primer (2)

Eric Chassande-Mottin
AstroParticule et Cosmologie (APC)
CNRS Université de Paris

Summary of Part 1

- Needle in the haystack
- Laid down the observation equation
  - Signal: 2 polarizations, antenna pattern, time delay

$$h_{\mathrm{signal}}(t) = F_+ h_+(t-\tau) + F_\times h_\times(t-\tau)$$

  - Noise: additive, Gaussian, colored
- Deduced the likelihood probability distribution and related statistics

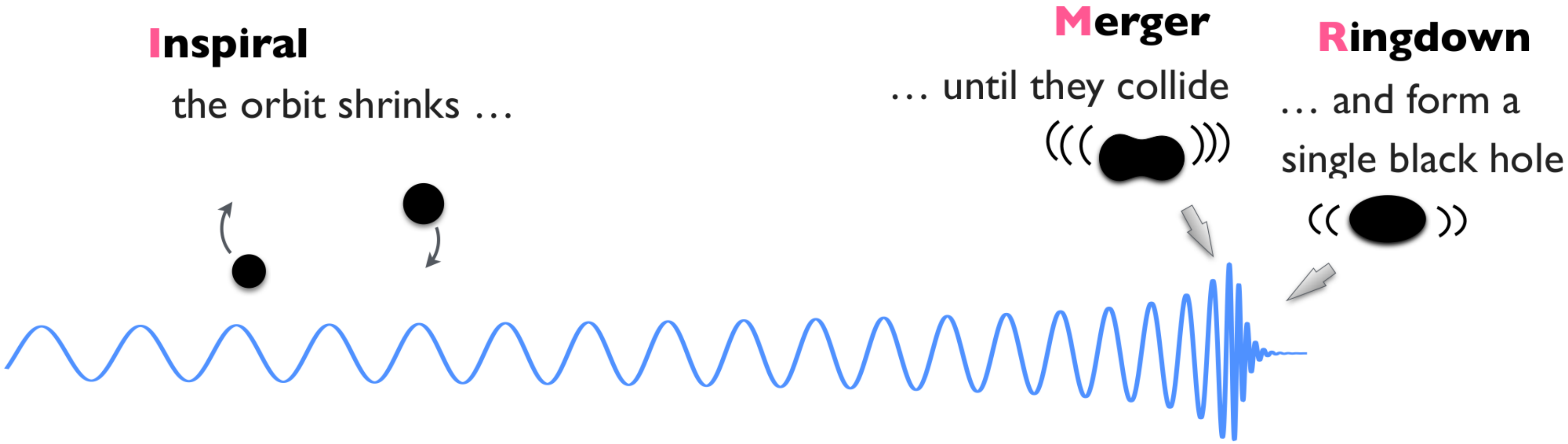
$$p(d|h) = \frac{\exp -\frac{1}{2}(d_i - h_i) C^{-1}_{ij} (d_j - h_j)}{\sqrt{|2\pi\, C|}}$$

- Compact binary coalescences (Detection)
  - Waveform approximants and source parameters
  - Matched filtering: theory and practice. Grid search
  - Noise glitches and their mitigation
  - Event significance and background estimation

Outline – Part 2

- From general principles to transient searches
  - Compact binary coalescences – Part 2 (Estimation)
    - General problem statement
    - The Bayesian viewpoint
    - Bayesian samplers
  - Unmodelled transient sources or "bursts"
    - Astrophysical targets and phenomenology
    - Standpoint on analysis
    - Coherent time-frequency searches

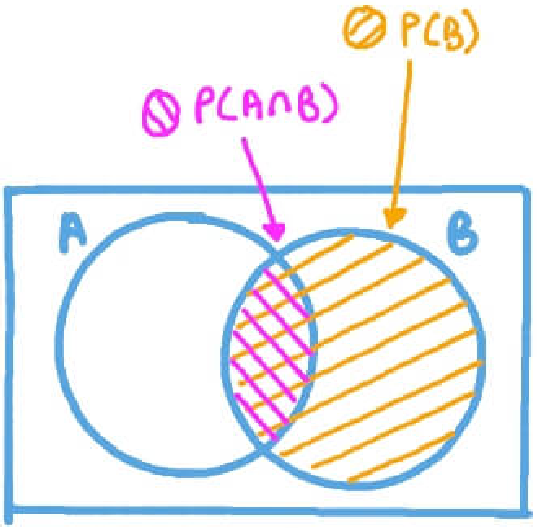
Compact binary coalescences – Estimation



Parameter estimation

- Search pipelines deliver candidate events
  - Very crude estimate of the source parameters (best fit template)
  - No uncertainty (error bar)
  - Some parameters are more relevant than others – Need to "marginalize" over irrelevant parameters
  - Non-trivial coupling between parameters
  - Model comparison and selection
- Bayesian inference provides a framework to address those issues

Forewords: conditional probabilities

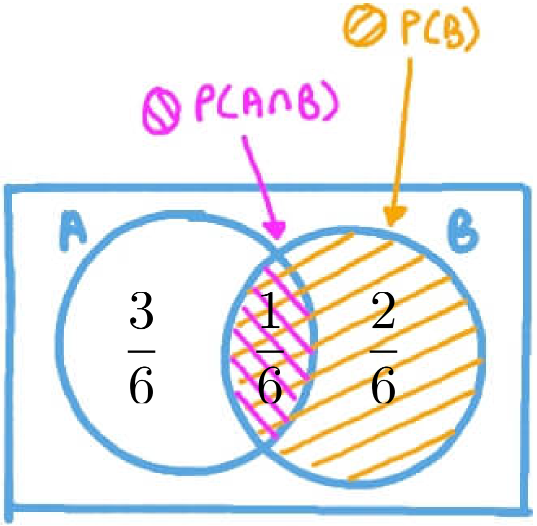


- **Probabilities measure "areas" in a given sample space**
- **P(A|B) :** probability of A given that B has occurred
  - B is the new "universe" where probabilities sum to 1
  - Initial sample space is irrelevant

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(requires P(B) ≠ 0)

Forewords: conditional probabilities



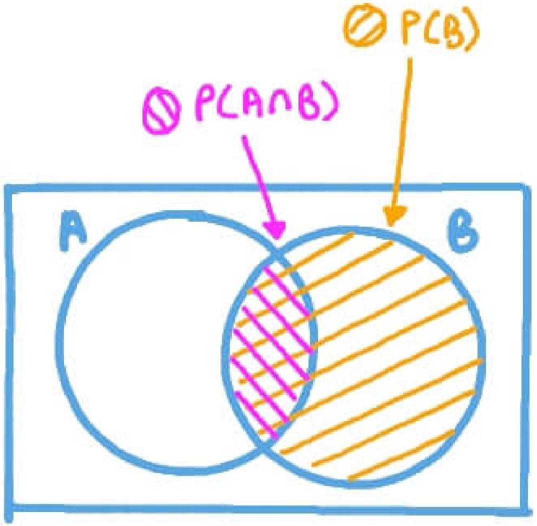
$$P(A|B) = \frac{1/6}{3/6} = \frac{1}{3}$$

- **Probabilities measure “areas” in a given sample space**
- **P(A|B) :** probability of A given that B has occurred
  - B is the new “universe” where probabilities sum to 1
  - Initial sample space is irrelevant

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(requires P(B) ≠ 0)

Forewords: conditional probabilities



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional probabilities are useful to:

- Build hierarchical or causal probabilistic models
  - Occurence of events under different scenarios, i.e. $P(A|B_k)$
- Include new information into probabilistic models

Bayesian inference (1): Principles

Bayes' rule:

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

Thomas Bayes
An Essay towards solving a Problem in the Doctrine of Chances (1763)

Pierre-Simon Laplace
Théorie analytique des probabilités (1812)

- **Idea: how can we learn from experience or experimental data in a systematic way?**
  - Revise initial beliefs based on observation
- **Revert conditional probabilities or causal probabilistic models**

Forward modelling

scenario B_k → event A with probability $P(A|B_k)$

Inference

event A → preferred scenario B_k with $P(B_k|A)$

Bayesian inference (2)

Initial beliefs on parameters λ

$\boldsymbol{\lambda} = \{m_1, m_2, \mathbf{S}_1, \mathbf{S}_2, \ldots\}$

Prior: $p(\boldsymbol{\lambda})$

+

Experiment that provides new data d

Likelihood: $p(d|\boldsymbol{\lambda})$

Update beliefs on λ given new data

Posterior: $p(\boldsymbol{\lambda}|d)$

likelihood weighted by prior belief

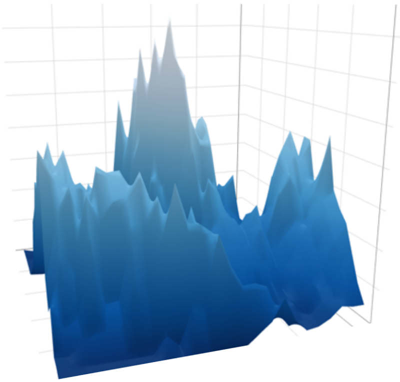
Bayes' rule

$$p(\boldsymbol{\lambda}|d) = \frac{p(\boldsymbol{\lambda})p(d|\boldsymbol{\lambda})}{p(d)}$$

$p(\boldsymbol{\lambda}|d)$ is a probability density – Should sum to 1

$p(d)$ is the normalization factor – Evidence

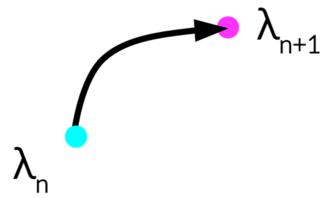
Bayesian inference (3): practical issues



- Come up with reasonable priors (subjective choice)
  - Strong priors may lead to bias
- Map out the posterior distribution in n=17 dimensions
  - Likelihood is a complex distribution with narrow peaks
  - Marginal (e.g, mass) distribution requires to integrate posterior over *n-1* parameters – Difficult to compute using standard numerical integration

    $\rightarrow$ Use stochastic integration: Monte-Carlo methods

Bayesian inference (4): Markov-chain Monte Carlo



Propose new sample with **q**
Decide to move with **H**:
always go up, sometime come down

Transition probability (Metropolis-Hastings)

$$H = \min\left(1, \frac{p(\boldsymbol{\lambda}_{n+1})}{p(\boldsymbol{\lambda}_n)} \frac{p(d|\boldsymbol{\lambda}_{n+1})}{p(d|\boldsymbol{\lambda}_n)} \frac{q(\boldsymbol{\lambda}_{n+1})}{q(\boldsymbol{\lambda}_n)}\right)$$

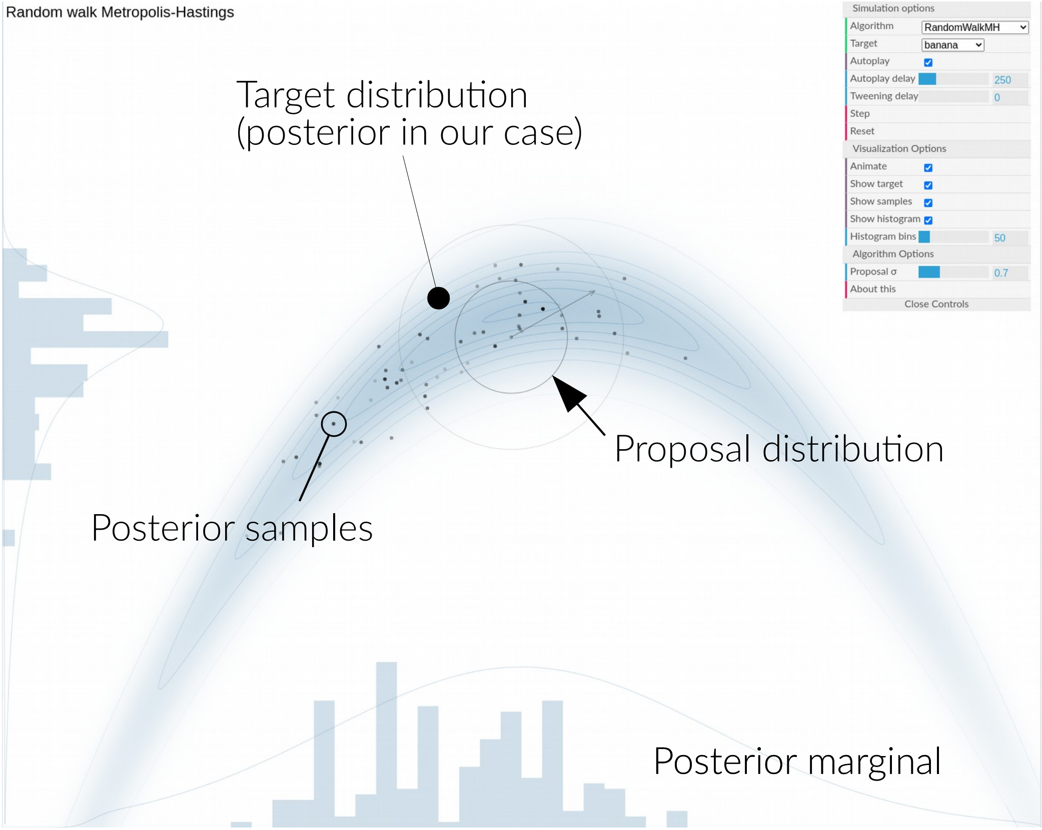
Prior Likelihood Proposal

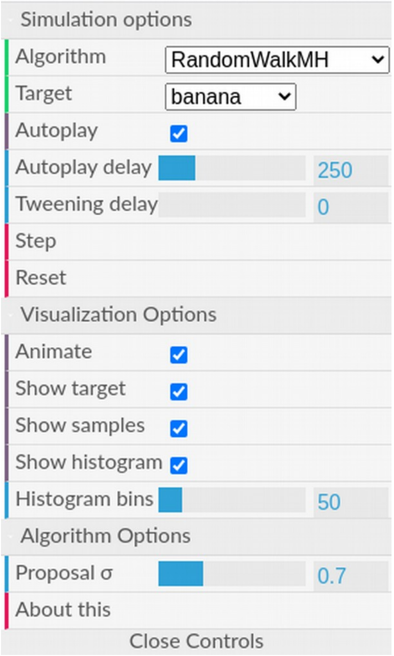
Note: ratios only. No need for properly normalized distributions!

- **Idea**: build a **iterative stochastic process** (random walks) driven to sample from the posterior distribution
  - Generate samples of **λ** drawn from p(**λ**|d)
  - **Importance sampling**: sample first regions with large posterior probabilities
- A fairly large variety of **Bayesian samplers**

Online demo

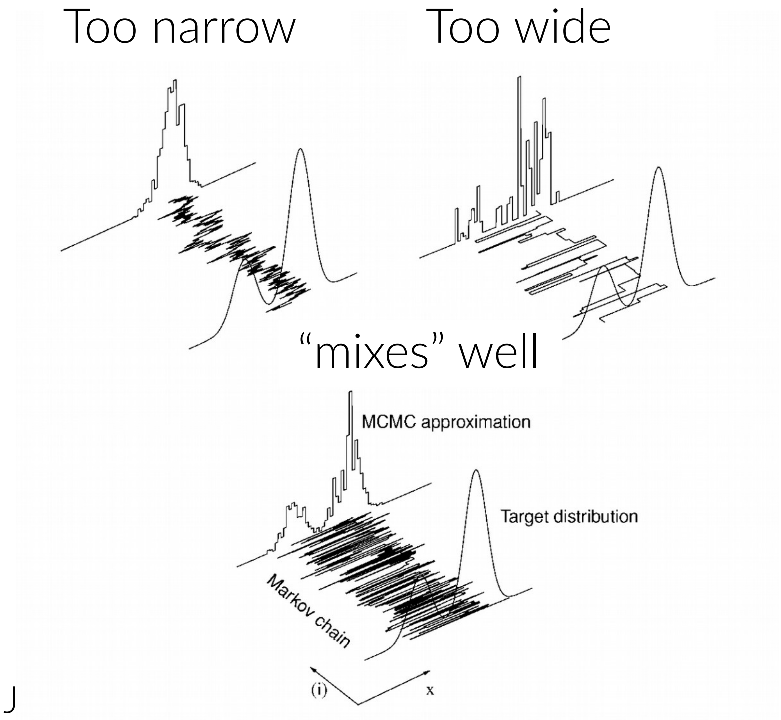
https://chi-feng.github.io/mcmc-demo/app.html?algorithm=RandomWalkMH&target=banana





MCMC recipes

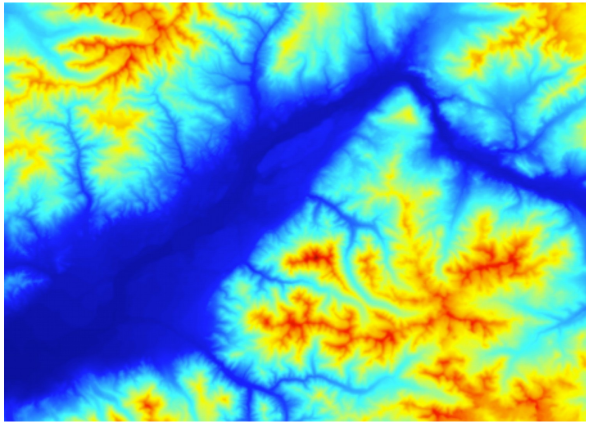
Good choice of proposal distribution matters for convergence



J

Certain posteriors can be very difficult to sample: multi-modal, etc and require other more advanced ingredients

Bayesian samplers are computationally demanding (dominant fraction of CPU budget related to CBC analysis)



Bayesian model selection

Posterior on model *M* $$p(M|d) \propto p(M)p(d|M)$$

Odds ratio: $$O_{n,m} = \frac{p(M_n|d)}{p(M_m|d)}$$
$$= \underbrace{\frac{p(M_n)}{p(M_m)}}_{\text{Prior odds ratio}} \underbrace{\frac{p(d|M_n)}{p(d|M_m)}}_{\text{Bayes factor}}$$

From posterior samples

$$\frac{\sum_j p(d|\boldsymbol{\lambda}_j, M_n)}{\sum_k p(d|\boldsymbol{\lambda}_k, M_m)}$$

Parameter couplings

- Component masses

  From the phase evolution, we primarily measure the chirp mass

$$\mathcal{M} = \frac{(m_1\, m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

- Inclination vs distance

  From the signal amplitude evolution, we primarily measure the amplitude of the first polarization

$$A \propto \frac{1 + \cos^2 \iota}{d_L}$$

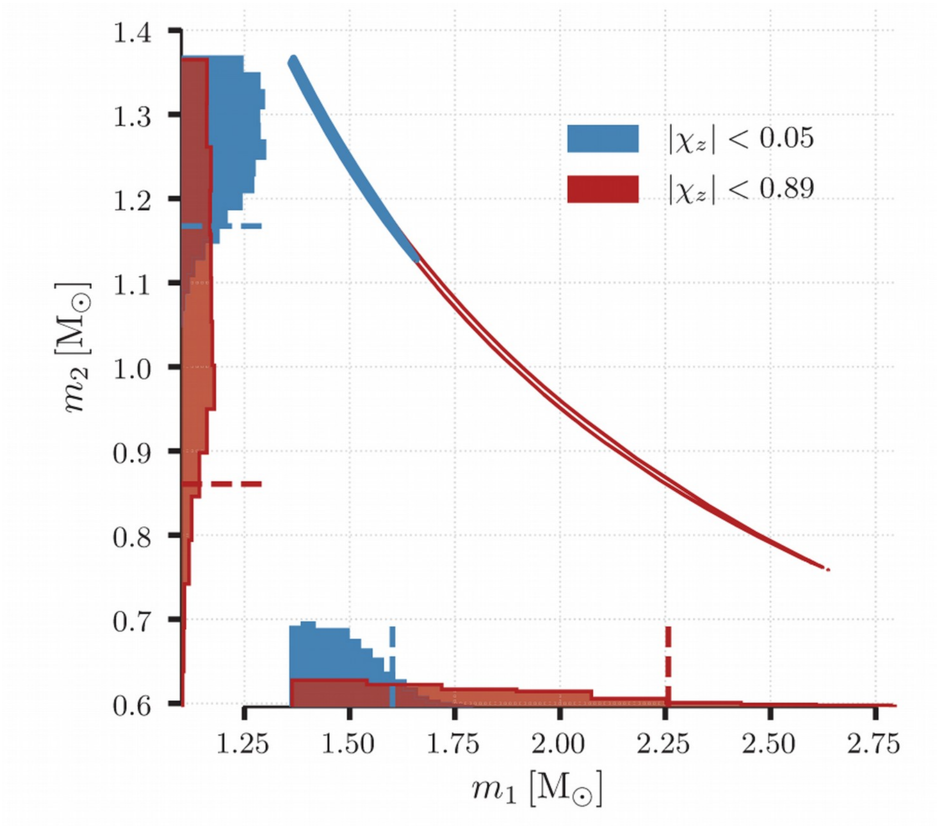
- Cosmology matters
  - Observed frequency of the signal is redshifted by a factor of (1+z)
  - Masses are inferred from phase/frequency measurement

    [Gmf/c^3 is dimension-less]

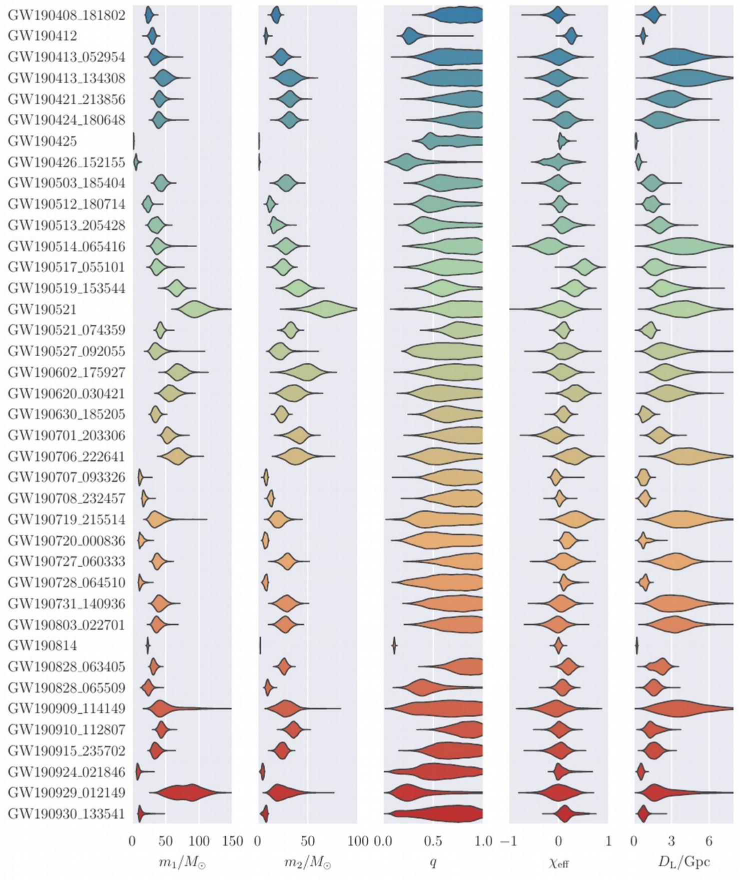
$$m^{(d)} = (1 + z) m^{(s)}$$

  - *z* is inferred from distance, thus $m^{(s)}$ is coupled to d_L

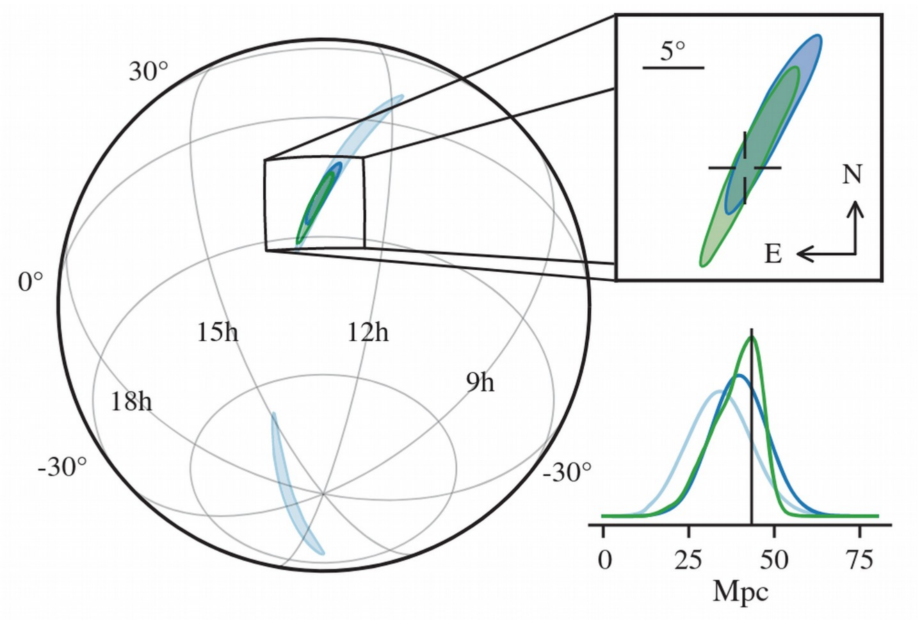
Marginal posterior distribution for the component masses of GW170817



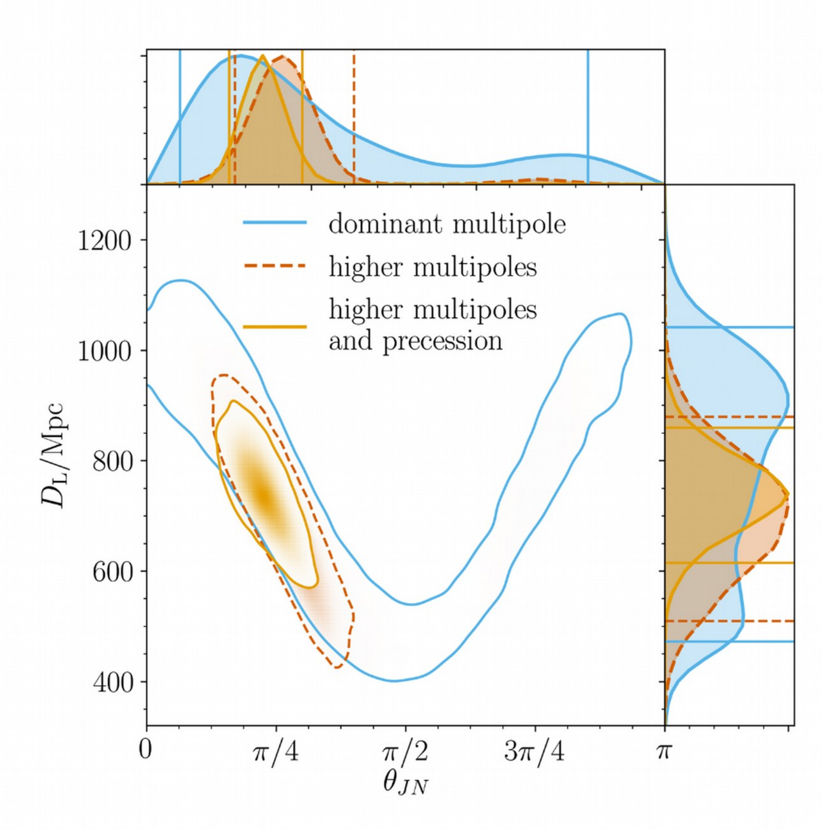
Marginal posterior distributions for a selection of astrophysical parameters – GWTC #2



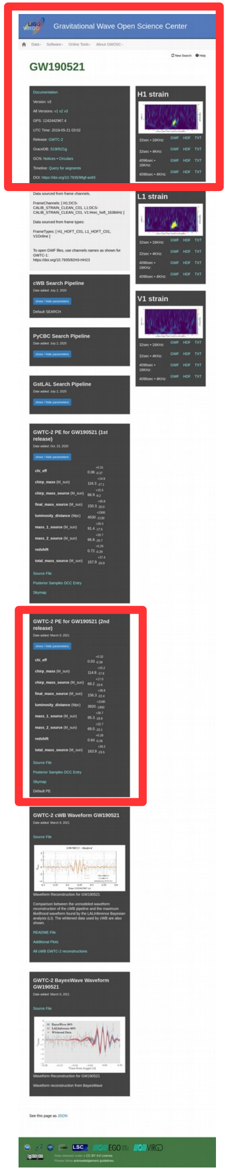
Marginal posterior distribution for the sky coordinates of GW170817



Marginal posterior distribution for the distance and inclination of GW190412



Posterior samples can be downloaded from gw-openscience.org



GW190521

Documentation

Version: v3

All Versions: v1 v2 v3

GPS: 1242442967.4

UTC Time: 2019-05-21 03:02

Release: GWTC-2

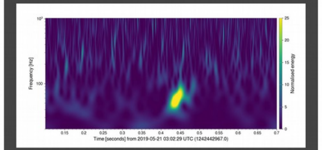
GraceDB: S190521g

GCN: Notices • Circulars

Timeline: Query for segments

DOI: https://doi.org/10.7935/99gf-ax93

H1 strain



32sec • 16KHz:	GWF	HDF	TXT
32sec • 4KHz:	GWF	HDF	TXT
4096sec • 16KHz:	GWF	HDF	TXT
4096sec • 4KHz:	GWF	HDF	TXT

GWTC-2 PE for GW190521 (2nd release)

Date added: March 9, 2021

show / hide parameters

chi_eff	$0.03^{+0.32}_{-0.39}$
chirp_mass (M_sun)	$114.8^{+15.2}_{-17.6}$
chirp_mass_source (M_sun)	$69.2^{+17.0}_{-10.6}$
final_mass_source (M_sun)	$156.3^{+36.8}_{-22.4}$
luminosity_distance (Mpc)	3920^{+2190}_{-1950}
mass_1_source (M_sun)	$95.3^{+28.7}_{-18.9}$
mass_2_source (M_sun)	$69.0^{+22.7}_{-23.1}$
redshift	$0.64^{+0.28}_{-0.28}$
total_mass_source (M_sun)	$163.9^{+39.2}_{-23.5}$

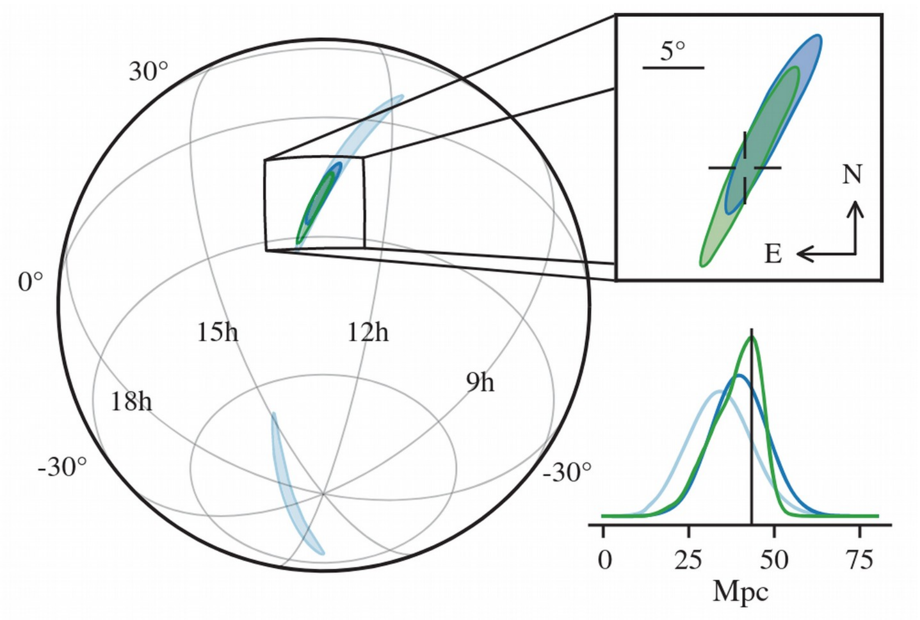
Source File

Posterior Samples DCC Entry

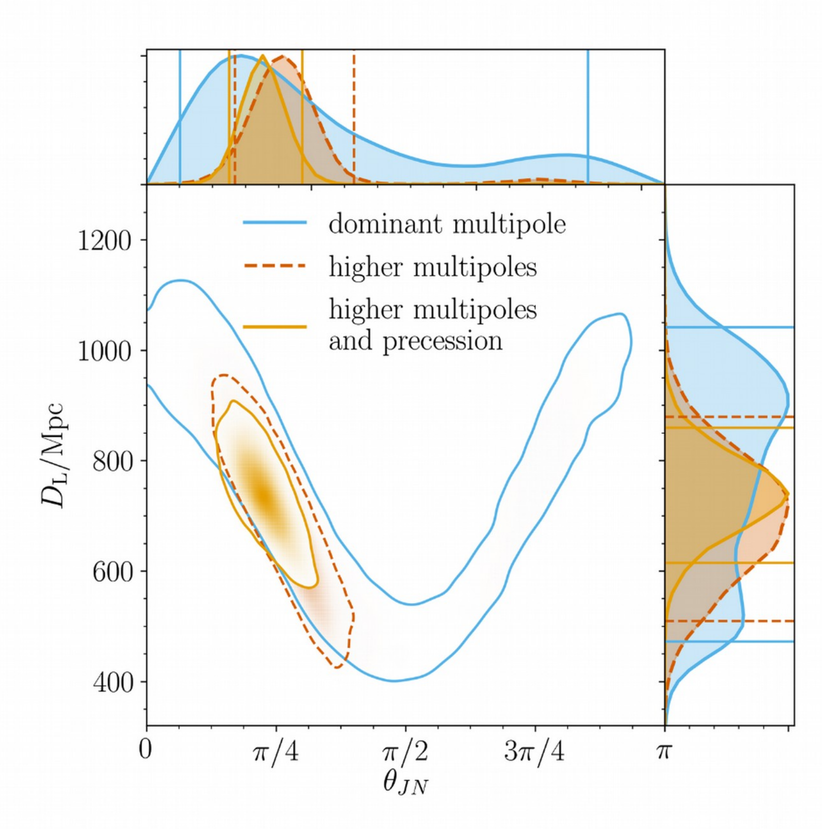
Skymap

Default PE

Marginal posterior distribution for the sky coordinates of GW170817

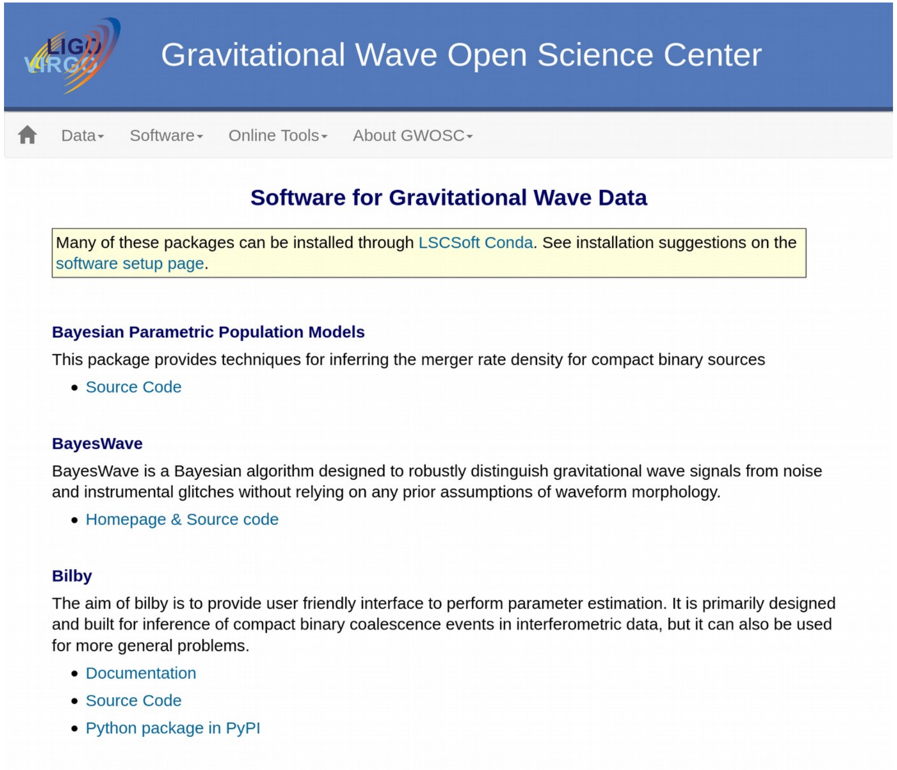


Marginal posterior distribution for the distance and inclination of GW190412



Resources

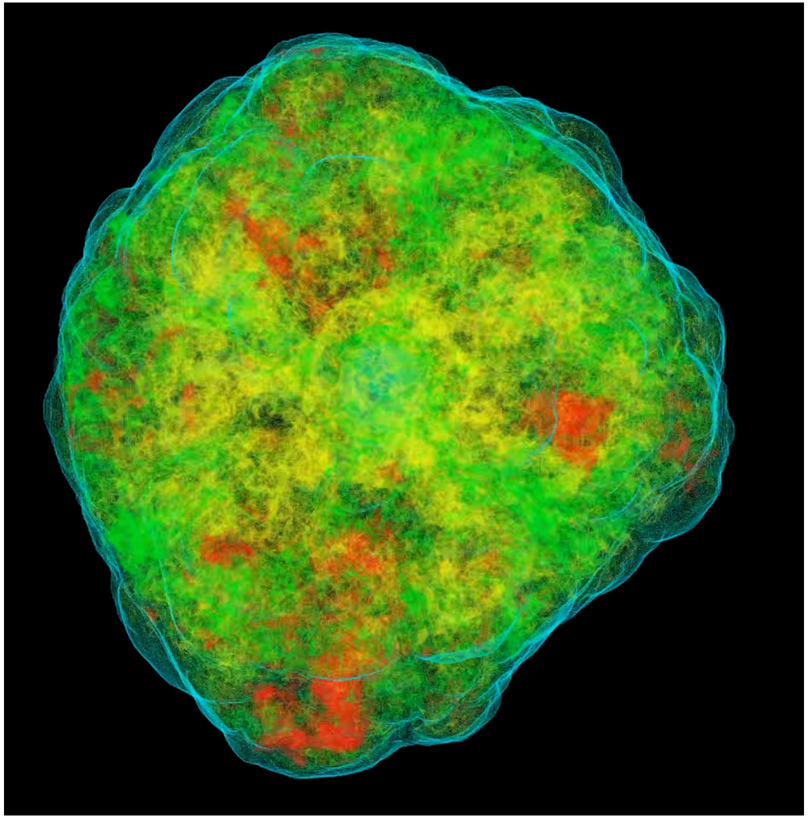
https://www.gw-openscience.org/software/



Software packages for Bayesian inference applied to compact binary coalescences

- LALInference – https://git.ligo.org/lscsoft/lalsuite
- Bilby – https://git.ligo.org/lscsoft/bilby
- Bayesian Parametric Population Models – https://git.ligo.org/daniel.wysocki/bayesian-parametric-population-models
- Bayeswave – https://git.ligo.org/lscsoft/bayeswave

Unmodelled transient sources or "bursts"



Simulating eXtreme Spacetimes (SXS) project. http://www.black-holes.org. The simulation work was led by Christian Ott at Caltech and the movie was rendered by Steve Drasco at Cal Poly San Luis Obsipo.

https://youtu.be/oxGajNoPz8c

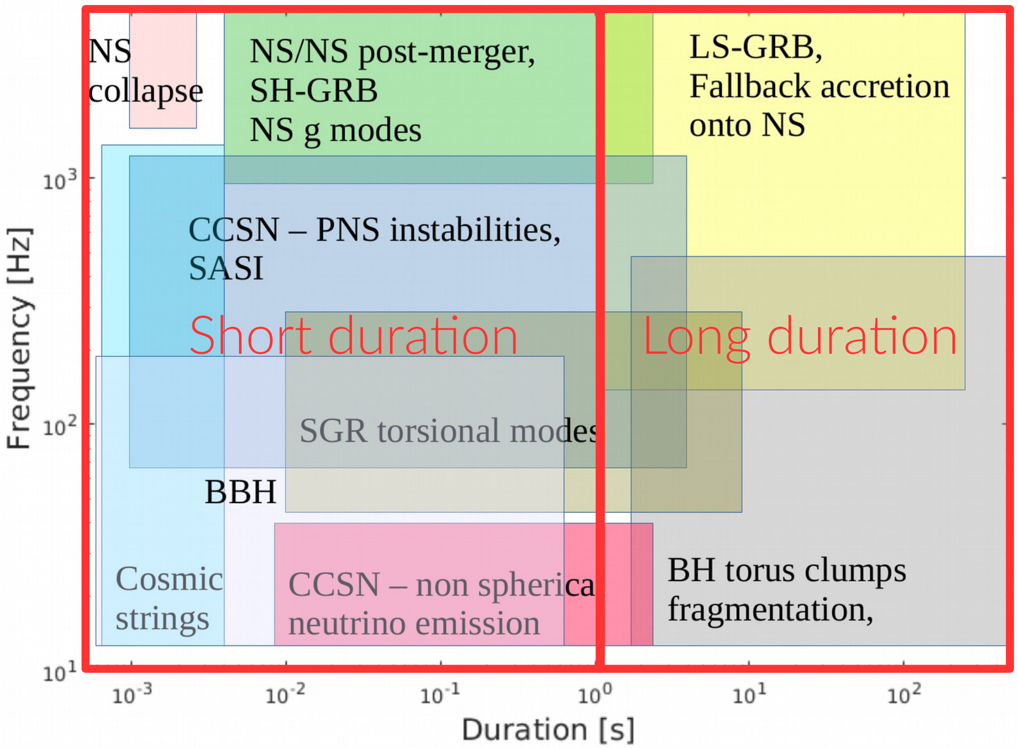
Things that go boom in the night!

- Under the term "**burst**" are included **transient sources** whose **expected GW signal is not completely known**
- Also includes **unexpected astrophysical scenarios** or **unknown physics**
- "Template" waveforms **not available or solely representative** of the true signal (not accurate enough to be used as a template)
- Motivate for other **"agnostic" analysis approaches**

https://youtu.be/8xCme9cKWWc

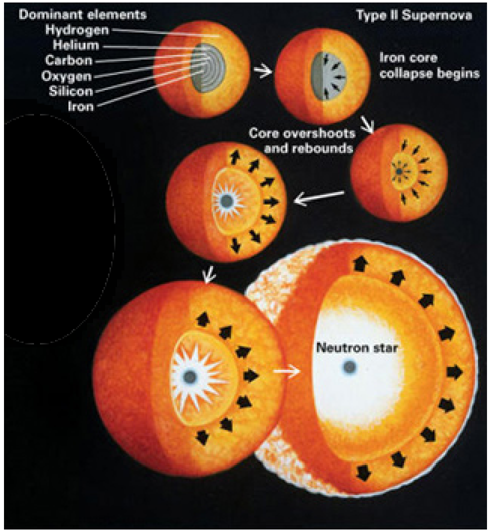
Phenomenology

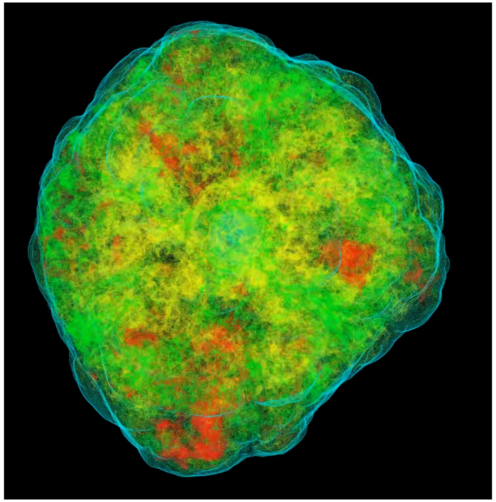
- Potential astrophysical sources relate mainly to **compact stars** (neutron star or black hole)
  - Core collapse supernova
  - Neutron star instabilities
  - Accretion onto a neutron star or black hole
  - Star quakes: magnetars and pulsar glitches
  - Afterglow after binary neutron star mergers
  - Etc…
- But also to other sources such as cosmic string cusps and kinks



Supernovae core-collapse

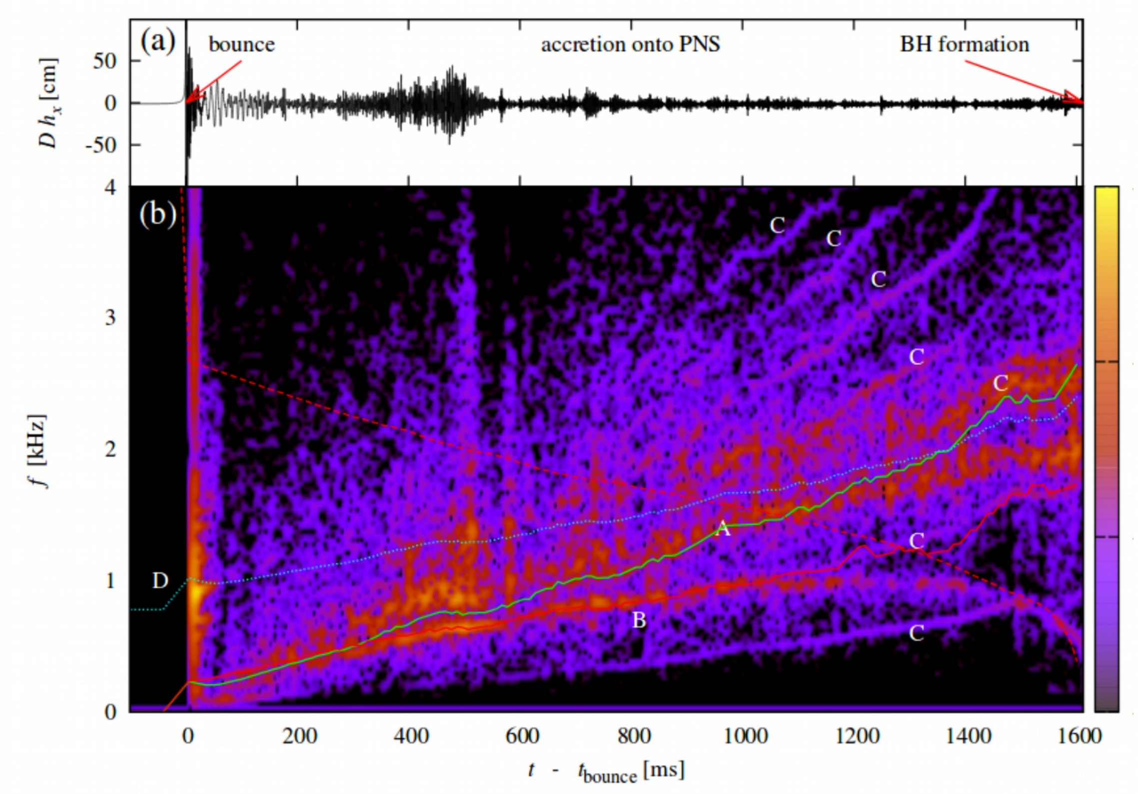
- Gravitational collapse of the core of a massive star after exhaustion of nuclear fuel
  - **Complex process** that involves many physical ingredients (relativistic hydrodynamics, microphysics, neutrino transport, etc)
  - Modelled through **large-scale numerical simulations**
  - *Key question*: what level of **non-axisymmetry** can develop during or after the collapse?
  - Different mechanisms or phases potentially leading to substantial GW emission
    - Core bounce, rotational instabilities, standing accretion shock instability (SASI) and PNS pulsations





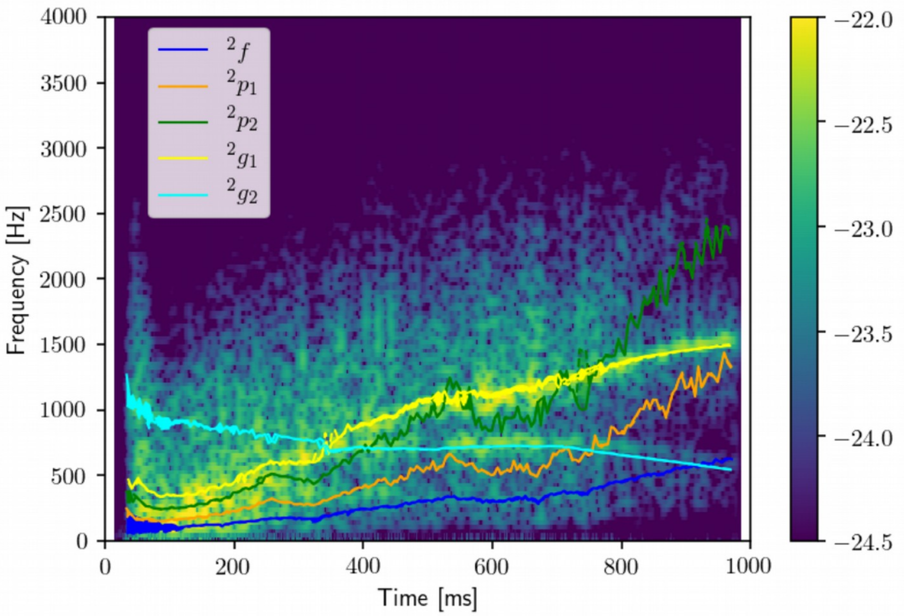
Simulating eXtreme Spacetimes (SXS)
http://www.black-holes.org.

Supernovae core-collapse (cont'd)



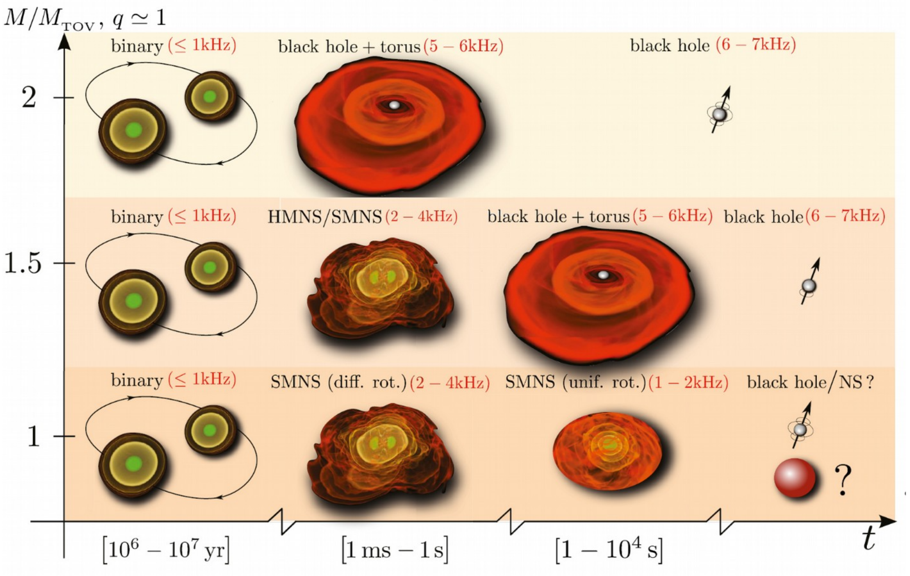
arXiv:1310.8290

Complex and composite waveform that includes a lot of physics about the collapse and remnant star, e.g., PNS pulsation modes

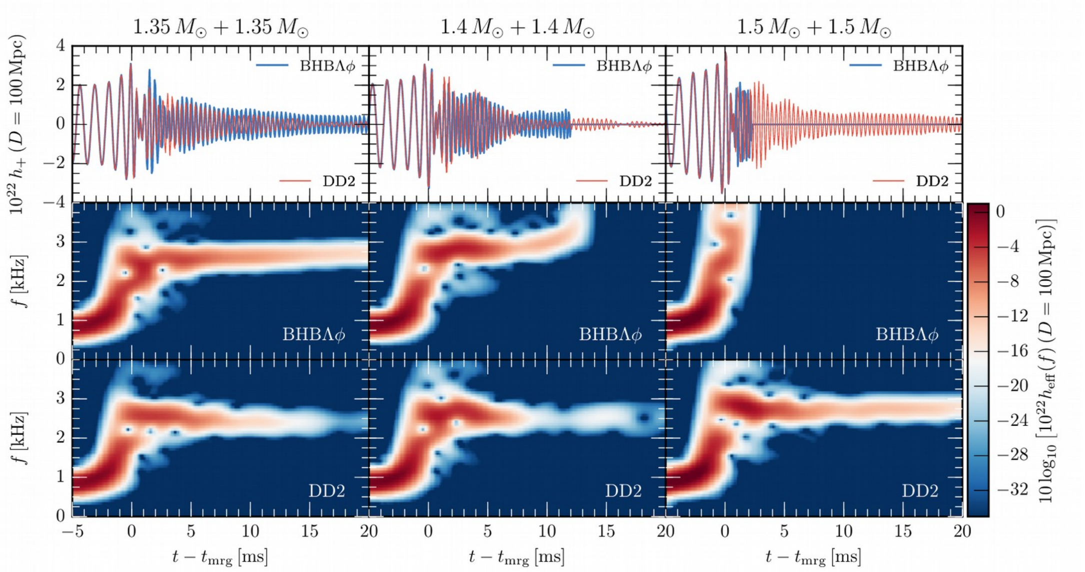


arXiv:1806.11366

“Post-merger” signal after binary neutron star coalescence



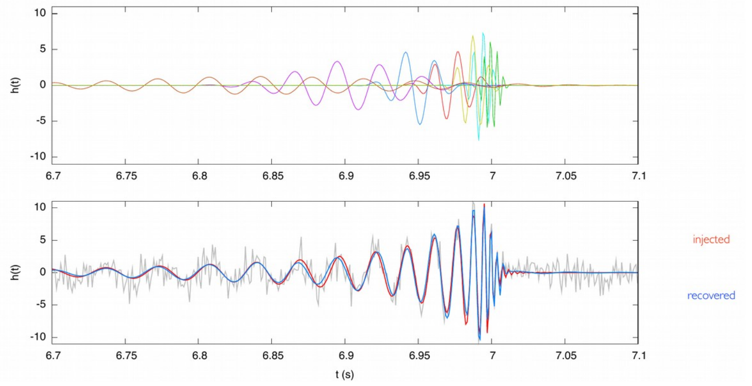
Rezzolla, Zanotti, 2013



arXiv:1612.06429

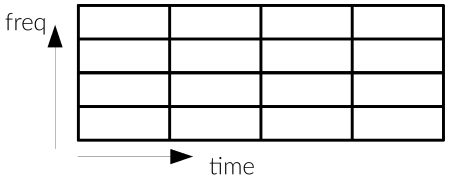
Standpoint on analysis

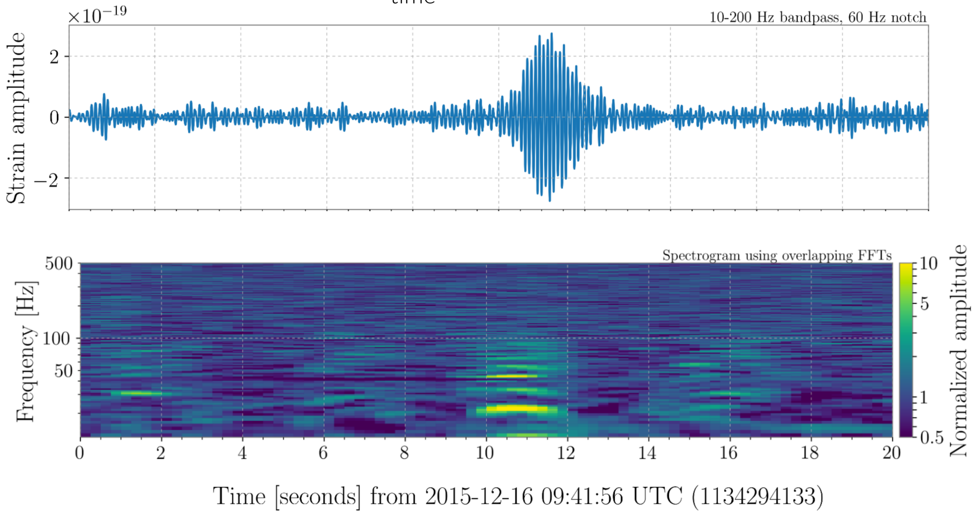
- Representative waveforms to rely on to guide the search algorithm design and tuning
  - E.g., simulations of core collapse supernovae
  - However, **cannot trust** 100 % and few examples are available and there is no full coverage of the physical parameter space → **templated search not a good idea**
- Rather build search pipelines on **generic assumptions** about the expected waveform features
  - E.g., expected signals can be reasonably well described by the sum of a **small number of wave packets**
  - Motivation for **time-frequency based searches**
  - Often, based on **coherent search** paradigm ("multiple detectors" likelihood) for better noise rejection



Time-frequency maps

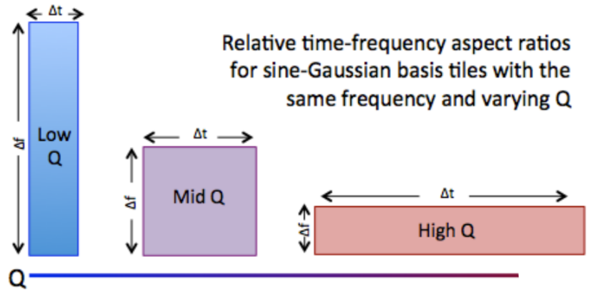
Spectrogram or
short-time Fourier transform

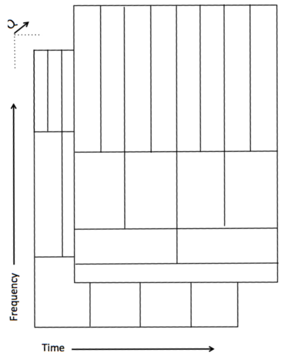


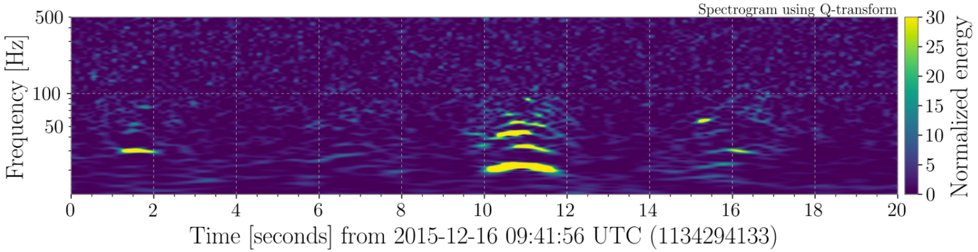


Q transform

S. Chatterji et al. CQG (2010)







Coherent WaveBurst (1)

$$\mathcal{L}(h) \equiv -\log\Lambda = \langle d|h\rangle - \frac{1}{2}\langle h|h\rangle$$

Using a vector-based formalism:

$$\mathcal{L}(h) = d^T h - \frac{1}{2}h^T h$$

Write $h = F\mathfrak{h}$ with

$$F = \begin{bmatrix} F_+ & F_\times \end{bmatrix} \qquad \mathfrak{h} = \begin{bmatrix} h_+ \\ h_\times \end{bmatrix}$$

Maximize $\dfrac{\mathrm{d}\mathcal{L}}{\mathrm{d}\mathfrak{h}} = 0$ and find

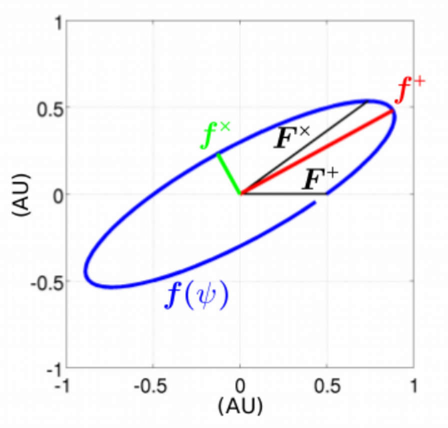
$$\mathfrak{h}^* = (FF^T)^{-1}F^T d$$

- **Generalized likelihood ratio test –** Maximize over h
- **Regularized when antenna pattern *F* is degenerate**

$$\mathcal{L}(F\mathfrak{h}^*) = (d^T f_+)^2 + \boxed{(d^T f_\times)^2}$$

Set to 0 or weighted when F is singular

Left-singular vectors of F

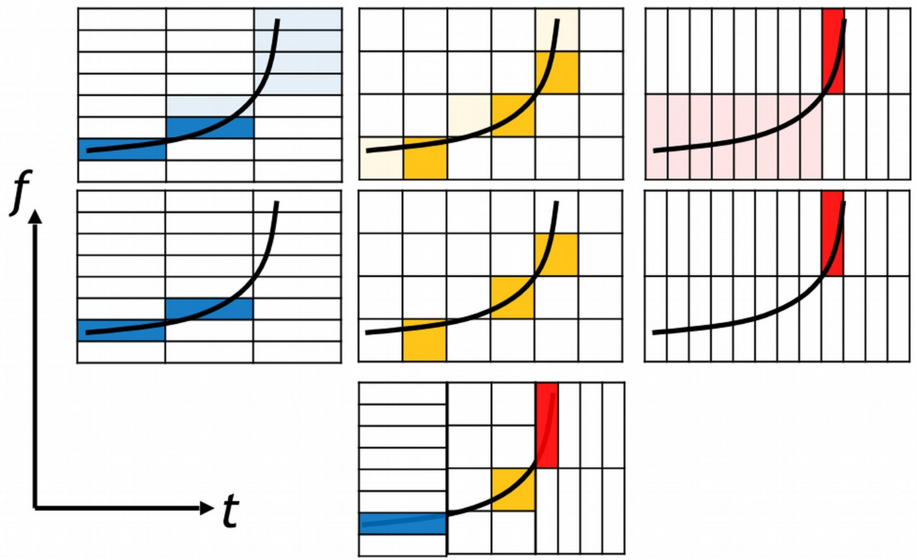


- **Compute $\mathcal{L}$ over the full sky and obtain a likelihood map**

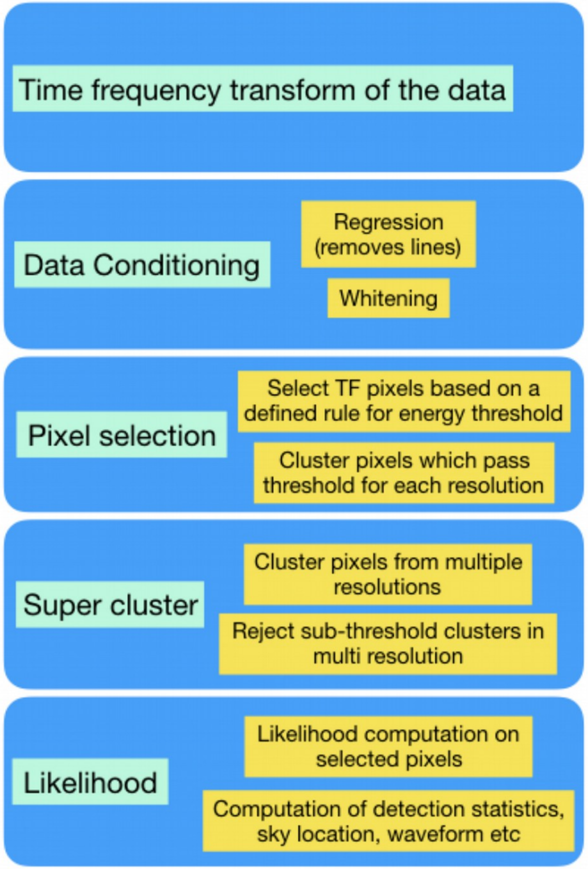
Coherent WaveBurst (2)

$$\mathcal{L}(F\mathfrak{h}^*) = (d^T f_+)^2 + (d^T f_\times)^2$$

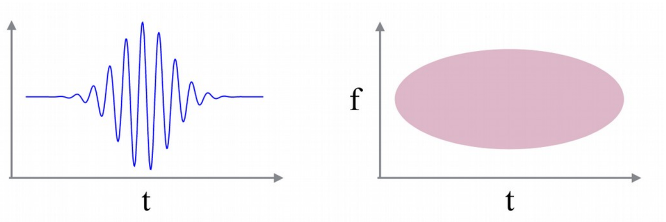
First project *d* onto a set of time-frequency bases and select bright pixels



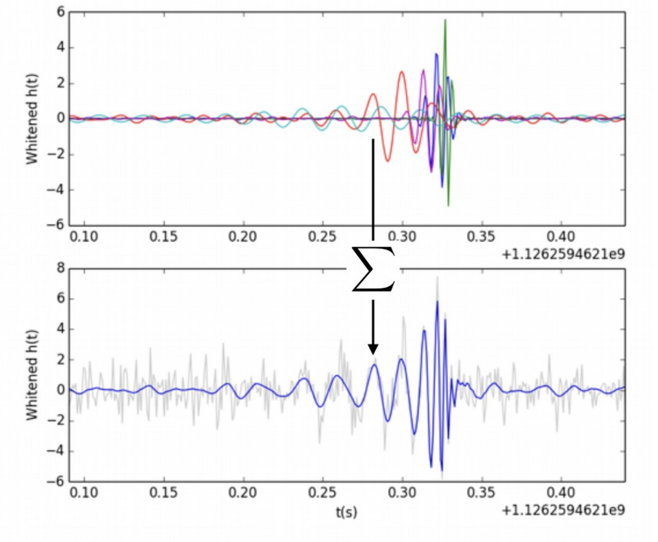
https://gwburst.gitlab.io/



Bayeswave



$$\Psi(t) = Ae^{-(t-t_0)^2/\tau^2}\cos(2\pi f_0(t-t_0)+\phi_0)$$



- **Three hypotheses**

  (1) signal + noise (2) glitch + noise (3) noise only
- **Signal and glitch waveform models**
  - Sum of Gabor wavelets $h = \sum_a^N \Psi(t;\theta_a)$
  - Signal: coherent model for all detectors
  - Glitch: incoherent (different wavelets for each detector)
- **Posterior on parameters obtained using a reversible jump MCMC algorithm**
  - Include model order *N*
- **Hypothesis testing using Bayes factor**

https://git.ligo.org/lscsoft/bayeswave

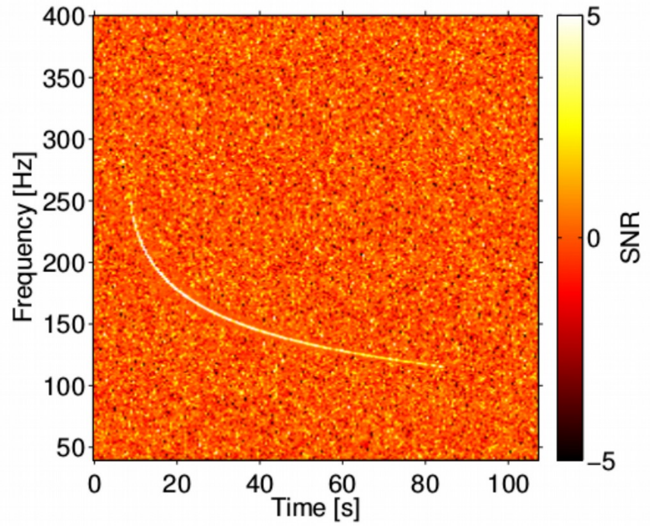
Long bursts (> 10 sec) – STAMP-AS

- Piggybacking on searches for stochastic GW background
  - Coherent combining of “whitened” time-frequency maps

$$C(t,f;\theta,\phi) = \mathrm{Re}\left\{Q_{lm}(t,f;\theta,\phi)\, s^l(t,f) s^{m*}(t,f)\right\}$$

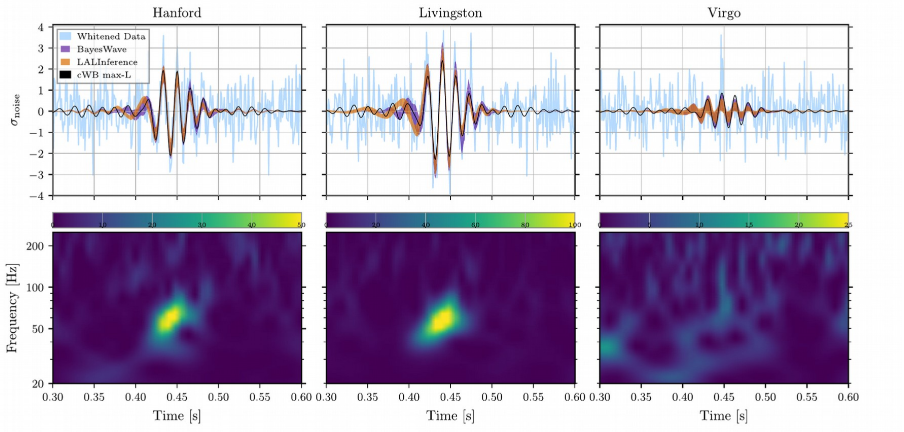
with $$Q_{lm}(t,f;\theta,\phi) = \frac{e^{2\pi i f(\vec{n}_{\theta,\phi}\cdot\vec{\delta}_{lm})/c}}{\mathcal{N}_{lm}(t;\theta,\phi)}$$

- Excess cross-power in pairs of detectors
  - Search for cluster in outlier map $\mathrm{SNR}(t,f;\theta,\phi) = \dfrac{C}{\sigma_C}$
  - Grid search over the full sky



arXiv:1108.1521

Burst-related observations ?



- No GW burst candidate so far
- Burst algorithms provides better significance with very massive compact binaries
  - Example: GW190521
  - Better noise rejection for short duration signals?
  - High-order modes leading to SNR loss in matched-filter searches?

References

- Papers/Reviews
  - LIGO/Virgo, "A guide to LIGO–Virgo detector noise and extraction of transient gravitational-wave signals", Class Quantum Grav 37, 055002 (2020)
- Books
  - Maggiore, "Gravitational Waves: Volume 1: Theory and Experiments"
  - Creighton & Anderson, "Gravitational-Wave Physics and Astronomy: An Introduction to Theory, Experiment and Data Analysis"