Gravitational wave theory

Danièle Steer
APC, University of Paris

GW theory

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[2] M. Maggiore, “Gravitational Waves: Volume 1: Theory and Experiments”, Oxford University Press, 2008.

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[4] B.Schutz, “Gravitational waves on the back of an envelope”, Am. J. Phys. **52** (5) 1984.

[5] Luc Blanchet, “Gravitational Radiation from Post-Newtonian Sources and Inspiralling Compact Binaries”, Living Rev. Relativity 17 (2014), `https://link.springer.com/article/10.12942/lrr-2014-2`

[6] C.Caprini and D.Figueroa, “Cosmological Backgrounds of Gravitational Waves”, `arXiv:1801.04268`

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[8] Bartolo et al, “Science with the space-based interferometer LISA. IV: probing inflation with gravitational waves”, JCAP 1612 (2016) no.12, 026, arXiv: 1610.06481

1) General introduction

2) From Einsteins equations to GWs: basic linearized theory
  - wave equation
  - polarizations, etc
  - generation of GWs
  - energy carried by GWs

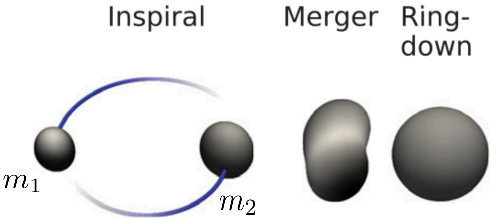
3) example: inspiral of compact binary system
  - wave form
  - characteristic scales, link with observations

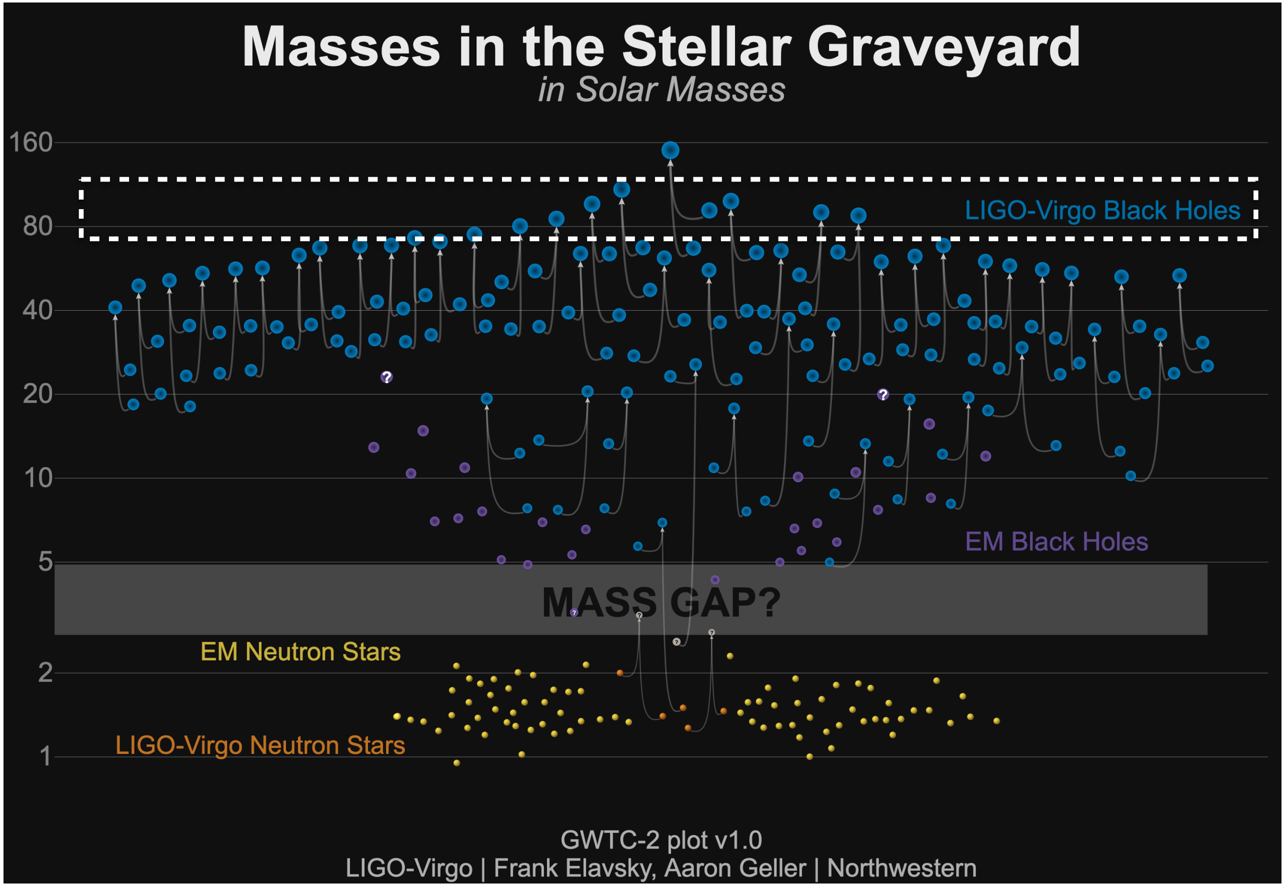
4) The sources: on black holes (BH) and neutron stars (NS)

5) GWs in an expanding universe; cosmology

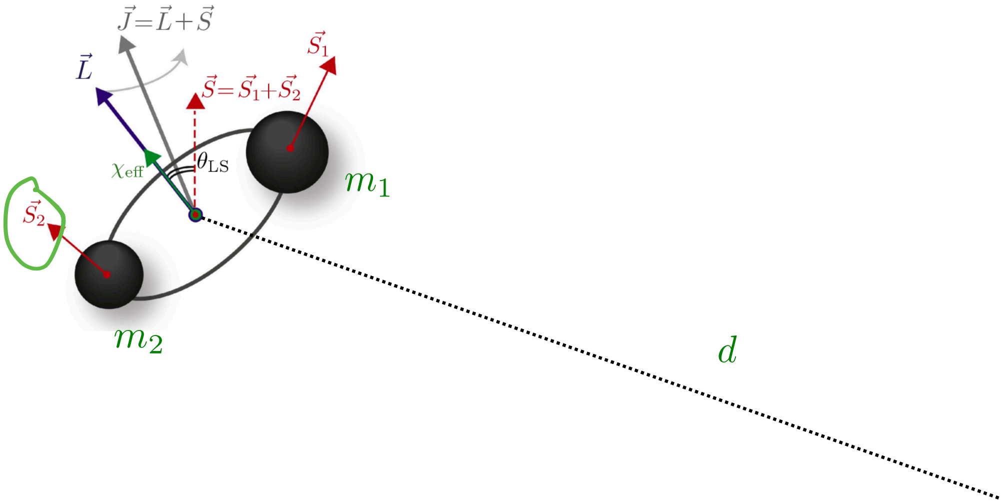
6) The post-newtonian expansion of Einsteins equations

Introduction

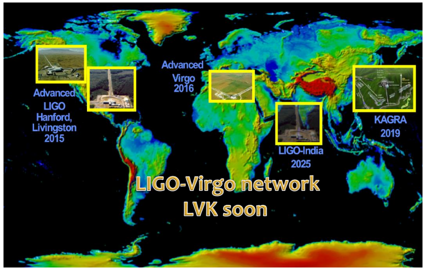


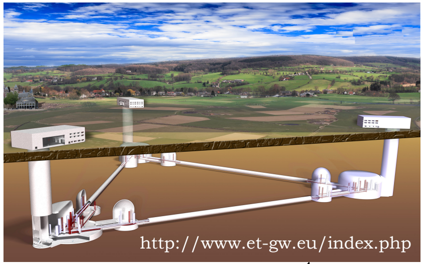


- Shape of the GW signal depends on the properties of the source

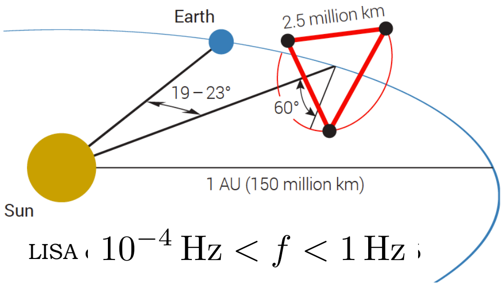


$10\,\mathrm{Hz} < f < 5\,\mathrm{kHz}$



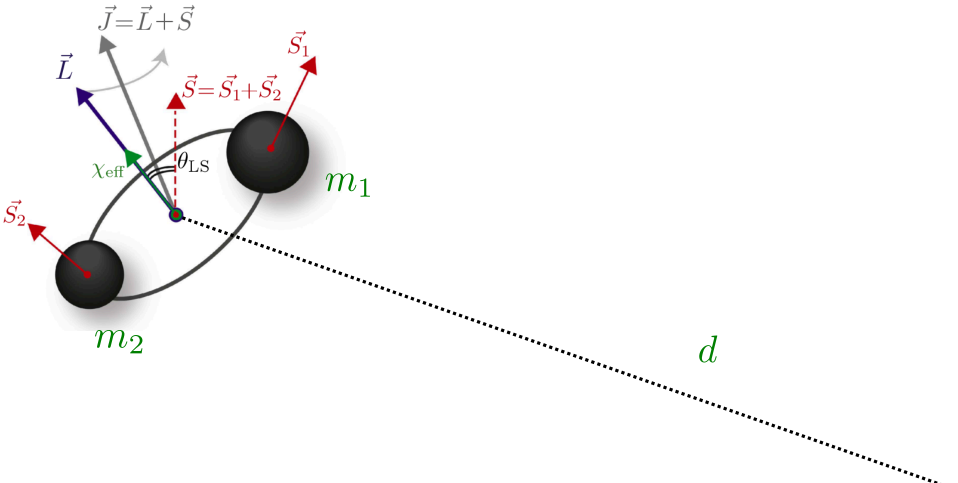

http://www.et-gw.eu/index.php

$1\,\mathrm{Hz} < f < 10^4\,\mathrm{Hz}$

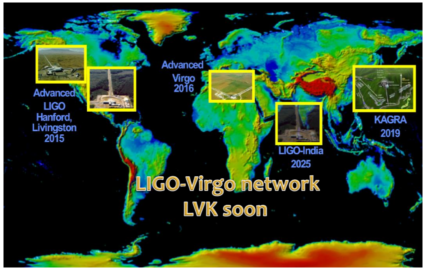


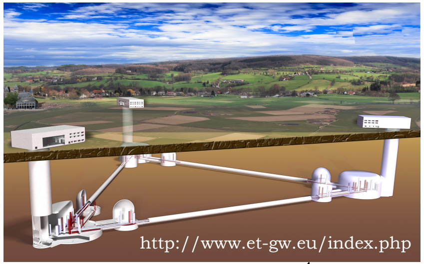
LISA $10^{-4}\,\mathrm{Hz} < f < 1\,\mathrm{Hz}$

- Shape of the GW signal depends on the properties of the source



$10\,\mathrm{Hz} < f < 5\,\mathrm{kHz}$





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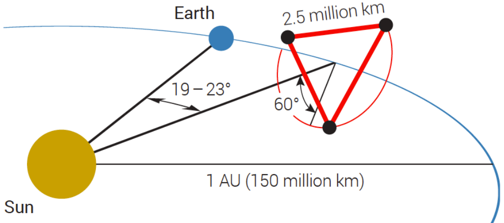

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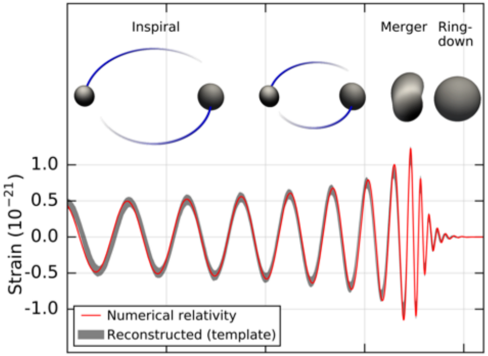
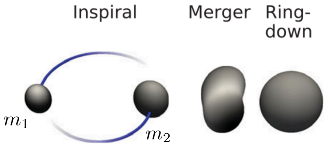
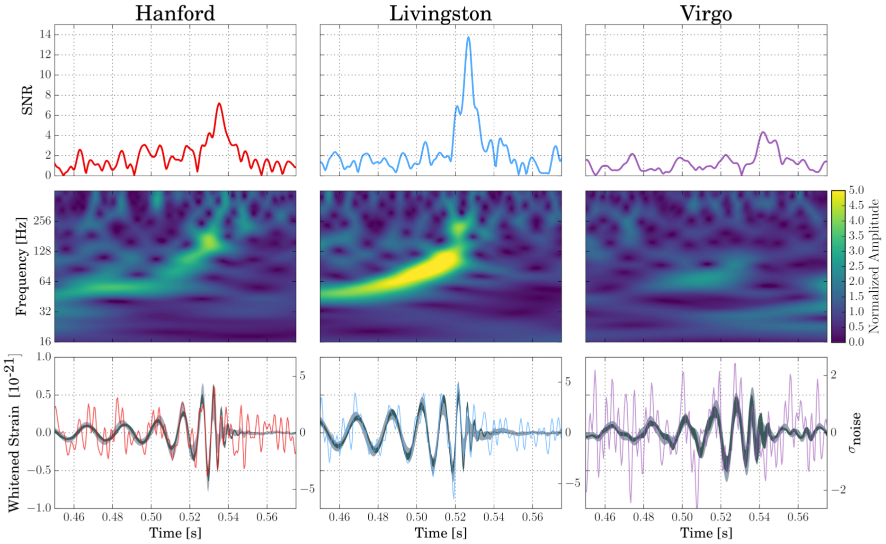
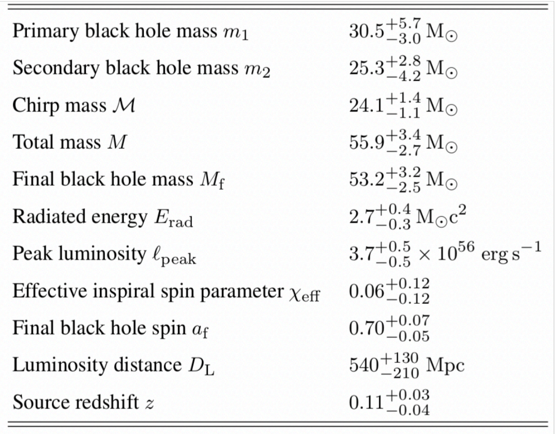


- From data, [See lectures by E. Chassande-Mottin] can extract properties of the source using the detailed theoretical predictions of expected signal.

- eg GW170814 (first event seen by Virgo)







Primary black hole mass m_1	$30.5^{+5.7}_{-3.0}\ \mathrm{M}_\odot$
Secondary black hole mass m_2	$25.3^{+2.8}_{-4.2}\ \mathrm{M}_\odot$
Chirp mass $\mathcal{M}$	$24.1^{+1.4}_{-1.1}\ \mathrm{M}_\odot$
Total mass M	$55.9^{+3.4}_{-2.7}\ \mathrm{M}_\odot$
Final black hole mass $M_{\rm f}$	$53.2^{+3.2}_{-2.5}\ \mathrm{M}_\odot$
Radiated energy $E_{\rm rad}$	$2.7^{+0.4}_{-0.3}\ \mathrm{M}_\odot \mathrm{c}^2$
Peak luminosity $\ell_{\rm peak}$	$3.7^{+0.5}_{-0.5} \times 10^{56}\ \mathrm{erg\,s^{-1}}$
Effective inspiral spin parameter $\chi_{\rm eff}$	$0.06^{+0.12}_{-0.12}$
Final black hole spin $a_{\rm f}$	$0.70^{+0.07}_{-0.05}$
Luminosity distance $D_{\rm L}$	540^{+130}_{-210} Mpc
Source redshift z	$0.11^{+0.03}_{-0.04}$

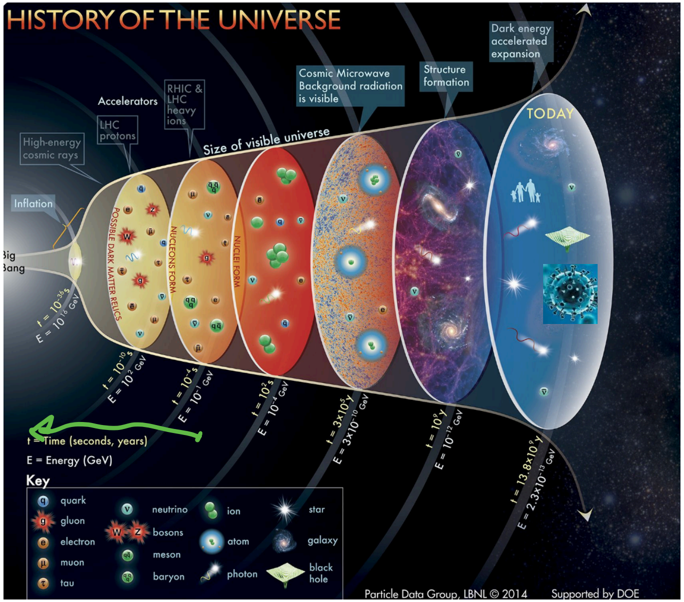
Observations: 1) Frequency and amplitude increase with t => initial phase cannot be due to a perturbed system returning back to stable equilibrium. (Oscillations about equilibrium have ~ const frequencies, and decaying amplitudes).

2) Must be dealing with **compact** objects, not stars whose radius is too large to explain data [Lectures by S. Bernuzzi].

Here: *analytic calculations:* focus on basic linearized perturbation theory, about a Minkowski and FLRW background assuming General Relativity.

Perturbation theory of a different kind (quasi-normal modes) can also be carried out to study the ringdown phase (see 0905.2975 for a review; and also 2103.14750 for recent progress on ringdown in modified gravity)

Gravitational waves and cosmology



Tested cosmology,

$$t \gtrsim 10^{-3}\mathrm{sec}$$

$$T \lesssim 100\,\mathrm{MeV}$$

Tested particle physics

$$T \lesssim 1\,\mathrm{TeV}$$

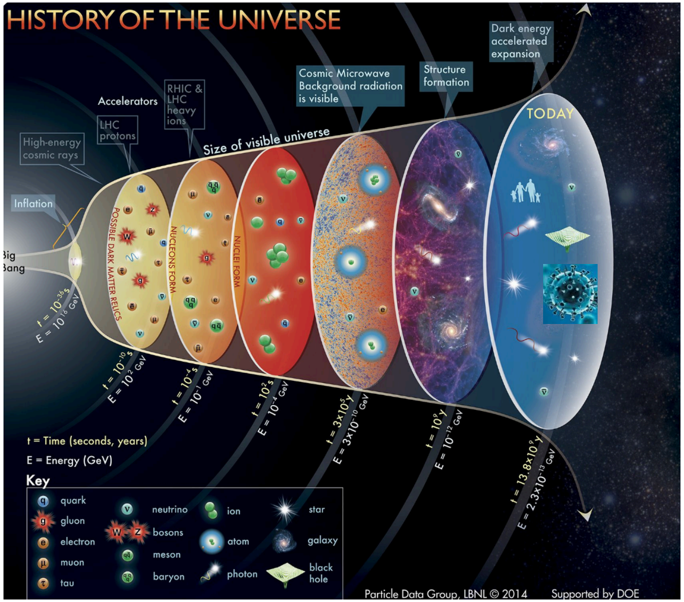
- FLRW universe:

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

- Hubble parameter: $H(t) = \dfrac{\dot{a}(t)}{a(t)}$
- redshift: $1 + z = \dfrac{a(t_0)}{a(t)}$
- Hubble constant today: H_0

Characterizes local time-scale in universe $c/H_0 \sim \mathrm{Gpc}$

Gravitational waves and cosmology



Particle Data Group, LBNL © 2014 Supported by DOE

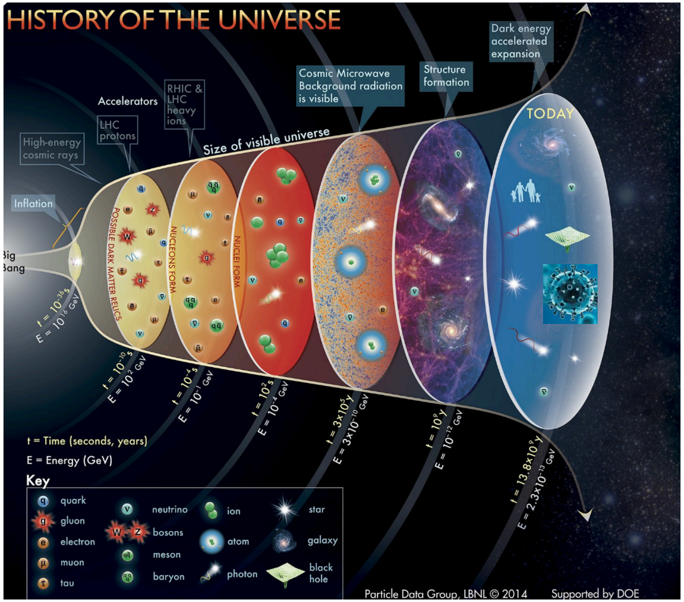
late-time universe

Individual sources
and populations of sources
at cosmological distances

e.g. binary neutron stars (BNS),
binary black holes (BBH),
neutron star- black-hole binary (NS-BH)…

– Expansion rate $H(z)$
– H_0, Hubble constant
– Ω_m
– beyond $\Lambda\mathrm{CDM}$
dark energy $w(z)$ and dark matter
– modified gravity (modified GW propagation)
– astrophysics; eg BH populations, PISN mass gap?

Gravitational waves and cosmology



late-time universe

↕

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Very early universe

$t \gtrsim t_{Pl}$

↕

Stochastic background of GWs
of cosmological origin

↓

- quantum processes during inflation
- primordial black holes
- Phase transitions in Early universe
- topological defects, eg cosmic strings
- …..

GWs & properties of the very early universe $t \gtrsim t_{Pl}$

- *not individual sources, but observation of a stochastic GW background (SGWB) of cosmological origin*
- SGWB: superposition of GWs arriving at random times and from random directions, overlapping so much that individual waves not detectable
- Analogue of the CMB of photons, but crucial difference due to the weakness of GW interactions

$T_{dec} \sim 3000\mathrm{K},\ z_{dec} \sim 1100$

– reminder: particles which decouple from primordial plasma at $t \sim t_{dec}$ or $T \sim T_{dec}$ give snapshot of state of universe at that time

$t < t_{dec}$
$T > T_{dec}$ they are coupled and interactions obliterate all information.

- In thermal equilibrium when

$$\Gamma \sim n\sigma|v| > H$$

rate of process maintaining thermal equilibrium ← Γ

n: Number density of particles

σ: X-section for interaction

$|v|$: typical velocity

For light/massless particles at temperature T

$n \sim T^3$
$v \sim 1$
$H^2 \sim T^4 M_{\rm Pl}^{-2}$

and drop out when $\Gamma \sim H$

- Neutrinos: $\sigma \sim G_F^2 T^2$

$$\left(\frac{\Gamma}{H}\right)_{\rm neutrino} \sim \left(\frac{T}{1\mathrm{MeV}}\right)^3$$

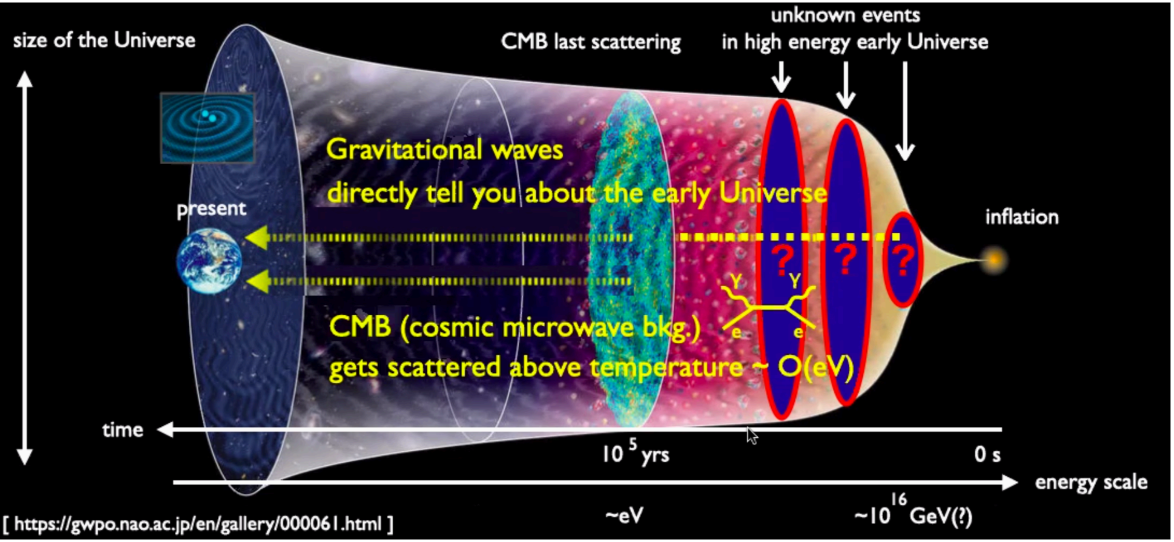
- Gravitons $\sigma \sim G_N^2 T^2 \sim \dfrac{T^2}{M_{\rm Pl}^4}$

$$\left(\frac{\Gamma}{H}\right)_{\rm graviton} \sim \left(\frac{T}{M_{\rm Pl}}\right)^3$$

– gravitons decoupled below Planck scale!
– do not loose memory of conditions when produced
– retain spectrum/shape/typical frequency & intensity of physics at corresponding high energy scales.

Stochastic GW Background:

=> Direct access to earliest stages in the evolution of the universe, which we cannot access through EM radiation.
=> Predictions based on physics beyond the standard model of particle physics (and possibly beyond GR)
=> Corollary: 1) predictions uncertain (based on untested physics)
2) if SGWB of cosmological origin detected, then huge discovery potential



[https://gwpo.nao.ac.jp/en/gallery/000061.html]

Sources?
– quantum processes during inflation
– primordial black holes
– Phase transitions in Early universe
– topological defects, eg cosmic strings
– …..

Tested cosmology, $T \lesssim 100\,\mathrm{MeV}$

Tested particle physics $T \lesssim 1\,\mathrm{TeV}$

1) General introduction

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Sections 2 and part of 3 will be handwritten

From E eq=s to GWs

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

- Approximations. Physics of weak grav. fields

$$\Phi = \frac{GM}{c^2 r}$$

M ← charac. mass

r ← char. dist.

$$\sim \frac{R_s}{2r}$$ Schw. radius $R_s \equiv \frac{2GM}{c^2}$.

$\Phi \ll 1$
weak field

$\Phi^{\text{surface}}_{\text{Sun}} \sim 10^{-5}$

$\Phi^{\text{surface}}_{\text{WD}} \sim 10^{-4}$

$\Phi_{NS} \sim 0.1$

$\Phi_{BH} \sim 1.$

- background metric : $\eta_{\mu\nu}$

$$g_{\mu\nu} = \overset{\swarrow \bar{g}_{\mu\nu}}{\eta_{\mu\nu}} + h_{\mu\nu} + \dots$$

with $h_{\mu\nu} \ll 1$.

- In linearized theory, expand E eq$^{\underline{n}}$ to linear order in $h_{\mu\nu}$.

- Expansion is not unique (depends on choice of coords used to describe the bkgd)

- Linearised approach assumes bkgd curvatu & velocity can be treated as independent expansion parameters.

a) weak field expansion. (exact, linear expressions)

Followed by

b) $\left(v/c\right)$ expansion

↳ multipolar expansions,

→ famous quadrupole formula.

- For self gravitating systems : not true

Virial theorem :

$$\left(\frac{v}{c}\right)^2 \sim \frac{GM}{dc^2}$$

$$\frac{v^2}{c^2} \sim \frac{R_s}{2d.}$$

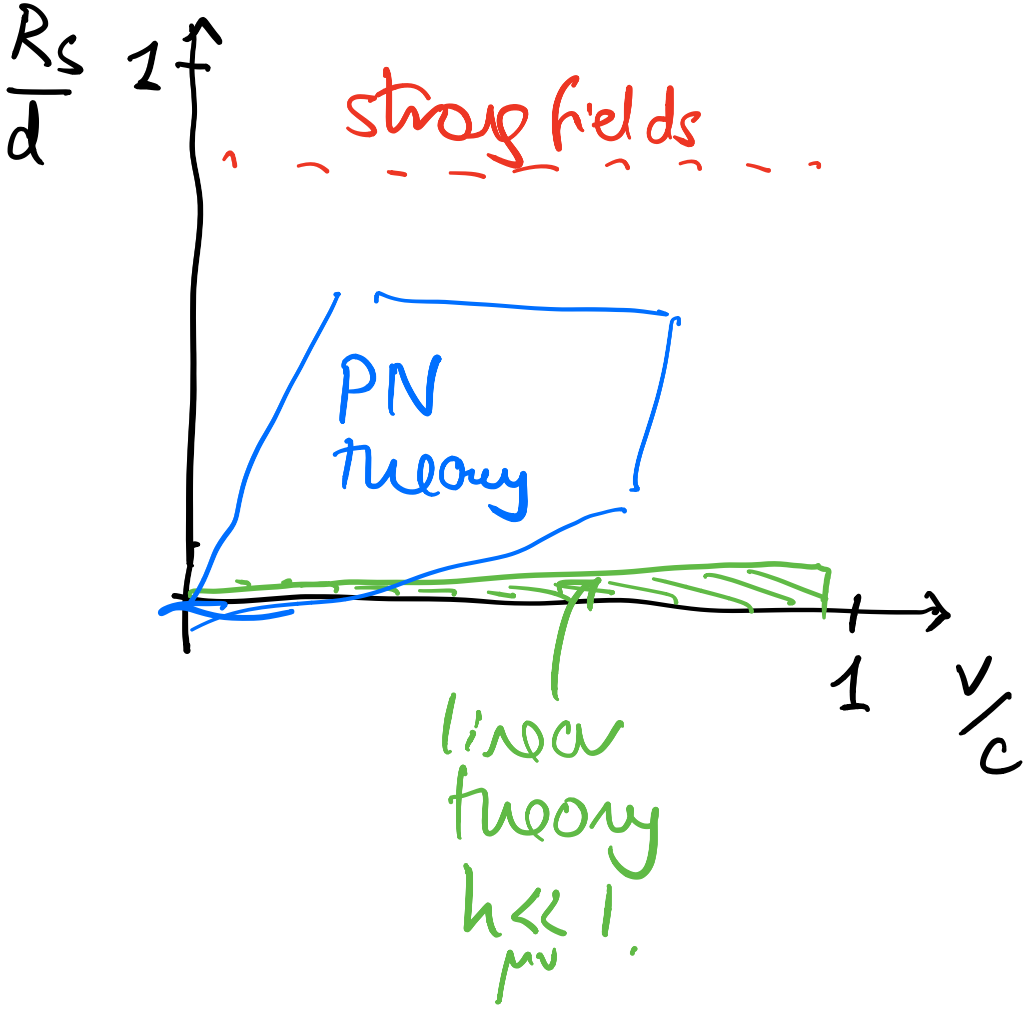
Weak grav. fields $\left(\frac{R_s}{d}\right) \ll 1$

$$\Leftrightarrow \left(v/c\right) \ll 1.$$

- Expansions not indep ⇒ as soon as switch on (v/c) corrections, must for consistency consider deviations from flat space-time.

↓ basis of the post-newtonian (PN) expansion

(systematic expansion in powers of (v/c))



Linearized theory

- $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- $g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$ $\quad (h^{\mu\nu} = h_{\alpha\beta}\,\eta^{\alpha\mu}\eta^{\beta\nu})$
- Subst. into E eq^{n} & expand:

$$G_{\mu\nu}[\eta + h] = {}^{(0)}G_{\mu\nu}[\eta] + {}^{(1)}G_{\mu\nu}[h] + \dots$$

$$= {}^{(0)}T_{\mu\nu} + {}^{(1)}T_{\mu\nu} + \dots$$

0^{th} order term:

$$^{(0)}G_{\mu\nu}[\eta] = 0 = {}^{(0)}T_{\mu\nu}$$

Stress energy tensor is 1^{st} order.

- Conservation of stress energy

$$\nabla_\mu T^{\mu\nu} = \partial_\mu T^{\mu\nu} + \cancel{\overset{①}{\Gamma}\, \overset{①}{T} + \Gamma T}$$

$$\boxed{\partial_\mu T^{\mu\nu} = 0}$$

↓ grav. fields has no effect on the motion the matter, in this linearised expansion.

- 1^{st} order term $\quad {}^{(1)}G_{\mu\nu} = {}^{(1)}T_{\mu\nu}$

  - rewritten in terms of the "trace reversed" perturbation $\bar{h}_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta}\, \underset{\substack{\uparrow \\ \text{trace}}}{h}$

- and using infinitesimal coordinate

transformation to fix the Lorentz gauge

$$\boxed{\partial_\rho \bar{h}^{\rho\mu} = 0}$$

- 1st order eq^n's reduce

$$\boxed{\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}} \quad \text{—①}$$

$$\Box = \eta_{\mu\nu} \partial^\mu \partial^\nu$$

Away from the source $\quad T_{\mu\nu} = 0$

$$\boxed{\Box \bar{h}_{\mu\nu} = 0} \qquad \text{wave eq}^n$$

- Lorentz gauge doesn't fix the gauge entirely, can impose 4 more conditions to simplify $\bar{h}_{\mu\nu}$:

Fix $\bar{h} = 0$ (traceless) $\qquad \bar{h}_{\mu\nu} = h_{\mu\nu}$.

$h_{0i} = 0$ transverse $\Leftrightarrow \partial_i h^{ij} = 0$

↑

Lorentz : $\partial_\mu \bar{h}^{\mu\nu} = 0$

$\nu = j$: $\cancel{\partial_0 h^{0j}} + \partial_i h^{ij} = 0$

Summarise ! impose the TT (transverse traceless) gauge

$$\boxed{\begin{aligned} h^{0\mu} &= 0 \\ h &= 0 \\ \partial_i h^{ij} &= 0 \end{aligned}}$$

just two physically prop. d of f.

Sol^n

$$\Box \bar{h}_{ij} = 0$$

$$\bar{h}_{ij} = \varepsilon_{ij} e^{ik\cdot x}$$

$$k^\mu = (\omega, \vec{k}) \quad \text{with} \quad k^2 = 0$$

$$\Rightarrow \omega^2 = \vec{k}^2$$

Wave prop. in the z dir^n

$$k^\mu = (\omega, 0, 0, \omega)$$

- $\partial^i h_{ij} = 0 \Rightarrow k^i h_{ij} = 0$

$$\Rightarrow \omega h_{3j} = 0$$

$$\Rightarrow \boxed{h_{3j} = 0}$$

- $h = 0 \Rightarrow h_{11} + h_{22} + h_{33} = 0$

but $h_{33} = 0 \Rightarrow h_{11} = -h_{22}$

$$\boxed{{}^{TT}h_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} e^{i(\omega)(t\pm z)}}$$

- Metric describing a wave prop in the z - dir^{n} :

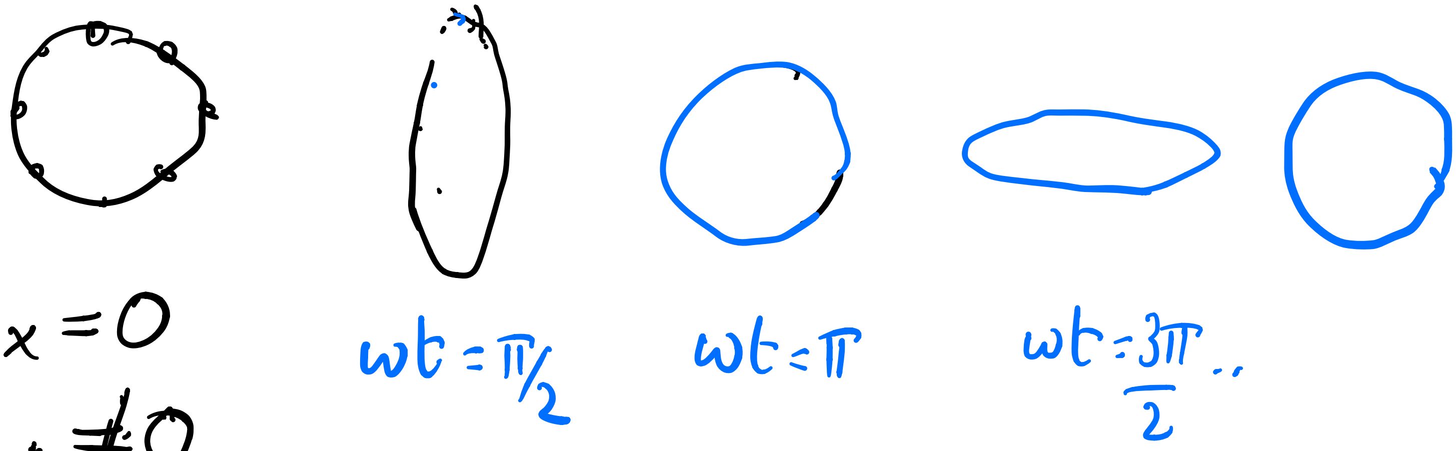
$$ds^2 = -dt^2 + dz^2 + \left(\delta_{ab} + {}^{TT}h_{ab}(t\pm z)\right) dx^a dx^b$$

$(a, b = 1, 2)$

- Effect of a prop. grav. wave on the pos^{n} of initially static particles.
  - δx^μ
  - geodesic deviation eq^{n}

→ no motion in the longitudinal direction (∥ to propag).

→ motion in the transverse directions.

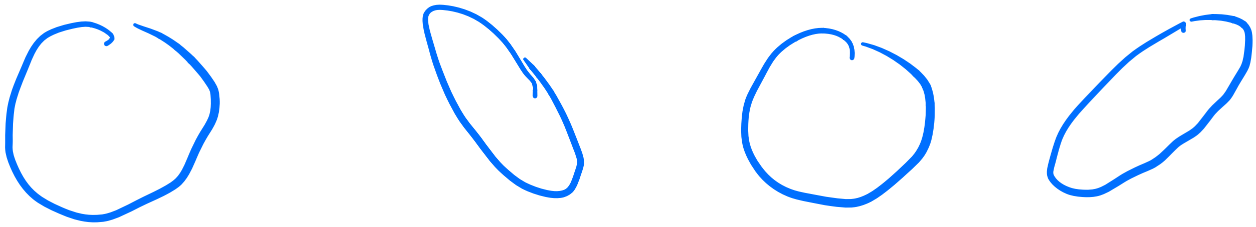


$h_\times = 0$

$h_+ \neq 0$

$h_\times \neq 0$

$h_+ = 0$



- eqⁿ ① is not in the TT gauge.
- in general, your coordinate system will not be aligned with the dirⁿ of propagation of the wave.
- Given a solⁿ of ①, $(\bar{h}_{ij})$; how does one determined $^{TT}h_{ij}$
- Wave prop. in dirⁿ $\vec{n} = \vec{k}$

$|\vec{k}|$

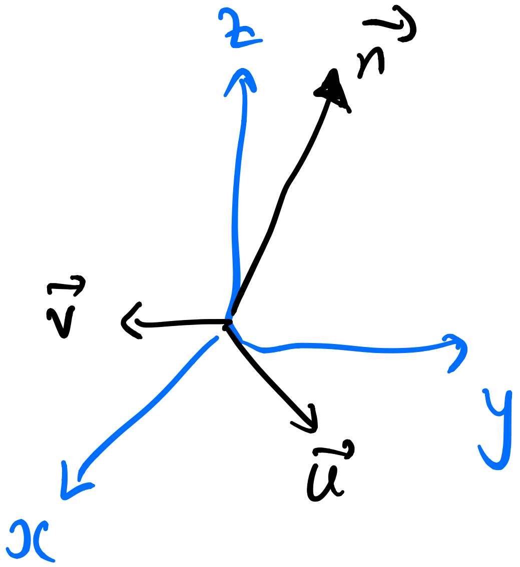
To go from a sol^{n} of ① to ${}^{TT}h_{ij}$:

$${}^{TT}h_{ij} = \Lambda_{ij}{}^{kl}(\vec{n})\, h_{kl} .$$

$$\Lambda_{ijkl} = P_{ik}(\vec{n})\, P_{jl}(\vec{n}) - \frac{1}{2} P_{ij} P_{kl}$$

$$\downarrow$$

$$\delta_{ik} - n_i n_k .$$

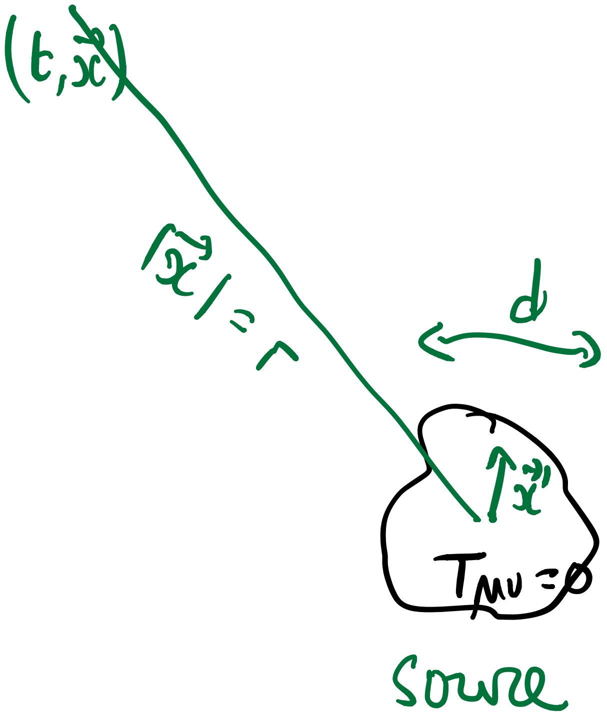


- $T_{\mu\nu} \neq 0$: <u>generation of GWs in linearized theory.</u>

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \qquad \partial^\mu \bar{h}_{\mu\nu} = 0$$

$$\Rightarrow \quad \bar{h}_{\mu\nu}(t,\vec{x}) = \frac{4G}{c^4}\int d^3\vec{x}' \, \frac{1}{|\vec{x}-\vec{x}'|} \times$$

$$T_{\mu\nu}\left(t - \frac{|\vec{x}-\vec{x}'|}{c}, \vec{x}'\right)$$



On distances $r \gg d$

$$|\vec{x} - \vec{x}'| \sim r - \vec{x}' \cdot \hat{\vec{n}} + \mathcal{O}\left(\frac{d^2}{r}\right)$$

$$\boxed{{}^{TT}h_{ij}(t,\vec{x}) = \frac{4G}{c^4 r}\Lambda_{ij,kl}(\vec{n}) \times \int d^3\vec{x}' \, T_{kl}\left(t - \frac{r}{c} + \frac{\vec{x}' \cdot \vec{n}}{c}, \vec{x}'\right) + \mathcal{O}(1/r^2)}$$

true,

$+\left(\frac{v}{c}\right)$ in linearized theory, provided $r \gg d$.

Taylor expand:

$$= T_{kl}\left(t - r/c\right) + \delta\, \partial_t T_{kl} + \delta^2\, \partial_t^2 T_{kl} + \cdots$$

Equivalent to a low velocity expansion:

$$\frac{\vec{x}' \cdot \vec{n}}{c} \ll t_c$$

$$\frac{d}{t_c} \ll c$$

$$v_c \ll c$$

Lowest order term:

$$ {}^{TT}h_{ij}(t,\vec{x}) = \frac{4G}{c^4 r}\Lambda_{ij,kl}\int d^3x'\, T_{kl}\left(t-\frac{r}{c},\vec{x}'\right) + \mathcal{O}\left(\frac{1}{r^2}\right) $$

$$ \partial_\mu T^{\mu 0} = 0 $$

$$ \boxed{ {}^{TT}h_{ij}(t,\vec{x}) = \frac{4G}{c^4 r}\Lambda_{ij,kl}\,\partial_t^2 \underbrace{\int d^3x'\, x'_i x'_j\, T_{00}\left(t-\frac{r}{c},\vec{x}'\right)}_{\text{Quadrupole moment of the source.}} } $$

- Application to a binary system of 2 masses m_1 & m_2 orbiting eachother.

  - ignore spins
  - assume point masses
  - dynamics descibed by Keplers laws

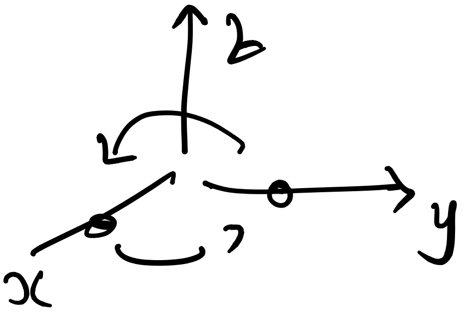
$$\vec{r} = \vec{x}'_1 - \vec{x}'_2$$

For circular orbits of radius a

$$\vec{r} = (a\cos\Omega t,\ a\sin\Omega t)$$

orbital angular freq

$$\Omega = \sqrt{\frac{GM}{a^3}}$$



$M = M_1 + M_2$ (total mass)

- $\vec{x}'_1 = \frac{M_2}{M}\vec{r}$, $\vec{x}'_2 = -\frac{M_1}{M}\vec{r}$

Calculate

$$I_{ij} \equiv \int d^3\vec{x}'\, x'_i x'_j\, T_{00}(\tau, \vec{x}')$$

$$T_{00} = M_1\delta(\vec{x}_1 - \vec{x}'_1) + M_2\delta(\vec{x}_2 - \vec{x}'_2)$$

$\Rightarrow I_{ij} = \mu r_i r_j$ μ = reduced mass $= \frac{M_1 M_2}{M}$

M.

=> I_{xx}, I_{xy}, ... & then their 2nd time derivative

$$\ddot{I}_{xx} = -2\mu \frac{GM}{a} \cos(2\Omega t)$$

=>

$$\bar{h}_{ij} = \frac{2G}{c^4 r} 2\mu \frac{GM}{a} \begin{pmatrix} -\cos(2\Omega t_R) & -\sin(2\Omega t_R) \\ -\sin(2\Omega t_R) & \cos(2\Omega t_R) \\ 0 & 0 \end{pmatrix}$$

$t_R = t - r/c$

- Waves have twice the angular freq of the source $f = 2\Omega$.

- Amplitude $h_0 \sim \frac{2G}{c^4 R} \cdot 2\mu \cdot \frac{GM}{a}$

which we can rewrite in terms of f

Given

$= \; -M \; - \; -$

$\overline{a^3} \quad - \quad 2$

$$\Rightarrow h_0 \sim \frac{1}{r} f^{2/3} \left(G\mathcal{M}\right)^{5/3}$$

$$\mathcal{M} = \text{chirp mass} = \frac{\left(m_1 m_2\right)^{3/5}}{\left(m_1 + m_2\right)^{1/2}}$$

- $f \sim 100\,\text{Hz}$
  $m_1 \sim 20\,M_\odot \approx m_1$
  $r \sim 100\,\text{Mpc}$
  $\Big\}\ h_0 = ?$

- GW carry away stress energy from the source.

$t^{GW}_{\mu\nu}$

$$t^{GW}_{\mu\nu} = \frac{c^2}{32\pi G} \left\langle \partial_\mu h_{\alpha\beta}\, \partial_\nu h^{\alpha\beta} \right\rangle$$

↑ average

over many periods.

$$\rightarrow t_{00}^{GW} \approx \langle \underline{\dot{h}_+^2 + \dot{h}_\times^2} \rangle$$

$$h_+ \sim h_0 \cos(ft) \qquad f = 2\Omega$$

$$\sim f^{2/3} \cos(ft).$$

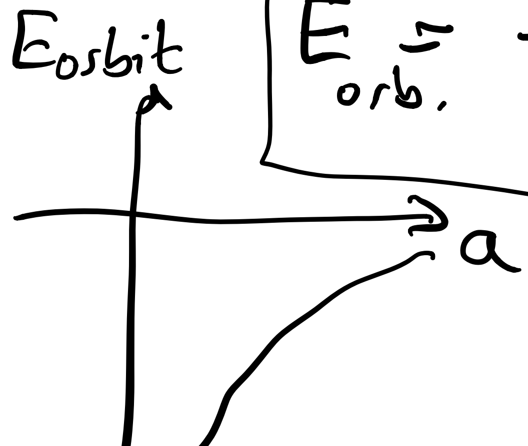
$$\dot{h}_+ \sim f^{2/3} \cdot f \sin(ft) \sim f^{5/3} (\sin ft)$$

$$t_{00}^{GW} \sim f^{10/3}$$

Power radiated in GW $\sim f^{10/3}$.

→ putting this together:

$$\boxed{E_{orb.} = - \frac{Gm\mu}{2a},}$$



GWs carry away energy!

consr$^{\underline{n}}$ of energy

$$\frac{dE_{orb.}}{dt} = -\underline{P_{GW}} \quad \swarrow \text{ power radiated in GW}$$

$$\downarrow$$

$$\frac{2m\mu\,\dot{a}}{2a^2} \qquad\qquad \frac{f}{2} = \sqrt{\frac{GM}{a^3}}$$

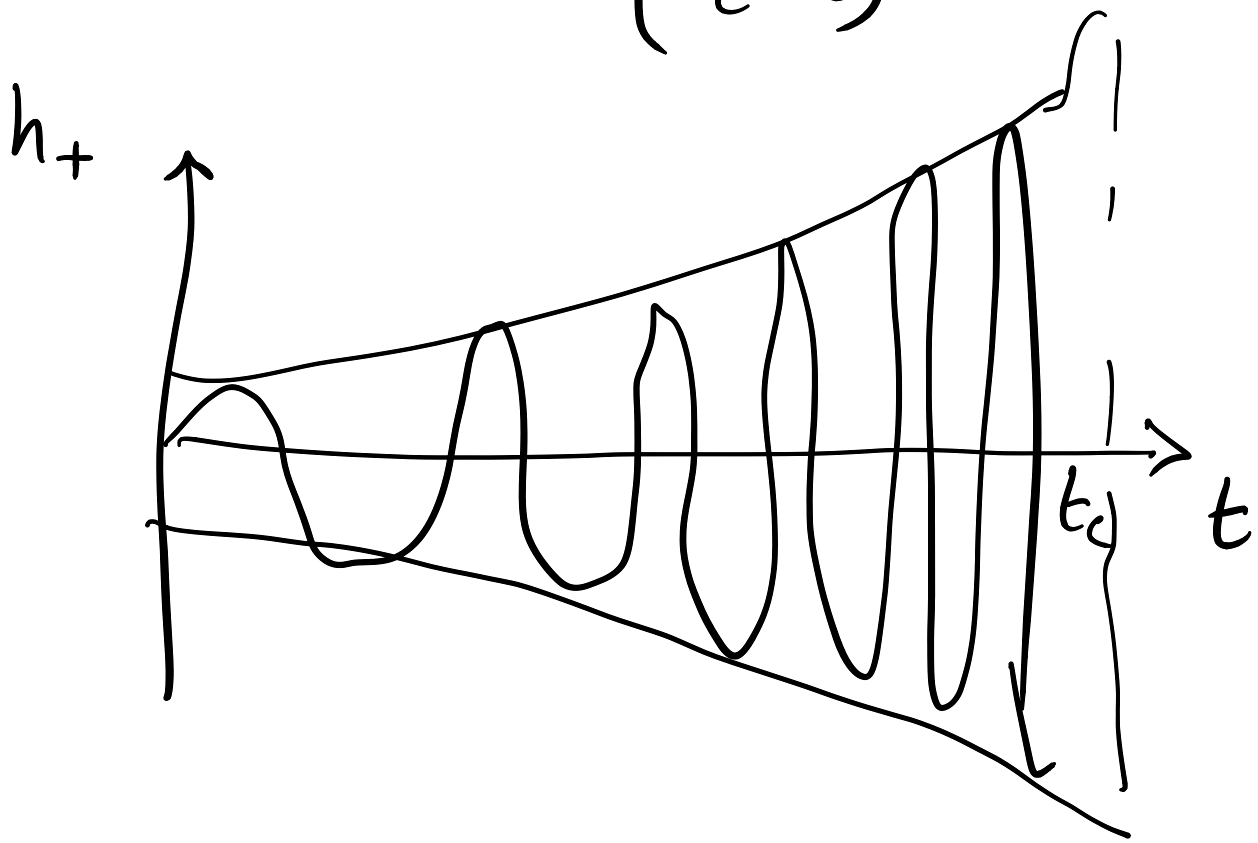
$$\hookrightarrow \frac{\dot{f}}{f^{1/3}}(\text{mass terms}) = \# f^{10/3}$$

$$\boxed{\dot{f} = \left(\frac{96}{5}\right)\pi^{8/3}\left(\frac{GM}{c^5}\right)^3 f^{11/3}}$$

$$f^{-8/3} \sim (t_c - t)$$

$$f \sim \frac{1}{(t_c - t)^{3/8}}$$

$$h_0 \sim f^{2/3} \sim \frac{1}{(t_c - t)^{1/4}}$$



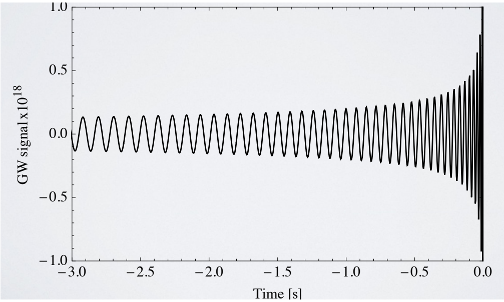
Summary: taking into account the energy loss into gravitational radiation,

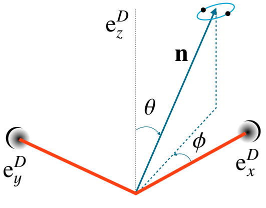
$$\boxed{\begin{aligned} h_+(t,i,\phi_c) &= \frac{4}{R}(G\mathcal{M})^{5/3}(\pi f(\tau))^{2/3}\frac{1+\cos^2(i)}{2}\cos(\phi(\tau)) \\ h_\times(t,i,\phi_c) &= \frac{4}{R}(G\mathcal{M})^{5/3}(\pi f(\tau))^{2/3}\cos\,(i)\sin(\phi(\tau)) \end{aligned}}$$

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

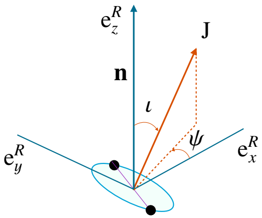
$$f(\tau) = \frac{1}{\pi}\left(\frac{5}{256\tau}\right)(G\mathcal{M})^{-5/8}$$

$$\phi = \phi_c + 2\pi\int_{t_c}^{t} dt' f(t') = -2\left(\frac{\tau}{5G\mathcal{M}}\right)^{5/8}$$

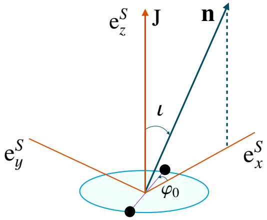




Detector frame



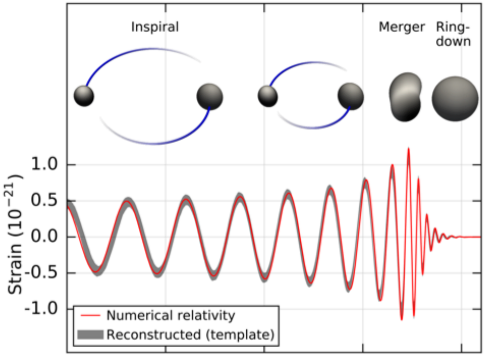
Radiation frame



Source frame

and the signal observed at at the detector is

$$\boxed{h(t-t_0) = F_+(\theta,\phi,\psi)h_+(t) + F_\times(\theta,\phi,\psi)h_\times(t)}$$



– Inspiral phase, neglecting expansion (z<<1)

$$f_{\rm GW} = \frac{1}{\pi}(GM_c)^{-5/8}\left(\frac{5}{256\,\tau}\right)^{3/8}$$

$$\tau = t - t_c$$

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$$

(dominant quadrupolar mode calculated to lowest order in the Newtonian expansion, for *point particles of mass m1 and m2*; no tidal effects, no spins,…, assuming circular orbit; and using quadrupole formula)

- *Merger frequency*: Assuming merger at innermost stable circular orbit (ISCO)

$$\boxed{f_{\rm merger} = \frac{1}{6^{3/2}\pi}\left(\frac{c^3}{GM}\right)}$$

M=total mass
Follows from Keplers laws