



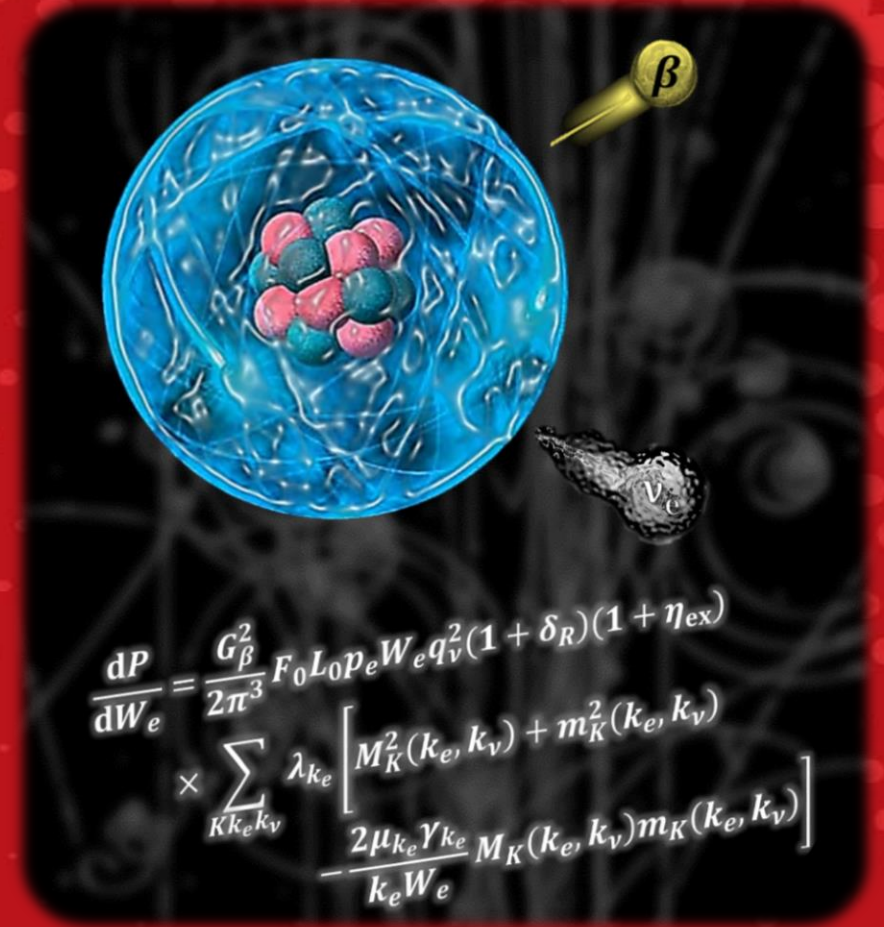
DE LA RECHERCHE  
À L'INDUSTRIE

# NACRE Workshop

## Beta spectra and nuclear structure

June 27-28, 2022

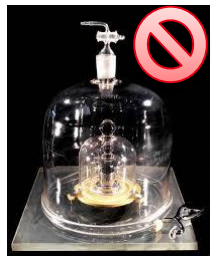
X. Mougeot





LNHB (National Laboratory Henri Becquerel) is the French Designated Institute for primary standards in ionizing radiation metrology.

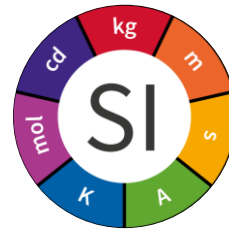
- **Definition of activity (Bq) and dose (Sv, Gy)** through international intercomparisons and transfer to users through standards. International and national traceability is ensured by BIPM referencing and COFRAC certification.



**Instrumentation + Methods = Primary standards**, reference uncertainty

**Instrumentation + Calibration = Secondary standards**, transfer

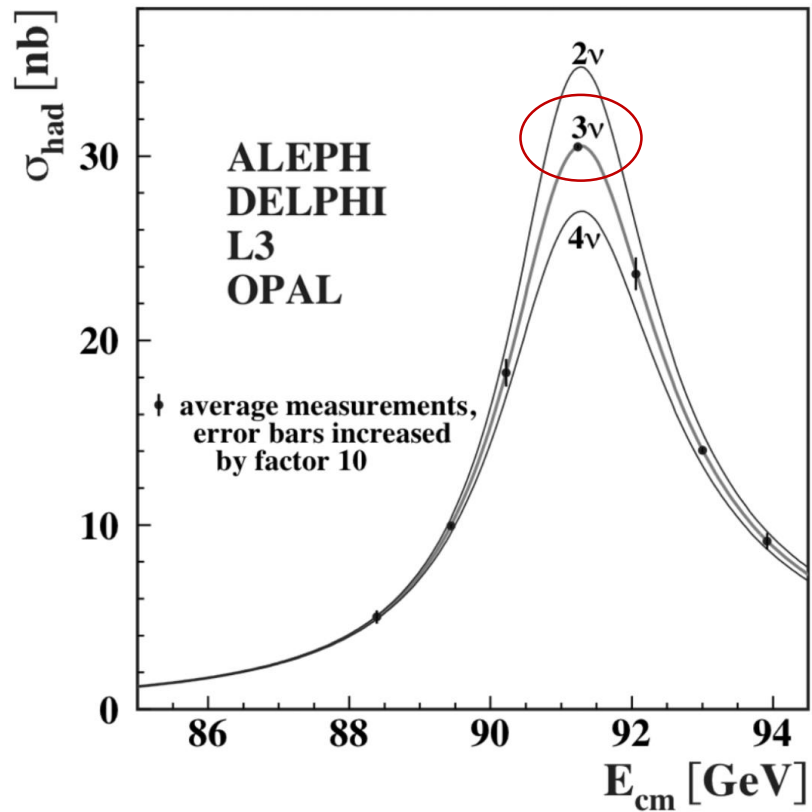
**The diversity of radioactive processes makes necessary a certain knowledge: decay schemes, atomic and nuclear data.**



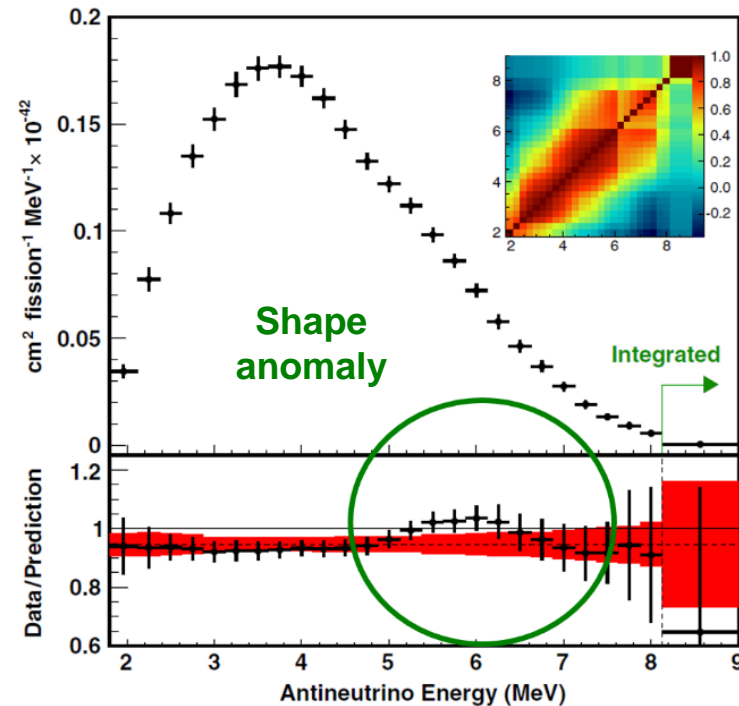
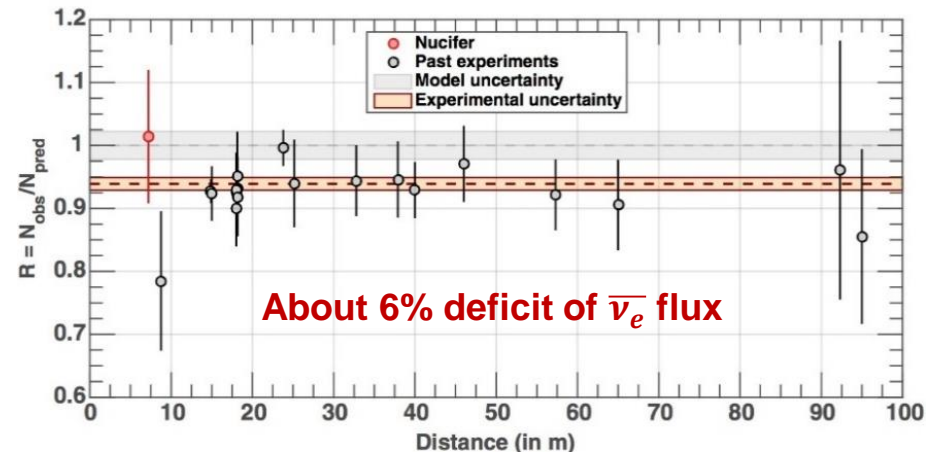
LNHB is highly involved in the evaluation of atomic and nuclear decay data and associated decay schemes, for metrology.

- **Coordination of the DDEP** (Decay Data Evaluation Project) **international collaboration**. Links with IAEA and ENSDF community.
- **Decay data recommended by the BIPM.**



Production cross section of  $Z^0$  boson

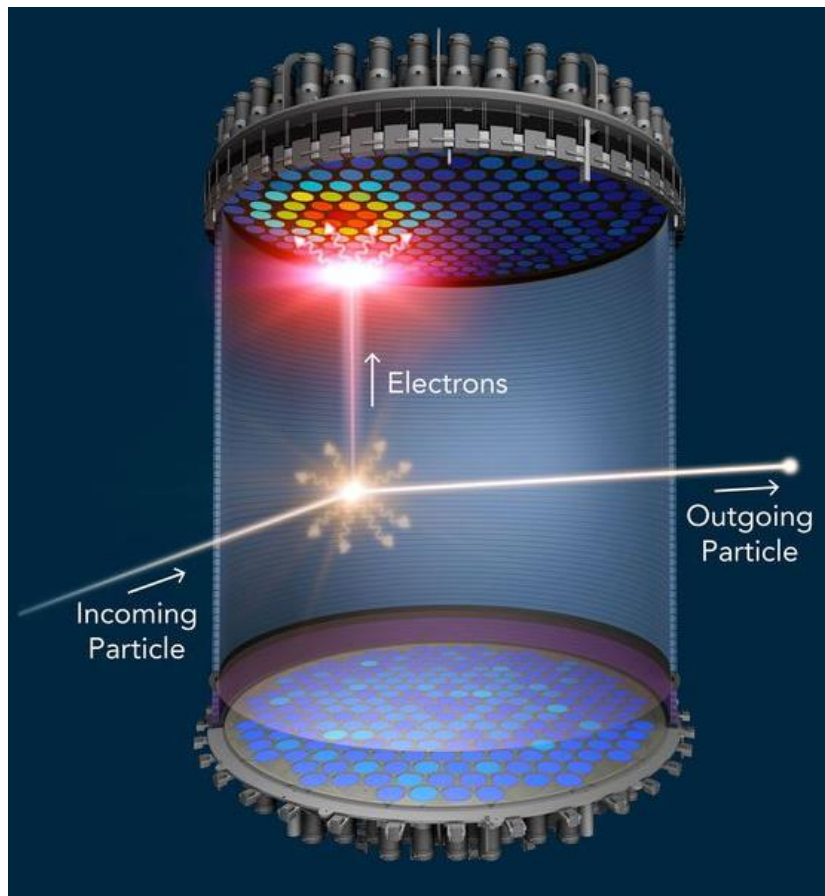
G. Mention et al., Phys. Rev. D 83, 073006 (2011)  
 F.P. An et al., Phys. Rev. Lett. 118, 251801 (2017)  
 G. Bak et al., Phys. Rev. Lett. 122, 232501 (2019)



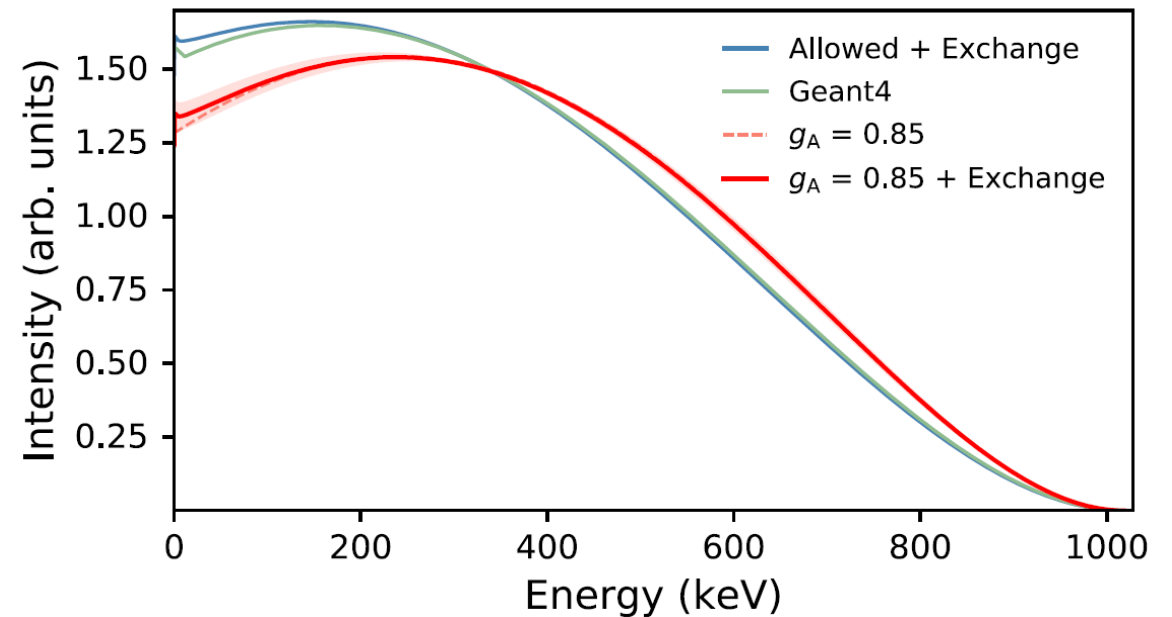
## XENON Collaboration

Accurate description of low energy radioactive background is essential.

Excess of electron events as a monoenergetic peak at low energy ( $\sim 2$  keV).



$^{214}\text{Pb}$  gs to gs decay: influence of nuclear structure



E. Aprile et al., Phys. Rev. D 102, 072004 (2020)

S.J. Haselschwardt et al., Phys. Rev. C 102, 065501 (2020)

$$N(W)dW \propto pW(W_0 - W)^2 \times \left( 1 + \frac{\gamma m_e}{W} b_{\text{Fierz}} \pm \frac{4W}{3M} b_{\text{wm}} \right) dW$$

### Weak magnetism

Point-like nucleons → Finite size nucleons with internal structure.

### Fierz interference

Additional interactions induced by exotic currents beyond the Standard Model.

- Their energy dependency allows an almost independent treatment through the analysis of both low and high energy transitions.

→ **ANR bSTILED**: Project led by O. Naviliat-Cuncic (LPC Caen).

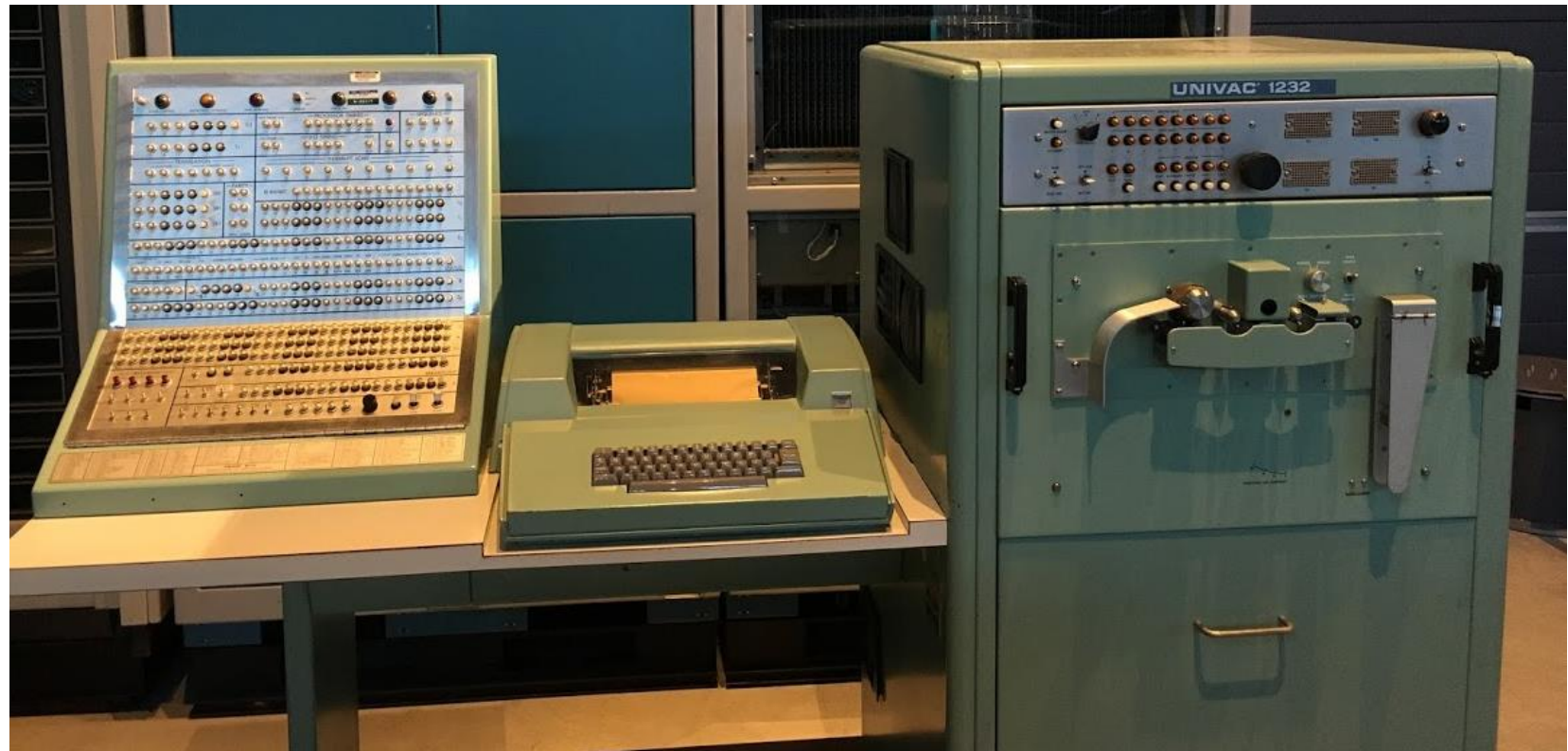
High-precision measurement of  ${}^6\text{He}$  beta spectrum at GANIL.

L. Hayen et al.,  
Rev. Mod. Phys.  
90, 015018 (2018)

Weak interaction properties in nuclear decay data are incomplete and come from calculation: mean energies, capture probabilities, log-*ft* values.

They have been determined for the last 50 years with the LogFT code, developed in the late 1960's. It is still the case in 2022.

Quite simple analytical models of beta transitions and electron captures. Limited in forbiddenness degree.



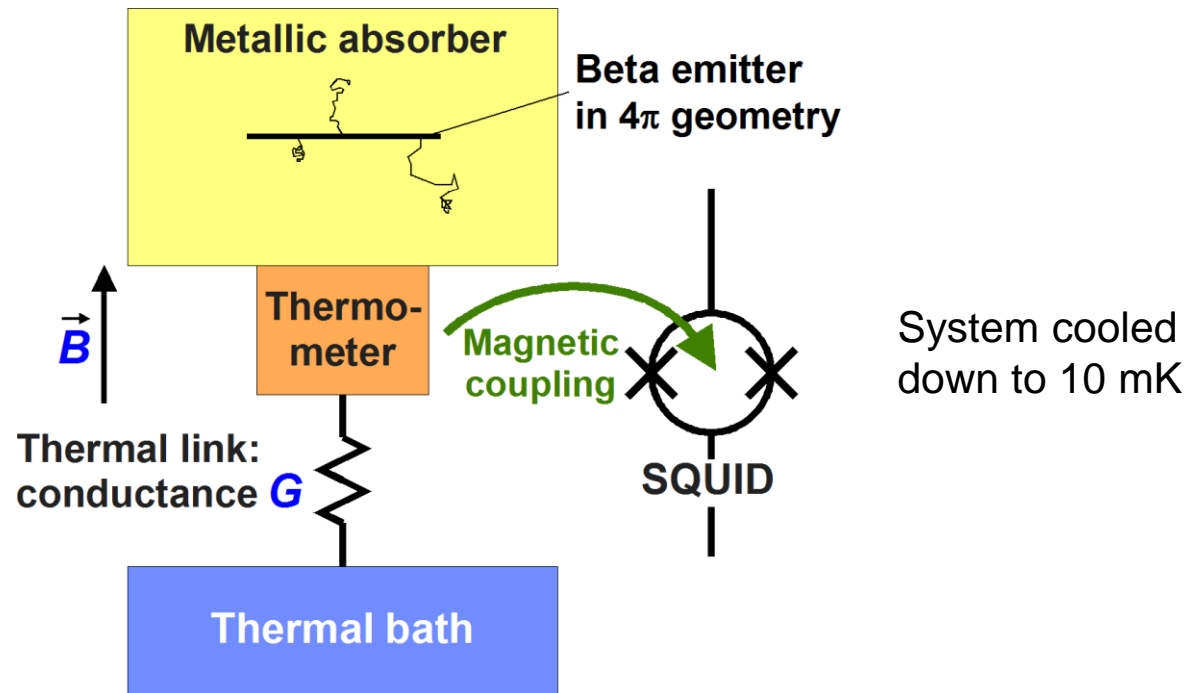
N.B. Gove, M.J. Martin, Nuclear Data Tables 10, 205-317 (1971)

- ✓ **Allowed and forbidden unique transitions** (no limitation): ~75% of 20,000 transitions in ENSDF database.
- ✓ Treatment of beta transitions and electron captures with improved theoretical models. Provision of :
  - ✓ Beta and neutrino spectra for each transition, and total spectrum of the decay.
  - ✓ Capture probabilities, capture probability ratios and capture-to-beta-plus ratios, for each subshell and gathered by shell; splitting of the branch when competition between capture and beta plus processes.
  - ✓ Mean energies of continuum spectra,  $\log-f$ ,  $\log-t$  and  $\log-ft$  values.
  - ✓ Experimental beta shape factors (database included).
- ✓ Reads and updates of ENSDF files. Uncertainty propagation of input parameters. Report files.
- ✓ Various options such as: switching on/off the corrections; fixing energy step in continuum spectra; automatic update of Q-values from AME2020; creation of CSV files; coupling with the Saisinuc software for DDEP evaluations.
- ✓ Latest version (2.2) released in June 2021. Executables are available for various platforms: Windows 10, macOS Big Sur (Intel and M1), Scientific Linux 6.7, Ubuntu 20.04 and Centos 8.

<http://www.lnhb.fr/rd-activities/spectrum-processing-software/>

→ **Nuclear structure required to treat forbidden non-unique transitions (~25% in ENSDF database)**

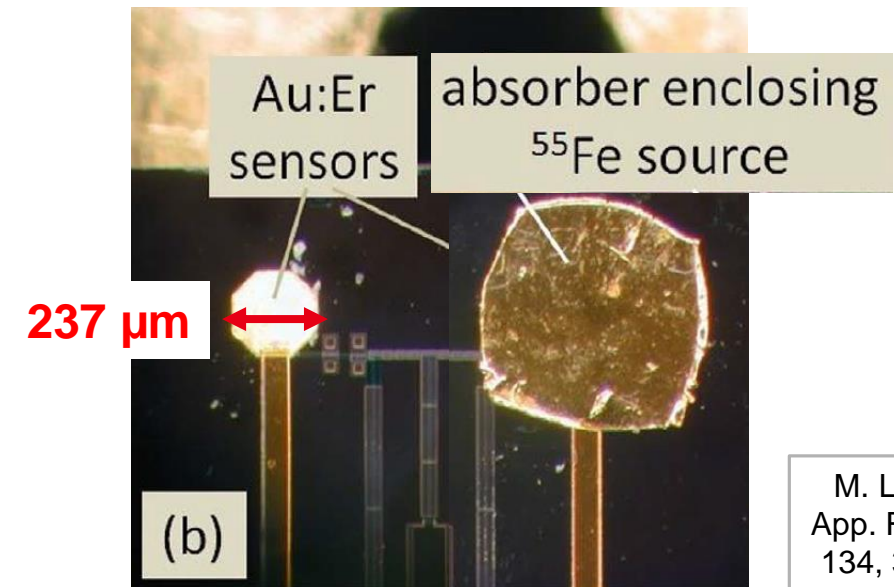
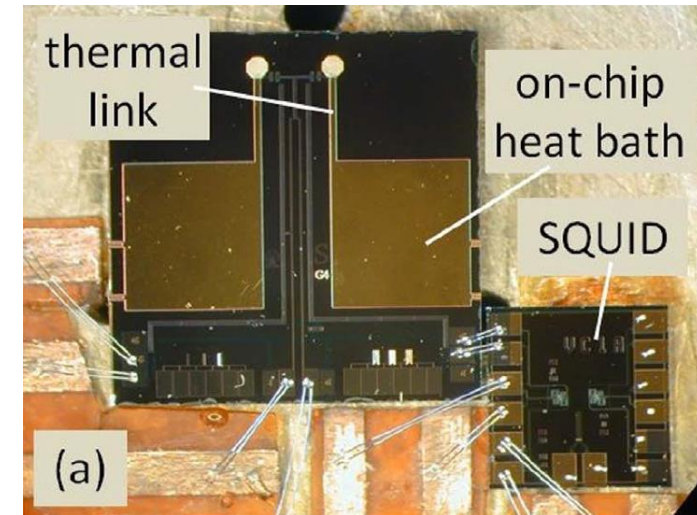
LNHB group led by Martin Loidl (Senior Expert)



MMC are the highest linear cryogenic detectors

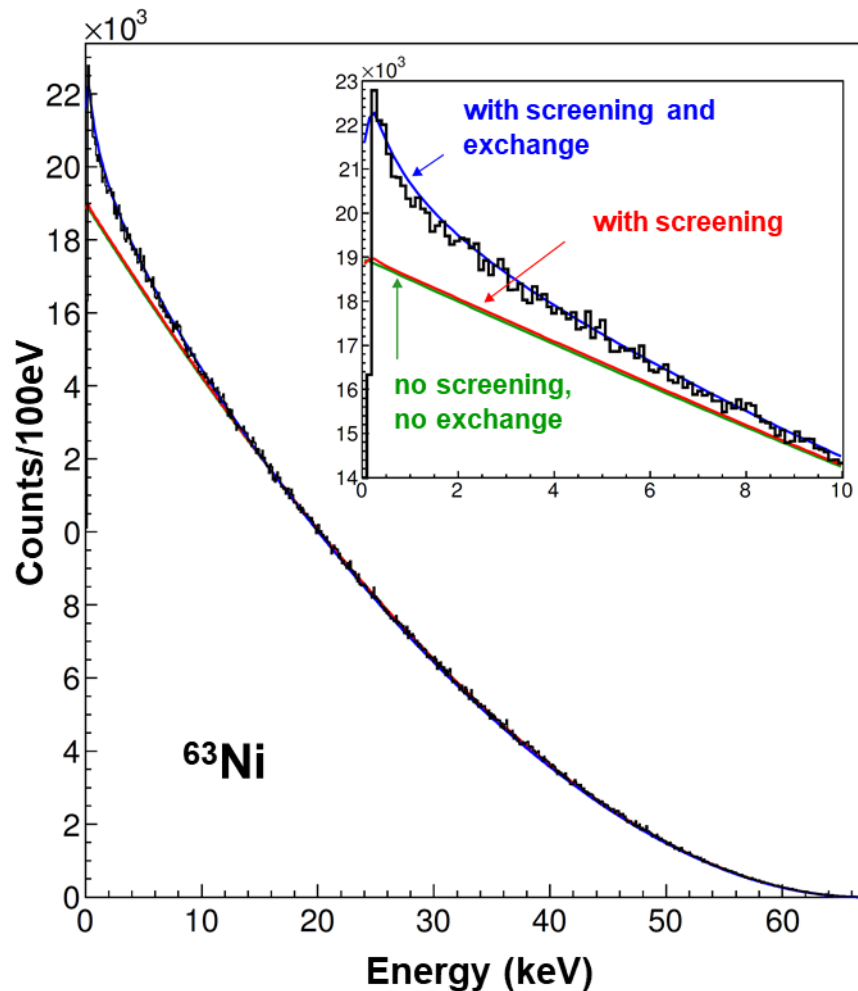
Typically in beta spectrometry:

- Energy threshold < 500 eV
- Energy resolution ~ 30 eV at 10 keV



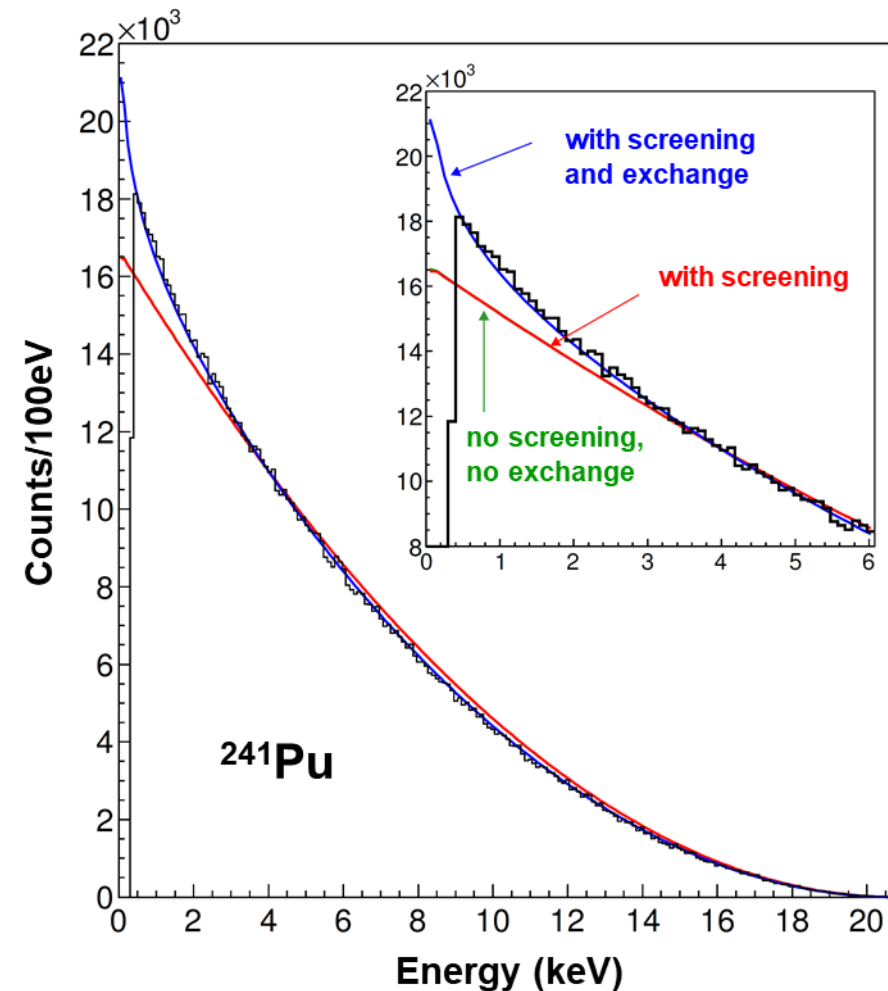
M. Loidl *et al.*,  
App. Radiat. Isot.  
134, 395 (2018)





Usual calculation:  $E_{\text{mean}} = 17.45 \text{ keV}$

Full calculation:  $E_{\text{mean}} = 17.06 \text{ keV}$



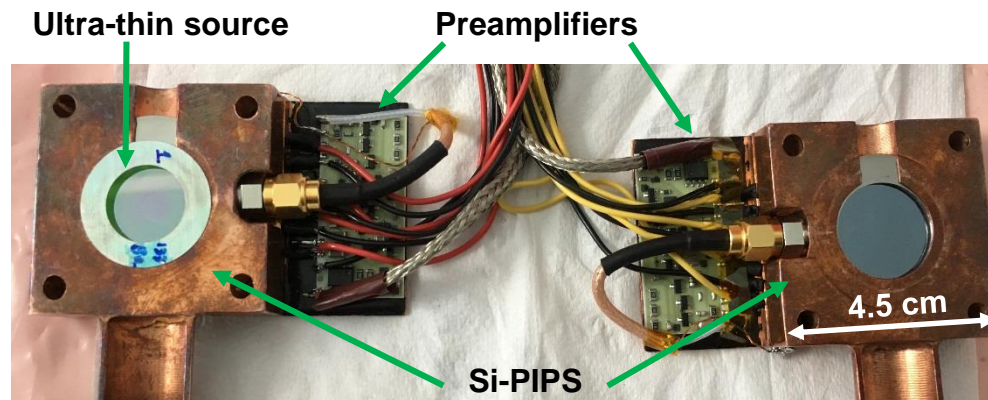
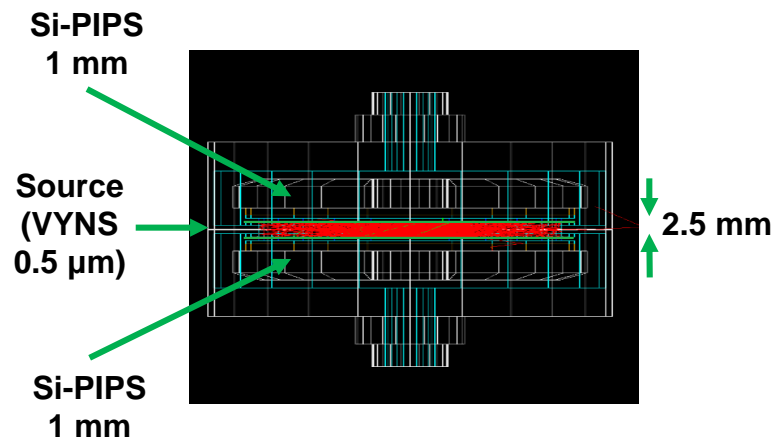
Usual calculation:  $E_{\text{mean}} = 5.24 \text{ keV}$

Full calculation:  $E_{\text{mean}} = 5.04 \text{ keV}$

M. Loidl *et al.*, App.  
Radiat. Isot. 68,  
1454 (2010)

X. Mougeot, C.  
Bisch, Phys. Rev. A  
90, 012501 (2014)

Abhilasha Singh (PhD 2017-2020)

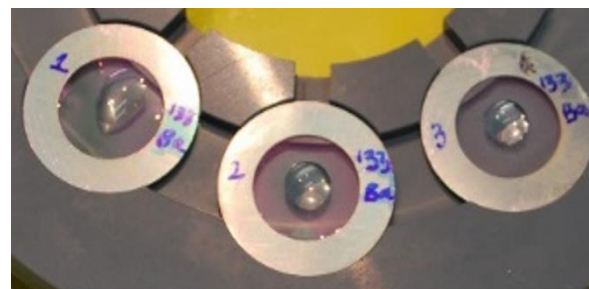


Configuration for measurement

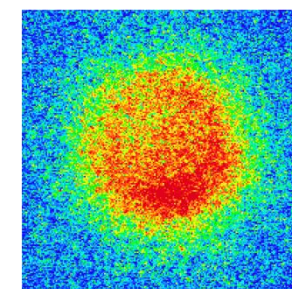


Cold finger for liquid nitrogen

Specific source preparation technique



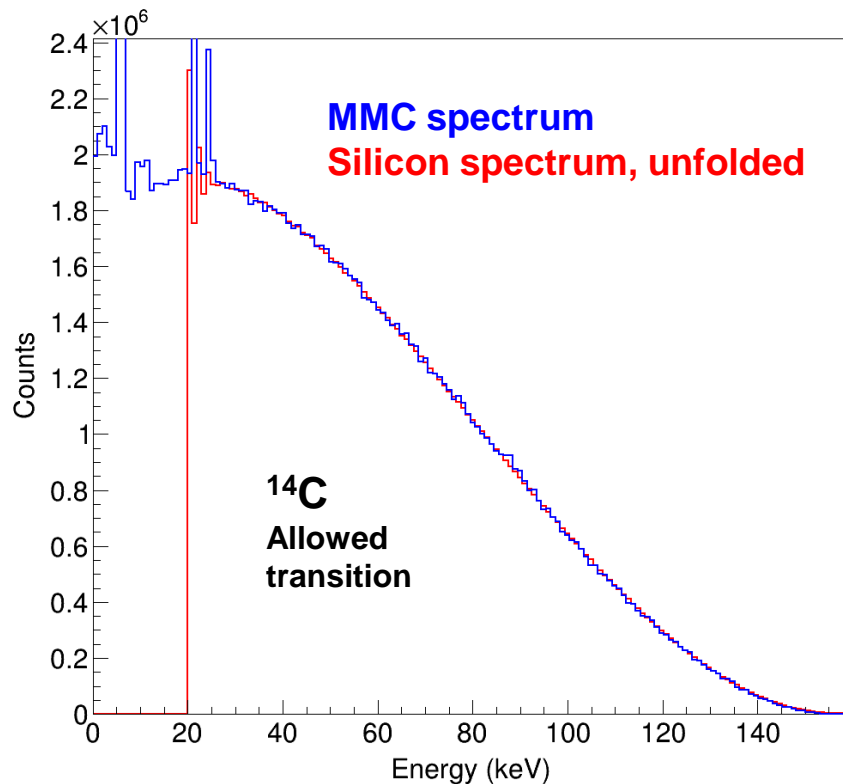
Radioactive deposit



Auroradiography

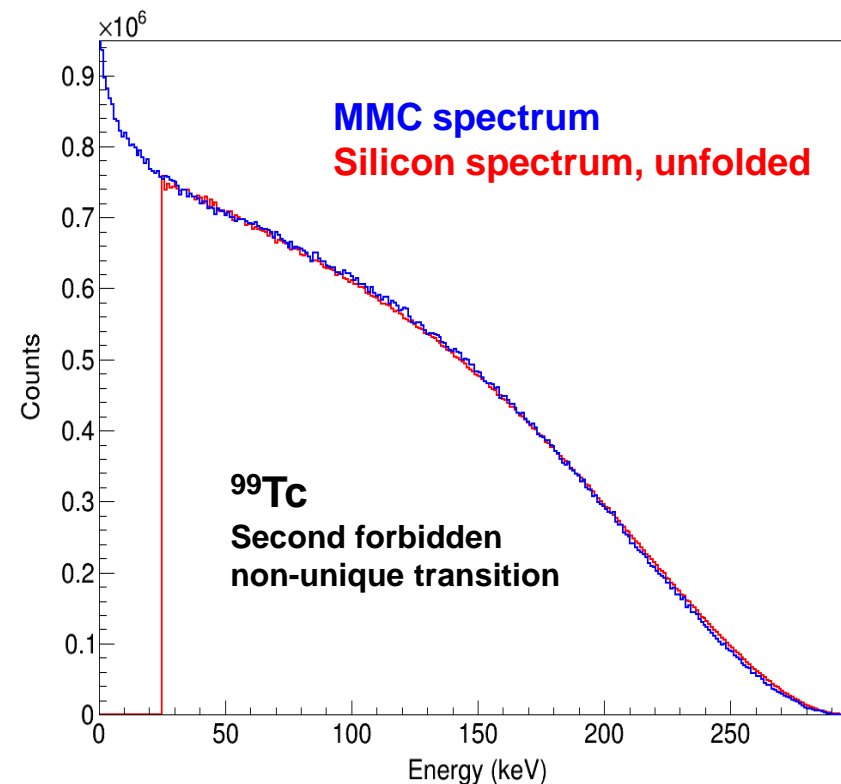
Unsealed sources: 0.5 to  $0.7 \mu\text{m}$  thick  
 Sealed sources: 1 to  $1.5 \mu\text{m}$  thick  
 Typical activity:  $\sim 1 \text{ kBq}$

Unfolding process for silicon measurements: response matrix built from precise Monte Carlo simulations.



→ Spectrum much closer to allowed shape than the reference spectrum.

Kuzminov, *Physics of Nuclei*, 63 (7) (2000)



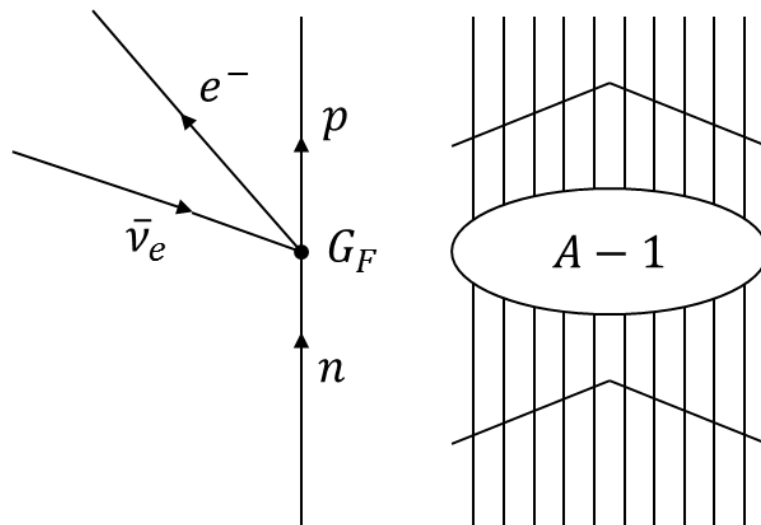
→ Not consistent below 100 keV with the existing reference measurement.

Reich, Schüpferling, *Z. Physik* 271, 107 (1974)

$$C(W_e) = \sum_{Kk_e k_\nu} \lambda_{k_e} \left[ M_K^2(k_e, k_\nu) + m_K^2(k_e, k_\nu) - \frac{2\mu_{k_e} \gamma_{k_e}}{k_e W_e} M_K(k_e, k_\nu) m_K(k_e, k_\nu) \right]$$

H. Behrens, W. Bühring,  
*Electron Radial Wave  
functions and Nuclear Beta  
Decay*, Oxford Science  
Publications (1982)

**Multipole expansion** of hadron and lepton currents. Calculation of shape factors, half-lives, branching ratios, log  $ft$  values.



### Fermi theory

- Vertex of the weak interaction is assumed to be point-like. No propagation of  $W^\pm$  boson.
- Effective coupling constant  $G_F$ .

### Impulse approximation

- The nucleon is assumed to feel only the weak interaction.
- Other nucleons are spectators.

$${}^V \mathcal{M}_{KK0}(q^2) = \frac{\sqrt{2}}{\sqrt{2J_i + 1}} \cdot \frac{(2K + 1)!!}{(qR)^K}$$

Components of the relativistic bound wave function of the nucleon

Geometrical coefficients

×

$G_{KK0}(\kappa_f, \kappa_i) \int_0^\infty$

$g_f(r, \kappa_f) j_K(qr) g_i(r, \kappa_i) r^2 dr$

+  $S_{\kappa_f} S_{\kappa_i}$

$G_{KK0}(-\kappa_f, -\kappa_i) \int_0^\infty$

$f_f(r, \kappa_f) j_K(qr) f_i(r, \kappa_i) r^2 dr$

]

Some 3-6-9-j symbols

Transition described as the transformation of a single nucleon

→ Single particle matrix elements in spherical symmetry

→ **Relativistic wave functions are necessary**

Nuclear state described as a **superposition of nucleon states**

$$\langle \xi_f J_f || T_\lambda || \xi_i J_i \rangle = \hat{\lambda}^{-1} \sum_{a,b} \langle a || T_\lambda || b \rangle \langle \xi_f J_f || [c_a^\dagger \tilde{c}_b]_\lambda || \xi_i J_i \rangle$$

transition matrix element      tensor rank      single particle matrix element      one-body transition density

**One-body transition densities** must be given by a nuclear structure model.

→ **NushellX@MSU**: spherical shell model, effective Hamiltonians fitted on nuclear data, can be used by non-experts.

### **Nuclear structure models are non-relativistic**

The small component of the nucleon wave function can be estimated from the large (non-relativistic) component.

→ Such an approximation has been demonstrated for decades not to be sufficiently accurate for beta transitions.

$$M_n(k_e, k_\nu) = K_n(pR)^{k_e-1}(qR)^{k_\nu-1} \left\{ -\sqrt{\frac{2n+1}{n}} \underline{V F_{n,n-1,1}^{(0)}} - \frac{\alpha Z}{2k_e+1} \underline{V F_{n,n,0}^{(0)}}(k_e, 1, 1, 1) \right. \\ \left. - \left[ \frac{WR}{2k_e+1} + \frac{qR}{2k_\nu+1} \right] \underline{V F_{n,n,0}^{(0)}} - \frac{\alpha Z}{2k_e+1} \sqrt{\frac{n+1}{n}} \underline{A F_{n,n,1}^{(0)}}(k_e, 1, 1, 1) - \left[ \frac{WR}{2k_e+1} - \frac{qR}{2k_\nu+1} \right] \sqrt{\frac{n+1}{n}} \underline{A F_{n,n,1}^{(0)}} \right\}$$

Relativistic  
matrix element

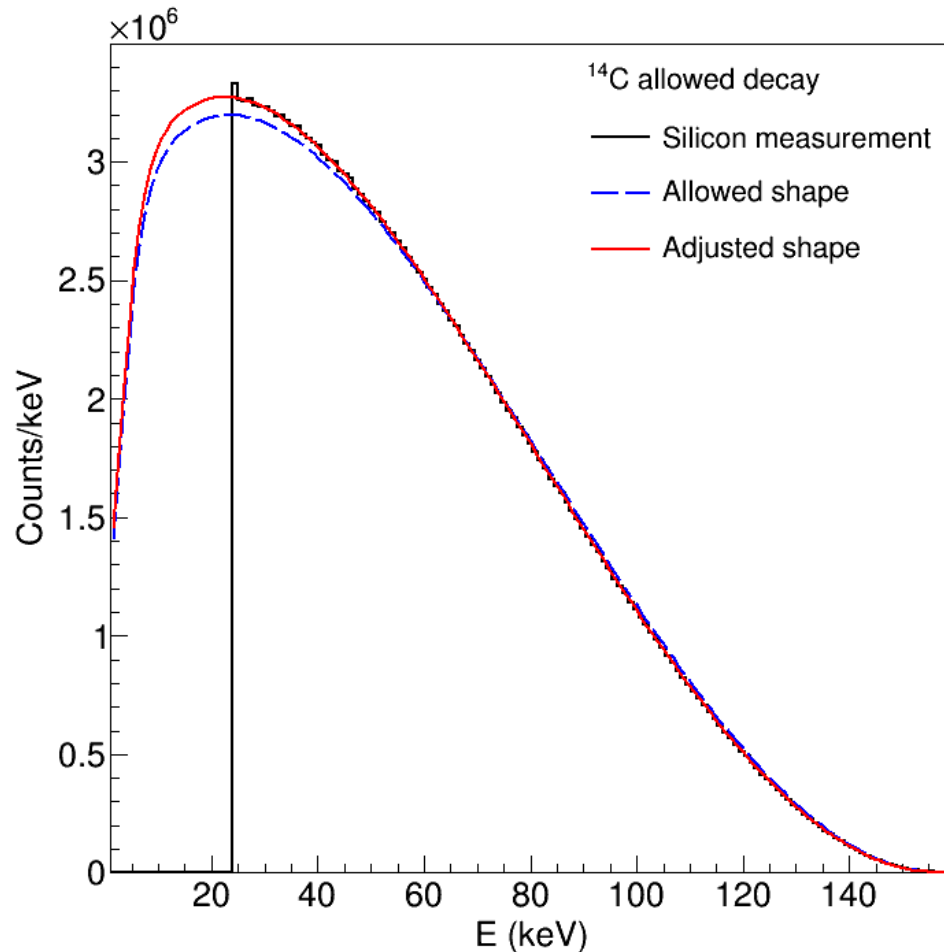
Non-relativistic  
matrix elements

- Form factor coefficients  $V/A F_{KLS}^{(N)}$  are directly related to single-particle matrix elements.
- Here, lepton current has been simplified and developed, keeping only the dominant terms. One can also use a complete, full numerical lepton current.

### Conserved Vector Current (CVC) hypothesis

- Comes from gauge invariance of the weak interaction.
  - Relationships between non-relativistic and relativistic vector matrix elements.
- **CVC could be used to quantify the accuracy of a relativistic nuclear model**

A controversy exists in the literature on  $^{14}\text{C}$  spectrum shape, experiments and theory being discrepant.



### Preliminary analysis of Silicon spectrum

- ✓ Excellent agreement with MMC spectrum.
- ✓ Extracted Q-value = 156.47 (43) keV in excellent agreement with AME2020 value of 156.476 (4) keV.
- !! Additional shape factor necessary:  $C(E) = 1 + aE$  with an extracted value  $a = -0.430$  (23)  $\text{MeV}^{-1}$ .

Detailed theoretical studies suggest accidental cancellation of nuclear matrix elements → **enhanced weak magnetism term.**

Additional shape factor  $C(E) = 1 + aE$  with predicted value:

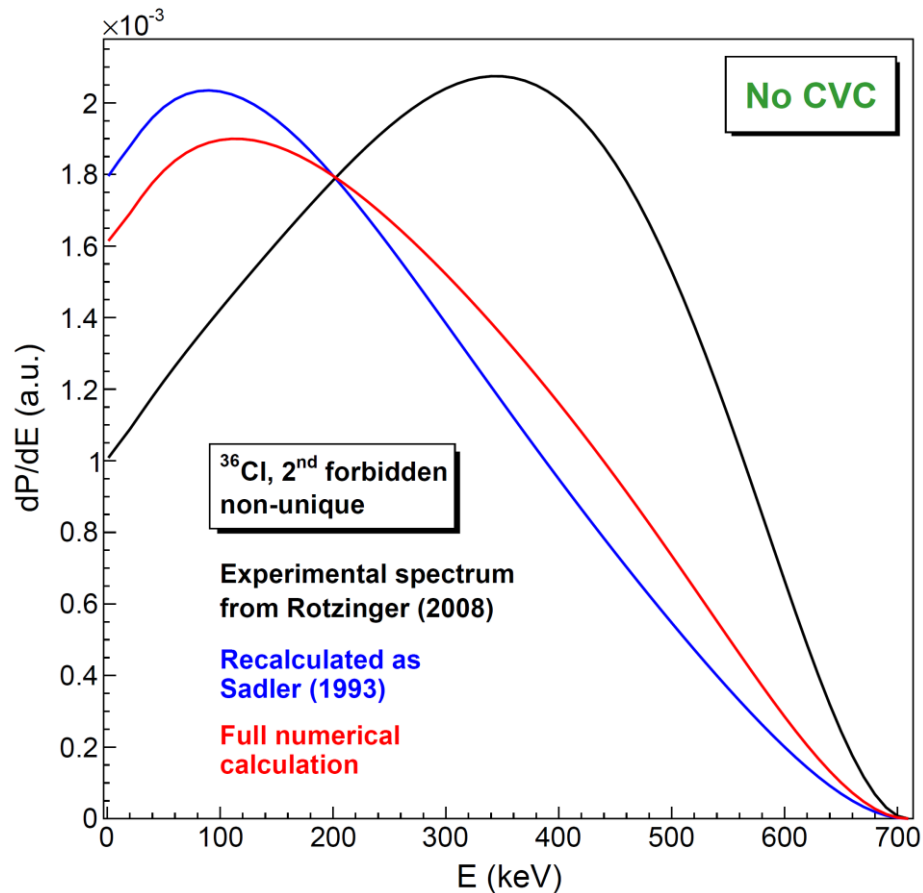
- [Calaprice and Holstein, NP 273, 301, 1976]:  $a = -0.386 \text{ MeV}^{-1}$
- [Garcia and Brown, PRC 52, 3416, 1995]:  $a = -0.37$  (4)  $\text{MeV}^{-1}$
- [Towner and Hardy, PRC 72, 055501, 2005]:  $a = -0.43 \text{ MeV}^{-1}$

→ **Our measurements are very consistent with the best theoretical predictions.**



## Precise measurement exists

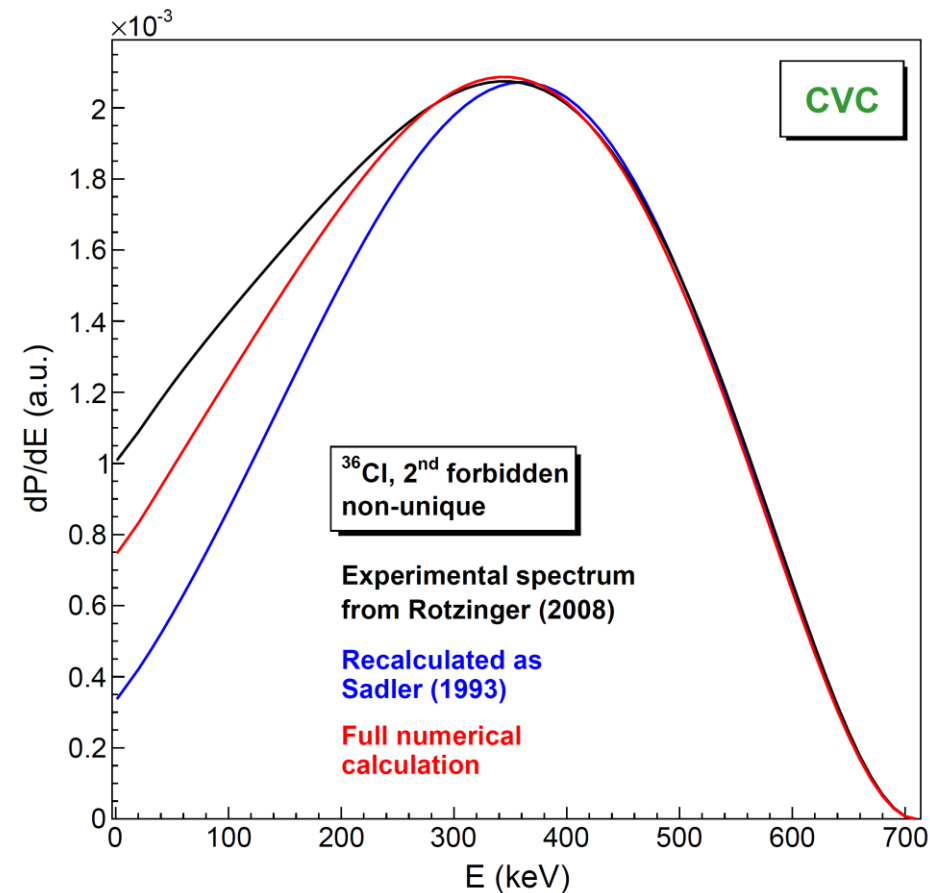
Rotzinger et al., J. Low Temp. Phys. 151, 1087 (2008)



## Detailed theoretical study (with approximations)

→ Matrix elements are correctly recalculated

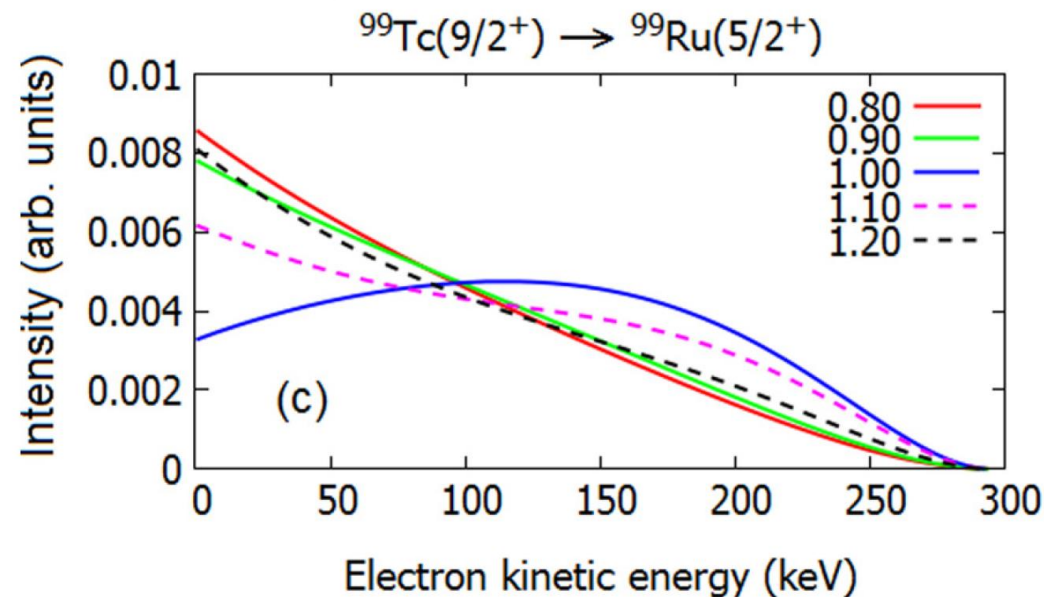
Sadler, Behrens, Z. Phys. A 346, 25 (1993)



→ CVC hypothesis mandatory + Influence of lepton current treatment

J. Kostensalo, J. Suhonen, PRC 96, 024317 (2017)

**$g_A$ -driven shapes of electron spectra of forbidden beta decays in the nuclear shell model**

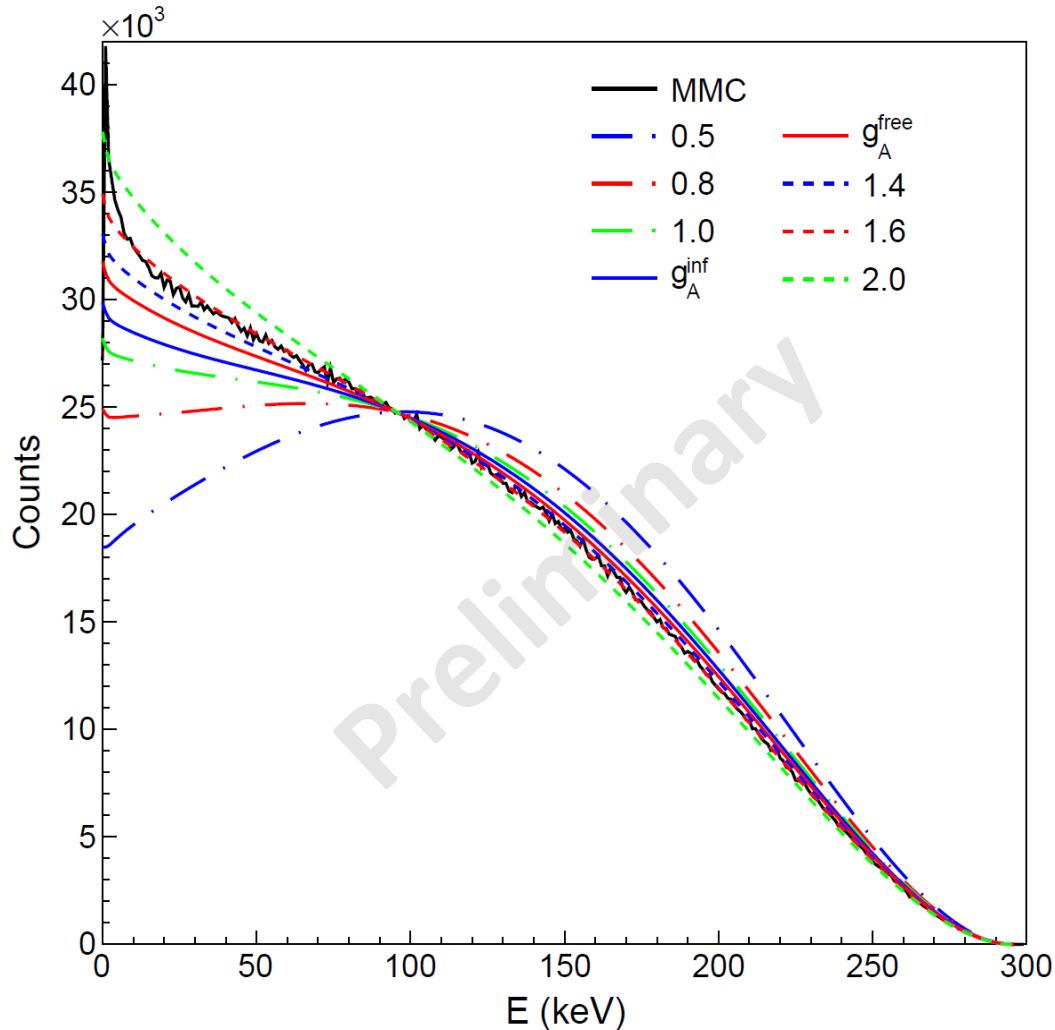


The value of  $g_A$  can be affected by several nuclear effects due to the necessary assumptions in nuclear models to deal with the many-body problem.

→  $^{99}\text{Tc}$  beta spectrum, second forbidden non-unique, has been predicted to be very sensitive to  $g_A$ .

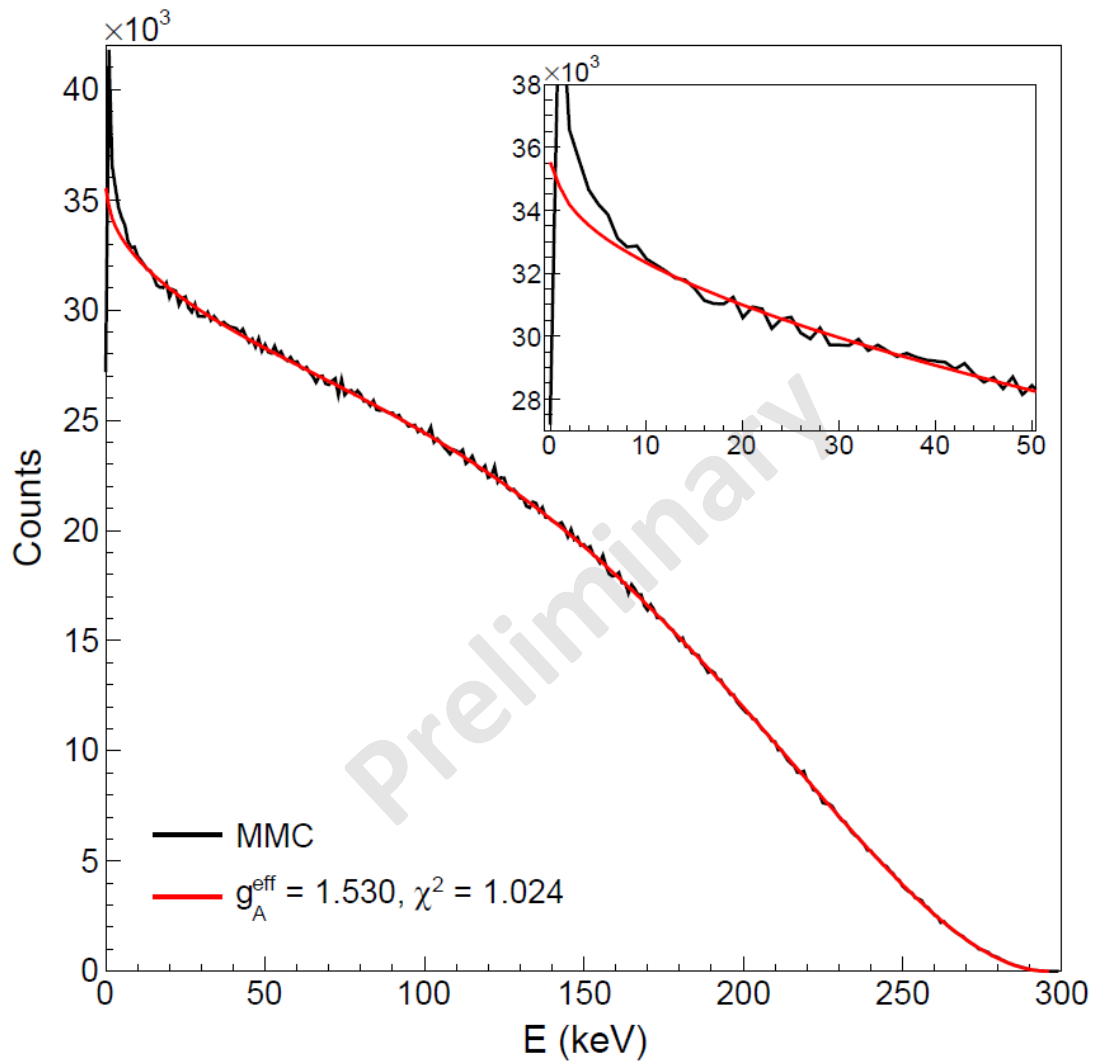
→ J. Suhonen proposed in a review [Front. Phys. 5, 55 (2017)] a formula that depends on a quenching factor in infinite nuclear matter and the free-nucleon value [PDG 2020]  $g_A^{\text{free}} = 1.2754(13)$ .

→ For  $^{99}\text{Tc}$ , one thus expects  $g_A^{\text{inf}} = 1.12$ .



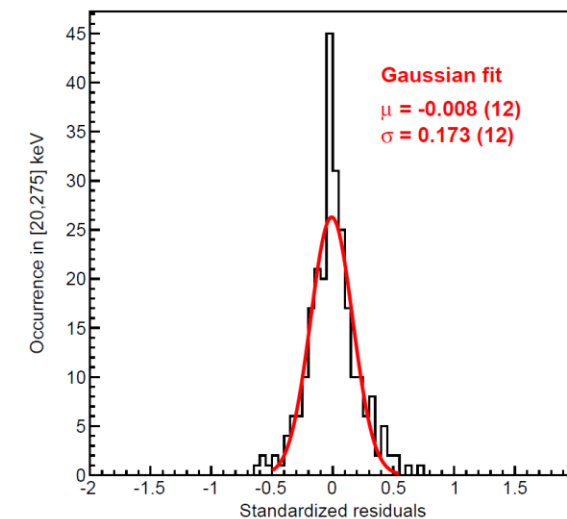
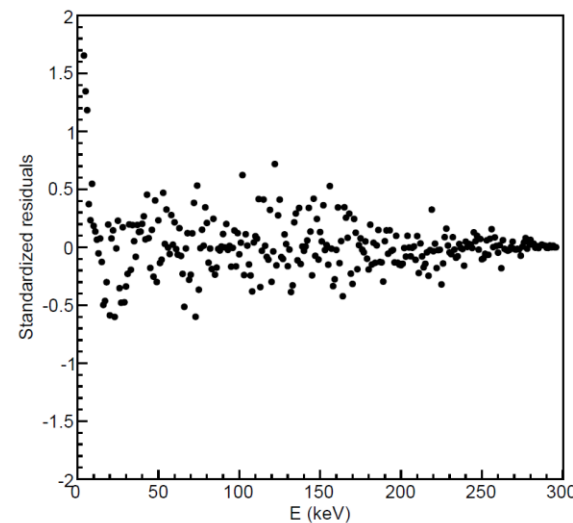
The value of  $g_A$  can be affected by several nuclear effects due to the necessary assumptions in nuclear models to deal with the many-body problem.

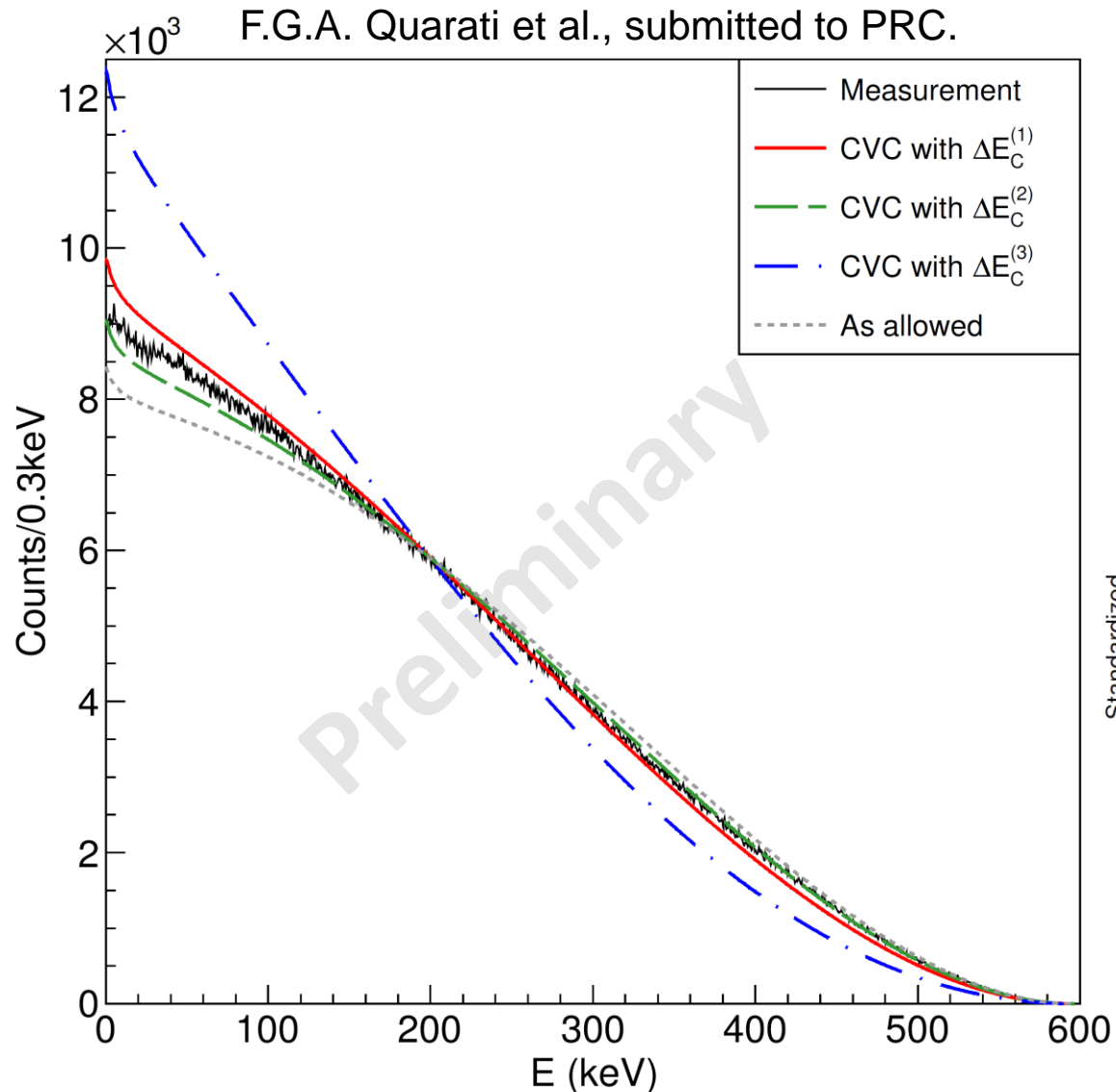
- ✓ High-precision measurements of  $^{99}\text{Tc}$  spectrum with MMC at LNHB and PTB, and with Silicon detectors at LNHB.
- ✓ Excellent agreement of all the three spectra.
- ✓ New Q-value = 295.82 (16) keV not consistent with AME2020 value of 297.5 (9) keV.
- ✓ Confirmation of the spectrum shape sensitivity to  $g_A$ .



The value of  $g_A$  can be affected by several nuclear effects due to the necessary assumptions in nuclear models to deal with the many-body problem.

- ✓ Best adjustment gives  $g_A^{\text{eff}} = 1.53$  (8)
- !! Result far from  $g_A^{\text{inf}} = 1.12$ , which cannot give an accurate spectrum shape in any of the considered assumptions.
- ✓ Residuals without any trend down to 6 keV.



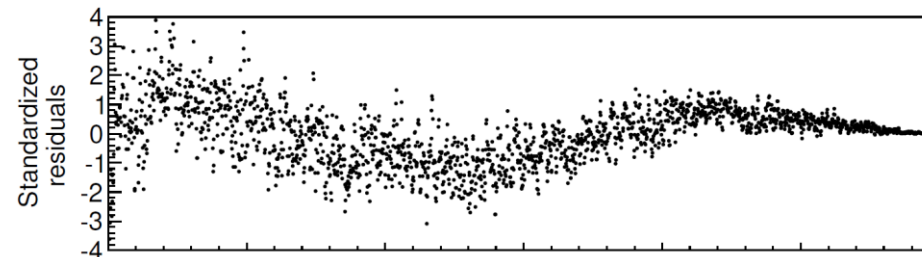


### First forbidden non-unique transition

- ✓ First high-precision spectrum measurement from self-scintillation of a LuAG:Pr crystal.
- Realistic shape only possible with CVC, found to be quite sensitive to the Coulomb displacement energy  $\Delta E_C$ .

Different methods to estimate  $\Delta E_C$ . Adjustments lead to similar residuals but very different  $g_A^{\text{eff}}$ .

→ **A non-linear trend remains.**



$\Delta E_C$ (MeV)	$g_A$
20.527(53)	$g_A^{\text{free}}$
$\Delta E_C^{(1)}$	1.057(5)
$\Delta E_C^{(2)}$	1.560(6)
$\Delta E_C^{(3)}$	0.834(4)

Nucleus deformation: hindered transition ( $\Delta K = 7$ )

!! Calculated half-life shorter by 13 orders of magnitude.

→ **Detailed analysis would require accurate modelling with nuclear deformation.**

Beta decays are of importance in many scientific areas: radionuclide metrology, nuclear medicine, nuclear reactors, neutrino physics, dark matter studies, etc.

**Our aim is to improve nuclear decay data** by providing precise measurements and theoretical predictions **for every possible beta transition** or electron capture.

- ✓ When no nuclear structure is required, the BetaShape code already improves the situation.
- ✓ We started to study of beta transitions that are sensitive to nuclear structure effects.

**We are seeking collaborations** about:

- High-precision measurements, which can be used as a probe of nuclear models.
- Extended nucleons → Induced current, weak magnetism.
- Treatment of the many-body problem → Effective axial-vector coupling constant.
- Relativistic nuclear models → CVC can be used to quantify their accuracy.
- Medium and heavy nuclei, nuclear deformation  
→ Ongoing collaboration with S. Péru (CEA/DAM) and M. Martini (IPSA) in the framework of the European metrology project PrimA-LTD.

Technical adaptations are necessary of both nuclear and beta decay modelling.



**Thank you for attention.**

