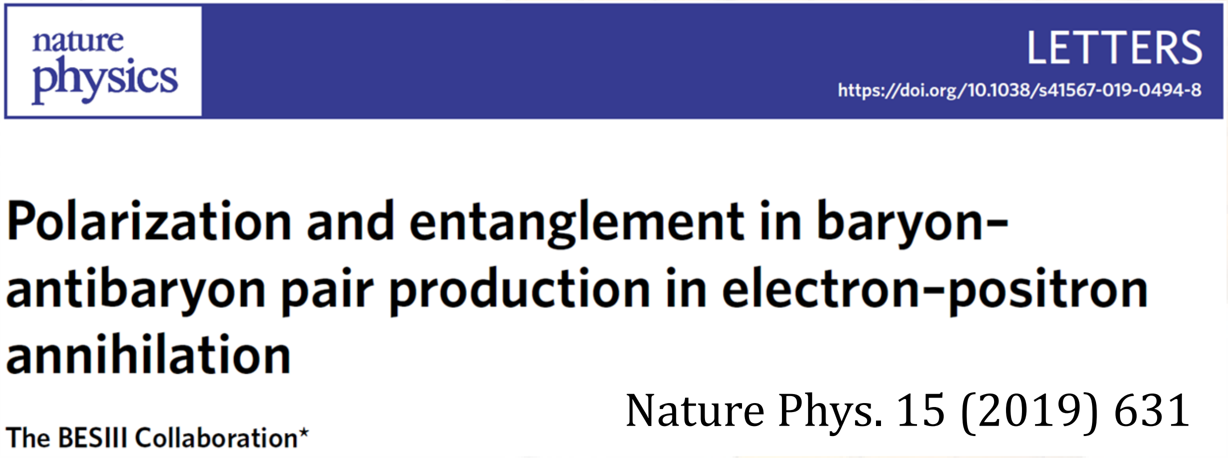
Measurement of α_- $in\ e^+e^- \to$ J/ψ→ $\Lambda\bar{\Lambda}$

Andrzej Kupsc

Uppsala Univ.

- ❑ Observation of Λ transverse polarization
- ❑ Determination of decay asymmetries



nature physics

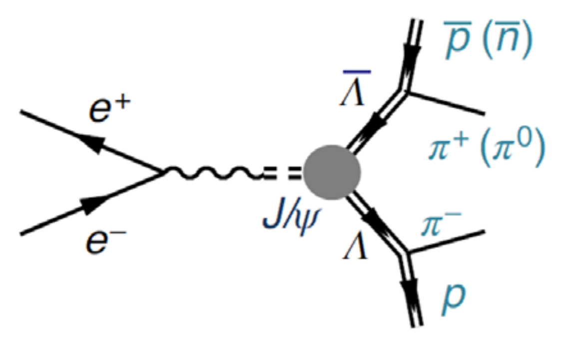
LETTERS

https://doi.org/10.1038/s41567-019-0494-8

Polarization and entanglement in baryon-antibaryon pair production in electron-positron annihilation

The BESIII Collaboration*

Nature Phys. 15 (2019) 631

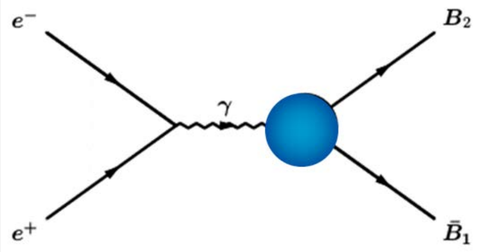


UPPSALA
UNIVERSITET

Methods(UU):
1. G.Fäldt, AK PLB772 (2017) 16
2. E.Perotti,G.Fäldt,AK,S.Leupold,JJ.Song PRD99 (2019)056008

b-baryon fest (zoom) 5-6 November 2020

Outline:



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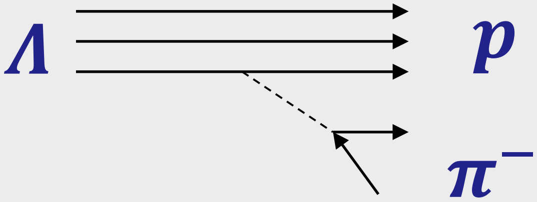

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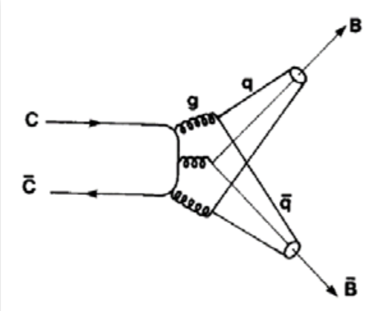
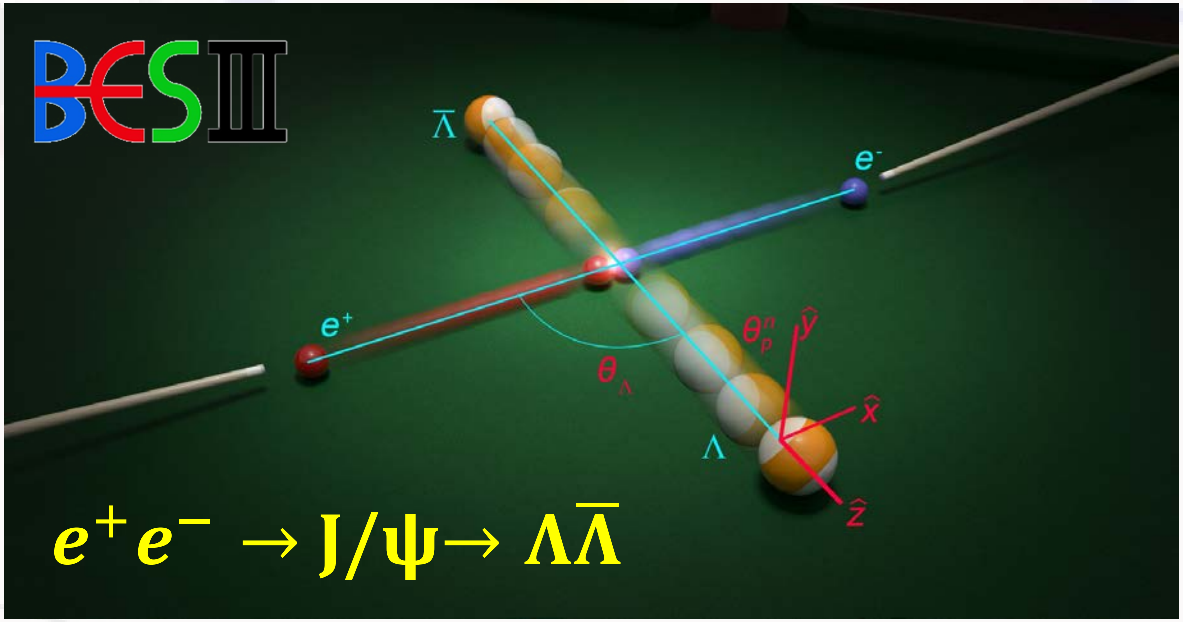


$e^+e^- \to f\bar{f}, \mathrm{B}_1\bar{\mathrm{B}}_2$ $\Lambda \to p\pi^-$ $\mathrm{J}/\psi \to \mathrm{B}_1\bar{\mathrm{B}}_2$



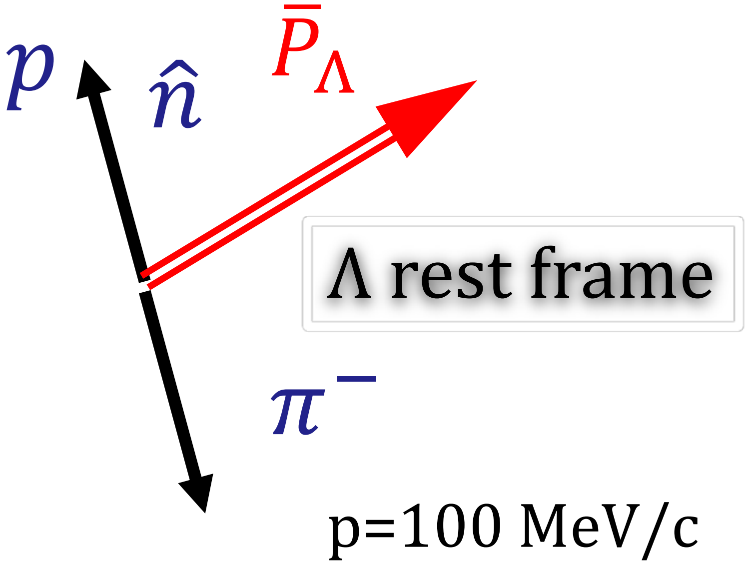
$e^+e^- \to \mathrm{J}/\psi \to \Lambda\bar{\Lambda}$

CP tests

$e^+e^- \to \mathrm{J}/\psi \to \Xi\bar{\Xi}$

Picture:Piotr Kupsc

Weak decay $\Lambda \to p\pi^-$



$$\boxed{\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi}(1 + \alpha_- \hat{n}\,\bar{P}_\Lambda)}$$

$\alpha_-=0.642\pm0.013$

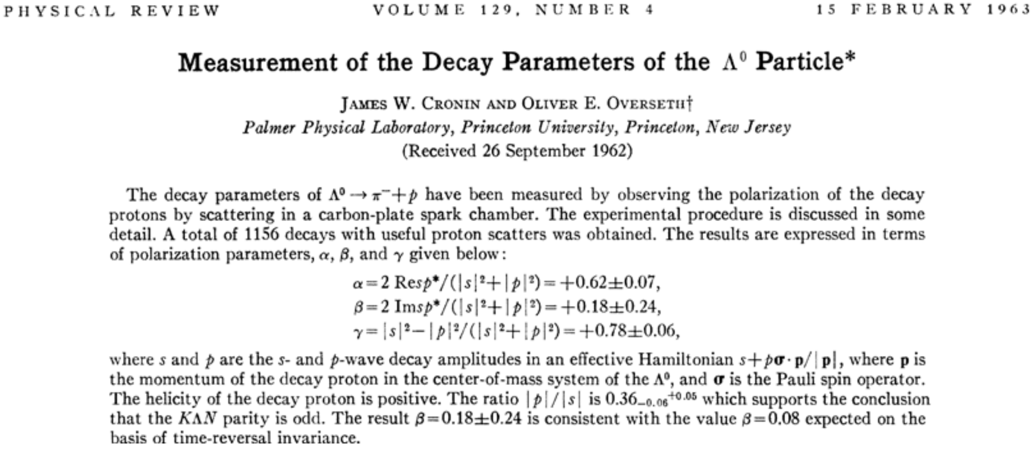
Value in PDG≤2018 established in 1978 based on 1963-75 experiments

**It was used/assumed in all experiments where Λ polarization is measured.
Also decay parameters of all baryons decays into final states with Λ: $\Xi \to \Lambda\pi$, $\Omega \to \Lambda K$, …**

Measuring α, β, γ in the 20th century

James Cronin
1931-2016

Oliver Overseth
1928-2008



PHYSICAL REVIEW VOLUME 129, NUMBER 4 15 FEBRUARY 1963

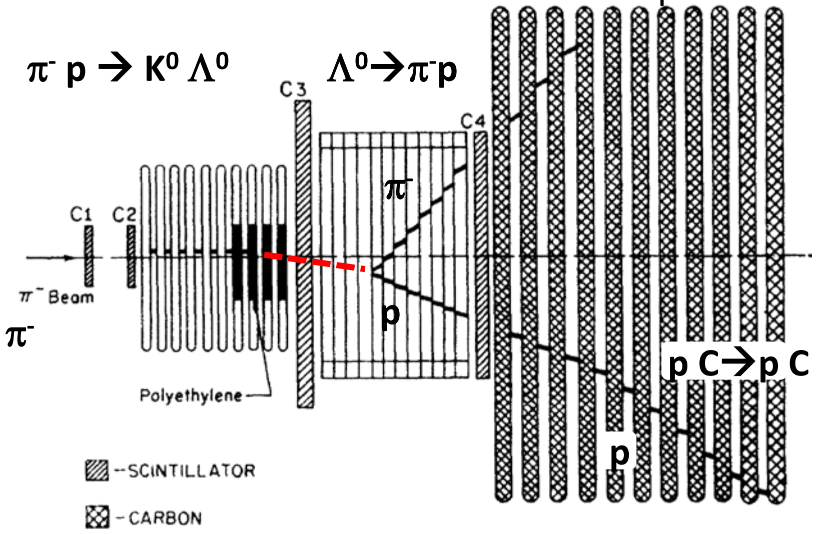
Measurement of the Decay Parameters of the Λ^0 Particle*

James W. Cronin and Oliver E. Overseth†
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 26 September 1962)

The decay parameters of $\Lambda^0 \to \pi^- + p$ have been measured by observing the polarization of the decay protons by scattering in a carbon-plate spark chamber. The experimental procedure is discussed in some detail. A total of 1156 decays with useful proton scatters was obtained. The results are expressed in terms of polarization parameters, α, β, and γ given below:

$$\alpha = 2\,\mathrm{Re}\,sp^*/(|s|^2+|p|^2) = +0.62\pm0.07,$$
$$\beta = 2\,\mathrm{Im}\,sp^*/(|s|^2+|p|^2) = +0.18\pm0.24,$$
$$\gamma = |s|^2-|p|^2/(|s|^2+|p|^2) = +0.78\pm0.06,$$

where s and p are the s- and p-wave decay amplitudes in an effective Hamiltonian $s+p\boldsymbol{\sigma}\cdot\mathbf{p}/|\mathbf{p}|$, where $\mathbf{p}$ is the momentum of the decay proton in the center-of-mass system of the Λ^0, and $\boldsymbol{\sigma}$ is the Pauli spin operator. The helicity of the decay proton is positive. The ratio $|p|/|s|$ is $0.36_{-0.06}^{+0.05}$ which supports the conclusion that the $K\Lambda N$ parity is odd. The result $\beta = 0.18\pm0.24$ is consistent with the value $\beta = 0.08$ expected on the basis of time-reversal invariance.



1.25 cm-thick carbon plates

$\pi^- p \to K^0 \Lambda^0$

$\Lambda^0 \to \pi^- p$

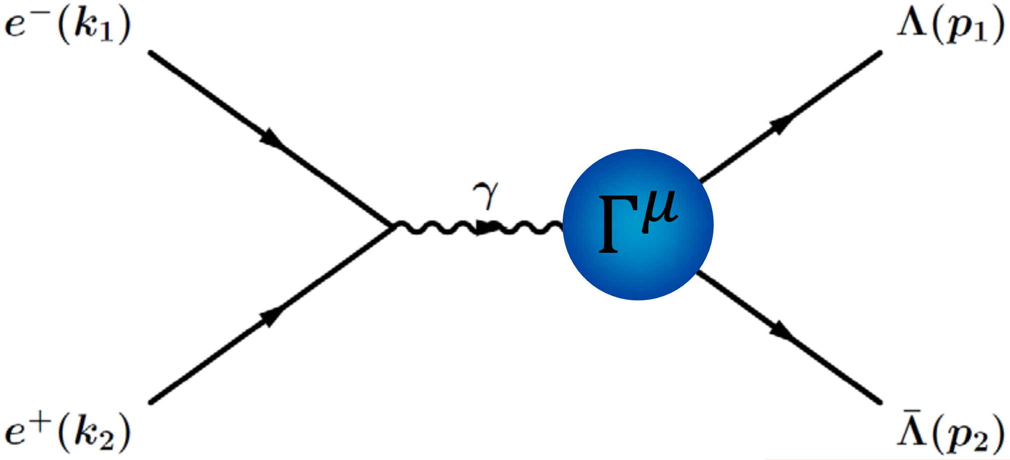
p C→p C

no H_2 target, no magnet; use kinematics and proton's range in carbon to infer E_p

$$P_p = \frac{(\alpha + P_\Lambda \cos\theta)\dot{z}' + \beta P_\Lambda \dot{x}' + \gamma P_\Lambda \dot{y}'}{1+\alpha P_\Lambda \cos\theta}$$

Slide from Steve Olsen

$e^+e^- \to \gamma^* \to B\overline{B}$ (spin 1/2)



$$s = (p_1 + p_2)^2$$
$$q = p_1 - p_2$$

$$\Gamma^\mu(p_1, p_2) = -ie\left[\gamma^\mu F_1(s) + i\frac{\sigma^{\mu\nu}}{2M_B}q_\nu F_2(s)\right]$$

F_1 (Dirac) and F_2 (Pauli) Form Factors

Sachs Form Factors (FFs) ⇔ helicity amplitudes:

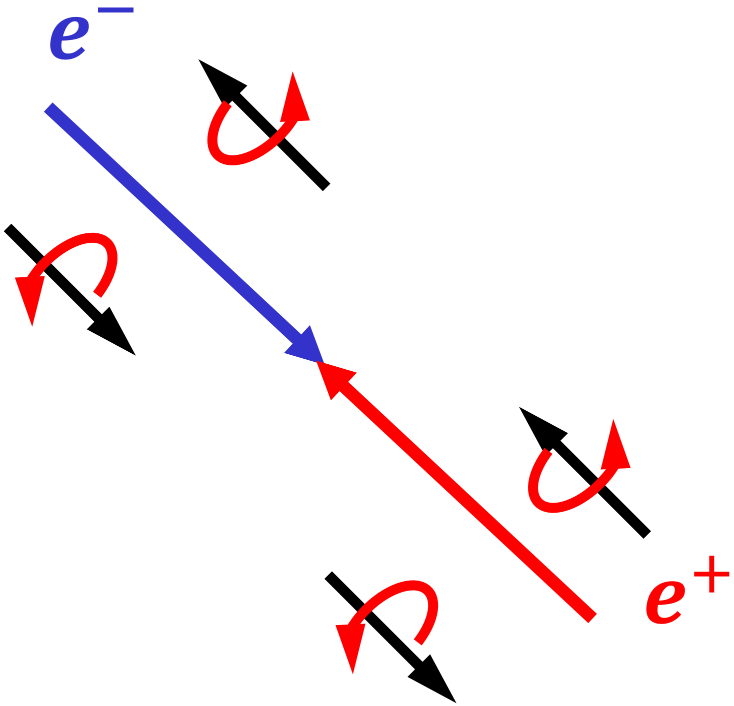
$$G_M(s) = F_1(s) + F_2(s), \quad G_E(s) = F_1(s) + \tau F_2(s)$$

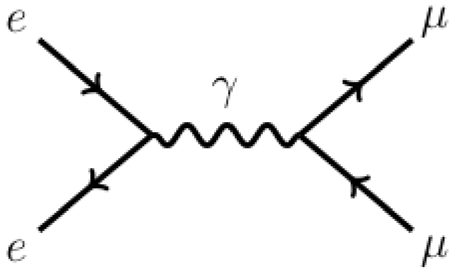
helicity non-flip helicity flip

$$\tau = \frac{s}{4M_B^2}$$

$e^+e^- \to \mu^+\mu^-$

At high energies annihilating e+e- have opposite helicities.





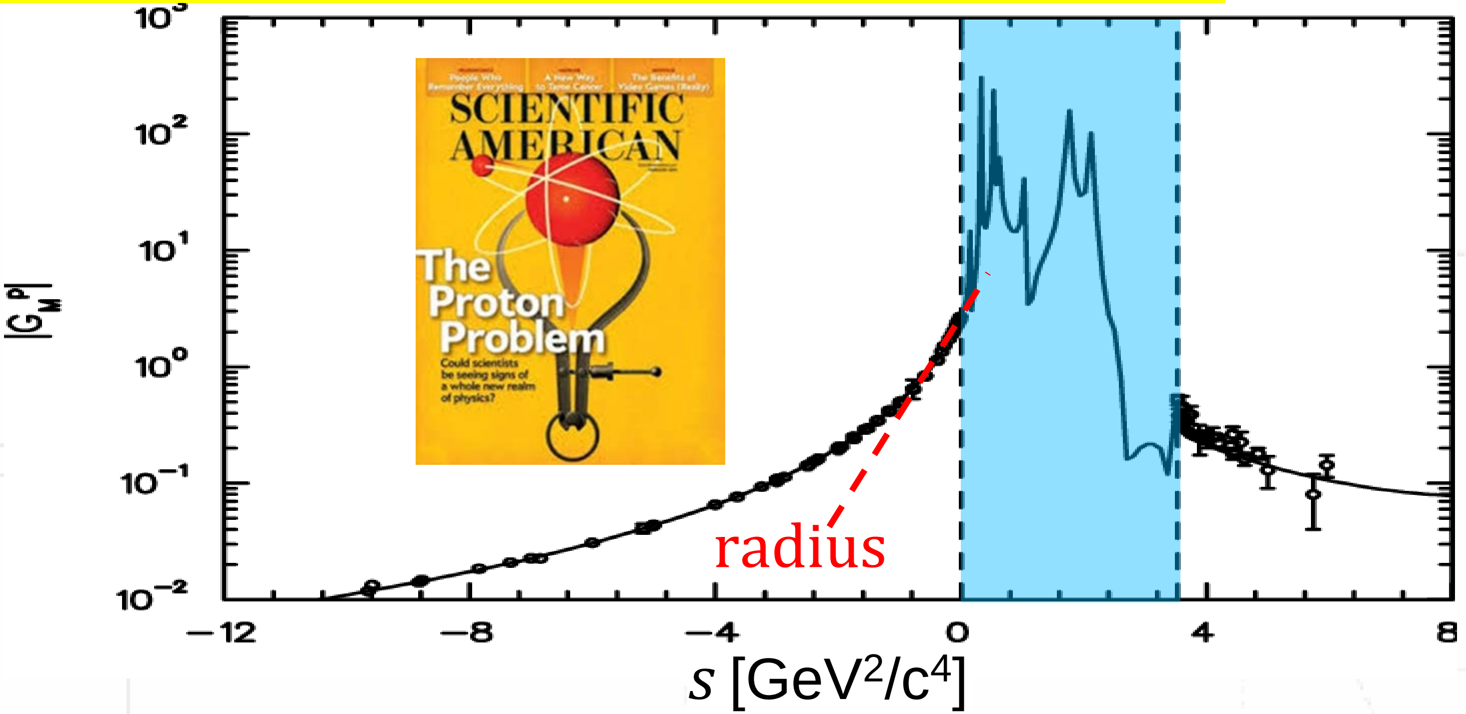
$$F_1(0) = 1, \quad F_2(0) = a_\mu$$

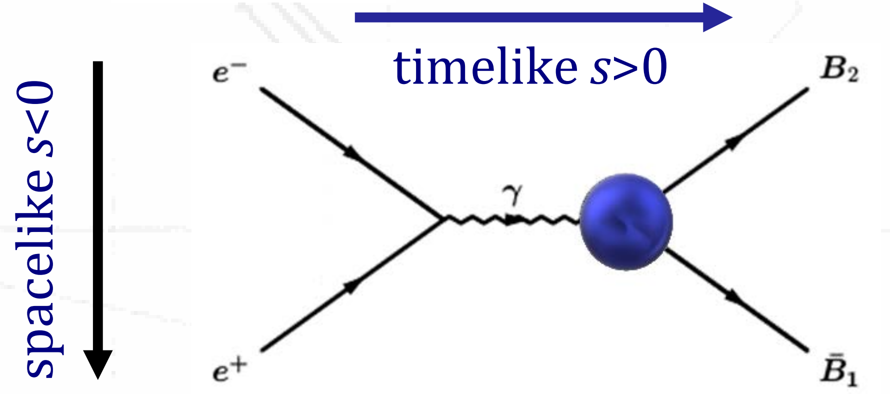
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2\theta)$$

γ^* has ± 1 helicity

$$\rho_1(\theta) = \begin{pmatrix} \frac{1+\cos^2\theta}{2} & -\frac{\cos\theta\sin\theta}{\sqrt{2}} & \frac{\sin^2\theta}{2} \\ -\frac{\cos\theta\sin\theta}{\sqrt{2}} & \sin^2\theta & \frac{\cos\theta\sin\theta}{\sqrt{2}} \\ \frac{\sin^2\theta}{2} & \frac{\cos\theta\sin\theta}{\sqrt{2}} & \frac{1+\cos^2\theta}{2} \end{pmatrix}$$

Baryon Electromagnetic Form Factors





$$\gamma^* \to B_1\bar{B}_2$$

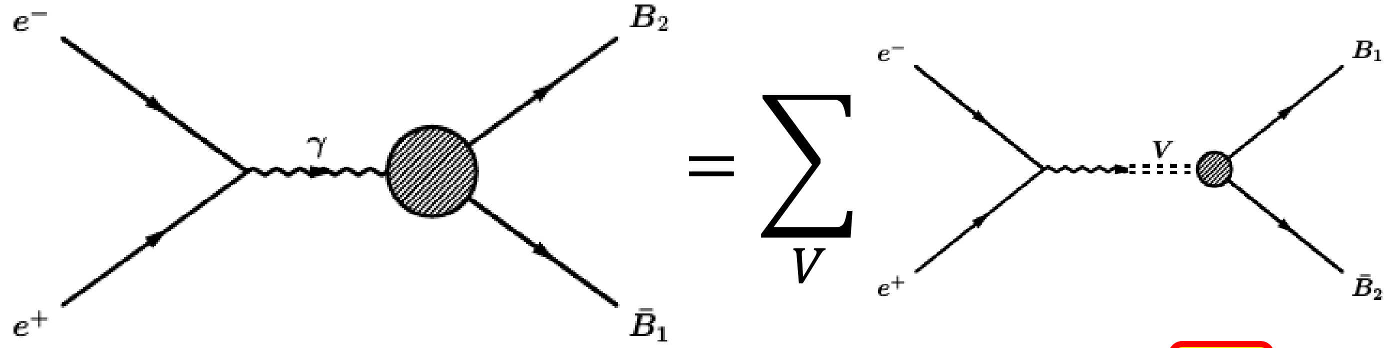
$$B_1 \to B_2 e{+}e{-}$$

$$p\bar{p} \to e{+}e{-}$$

elastic FFs: B_2=B_1
(B_2≠B_1 transition)

FFs are complex for $s > 4m_\pi^2$

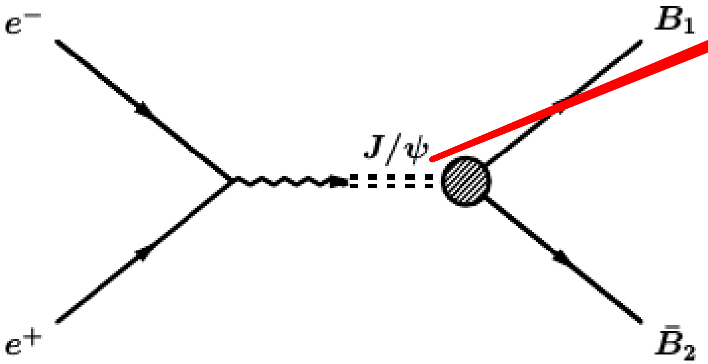
Baryon FFs (continuum):



Baryons B_1 and $\bar{B}_2$
(spin 1/2, ...)

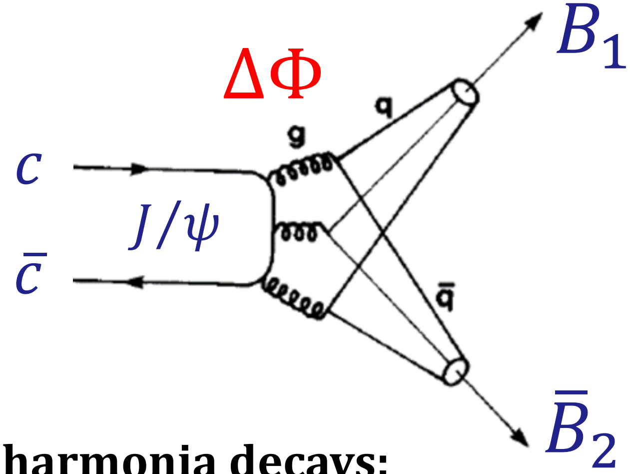
$V = \gamma, \rho, \omega, \varphi, ..., J/\psi, \psi(2S), ...$

vs J/ψ decay:



Both processes described by two complex FFs: relative phase $\Delta\Phi$

Cabibbo, Gatto PR124 (1961)1577



Time like spin ½ baryon FFs:
Dubnickova, Dubnicka, Rekalo
Nuovo Cim. A109 (1996) 241
Gakh, Tomasi-Gustafsson Nucl.Phys. A771 (2006) 169
Czyz, Grzelinska, Kuhn PRD75 (2007) 074026
Fäldt EPJ A51 (2015) 74; EPJ A52 (2016)141

Charmonia decays:
Fäldt, Kupsc PLB772 (2017) 16

$e^+e^- \rightarrow \gamma^* \rightarrow B\overline{B}$

For spin ½ $B\overline{B}$ production two complex FFs: $G_M(s)$, $G_E(s)$

⇨ process described by three parameters at fixed $\sqrt{s}$:

- ☐ cross section (σ)
- ☐ FFs ratio R or angular distribution parameter α_ψ
- ☐ relative phase between FFs (ΔΦ)

$$R = \left|\frac{G_E}{G_M}\right| \quad \left(\alpha_\psi = \frac{\tau - R^2}{\tau + R^2}\right) \qquad G_E = R G_M e^{i\Delta\Phi}$$

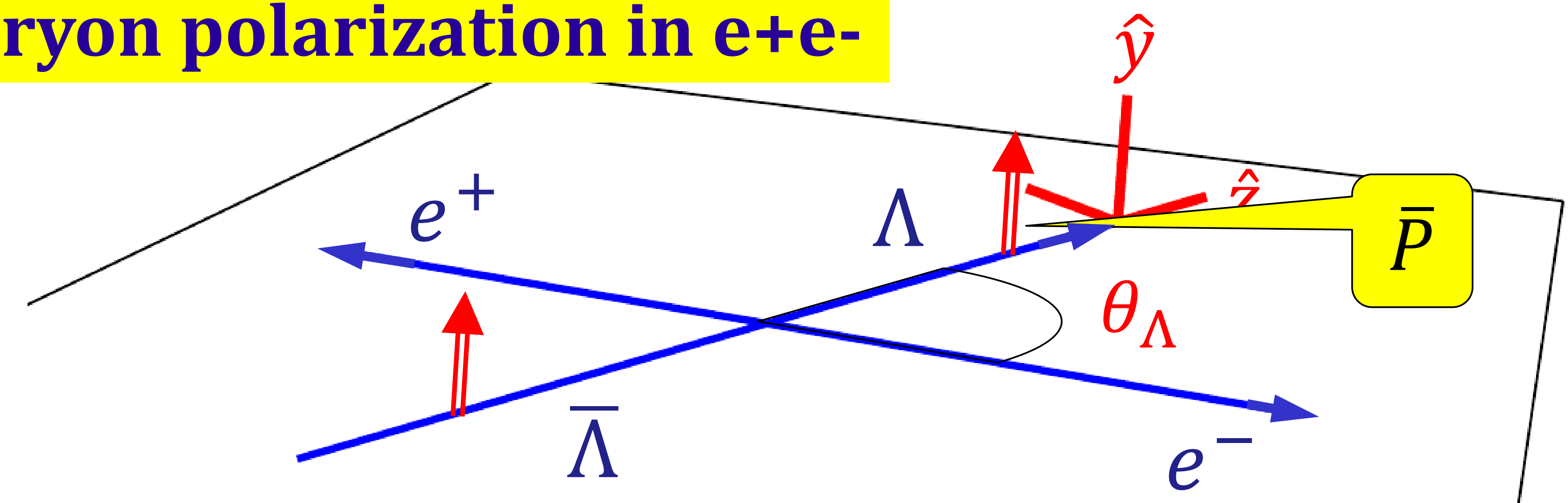
$$\tau = \frac{s}{4M_B^2}$$

Angular distribution:

$$\boxed{\frac{d\Gamma}{d\Omega} \propto 1 + \alpha_\psi \cos^2\theta \qquad -1 \leq \alpha_\psi \leq 1}$$

Phase ***ΔΦ*** expected/predicted for continuum but neglected/not expected for the decays

Baryon polarization in e+e-

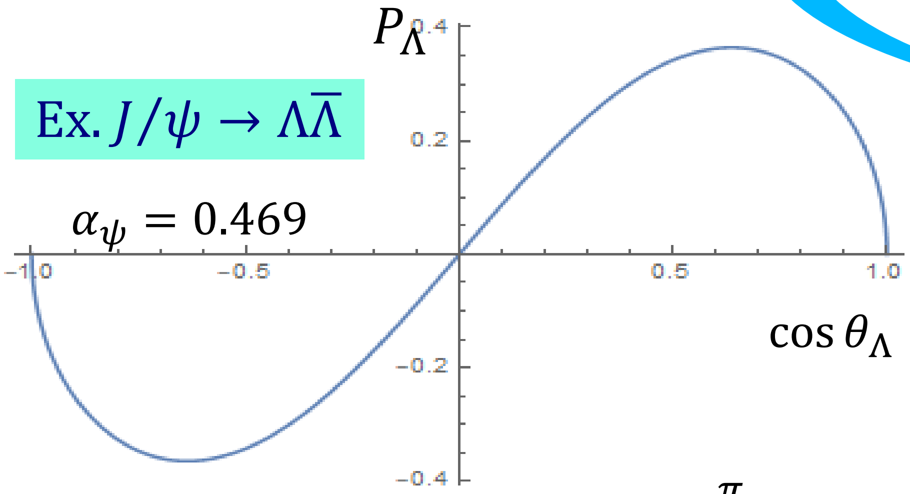


Unpolarized e+e- beams ⇨ transverse polarization:

$$P_y(\cos\theta_\Lambda) = \frac{\sqrt{1-\alpha_\psi{}^2}\cos\theta_\Lambda\sin\theta_\Lambda}{1+\alpha_\psi\cos^2\theta_\Lambda}\sin(\Delta\Phi)$$

$\Delta\Phi \neq 0$

Ex. $J/\psi \to \Lambda\bar{\Lambda}$



$\alpha_\psi = 0.469$

Max P_Λ = 36.4%

if $\Delta\Phi = \frac{\pi}{2}$

Baryon-antibaryon spin density matrix

$$e^+e^- \rightarrow B_1\bar{B}_2$$

General two spin ½ particle state:

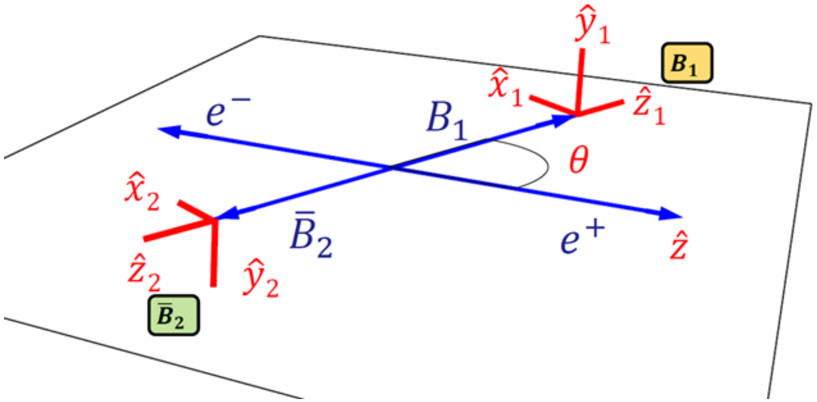
$$\rho_{1/2,\overline{1/2}} = \frac{1}{4}\sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}}\, \sigma_\mu^{B_1} \otimes \sigma_{\bar{\nu}}^{\bar{B}_2}$$

$(\sigma_0 = \mathbf{1}_2, \sigma_1 = \sigma_x, \sigma_2 = \sigma_y, \sigma_3 = \sigma_z)$

P_y

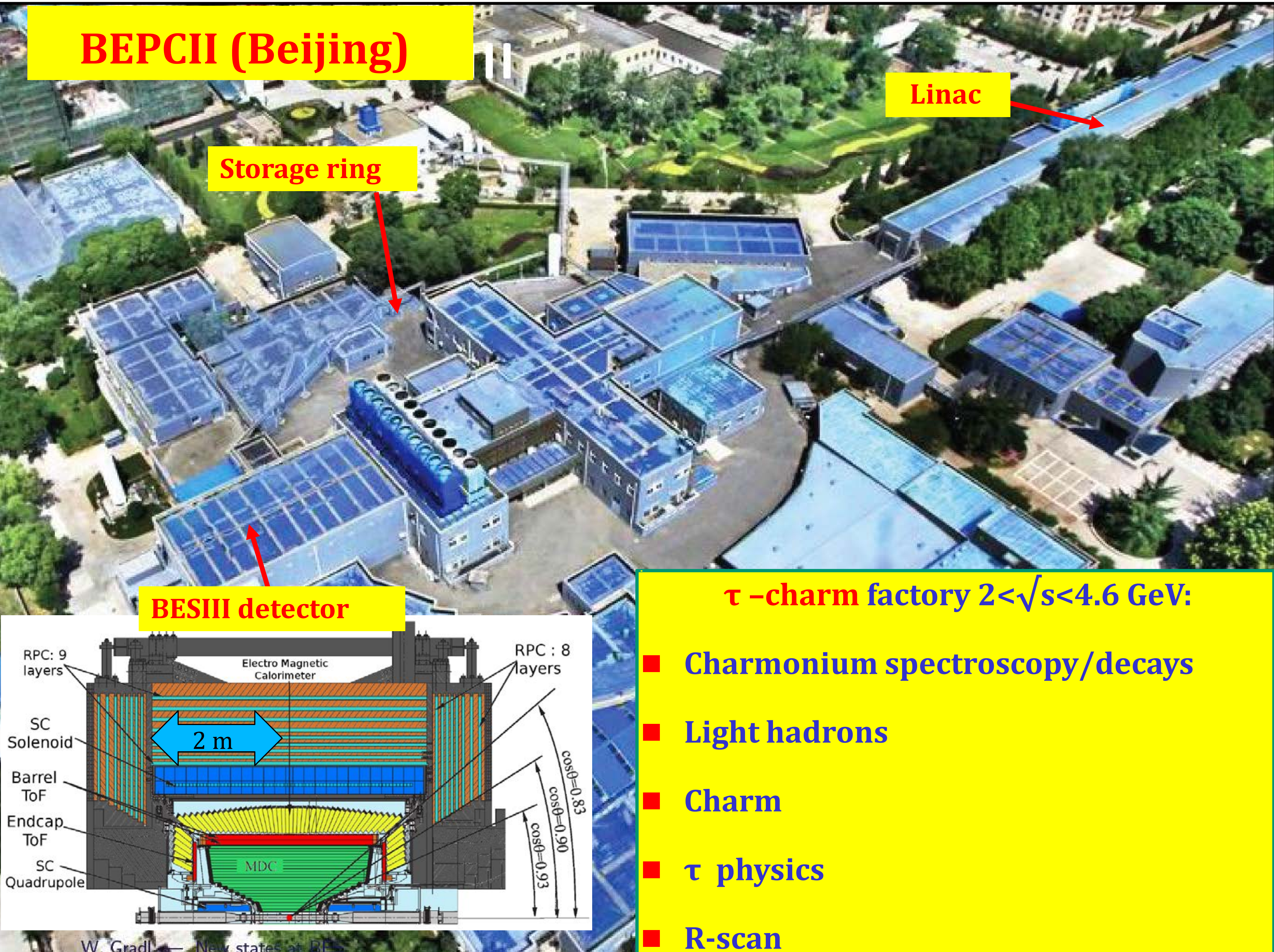
$$C_{\mu\bar{\nu}} = \begin{pmatrix} 1+\alpha_\psi\cos^2\theta & 0 & \boxed{\beta_\psi\sin\theta\cos\theta} & 0 \\ 0 & \sin^2\theta & 0 & \gamma_\psi\sin\theta\cos\theta \\ -\beta_\psi\sin\theta\cos\theta & 0 & \alpha_\psi\sin^2\theta & 0 \\ 0 & -\gamma_\psi\sin\theta\cos\theta & 0 & -\alpha_\psi-\cos^2\theta \end{pmatrix}$$

$$\beta_\psi = \sqrt{1-\alpha_\psi^2}\,\sin(\Delta\Phi) \qquad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\,\cos(\Delta\Phi)$$



E.Perotti, G.Faldt, AK, S.Leupold,JJ.Song PRD99 (2019)056008

BEPCII (Beijing)

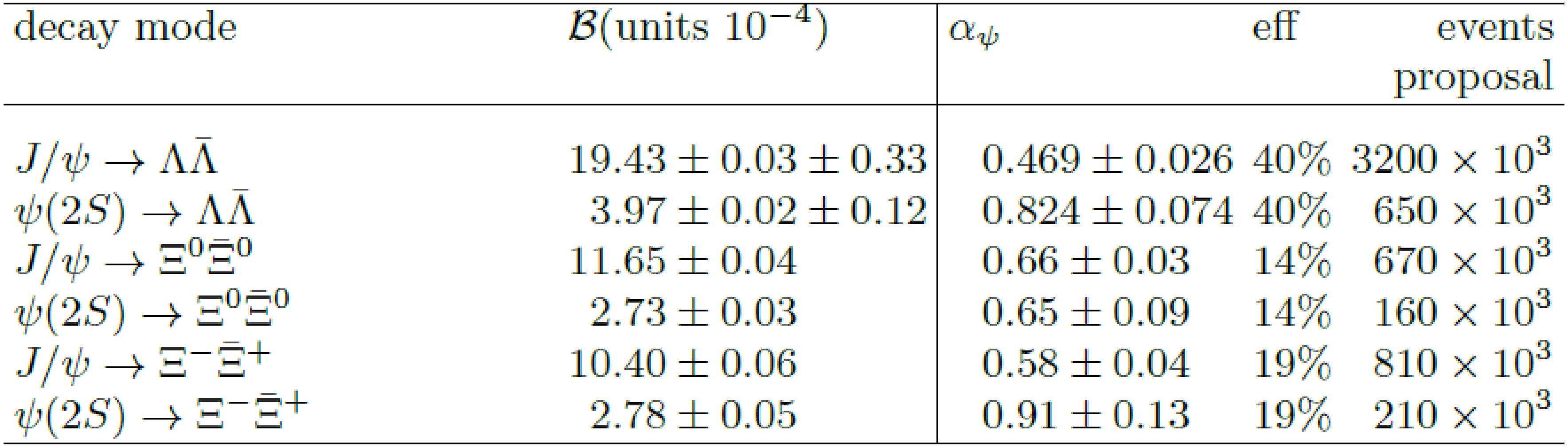


τ –charm factory 2<√s<4.6 GeV:

- Charmonium spectroscopy/decays
- Light hadrons
- Charm
- τ physics
- R-scan

$J/\psi, \psi' \to B\bar{B}$

$$\mathcal{B}(J/\psi \to p\bar{p}) = (21.21 \pm 0.29) \times 10^{-4}$$



decay mode	$\mathcal{B}$(units 10^{-4})	α_ψ	eff	events proposal
$J/\psi \to \Lambda\bar{\Lambda}$	$19.43 \pm 0.03 \pm 0.33$	0.469 ± 0.026	40%	3200×10^3
$\psi(2S) \to \Lambda\bar{\Lambda}$	$3.97 \pm 0.02 \pm 0.12$	0.824 ± 0.074	40%	650×10^3
$J/\psi \to \Xi^0\bar{\Xi}^0$	11.65 ± 0.04	0.66 ± 0.03	14%	670×10^3
$\psi(2S) \to \Xi^0\bar{\Xi}^0$	2.73 ± 0.03	0.65 ± 0.09	14%	160×10^3
$J/\psi \to \Xi^-\bar{\Xi}^+$	10.40 ± 0.06	0.58 ± 0.04	19%	810×10^3
$\psi(2S) \to \Xi^-\bar{\Xi}^+$	2.78 ± 0.05	0.91 ± 0.13	19%	210×10^3

(Feb 2019): 10^{10} J/ψ

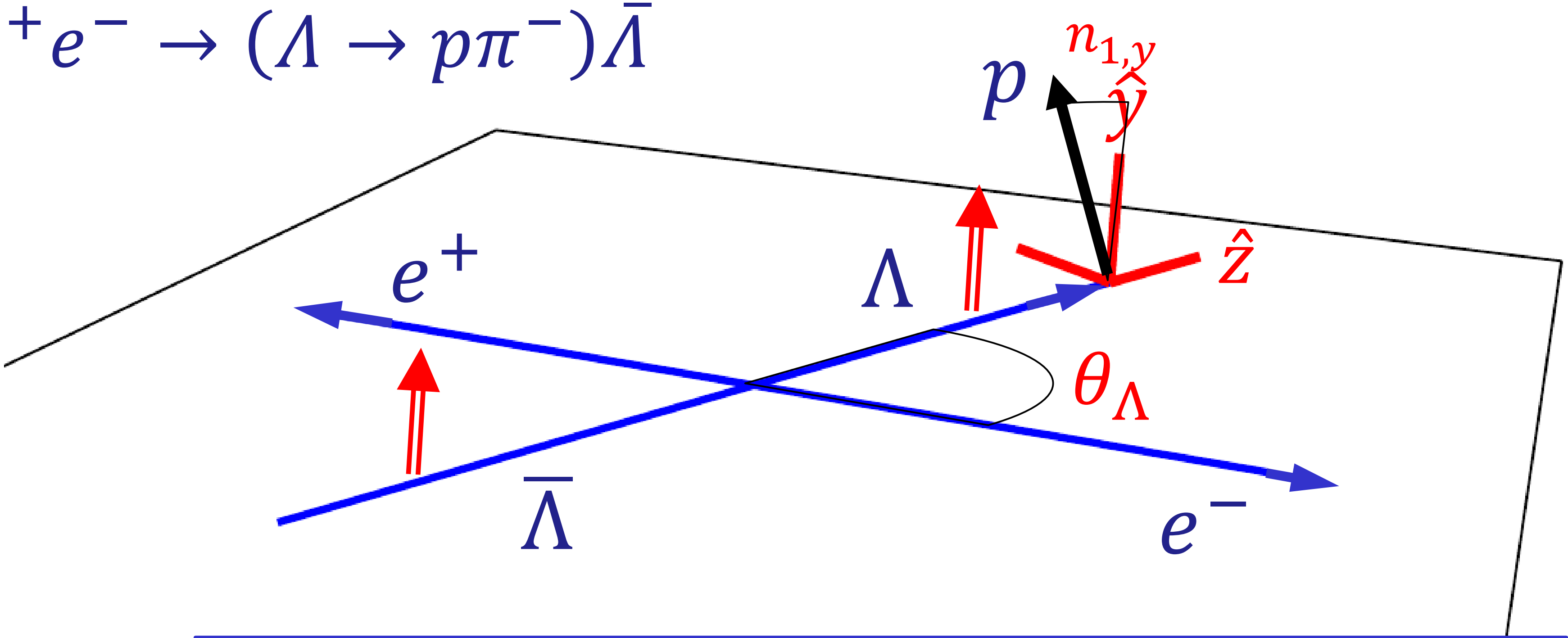
PRD 93, 072003 (2016)
PLB770,217 (2017)

PRD 95, 052003 (2017)

BESIII proposal:3.2x10^9 ψ(2S)

Inclusive decay angular distribution

$e^+e^- \to (\mathit{\Lambda} \to p\pi^-)\bar{\mathit{\Lambda}}$



$$\frac{d\Gamma}{d\cos\theta_\Lambda d\Omega_1} \propto (1 + \boldsymbol{\alpha_\psi} \cos^2\theta_\Lambda)\{1 + \boldsymbol{\alpha_-} P_y n_{1,y}\}$$

$\Lambda \to p\pi^-$: $\hat{\mathbf{n}}_1 \to \Omega_1 = (\cos\theta_1, \phi_1)$: α_-

⇒Determine product: $\alpha_- P_y \sim \alpha_- \sin(\boldsymbol{\Delta\Phi})$

Exclusive joint angular distribution (modular form)

$$e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$$

General two spin ½ particle state: $\rho_{1/2,\overline{1/2}} = \frac{1}{4}\sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}}\, \sigma_\mu^\Lambda \otimes \sigma_{\bar{\nu}}^{\bar{\Lambda}}$

$(\sigma_0 = \mathbf{1}_2, \sigma_1 = \sigma_x, \sigma_2 = \sigma_y, \sigma_3 = \sigma_z)$

$$C_{\mu\bar{\nu}} = \begin{pmatrix} 1+\alpha_\psi\cos^2\theta & 0 & \beta_\psi\sin\theta\cos\theta & 0 \\ 0 & \sin^2\theta & 0 & \gamma_\psi\sin\theta\cos\theta \\ -\beta_\psi\sin\theta\cos\theta & 0 & \alpha_\psi\sin^2\theta & 0 \\ 0 & -\gamma_\psi\sin\theta\cos\theta & 0 & -\alpha_\psi-\cos^2\theta \end{pmatrix}$$

$$\beta_\psi = \sqrt{1-\alpha_\psi^2}\,\sin(\Delta\Phi) \qquad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\,\cos(\Delta\Phi)$$

Apply decay matrices:

$$\sigma_\mu^\Lambda \to \sum_{\mu'=0}^{3} a_{\mu,\mu'}^\Lambda\, \sigma_{\mu'}^p$$

The angular distribution:

$$\boxed{W = Tr\rho_{p,\bar{p}} = \sum_{\mu,\bar{\nu}=0}^{3} C_{\mu\bar{\nu}} a_{\mu,0}^\Lambda a_{\bar{\nu},0}^{\bar{\Lambda}}}$$

E.Perotti, G.Faldt, AK, S.Leupold,JJ.Song PRD99 (2019)056008

Exclusive joint angular distribution

$$e^+e^- \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$$

$\Lambda \to p\pi^-$: $\hat{\mathbf{n}}_1 \to (\cos\theta_1, \phi_1)$: α_-

$\bar{\Lambda} \to \bar{p}\pi^+$: $\hat{\mathbf{n}}_2 \to (\cos\theta_2, \phi_2)$: α_+

$\xi : (\cos\theta_\Lambda, \hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2)$ 5D PhSp

$$d\Gamma \propto W(\xi; \alpha_\psi, \Delta\Phi, \alpha_-, \alpha_+) =$$

$$1 + \alpha_\psi \cos^2\theta_\Lambda$$

Cross section

$$+\alpha_-\alpha_+\{\sin^2\theta_\Lambda(n_{1,x}n_{2,x} - \alpha_\psi n_{1,y}n_{2,y}) + (\cos^2\theta_\Lambda + \alpha_\psi)n_{1,z}n_{2,z}\}$$

$$+\alpha_-\alpha_+\sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)\sin\theta_\Lambda\cos\theta_\Lambda(n_{1,x}n_{2,z} + n_{1,z}n_{1,x})$$

Spin correlations

$$+\sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi)\sin\theta_\Lambda\cos\theta_\Lambda(\alpha_-n_{1,y} + \alpha_+n_{2,y})$$

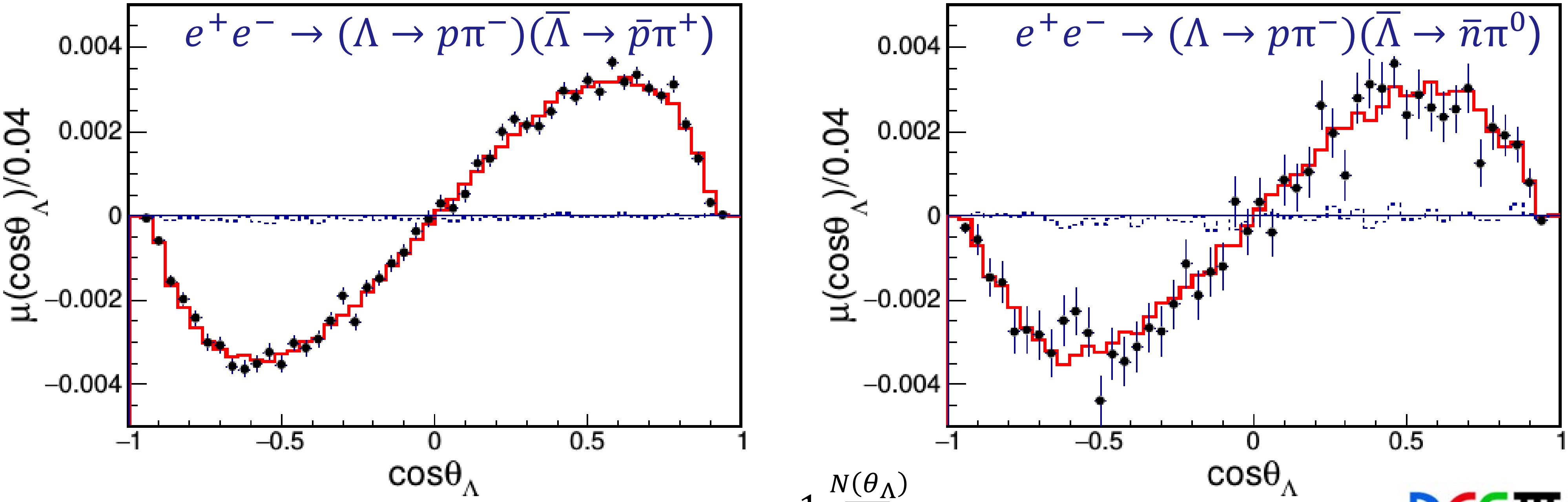
Polarization

$\Delta\Phi \neq 0$ $\Rightarrow$ independent determination of α_- and α_+

Fäldt, Kupsc PLB772 (2017) 16

Fit results

$\Delta\Phi = 42.3^{\circ} \pm 0.6^{\circ} \pm 0.5^{\circ}$



moment: $$\mu(\cos\theta_\Lambda) = \frac{1}{N}\sum_{i=1}^{N(\theta_\Lambda)}\left(n_{1,y}^{(i)} - n_{2,y}^{(i)}\right)$$

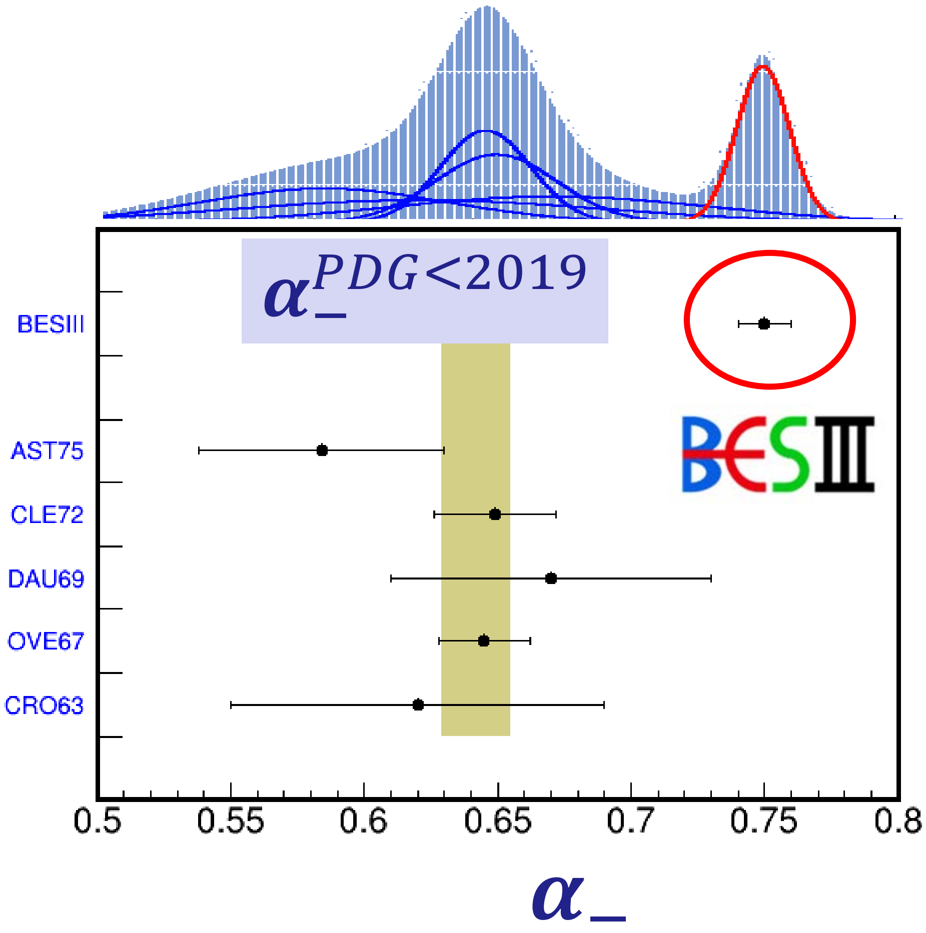
(uncorrected for acceptance)

Parameters	This work	Previous results
α_ψ	**0.461 ± 0.006 ± 0.007**	0.469 ± 0.027 BESIII
$\Delta\Phi$ (rad)	**0.740 ± 0.010 ± 0.008**	–
α_-	**0.750 ± 0.009 ± 0.004**	0.642 ± 0.013 PDG
α_+	**−0.758 ± 0.010 ± 0.007**	−0.71±0.08 PDG
$\bar{\alpha}_0$	**−0.692 ± 0.016 ± 0.006**	–

BESIII Nature Phys. (2019)

Implications of the BESIII result

$\Lambda \to p\pi^-$: α_=0.750±0.009±0.004



17(3)% larger

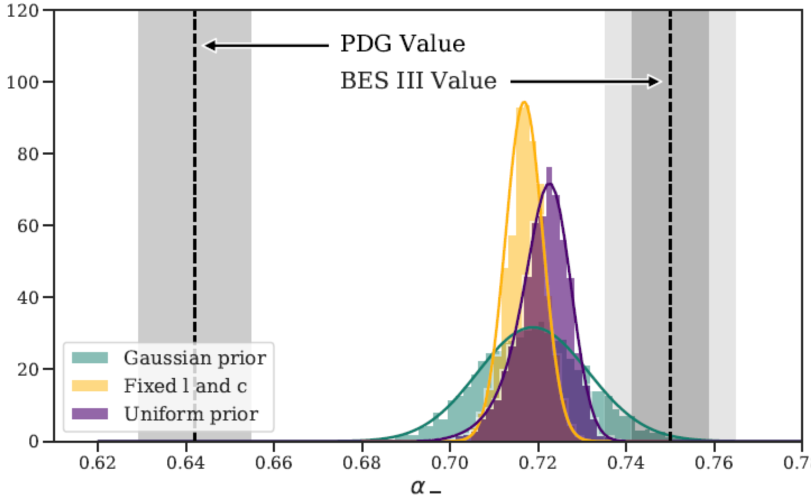
Reset of α_- value in PDG

Verification of the result?

$$\vec{\gamma}p \rightarrow K^{+}\Lambda$$

$$\alpha_{-} = 0.721(6)(5)$$

- D. Ireland et al *PRL* 123 (2019) 182301



$$\langle\alpha_{-}\rangle_{\text{BESIII}} = \frac{\alpha_{-}-\alpha_{+}}{2} = 0.754(3)(2)$$

Since $\rho(stat) = 0.82!$ and using quoted syst uncertainties for $\alpha_{-}, \alpha_{+}, A_{\Lambda}$ to deduce $\rho(syst) = 0.835$

ie 4% difference with 3.8 σ
new puzzle?...

Independent verification at BESIII eg:

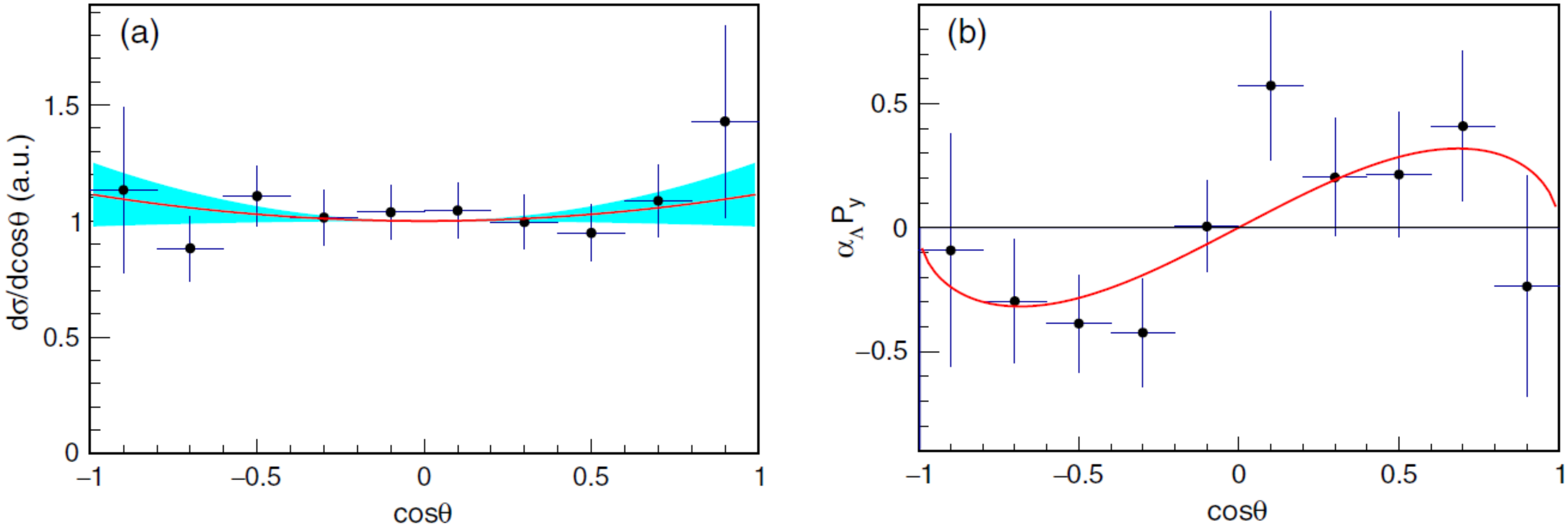
$$J/\psi \rightarrow \gamma\eta_c \rightarrow \gamma\Lambda\bar{\Lambda} \qquad \eta_c \rightarrow \Lambda\bar{\Lambda}$$

$$W = (1 - \alpha_{-}\alpha_{+}\cos\theta_{p\bar{p}})$$

$$e^{+}e^{-} \rightarrow J/\psi \rightarrow \Xi^{-}\bar{\Xi}^{+} \rightarrow \Lambda\pi^{-}\bar{\Lambda}\pi^{+} \rightarrow \mathrm{p}\pi^{-}\pi^{-}\bar{\mathrm{p}}\pi^{+}\pi^{+}$$

$e^+e^- \to \gamma^* \to \Lambda\bar{\Lambda}$ (continuum: 2.396 GeV)

PHYSICAL REVIEW LETTERS **123,** 122003 (2019)



555 events selected

("α_ψ"=0.13±0.16)

$\Delta\Phi = 37^o \pm 12^o \pm 6^o$

R = 0.94 ± 0.16(stat.) ± 0.03(sys.) ± 0.02(α_)

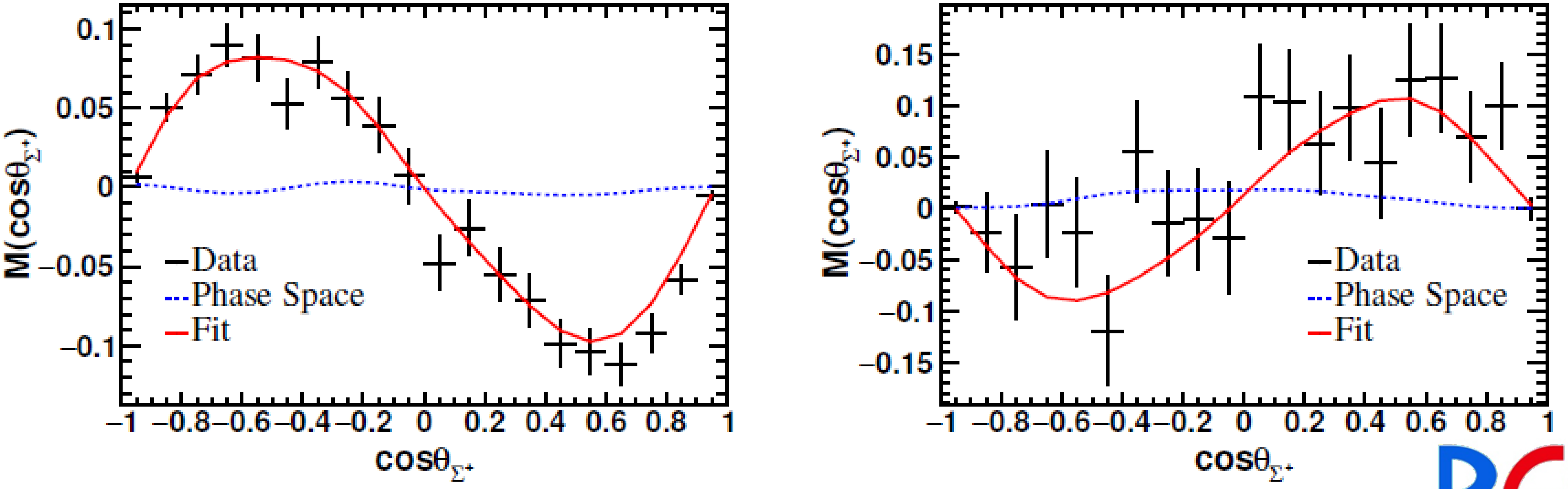
The same fit as for $J/\psi \to (\Lambda \to p\pi^-)(\bar{\Lambda} \to \bar{p}\pi^+)$ but $\alpha_- = \alpha_+$ and fixed

$e^+e^- \to J/\psi, \psi' \to \Sigma^+\bar{\Sigma}^- \to p\pi^-\bar{p}\pi^+$

The same formalism as for $J/\psi \to \Lambda\bar{\Lambda}$

$J/\psi, \to \Sigma^+\bar{\Sigma}^-$

$\psi' \to \Sigma^+\bar{\Sigma}^-$



$\alpha_{J/\psi}/\alpha_{\psi}$ = -0.507±0.006±0.002 / 0.676±0.030±0.006

$\Delta\Phi(J/\psi, \psi)$ = (-15.4±0.7±0.3)° / (21.5 ± 0.4 ± 0.5)°

α_0 = -0.999±0.037±0.010

$\bar{\alpha}_0$ = 0.992±0.037±0.008

A_{CP} = -0.015±0.037±0.008

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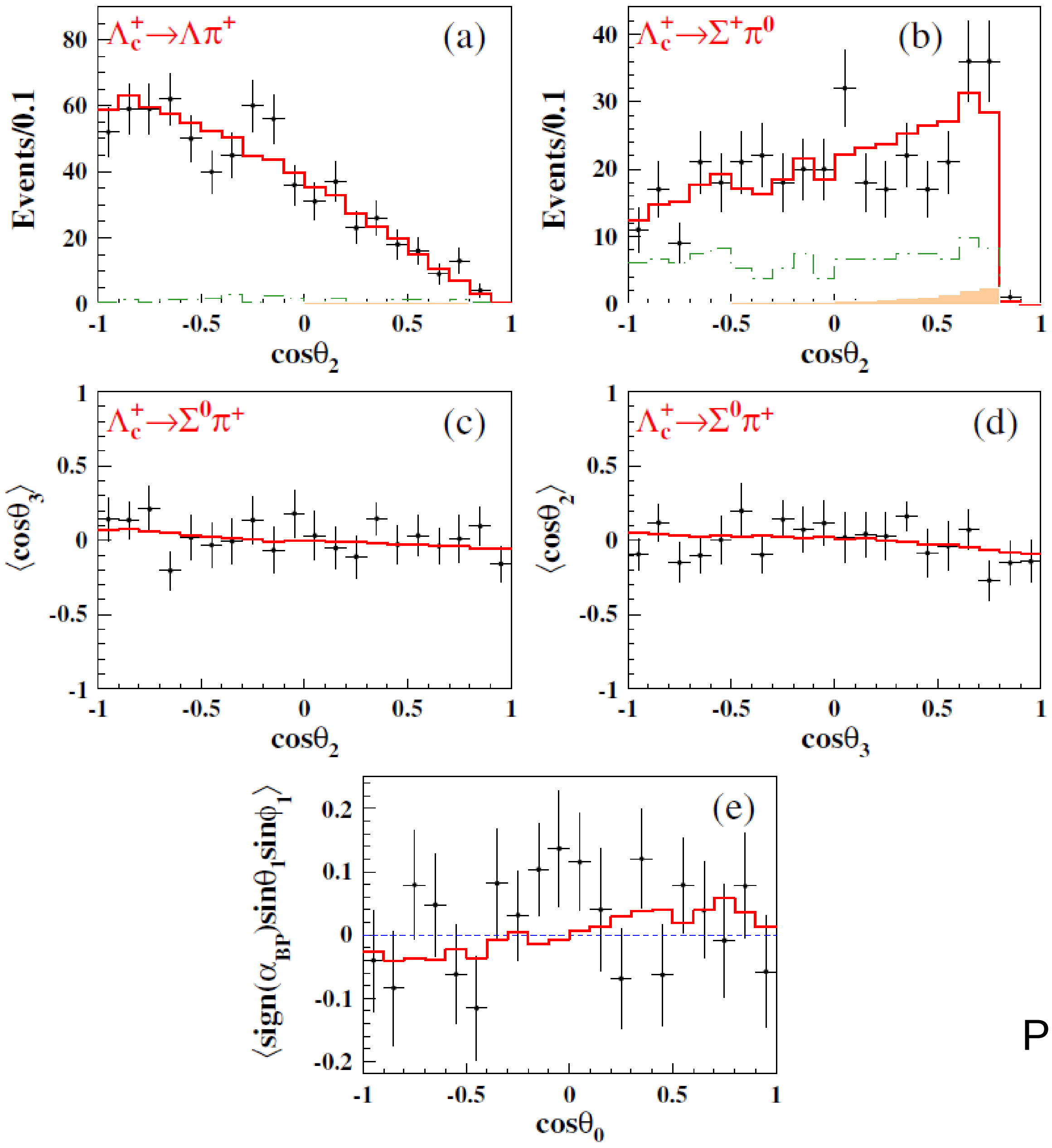
$e^+e^- \to (\Lambda_c^+ \to B\pi)(\overline{\Lambda}_c^- \to X)$



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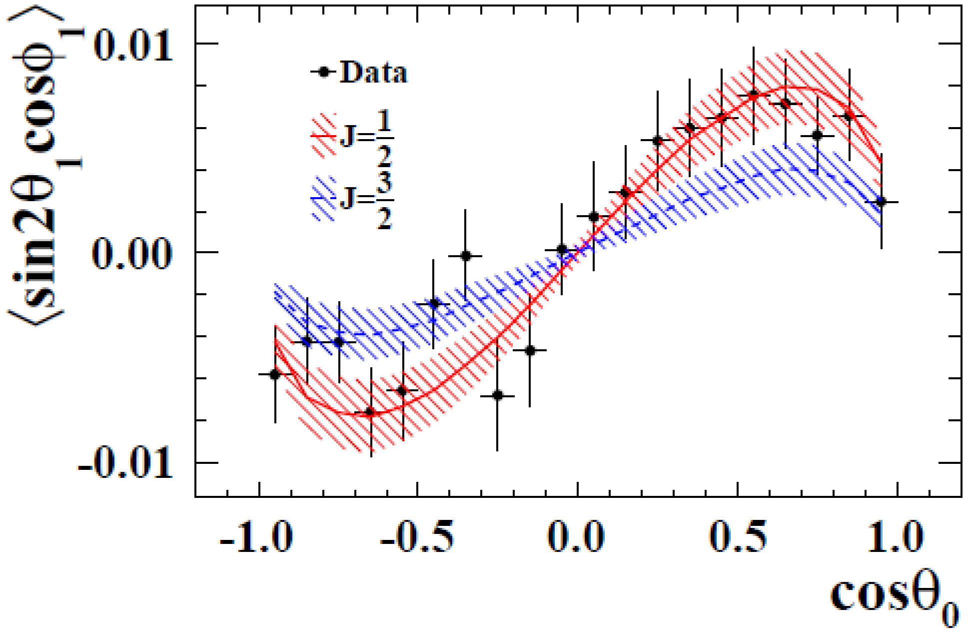
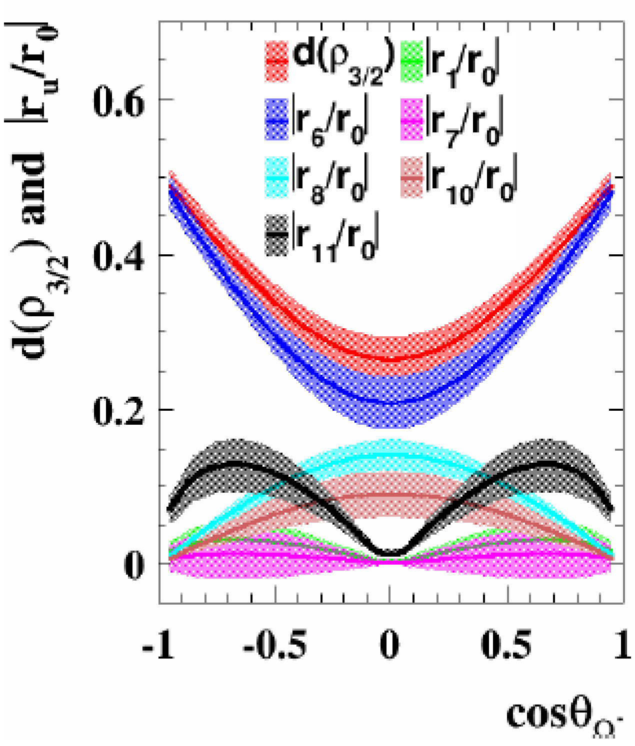
Λ_c^+ spin determination in $e^+e^- \to (\Lambda_c^+)(\bar{\Lambda}_c^-)$



BESIII arxiv:2011.00396

Helicity Amplitude Analysis of $\psi' \to \Omega^-\bar{\Omega}^+$

BESIII arxiv:2007.03679



Degree of polarization $d(\varrho_{3/2})$

dipole polarization

Conclusions:

$\boldsymbol{e^+e^- \rightarrow J/\psi, \psi' \rightarrow B\bar{B}}$ process

$\boldsymbol{B\bar{B}}$ polarized and spin correlated, process described by few global parameters (two for spin $^1/_2$, $^1/_2$)

Method to determine decay asymmetries of hyperons and antihyperons: revision of decay parameter for Λ

Potential to test CP-violation in hyperon decays

Thank you!

Spin density matrix for $e^+e^- \to \Omega^-\Omega^+$

$$\rho^{\lambda_1\lambda_2,\lambda_{1'}\lambda_{2'}}_{3/2,\overline{3/2}} = \sum_{\kappa=\pm1} D^{1*}_{\kappa,\lambda_1-\lambda_2}(0,\theta_\Omega,0)\,D^{1}_{\kappa,\lambda_{1'}-\lambda_{2'}}(0,\theta_\Omega,0)\,A_{\lambda_1\lambda_2}A^*_{\lambda_{1'}\lambda_{2'}}$$

$$A = \begin{pmatrix} \mathbf{h}_4 & \mathbf{h}_3 & 0 & 0 \\ \mathbf{h}_3 & \mathbf{h}_1 & \mathbf{h}_2 & 0 \\ 0 & \mathbf{h}_2 & \mathbf{h}_1 & \mathbf{h}_3 \\ 0 & 0 & \mathbf{h}_3 & \mathbf{h}_4 \end{pmatrix}$$

(Complex) Form Factors

$$\mathbf{h}_k \rightarrow h_k \exp(i\phi_k)$$

Using base 3/2 spin matrices Q:

M.G.Doncel, L.Michel, P.Minnaert Nucl. Phys. B38, 477(1972)

$$\rho_{3/2} = r_0\left(Q_0 + \frac{3}{4}\sum_{M=-1}^{1} \overbrace{r^1_M}^{r^1_{-1} \rightarrow P_y \;\; r^1_0 \rightarrow P_x \;\; r^1_1 \rightarrow P_z} Q^1_M + \frac{3}{4}\sum_{M=-2}^{2} r^2_M Q^2_M + \frac{3}{4}\sum_{M=-3}^{3} r^3_M Q^3_M\right)$$

$$\frac{3}{4}Q^L_M \rightarrow Q_\mu\,, \mu = 1, \ldots, 15$$

$$Q_0 = \frac{1}{4}I \qquad \rho_{3/2} = \sum_{\mu=0}^{15} r_\mu Q_\mu$$

$$\rho_{3/2,\overline{3/2}} = \sum_{\mu=0}^{15}\sum_{\bar\nu=0}^{15} C_{\mu,\bar\nu}\, Q_\mu \otimes Q_{\bar\nu}$$

Single tag $e^+e^- \rightarrow \Omega^-\overline{\Omega}^+$

Single 3/2-spin baryon density matrix is

$$\rho_{3/2} = \sum_{\mu=0}^{15} r_\mu Q_\mu = \sum_{\mu=0}^{15} C_{\mu,0} Q_\mu$$

Angular distribution (using decay matrices in helicity frames):

$$W = \sum_{\mu=0}^{15}\sum_{\kappa=0}^{3} C_{\mu,0} \boxed{b^{\Omega}_{\mu,\kappa}} \boxed{a^{\Lambda}_{\kappa,0}}$$

decay 1/2->1/2 0 (Λ->pπ)

decay 3/2->1/2 0 (Ω->ΛK)

$$r_0 = (1+\cos^2\theta_\Omega)(h_2^2 + 2h_3^2) + 2\sin^2\theta_\Omega(h_1^2 + h_4^2)$$

$$r_1 = 2\sin 2\theta_\Omega \frac{2\Im(\mathbf{h}_1\mathbf{h}_2^*) + \sqrt{3}\Im(\mathbf{h}_3^*(\mathbf{h}_1 + \mathbf{h}_4))}{\sqrt{30}}$$

$$r_6 = -\frac{2\sin^2\theta_\Omega(h_1^2 - h_4^2) + h_2^2(\cos^2\theta + 1)}{\sqrt{3}}$$

$$r_7 = \sqrt{2}\sin 2\theta_\Omega \frac{\Re(\mathbf{h}_3^*(\mathbf{h}_4 - \mathbf{h}_1))}{\sqrt{3}}$$

$$r_8 = 2\sin^2\theta_\Omega \frac{\Re(\mathbf{h}_3\mathbf{h}_2^*)}{\sqrt{3}}$$

$$r_{10} = 2\sin^2\theta_\Omega \frac{\Im(\mathbf{h}_3\mathbf{h}_2^*)}{\sqrt{3}}$$

$$r_{11} = 2\sin 2\theta_\Omega \frac{\Im(\sqrt{3}\mathbf{h}_2\mathbf{h}_1^* + \mathbf{h}_3^*(\mathbf{h}_1 + \mathbf{h}_4))}{\sqrt{15}}$$

Degree of polarization

$$d(\rho_{3/2}) = \sqrt{\sum_{L=1}^{3}\sum_{M=-L}^{L} (r_M^L)^2}$$

At threshold: d(3/2)=23%

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$e^+e^- \to J/\psi \to \Xi^-\bar{\Xi}^+ \to \Lambda\pi^-\bar{\Lambda}\pi^+ \to p\pi^-\pi^-\bar{p}\pi^+\pi^+$

$d\Gamma \propto W(\xi;\boldsymbol{\omega})$ $\quad \xi$ 9 kinematical variables 9D PhSp

Parameters: 2 production + 6 for decay chains

$$\boldsymbol{\omega} = (\alpha_\psi, \Delta\Phi, \overbrace{\alpha_\Xi, \phi_\Xi, \alpha_\Lambda, \bar{\alpha}_\Xi, \bar{\phi}_\Xi, \bar{\alpha}_\Lambda})$$

$$W = \sum_{\mu,\bar{\nu}} C_{\mu\bar{\nu}} \sum_{\mu',\bar{\nu}'} a^{\Xi}_{\mu,\mu'} a^{\bar{\Xi}}_{\bar{\nu},\bar{\nu}'} a^{\Lambda}_{\mu',0} a^{\bar{\Lambda}}_{\bar{\nu}',0}$$

$\Delta\Phi \neq 0$ is not needed!

Variables and parameters factorize:

$$W(\xi;\boldsymbol{\omega}) = \sum_{k=1}^{M} f_k(\boldsymbol{\omega}) T_k(\xi)$$

	$\Xi^-\bar{\Xi}^+$	$\Lambda\bar{\Lambda}$
$\Delta\Phi \neq 0$:	$M = 72$	(7)
$\Delta\Phi = 0$:	$M = 56$	(5)

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