Is there a road to $\Lambda_b \to \Lambda\mu^+\mu^-$ at small q^2?

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in collaboration with

N. Gubernari & J. Virto (bounds, 2011.abcde)

M. Rahimi ($\Lambda_b \to \Lambda\mu^+\mu^-$, work in progress)

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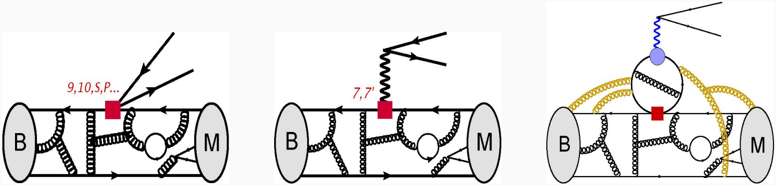
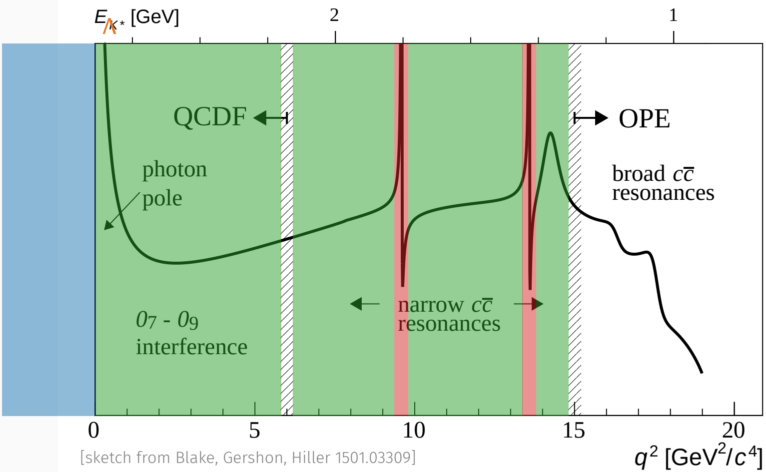
Anatomy of the $\Lambda_b \to \Lambda\mu^+\mu^-$ Amplitudes



$$\mathcal{A}_\lambda^\chi = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

for $\Lambda_b \to \Lambda\mu^+\mu^-$

- eight complex amplitudes for $m_\ell = 0$ $(\lambda = \perp_0, \perp_1, \|_0, \|_1, \chi = L, R)$
- local matrix elements: $\mathcal{F}_\lambda^{(T)}$ known from lattice QCD
  - can be systematically improved — see talk by Stefan Meinel
- non-local matrix elements dominated by time-ordered product $T\{J^\mu_{\text{em}}(x), [\bar{c}\ldots c\,\bar{s}\ldots b](0)\}$
  - focus of this talk



[sketch from Blake, Gershon, Hiller 1501.03309]

strategy

- compute $\mathcal{H}$ at spacelike q^2
- extrapolate to timelike $q^2 \leq 4M_D^2$
- include information from hadronic decays $\Lambda_b \to \Lambda\psi_n$

contribution to $\mathcal{H}_\lambda$	$B \to K^{(*)}$	$\Lambda_b \to \Lambda$
local OPE ($q^\mu \to \infty$)	✓	✓
form factors	✓ (LQCD+LCSR)	✓ (LQCD, large unc.)
hard spectator inter.	✓ (QCDF)	—
subleading OPE ($\|q^2\| \to \infty$)	✓	✓
soft-gluon matrix elem.	✓ (LCSR)	—

LQCD: lattice QCD LCSR: light-cone sum rule QCDF: QCD factorization

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LQCD: lattice QCD LCSR: light-cone sum rule QCDF: QCD factorization at subleading power in the OPE, need matrix elements of a non-local operator

$$\bar{s}(0)\gamma^{\rho}P_L G^{\alpha\beta}(-un^{\mu})b(0)$$

similar what is needed for $\Lambda_b^0 \to \Lambda_c^+ K^-$; see talk by Tobias Huber

for $B \to K^{(*)}$ transitions

- matrix element has been calculated in light-cone sum rules
  [Khodjamirian et al, 1006.4945]
- depends crucially on three-particle (i.e. $b\bar{u}G$) light-cone distribution amplitudes [Gubernari,Virto,DvD 2011.abcde]

can we apply this to $\Lambda_b \to \Lambda$ transitions?

- light-cone sum rule *starts* with four-particle *budG* light-cone distribution amplitudes
- these have not even been classified yet! see talk by Thorsten Feldmann
  volunteers?
- unlikely to be computed any time soon...

based on preliminary work [Gubernari/Virto/DvD 2011.abcde]

matrix elements $\mathcal{H}$ arise from non-local operator

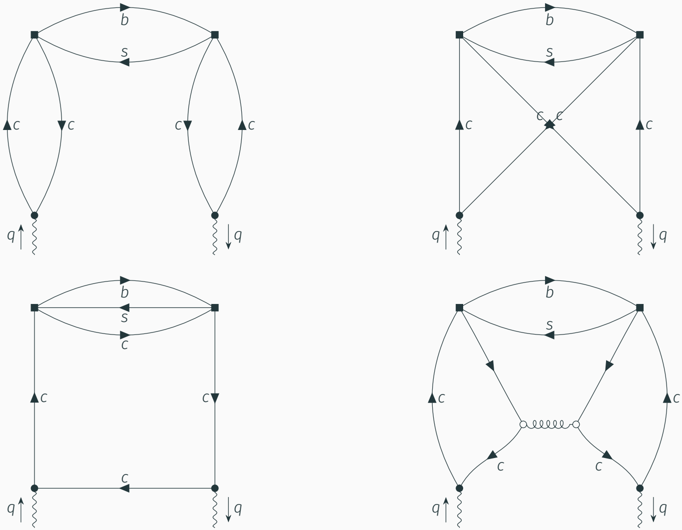
$$O^\mu(q;x) \sim \int e^{iq\cdot y}\, T\{J^\mu_{\rm em}(x+y), \mathcal{O}_{\bar{s}b\bar{c}c}(x)\}$$

construct four-point operator to derive a dispersive bound

- define matrix element of “square“ operator

$$\left[\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right]\Pi(q^2) \equiv \int e^{iq\cdot x}\,\langle 0|\, T\{O^\mu(q;x) O^{\dagger,\nu}(q;0)\}\,|0\rangle$$

- for $q^2 < 0$ we find that $\Pi(q^2)$ has two types of discontinuities
  - from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
  - from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)



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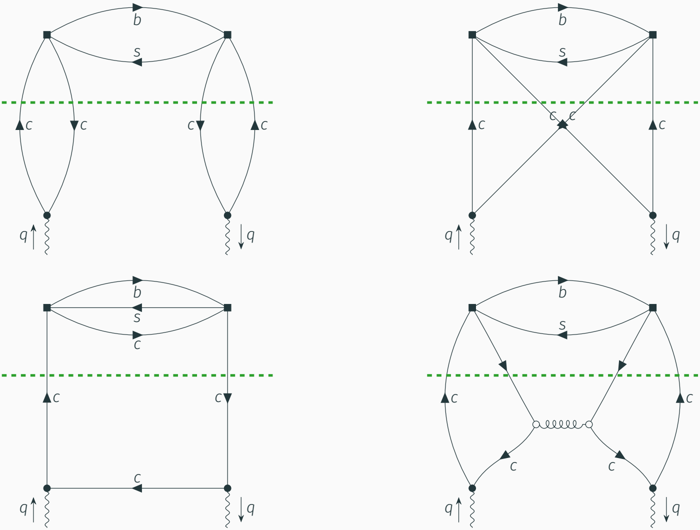
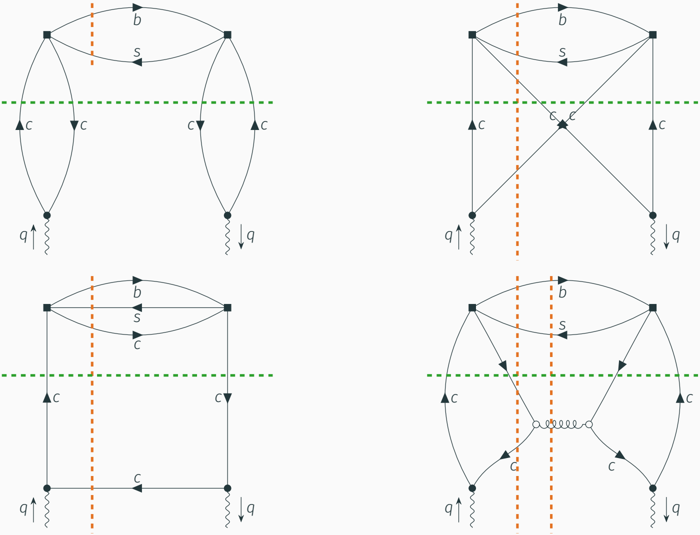


- from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)



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dispersive representation of the $b\bar{s}$ contribution to derivative of Π

$$\chi(Q^2) \equiv \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \frac{1}{2i\pi} \int_{(m_b+m_s)^2}^{\infty} ds\, \frac{\mathrm{Disc}_{b\bar{s}}\, \Pi(s)}{s - Q^2}$$

- can be computed in the local OPE $\to \chi^{\mathrm{OPE}}$
- can be expressed in terms of the matrix elements $\mathcal{H}_\lambda \to \chi^{\mathrm{had}}$
- for $Q^2 < 0$ the object χ is a sum / integral of positive definite terms
- equate the two to derive a dispersive bound

a suitable paramtrisation with outer functions ϕ_λ and orthonormal functions $f_n(q^2)$

$$\hat{\mathcal{H}}_\lambda(q^2) \propto \phi_\lambda(q^2)\mathcal{H}_\lambda(q^2) \propto \sum_n a_{\lambda,n} f_n(q^2)$$

leads to a diagonal bound

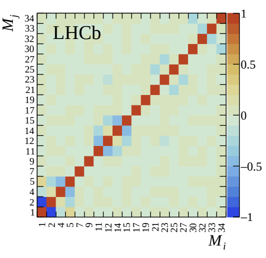
$$\sum_\lambda \sum_n |a_{\lambda,n}|^2 \leq 1$$

why/how is this relevant to $\Lambda_b \to \Lambda$?

1. the $\Lambda_b \to \Lambda$ matrix elements are bounded, giving us control of the truncation error in their parametrisation
2. even if we do not know the theory predictions of the non-local contributions beyond leading-power ($\to$ compute part), we can reliably connect the spacelike and timelike q^2 regions with each other
3. will likely depend more strongly on data-driven information in $\Lambda_b \to \Lambda$ than in $B \to K^{(*)}$
4. combining $\Lambda_b \to \Lambda$, $B \to K^{(*)}$ and $B_s \to \phi$ in one analysis will yield stronger constraints on the parameters than the individual transitions would

- full angular distribution of $\Lambda_b \to \Lambda(\to p\pi)J/\psi$ recently measured for the first time [LHCb PAPER-2020-005 2004.10563]
- measurements constrain residues of the non-local matrix elements

	7 TeV	8 TeV	13 TeV
M_1	$0.374 \pm 0.007 \pm 0.003$	$0.373 \pm 0.004 \pm 0.002$	$0.380 \pm 0.003 \pm 0.001$
M_2	$0.253 \pm 0.014 \pm 0.005$	$0.254 \pm 0.008 \pm 0.003$	$0.239 \pm 0.006 \pm 0.002$
M_4	$-0.286 \pm 0.017 \pm 0.008$	$-0.268 \pm 0.011 \pm 0.009$	$-0.273 \pm 0.008 \pm 0.006$
M_5	$-0.157 \pm 0.025 \pm 0.008$	$-0.181 \pm 0.015 \pm 0.007$	$-0.179 \pm 0.011 \pm 0.005$
M_7	$0.051 \pm 0.029 \pm 0.005$	$0.025 \pm 0.018 \pm 0.003$	$0.022 \pm 0.013 \pm 0.002$
M_9	$-0.017 \pm 0.029 \pm 0.005$	$-0.011 \pm 0.018 \pm 0.003$	$-0.027 \pm 0.013 \pm 0.002$
M_{11}	$0.005 \pm 0.014 \pm 0.004$	$0.003 \pm 0.009 \pm 0.004$	$-0.005 \pm 0.006 \pm 0.002$
M_{12}	$-0.004 \pm 0.018 \pm 0.005$	$0.010 \pm 0.011 \pm 0.004$	$0.006 \pm 0.008 \pm 0.003$
M_{14}	$0.007 \pm 0.025 \pm 0.007$	$-0.015 \pm 0.016 \pm 0.007$	$-0.009 \pm 0.012 \pm 0.003$
M_{15}	$-0.027 \pm 0.032 \pm 0.008$	$0.009 \pm 0.021 \pm 0.008$	$-0.006 \pm 0.016 \pm 0.005$
M_{17}	$0.008 \pm 0.039 \pm 0.006$	$-0.002 \pm 0.025 \pm 0.004$	$0.011 \pm 0.018 \pm 0.003$
M_{19}	$-0.006 \pm 0.038 \pm 0.004$	$-0.015 \pm 0.025 \pm 0.004$	$-0.003 \pm 0.018 \pm 0.002$
[illegible]	[illegible]	[illegible]	[illegible]



see talk by Tom Blake

Summary

- I think there is a clear road toward a reliable description of the non-local matrix elements in $\Lambda_b \to \Lambda\mu^+\mu^-$
- key is a combined theory + data driven approach, since theory calculations still do and will continue to lack behind the simpler case of $B \to K^{(*)}\mu^+\mu^-$ for the forseeable future
- nevertheless, $\Lambda_b \to \Lambda\mu^+\mu^-$ measurements are important
  - to cross check of the b anomalies
  - to provide complementary constraints on the Wilson coefficients
  - to have more powerful constraints on the hadronic nuisance parameters in $B \to K^{(*)}\mu^+\mu^-$