Light-cone distribution amplitudes for Λ_b baryons

and applications in QCDF, SCET and sum rules

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b-baryon Fest [virtual], 5.-6. November 2020

DFG **TRR 257**
particle **p**hysics **p**henomenology
after the **H**iggs discovery

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Motivation

- Many precision tests of flavour sector from *B*-meson decays:
  - CKM unitarity triangle
  - CP violation in quark sector
  - constraints on NP in rare decays and meson mixing
  - hints on LFU violation
  - . . .
  - help to sharpen theory tools (QCDF, SCET, LCSRs, Lattice, . . .)

- Why could baryonic decay modes (still) be interesting?
  - *b*-baryons produced and analyzed at hadron colliders:
    increasing information from experiment (LHCb, . . .)
  - baryons are fermions with half-integer spin:
    complementary observables for phenomenology (angular observables,. . .)
  - baryons contain (at least) two spectators:
    independent input from theory (hadronic structure of baryons, . . .)

  But: Baryons are more challenging for theory than Mesons!

This Talk:

- focus on properties of the Λ_b baryon
- discuss $\Lambda_b \to \Lambda$ form factors as an example
  (relevant for $\Lambda_b \to \Lambda\, \ell^+\ell^-$ analyses)

LCDAs for Λ_b baryons

[Ball/Braun/Gardi 2008] [Ali et al. 2012] [Parkhomenko 2017]

- "chiral-odd" LCDAs:

$$\epsilon^{abc} \langle 0|u^a(\tau_1 n) \, C\gamma_5 \, \not{n} \, d^b(\tau_2 n) \, h_v^c(0)|\Lambda_b(v,s)\rangle = f_{\Lambda_b}^{(2)} \, \tilde{\phi}_2(\tau_1,\tau_2) \, u_{\Lambda_b}(v,s)$$

$$\epsilon^{abc} \langle 0|u^a(\tau_1 n) \, C\gamma_5 \, \not{\bar{n}} \, d^b(\tau_2 n) \, h_v^c(0)|\Lambda_b(v,s)\rangle = f_{\Lambda_b}^{(2)} \, \tilde{\phi}_4(\tau_1,\tau_2) \, u_{\Lambda_b}(v,s)$$

- "chiral-even" LCDAs:

$$\epsilon^{abc} \langle 0|u^a(\tau_1 n) \, C\gamma_5 \, d^b(\tau_2 n) \, h_v^c(0)|\Lambda_b(v,s)\rangle = f_{\Lambda_b}^{(1)} \, \tilde{\phi}_3^s(\tau_1,\tau_2) \, u_{\Lambda_b}(v,s)$$

$$\epsilon^{abc} \langle 0|u^a(\tau_1 n) \, C\gamma_5 \, \frac{i\sigma_{\mu\nu}\bar{n}^\mu n^\nu}{4} \, d^b(\tau_2 n) \, h_v^c(0)|\Lambda_b(v,s)\rangle = f_{\Lambda_b}^{(1)} \, \tilde{\phi}_3^\sigma(\tau_1,\tau_2) \, u_{\Lambda_b}(v,s)$$

- depend on two light-like separations $\tau_{1,2}$ along n^μ (gauge-links not shown)
- distinguished by the Dirac projections of the diquark system   ($n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$)
- normalized to two scale-dependent $f_{\Lambda_b}^{(1,2)}(\mu)$ in HQET

             (not directly accessible in experiment)

[Ball/Braun/Gardi 2008] [Ali et al. 2012] [Parkhomenko 2017]

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Definition of 3-particle LCDAs for Λ_b

[Ball/Braun/Gardi 2008] [Ali et al. 2012] [Parkhomenko 2017]

- **"chiral-odd" LCDAs:**

$$\epsilon^{abc} \langle 0| u^a(\tau_1 n) \, C\gamma_5 \, \slashed{n} \, d^b(\tau_2 n) \, h_v^c(0)|\Lambda_b(v,s)\rangle = f_{\Lambda_b}^{(2)} \, \tilde{\phi}_2(\tau_1, \tau_2) \, u_{\Lambda_b}(v,s)$$

$$\epsilon^{abc} \langle 0| u^a(\tau_1 n) \, C\gamma_5 \, \slashed{\bar{n}} \, d^b(\tau_2 n) \, h_v^c(0)|\Lambda_b(v,s)\rangle = f_{\Lambda_b}^{(2)} \, \tilde{\phi}_4(\tau_1, \tau_2) \, u_{\Lambda_b}(v,s)$$

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- depend on two light-like separations $\tau_{1,2}$ along n^μ (gauge-links not shown)
- **distinguished by the Dirac projections of the diquark system**   ($n^2 = \bar{n}^2 = 0, \ n\cdot\bar{n} = 2$)
- normalized to two scale-dependent $f_{\Lambda_b}^{(1,2)}(\mu)$ in HQET

         (not directly accessible in experiment)

[Ball/Braun/Gardi 2008] [Ali et al. 2012] [Parkhomenko 2017]

- "chiral-odd" LCDAs:

$$\epsilon^{abc} \langle 0| u^a(\tau_1 n)\, C\gamma_5\, \slashed{n}\, d^b(\tau_2 n)\, h_v^c(0)|\Lambda_b(v,s)\rangle \;=\; f^{(2)}_{\Lambda_b}\, \tilde{\phi}_2(\tau_1,\tau_2)\, u_{\Lambda_b}(v,s)$$

$$\epsilon^{abc} \langle 0| u^a(\tau_1 n)\, C\gamma_5\, \slashed{\bar{n}}\, d^b(\tau_2 n)\, h_v^c(0)|\Lambda_b(v,s)\rangle \;=\; f^{(2)}_{\Lambda_b}\, \tilde{\phi}_4(\tau_1,\tau_2)\, u_{\Lambda_b}(v,s)$$

- "chiral-even" LCDAs:

$$\epsilon^{abc} \langle 0| u^a(\tau_1 n)\, C\gamma_5\, d^b(\tau_2 n)\, h_v^c(0)|\Lambda_b(v,s)\rangle \;=\; f^{(1)}_{\Lambda_b}\, \tilde{\phi}_3^s(\tau_1,\tau_2)\, u_{\Lambda_b}(v,s)$$

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- depend on two light-like separations $\tau_{1,2}$ along n^μ (gauge-links not shown)
- distinguished by the Dirac projections of the diquark system   ($n^2 = \bar{n}^2 = 0, \;\; n\cdot\bar{n} = 2$)
- normalized to two scale-dependent "decay constants" $f^{(1,2)}_{\Lambda_b}(\mu)$ in HQET
    (not directly accessible in experiment)

Simple(st) Models for LCDAs in momentum space

- Fourier transform $\tau_{1,2} \to \omega_{1,2} = n \cdot k_{1,2}$ (light-cone momenta of spectator quarks)

- exponential fall-off for $\omega \equiv (\omega_1 + \omega_2) \to \infty$, trivial shape in $u \equiv \frac{\omega_1}{\omega}$:

$$\phi_2(\omega_1, \omega_2) \to \frac{u(1-u)\,\omega^2}{\omega_0^4}\, e^{-\omega/\omega_0}\,, \qquad \phi_4(\omega_1, \omega_2) \to \frac{1}{\omega_0^4}\, e^{-\omega/\omega_0}$$

and

$$\phi_3^s(\omega_1, \omega_2) \to \frac{\omega}{2\omega_0^3}\, e^{-\omega/\omega_0}\,, \qquad \phi_3^\sigma(\omega_1, \omega_2) \to \frac{(2u-1)\,\omega}{2\omega_0^3}\, e^{-\omega/\omega_0}$$

- with a free parameter $\omega_0 \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$ (reference scale for spectator momenta)

- Estimates for the decay constants from sum rules [Groote/Körner/Yakovlev 1997]

$$f_{\Lambda_b}^{(1)} \simeq f_{\Lambda_b}^{(2)} \approx 0.032\,\mathrm{GeV}^3 \qquad (\text{at } \mu_0 = 1\ \mathrm{GeV})$$

Renormalization Group Evolution

1-loop RG equation for the LCDA ϕ_2: [Ball/Braun/Gardi 2008]

$$
\begin{aligned}
\frac{d\phi_2(\omega_1, \omega_2; \mu)}{d \ln \mu} &= -\left[\Gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu}{\sqrt{\omega_1 \omega_2}} + \gamma_+(\alpha_s) \right] \phi_2(\omega_1, \omega_2; \mu) \\
&\quad - \frac{\omega_1}{2} \int_0^\infty d\eta_1 \, \Gamma_+(\omega_1, \eta_1, \alpha_s) \, \phi_2(\eta_1, \omega_2; \mu) \\
&\quad - \frac{\omega_2}{2} \int_0^\infty d\eta_2 \, \Gamma_+(\omega_2, \eta_2, \alpha_s) \, \phi_2(\omega_1, \eta_2; \mu) \\
&\quad + \frac{\alpha_s C_F}{2\pi} \int_0^1 dv \, V^{ERBL}(u, v) \, \phi_2(v\omega, \bar{v}\omega; \mu)
\end{aligned}
$$

where $\bar{u} = 1 - u$ and $\bar{v} = 1 - v$.

- first 3 terms generalize the Lange-Neubert kernel, known from the *B*-meson LCDAs
- last term contains the Brodsky-Lepage kernel, known from light meson LCDAs

LN and ERBL kernel cannot simply be diagonalized simultaneously!

(. . . after all, it's a genuine 3-body problem)

Primary effect from $\Gamma_{\text{cusp}}, \Gamma_+ \Rightarrow$ treat $V_{ERBL}(u, v)$ as a perturbation:

- LN kernel is diagonalized by modified Bessel functions: [Bell/TF/Wang/Yip 2013]

$$\eta_2(s_1, s_2) \equiv \int\limits_0^\infty \frac{d\omega_1}{\omega_1} \int\limits_0^\infty \frac{d\omega_2}{\omega_2} \sqrt{\frac{\omega_1 \omega_2}{s_1 s_2}} \, J_1(2\sqrt{\omega_1 s_1}) \, J_1(2\sqrt{\omega_2 s_2}) \, \phi_2(\omega_1, \omega_2)$$

such that

$$\frac{d\eta_2(s_1, s_2; \mu)}{d\ln\mu}\Big|_{\text{LN}} = -\left[\Gamma_{\text{cusp}}(\alpha_s) \ln(\mu\sqrt{s_1 s_2}) + \gamma_+(\alpha_s)\right] \eta_2(s_1, s_2; \mu)$$

(A) Use <u>truncated</u> expansion of V_{ERBL} in terms of its eigenfunctions (Gegenbauer polynomials), and diagonalize a finite-dim. matrix. [Belle/TF/Wang/Yip 2013]

(B) Use conformal symmetry techniques to turn the RGE into a Hamiltonian problem that can be studied semi-numerically. [Braun/Derkachov/Manashov 2014]

QCD factorization for $\Lambda_b \to \Lambda$

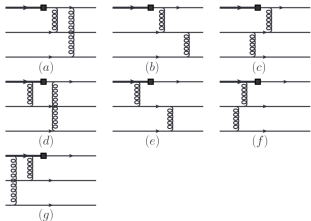# Application of Λ_b LCDAs in QCD Factorization

Consider $\Lambda_b \to \Lambda$ form factors in the large recoil limit: $\quad (E_\Lambda \sim m_b/2,\; q^2 \ll m_b^2)$

- Only one single form factor (instead of a priori 10)   [TF/Yip 2011, Mannel/Wang 2011]

$$\langle \Lambda(p') | \bar{s}\, \Gamma_i\, b | \Lambda_b(p) \rangle \quad \simeq \quad C_i\, \xi_\Lambda(E) \left[\bar{u}_\Lambda(p')\, \Gamma_i\, u_{\Lambda_b}(p) \right] + \text{(power corr.)}$$

- leading-power contribution is factorizable, but small: [W. Wang 2011]

- factorizable contribution involves LCDAs

$$\left. \xi_{\Lambda_b}(E) \right|_{\text{fact.}} \quad = \quad \alpha_s^2\, f_{\Lambda_b}\, f_\Lambda \left[\phi_{\Lambda_b} \otimes T_h \otimes \phi_\Lambda \right]$$
$$\simeq \quad \mathcal{O}(0.01)$$

- at sub-leading power, endpoint-divergent convolution integrals appear
  $\to$ soft (non-factorizable) contributions suppressed by $1/m_b$, but enhanced by $1/\alpha_s^2$

Sum rule for $\Lambda_b \to \Lambda$ in SCET

[TF/Yip 2011]

Calculation of soft contribution to $\xi_\Lambda(E)$:
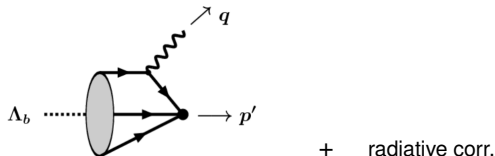

+ radiative corr.
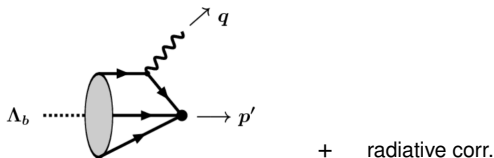
- study correlation function with interpolating current for light Λ baryon in SCET
- LO expression factorizes as

$$\Pi_\Lambda(n \cdot p') \quad \simeq \quad \int_0^\infty \frac{1}{\omega_1 + \omega_2 - n \cdot p' - i\epsilon} \, \phi_4(\omega_1, \omega_2) \, d\omega_1 \, d\omega_2$$

$\uparrow$ "hard-collinear" $\uparrow$ "soft"

where $(p')^2 = (n \cdot p')(\bar{n} \cdot p') < 0$ with $(\bar{n} \cdot p') = 2E$ and $(n \cdot p') \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$

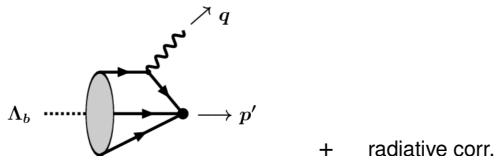[TF/Yip 2011]

Calculation of soft contribution to $\xi_\Lambda(E)$:

+ radiative corr.

- dispersion relations $\leftrightarrow$ model for the spectral density:
  Λ-pole $\oplus$ hadronic continuum above effective threshold s_0
- reduce sensitivity to continuum model $\rightarrow$ Borel trafo with parameter M^2

  LO sum rule: $e^{-m_\Lambda^2/M^2} (2E) f_\Lambda \xi_\Lambda(E) \approx f_{\Lambda_b}^{(2)} \int\limits_0^{s_0/2E} d\omega\, \omega \int_0^1 du\, \phi_4(\omega_1, \omega_2)$

[TF/Yip 2011]

Calculation of soft contribution to $\xi_\Lambda(E)$:

+   radiative corr.

- numerical analysis requires
  - values for the Λ and Λ_b "decay constants"
  - hadronic parameter $\omega_0 \sim$ few 100 MeV
    that controls endpoint behaviour of ϕ_4 for $\omega \to 0$
  - "reasonable windows" for threshold s_0 and Borel param. M^2
  - uncertainties from higher-order radiative corrections
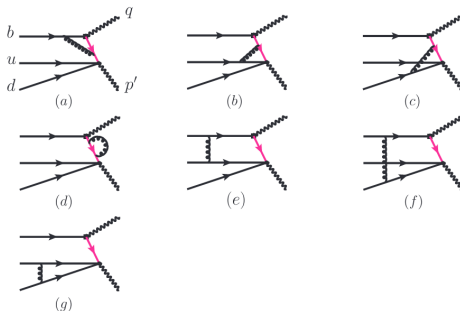- first-order estimate:

$$\xi_\Lambda(E_{max})\Big|_{soft} \approx 0.2 - 0.5$$

yields larger values than factorizable contribution!

[Wang/Shen 2015]

Perturbative corrections to hard-collinear function in SCET correlator:

- factorization scale-dependence under control
- resummation of Sudakov logs associated to radiation from hard-collinear quark
- numerical effect tends to decrease the value of ξ_Λ compared to LO estimate by about 50%

(analogous discussion for $B \to \pi$ and $B \to \rho$ sum rules in [De Fazio/TF/Hurth 2007])

Comparison with Lattice-QCD

[Detmold/Meinel 2016]

- form factors extracted at relatively small recoil ($q^2 > 0.6 \cdot q^2_{max}$)
- extrapolated to $q^2 = 0$ using "z-expansion"
- find consistent results with sum-rule estimates, with values for all 10 form factors around

$$f_i(q^2 = 0) \simeq 0.2$$

$f_+(\Lambda_b \to \Lambda)$
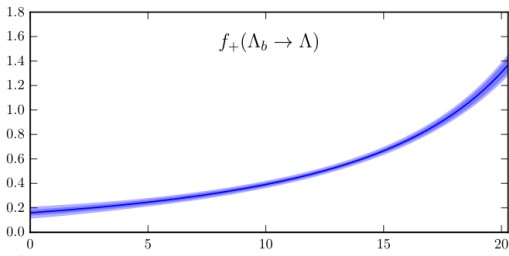
Summary/Conclusions

LCDAs and Form Factors of Λ_b-Baryon

- Λ_b described in terms of four 3-particle LCDAs
  (*B*-meson: two 2-particle LCDAs)
- normalization ("decay constants") and shape rather uncertain
  (*B*-meson: f_B well studied, information on shape from $B \to \gamma\ell\nu$)
- approximate solutions for RG evolution at 1-loop
  (*B*-meson: exact solutions at 2-loop)

- $\Lambda_b \to \Lambda$ form factors factorize for large recoil at leading power in $1/m_b$, but numerically sub-leading ($\mathcal{O}(\alpha_f^\in)$)
  (*B* → π FF: endpoint div. in QCDF at leading power)
- form factors can be estimated from (SCET) sum rules for large recoil
- comparison with lattice QCD for small recoil via "*z*-expansion"

Outlook

- improve knowledge on Λ_b LCDAs from phenomenological studies
- estimate contributions from hadronic operators in rare $\Lambda_b \to \Lambda\ell^+\ell^-$ decays

Backup Slides

Symmetry-breaking corrections to $\Lambda_b \to \Lambda$ FFs

\+ radiative corr.
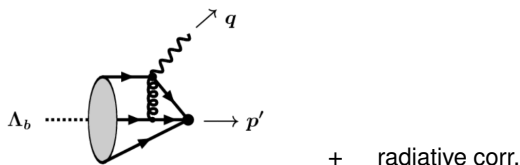
- Defined by SCET-1 matrix elements:

$$\langle \Lambda(p', s') | \bar{\xi}_{hc} \tilde{\Gamma} \, gA_\mu^\perp \, h_v | \Lambda_b(v, s) \rangle \quad \equiv \quad M_{\Lambda_b} \, \Delta\xi_\Lambda(E) \, \bar{u}_\Lambda(p', s') \gamma_\mu^\perp \tilde{\Gamma} \, u_{\Lambda_b}(v, s)$$

where transition current contains an extra hard-collinear gluon field

- Perform the same analysis as for soft FF – ratio scales as

$$\frac{\Delta\xi_\Lambda(E)}{\xi_\Lambda(E)} \sim \alpha_s \, \frac{\omega_0}{M_\Lambda} \, \frac{2E}{M_{\Lambda_b}}$$

numerically suppressed by α_s and $\omega_0/m_\Lambda \longrightarrow$ negligible for phenomenology