Relativistic redshift-space distortions at quasi-linear scales

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(Observatoire de Paris, LUTh, 01/09/2020 ~)

Collaborators

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M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, **S.Saga** [1803.04294]
A.Taruya, **S.Saga**, M-A.Breton, Y.Rasera, T.Fujita [1908.03854]
S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

Colloque national Action Dark Energy 2020 - 4ème édition

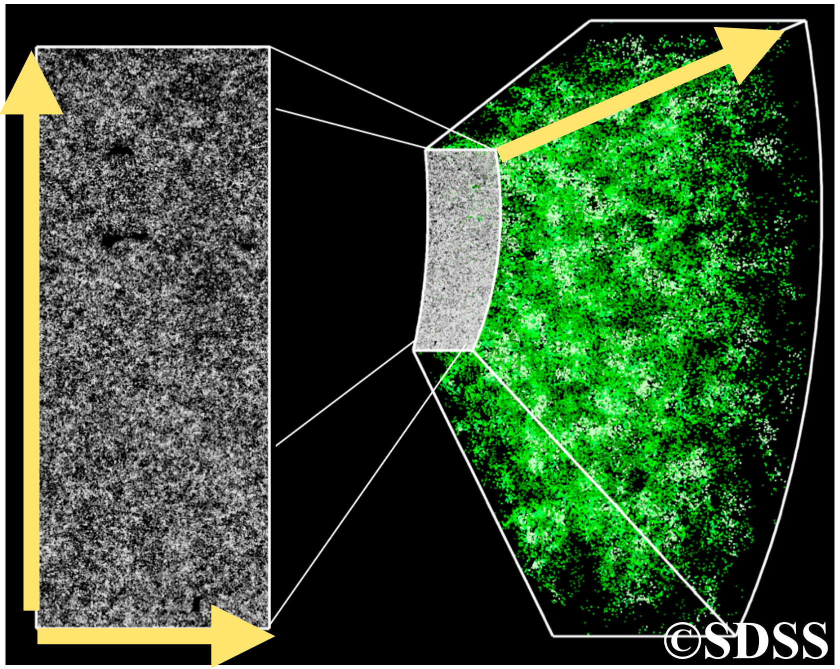
1. Relativistic effects on large-sale structure
2. Relativistic Redshift Space Distortions
3. Simulations (RayGalGroupSims)
4. Results: Quasi-linear modelling
5. Summary

1.1 Redshift space distortions (RSD)

Galaxy redshift surveys map the universe by measuring

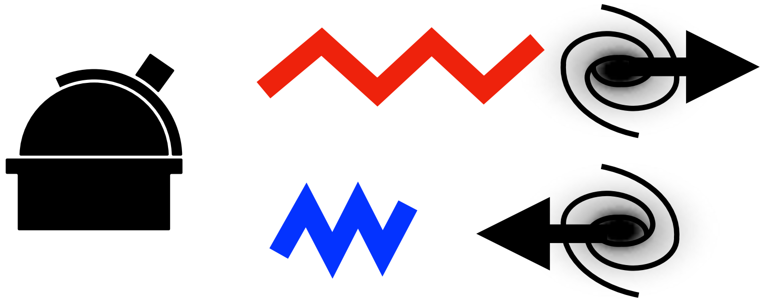
redshift $z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}}$

angular position $(\theta,\ \phi)$



Observed redshift

Cosmological redshift (Hubble flow) + **Doppler effect** (peculiar velocity)



Observed position (inferred from redshift) ≠ Actual position

Observed galaxy distribution appears distorted

= Redshift space distortions (RSD)

1.2 Classical Doppler effect

Primary source of RSD:

Doppler effect induced by peculiar velocity of galaxy

redshift space

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H(z)}\left(\boldsymbol{v}\cdot\hat{\boldsymbol{z}}\right)\hat{\boldsymbol{z}}$$

real space (special relativity, $v \ll 1$)

conservation law:

$$\Big(1+\delta^{(\mathrm{S})}(\boldsymbol{s})\Big)\mathrm{d}^3 s = \Big(1+\delta(\boldsymbol{r})\Big)\mathrm{d}^3 r$$

continuity equation (linear):

$$\dot{\delta}_{\mathrm{L}} + \frac{1}{a}\boldsymbol{\nabla}\cdot\boldsymbol{v} \simeq 0$$

Kaiser formula (Fourier space) N. Kaiser (1987)

$$\delta^{(\mathrm{S})}(\boldsymbol{k}) = \Big(b + f(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{z}})^2\Big)\delta_{\mathrm{L}}(\boldsymbol{k}) \qquad f \equiv \frac{\mathrm{d}\ln\delta_{\mathrm{L}}}{\mathrm{d}\ln a}$$: linear growth rate

1.3 Probe of gravity theory

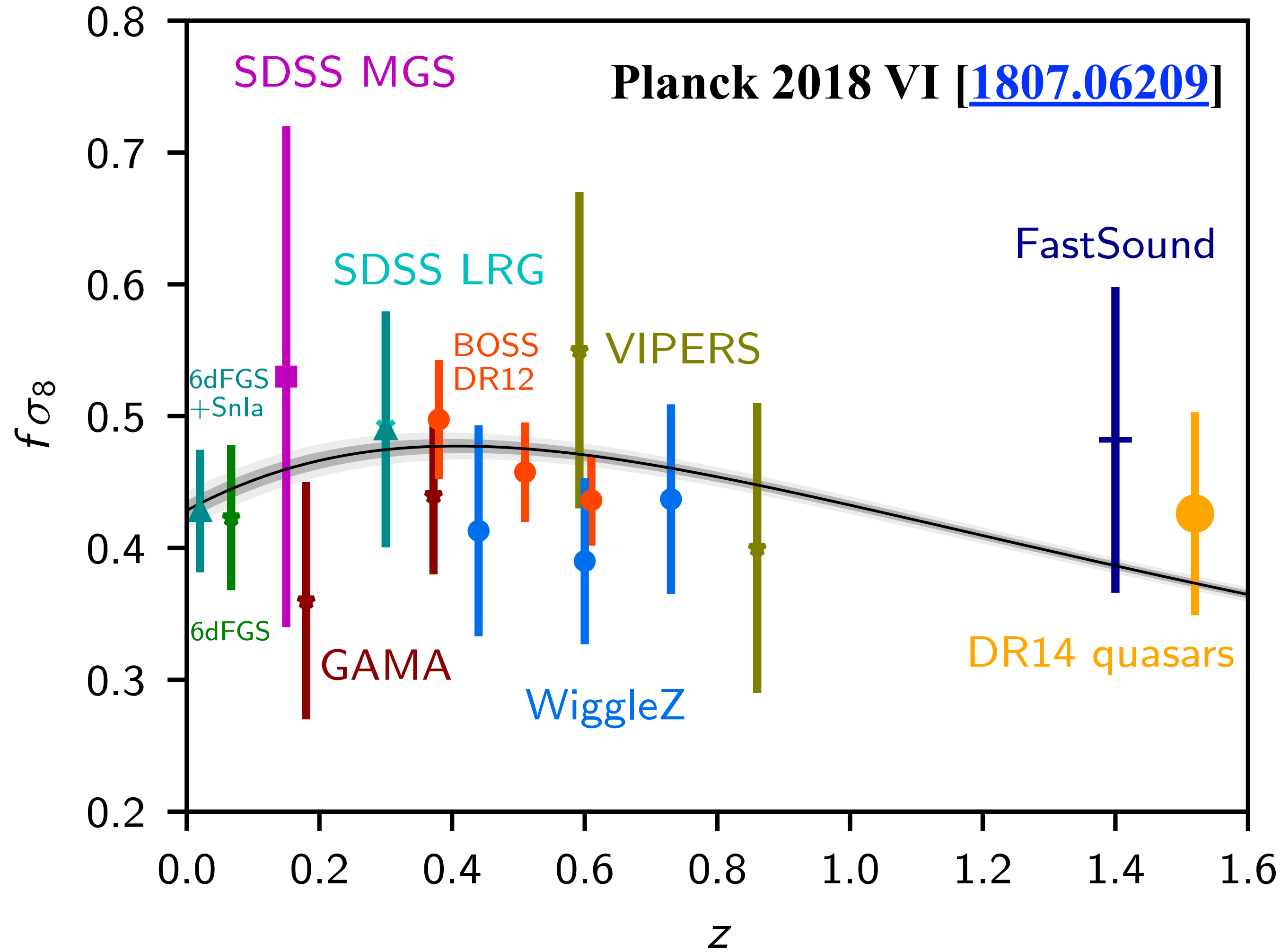
Kaiser formula (Fourier space)

$$\delta^{(\mathrm{S})}(\boldsymbol{k}) = \left(b + f(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{z}})^2\right)\delta_{\mathrm{L}}(\boldsymbol{k}) \qquad f \equiv \frac{\mathrm{d}\ln\delta_{\mathrm{L}}}{\mathrm{d}\ln a} \text{ : linear growth rate}$$

Linear growth rate depends on the gravity theory

→ RSD can be a *probe of gravity* on cosmological scales

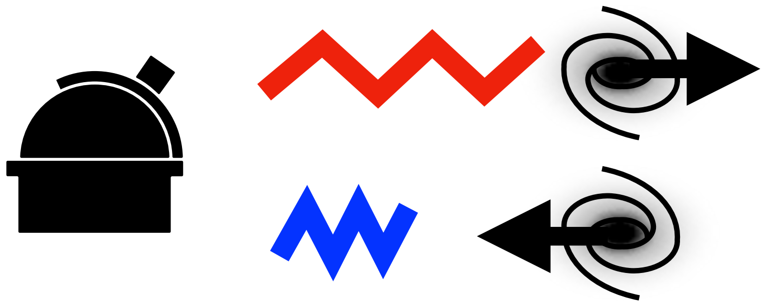
For ΛCDM: $f \approx (\Omega_{\mathrm{m}}(z))^{0.55}$



1.4 General relativistic effects

Observed redshift

Cosmological redshift (Hubble flow) + **Doppler effect** (peculiar velocity)



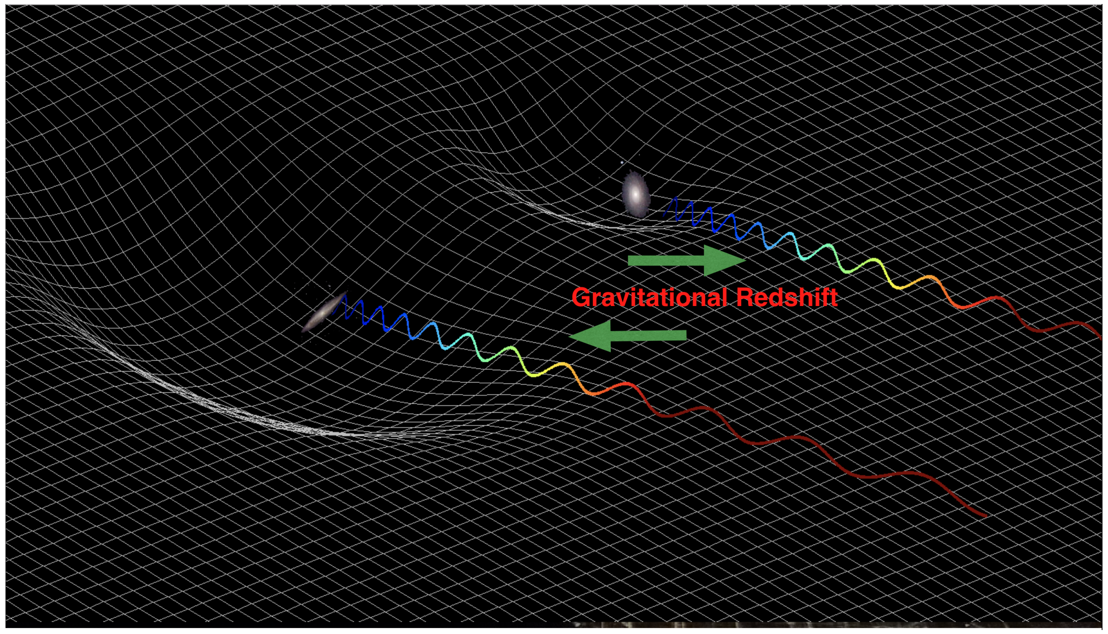
+ gravitational redshift (Sachs-Wolfe)

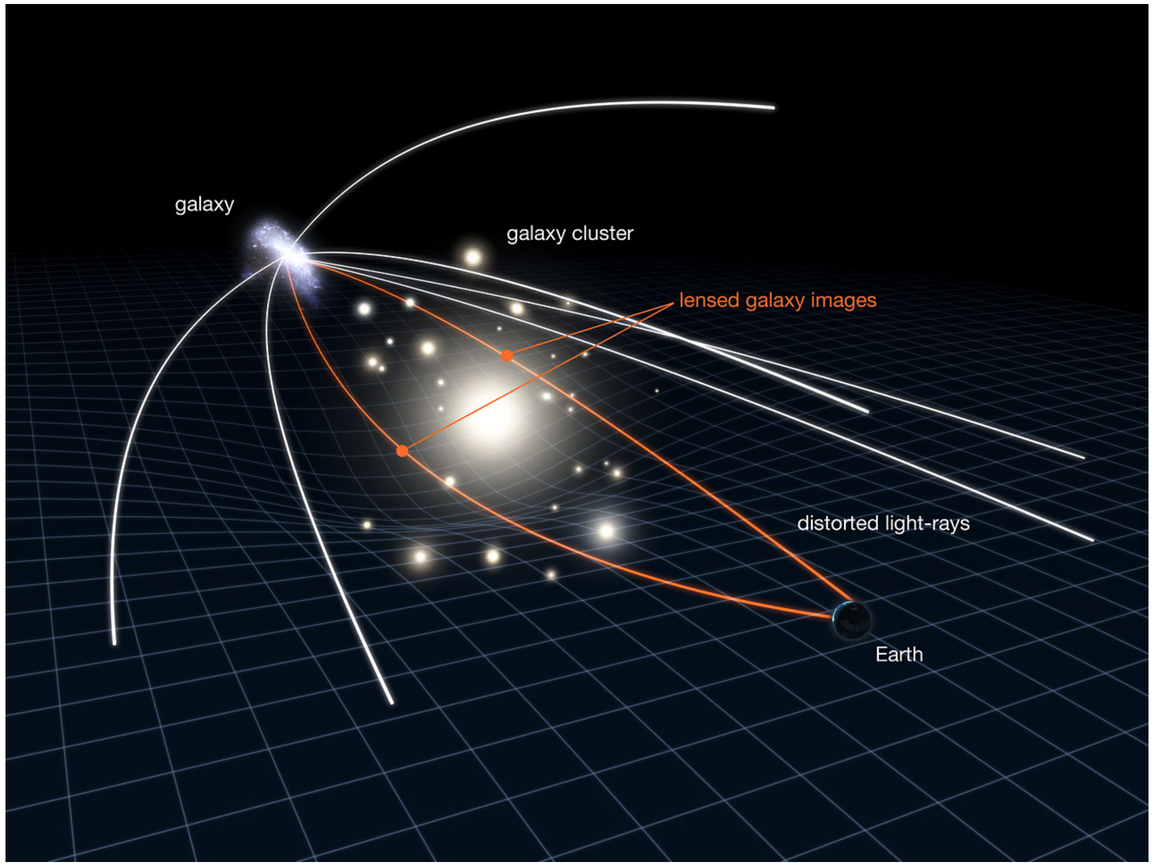
+ integrated Sachs-Wolfe

+ Shapiro time delay

+ gravitational lensing

+ ...





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2.1 Relativistic RSD

A.Challinor and A.Lewis [1105.5292]
C.Bonvin and R.Durrer [1105.5280]
C.Bonvin et al. [1309.1321]
J.Yoo [1409.3223],
and many works

How do relativistic effects imprint on redshift space?

Perturbed FLRW $ds^2 = \left[-(1+2\Phi)dt^2 + a^2(1-2\Psi)dx^2\right]$

Solve the geodesic eq. $\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta}k^\alpha k^\beta = 0$

Define observed redshift including all effects $1+z = \frac{(k_\mu u^\mu)_S}{(k_\mu u^\mu)_O}$

$s = r + \frac{1+z}{H}(v\cdot\hat{r})\hat{r}$ **(Classical) Doppler effect**

Relativistic effects (weak field approx.)

$$+\frac{1+z}{H}\left(-\Phi + \frac{1}{2}v^2 - \int_t^{t_0}\left(\dot{\Phi}+\dot{\Psi}\right)dt'\right)\hat{r} - \int_0^\chi (\Psi+\Psi)d\chi'\,\hat{r} - \int_0^\chi (\chi-\chi')\nabla_\perp(\Phi+\Psi)d\chi'$$

- gravitational redshift
- Transverse Doppler
- Shapiro time delay
- integrated Sachs-Wolfe
- gravitational lensing

High-precision future experiments might be possible to detect

➡ **which is the unique signature of relativistic effects ?**

2.2 Linear theory of relativistic RSD

(Classical) Doppler effect

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}$$

Relativistic effects

$$+\frac{1+z}{H}\left(-\Phi + \frac{1}{2}v^2 - \int_t^{t_0}\left(\dot{\Phi}+\dot{\Psi}\right)\mathrm{d}t'\right)\hat{\boldsymbol{r}} - \int_0^{\chi}(\Psi+\Psi)\mathrm{d}\chi'\,\hat{\boldsymbol{r}} - \int_0^{\chi}(\chi-\chi')\nabla_{\perp}(\Phi+\Psi)\mathrm{d}\chi'$$

conservation law

$$\left(1+\delta^{(\mathrm{S})}(s)\right)\mathrm{d}^3 s = \left(1+\delta(r)\right)\mathrm{d}^3 r$$

c.f. Kaiser formula

(linear approximation)

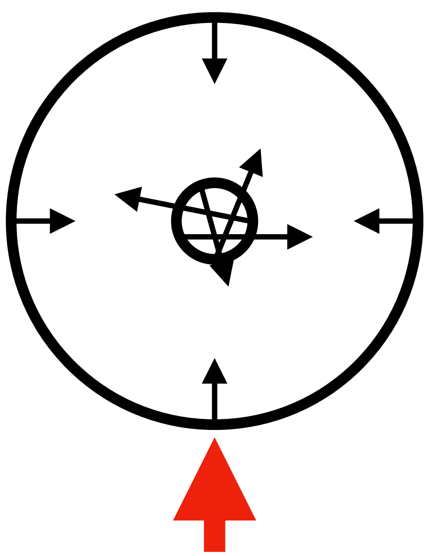
Linear density field with relativistic effects

$$\delta^{(\mathrm{s})} = b\delta - \frac{1}{\mathcal{H}}\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial\boldsymbol{r}}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{v}\right)$$
$$-\left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\hat{\boldsymbol{r}}\cdot\boldsymbol{v} + \frac{1}{\mathcal{H}}\left(\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial\boldsymbol{r}}\Psi + \mathcal{H}\hat{\boldsymbol{r}}\cdot\boldsymbol{v} + \hat{\boldsymbol{r}}\cdot\dot{\boldsymbol{v}}\right)$$
$$-2\Phi + \Psi + \frac{\dot{\Phi}}{\mathcal{H}} + \frac{1}{r}\int_0^r \mathrm{d}r'\left(2 - \frac{r-r'}{r'}\Delta_{\Omega}\right)(\Phi+\Psi) + \left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\left(\Psi + \int_0^r \mathrm{d}r'\left(\dot{\Psi}+\dot{\Phi}\right)\right)$$

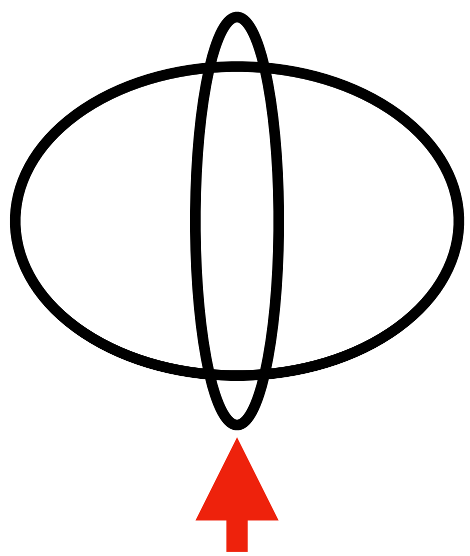
2.3 Dipole anisotropies

Linear density field with relativistic effects

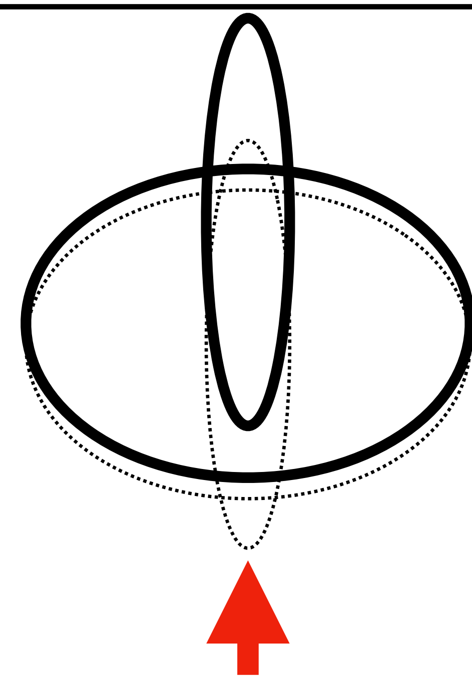
$$\delta^{(\mathrm{s})} = b\delta - \frac{1}{\mathcal{H}}\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial \boldsymbol{r}}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{v}\right) - \left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\hat{\boldsymbol{r}}\cdot\boldsymbol{v} + \frac{1}{\mathcal{H}}\left(\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial \boldsymbol{r}}\Psi + \mathcal{H}\hat{\boldsymbol{r}}\cdot\boldsymbol{v} + \hat{\boldsymbol{r}}\cdot\dot{\boldsymbol{v}}\right) - 2\Phi + \Psi + \frac{\dot{\Phi}}{\mathcal{H}} + \frac{1}{r}\int_0^r \mathrm{d}r'\left(2 - \frac{r-r'}{r'}\Delta_\Omega\right)(\Phi+\Psi) + \left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\left(\Psi + \int_0^r \mathrm{d}r'\left(\dot{\Psi}+\dot{\Phi}\right)\right)$$



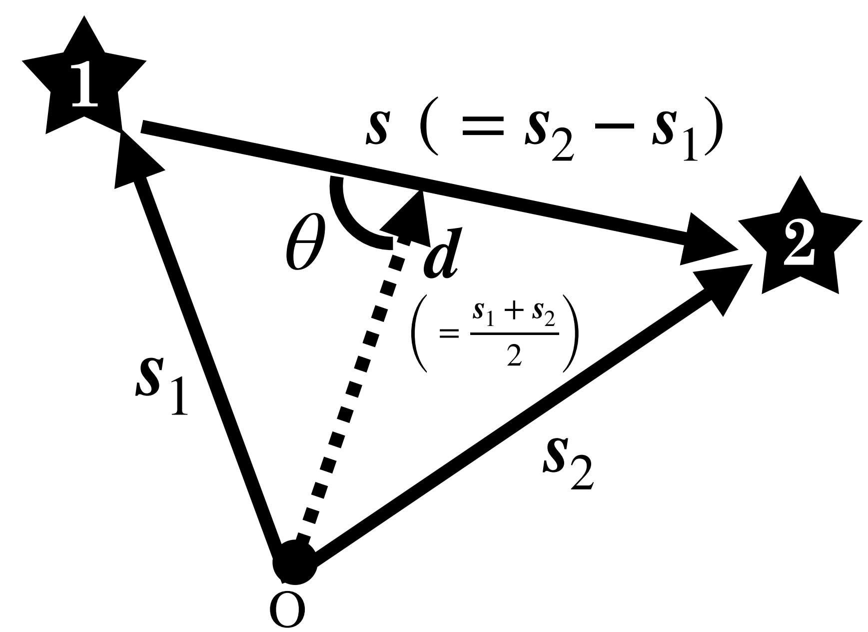
Real space



Redshift space



Redshift space w/ relativistic effects



Dipole cross-correlation: $\xi_1 = \frac{3}{2}\int_{-1}^{1}\mathrm{d}\mu\,\xi^{(\mathrm{S})}(\boldsymbol{s}_1,\boldsymbol{s}_2)$ $\quad \xi_1 \propto (b_1 - b_2)$

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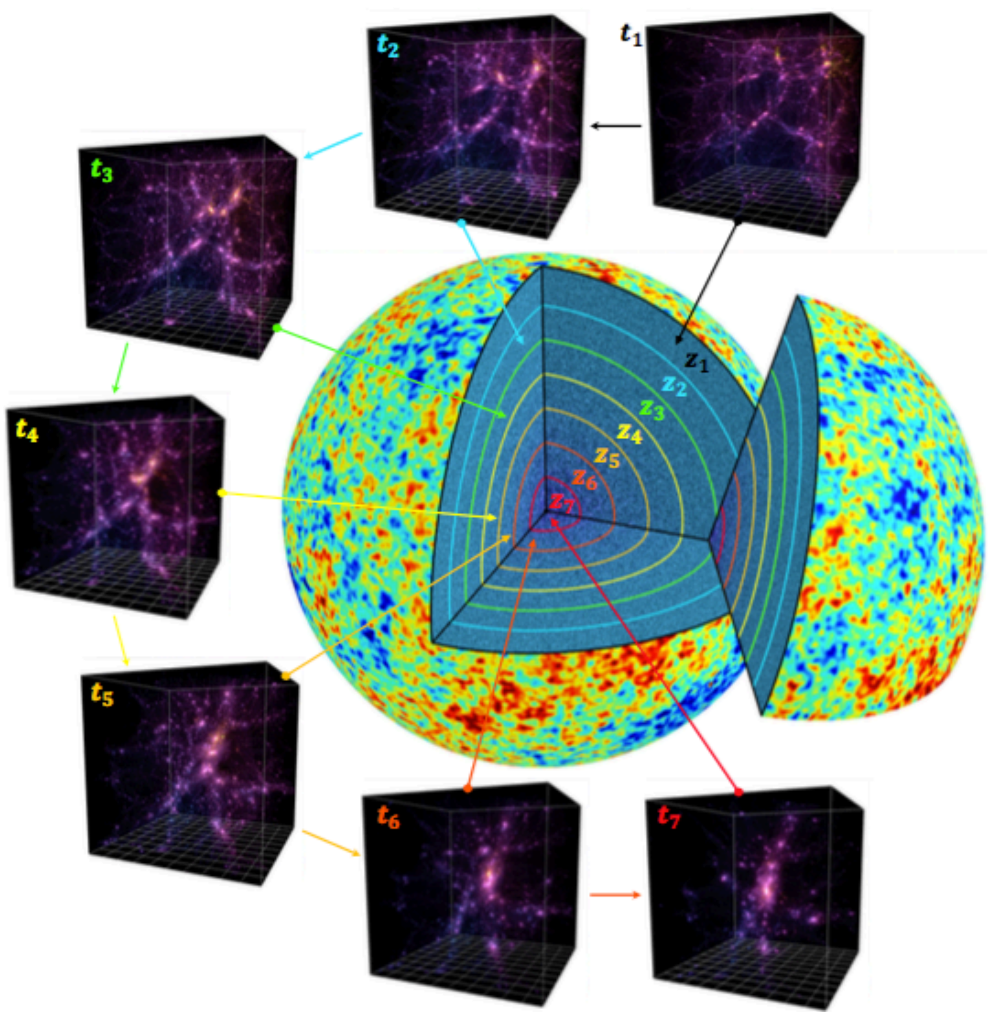
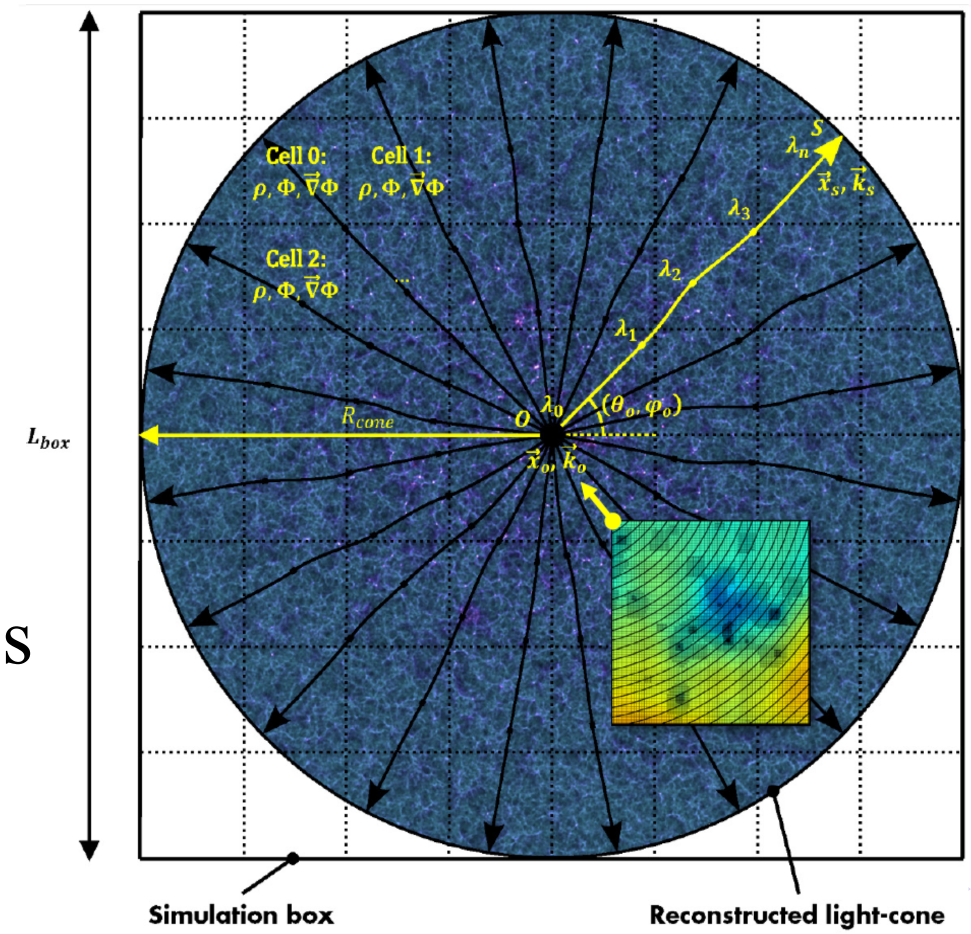
3.1 N-body simulations

M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, S.Saga [1803.04294]

- Storing potential data on light cone
- Tracing back the light ray to the source by direct integration of geodesic equation
- "Observed" position and redshift





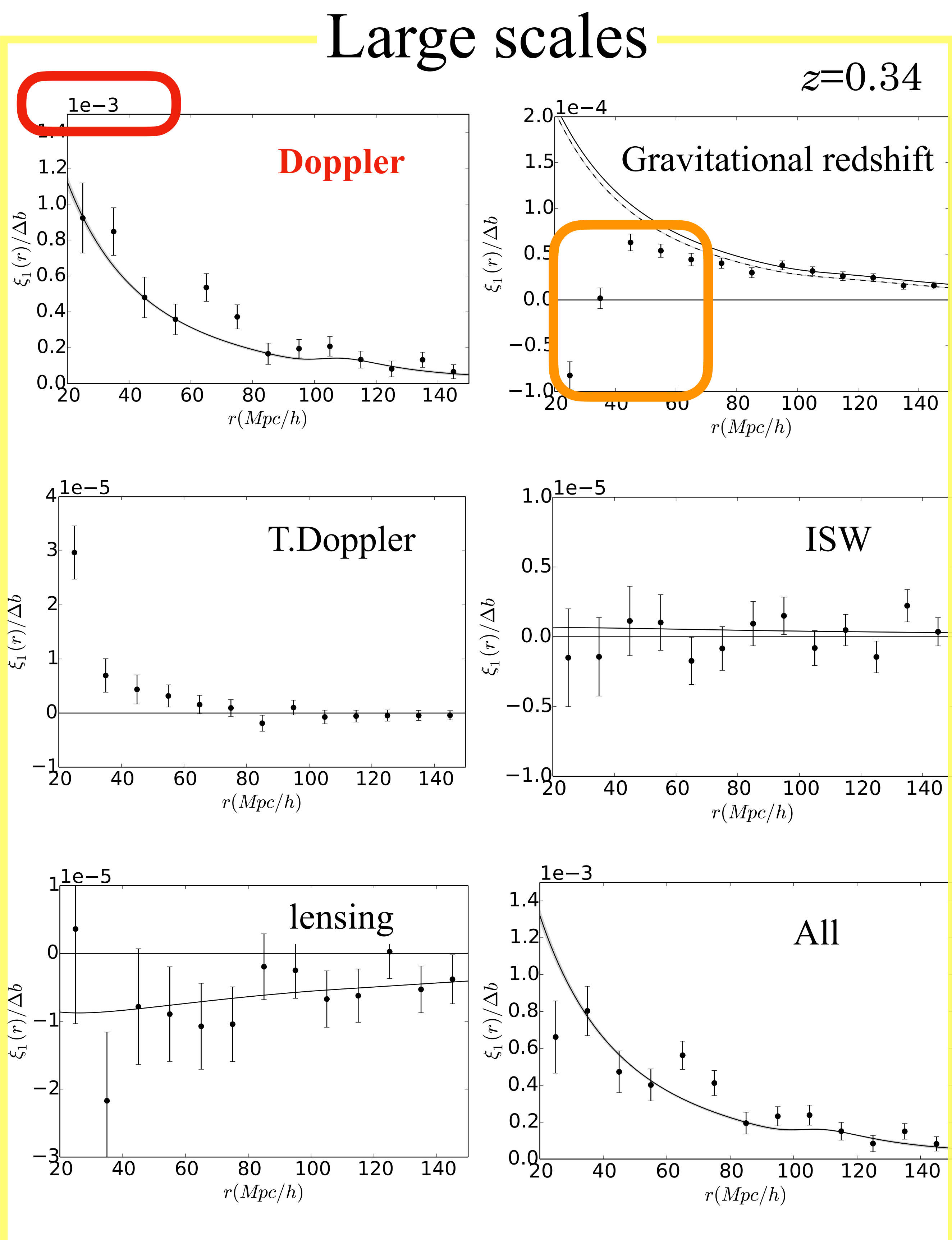
(2.625 h^{-1}Mpc)^3
4096^3 DM particles
(assumed Φ=Ψ)

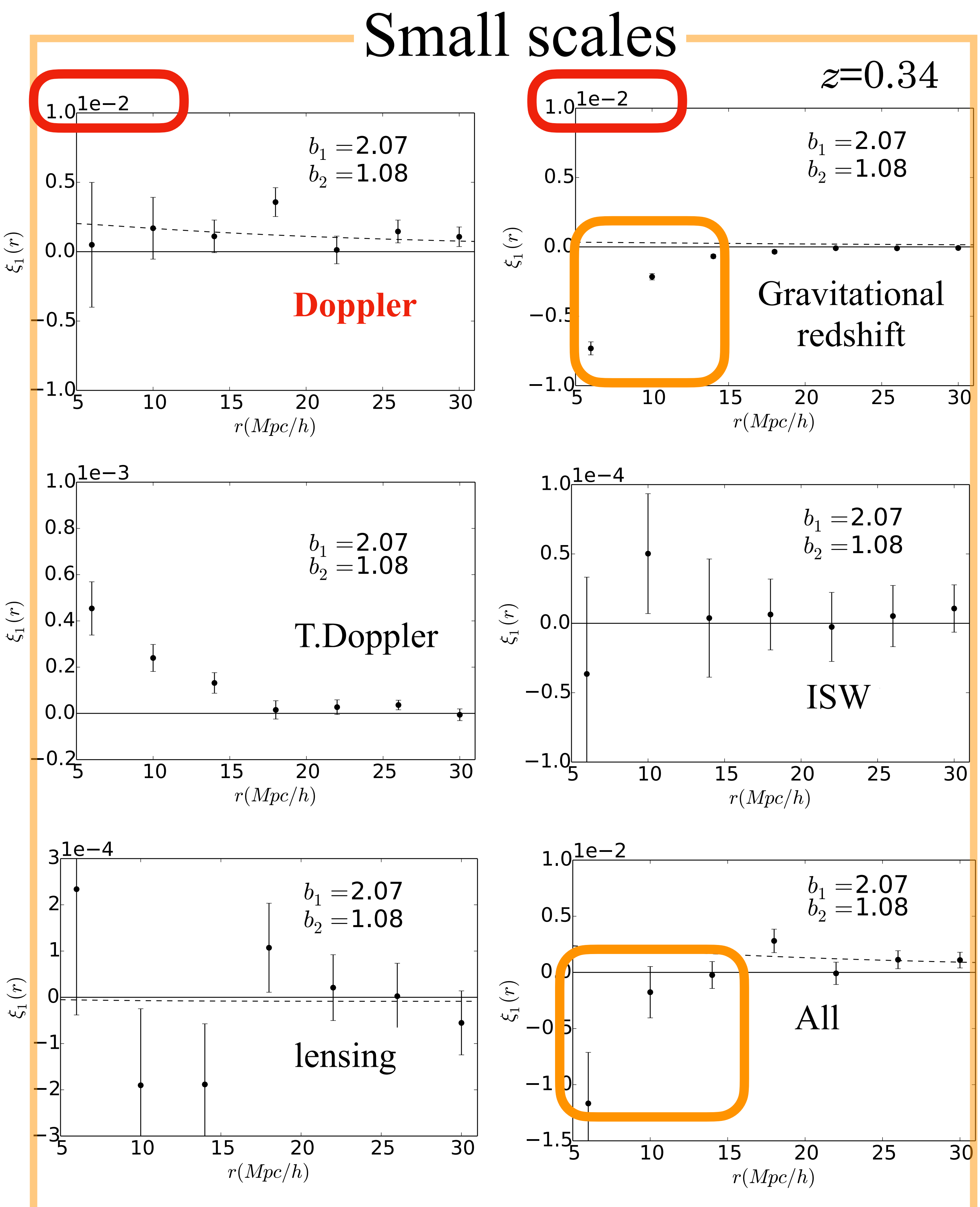
RayGalGroupSims
By Michel-Andres Breton and Yann Rasera

Then, all possible relativistic effects are taking into account

3.2 Measurements in RayGalGroupSims

M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, S.Saga [1803.04294]





3.3 Short summary

c.f. Detection of relativistic effects?
S.Alam et al(2017)

Based on RayGalGroupSims and/or linear theory...

	large scales	small scales
Dominated by	Doppler effect (Wide-angle effect)	Gravitational redshift (+ Doppler effect)
Related works	Castorina & White [1803.08185]	E. Di Dio & Seljak [1811.03054] F.Beutler & E. Di Dio [2004.08014]
Method	Zel'dovich approximation + linear bias	1-loop Standard PT + EFT like parameter + nonlinear bias

We make a quasi-linear model taking into account both effects based on Lagrangian PT (Zel'dovich approx.)

A.Taruya, S.Saga, M-A.Breton, Y.Rasera, T.Fujita [1908.03854]
S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

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4.1 Quasi-linear modelling

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} + \frac{1+z}{H}\left(-\Phi + \frac{1}{2}v^2 - \int_t^{t_0}\left(\dot{\Phi}+\dot{\Psi}\right)\mathrm{d}t'\right)\hat{\boldsymbol{r}} - \int_0^{\chi}(\Psi+\Psi)\mathrm{d}\chi'\,\hat{\boldsymbol{r}} - \int_0^{\chi}(\chi-\chi')\nabla_{\perp}(\Phi+\Psi)\mathrm{d}\chi'$$

Doppler effect (wide-angle effect)

Relativistic effects

Pick up the dominant contributions

- ✓ Doppler effect
- ✓ Gravitational redshift effect

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} + \frac{1+z}{H}\left(-\Phi + \frac{1}{2}v^2 - \int_t^{t_0}\left(\dot{\Phi}+\dot{\Psi}\right)\mathrm{d}t'\right)\hat{\boldsymbol{r}} - \int_0^{\chi}(\Psi+\Psi)\mathrm{d}\chi'\,\hat{\boldsymbol{r}} - \int_0^{\chi}(\chi-\chi')\nabla_{\perp}(\Phi+\Psi)\mathrm{d}\chi'$$

Lagrangian PT (Zel'dovich approximation)

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \frac{1+z}{H}\Phi$$

$$\boldsymbol{r} = \boldsymbol{q} + \boldsymbol{\Psi}_{\mathrm{ZA}}(\boldsymbol{q},t) \qquad \nabla_q\cdot\boldsymbol{\Psi}_{\mathrm{ZA}}(\boldsymbol{q},t) = -D_+(t)\delta_{\mathrm{L}}(\boldsymbol{q})$$

$D_+(t)$: Linear growth factor

4.2 Modelling gravitational potential

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

$$s = r + \frac{1+z}{H}(v \cdot \hat{r})\hat{r} - \frac{1+z}{H}\Phi$$

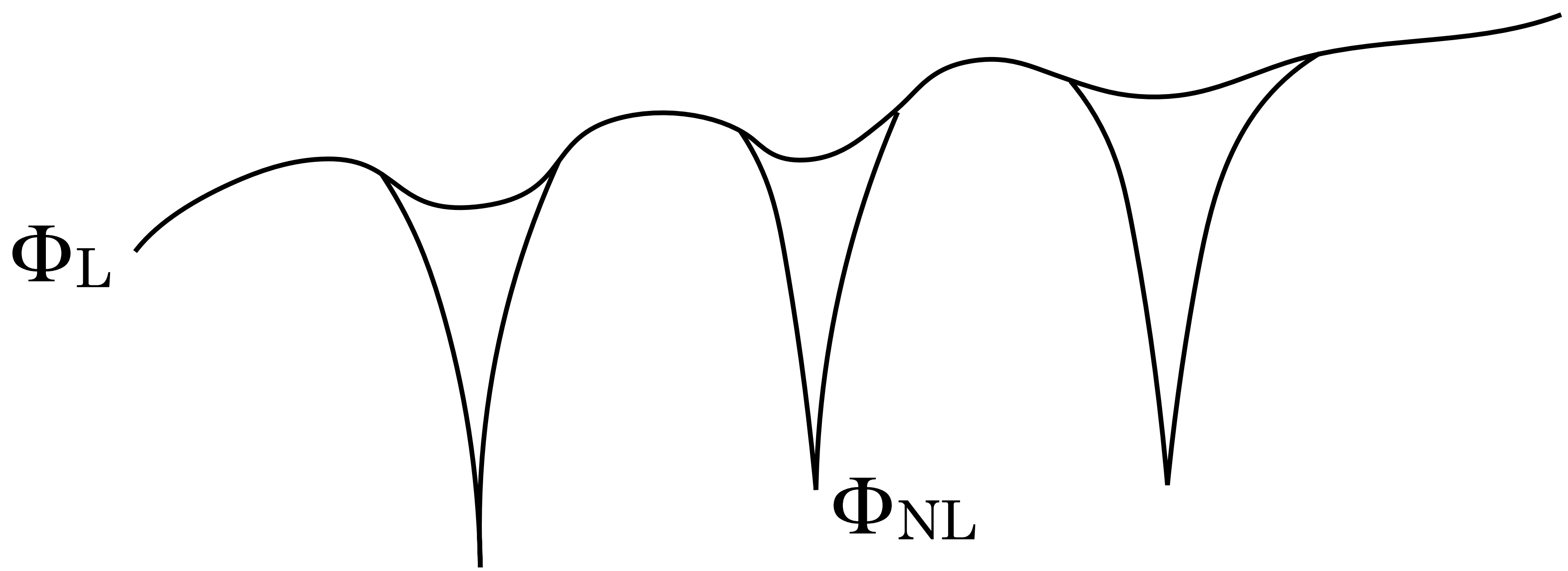
Φ should be modified by the gravitational potential of haloes

linear potential (computed based on Zel'dovich approximation)

$$\Phi = \Phi_{\rm L} + \underline{\Phi_{\rm NL}}$$

$$\nabla_x^2 \Phi_{\rm L}(x) = 4\pi G a^2 \bar{\rho} \delta_{\rm L} = -4\pi G a^2 \bar{\rho}(\nabla \cdot \Psi_{\rm ZA})$$

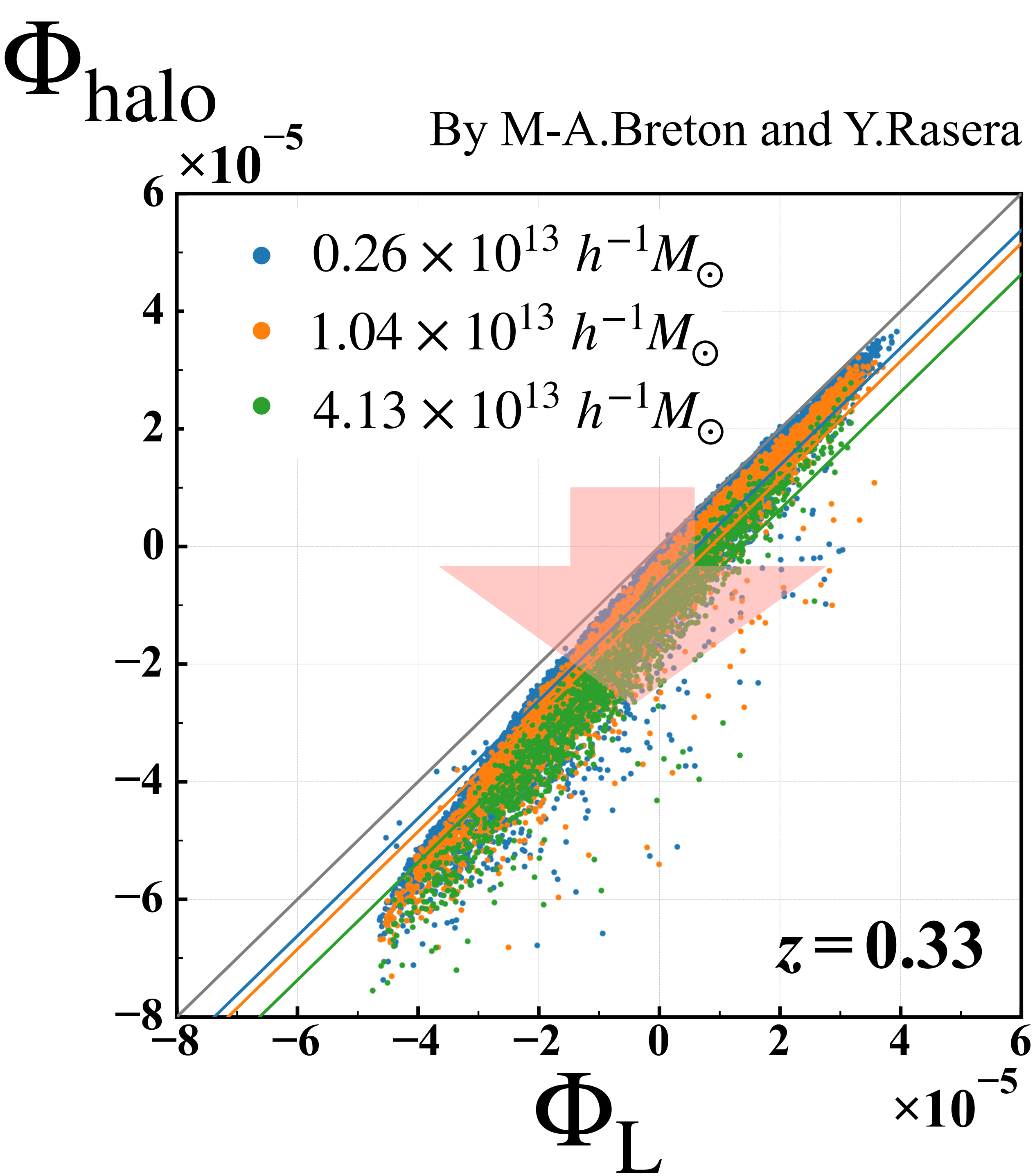
non-linear halo potential



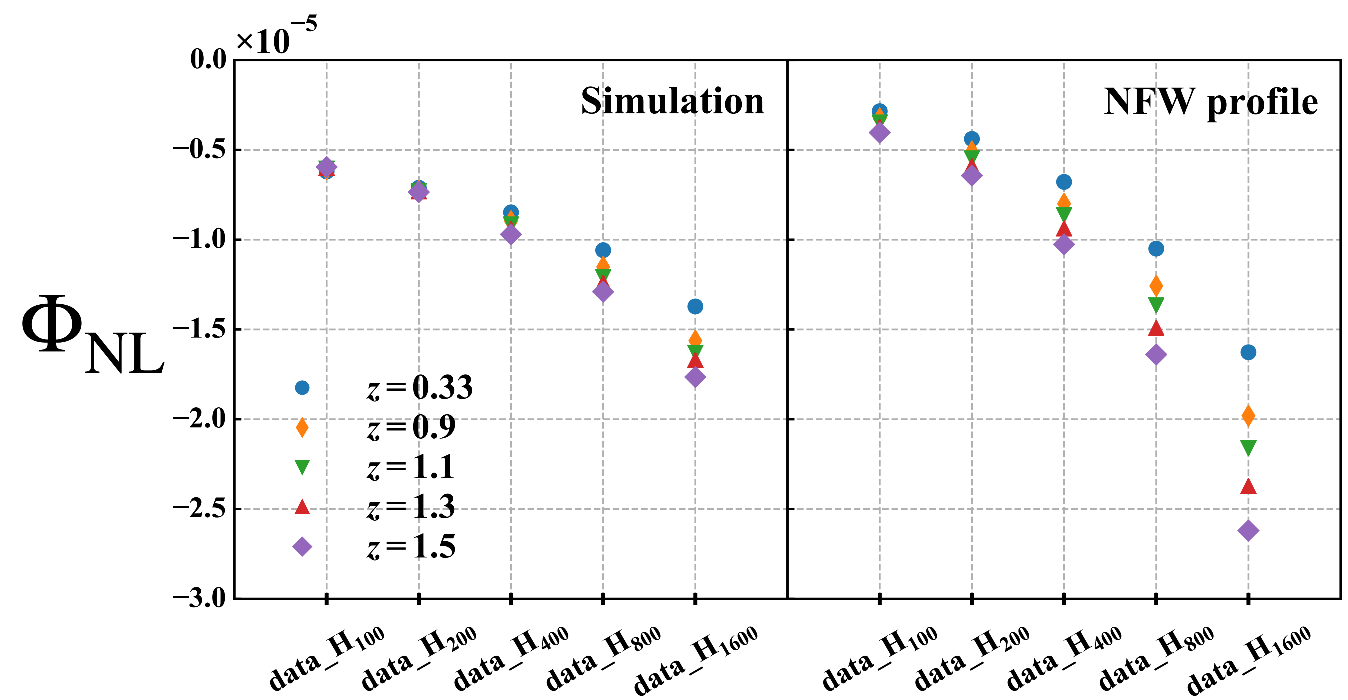
Assumptions: $\Phi_{\rm NL}$ is a constant value determined by halo masses and redshifts

4.3 Non-linear halo potential

Gravitational potential at the center of haloes is systematically deeper than linear potential



$$\Phi_{\rm halo} = \Phi_{\rm L} + \Phi_{\rm NL}$$

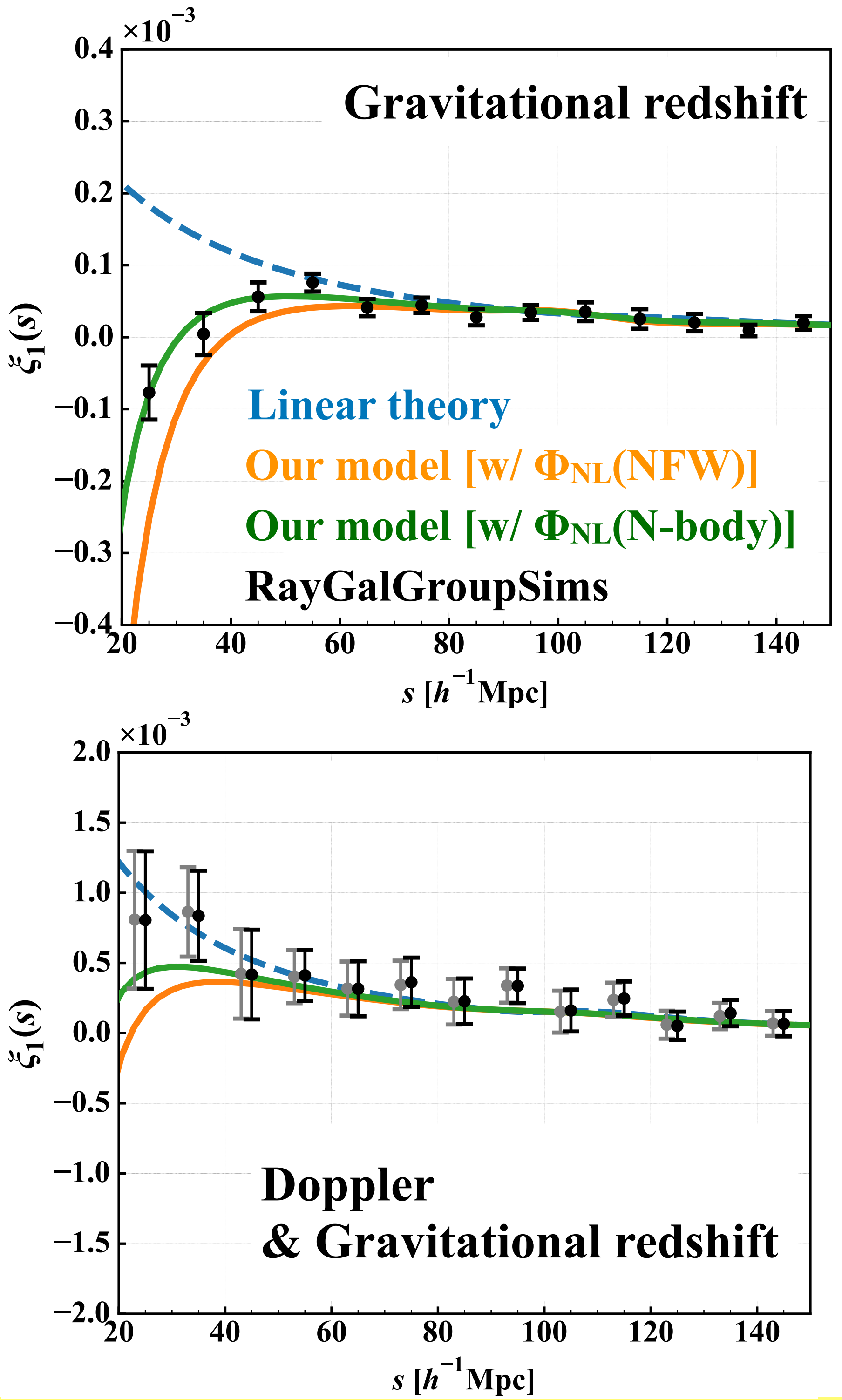


S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

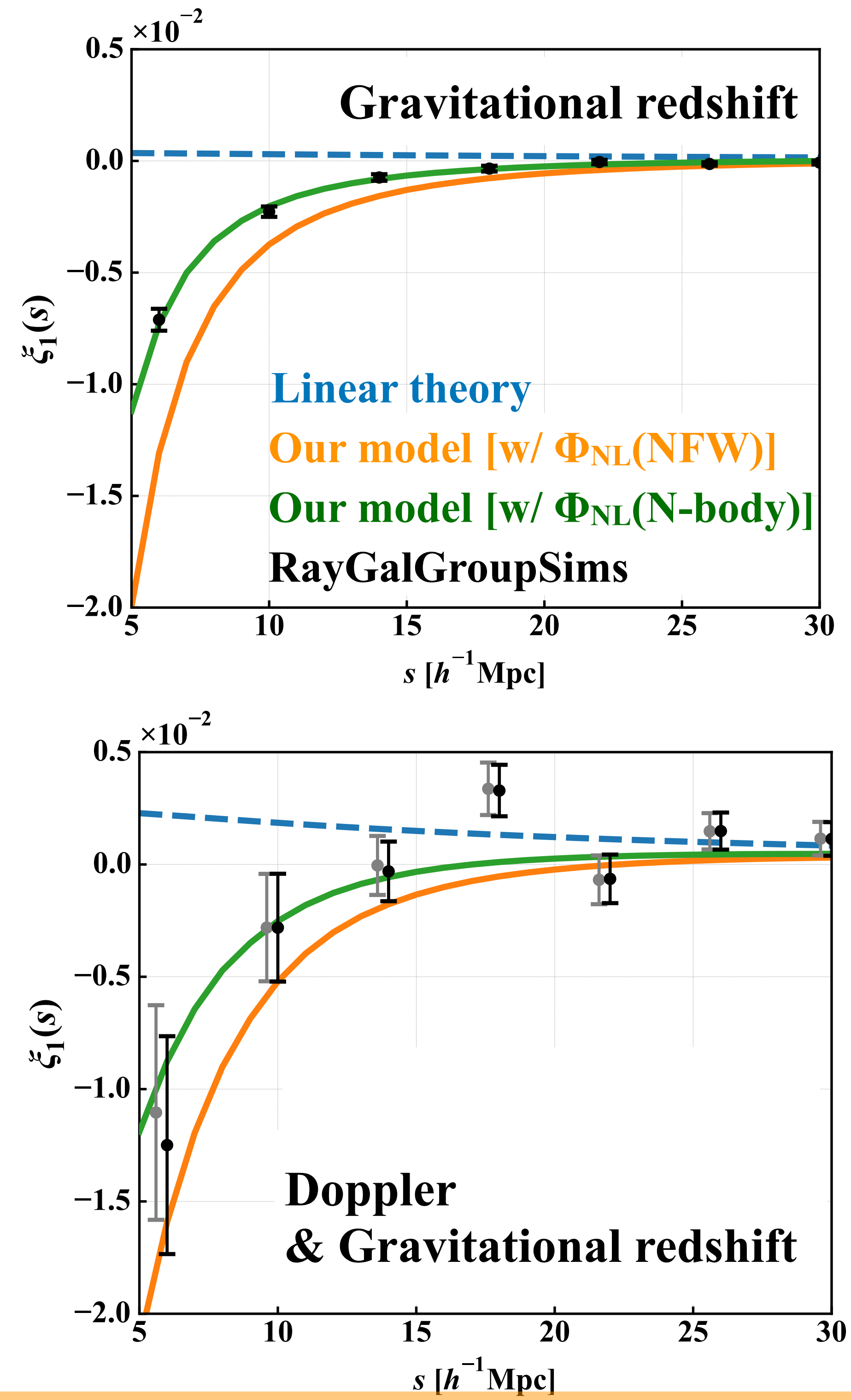
4.4 low-redshift (z = 0.33)

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

Large scales



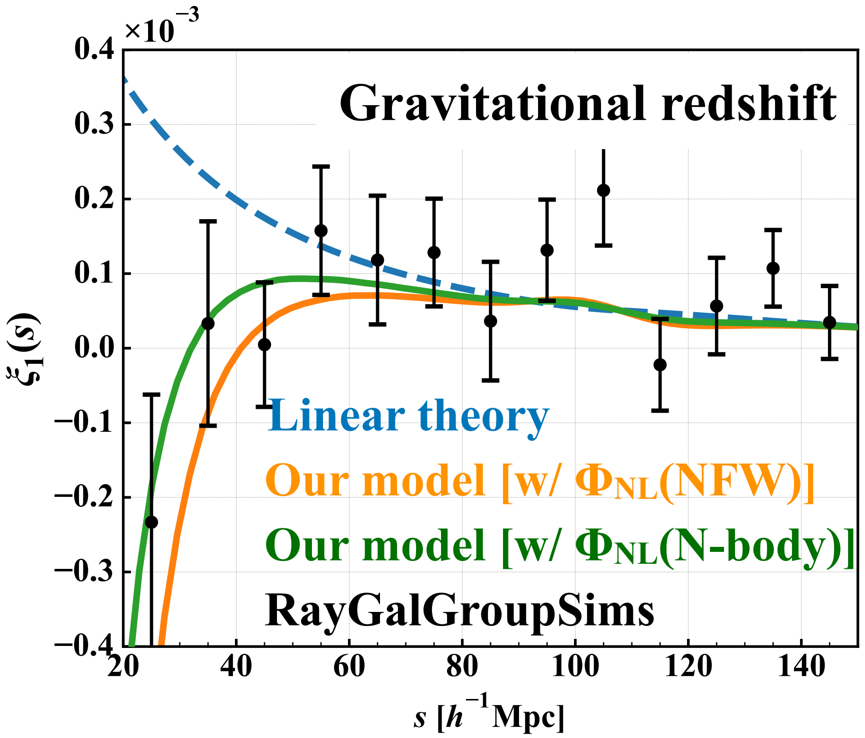
Small scales

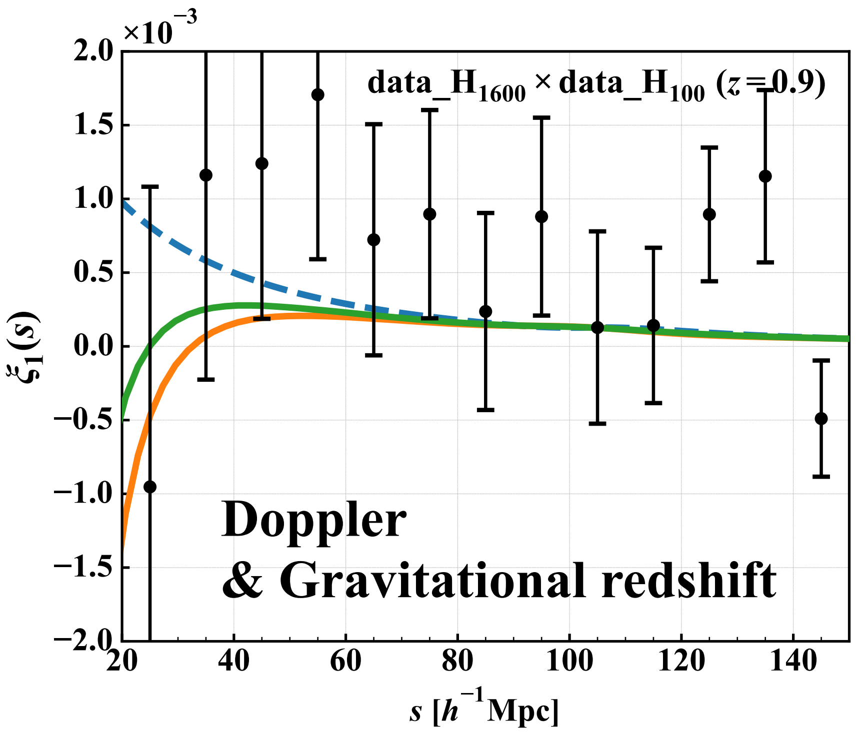


4.5 high-redshift (z = 0.90)

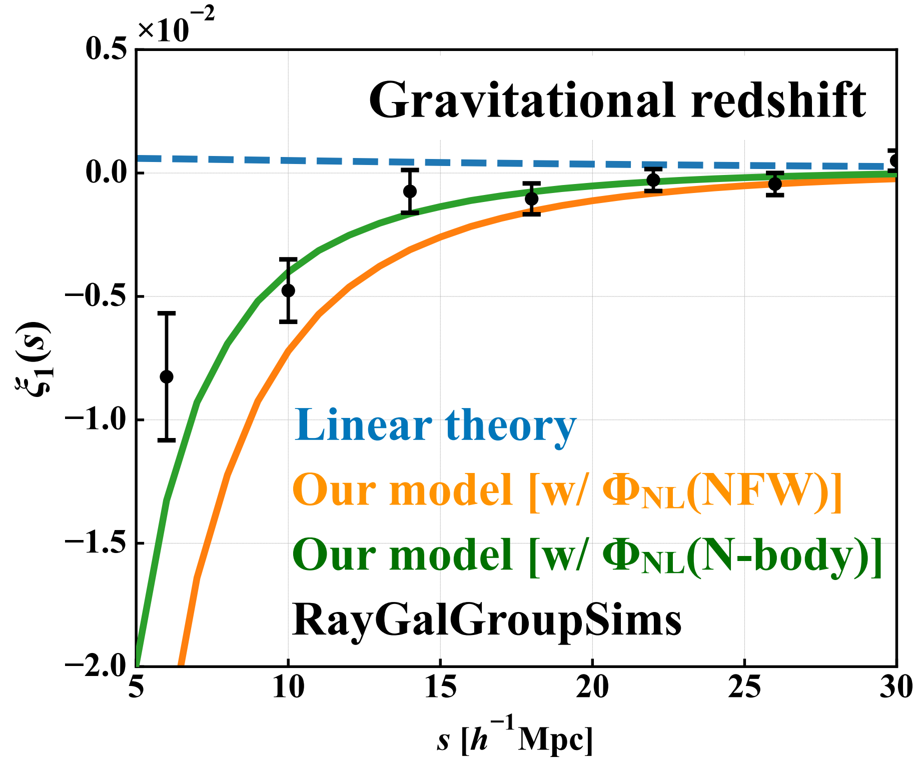
S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

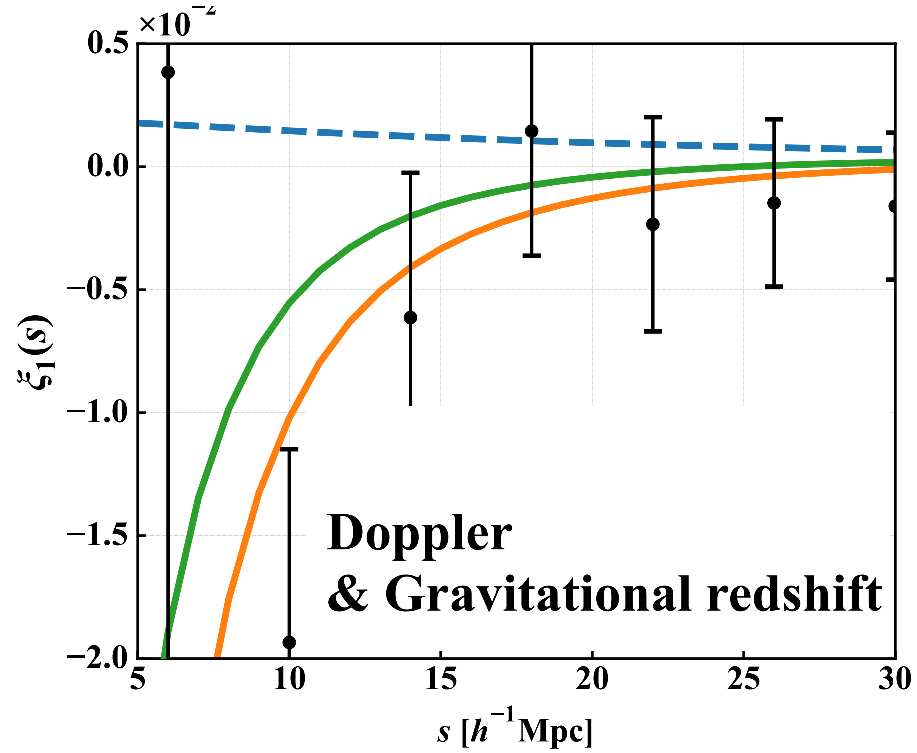
Large scales





Small scales





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Based on Zel'dovich approximation + non-linear potential

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \frac{1+z}{H}\Phi$$

$$\Phi = \Phi_{\mathrm{L}} + \Phi_{\mathrm{NL}}$$

We construct the **quasi-linear model** taking into account both **Doppler** and **Gravitational redshift** effects

- ✓ Our model describes RayGalGroupSims results
- ✓ Linear theory is recovered at large scales.
- ✓ Non-linear halo potential plays important role at small scales
- ✤ New probe of gravity?
- ✤ Detectability e.g., Euclid? **S.Saga et al. [in prep.]**