Precision calculations for dark matter

within the DM@NLO project

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CPTGA workshop on precision calculation — 19th november 2019 — Annecy

Dark matter relic abundance — freeze-out picture

Time evolution of number density of the relic particle described by Boltzmann equation
— key ingredient from particle physics: **(co-)annihilation cross-section**

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3Hn - \langle \sigma_{\mathrm{ann}} v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right)$$

$$\Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_{\mathrm{crit}}} \sim \frac{1}{\langle \sigma_{\mathrm{ann}} v \rangle}$$

$$\langle \sigma_{\mathrm{ann}} v \rangle = \sum_{i,j} \sigma_{ij} v_{ij} \frac{n_i^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}} \frac{n_j^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}}$$

(dis)favoured parameter regions…?

$$\Omega_{\mathrm{CDM}} h^2 = 0.1200 \pm 0.0012$$

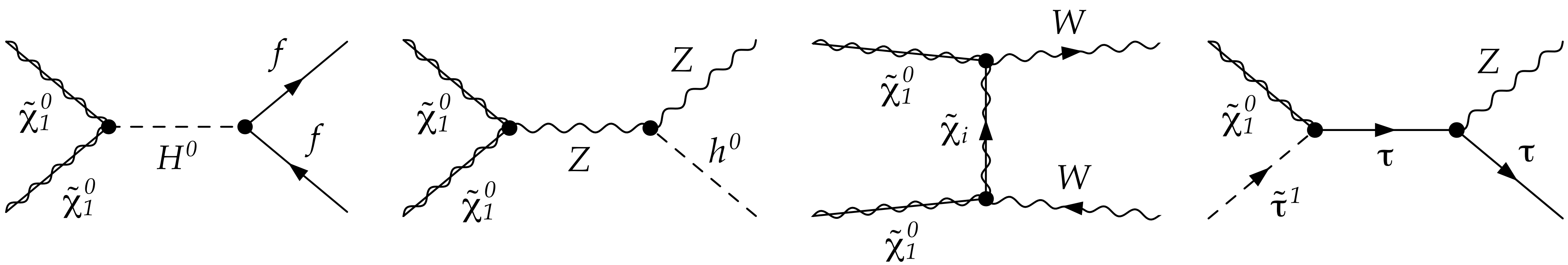
$$\frac{n_i^{\mathrm{eq}}}{n_\chi^{\mathrm{eq}}} \sim \exp\left\{ -\frac{m_i - m_\chi}{T} \right\}$$

Relic abundance known to 1% accuracy
Planck collaboration 2018

Computational tools allow an efficient calculation of the (neutralino) relic density:
DarkSUSY Bergström, Edsjö, Gondolo *et al.* 2004-2019, **micrOMEGAs** Bélanger, Boudjema, Pukhov *et al.* 2003-2019, **SuperIsoRelic** Arbey, Mahmoudi 2008, **MadDM** Arina, Backovic, Maltoni, Mantani, Mattelaer *et al.* 2015-2019, …

Motivation for higher order corrections

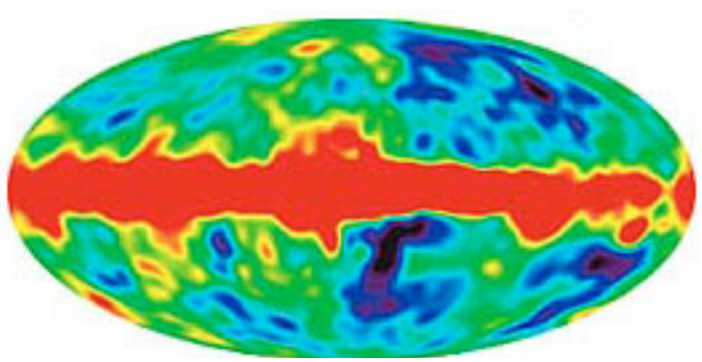
All processes implemented in public codes — **but only at the (effective) tree-level**



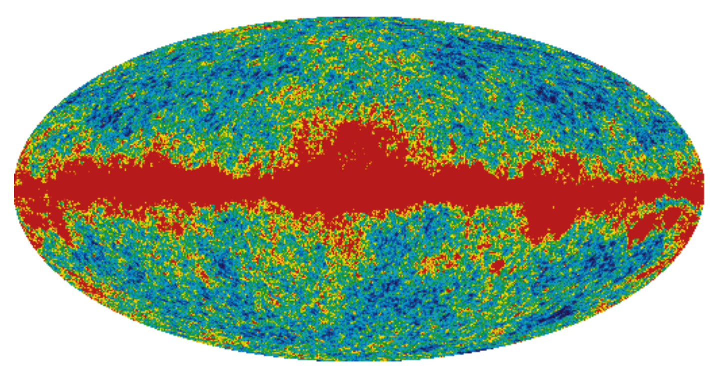
Higher-order loop corrections can give important contributions to cross-sections

In particular, sizeable impact from QCD corrections due to strong coupling constant

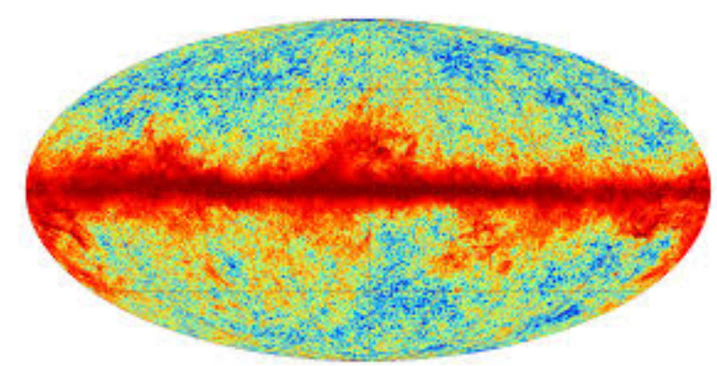
More precise theoretical predictions needed to keep up with experimental improvements



COBE 1989



WMAP 2002

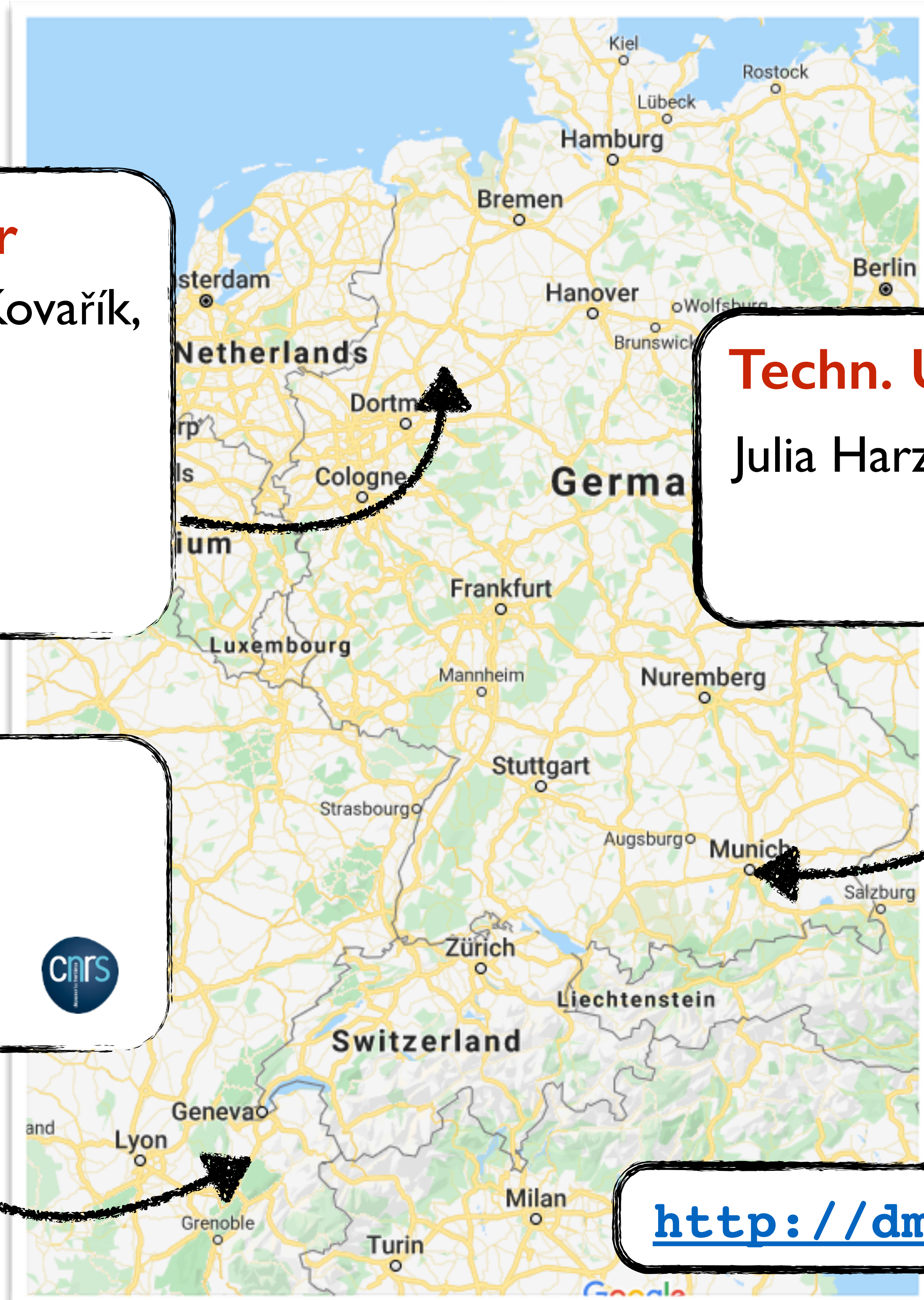


Planck 2013

DM@NLO project — **Provide calculation of σ_{ann} including QCD corrections**

— Extension to public codes (e.g. micrOMEGAs, DarkSUSY)…

DM@NLO — Collaboration



Universität Münster

Michael Klasen, Karol Kovařík, Johannes Branahl, …

Westfälische Wilhelms-Universität Münster

Graduiertenkolleg 2149 Research Training Group

Techn. Universität München

Julia Harz, …

TUM

Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

LAPTh Annecy

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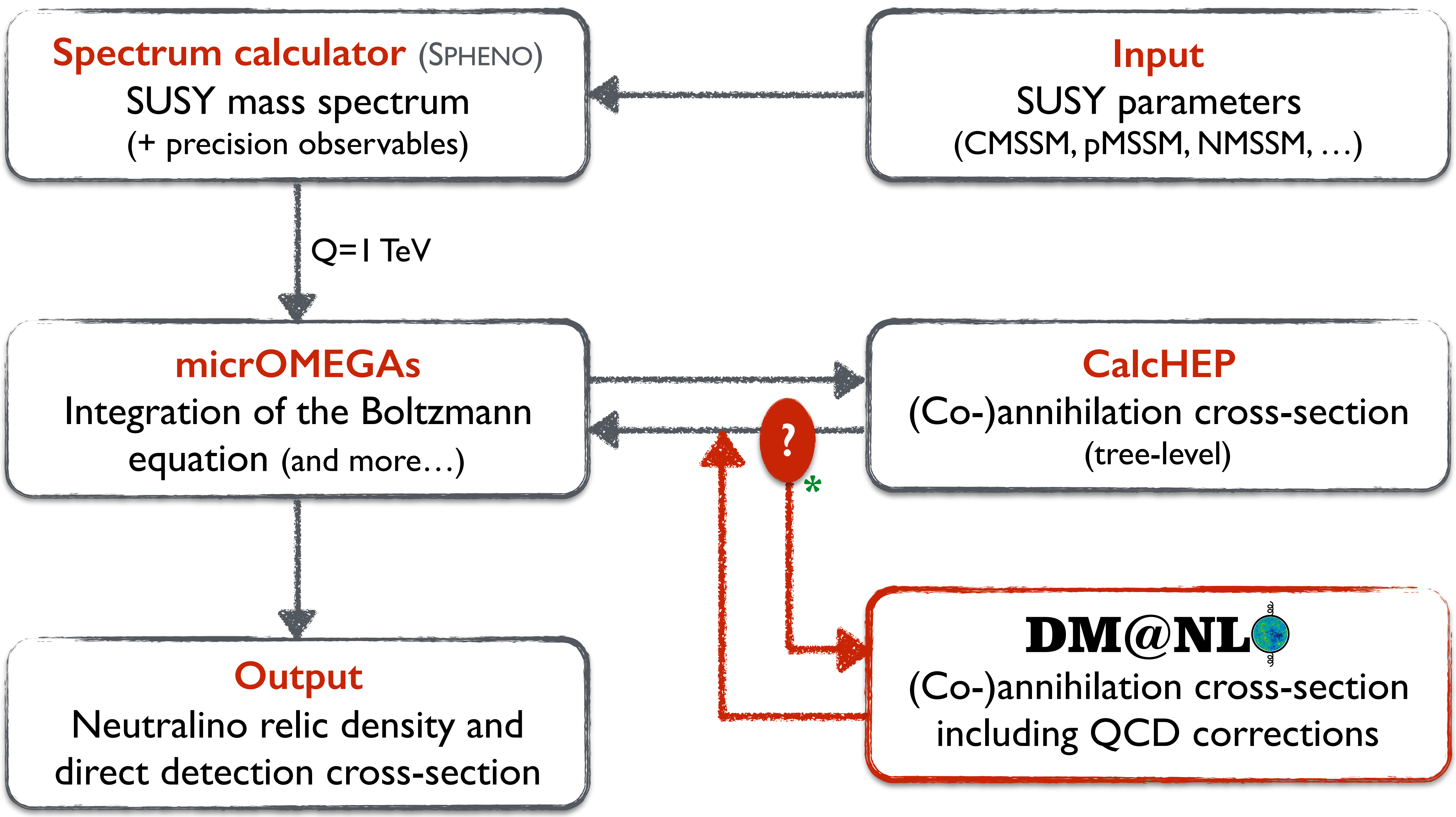
Université Savoie Mont Blanc

cnrs

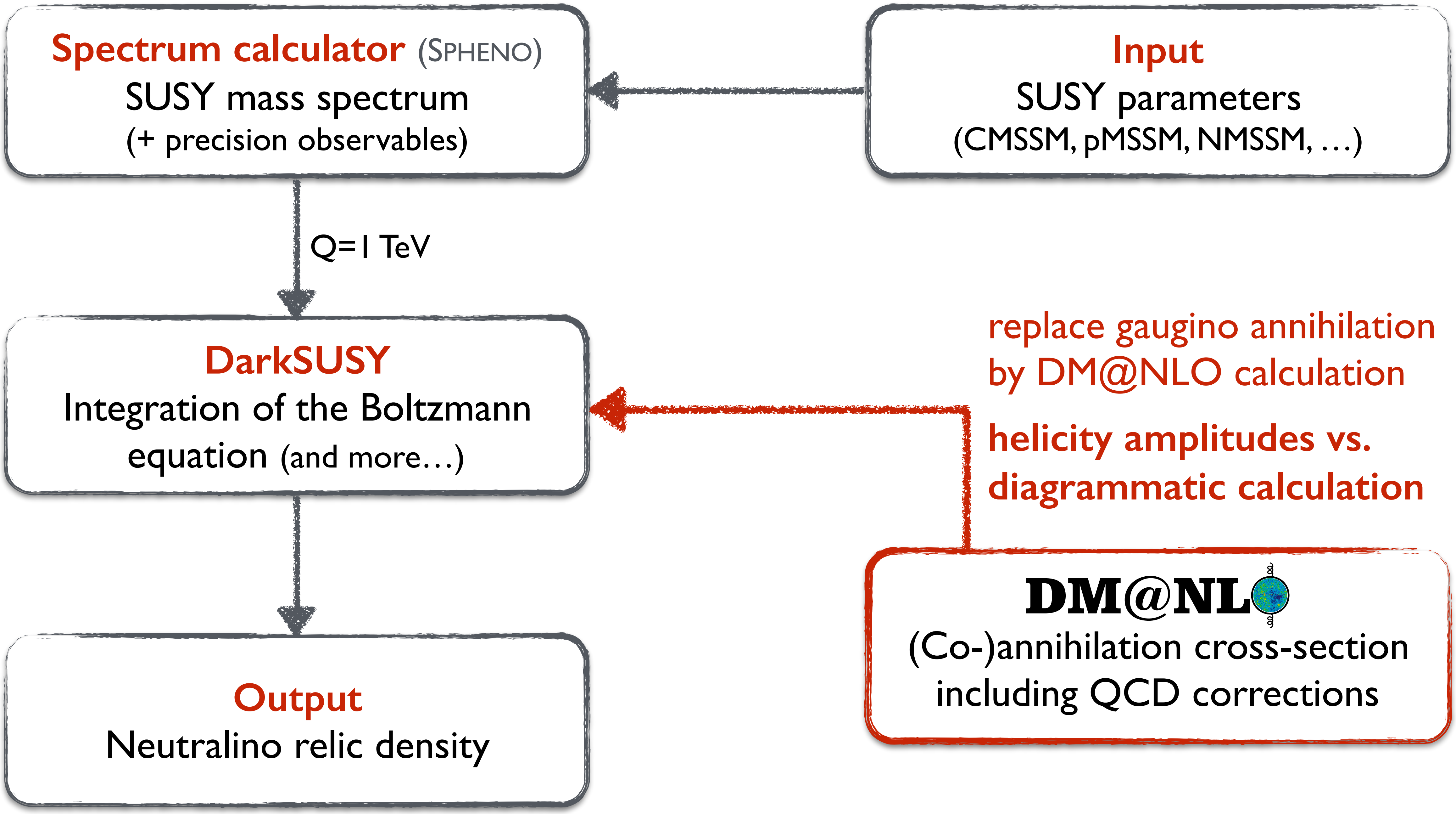
http://dmnlo.hepforge.org

DM@NLO — Setup



* Thanks to A. Pukhov!

DM@NLO — Setup



Ultimate goal: use within global fitting studies…

C. Niblaeus, J. Harz, B. Herrmann, J. Edsjö — *ongoing…*

DM@NLO — Status

Provide a **next-to-leading order calculation** (in QCD) for the following (co-)annihilation cross sections (and thus for the dark matter relic density)

$\tilde{\chi}\tilde{\chi}' \to q\bar{q}'$	**implemented micrOMEGAs + DarkSUSY**
$\tilde{\chi}\tilde{q} \to q'H/q'V$	**implemented micrOMEGAs**
$\tilde{q}\tilde{q}^* \to HH/HV/VV$	**implemented micrOMEGAs**
$\tilde{q}\tilde{q}^* \to q\bar{q}'$	**implemented micrOMEGAs *(to be published soon)***
$\tilde{q}\tilde{q} \to qq$	**implemented micrOMEGAs**
$\tilde{\tau}\tilde{\tau}^* \to qq'$	**implemented micrOMEGAs**
$\tilde{q}\tilde{q}^* \to gg$	**to be done…**
$\tilde{\chi}\tilde{\chi}' \to gg/\gamma\gamma$	**done but not fully checked — relevant…?**

Definition and implementation of a dedicated **renormalization scheme**

Infrared treatment — phase space slicing and dipole subtraction à la Catani-Seymour

Resummation of **Coulomb corrections** for scalar-scalar annihilation

Outline

Motivation and introduction

Brief overview of technical details

Some numerical examples

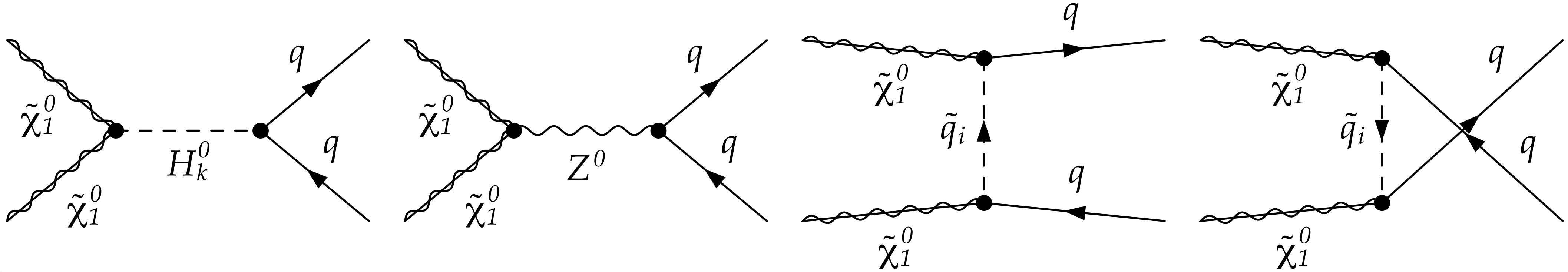
Theoretical uncertainty

Conclusion and Outlook

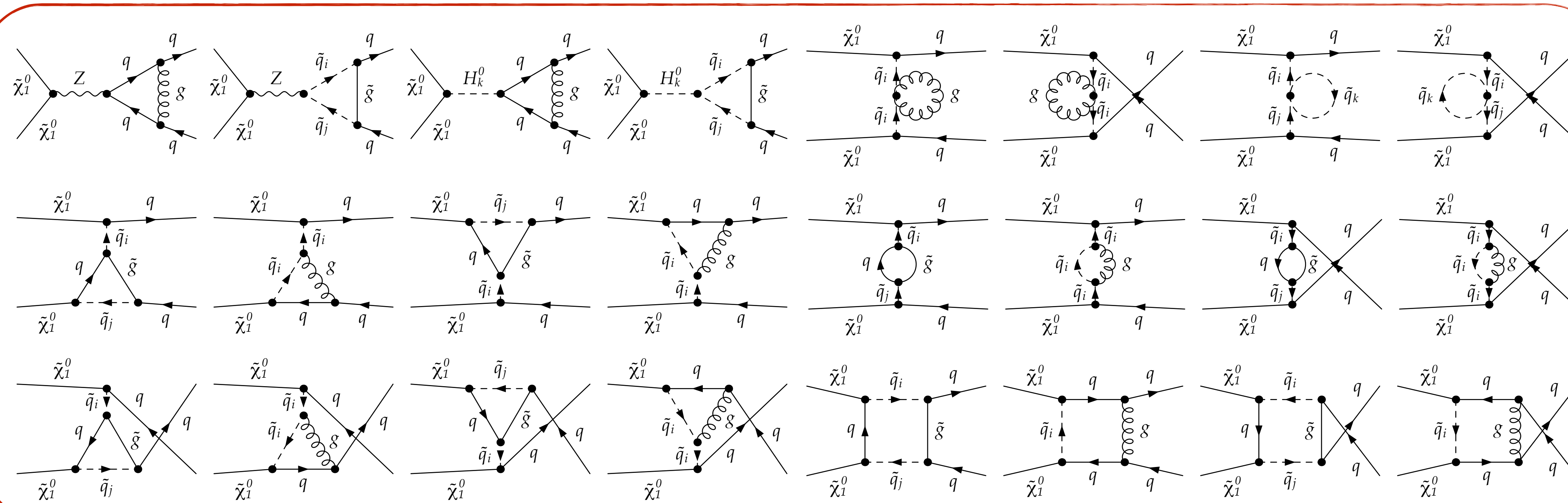
J. Branahl, J. Harz, B. Herrmann, M. Klasen, K. Kovařík, S. Schmiemann — Phys. Rev. D accepted — arXiv:1909.09527 [hep-ph]
S. Schmiemann, J. Harz, B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D99: 095015 — arXiv:1903.10998 [hep-ph]
M. Klasen, K. Kovařík, P. Steppeler — Phys.Rev. D94: 095002 (2016) — arXiv:1607.06396 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, P. Steppeler — Phys. Rev. D 93: 114023 (2016) — arXiv:1602.08103 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke — Phys. Rev. D 91: 034012 (2015) — arXiv:1410.8063 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 91: 034028 (2015) — arXiv:1409.2898 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke, P. Steppeler — Phys. Rev. D 89: 114012 (2014) — arXiv:1404.2931 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, Q. Le Boulc'h — Phys. Rev. D 87: 054031 (2013) — arXiv:1212.5241 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 79: 061701 (2009) — arXiv:0901.0481 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 80: 085025 (2009) — arXiv:0907.0030 [hep-ph]
B. Herrmann, M. Klasen — Phys. Rev. D 76: 117704 (2007) — arXiv:0709.0043 [hep-ph]

Technical details

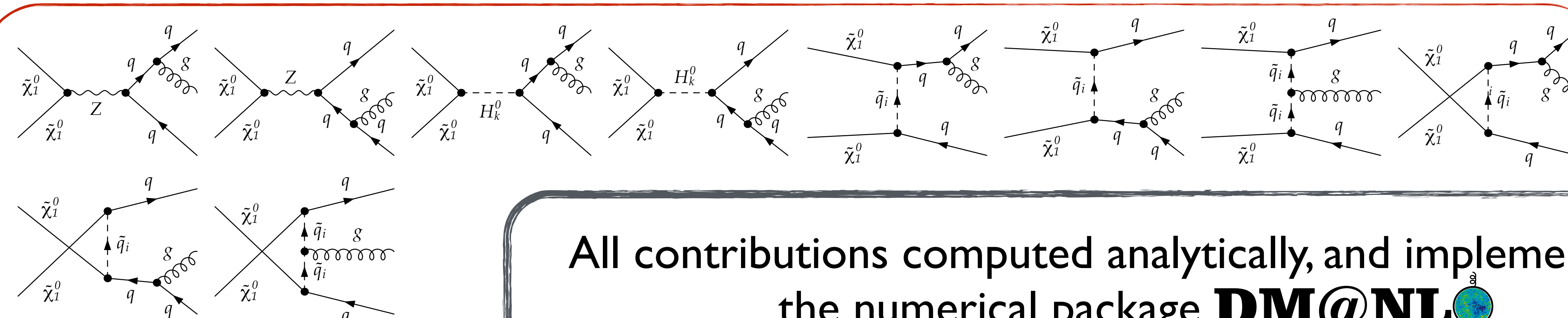
Neutralino pair annihilation into quarks



Loop corrections



Real emission



Herrmann, Klasen (2007); Herrmann, Klasen, Kovarik (2009),
Herrmann, Klasen, Kovarik, Meinecke, Steppeler (2013)

All contributions computed analytically, and implemented in the numerical package **DM@NLO**
(goal: extension to existing dark matter codes)

Ultraviolet and infrared treatment

Loop diagrams include UV-divergent integrals → **Renormalization!**

Hybrid on-shell/$\overline{\text{DR}}$ renormalization scheme for the squark sector (3rd generation), which is applicable to all (co)annihilation processes

$$A_t, A_b \qquad m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$$

Independent "input" parameters

→

$$M^2_{\tilde{Q}}, M^2_{\tilde{U}}, M^2_{\tilde{D}} \qquad m_{\tilde{t}_2}, \theta_{\tilde{t}}, \theta_{\tilde{b}}$$

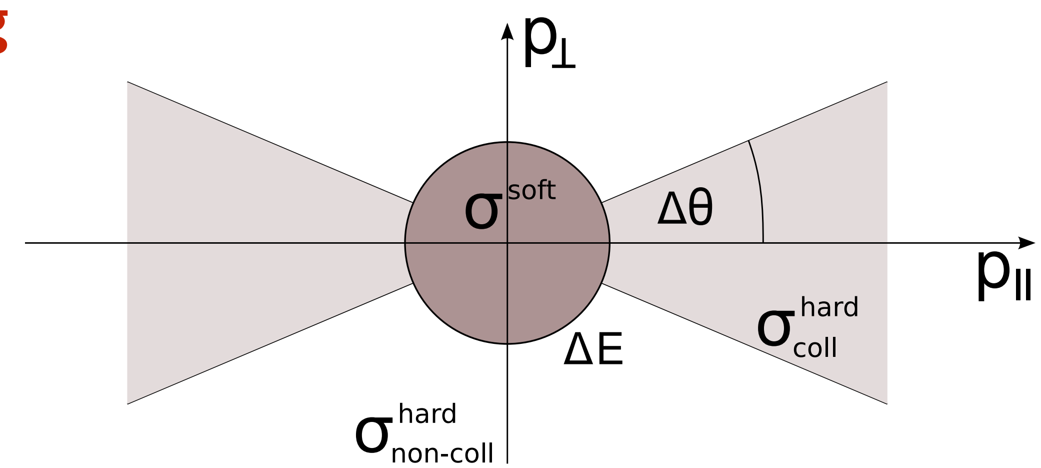
Dependent parameters

Loop diagrams contain **IR-divergencies** (soft and/or collinear), which vanish when taking into account the real emission of a gluon (2→3 processes)

Dipole Subtraction Method and **Phase Space Slicing**

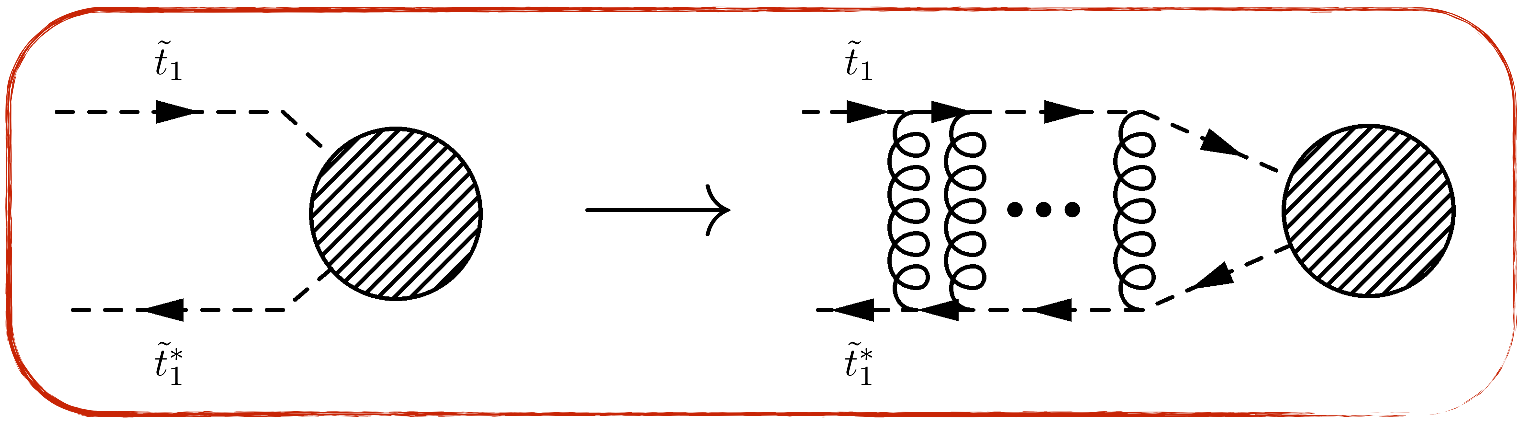
Catani, Seymour (2001)

$$\sigma_{\text{NLO}} = \int_3 \left[\mathrm{d}\sigma^{\mathrm{R}} \Big|_{\epsilon=0} - \mathrm{d}\sigma^{\mathrm{A}} \Big|_{\epsilon=0} \right] + \int_2 \left[\mathrm{d}\sigma^{\mathrm{V}} + \int_1 \mathrm{d}\sigma^{\mathrm{A}} \right]_{\epsilon=0}$$



Coulomb corrections and loop integrals

Exchange of multiple gluons in the initial state (in addition to one-loop diagrams)
— **resummation to all orders using non-relativistic QCD**



$$\sigma^{\text{Coul}} = \frac{4\pi}{v m_{\tilde{t}}^2} \Im\left\{ G^{[1]}\left(\mathbf{r}=0; \sqrt{s} + i\Gamma_{\tilde{t}}\right)\right\} \sigma^{\text{LO}}$$

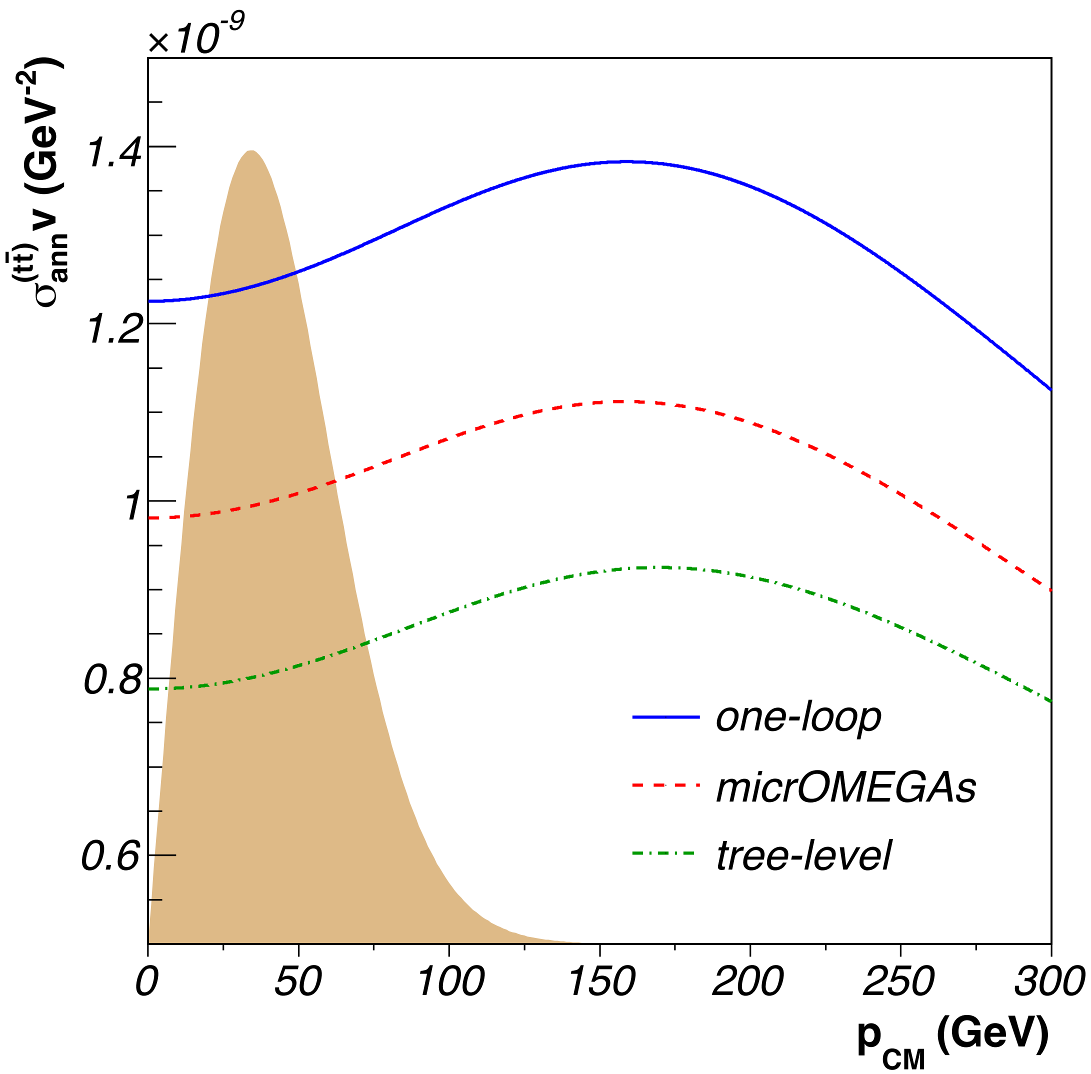
Similar treatment for **QED Sommerfeld enhancement** (e.g., stau-antistau annihilation, …)

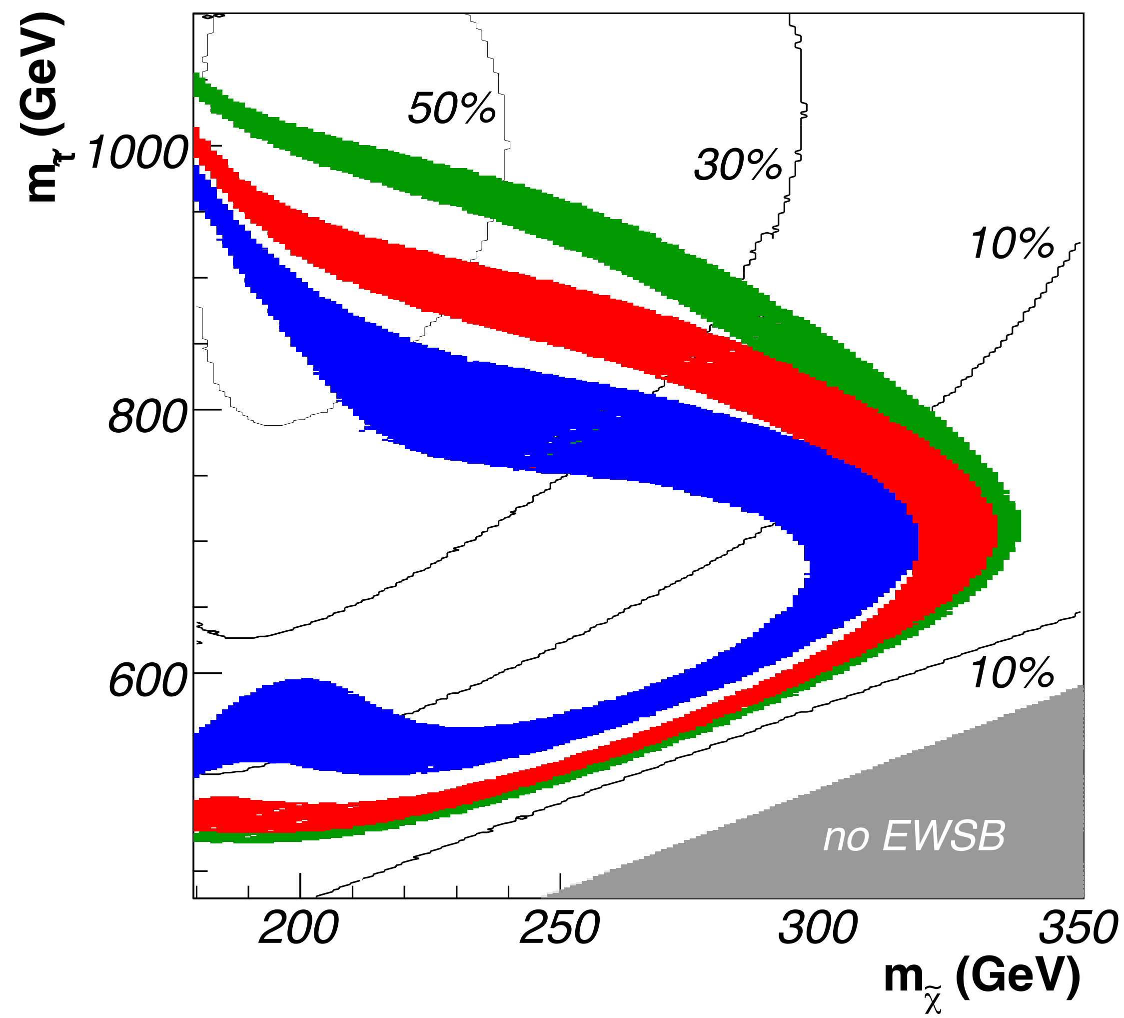
Avoid double counting of NLO corrections contained in Green's function and one-loop result!

DM@NLO code includes **independent library for numerical evaluation of loop integrals**
— Passarino-Veltman reduction including numerical special cases (i.e. special argument sets)
— special reduction for **vanishing Gram determinants** ($v \to 0$, application to direct/indirect detection)

Numerical examples

Neutralino pair annihilation into top quarks





Annihilation cross-section enhanced by up to 50% by radiative corrections

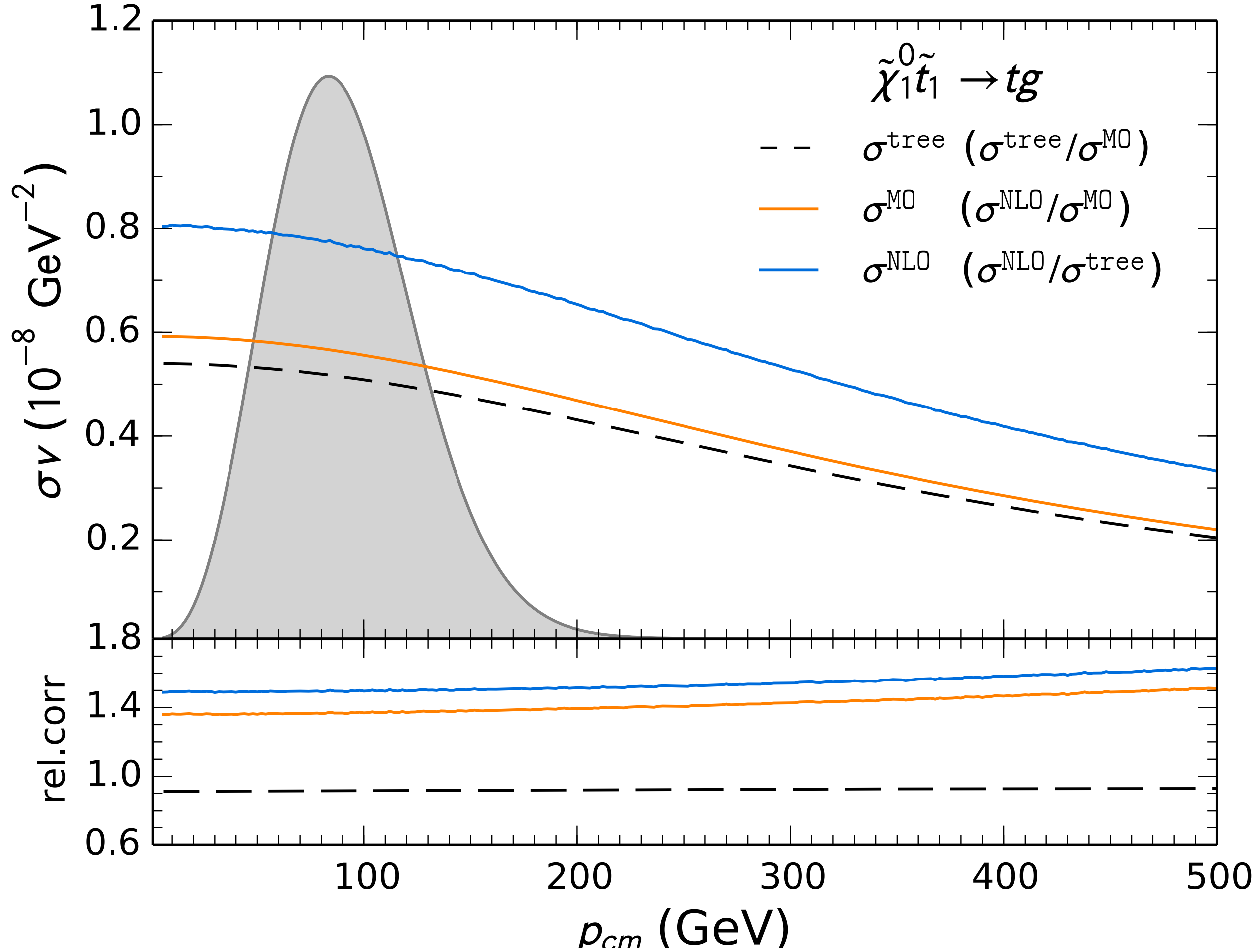
Corrections can lead to **important shifts for preferred regions** (e.g. ~200 GeV for m_{stop})

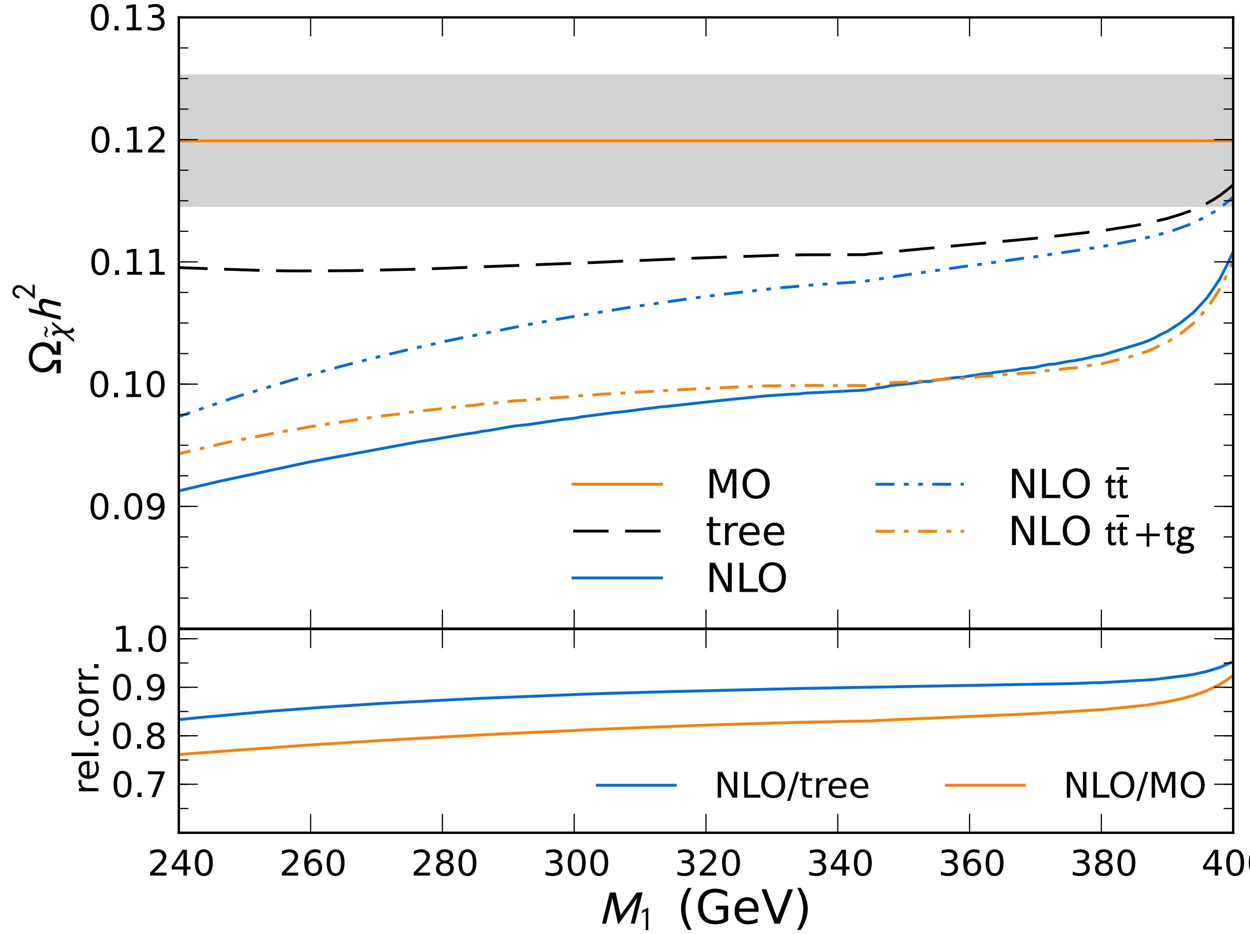
Effective Yukawa couplings (as e.g. in micrOMEGAs) very good approximation around Higgs-resonances, **but other sub-channels can be dominant** (here: Z^0/squark-exchange)

B. Herrmann, M. Klasen, K. Kovařík — Phys. Rev. D 80: 085025 (2009) — arXiv:0907.0030 [hep-ph]

B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke, P. Steppeler — Phys. Rev. D 89: 114012 (2014) — arXiv:1404.2931 [hep-ph]

Neutralino-stop co-annihilation



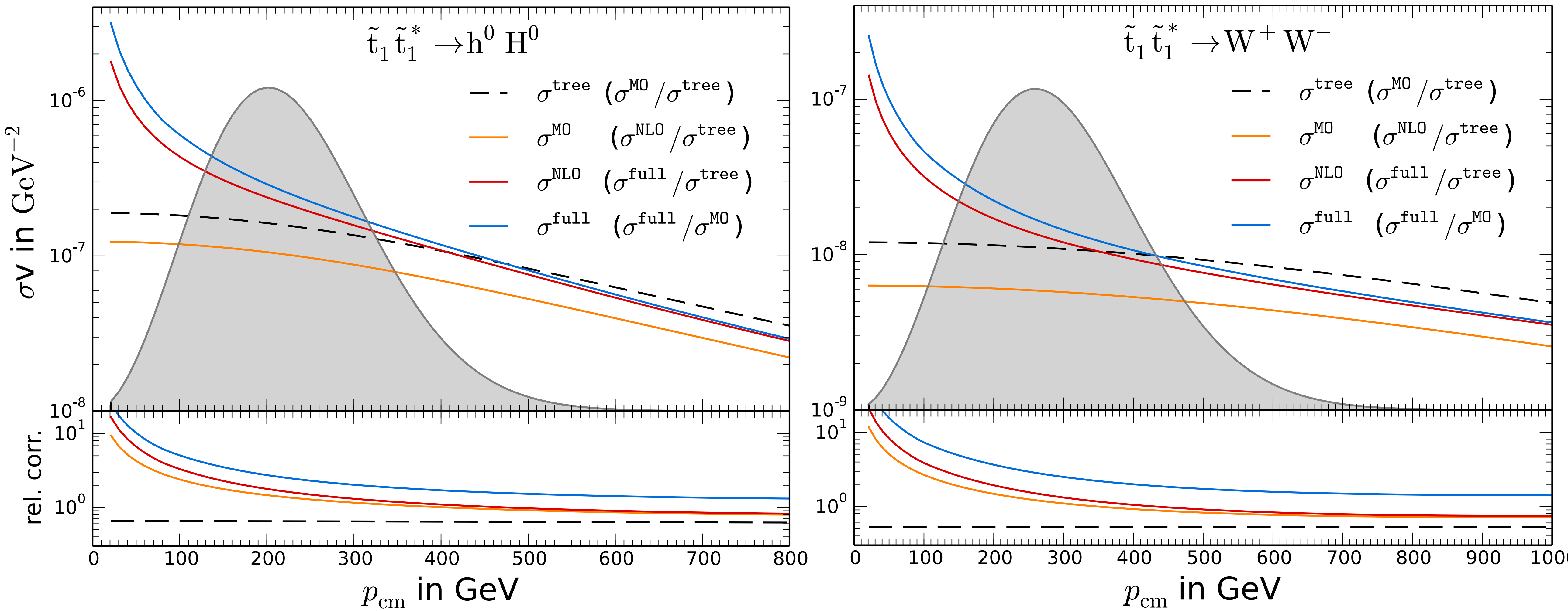


Relative corrections of up to 40-50% observed for the co-annihilation cross-section, leading to a **numerically important shift** for the predicted **neutralino relic density** (up to about 25% — more than Planck uncertainty!)

Co-annihilation into **SM-like Higgs** and gluon most important (other final states generally subdominant)

Harz, Herrmann, Klasen, Kovařík, Le Boulc'h — Phys. Rev. D 87: 054031 (2013) — arXiv:1212.5241 [hep-ph]
Harz, Herrmann, Klasen, Kovařík — Phys. Rev. D 91: 034028 (2015) — arXiv:1409.2898 [hep-ph]

Stop pair annihilation — electroweak final states

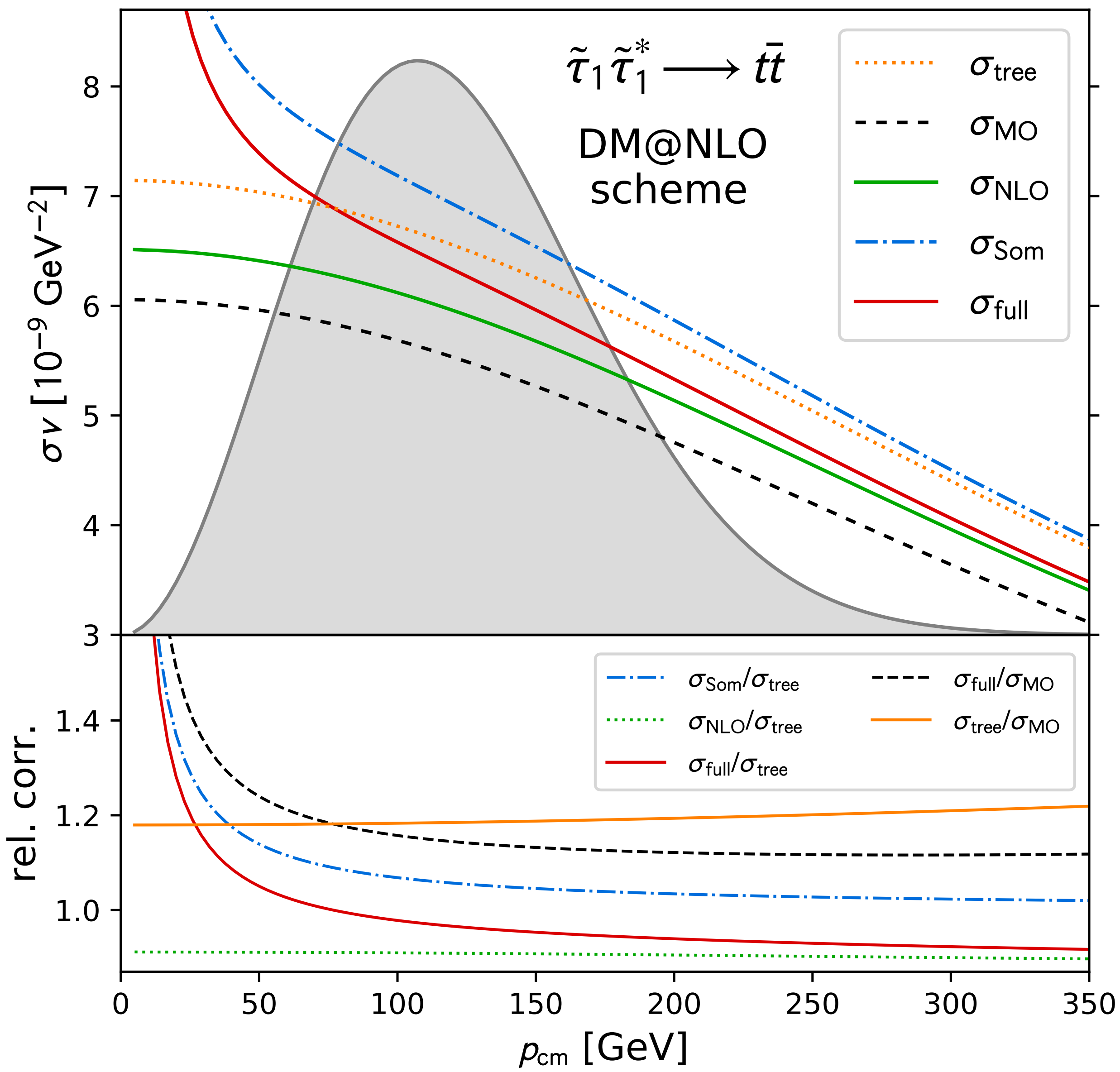


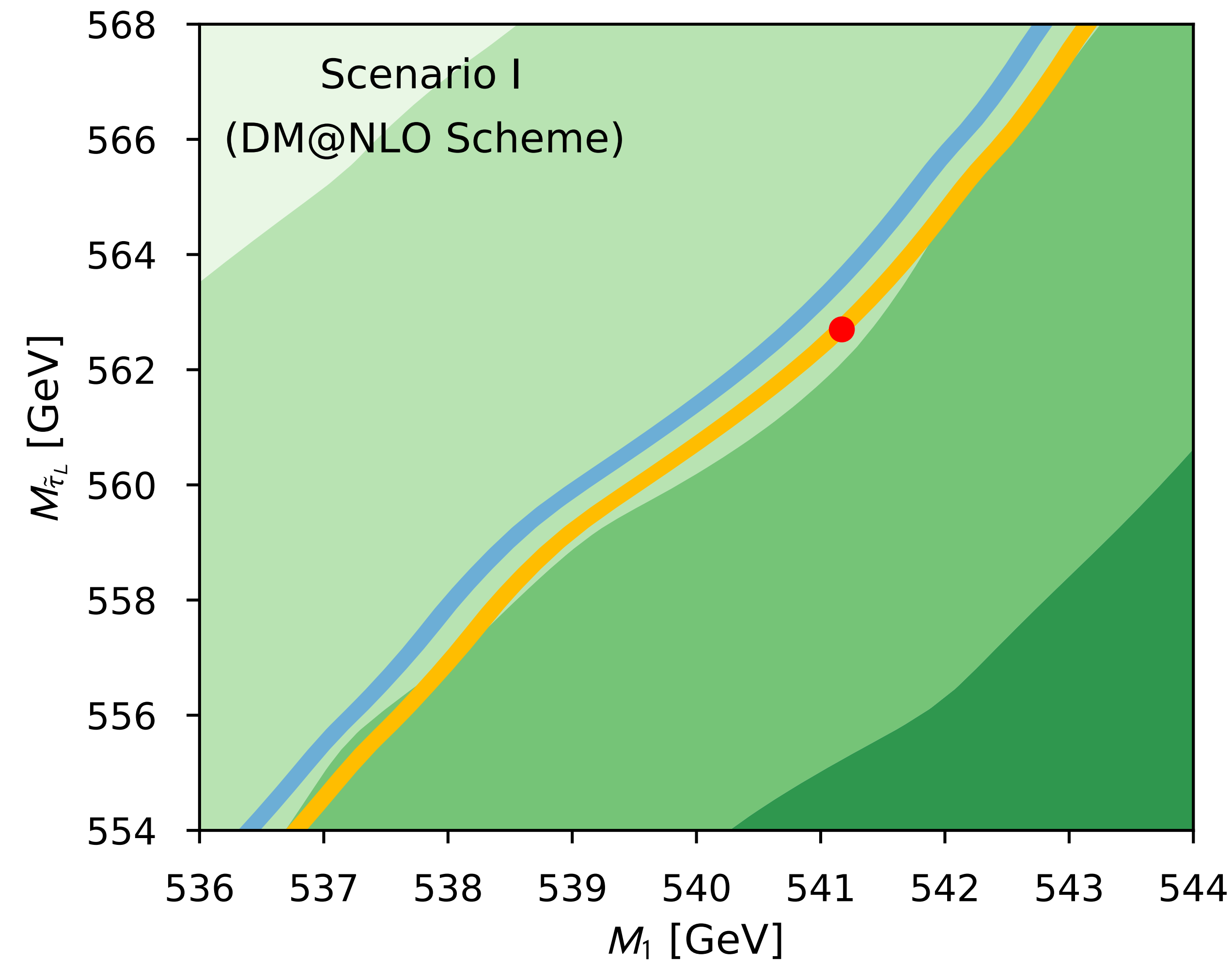
Coulomb corrections **dominant for small values of p_{cm}** (Coulomb singularity), while fixed-order corrections dominant for high-momentum region

Resulting relic density receives corrections of up to 40% (more important than Planck uncertainty!)

Harz, Herrmann, Klasen, Kovařík, Meinecke — Phys. Rev. D 91: 034012 (2015) — arXiv:1410.8063 [hep-ph]
Harz, Herrmann, Klasen, Kovařík, Schmiemann — *to be published…*

Stau pair annihilation into quarks





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J. Branahl, J. Harz, B. Herrmann, M. Klasen, K. Kovařík, S. Schmiemann — Phys. Rev. D accepted — arXiv:1909.09527 [hep-ph]

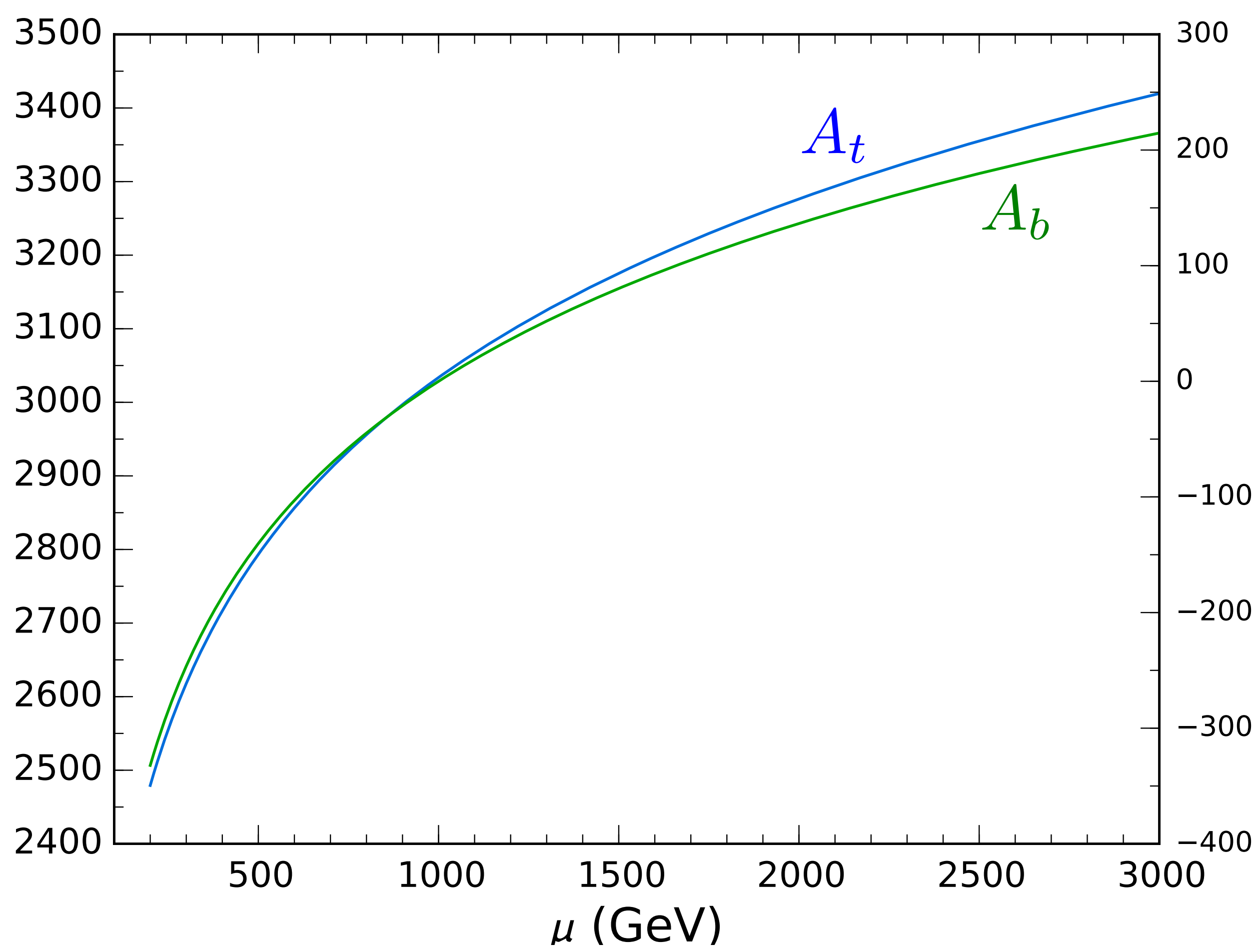
Theoretical uncertainty
Scheme and scale dependence

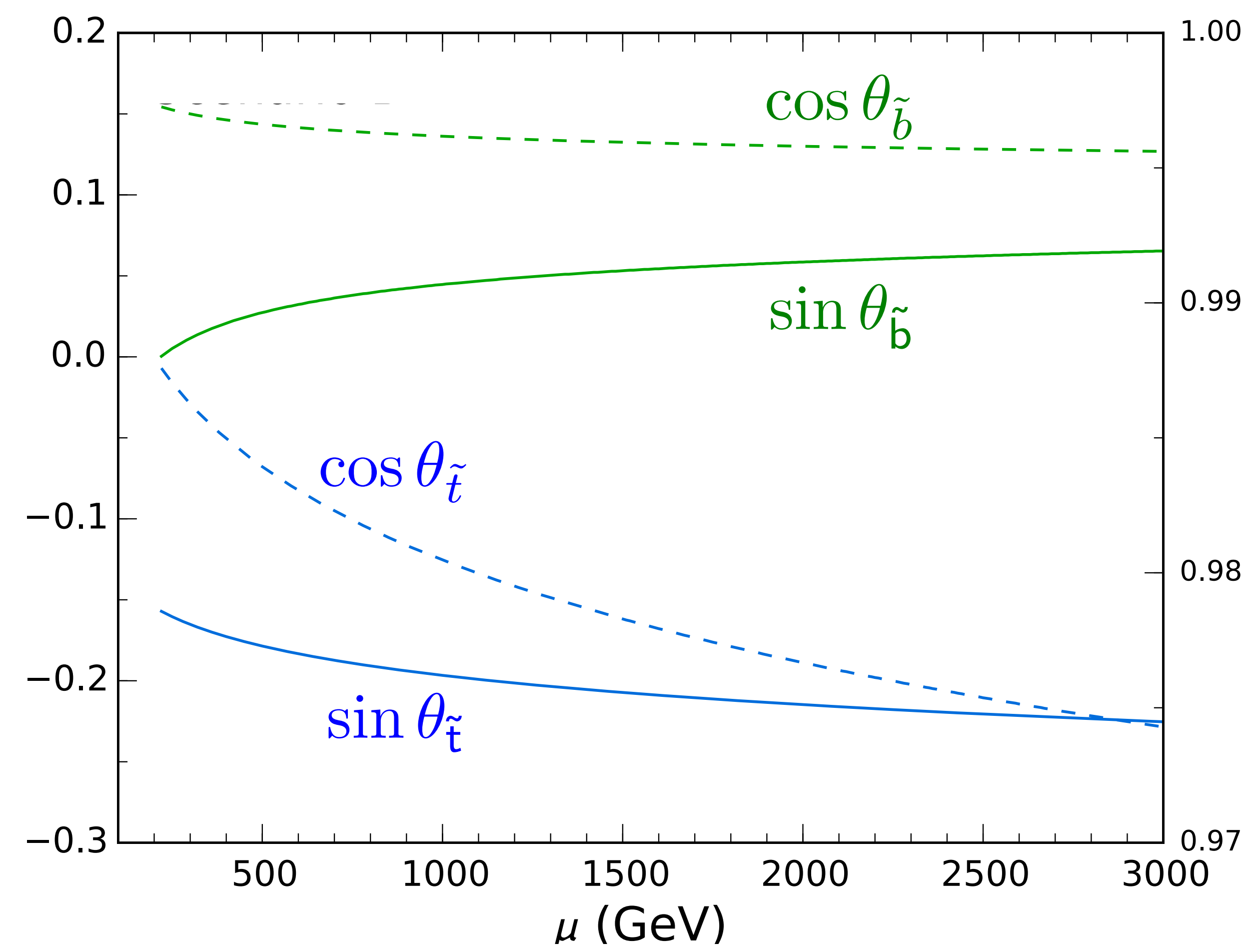
Scale dependence and theoretical uncertainty

Evaluation of theoretical uncertainty by **varying** (unphysical) **renormalization scale**
— hybrid on-shell / DRbar renormalization scheme designed for neutralino (co-)annihilation

$\mu_R = 500 \ldots 2000$ GeV → A_t, A_b, $\theta_{\tilde{t}}$, $\theta_{\tilde{t}}$, α_s, m_b

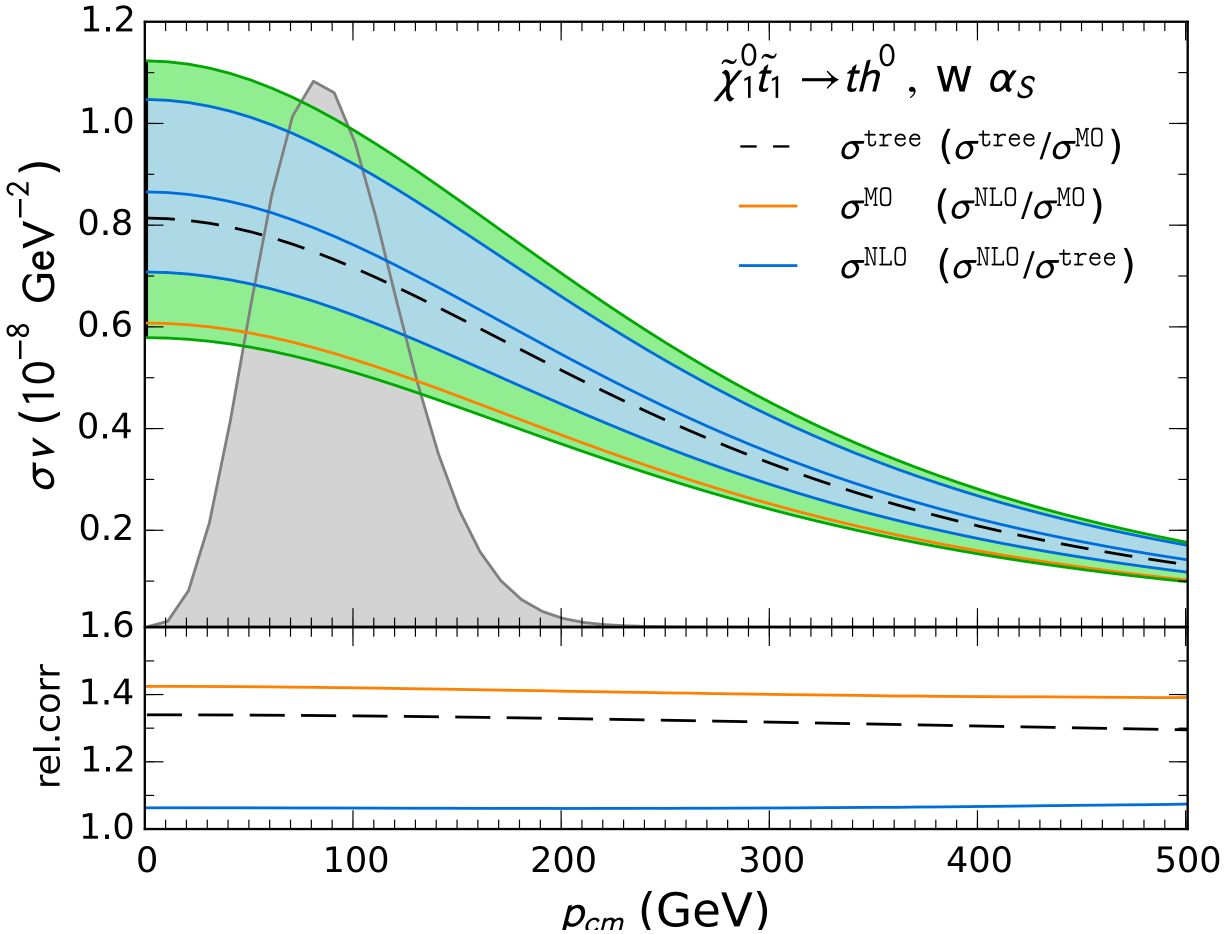
scale-dependent parameters

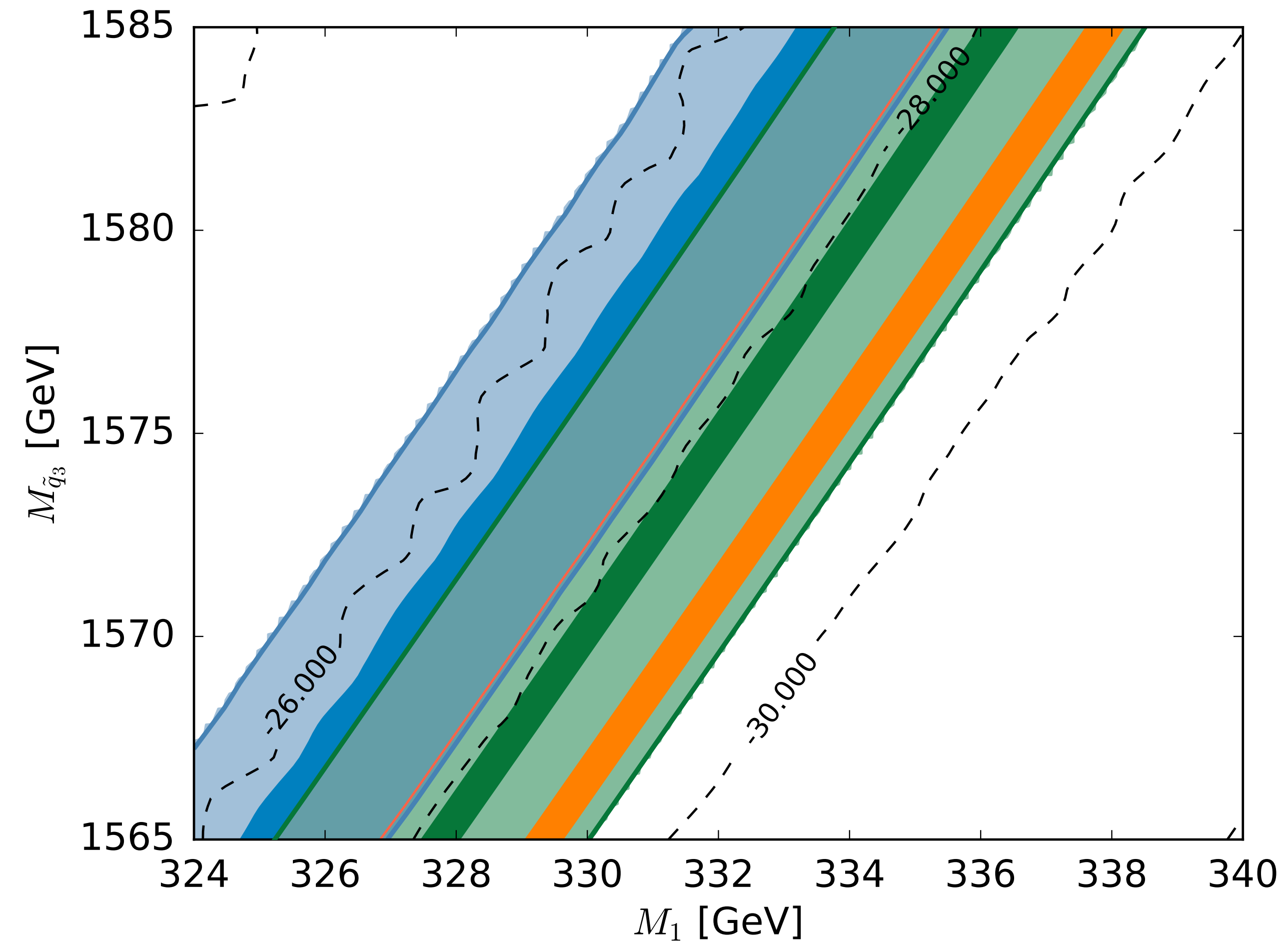




J. Harz, B. Herrmann, M. Klasen, K. Kovařík, P. Steppeler — Phys. Rev. D 93: 114023 (2016) — arXiv:1602.08103 [hep-ph]

Scale dependence and theoretical uncertainty



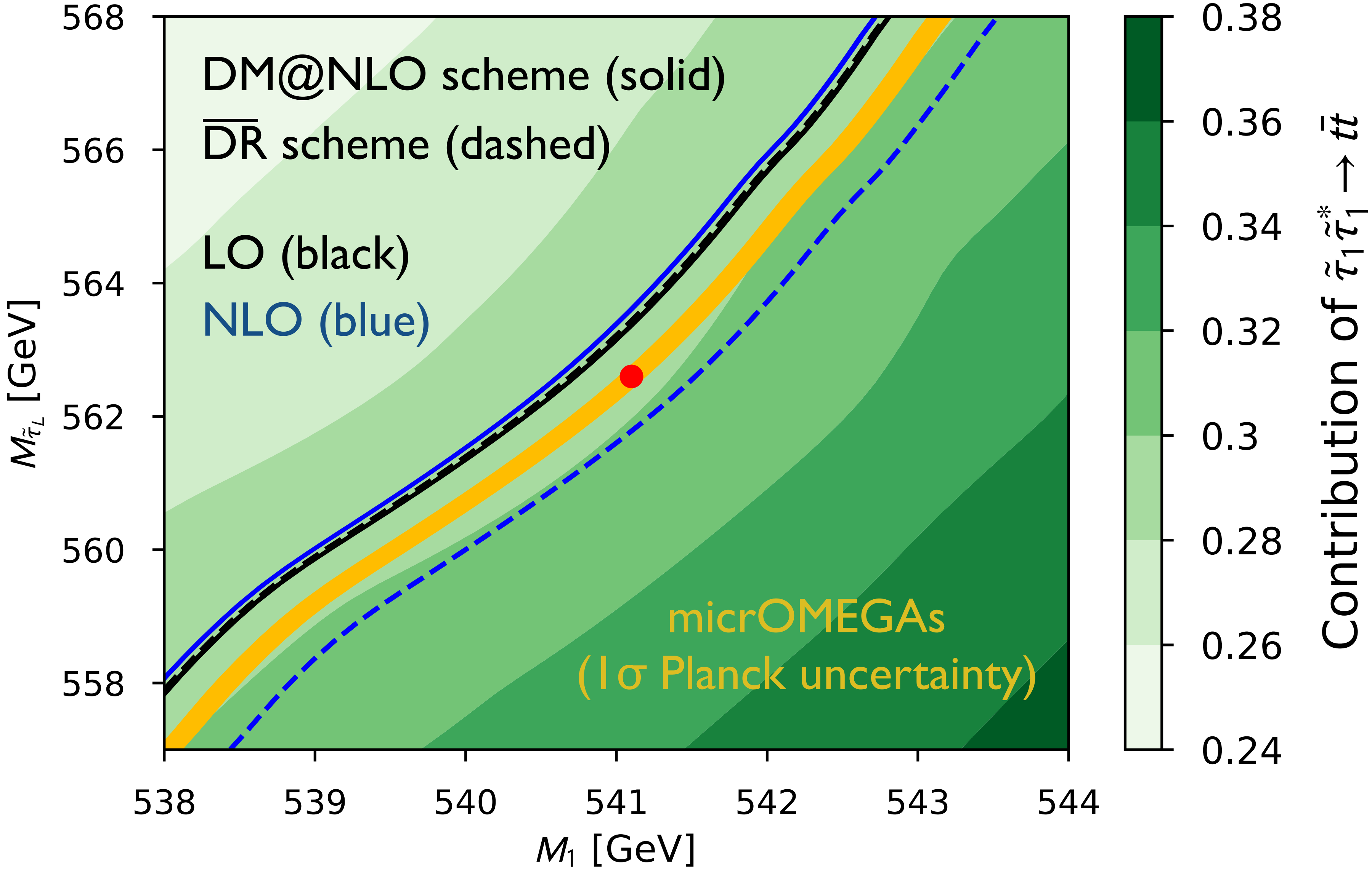


Within the scale uncertainty, the **tree-level result agrees** with the NLO calculation and the micrOMEGAs value

Scale uncertainty reduced at the one-loop level w.r.t. to tree-level result (as expected)
— main effect from **mixing angle** and **trilinear coupling**
— dependence of α_s subdominant

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, P. Steppeler — Phys. Rev. D 93: 114023 (2016) — arXiv:1602.08103 [hep-ph]

Scheme variation and theoretical uncertainty



$\overline{\text{DR}}$ scheme more sensitive to changes in the renormalization scale (not shown here)
Scheme uncertainty corresponds to the distance between two corresponding bands
— **NLO predictions very consistent**
— **theoretical uncertainty** (from scheme variation) **reduced at the NLO level**

J. Branahl, J. Harz, B. Herrmann, M. Klasen, K. Kovařík, S. Schmiemann — Phys. Rev. D accepted — arXiv:1909.09527 [hep-ph]

Conclusion

Summary and perspectives

Experimental improvements require more precise theory predictions for dark matter

DM@NLO — **calculation of neutralino (co-)annihilation including QCD corrections**

Impact of corrections on the relic density more important than current exp. uncertainty

— Higher-order corrections important when extracting parameters from cosmological data

Analysis of the theory uncertainty shows that the **relic density cannot always be predicted with a precision of 1-2%** similarly to the experimental result

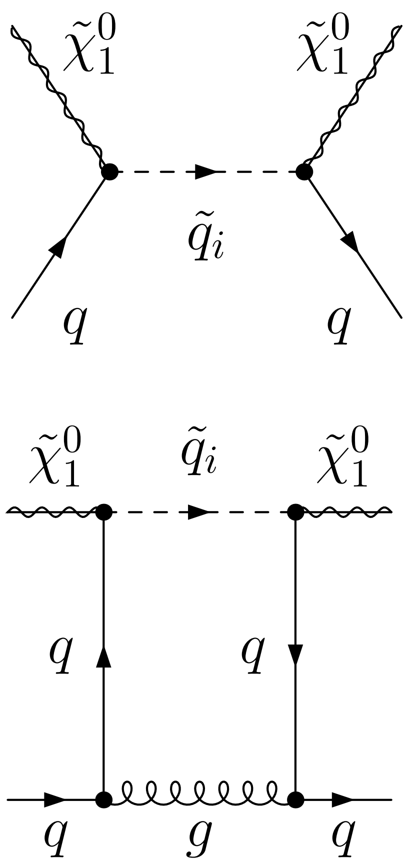
Next possible and interesting steps…

- — complete code with missing processes
- — extend to other new physics' models
- — provide some public form of the code…?
- — implement dipole subtraction scheme for all process classes
- — include calculation of the indirect detection cross-section…?

谢谢

Backup — application to direct detection

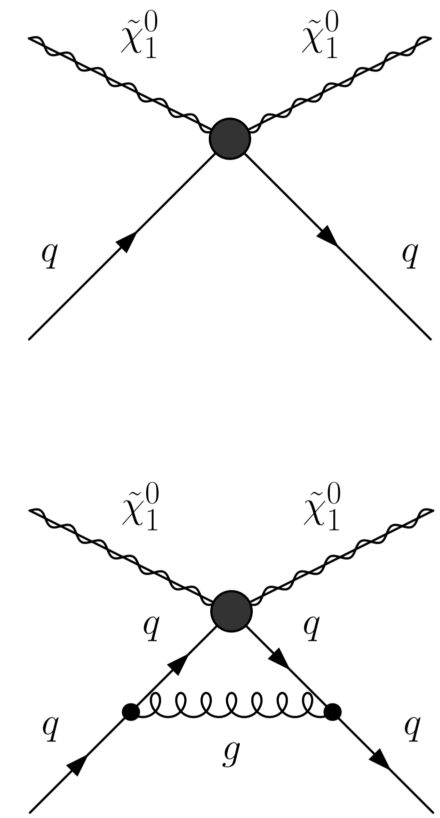
Corrections to direct dark matter detection



Full theory
(LO + NLO)
valid at Q~1 TeV

Matching
at Q~1 TeV

Effective theory
(LO + NLO)
valid at Q~5 GeV



Renormalization (same scheme as before) in order to treat ultraviolet divergencies

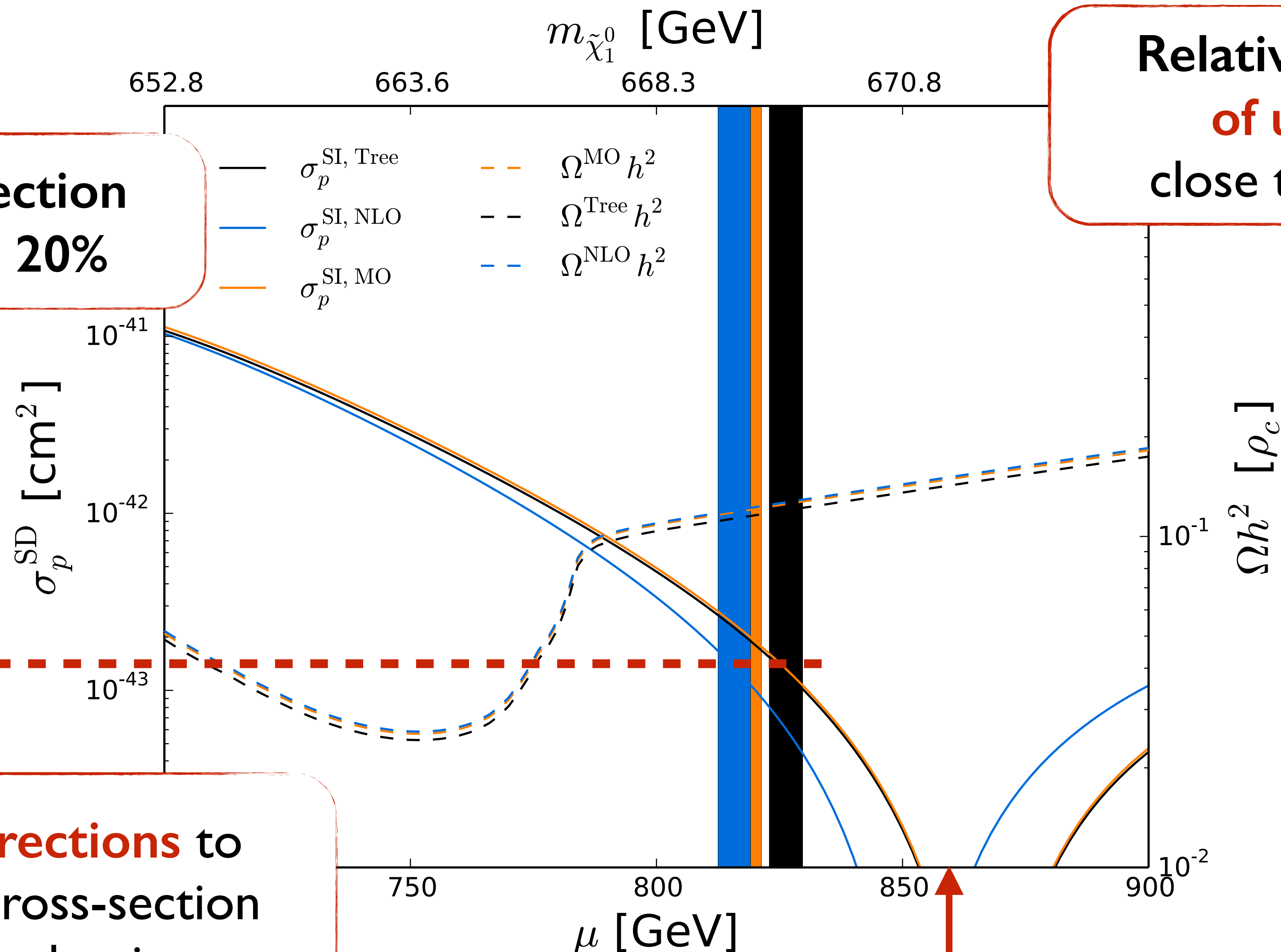
Infrared divergencies cancel between the different contributions

Dedicated **integral reduction procedure applicable to zero-velocity limit**

Renormalization group running of effective theory from Q~1 TeV to Q~5 GeV

M. Klasen, K. Kovařík, P. Steppeler — Phys.Rev. D94: 095002 (2016) — arXiv:1607.06396 [hep-ph]

Corrections to direct dark matter detection



Relative correction of about 10 - 20%

Relative correction
of up to 50%
close to the "drop"

Effect of corrections to DM-nucleon cross-section and to relic density **may compensate each other**

Destructive interference "higgsino-bino switch"

M. Klasen, K. Kovařík, P. Steppeler — Phys.Rev. D94: 095002 (2016) — arXiv:1607.06396 [hep-ph]