One-loop calculator in practice: The IDM example

Shankha Banerjee
IPPP, Durham University

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Based on

JHEP 1905 (2019) 150; arXiv:1612.01973 (with N. Chakrabarty)
PRD; arXiv:1906.11269 (with F. Boudjema, N. Chakrabarty, G. Chalons and H. Sun)

Plan of my talk

- Motivations
- The Inert Higgs Doublet Model
- The Renormalisation Schemes
- The Large Mass Regime
- Annihilation processes
- Relic abundance calculation
- Summary and Outlook

Motivations

- The Relic Density of Dark Matter (DM) in the ΛCDM framework has been measured precisely $\rightarrow$ $\boxed{\Omega_{\rm DM}h^2 = 0.1199 \pm 0.0027}$ $\rightarrow \sim 2.3\%$ accuracy
- This necessitates precise theoretical predictions to compare with experiments
- For thermal DM, *i.e.*, abundance computed through solving Boltzmann equation, precise calculations involve: Higher Order Corrections, Theoretical Uncertainty, Sommerfeld Enhancement, Bound State Formation
- Assuming WIMP paradigm, extended Higgs sectors upon assuming discrete symmetry(ies) $\rightarrow$ good DM candidate $\rightarrow$ testable at colliders as well as from astrophysics/cosmology

The Inert Higgs Doublet Model

- The SM is augmented by a second scalar doublet (under $SU(2)_L$), $\Phi_2 \to$ Odd under a discrete $\mathbb{Z}_2$ symmetry $\to$ Does not get a *vev*

$$\mathcal{L}_{IDM} = \mathcal{L}_{SM} + (D^\mu \Phi_2)^\dagger D_\mu \Phi_2 + \mathscr{V}_{IDM}(\Phi_1, \Phi_2)$$

$$\begin{aligned} \mathscr{V}_{IDM}(\Phi_1, \Phi_2) &= \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 + \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 \\ &\quad + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 (\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \left(\frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c} \right) \end{aligned}$$

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG) \end{pmatrix} \text{ and } \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}$$

The Inert Higgs Doublet Model

- No mixing between Φ_1 and Φ_2, Lightest $\mathbb{Z}_2$ odd particle, $H/A \to$ DM, Couplings of Φ_1 to fermions/gauge-bosons SM-like
- Higgs is aligned
- Examples of trilinear couplings

$$(H^+H^-\gamma,\ H^+H^-Z,\ HH^\pm W^\mp,\ iAH^\pm W^\mp,\ iAHZ) = i\frac{g}{2}(2s_W,\ c_{2W}/c_W,\ \mp 1,\ -1,\ -1/c_W)$$

- Similarly, we also have quartic couplings involving the second doublet $\to$ Annihilations of DM to gauge bosons happen either through such gauge interactions or through scalar potential coupling

The Inert Higgs Doublet Model: Mass Spectrum

- Minimisation of potential $\rightarrow$ vanishing of tadpoles for Φ_1

$$\frac{T}{v} = \mu_1^2 + \lambda_1 v^2 \equiv 0$$

- No corresponding tadpole for Φ_2 because of unbroken $\mathbb{Z}_2$ symmetry $\rightarrow$ condition met at all orders
- Mass spectrum

$$M_h^2 = \frac{T}{v} + 2\lambda_1 v^2, M_{H^\pm}^2 = \mu_2^2 + \lambda_3 \frac{v^2}{2},$$
$$M_H^2 = \mu_2^2 + \lambda_L \frac{v^2}{2} = M_{H^\pm}^2 + (\lambda_4 + \lambda_5)\frac{v^2}{2},$$
$$M_A^2 = \mu_2^2 + \lambda_A \frac{v^2}{2} = M_{H^\pm}^2 + (\lambda_4 - \lambda_5)\frac{v^2}{2} = M_H^2 - \lambda_5 v^2$$

where $\lambda_{L/A} = \lambda_3 + \lambda_4 \pm \lambda_5$

The Inert Higgs Doublet Model: Scalar Couplings

- H and A are on equal footing and either can be the DM candidate
- For $m_H < m_A/m_H^{\pm}$, $\lambda_4 + \lambda_5 < 0$ and $\lambda_5 < 0$
- For $m_A < m_H/m_H^{\pm}$, $\lambda_4 - \lambda_5 < 0$ and $\lambda_5 > 0$

$$\lambda_{hHH} = \lambda_L v, \quad \lambda_{hAA} = \lambda_A v, \lambda_{hH^+H^-} = \lambda_3 v$$

$$\lambda_{hhHH,hhAA,hhH^+H^-} = \lambda_L, \lambda_A, \lambda_3$$

- λ_2 controls all quartic couplings within the dark sector, *viz.*, $HHHH, AAAA, HHAA, HHH^+H^-, AAH^+H^-, H^+H^-H^+H^-$

The Inert Higgs Doublet Model: Counting Parameters

- Apart from the tadpole condition and the 125 GeV Higgs mass criteria, there are 5 extra parameters

$$(\mu_2, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$$

- 3 of these parameters can be traded for physical masses

$$(\mu_2, \lambda_3, \lambda_4, \lambda_5; \lambda_2) \to (M_H, M_A, M_{H^\pm}, \lambda_{L/A}; \lambda_2),$$

or equivalently

$$(\mu_2, \lambda_3, \lambda_4, \lambda_5; \lambda_2) \to (M_H, M_A, M_{H^\pm}, \mu_2; \lambda_2)$$

- λ_2's renormalisation not necessary as it does not enter the $2 \to 2$ processes at tree-level

$$\lambda_4 = \frac{1}{v^2}\left(M_H^2 + M_A^2 - 2M_{H^\pm}^2\right), \lambda_5 = \frac{1}{v^2}\left(M_H^2 - M_A^2\right)$$

$$\lambda_3 = \frac{2}{v^2}\left(M_{H^\pm}^2 - \mu_2^2\right) = \frac{2}{v^2}\left(M_{H^\pm}^2 - M_H^2\right) + \lambda_L$$

The Inert Higgs Doublet Model: Renormalisation of the model

- $\mathbb{Z}_2$ symmetry eases the renormalisation $\rightarrow$ No mixing between SM and other fields at any order
- Tadpole condition only for SM part
- SM part including Higgs, Goldstones renormalised exactly as in SM
- Follow the On-Shell (OS) scheme for renormalisation
- OS approach for all three physical fields (mass as input parameters)
- 2 parameters still need to be defined $\rightarrow$ λ_2 renormalisation not needed for annihilation to SM particles
- Renormalisation of μ_2 or $\lambda_{L/A}$ or a combination is necessary

The Inert Higgs Doublet Model: Renormalisation of the model

- Shifts introduced (counterterms) for Lagrangian parameters and fields

  $$X_0 \to X + \delta X, \quad X = \mu_2, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \qquad \phi_0 \to \phi + \frac{1}{2}\delta Z_\phi, \quad \phi = (h, H, A, H^\pm)$$

- OS conditions on physical masses require masses to be defined as pole masses of the renormalised one-loop propagator and that residue at pole be unity

  $$\delta M_\phi^2 = \Sigma_{\phi\phi}(M_\phi^2)$$

  $$\delta Z_\phi = -\left.\frac{\partial \Sigma_{\phi\phi}(p^2)}{\partial p^2}\right|_{p^2 = M_\phi^2}$$

- $\Sigma_{\phi\phi}(p^2)$ is the scalar 2-point function with momentum p ($\phi = h, H, A, H^\pm$)
- $\delta M_H, \delta M_A, \delta M_H^\pm$ give OS definitions for λ_4, λ_5
- With 3 physical masses, 5 counterterms can't be constructed from 2-point functions

The Inert Higgs Doublet Model: Renormalisation of the model

- λ_L measures the hHH coupling and hence $\delta\lambda_L$ can be obtained from hHH coupling measurement
- For $M_h > 2M_H$, $\Gamma(h \to HH)$ (unaffected by infrared divergence) is the variable
- One denotes the amplitude $\mathcal{A}(h \to HH) \equiv \mathcal{A}_{hHH}$ and at tree level
  $\mathcal{A}^0_{hHH} = -\lambda_L v$
- The full one-loop renormalisation amplitude for $h(p^2) \to H(p_1^2)H(p_2^2)$ as
  $\mathcal{A}^{\rm ren}_{hHH}(p^2, p_1^2, p_2^2) = -\lambda_L v \left(\frac{\delta\lambda_L}{\lambda_L} + \frac{\delta v}{v} + \frac{1}{2}\delta Z_h + \delta Z_H \right) + \mathcal{A}^{\rm 1PI}_{HHh}(p^2, p_1^2, p_2^2)$
- $\mathcal{A}^{\rm 1PI}(p^2, p_1^2, p_2^2)$ is the full one-loop 1PI vertex
- When threshold is open, one sets $p^2 = M_h^2$, $p_1^2 = p_2^2 = M_H^2$, one defines the gauge-invariant counterterm for λ_L as
  $\frac{\delta^{\rm OS}\lambda_L}{\lambda_L} = \frac{\mathcal{A}^{\rm 1PI}_{HHh}(m_h^2, M_H^2, M_H^2)}{\lambda_L v} - \frac{\delta v}{v} - \frac{1}{2}\delta Z_h - \delta Z_H$

The Inert Higgs Doublet Model: Renormalisation of the model

- Another gauge invariant but scale-dependent choice is to use the $\overline{\mathrm{MS}}$ scheme where the mass-independent UV divergent part is kept

$$\frac{\delta^{\overline{\mathrm{MS}}}\lambda_L}{\lambda_L} = \left(\frac{\mathcal{A}^{\mathrm{1PI}}_{HHh}(m_h^2, M_H^2, M_H^2)}{\lambda_L v} - \frac{\delta v}{v} - \frac{1}{2}\delta Z_h - \delta Z_H\right)_\infty$$

- Coefficient of UV divergent part is the one-loop β constant (β_{λ_L}) of λ_L

$$\frac{\delta^{\overline{\mathrm{MS}}}\lambda_L}{\lambda_L} = \beta_{\lambda_L} C_{\mathrm{UV}}, \quad C_{\mathrm{UV}} = \frac{2}{\varepsilon} - \gamma_E + \ln(4\pi)$$

with $\epsilon = 4 - d$ with d being number of dimensions in dimensional regularisation and γ_E is the Euler constant. A general renormalisation scheme can be defined as

$$\frac{\delta\lambda_L}{\lambda_L} = \beta_{\lambda_L}\left(C_{\mathrm{UV}} + \ln(\bar{\mu}^2/Q_\lambda^2)\right)$$

- Q_λ $\rightarrow$ effective scale depending on external momenta and internal masses used to define counterterms
- $\bar{\mu}$ $\rightarrow$ scale introduced by dimensional reduction. In $\overline{\mathrm{MS}}$, $Q_\lambda = \bar{\mu}$
- For $M_h < 2M_H$ $\rightarrow$ difficult to come up with straightforward OS scheme for λ_L (or μ_2) $\rightarrow$ Hence choosing an $\overline{\mathrm{MS}}$, $Q_\lambda = \bar{\mu}$ for λ_L
- Counterterm for μ_2

$$\delta\mu_2^2 = \delta M_H^2 - \frac{v^2}{2}\delta\lambda_L - \lambda_L v^2 \frac{\delta v}{v}$$

The Inert Higgs Doublet Model: High mass benchmark

- Collider and astrophysical studies delineates two regimes with viable DM candidate with correct tree-level relic density, *i.e.* for $M_H \approx M_h/2$ (low mass regime) and $M_H \gtrsim 500$ GeV (high mass regime)
- The benchmark chosen for this talk

$$M_H = 550 \text{ GeV}, \quad M_A = 551 \text{ GeV}, \quad M_H^{\pm} = 552 \text{ GeV},$$
$$\lambda_L = 0.0193, \quad \lambda_2 = 0.01$$
$$(\lambda_3 = 0.0926, \quad \lambda_4 = -0.0545, \quad \lambda_5 = -0.0181 \text{ and } \mu_2 = 549.45 \text{ GeV})$$

- For SM values, $M_h = 125$ GeV, $M_W = 80.45$ GeV, $M_Z = 91.19$ GeV, $\alpha = 1/137$
- One obtains $\Omega h^2 = 0.117$ at tree level
- At tree-level, cross-section doesn't depend on λ_2
- Viability of point depends on almost degenerate IDM scalar spectrum and small values of λs $\rightarrow$ vacuum stability holds and EW precision measurements are evaded

$$\Delta T \simeq \frac{1}{24\pi^2 \alpha v^2} \left(M_{H^\pm} - M_A\right)\left(M_{H^\pm} - M_H\right)$$

The Inert Higgs Doublet Model: Relic contributions

- Stringent constraint comes from spin-independent DM-nucleon cross-section for direct detection
- Here, the one-loop EW gauge contribution to H nucleon cross-section, $\sigma_{HN}^{(g)}$ is almost an order of magnitude larger than tree-level Higgs exchange contribution triggered by λ_L, $\sigma_{HN}^{(\lambda_L)}$
- In the regime $M_H \gg M_W$

$$\mathcal{R}(\lambda_L) = \frac{\sigma_{HN}^{(g)}}{\sigma_{HN}^{(\lambda_L)}} \sim \left(6\pi \frac{\alpha^2}{\lambda_L s_W^4}\right)^2 \left(\frac{M_H}{8M_W}\right)^2 \left(1 + \frac{M_h^2}{M_W^2}\right)^2$$
$$\sim 9 \quad \text{for } \lambda_L = 0.019 \text{ and } M_H = 550 \text{ GeV}.$$

- At present, this point still passes the current XENON1T constraints
- Degeneracy in scalar masses $\longrightarrow$ relic built by few co-annihilation channels $\longrightarrow$ main ones into vector bosons

$$\begin{cases} HH \to W^+W^- \quad (18\%), \\ HH \to ZZ \quad (14\%), \\ H^+H^- \to W^+W^- \quad (13\%), \\ AA \to W^+W^- \quad (9\%), \\ H^+H \to W^+\gamma \quad (8\%), \\ AA \to ZZ \quad (7\%), \\ H^+A \to W^+\gamma \quad (6\%). \end{cases}$$

The Inert Higgs Doublet Model: Relic contributions

- Out of these 7 cross-sections, the ones with annihilations to photons, *viz.*, $H^+A/H^+H \to W^+\gamma$ are driven solely by gauge couplings and at tree-level are independent of λ_L
- Considering the small value of λ_L in our BP, $\sigma_{HH\to W^+W^-} \sim \sigma_{AA\to W^+W^-} \sim \sigma_{H^+H^-\to W^+W^-} = 2c_W^4\sigma_{HH\to ZZ} = 2c_W^4\sigma_{AA\to ZZ}$
- The aforementioned weights are a measure of the relative importance of the cross-sections diluted by the Boltzmann factor

Annihilation cross-sections at one-loop: Technicalities

- Calculation of relic density requires dependence of different relevant cross-sections on relative velocity, v of annihilating particles times v, $\sigma_{ij} v_{ij}$, where i, j denote annihilating/co-annihilating particles before thermal averaging
- `micrOMEGAs` computes the latter within the Standard Cosmological model and assuming freeze-out
- For two annihilating particles with masses $m_{1/2}$ and momenta $p_{1/2}$, relative velocity defined as

$$v = 2s\frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{s^2-(m_1^2-m_2^2)^2}, \quad s=(p_1+p_2)^2,$$
$$v = 2\sqrt{1-4M_{\rm DM}^2/s} = 2\beta \quad \text{for} \quad m_1=m_2=M_{\rm DM}.$$

- Possible to replace the cross-section by loop-corrected one
- We depend on `SloopS` which depends on `LANHEP` that generates Feynman rules with shifts on fields and parameters $\longrightarrow$ interfaced with `FeynArts, FormCalc, LoopTools`
- For each process/velocity point, we check UV finiteness for the virtual corrections $\longrightarrow$ vary C_{UV} by 7 orders of magnitude and check stability of result within machine precision
- For processes involving charged particles $\longrightarrow$ bremsstrahlung processes $2 \longrightarrow 2+\gamma$ are generated $\longrightarrow$ latter split into two parts $\longrightarrow$ soft and hard photon radiation
- Soft photon radiation for photon energies $E_\gamma < k_c$ with small enough k_c $\longrightarrow$ generated automatically through factorisation formula which eliminates one-loop IR divergence that is regularised with small finite photon mass
- Hard photon radiation is computed numerically $\longrightarrow$ looping over few values of k_c $\longrightarrow$ ensuring soft plus hard contribution insensitive to k_c $\longrightarrow$ this is automated

Processes at one-loop: $H^+H^- \to W^+W^-$

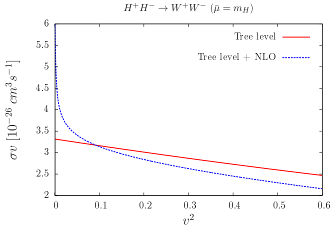
- Contributes to 13% of the total relic (at tree level) but brings about an important feature
- At tree-level, σv slowly and linearly varies with v^2 $\to$ holds for full one-loop corrected σv around $v^2 \sim 0.2$ where correction is around -10% $\to$ correction shoots up for extremely low values of v^2 $\to$ understood in terms of electromagnetic Sommerfeld effect
- Photon exchange between the electrically charged co-annihilating particles at very low relative velocities leads to relative correction which at one-loop reads as

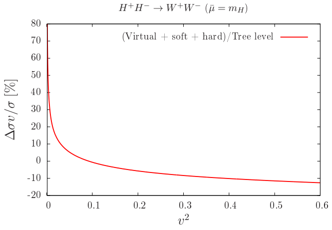
$$\frac{\Delta\sigma^{1-\text{loop Somm.}} v}{\sigma^{\text{tree}} v} = \frac{\pi\alpha}{v}$$

- The one-loop Sommerfeld contribution can be resummed with the result that the tree-level cross-section is turned into

$$\sigma^{\text{resummed}} = S_{\text{nr}}\,\sigma^{\text{tree}}, \quad S_{\text{nr}} = \frac{X_{\text{nr}}}{1 - e^{-X_{\text{nr}}}} \quad \text{and } X_{\text{nr}} = 2\pi\alpha/v$$

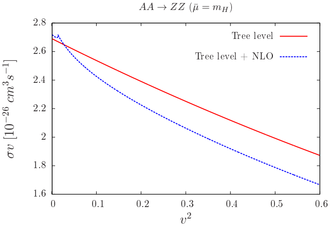
- Characteristic velocities for the calculation of relic density are typically in the range $v \sim 0.2 - 0.3$, the Sommerfeld enhancement taken either at one-loop or resummed to all orders does not have much of an impact on the relic density

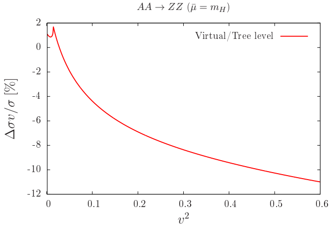


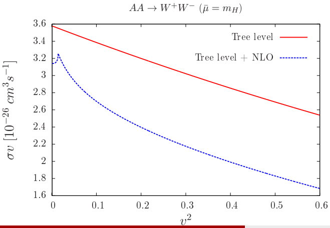


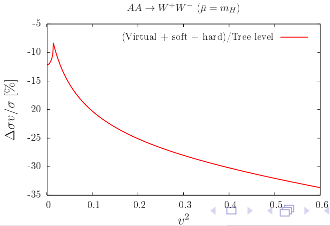
Processes at one-loop: $AA \to ZZ/W^+W^-$

- $AA \to ZZ$ and $AA \to W^+W^-$ respectively contribute to 7% and 10% to the relic
- Velocity dependence of cross-sections at tree-level decreases slowly with an almost similar rate
- At one loop, photon final state radiation affects W^+W^- channel
- For $v = 0.3$, correction in $ZZ(W^+W^-)$ is $\sim -5\%(-20\%)$
- No Z exchange such processes and these proceed through W exchange only



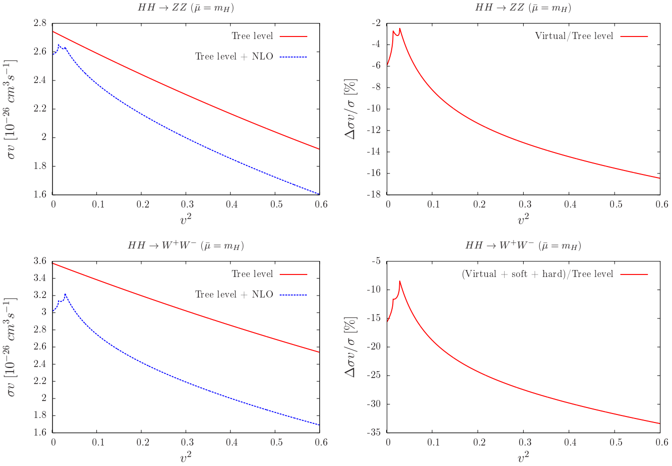






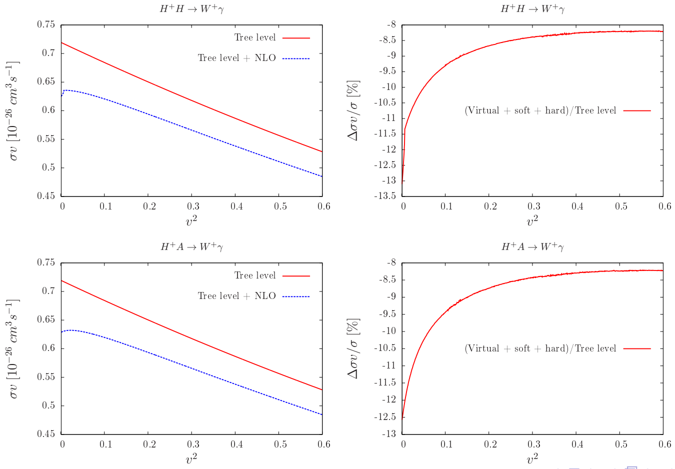
Processes at one-loop: $HH \to ZZ/W^+W^-$

- The one-loop corrections show two dents as now the Z-exchange diagrams are also open



Processes at one-loop: $H^+H/H^+A \to W^+\gamma$

- Practically interchangeable both at LO and NLO
- Relative one-loop corrections decrease as relative velocity decreases with a correction of about -10% for $v \sim 0.3$
- Since at tree-level these processes do not depend on $\lambda_{L/A}$, fully on-shell renormalisation possible with $\bar{\mu}$-independent one-loop cross-sections



Correction to relic abundance and impact on λ_2

Process	LO	$\mu = m_X$	$\mu = m_X/2$	$\mu = 2 \times m_X$
$HH \to W^+W^-$	18%	16%	18%	14%
$HH \to ZZ$	14%	14%	14%	14%
$H^+H^- \to W^+W^-$	13%	15%	15%	15%
$AA \to W^+W^-$	9%	8%	9%	7%
$H^+H \to W^+\gamma$	8%	7%	7%	8%
$AA \to ZZ$	7%	7%	7%	8%
$H^+A \to W^+\gamma$	6%	6%	6%	7%
$\bullet H^+H^- \to \gamma\gamma$	5%	5%	5%	6%
$\bullet H^+H^- \to \gamma Z$	4%	5%	4%	5%
$\bullet H^+H \to ZW^+$	3%	3%	3%	3%
$\bullet H^+A \to ZW^+$	3%	3%	2%	3%
$\bullet H^+H^- \to ZZ$	2%	2%	2%	2%

λ_2	$\mu = M_H$	$\mu = M_H/2$	$\mu = 2M_H$
0.01	0.12494 (6.9%)	0.11652 (-0.3%)	0.13469 (15.3%)
0.1	0.12210 (4.5%)	0.11843 (1.3%)	0.12601 (7.8%)
1	0.09950 (-14.9%)	0.14163 (21.2%)	0.07683 (-34.3%)

Summary and Conclusions

- We discuss the OS and $\overline{\text{MS}}$ schemes for the renormalisation of the IDM
- This example shows the necessity for higher order calculations to relic density in lieu of experimental accuracy
- We dealt with the high mass regime of the IDM parameter space and showed the dependence of the cross-sections and the relic on various parameters including λ_2
- The one-loop corrections range between -34% and 15% depending on the renormalisation scale chosen and the value of λ_2
- The effect of λ_2 can be larger than the renormalisation scale uncertainty and hence must be studied carefully
- Preliminary investigations for DM masses beyond 750 GeV show that EW Sommerfeld effects become important and that some resummation needs to be performed and merged with perturbative purely one-loop effects like those triggered by rescattering in the dark sector
- Low mass regime featuring $M_H \approx M_h/2$ which has $2 \to 3$ processes need to be studied