The SloopS Project

How did the code see the light? and what was required from the code

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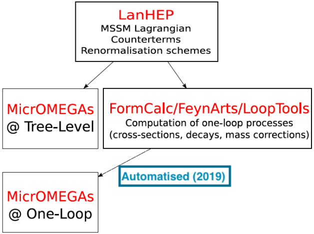
Within Japanese Collaboration, EW corrections within `GraceLoop`. Collaborate on automatic calculations of 1-loop supersymmetric model

N. Baro, FB, G. Chalons, G. Drieu La Rochelle, S. Hao, Ninh Le Duc, A. Semenov, (D. Temes)

- Need for an automatic tool for susy calculations
- handles large numbers of diagrams both for tree-level
- and loop level
- able to compute loop diagrams at $v = 0$: dark matter, LSP, move at galactic velocities, $v = 10^{-3}$
- ability to check results: UV and IR finiteness but also gauge parameter independence for example
- ability to include different models easily and switch between different renormalisation schemes
- Used for SM one-loop multi-leg: new powerful loop libraries (with Ninh Le Duc, Sun Hao)

General Structure



SLOOPS

An automatic code for calculation of loops diagrams for $\mathcal{SM}$ and $\mathcal{BSM}$ processes with application to colliders, astrophysics and cosmology.

- Automatic derivation of the CT Feynman rules and computation of the CT's
- Models renormalized: SM, MSSM, NMSSM, **xSM** (w/ & w/o singlet vev)
- Modularity between different renormalisation schemes.
- Non-linear gauge fixing.
- Checks: results UV,IR finite and gauge independent.

General Structure

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Strategy: Exploiting and interfacing modules from different codes

Lagrangian of the model defined in LanHEP

- particle content
- interaction terms
- shifts in fields and parameters
- ghost terms constructed by BRST

↓ ↓

Generic Model -kinematical structures

Classes Model -Feynman rules, including CT

⇓

Evaluation via FeynArts-FormCalc

LoopTools modified!!
tensor reduction inappropriate for small relative velocities
(Zero Gram determinants)

⇑

Renormalisation scheme

- definition of renorm. const. in the classes model

Non-Linear gauge-fixing constraints, gauge parameter dependence checks

Defining the model: extension of `LanHEP`. 1

- *FeynArts* and *FormCalc* are used for matrix element calculation: *FeynArts* model format output implemented in LanHEP.
- Shifts in fields and parameters to produce counterterms by LanHEP:

```
infinitesimal dMHsq, dMZsq, dMWsq,dZAA, dZAZ, dZZA, dZZZ, dZW,
dZH. infinitesimal  dEE= -(dZAA - SW/CW*dZZA)/2. transform
A->A*(1+dZAA/2)+dZAZ*Z/2, Z->Z*(1+dZZZ/2)+dZZA*A/2,
      'W+'->'W+'*(1+dZW/2),'W-'->'W-'*(1+dZW/2), H->H*(1+dZH/2).
```

  Different normalization schemes can be used, easy to switch between different RS
- Non-linear gauge fixing (see later)

Defining the model: extension of `LanHEP`. 2

From the Lagrangian to the Feynman Rules

```
vector
    A/A: (photon, gauge),
    Z/Z:('Z boson', mass MZ = 91.1875, gauge),
    'W+'/'W-': ('W boson', mass MW = MZ*CW, gauge).
scalar  H/H:(Higgs, mass MH = 115).
```

```
transform A->A*(1+dZAA/2)+dZAZ*Z/2, Z->Z*(1+dZZZ/2)+dZZA*A/2,
    'W+'->'W+'*(1+dZW/2),'W-'->'W-'*(1+dZW/2).
transform H->H*(1+dZH/2), 'Z.f'->'Z.f'*(1+dZZf/2),
    'W+.f'->'W+.f'*(1+dZWf/2),'W-.f'->'W-.f'*(1+dZWf/2).

let pp = { -i*'W+.f',  (vev(2*MW/EE*SW)+H+i*'Z.f')/Sqrt2 },
PP=anti(pp).
```

```
lterm -2*lambda*(pp*anti(pp)-v**2/2)**2
     where
    lambda=(EE*MH/MW/SW)**2/16, v=2*MW*SW/EE .

let Dpp^mu^a = (deriv^mu+i*g1/2*B0^mu)*pp^a +
     i*g/2*taupm^a^b^c*WW^mu^c*pp^b.
let DPP^mu^a = (deriv^mu-i*g1/2*B0^mu)*PP^a
    -i*g/2*taupm^a^b^c*{'W-'^mu,W3^mu,'W+'^mu}^c*PP^b.
lterm DPP*Dpp.
```

Gauge fixing and BRS transformation

```
let G_Z = deriv*Z+(MW/CW+EE/SW/CW/2*nle*H)*'Z.f'.

lterm -G_A**2/2 - G_Wp*G_Wm - G_Z**2/2.

lterm -'Z.C'*brst(G_Z).
```

Defining the model: extension of `LanHEP`. Counterterms and renormalisation

```
RenConst[ dMHsq ] := ReTilde[SelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZH ] := -ReTilde[DSelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZZf ] := -ReTilde[DSelfEnergy[prt["Z.f"] -> prt["Z.f"],
MZ]] RenConst[ dZWf ] := -ReTilde[DSelfEnergy[prt["W+.f"] ->
prt["W+.f"], MW]]
```

Defining the model: extension of `LanHEP` Interface with `FeynArts`

From the Lagrangian to the Feynman Rules

```
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    A/A: (photon, gauge),
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transform H->H*(1+dZH/2), 'Z.f'->'Z.f'*(1+dZZf/2),
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let pp = { -i*'W+.f',  (vev(2*MW/EE*SW)+H+i*'Z.f')/Sqrt2 },
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lterm -2*lambda*(pp*anti(pp)-v**2/2)**2
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let Dpp^mu^a = (deriv^mu+i*g1/2*B0^mu)*pp^a +
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let DPP^mu^a = (deriv^mu-i*g1/2*B0^mu)*PP^a
    -i*g/2*taupm^a^b^c*{'W-'^mu,W3^mu,'W+'^mu}^c*PP^b.
lterm DPP*Dpp.
```

Gauge fixing and BRS transformation

```
let G_Z = deriv*Z+(MW/CW+EE/SW/CW/2*nle*H)*'Z.f'.

lterm -G_A**2/2 - G_Wp*G_Wm - G_Z**2/2.

lterm -'Z.C'*brst(G_Z).
```

Output of Feynman Rules with Counterterms !!

```
M$CouplingMatrices = {

  (*------    H  H  ------*)
   C[ S[3], S[3] ] == - I *
{
 { 0 , dZH },
 { 0 , MH^2 dZH + dMHsq }
},
  (*------    W+.f  W-.f  ------*)
   C[ S[2], -S[2] ] == - I *
{
 { 0 , dZWf },
 { 0, 0 }
}, (*------    A  Z  ------*)
   C[ V[1], V[2] ] == 1/2 I / CW^2 MW^2 *
{
 { 0, 0 },
 { 0 , dZZA },
 { 0, 0 }
},

(*------    H  H  H  ------*)
   C[ S[3], S[3], S[3] ] == -3/4 I EE / MW / SW *
{
 { 2 MH^2 , 3 MH^2 dZH -2 MH^2 / SW dSW - MH^2 / MW^2 dMWsq
},
  (*------    H  W+.f  W-.f  ------*)
   C[ S[3], S[2], -S[2] ] == -1/4 I EE / MW / SW *
{
 { 2 MH^2 , MH^2 dZH + 2 MH^2 dZWf -2 MH^2 / SW dSW - MH^2 ,
},

  (*------    W-.C  A.c  W+  ------*)
   C[ -U[3], U[1], V[3] ] == - I EE *
{
 { 1 },
 { - nla }
},
```

Gauge fixing. Usual Linear GF

$$\mathcal{L}_{GF} = -\frac{1}{\xi_W}|\partial.W^+ + \xi_W\frac{g}{2}vG^+|^2$$
$$-\frac{1}{2\xi_Z}(\partial.Z + \xi_Z\frac{g}{2c_W}v + G^0)^2 - \frac{1}{2\xi_\gamma}(\partial.A)^2$$

This only affects the propagators. Usually calculations done with $\xi = 1$, otherwise large expressions, higher rank tensors, unphysical thresholds,..

$$\frac{1}{k^2 - M_W^2}\left(g_{\mu\nu} - (1-\xi_W)\frac{k_\mu k_\nu}{k^2 - \xi_W M_W^2}\right)$$

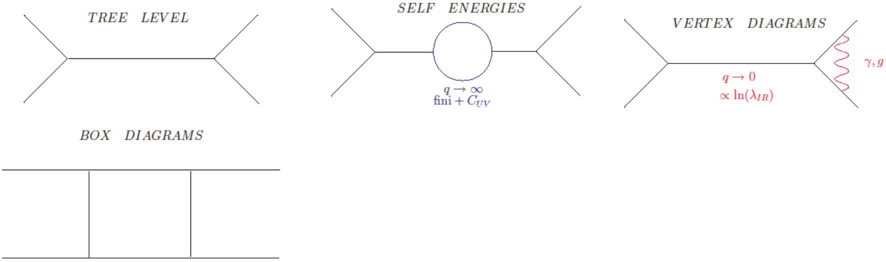
how to have $\xi = 1$ and still check for gauge parameter independence?

Power of non-linear GF

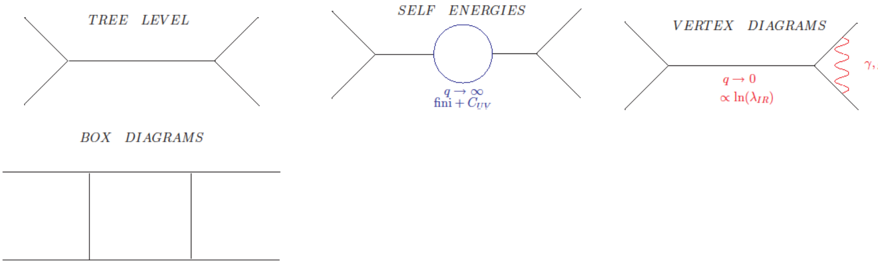
$$\begin{aligned}\mathcal{L}_{GF} &= -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu\,+} + \xi_W\frac{g}{2}(v+\tilde{\delta}h+\tilde{\omega}H+i\tilde{\rho}A^0+i\tilde{\kappa}G^0)G^+|^2\\ &\quad -\frac{1}{2\xi_Z}(\partial.Z+\xi_Z\frac{g}{2c_W}(v+\tilde{\epsilon}h+\tilde{\gamma}H)G^0)^2 - \frac{1}{2\xi_\gamma}(\partial.A)^2\end{aligned}$$

- quite a handful of gauge parameters, but with $\xi_i = 1$, no “unphysical threshold”, no higher rank tensors, gauge parameter dependence in gauge/Goldstone/ghosts vertices.
- more important: no need for higher (than the minimal set)for higher rank tensors and tedious algebraic manipulations

Different ingredient of a NLO calculation. 1



Different ingredient of a NLO calculation. 2

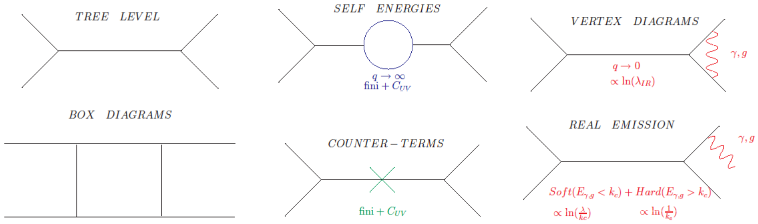


REGULARISATION

Isolate infinite parts in loops

- UV: $\ln \Lambda_{UV}$ with cut-off, $1/\epsilon_{UV}$ poles in DR.
- IR: $\ln \lambda_{IR}$ with cut-off, $1/\epsilon_{IR}$ poles in DR.

Different ingredient of a NLO calculation. 3

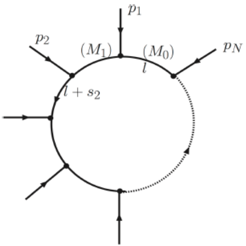


REGULARISATION

Isolate infinite parts in loops

- UV: $\ln \Lambda_{UV}$ with cut-off, $1/\epsilon_{UV}$ poles in DR.
- IR: $\ln \lambda_{IR}$ with cut-off, $1/\epsilon_{IR}$ poles in DR.

Loop Integrals. Gram Determinants. Good loop library. 1



$$T^{(N)}_{\underbrace{\mu\nu\cdots\rho}_{M}} = \int \frac{d^n l}{(2\pi)^n} \frac{l_\mu l_\nu \cdots l_\rho}{D_0 D_1 \cdots D_{N-1}}, \quad M \leq N,$$

where

$$D_i = (l + s_i)^2 - M_i^2, \quad s_i = \sum_{j=1}^{i} p_j, \quad s_0 = 0.$$

M_i are the internal masses, p_i the incoming momenta and l the loop momentum.

The tensor integral of rank M corresponding to a N-point graph, $\{M, N\}$, that we encounter in the general non-linear gauge but with Feynman parameters $\xi = 1$ are such that $M \leq N$

Loop Integrals. Gram Determinants. Good loop library. 2

Avoiding zero Gram determinants: Segmentation (FB, A. Semenov, D. Temes ,hep-ph/0507127, PRD...)

For the problems at hand:

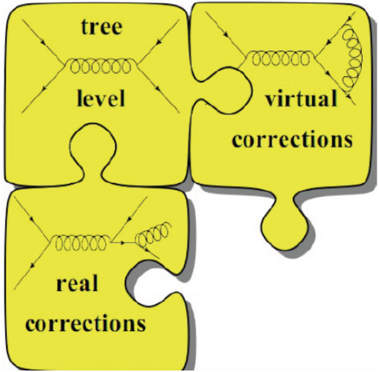
$$
\begin{aligned}
DetG &= M^6_{\tilde{\chi}^0_1} v^2 \frac{\sin^2\theta}{(1-v^2/4)^3}(1-z^2), \quad z^2 = \frac{M_Z^2}{4M^2_{\tilde{\chi}^0_1}}(1-v^2/4) \\
DetG(p_1,p_2) &= -M^4_{\tilde{\chi}^0_1} v^2 \frac{1}{(1-v^2/4)^2}.
\end{aligned}
$$

Segmentation

$$
\begin{aligned}
\frac{1}{D_0D_1D_2D_3} &= \left(\frac{1}{D_0D_1D_2} - \alpha\frac{1}{D_0D_2D_3} - \beta\frac{1}{D_0D_1D_3} + (\alpha+\beta-1)\frac{1}{D_1D_2D_3}\right) \times \\
&\quad \frac{1}{A+2l.(s_3-\alpha s_1-\beta s_2)} \\
A &= (s_3^2-M_3^2)-\alpha(s_1^2-M_1^2)-\beta(s_2^2-M_2^2)-(\alpha+\beta-1)M_0^2. \\
&\quad (D_i=(l+s_i)^2-M_i^2, \quad s_i=\sum_{j=1}^{i} p_j)
\end{aligned}
$$

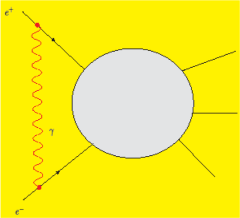
For any graph if $DetG(s_1,s_2,s_3)=0$ (or "small"), construct all 3 sub-determinants $DetG(s_i,s_j)$and take the couple s_i,s_j (as independent basis) that corresponds to $Max\,|Det(s_i,s_j)|$, then write

Real Corrections



Real Corrections

Infrared and Collinear Divergences

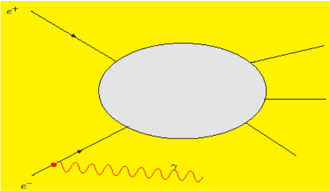


infrared divergent needs photon mass λ

$d\sigma_V(\lambda)$

collinear sing. need $m_f = m_e, \ldots$

Dim Reg: $\lambda, m_e \to 1/\epsilon^2, 1/\epsilon$.



must include bremsstrahlung $d\sigma_S(\lambda, E_\gamma < k_c) + d\sigma_H(\lambda, E_\gamma > k_c)$

$d\sigma_S \to$ analytical: factorisation (automatised) ;

$d\sigma_H \to$ adaptive MC

Dim Reg: $d\sigma_S$ part of $d\sigma_H \to$ Integrate in d-dim.

$$\sigma_{\mathcal{O}(\alpha)} = \underbrace{\int d\sigma_0 \left(1 + \delta_V^{EW}\right)}_{\sigma_0(1+\delta_W)} + \underbrace{\int d\sigma_0 \left(\delta_V^{QED}(\lambda) + \delta_S(\lambda, k_c)\right)}_{\sigma_{V+S}^{QED}(k_c)} + \underbrace{\int d\sigma_H(k_c)}_{\sigma_H(k_c)} .$$

Subtraction based on factorisation of collinear sing., more involved: dipoles, antennas,..but much more efficient (QCD)

Real Correction

In addition to the virtual corrections we also have to consider real photon emission. The corresponding amplitudes are divergent in the soft and collinear limits. The soft singularities cancel against the ones in the virtual corrections while the collinear singularities are regularised by the massive charged particle(s) that emit(s) the photon.

Dipole Subtraction

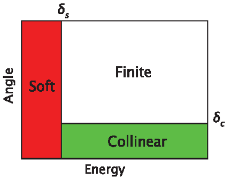
$$\sigma_{\text{real}} = \underbrace{\int_{m+\gamma} (\mathrm{d}\sigma_{\text{real}} - \mathrm{d}\sigma_{\text{sub}})}_{\text{numerically}} + \underbrace{\int_{m+\gamma} \mathrm{d}\sigma_{\text{sub}}}_{\text{photon mass reg.}} \;.$$

$$\int_{m+\gamma} \mathrm{d}\sigma_{\text{sub}} = \underbrace{-\frac{\alpha}{2\pi} \int \mathrm{d}x \sum_{i \neq j} Q_i Q_j \, \mathcal{G}_{ij}(x) \int_m \mathrm{d}\sigma_{\text{Born}}}_{\text{only mass sing.}} + \underbrace{\sigma_{\text{endpoint}}}_{\text{soft+coll. sing.}} \;,$$

$$\sigma_{\text{endpoint}} = -\frac{\alpha}{2\pi} \int_m \mathrm{d}\sigma_{\text{Born}} \sum_{i \neq j} Q_i Q_j \, G_{ij} \,.$$

Phase-space slicing

In the soft and collinear regions the real amplitude approximately factorizes into universal soft and collinear functions and the Born amplitude. In addition the phase space splits into the leading order phase space and a soft or collinear part. The phase-space integration over the photon degrees of freedom can then be performed analytically resulting in infrared and mass singular contributions. In the remaining part of phase space the amplitude is regular and the integration can be performed using numerical integration.



$$
\begin{aligned}
d\sigma_{real} &= d\sigma_{soft}(\delta_s) + d\sigma_{hard}(\delta_s), \\
d\sigma_{hard}(\delta_s) &= d\sigma_{coll}(\delta_s, \delta_c) + d\sigma_{fin}(\delta_s, \delta_c)
\end{aligned}
$$

Real Correction: Slicing vs Dipole

FB, Le Duc Ninh, Sun Hao 0912.4234