# *Testing Bell inequalities in W boson pair production at Higgs factory*

Based on Phys. Rev. D **109**, 036022
in collaboration with Qi Bi, Kun Cheng and Qing-Hong Cao

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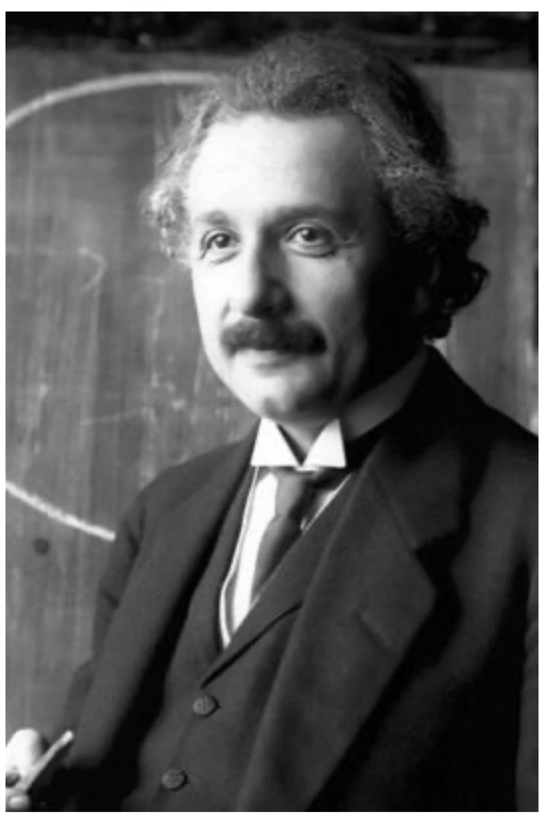
# An “Old” Physics

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

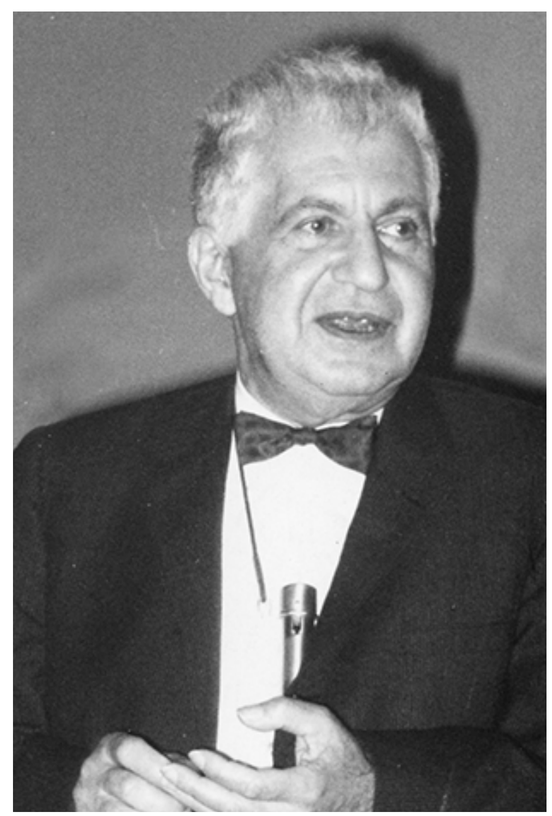
**Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?**

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.



Albert Einstein
(1879/03/14-1955/04/18)

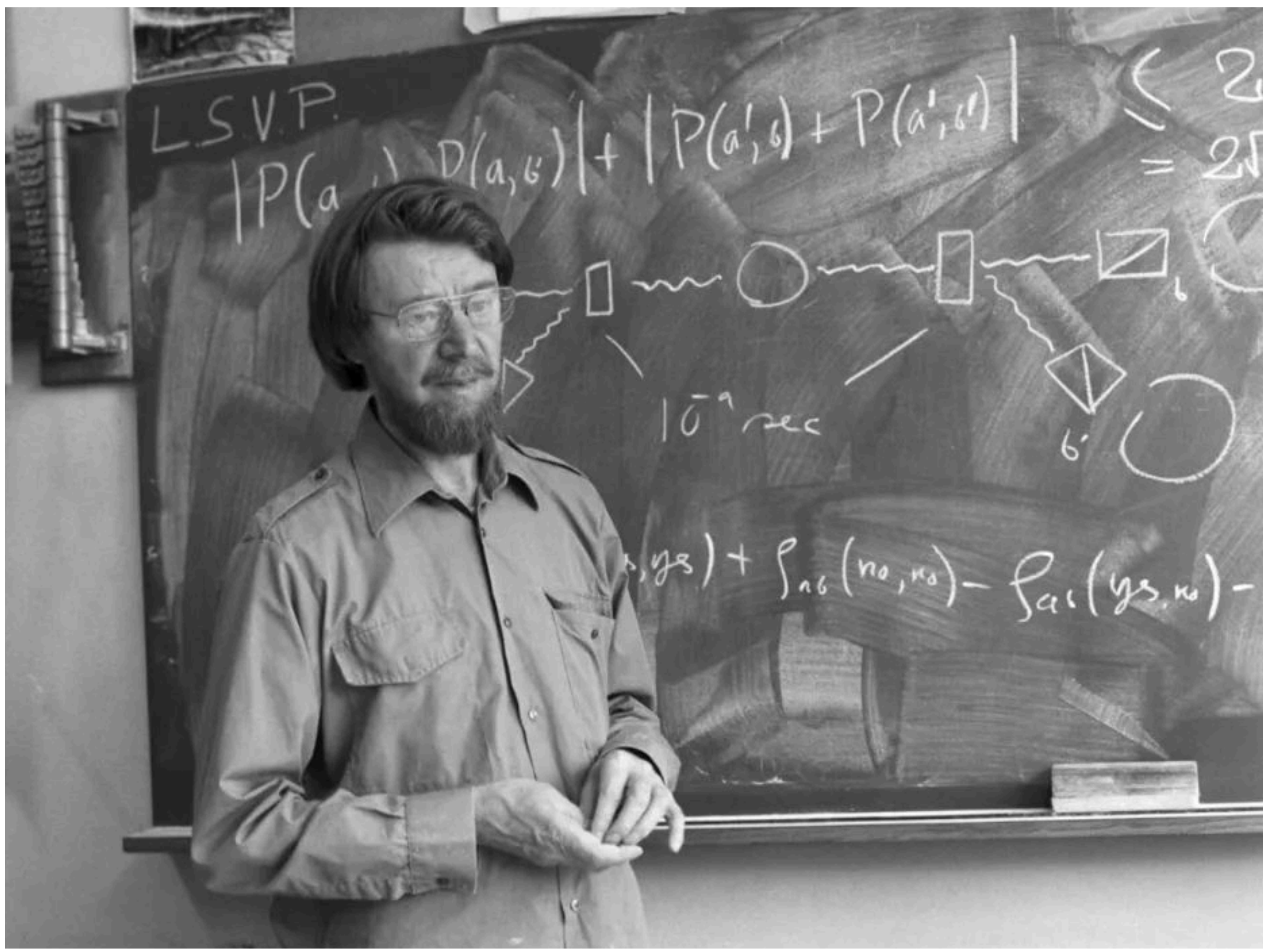


Boris Yakovlevich Podolsky
(1896/06/29-1966/11/28)

Nathan Rosen
(1909/03/22-1995/12/18)

# An “Old” Physics

***What is the essential different between a quantum theory and a theory based on determinism?***



John Stewart Bell (1928/07/28-1990/10/01)

# An "Old" Physics

***Entanglement: the more we know about the parts, the less we know about the whole system!***

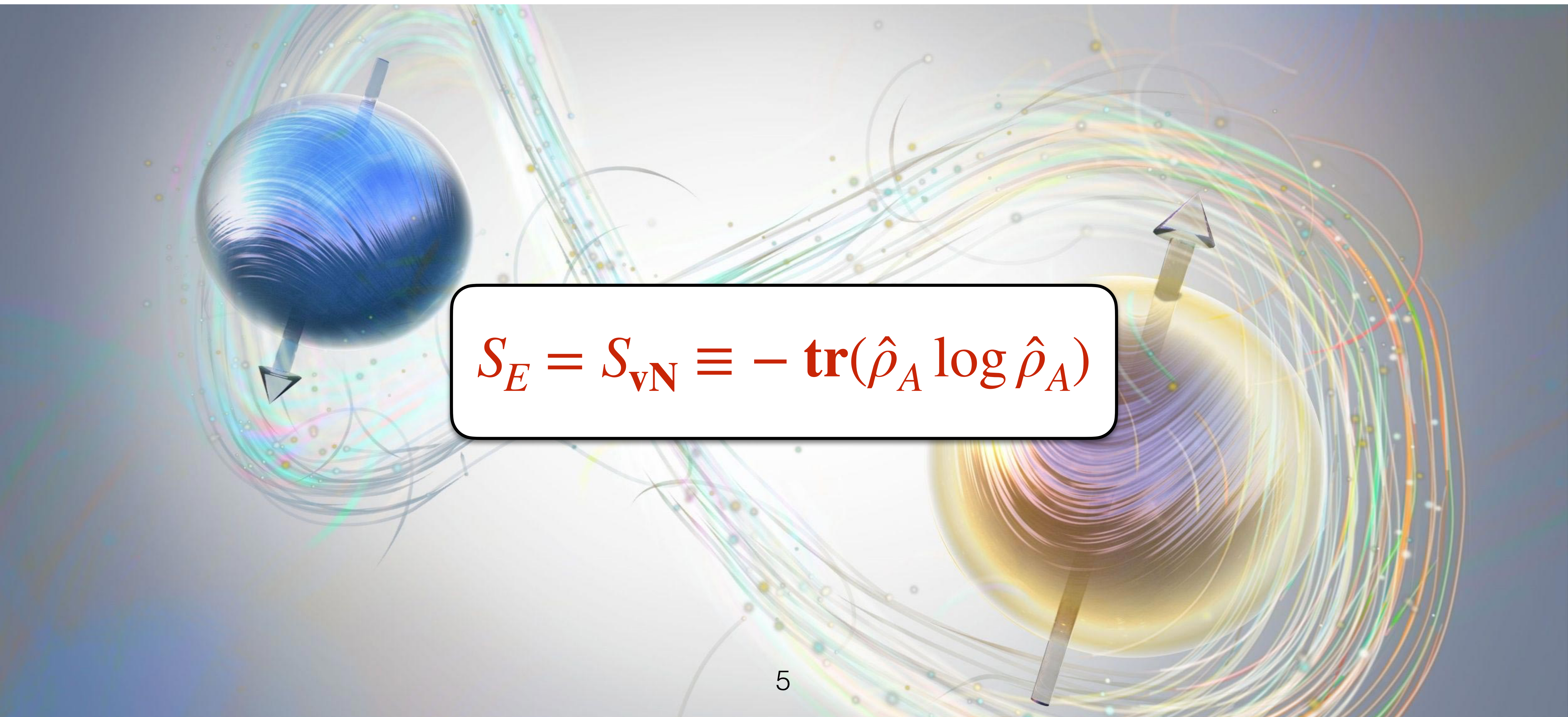

$$\mathbf{E}(QS) + \mathbf{E}(RS) + \mathbf{E}(RT) - \mathbf{E}(QT) \leq 2.$$

(for **L**ocal **H**idden **V**ariable **M**odels)

CHSH (John Clauser, Michael Horne, Abner Shimony, Richard Holt) inequality

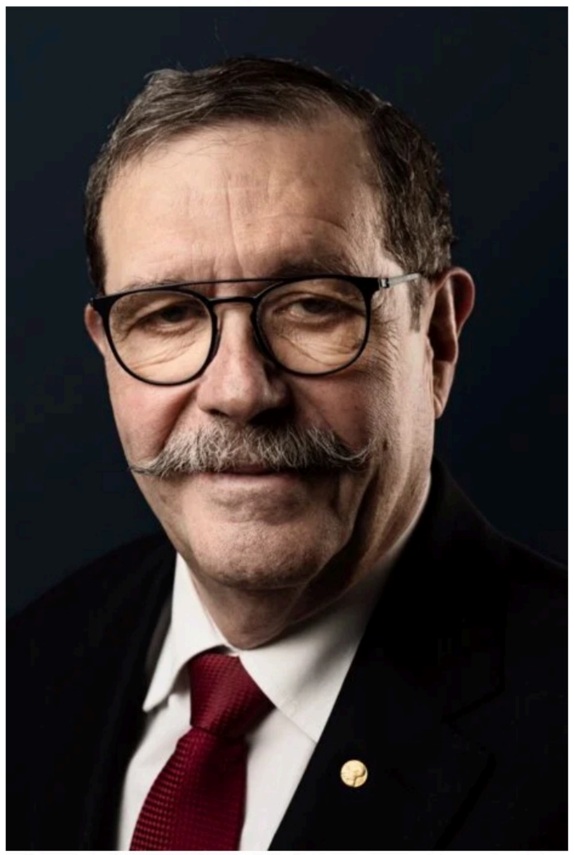
# An "Old" Physics

***Entanglement entropy: a description of the degree of the entanglement between subsystems.***



$$S_E = S_{\mathbf{vN}} \equiv -\,\mathbf{tr}(\hat{\rho}_A \log \hat{\rho}_A)$$

# The Verification



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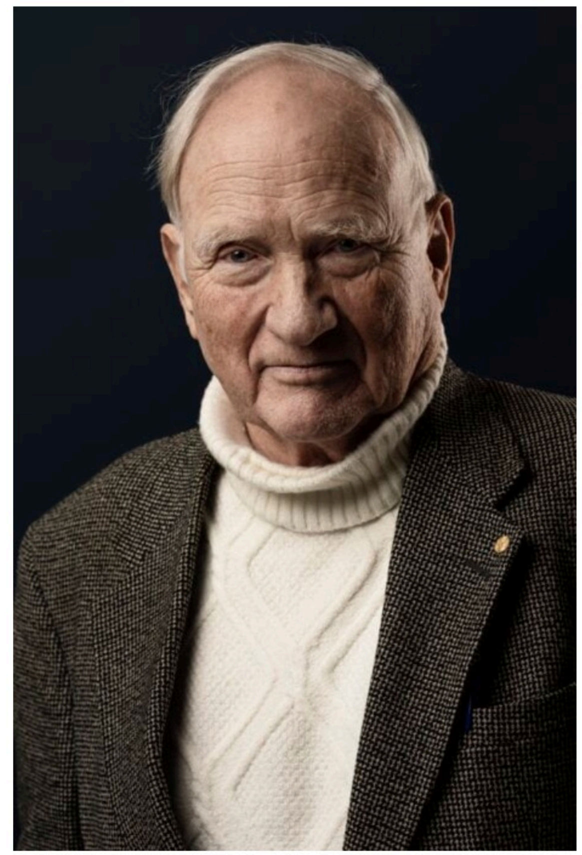
Alain Aspect

Prize share: 1/3



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John F. Clauser

Prize share: 1/3

© Nobel Prize Outreach. Photo: Stefan Bladh

Anton Zeilinger

Prize share: 1/3

*The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"*

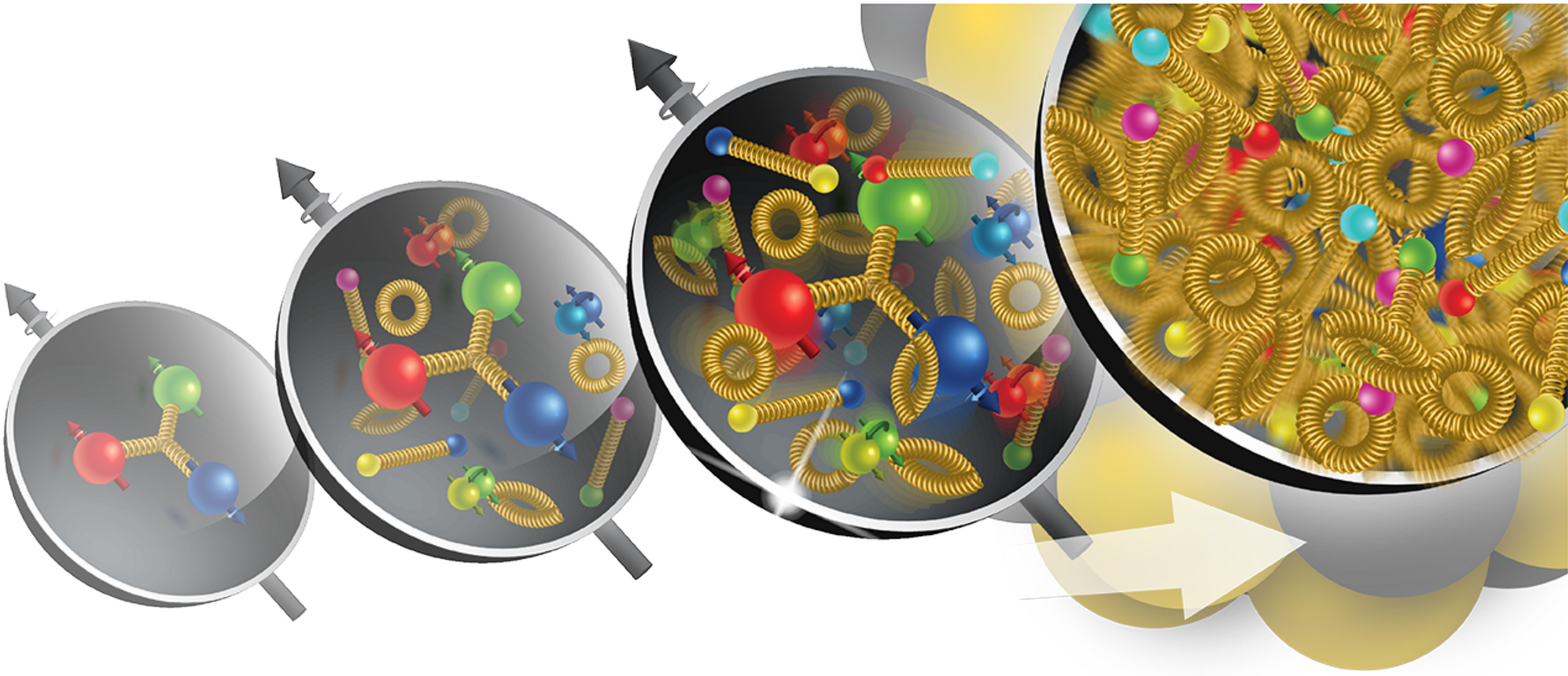
# The Verification in particle physics

- Neutral pion system
- Neutral Kaon system
- Other hadron systems
- …
- Testing with higher energy?

Many references…

# The Verification at the EW scale

- It is not easy, why?



# The Verification at the EW scale

***"A quantitatively characterization of the degree of the entanglement between the subsystems of a system in a mixed state, is not unique! "***

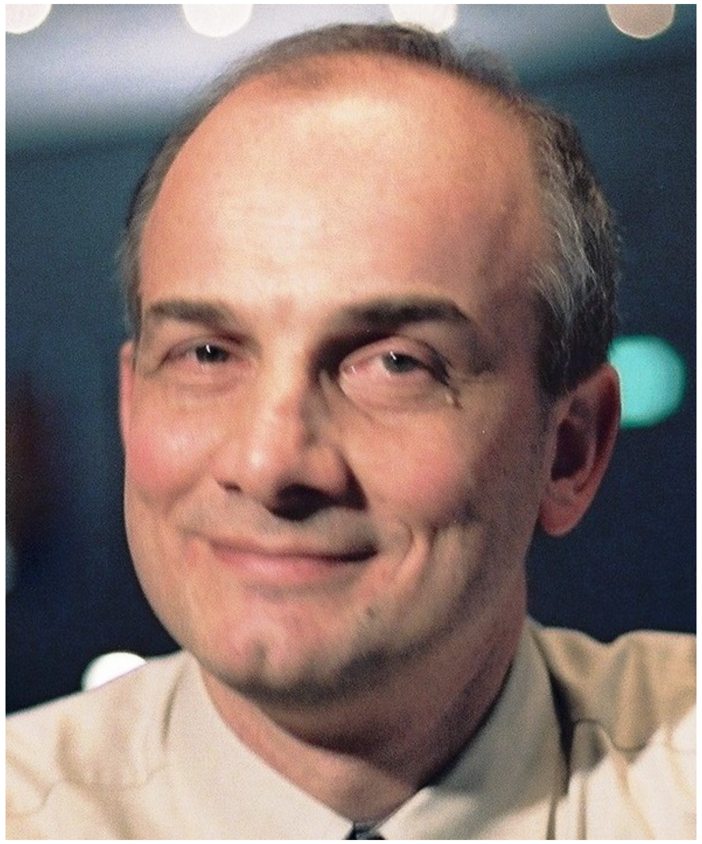
$$\sigma = p_1 \rho_{A1} \otimes \rho_{B1} + p_2 \rho_{A2} \otimes \rho_{B2} + \dots \quad \textbf{?}$$

# The Verification at the EW scale

***"A quantitatively characterization of the degree of the entanglement between the subsystems of a system in a mixed state, is not unique! "***

"Finally , we prove that the weak membership problem for the convex set of separable bipartite density matrices is ***NP-HARD***. "

——Leonid Gurvits



# The Verification at the EW scale

***Even the violation of the CHSH inequalities, is not equivalent to the non-existence of LHVM!***

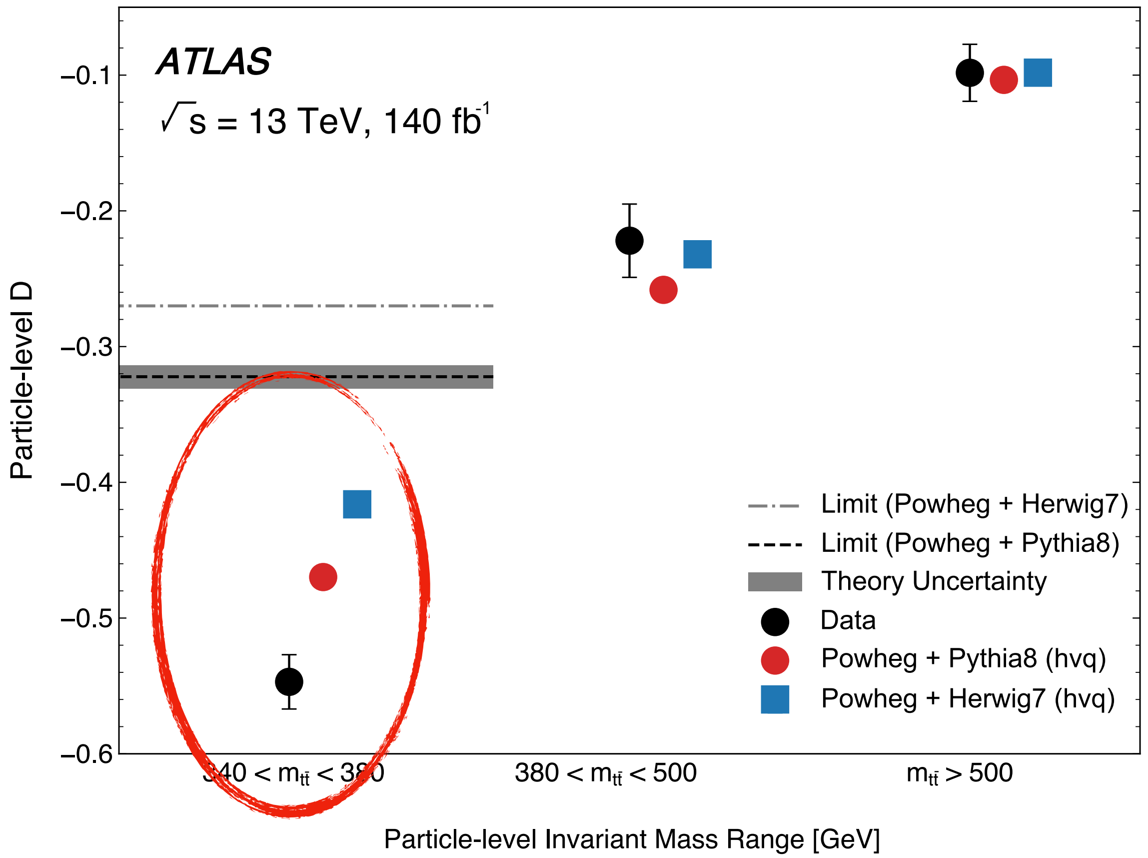
- To “solve” the problem, people introduce some smart criterions such as the CONCURRENCE. (For a nice review, see R. Herodecki, P. Herodecki, M. Herodecki, K. Herodecki, Rev. Mod. Phys. 81 (2009) 865)
- On the other hand, it is shown that asymptotic violation of the CHSH inequality is equivalent to **distillability**. (Means that some pure-state entanglement could be extracted by LOCC)

L. Masanes, Phys. Rev. Lett. 97 (2006) 050503.

# The Verification at the EW scale

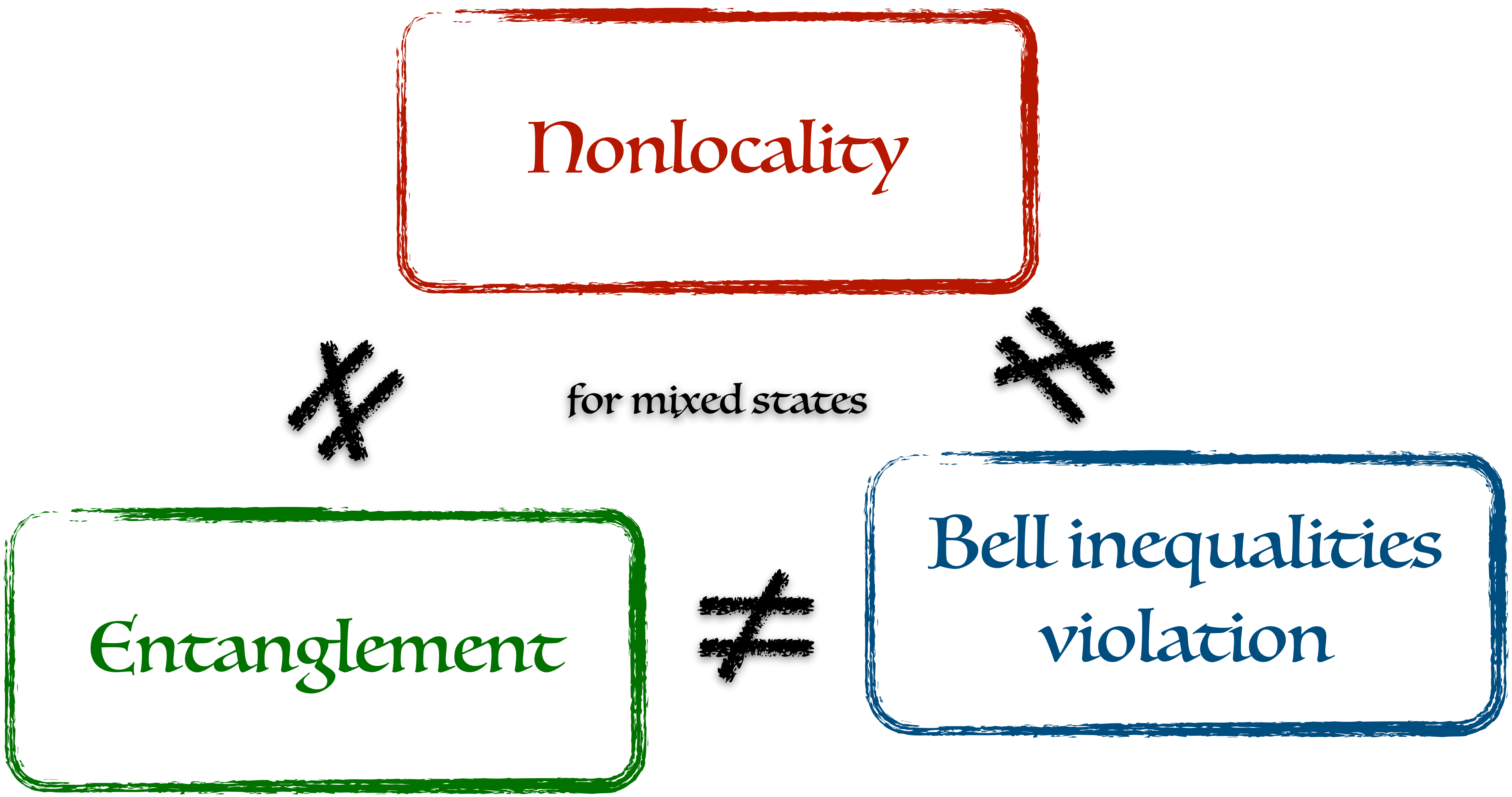
- For 2-qubit system, it is solved by Horodecki et al 1995.
- The most popular topic: $t\bar{t}$ production at the LHC.
- The result from the ATLAS collaboration.

$$D \equiv -3\langle \cos\varphi(\ell_t^+ \ell_{\bar{t}}^-)\rangle$$




ATLAS Collaboration, arXiv:2311.07288[hep-ex];
Y. Afik and J. R. M. de Nova, Eur. Phys. J. Plus **136** (2021) 907.

# The Verification at the EW scale



# WW production at Higgs factory

- The initial state is a mixed state

➡ (Generalized) Bell inequality as a test

Observables: $\hat{A}_1$ (Alice 1), $\hat{A}_2$ (Alice 2), $\hat{B}_1$ (Bob 1), $\hat{B}_2$ (Bob 2)

The results of measurement $\in \mathbb{Z}_3$

$$\max_{\hat{A}_1,\hat{A}_2,\hat{B}_1,\hat{B}_2} \mathcal{I}_3(\hat{A}_1, \hat{A}_2; \hat{B}_1, \hat{B}_2) > 2$$

$$\begin{aligned}\mathcal{I}_3 \equiv &+ \left[P\left(A_1 = B_1\right) + P\left(B_1 = A_2 + 1\right)\right. \\ &\quad \left. + P\left(A_2 = B_2\right) + P\left(B_2 = A_1\right)\right] \\ &- \left[P\left(A_1 = B_1 - 1\right) + P\left(B_1 = A_2\right)\right. \\ &\quad \left. + P\left(A_2 = B_2 - 1\right) + P\left(B_2 = A_1 - 1\right)\right]\end{aligned}$$

Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequality

D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).

# WW production at Higgs factory

- The initial state is a mixed state
  - ➡ (Generalized) Bell inequality
- 9-dim but not 4-dim Hilbert space.

**3x3>2x2**

- QuNit vs. qubit?
  - ➡ “the results for large N are shown to be more resistant to noise with a suitable choice of the observables”

D. Kaszlikowski, P. Gnaciński, M. Żukowski, W. Miklaszewski, A. Zeilinger, Phys. Rev. Lett. **85**, 4418 (2000);
T. Durt, D. Kaszlikowski, M. Żukowski, Phys. Rev. A **64**, 024101 (2001);
J.-L. Chen, D. Kaszlikowski, L. C. Kwek, C. H. Oh, M. Żukowski, Phys. Rev. A **64**, 052109 (2001);
D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).

# WW production at Higgs factory

- The density matrix (some technical details…)

$$\hat{\rho}_{WW} \propto \mathcal{M}(e^+e^- \to W^+W^-)\hat{\rho}_{e^+e^-}\mathcal{M}(e^+e^- \to W^+W^-)^\dagger$$

$$\hat{\rho}_W = \frac{1}{3}\hat{I}_3 + d^i\hat{S}_i + q^{ij}\hat{S}_{\{ij\}}, \;\; i,j = 1,2,3$$

$$\begin{aligned}\hat{\rho}_{WW} \;\; = \;\; & \frac{1}{9}\hat{I}_9 + \frac{1}{3}d^i_+\hat{S}^+_i \otimes \hat{I}_3 + \frac{1}{3}d^i_-\hat{I}_3 \otimes \hat{S}^-_i \\ & + \frac{1}{3}q^{ij}_+\hat{S}^+_{\{ij\}} \otimes \hat{I}_3 + \frac{1}{3}q^{ij}_-\hat{I}_3 \otimes \hat{S}^-_{\{ij\}} \\ & + C^{ij}_d\hat{S}^+_i \otimes \hat{S}^-_j + C^{i,jk}_{d,q}\hat{S}^+_i \otimes \hat{S}^-_{\{jk\}} \\ & + C^{ij,k}_{q,d}\hat{S}^+_{\{ij\}} \otimes \hat{S}^-_k + C^{ij,k\ell}_q\hat{S}^+_{\{ij\}} \otimes \hat{S}^-_{\{k\ell\}}\end{aligned}$$

# WW production at Higgs factory

- The density matrix (some technical details…) is “easy” to calculate

$$
\begin{aligned}
& d_+^2 = d_-^2 = q_+^{12} = q_+^{23} = q_-^{12} = q_-^{23} = C_d^{12} = C_d^{21} \\
& = C_d^{23} = C_d^{32} = C_{d,q}^{1,12} = C_{d,q}^{1,23} = C_{d,q}^{2,31} = C_{d,q}^{2,11} = C_{d,q}^{2,22} \\
& = C_{d,q}^{2,33} = C_{d,q}^{3,12} = C_{d,q}^{3,23} = C_{q,d}^{12,1} = C_{q,d}^{23,1} = C_{q,d}^{31,2} \\
& = C_{q,d}^{11,2} = C_{q,d}^{22,2} = C_{q,d}^{33,2} = C_{q,d}^{12,3} = C_{q,d}^{23,3} = C_q^{12,31} \\
& = C_q^{12,11} = C_q^{12,22} = C_q^{12,33} = C_q^{23,31} = C_q^{23,11} = C_q^{23,22} \\
& = C_q^{23,33} = C_q^{31,12} = C_q^{31,23} = C_q^{11,12} = C_q^{11,23} = C_q^{11,22} \\
& = C_q^{22,12} = C_q^{22,23} = C_q^{22,11} = C_q^{33,12} = C_q^{33,23} = C_q^{33,31} \\
& = 0. \qquad (46)
\end{aligned}
$$

# WW production at Higgs factory

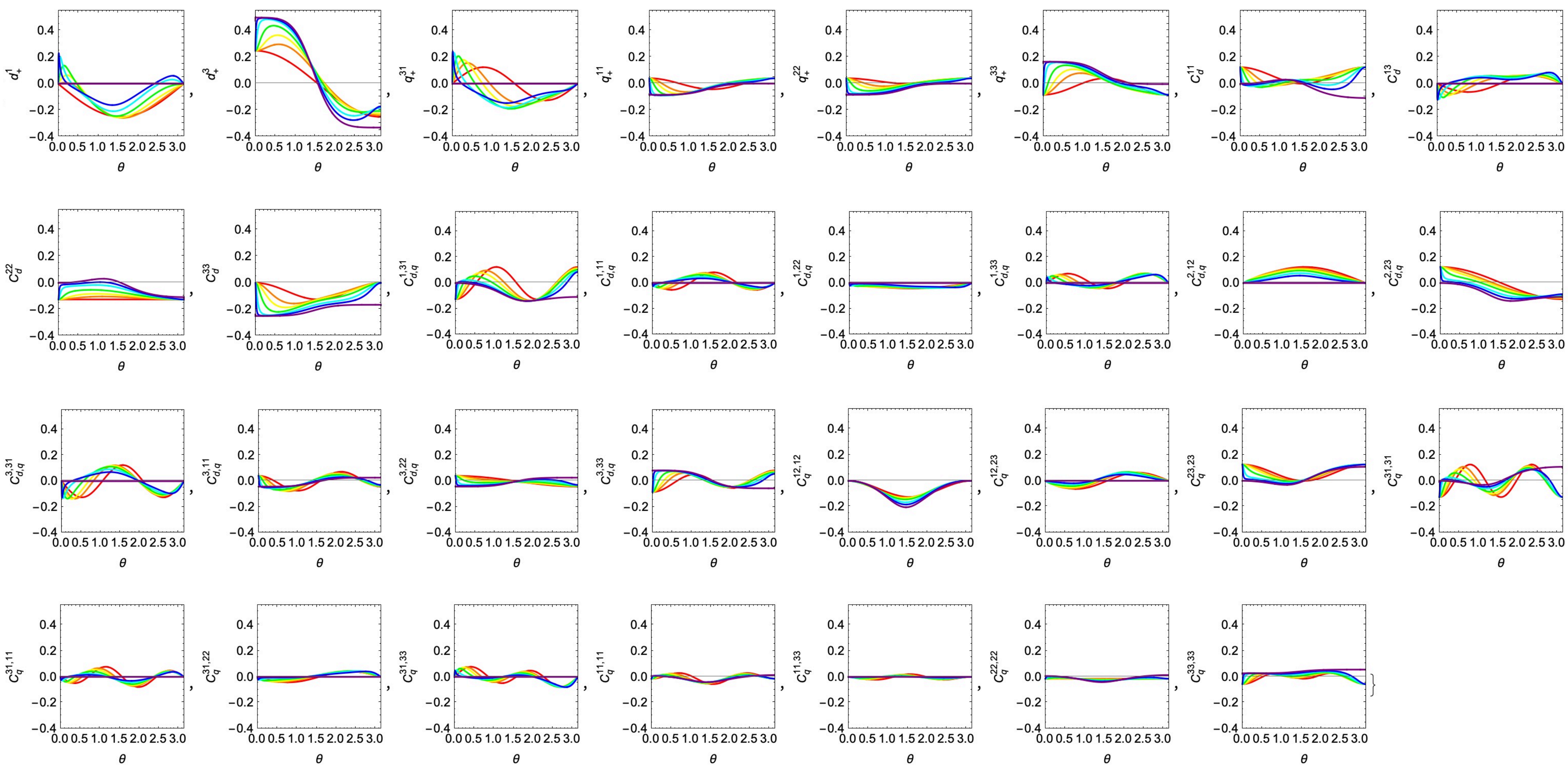
- The density matrix (some technical details…) is “easy” to calculate

$$d_+^1 = d_-^1,\ d_+^3 = -d_-^3,\ q_+^{31} = -q_-^{31},\ q_+^{11} = q_-^{11},\ q_+^{22} = q_-^{22},$$
$$q_+^{33} = q_-^{33},\ C_d^{13} = -C_d^{31},\ C_{d,q}^{1,31} = -C_{q,d}^{31,1},$$

$$\begin{aligned}
&C_{d,q}^{1,11} = C_{q,d}^{11,1},\ C_{d,q}^{1,22} = C_{q,d}^{22,1},\ C_{d,q}^{1,33} = C_{q,d}^{33,1},\\
&C_{d,q}^{2,12} = C_{q,d}^{12,2},\ C_{d,q}^{2,23} = -C_{q,d}^{23,2},\ C_{d,q}^{3,31} = C_{q,d}^{31,3},\\
&C_{d,q}^{3,11} = -C_{q,d}^{11,3},\ C_{d,q}^{3,22} = -C_{q,d}^{22,3},\ C_{d,q}^{3,33} = -C_{q,d}^{33,3},\\
&C_q^{12,23} = -C_q^{23,12},\ C_q^{31,11} = -C_q^{11,31},\ C_q^{31,22} = -C_q^{22,31},\\
&C_q^{11,33} = C_q^{33,11} = -C_q^{22,33} = -C_q^{33,22}.
\end{aligned} \tag{47}$$

# WW production at Higgs factory

- The density matrix (some technical details…) is “easy” to calculate



# WW production at Higgs factory

- The density matrix (some technical details…) is “easy” to calculate
- $\beta \to 0$ ?

# WW production at Higgs factory

- The density matrix (some technical details…) is “easy” to calculate
- $\beta \to 0$ ?
- *s*-channel is *p*-wave and suppressed by a factor of $\beta$.

# WW production at Higgs factory

- The density matrix (some technical details…) is “easy” to calculate
- $\beta \to 0$ ?
- *s*-channel is *p*-wave and suppressed by a factor of $\beta$.
- *t*-channel is purely left-handed current so that the initial state is selected by the (weak) interaction to be a pure state $|e_L^-\rangle \otimes |e_L^+\rangle$.

# WW production at Higgs factory

- The density matrix (some technical details…) is easy to calculate

- $\beta \to 0$ ?

$$\mathscr{M} \propto \frac{e^2}{2\sin^2\theta_W}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sin 2\theta & -\sqrt{2}(\cos 2\theta + \cos\theta) & -\sin 2\theta - 2\sin\theta & \sqrt{2}(\cos 2\theta - \cos\theta) & 2\sin 2\theta & -\sqrt{2}(\cos 2\theta + \cos\theta) & -\sin 2\theta + 2\sin\theta & \sqrt{2}(\cos 2\theta - \cos\theta) & \sin 2\theta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

# WW production at Higgs factory

- The density matrix (some technical details…) is easy to calculate
- $\beta \to 0$ ?

$$\rho_{W^+} \propto \begin{pmatrix} \frac{(3-\cos\theta)}{4}\cos^2\frac{\theta}{2} & -\frac{(1-\cos\theta)\sin\theta}{4\sqrt{2}} & -\frac{1}{8}\sin^2\theta \\ -\frac{(1-\cos\theta)\sin\theta}{4\sqrt{2}} & \frac{1}{8}(3+\cos 2\theta) & -\frac{(1+\cos\theta)\sin\theta}{4\sqrt{2}} \\ -\frac{1}{8}\sin^2\theta & -\frac{(1+\cos\theta)\sin\theta}{4\sqrt{2}} & \frac{(3+\cos\theta)}{4}\sin^2\frac{\theta}{2} \end{pmatrix} = \frac{1}{2}|v_1\rangle\langle v_1| + \frac{1}{2}|v_2\rangle\langle v_2|$$

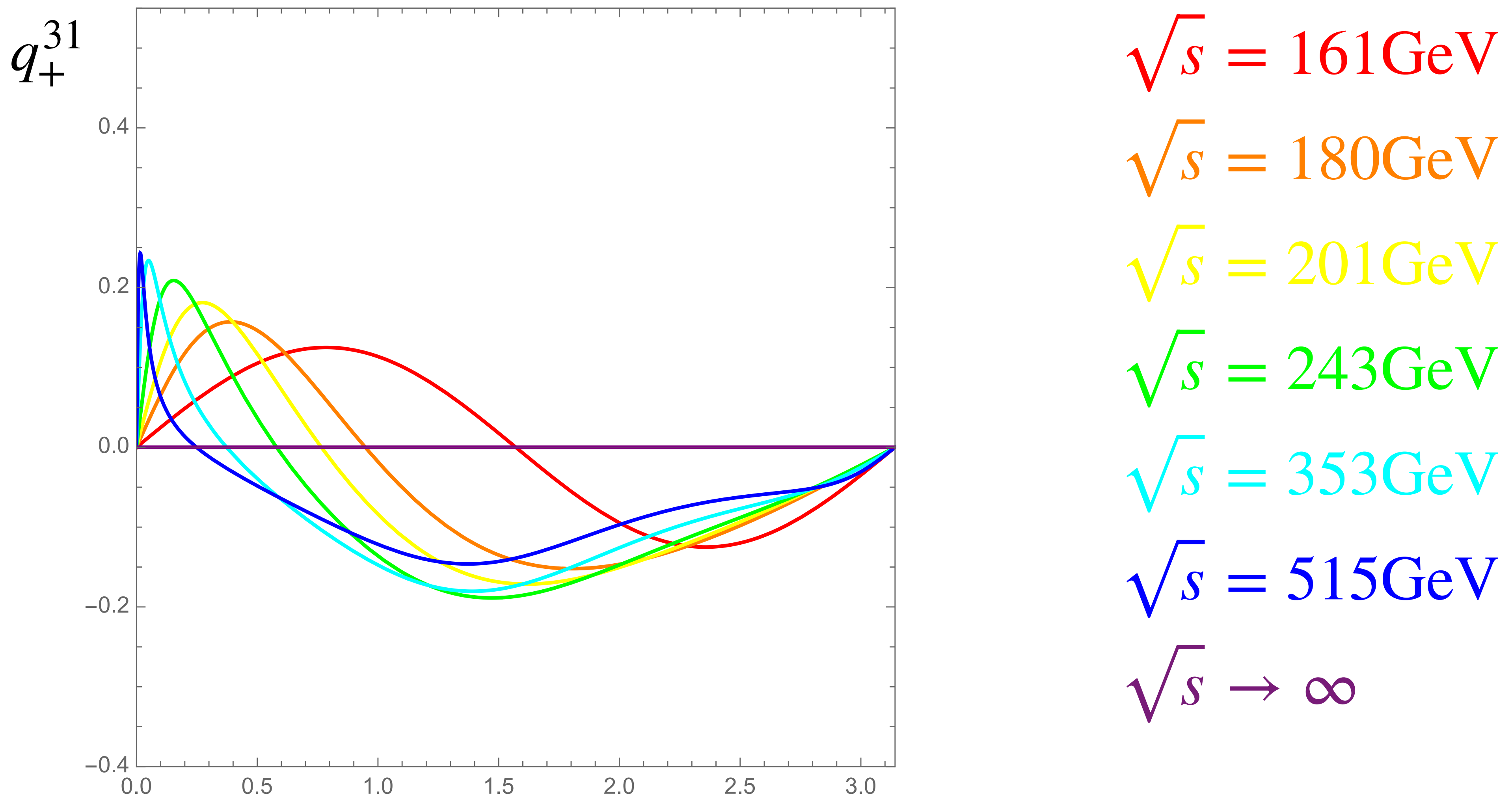
$$|v_1\rangle = \frac{1+\cos\theta}{\sqrt{3+\cos 2\theta}}|+\rangle - \frac{1-\cos\theta}{\sqrt{3+\cos 2\theta}}|-\rangle$$

$$|v_2\rangle = -\frac{(1-\cos\theta)\sin\theta}{\sqrt{2}\sqrt{3+\cos 2\theta}}|+\rangle + \frac{\sqrt{3+\cos 2\theta}}{2}|0\rangle - \frac{(1+\cos\theta)\sin\theta}{\sqrt{2}\sqrt{3+\cos 2\theta}}|-\rangle$$

$$|v_3\rangle = \left(\frac{1-\cos\theta}{2}\right)|+\rangle + \frac{\sin\theta}{\sqrt{2}}|0\rangle + \left(\frac{1+\cos\theta}{2}\right)|-\rangle$$

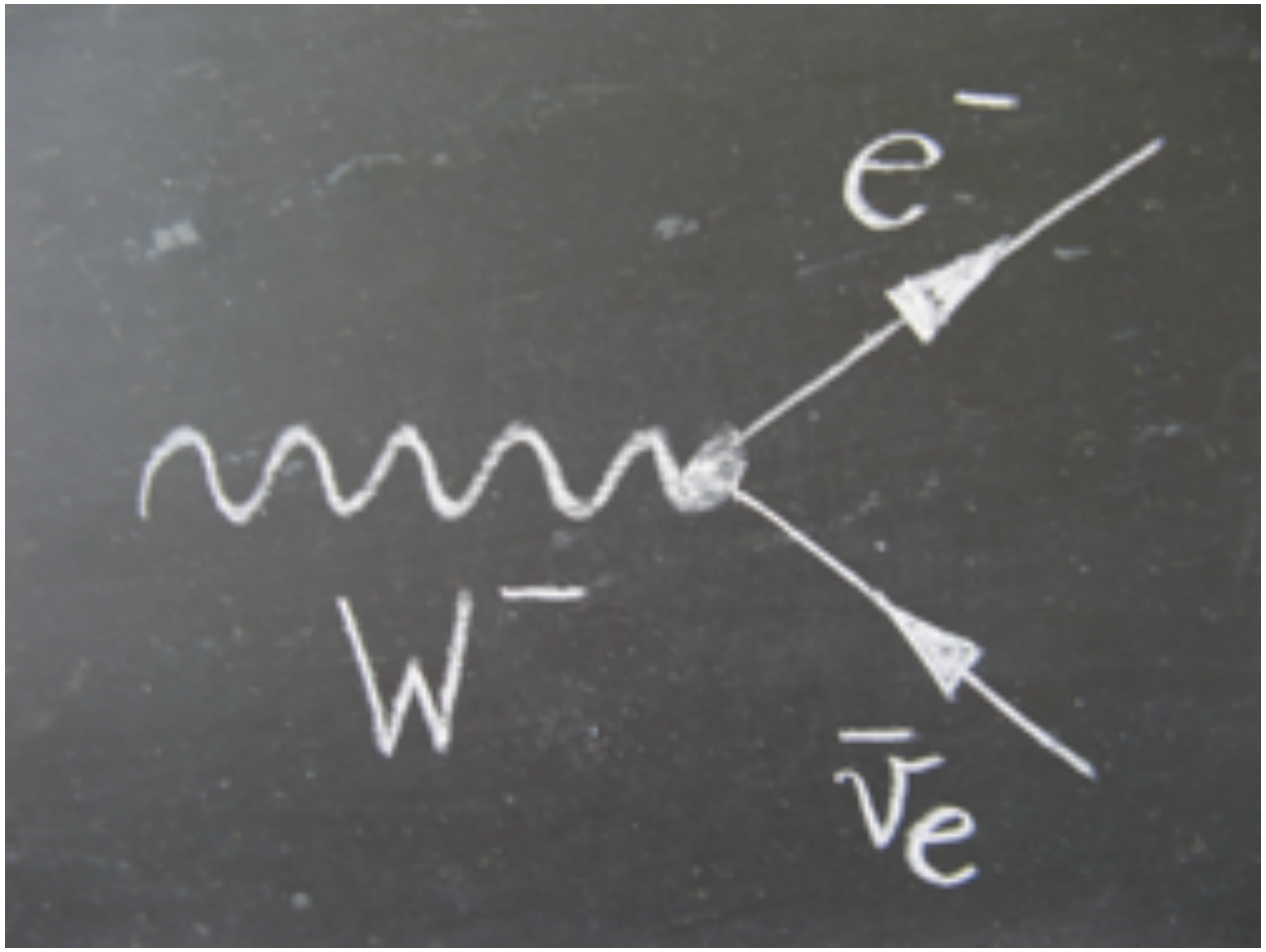
# WW production at Higgs factory

- The density matrix (some technical details…) is easy to calculate
- $\beta \to 0, \infty$ are not very good approximations.



# WW production at Higgs factory

- How to measure it at Higgs factory???
- “Measuring” the polarization direction of the W boson.



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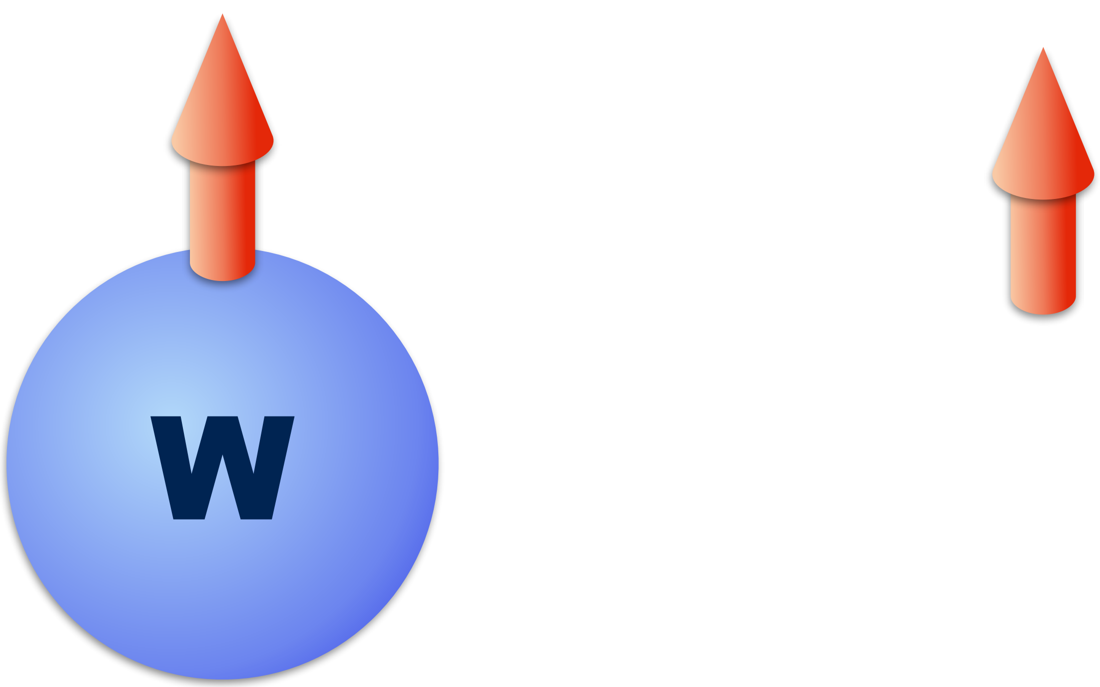

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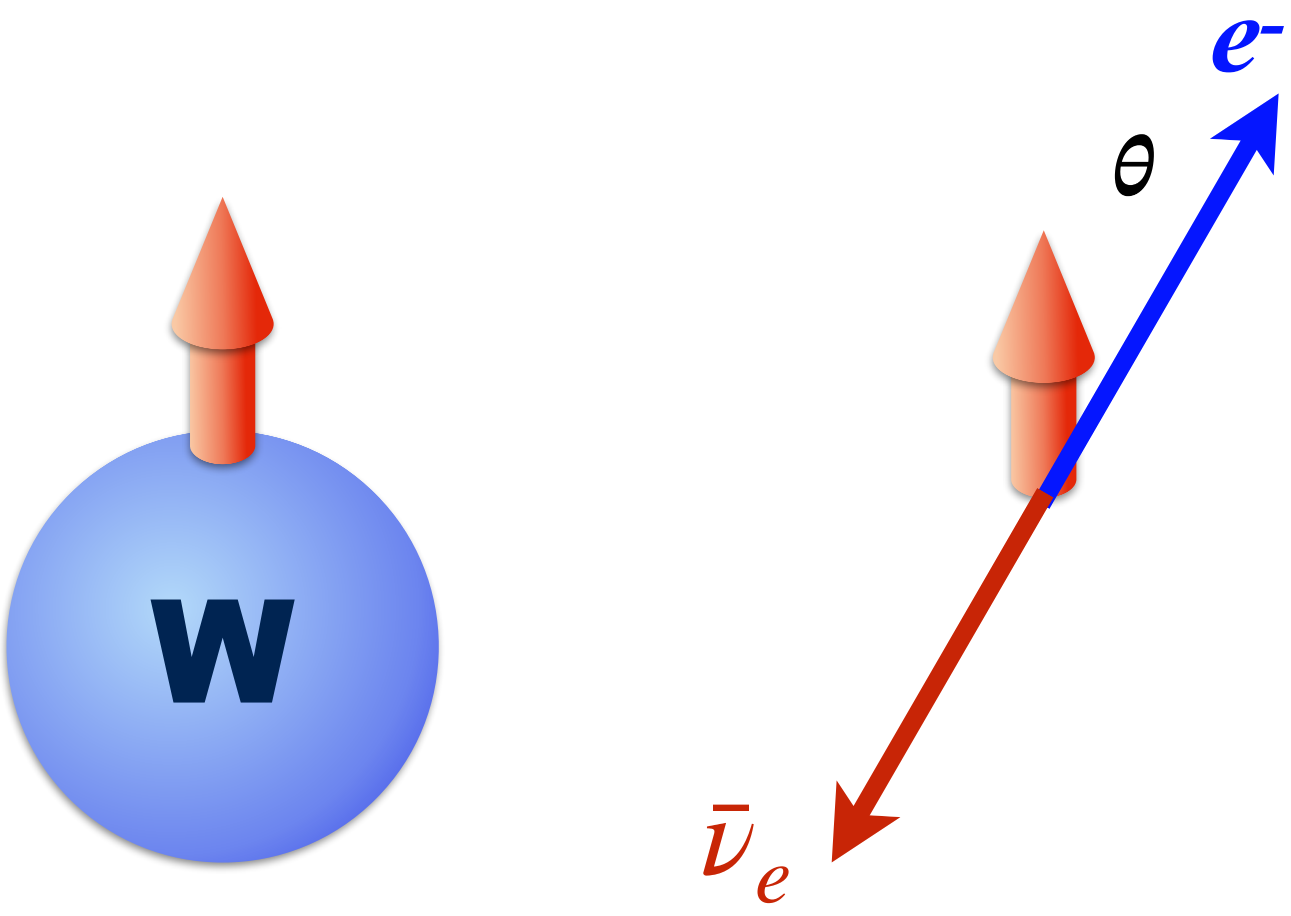

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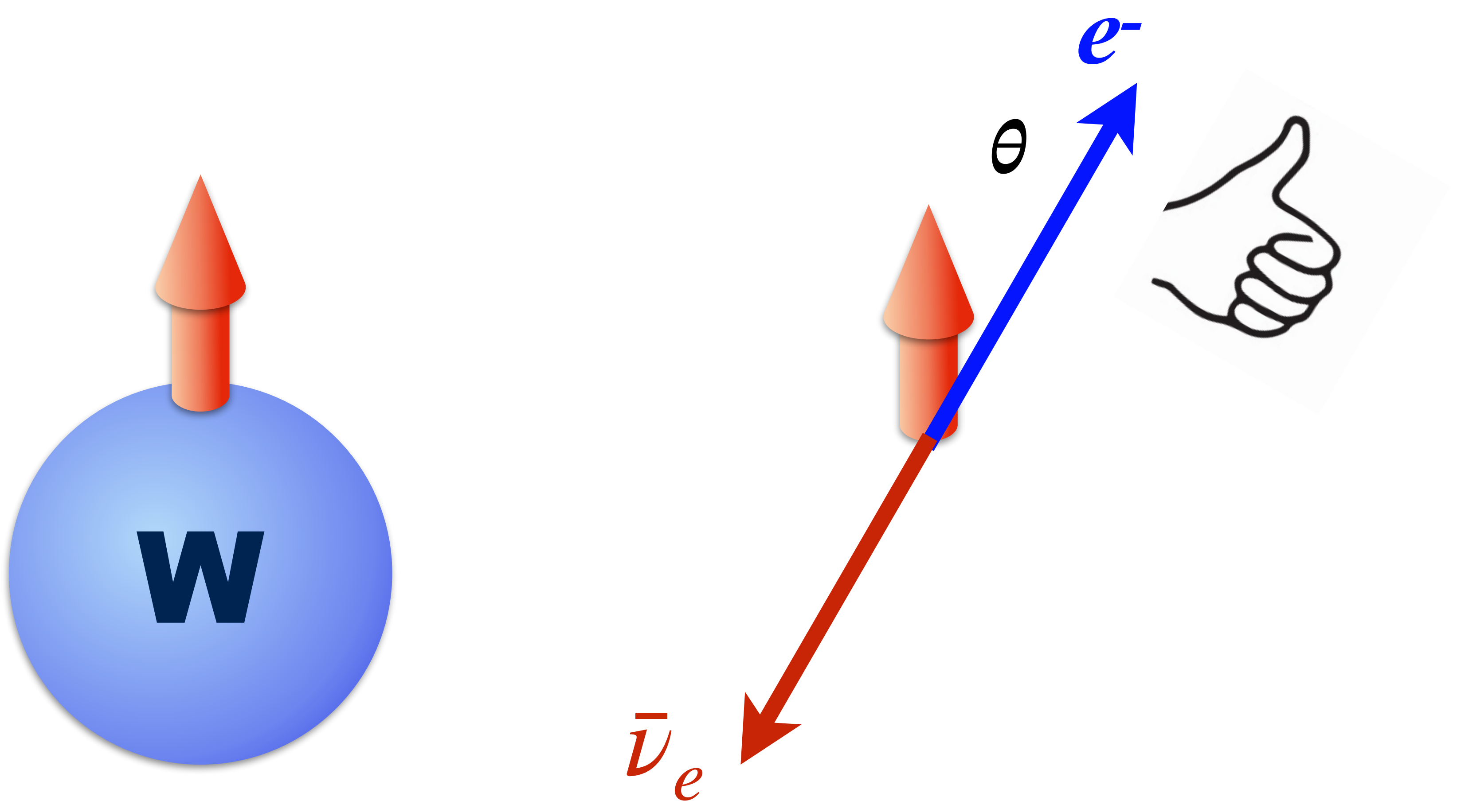

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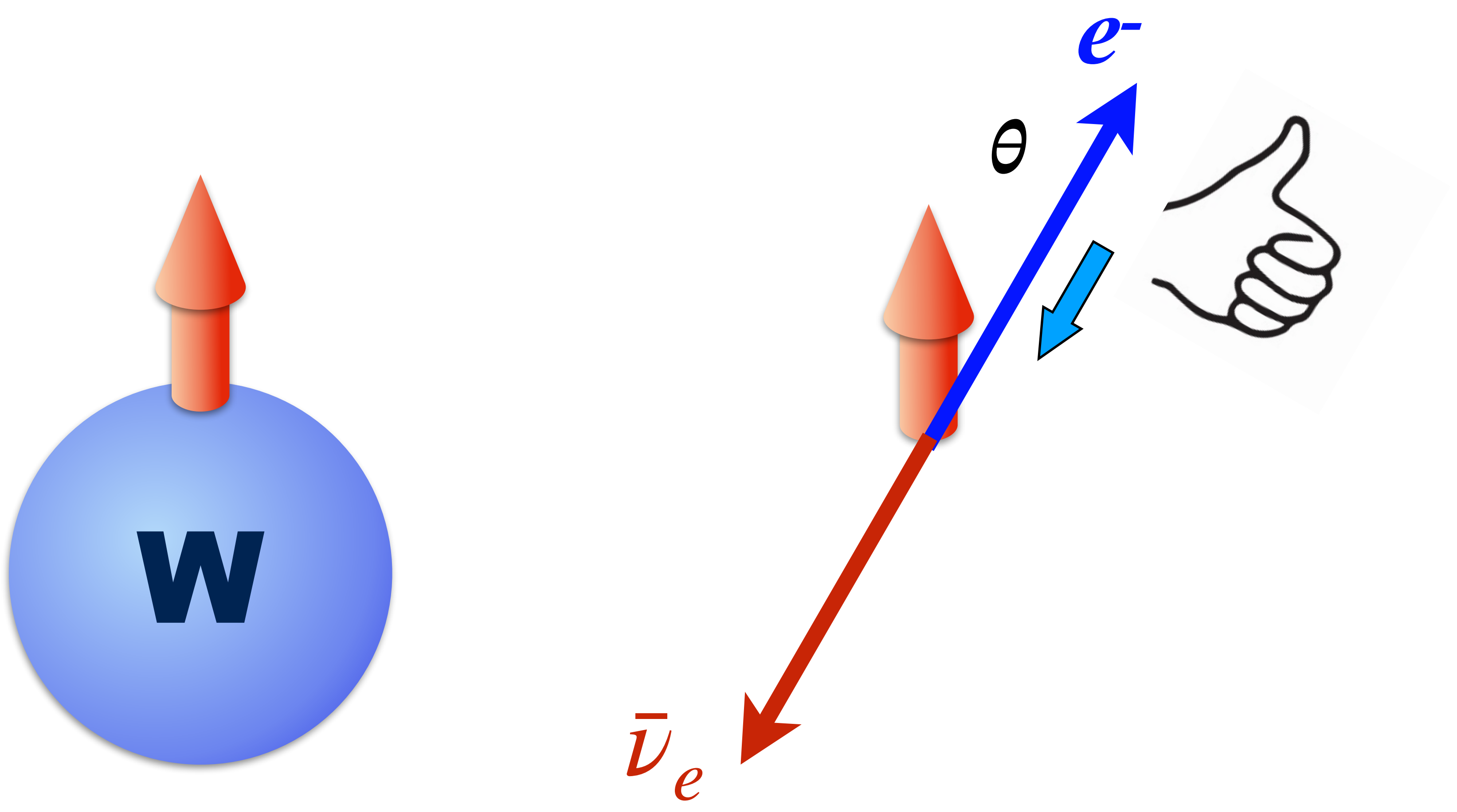

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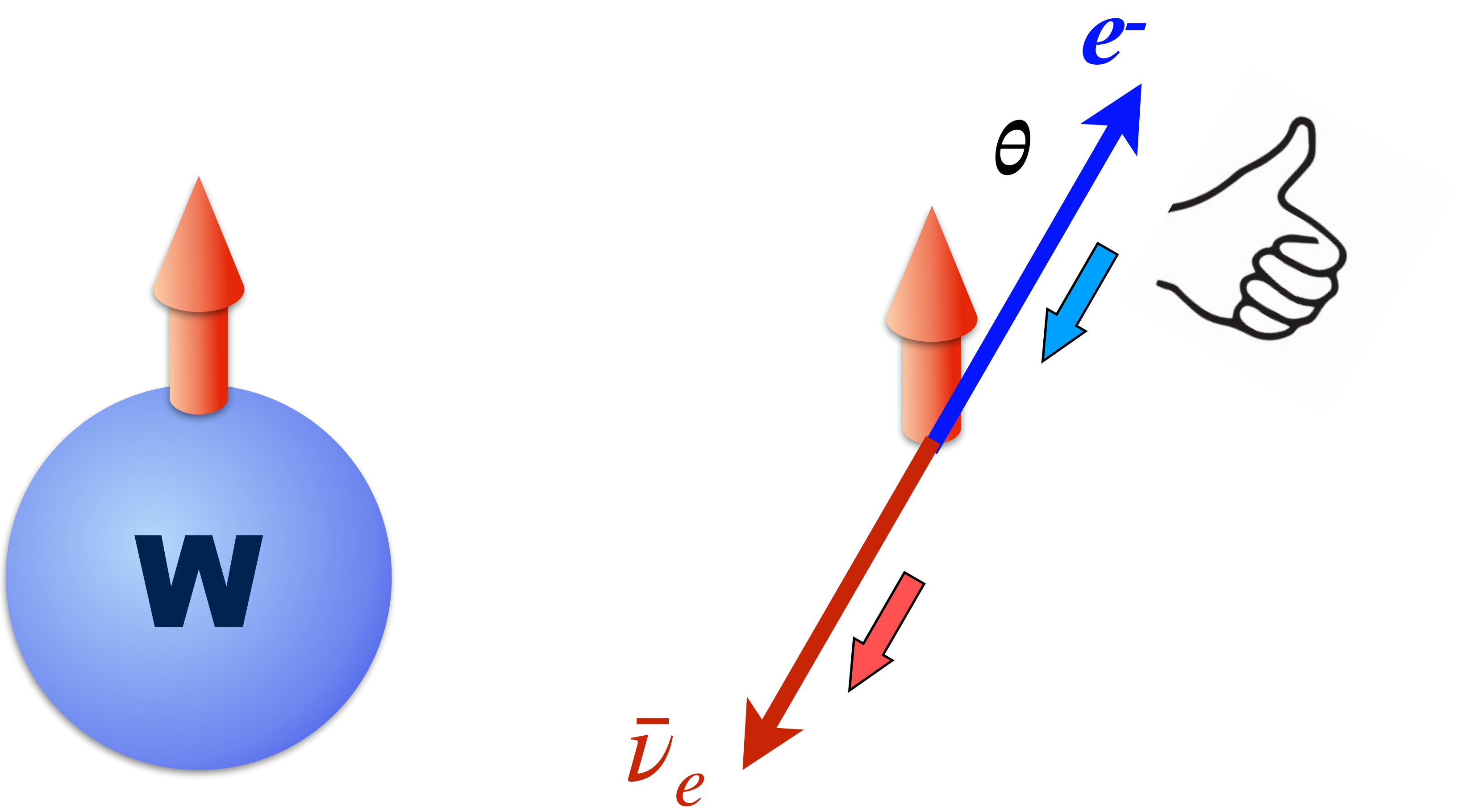

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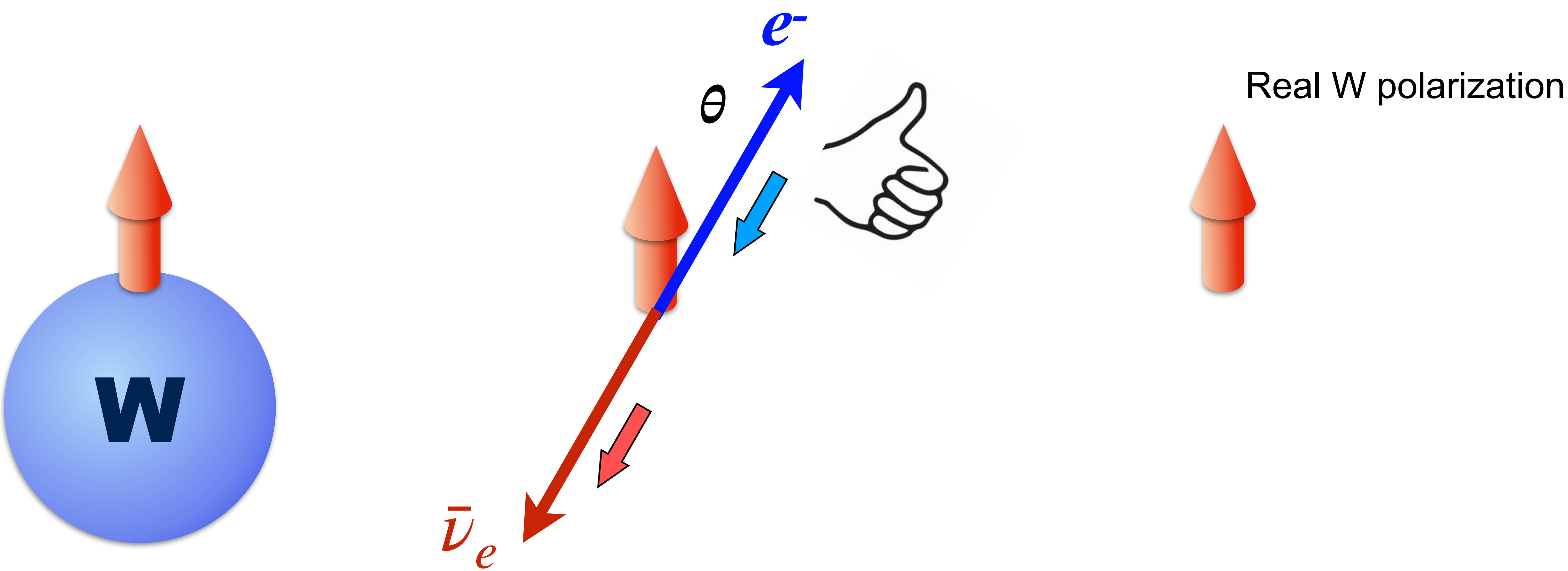

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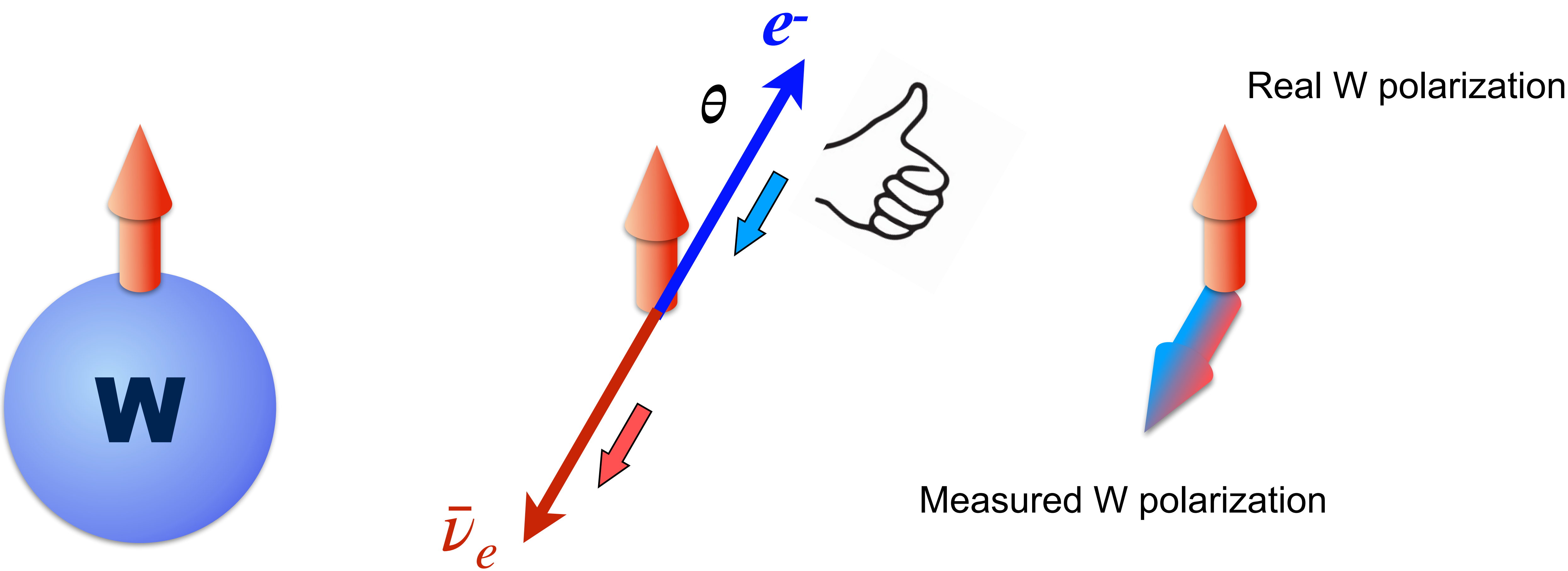


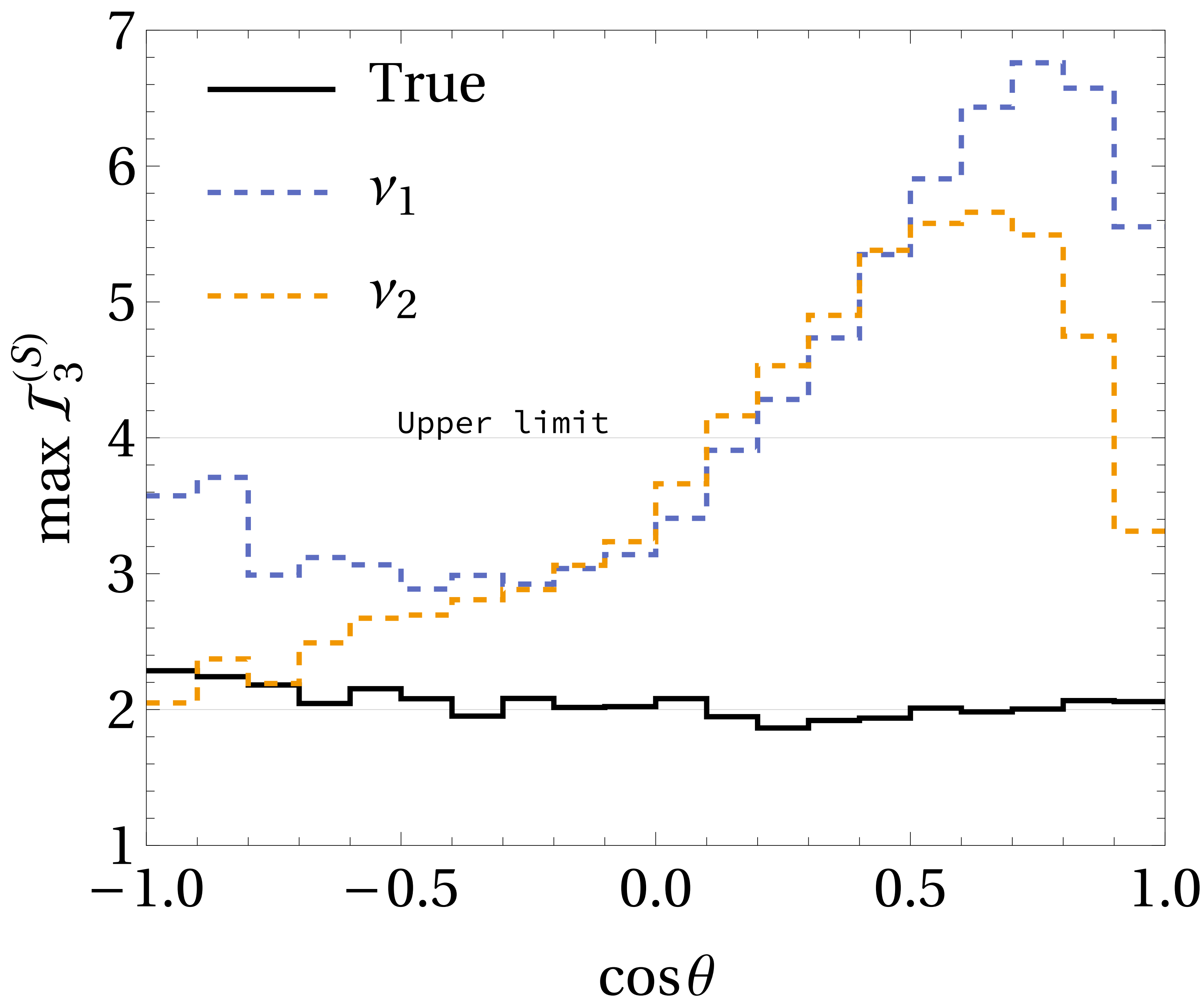
# WW production at Higgs factory

- How to measure it at Higgs factory???
- "Measuring" the polarization direction of the W boson.
- Projection operators of the spin eigenstates:

$$\hat{\Pi}_{\mathbf{n}} = \frac{1}{2}(\hat{S}_{\mathbf{n}} + \hat{S}_{\mathbf{n}}^2)$$

# WW production at Higgs factory

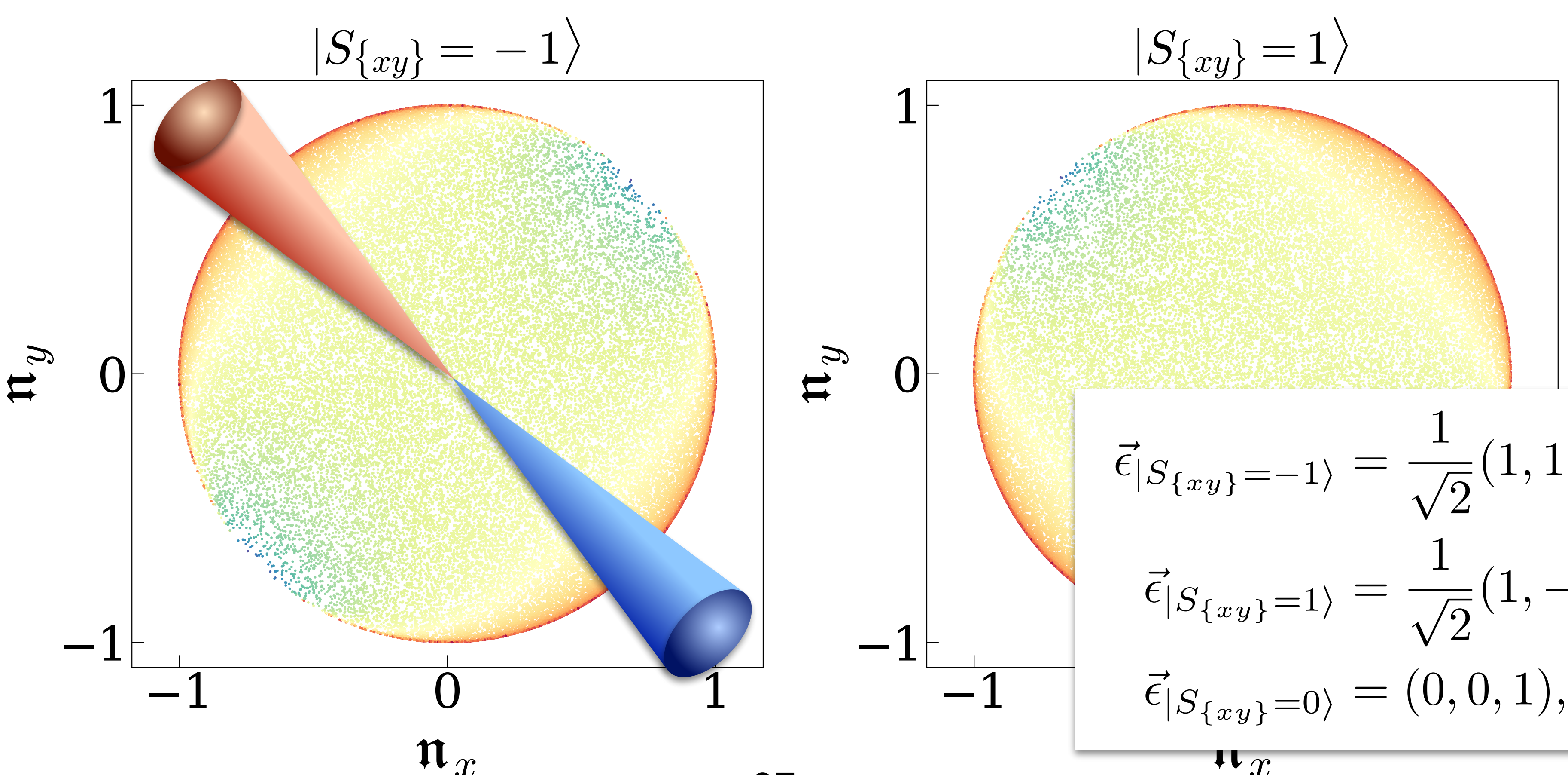
- Collider phenomenology



# WW production at Higgs factory

- Collider phenomenology: from dilepton channel to semi-leptonic channel.
- Circular polarization → linear polarization.

$$\hat{\Pi}_{\mathbf{n}} = \hat{I}_3 - \hat{S}^2_{\mathbf{n}}$$




$$\vec{\epsilon}_{|S_{\{xy\}}=-1\rangle} = \frac{1}{\sqrt{2}}(1,1,0),$$
$$\vec{\epsilon}_{|S_{\{xy\}}=1\rangle} = \frac{1}{\sqrt{2}}(1,-1,0),$$
$$\vec{\epsilon}_{|S_{\{xy\}}=0\rangle} = (0,0,1),$$

# WW production at Higgs factory

- Collider phenomenology: from dilepton channel to semi-leptonic channel.
- Circular polarization → linear polarization.

$$\mathscr{I}_3(\hat{S}_{\vec{a}_1}, \hat{S}_{\vec{a}_2}; \hat{S}_{\{x_3y_3\}}, \hat{S}_{\{x_4y_4\}}) \equiv + \left[P(S_{\vec{a}_1} = S_{\{x_3y_3\}}) + P(S_{\{x_3y_3\}} = S_{\vec{a}_2} + 1) + P(S_{\vec{a}_2} = S_{\{x_4y_4\}}) + P(S_{\{x_4y_4\}} = S_{\vec{a}_1})\right]$$
$$- \left[P(S_{\vec{a}_1} = S_{\{x_3y_3\}} - 1) + P(S_{\{x_3y_3\}} = S_{\vec{a}_2}) + P(S_{\vec{a}_2} = S_{\{x_4y_4\}} - 1) + P(S_{\{x_4y_4\}} = S_{\vec{a}_1} - 1)\right]$$

# WW production at Higgs factory

- Calculating the generalized Bell observable

$$P(S_{\vec{a}_1} = S_{\{x_3y_3\}}) = \sum_{\lambda=-1}^{1} \mathrm{Tr}\left[\hat{\rho}_{WW}\hat{\Pi}_{|S_{\vec{a}_1}=\lambda, S_{\{x_3y_3\}}=\lambda\rangle}\right]$$
$$= \mathrm{Tr}\left[\hat{\rho}_{WW}\cdot\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=-1)\otimes\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=-1)\right]$$
$$+\mathrm{Tr}\left[\hat{\rho}_{WW}\cdot\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=1)\otimes\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=1)\right]$$
$$+\mathrm{Tr}\left[\hat{\rho}_{WW}\cdot\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=0)\otimes\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=0)\right]$$
$$= 1 - 2q^{-}_{ij}\epsilon_{3i}\epsilon_{3j} - 2C^{dq}_{i,jk}a_{1i}(\epsilon_{1j}\epsilon_{1k} - \epsilon_{2j}\epsilon_{2k})$$
$$+2C^{q}_{ij,kl}a_{1i}a_{1j}(-\epsilon_{1k}\epsilon_{1l} - \epsilon_{2k}\epsilon_{2l} + 2\epsilon_{3k}\epsilon_{3l}).$$

$$\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=-1) = \frac{1}{2}(-\hat{S}_{\vec{a}_1}+\hat{S}^2_{\vec{a}_1}),$$
$$\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=1) = \frac{1}{2}(\hat{S}_{\vec{a}_1}+\hat{S}^2_{\vec{a}_1}),$$
$$\hat{\Pi}_{\vec{a}_1}(S_{\vec{a}_1}=0) = \hat{I}_3 - \hat{S}^2_{\vec{a}_1}.$$

$$\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=-1) = \hat{I}_3 - \hat{S}^2_{\vec{\epsilon}_1},\quad \vec{\epsilon}_1 = \frac{\hat{x}_3+\hat{y}_3}{\sqrt{2}},$$
$$\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=1) = \hat{I}_3 - \hat{S}^2_{\vec{\epsilon}_2},\quad \vec{\epsilon}_2 = \frac{\hat{x}_3-\hat{y}_3}{\sqrt{2}},$$
$$\hat{\Pi}_{x_3y_3}(S_{\{x_3y_3\}}=0) = \hat{I}_3 - \hat{S}^2_{\vec{\epsilon}_3},\quad \vec{\epsilon}_3 = \hat{x}_3\times\hat{y}_3.$$

# WW production at Higgs factory

- Calculating the generalized Bell observable

$$
\begin{aligned}
&\mathcal{I}_3(\hat{S}_{\vec{a}_1}, \hat{S}_{\vec{a}_2}; \hat{S}_{\{x_3y_3\}}, \hat{S}_{\{x_4y_4\}}) \\
&= 2q^{-}_{ij}(\omega_{1i}\omega_{1j} + \omega_{2i}\omega_{2j} - 2\omega_{3i}\omega_{3j}) \\
&+ 2C^{dq}_{i,jk}a_{1i}(2\epsilon_{1j}\epsilon_{1k} - \epsilon_{2j}\epsilon_{2k} - \epsilon_{3j}\epsilon_{3k} + \omega_{1j}\omega_{1k} \\
&\quad - 2\omega_{2j}\omega_{2k} + \omega_{3j}\omega_{3k}) \\
&+ 2C^{dq}_{i,jk}a_{2i}(-2\epsilon_{1j}\epsilon_{1k} + \epsilon_{2j}\epsilon_{2k} + \epsilon_{3j}\epsilon_{3k} + 2\omega_{1j}\omega_{1k} \\
&\quad - \omega_{2j}\omega_{2k} - \omega_{3j}\omega_{3k}) \\
&+ 6C^{q}_{ij,kl}a_{1i}a_{1j}(-\epsilon_{2k}\epsilon_{2l} + \epsilon_{3k}\epsilon_{3l} - \omega_{1k}\omega_{1l} + \omega_{3k}\omega_{3l}) \\
&+ 6C^{q}_{ij,kl}a_{2i}a_{2j}(\epsilon_{2k}\epsilon_{2l} - \epsilon_{3k}\epsilon_{3l} - \omega_{2k}\omega_{2l} + \omega_{3k}\omega_{3l})
\end{aligned}
$$

$$
\begin{aligned}
\langle \mathfrak{n}^{\pm}_i \rangle &= d^{\pm}_i, \\
\langle \mathfrak{q}^{\pm}_{ij} \rangle &= \frac{2}{5} q^{\pm}_{ij}, \\
\langle \mathfrak{n}^{+}_i \mathfrak{n}^{-}_j \rangle &= C^{d}_{ij}, \\
\langle \mathfrak{q}^{+}_{ij} \mathfrak{q}^{-}_{kl} \rangle &= \frac{4}{25} C^{q}_{ij,kl}, \\
\left\langle \mathfrak{n}^{+}_i \mathfrak{q}^{-}_{jk} \right\rangle &= \frac{2}{5} C^{dq}_{i,jk}, \\
\langle \mathfrak{q}^{+}_{ij} \mathfrak{n}^{-}_k \rangle &= \frac{2}{5} C^{qd}_{ij,k}.
\end{aligned}
$$

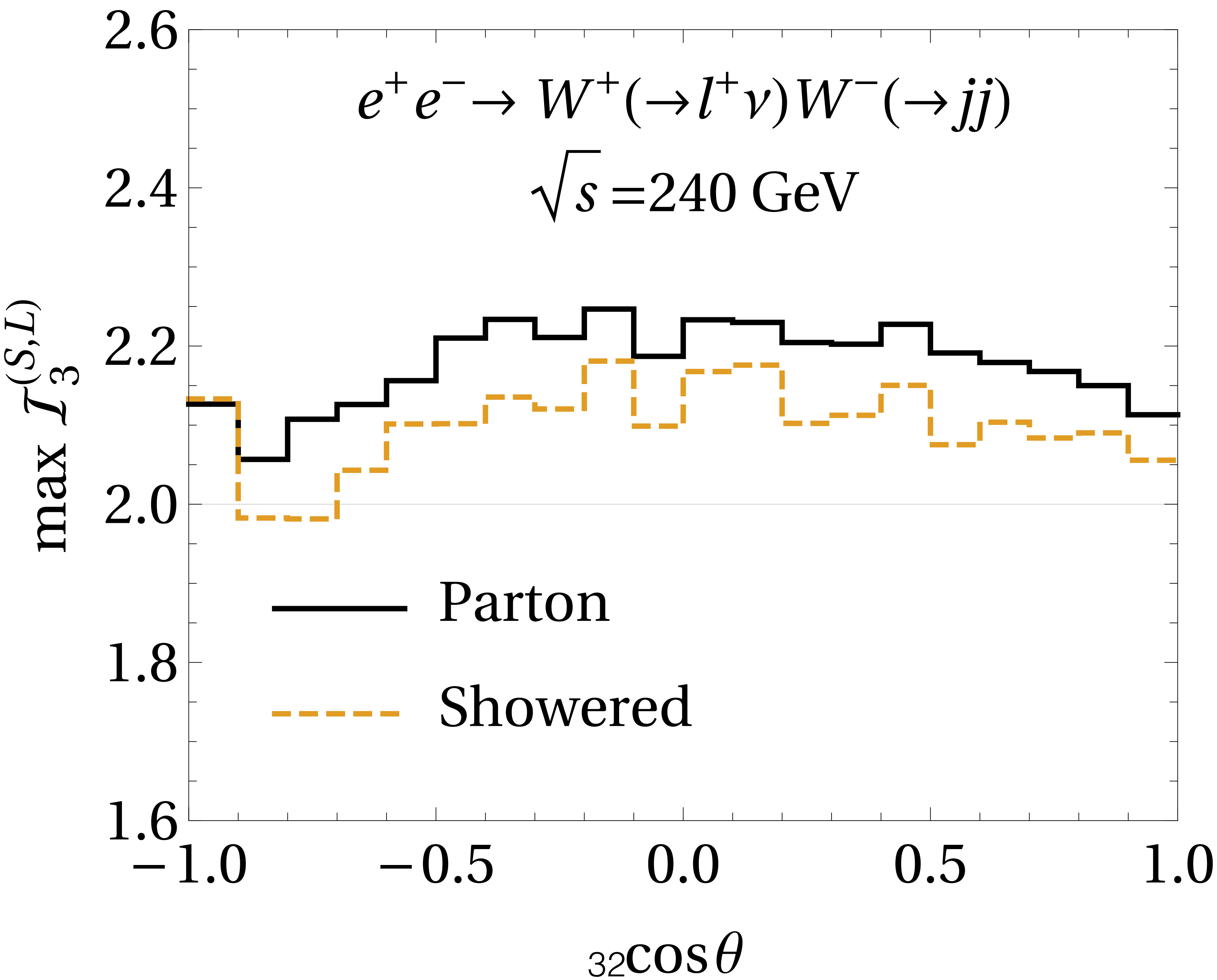
- We need to choose the directions according the coefficients to maximize the generalized Bell observable.

# WW production at Higgs factory

- Some details ($e^+e^- \to W^+W^- \to \ell^\pm \nu jj$)
    - 240GeV electron-positron collider
    - (LO) MADGRAPH5_AMC@NLO+PYTHIA8+FASTJET
    - 2 Exclusive jets with Durham algorithm ($E_j > 5\text{GeV}, |\eta_j| < 3.5$)
    - One isolated charged lepton ($e^\pm, \mu^\pm$) ($E_\ell > 15\text{GeV},\ |\cos\theta_\ell| < 0.98$)
    - Missing energy ($\cos\theta_{\ell\nu} < 0.2$)
    - Reconstructed *W* mass ($|m_{jj} - m_W| < 20\text{GeV}$)

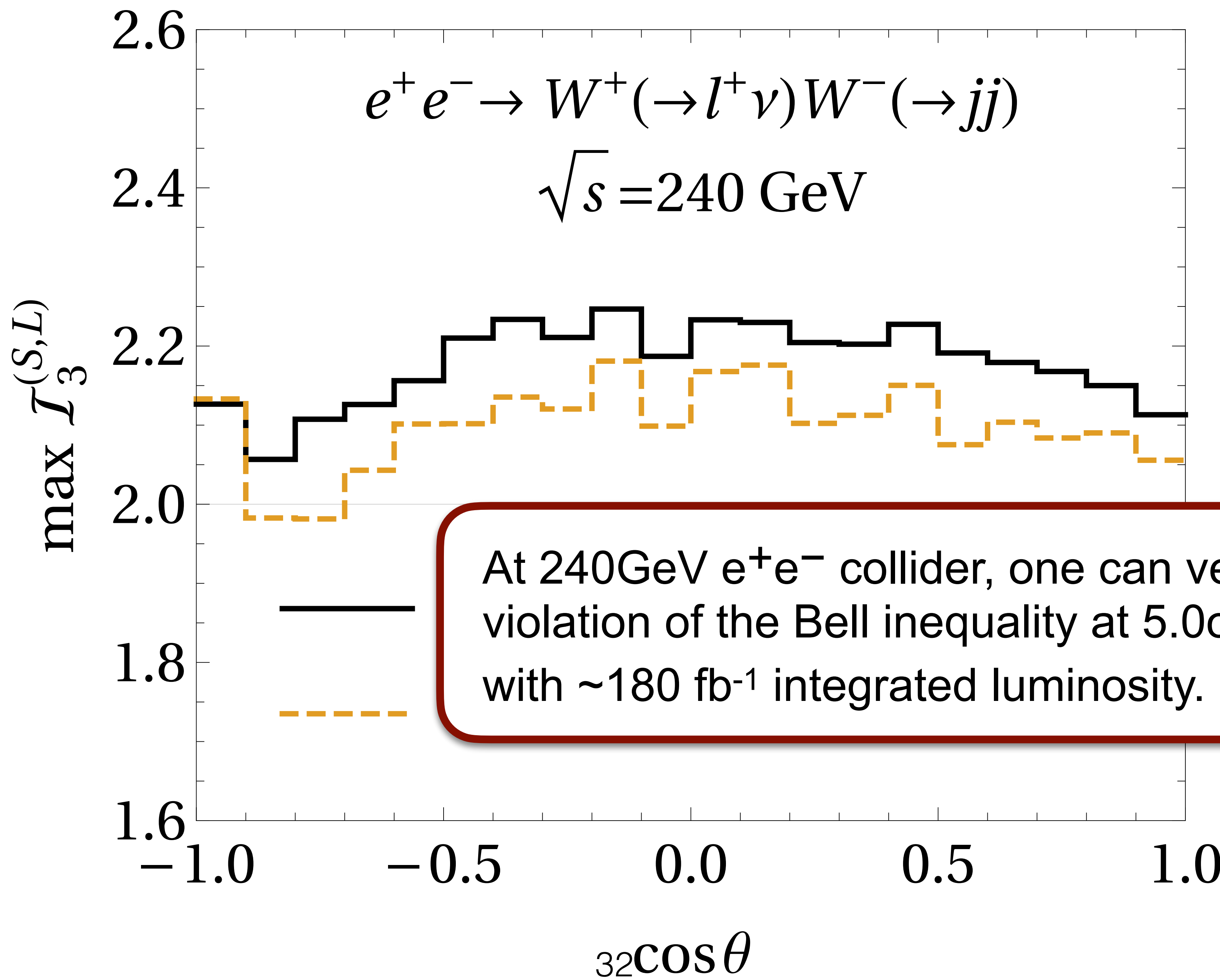
# WW production at Higgs factory

- The result



# WW production at Higgs factory

- The result




At 240GeV $e^+e^-$ collider, one can verify the violation of the Bell inequality at 5.0σ significance with ~180 $fb^{-1}$ integrated luminosity.

# Conclusion and Discussion

- We provide a realistic approach to test Bell inequalities in W pair systems using a new set of Bell observables based on measuring the linear polarization of W bosons.
- Our observables depend on only part of the density matrix that can be correctly measured in the semi-leptonic decay mode of W.
- To our best knowledge, this is the first attempt of testing Bell inequality in a basic qu3it system (**beyond qubit**).
- Loopholes?


Thank you!