Alpha particle distribution in nuclei

Where in nuclei are seeds of the alpha particles?

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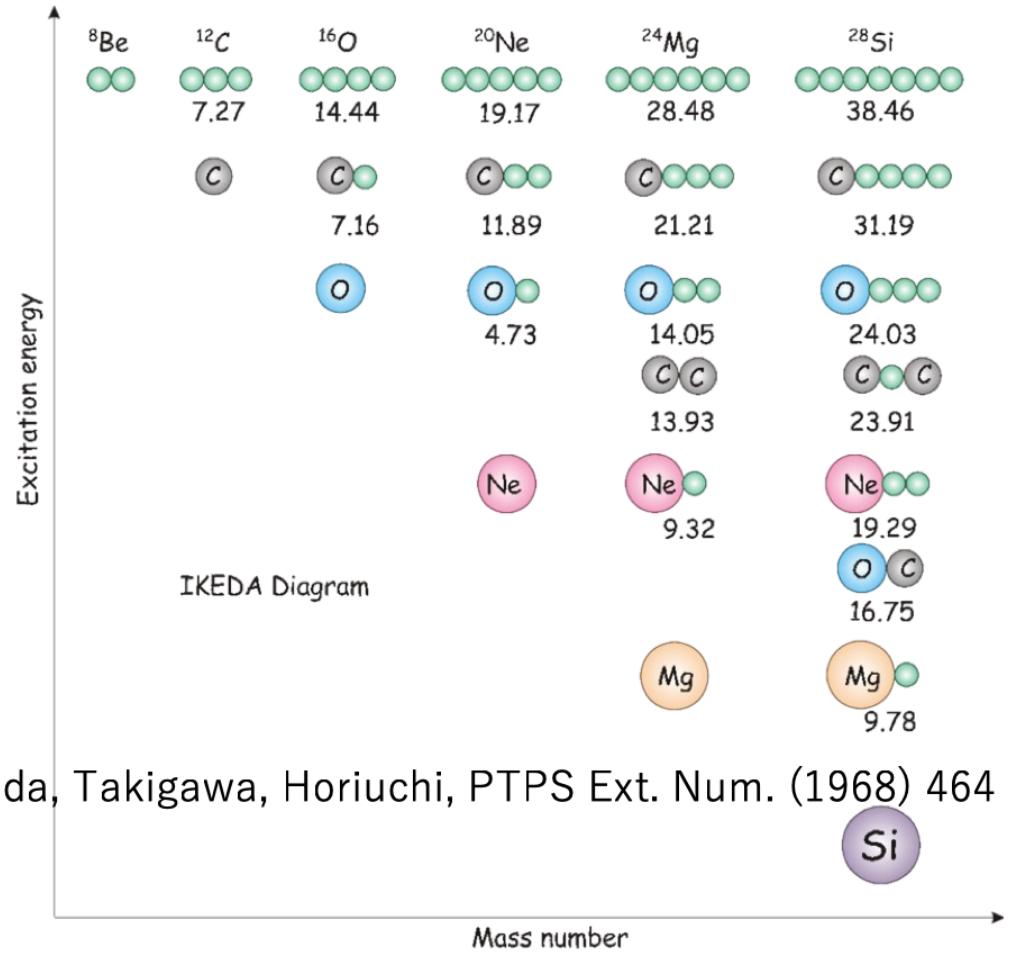
Alpha clusters in light nuclei



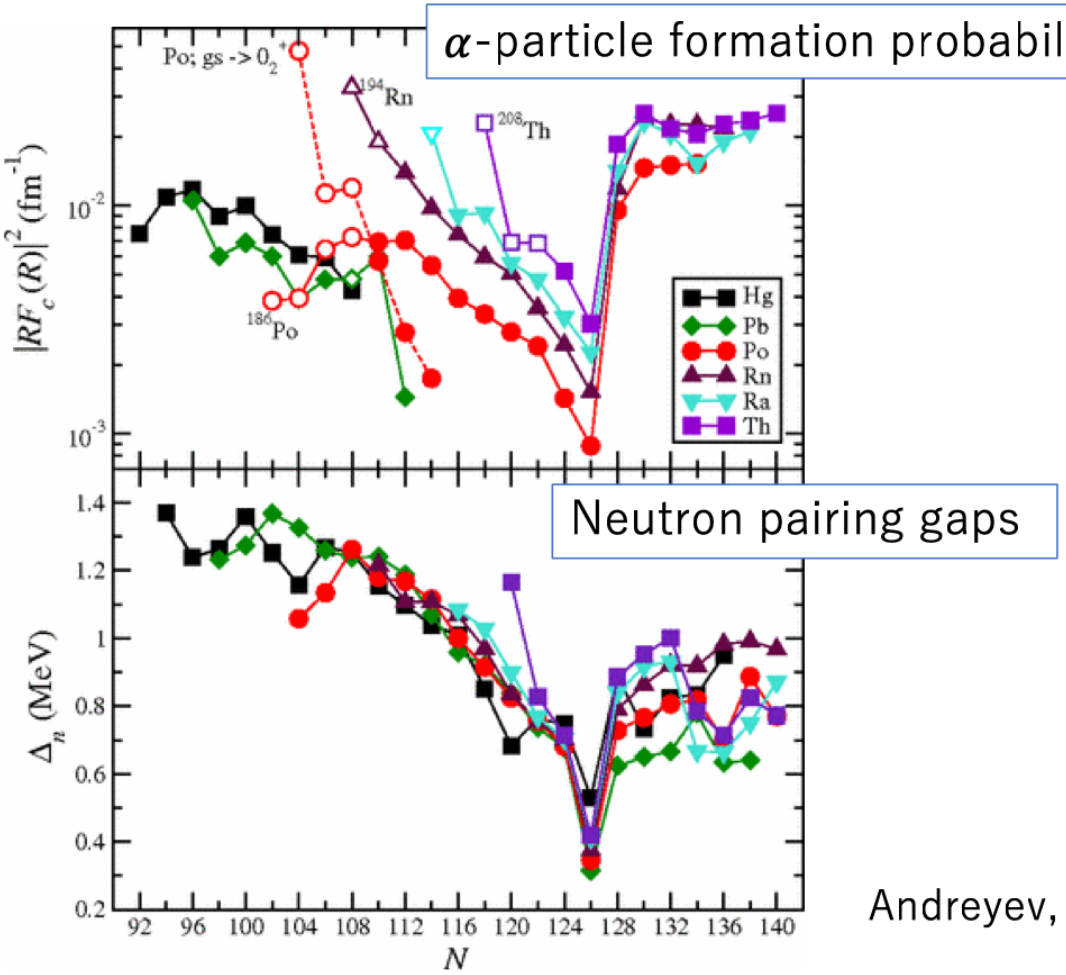
Ikeda, Takigawa, Horiuchi, PTPS Ext. Num. (1968) 464

The threshold rule

- Prominent clustering in states near the threshold
- Seeds of clustering in the ground state
- Significant impact on element synthesis (e.g., Hoyle state)

Figure from von Oertzen, Freer, Kanada-En'yo, PR **432**, 43 (2006)

Alpha clusters in heavy nuclei



Alpha decay

- Existence of cluster components in the ground state

$$\Psi = \Psi_{\mathbf{sh}} + \Psi_{\mathbf{cluster}}$$

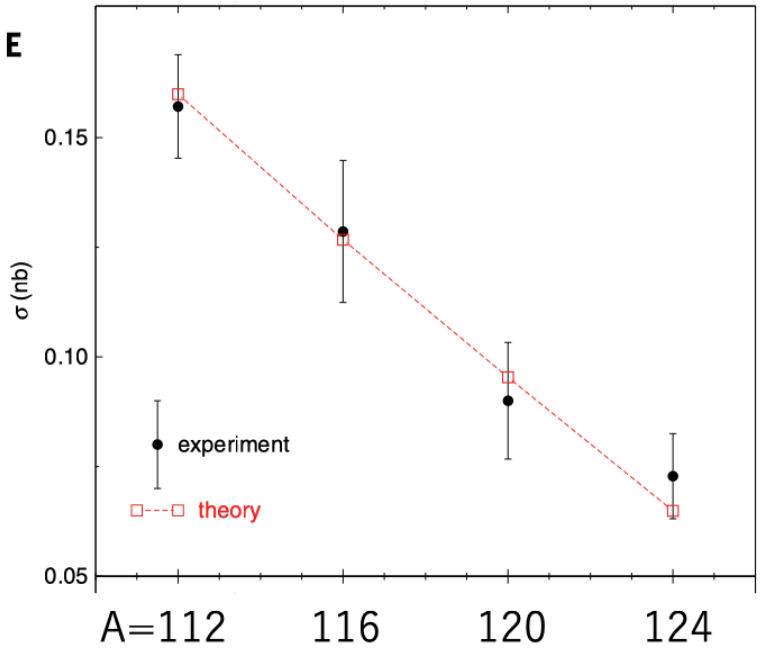
Varga, Lovas, Liotta, PRL 69, 37 (1992)

- Correlation with pairing

Andreyev, et al., PRL 110, 242502 (2013)

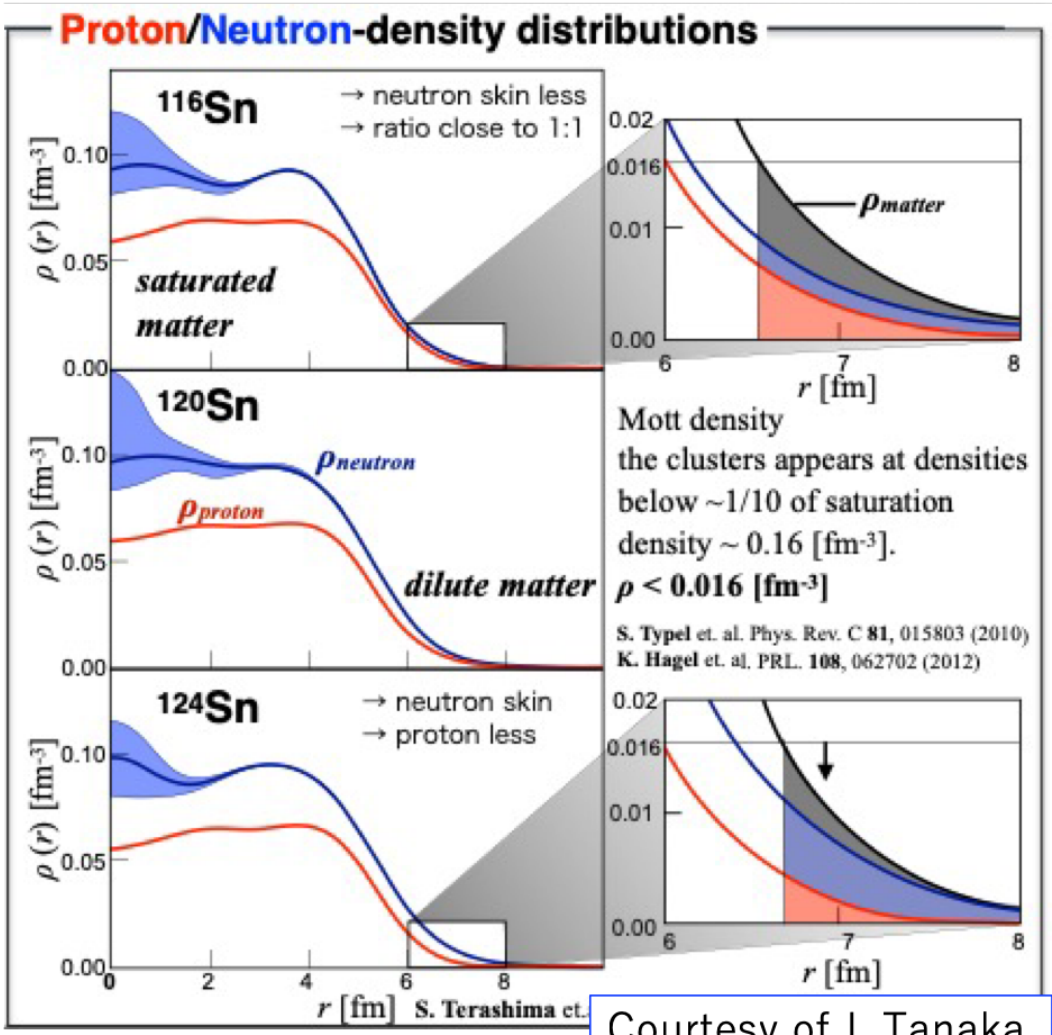
Alpha knockout in $^{116\text{-}124}$Sn

J. Tanaka, et al. Science 371, 260 (2021)



Nuclear matter with light clusters
Local density approx.

Q: To the g.s. state?



Courtesy of J. Tanaka

Typel, PRC 89, 064321 (2014)
Typel et al., PRC 81, 015803 (2010)

Localization function

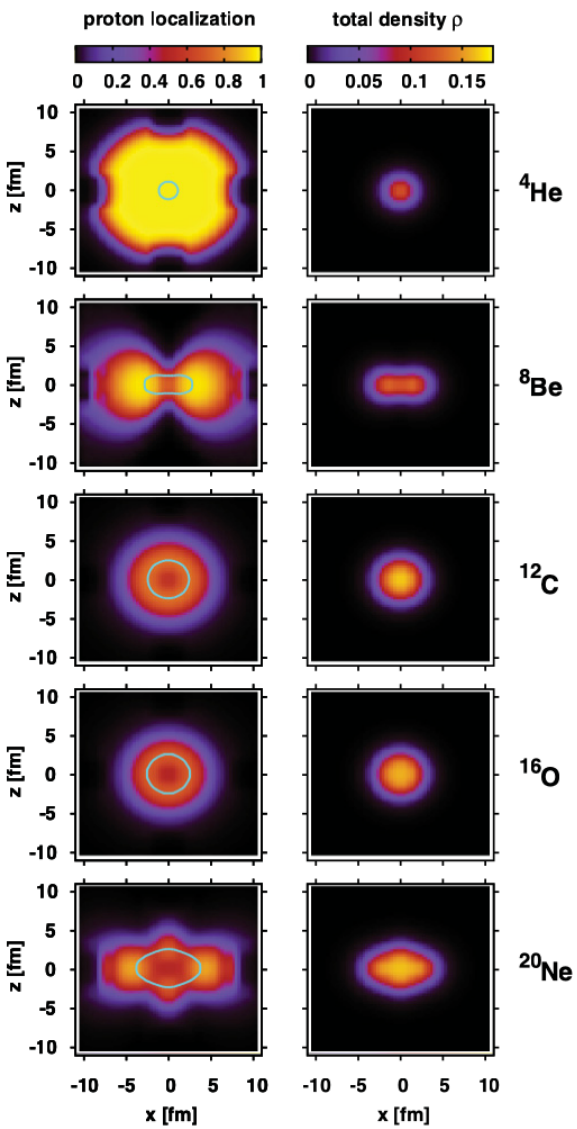
P.G.Reinhard et al., PRC 83, 034312 (2011)

$$C_{q\sigma}(\mathbf{r}) = \left[1 + \left(\frac{\tau_{q\sigma}\rho_{q\sigma} - \frac{1}{4}[\nabla\rho_{q\sigma}]^2 - \mathbf{j}_{q\sigma}^2}{\rho_{q\sigma}\tau_{q\sigma}^{\mathrm{TF}}}\right)^2\right]^{-1}$$

$C_{q\sigma}(r) \approx 1$ may imply possible existence α particles

but does not mean

Not localization of four particles of $(n\uparrow, n\downarrow, p\uparrow, p\downarrow)$



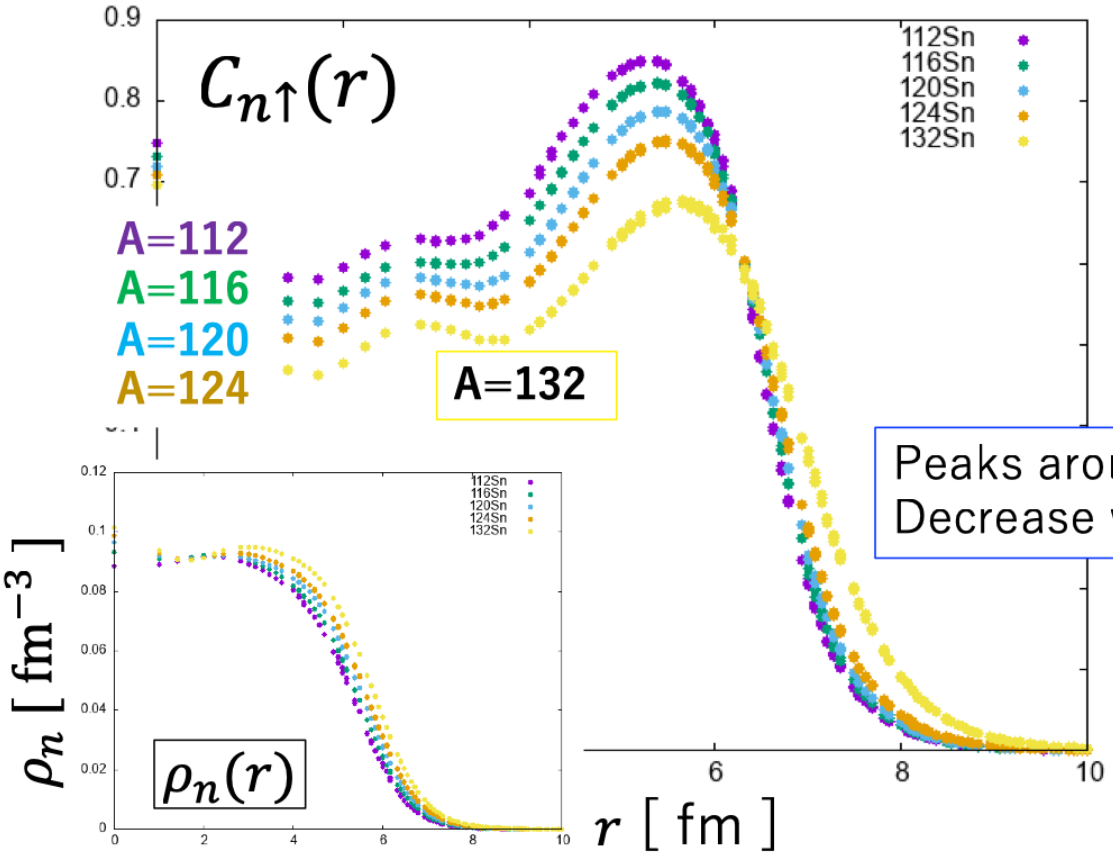
$\alpha + \alpha$

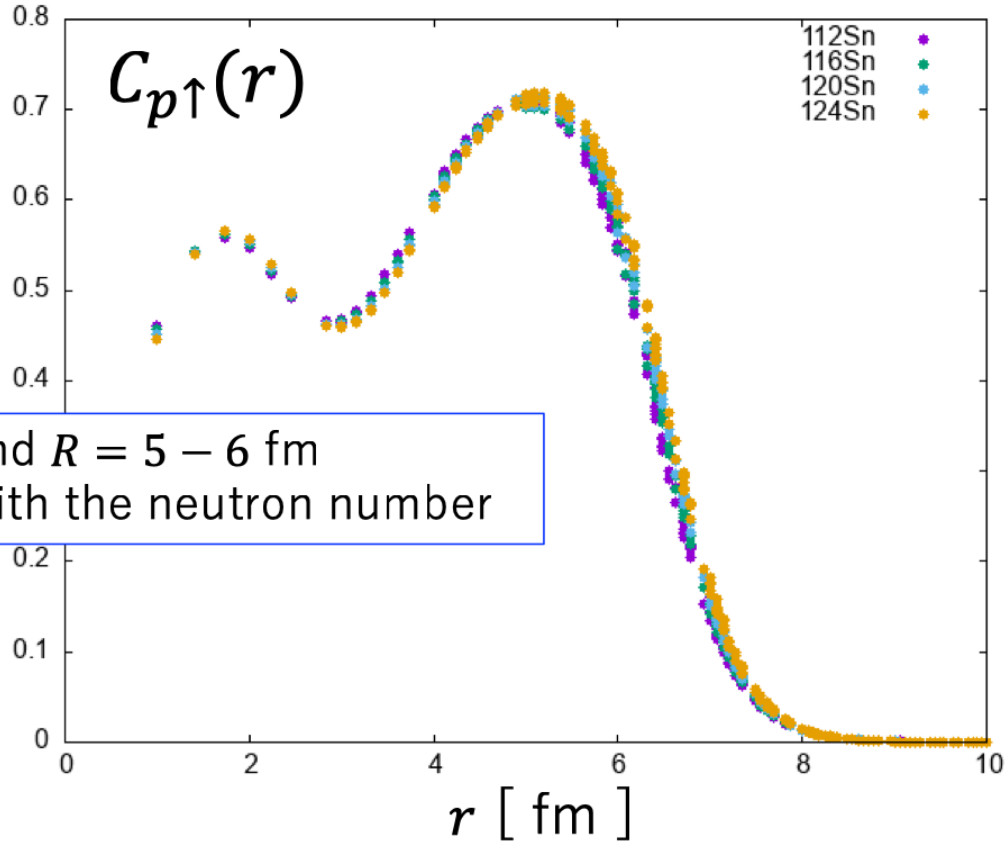
$\alpha + {}^{12}\mathrm{C} + \alpha$

Localization function

$$C_{q\sigma}(\mathbf{r}) = \left[1 + \left(\frac{\tau_{q\sigma}\rho_{q\sigma} - \frac{1}{4}[\nabla\rho_{q\sigma}]^2 - \mathbf{j}_{q\sigma}^2}{\rho_{q\sigma}\tau_{q\sigma}^{\mathrm{TF}}}\right)^2\right]^{-1}$$

Questions:
Really the α particles?



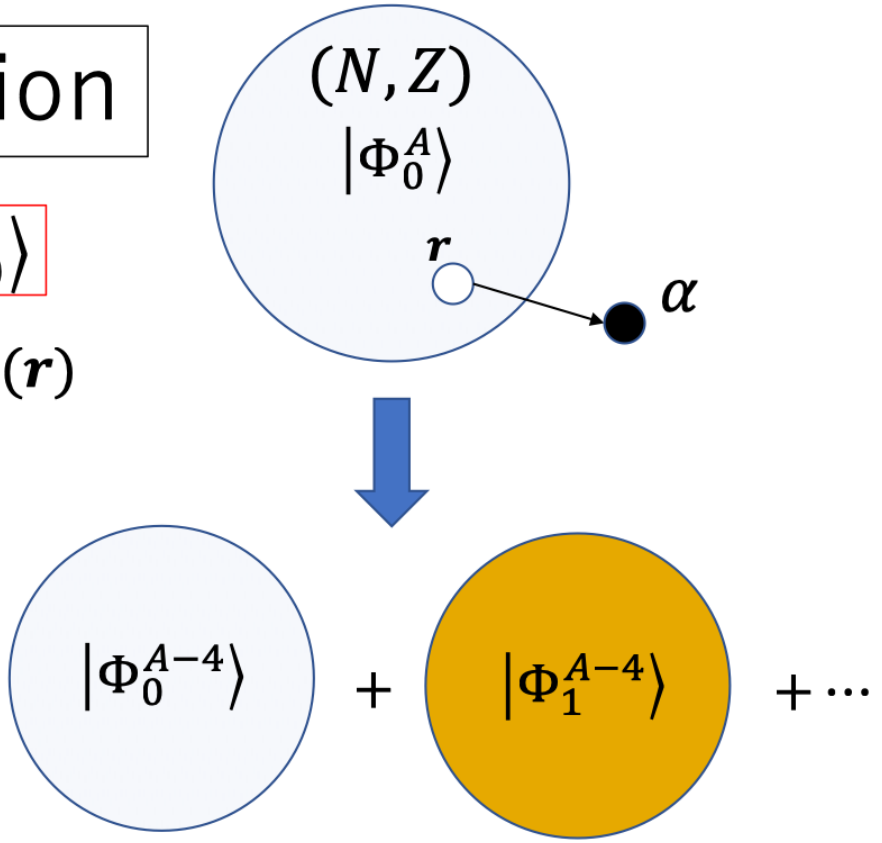


Peaks around $R = 5 - 6$ fm
Decrease with the neutron number

Local alpha strength function

$$a(\boldsymbol{r},E)=\left\langle\Phi_0\middle|\hat{\alpha}^{\dagger}(\boldsymbol{r})\delta\left(E-\hat{H}\right)\hat{\alpha}(\boldsymbol{r})\middle|\Phi_0\right\rangle$$

$$\hat{\alpha}(\boldsymbol{r})=\hat{\psi}_{n\uparrow}(\boldsymbol{r})\hat{\psi}_{n\downarrow}(\boldsymbol{r})\;\hat{\psi}_{p\uparrow}(\boldsymbol{r})\;\hat{\psi}_{p\downarrow}(\boldsymbol{r})$$



Approximation

- $|\Phi_0\rangle$: HF or HF+BCS state
- $\hat{H}$: Mean-field Hamiltonian
- No rearrangement of the mean fields
- No recoil

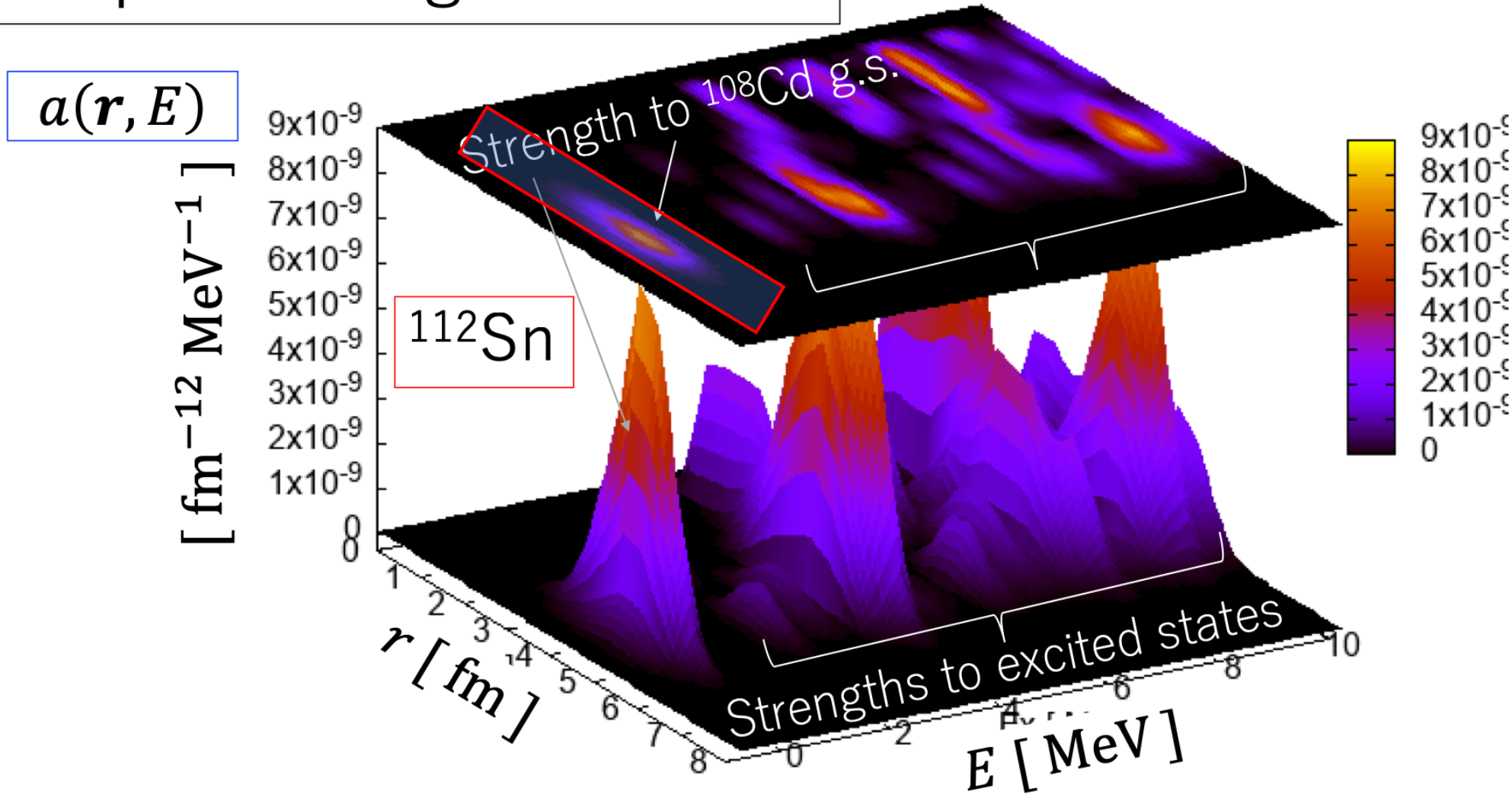
$$\hat{\alpha}(\boldsymbol{r})\left|\Phi_0^A\right\rangle=\sum_n C_n(\boldsymbol{r})|\Phi_n^{A-4}\rangle$$

Hamiltonian

Skyrme EDF (SkM*)+ monopole pairing
Pairing gap: odd-even mass diff.

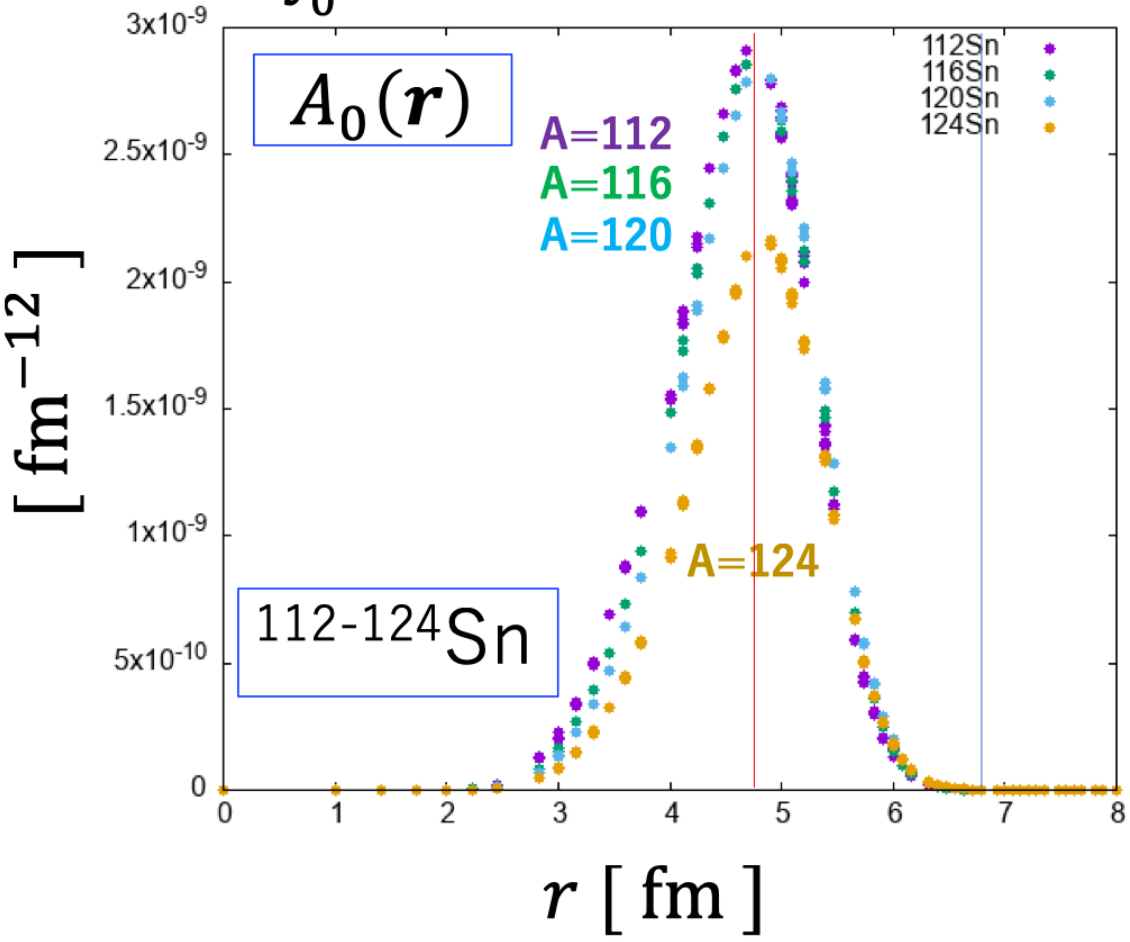
$$a(\boldsymbol{r},E_n)=\sum_n|C_n(\boldsymbol{r})|^2\delta(E-E_n)$$

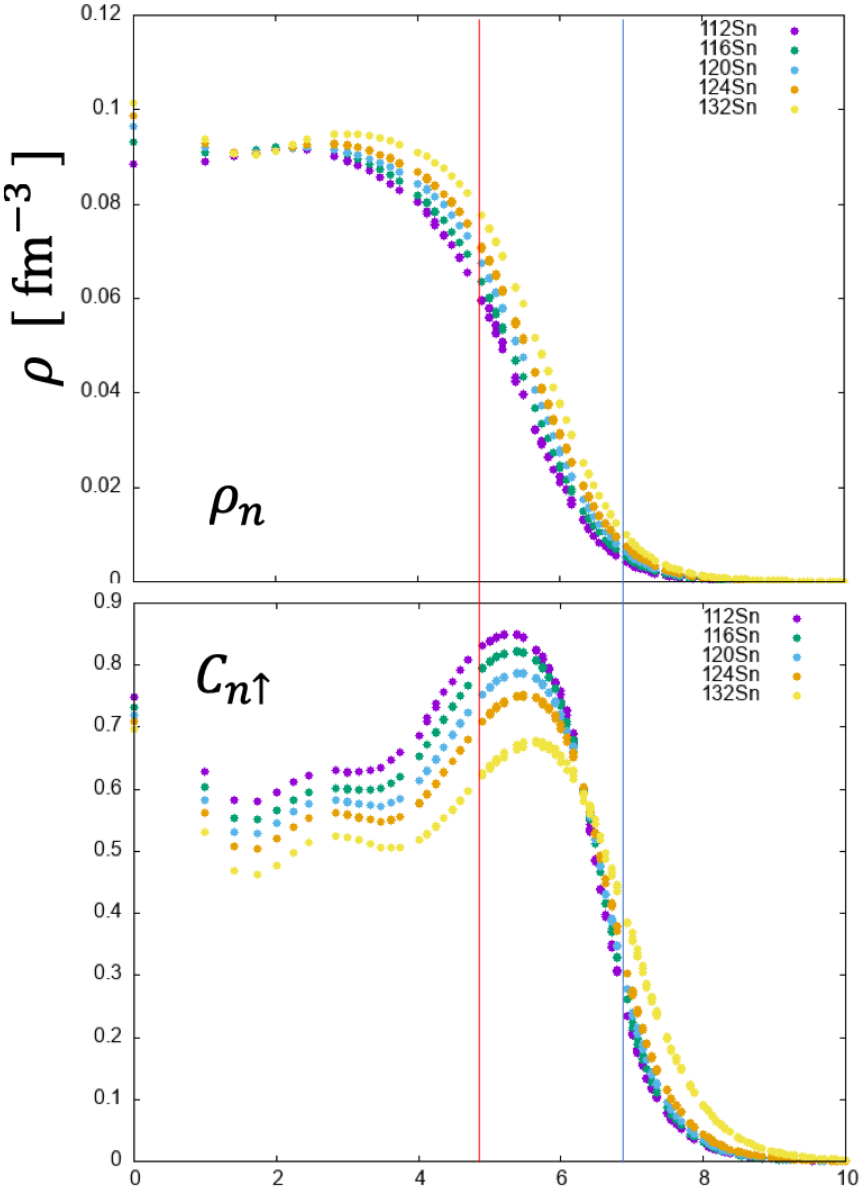
Local alpha strength function



Local alpha strength (g.s.)

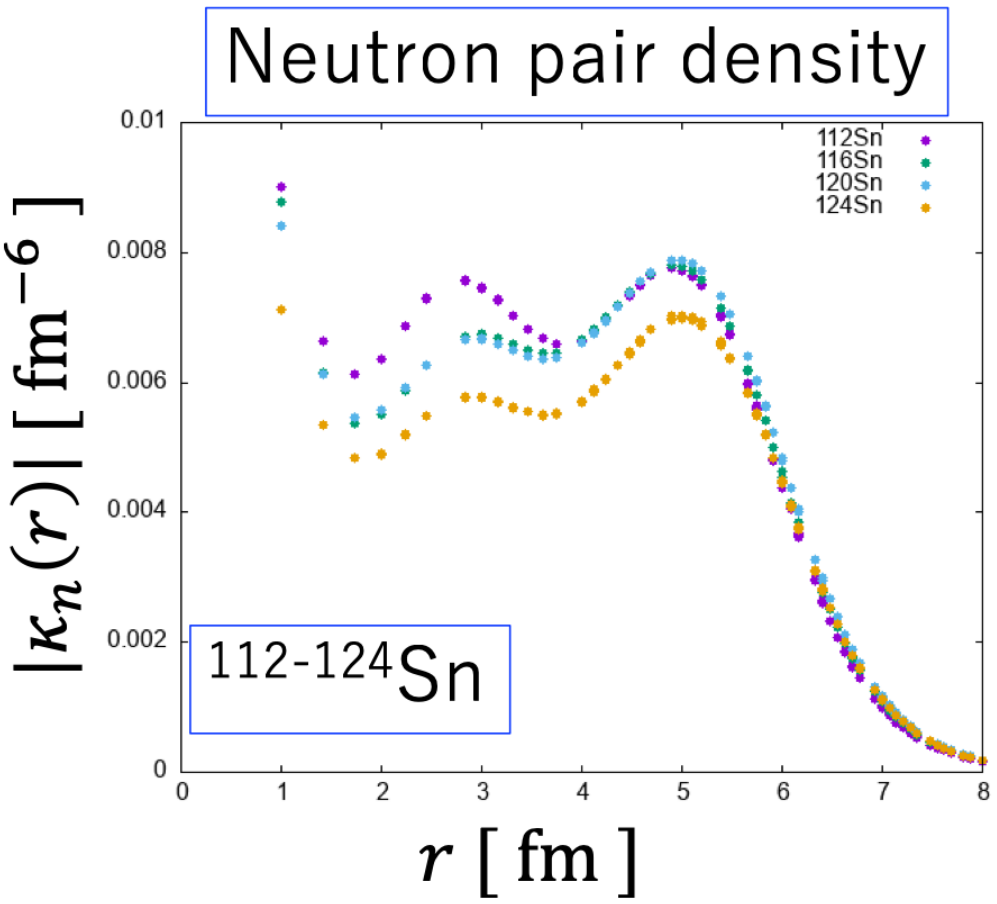
$$A_0(\boldsymbol{r}) \equiv \int_0^{\epsilon} a(\boldsymbol{r}, E)dE = \left|\left\langle \Phi_0^{A-4} | \hat{\alpha}(\boldsymbol{r}) | \Phi_0^{A} \right\rangle\right|^2$$



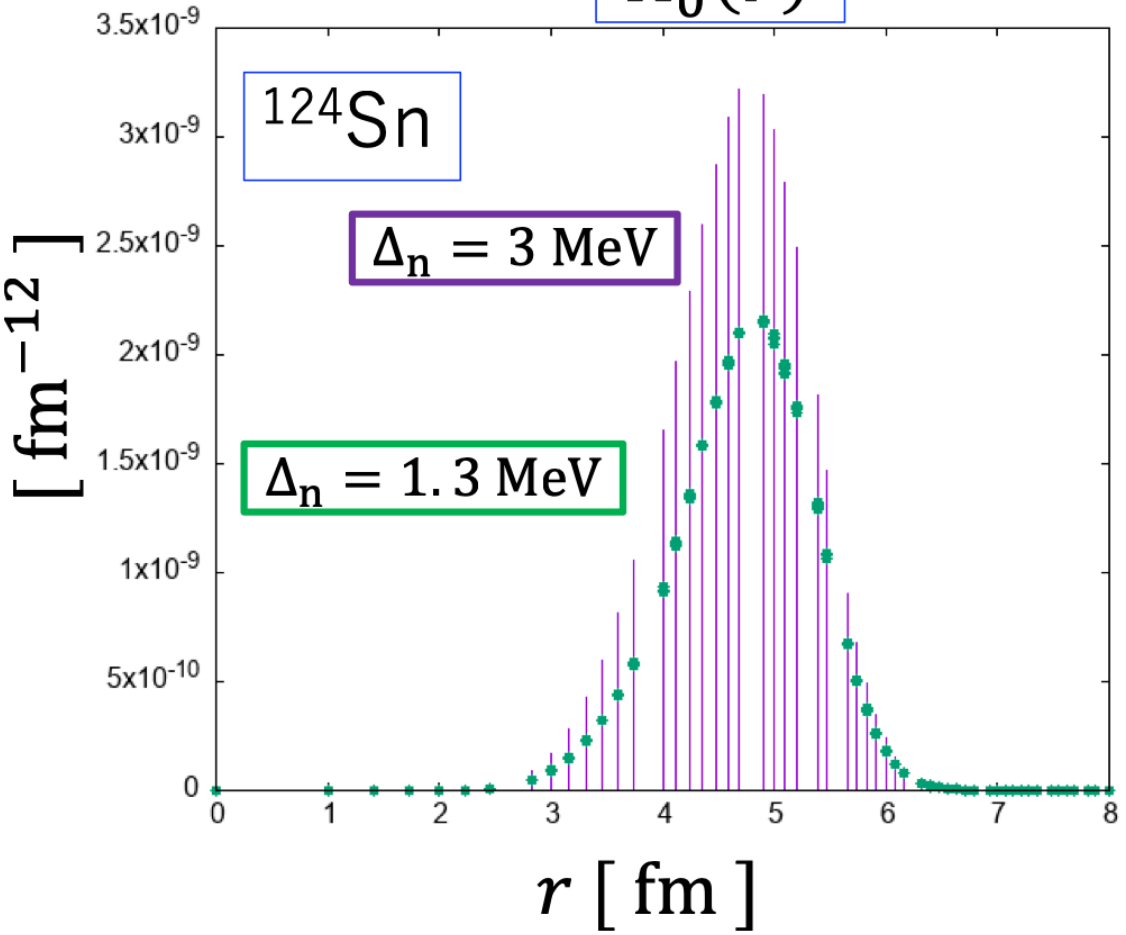


Effect of pairing correlations

Neutron pair density



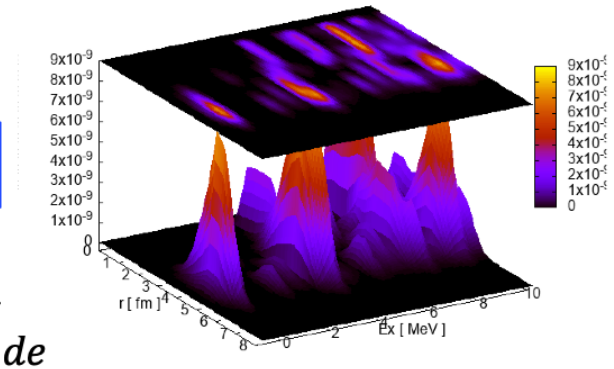
$A_0(r)$



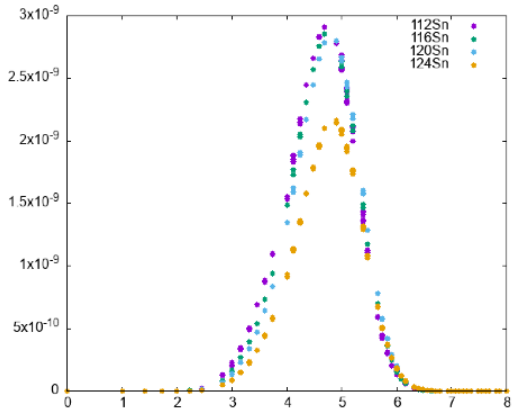
Local alpha probability

$a(\boldsymbol{r}, E)$

$$\int_0^{E_{gs}+\epsilon} de$$



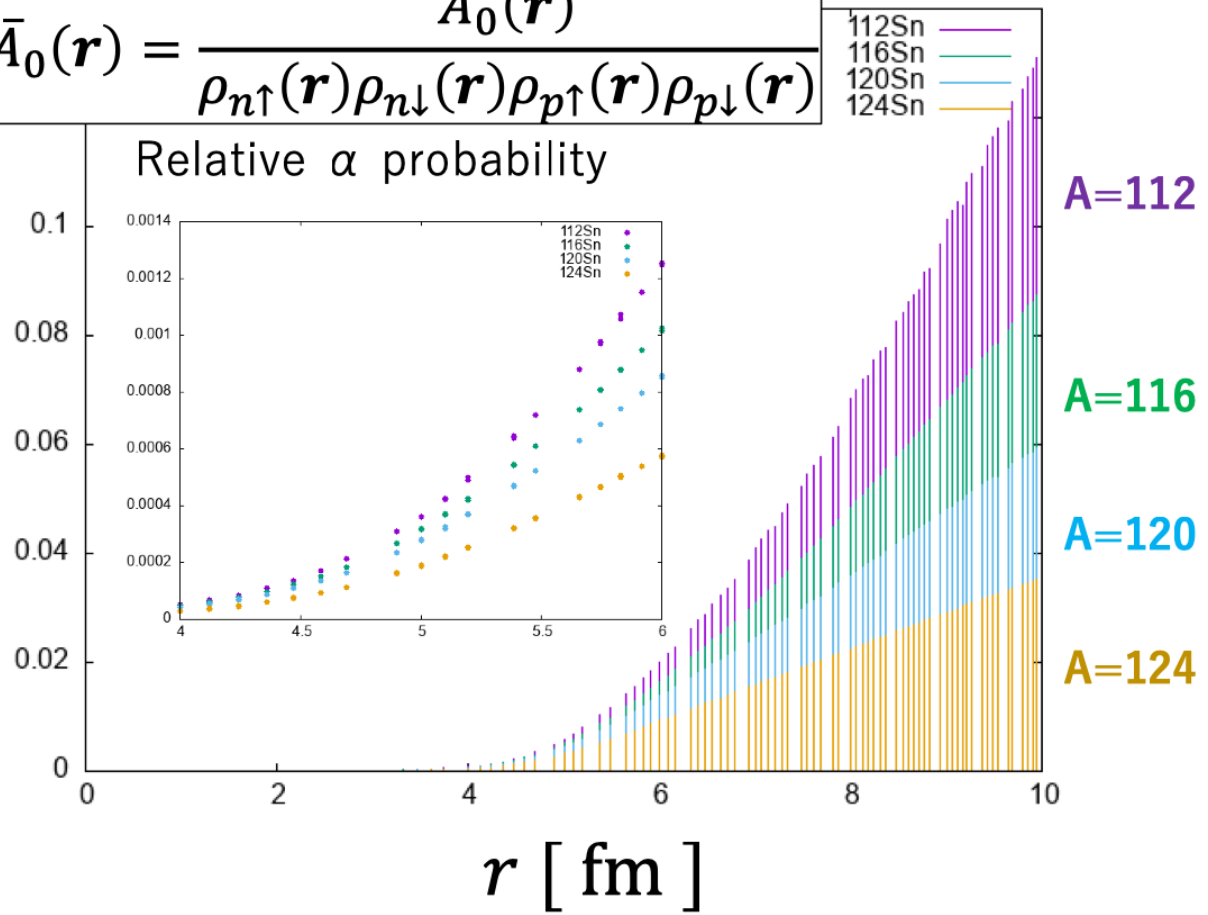
$A_0(\boldsymbol{r})$



$\bar{A}_0(\boldsymbol{r})$ Dimensionless probability

$$\bar{A}_0(\boldsymbol{r}) = \frac{A_0(\boldsymbol{r})}{\rho_{n\uparrow}(\boldsymbol{r})\rho_{n\downarrow}(\boldsymbol{r})\rho_{p\uparrow}(\boldsymbol{r})\rho_{p\downarrow}(\boldsymbol{r})}$$

Relative α probability



Consistent with exp.

Summary

- Local alpha strength
  - HF+BCS計算
  - $^{112\text{-}124}$Sn: g.s. → g.s.
    - Peak position: $\rho \approx \rho_0/2$
    - Sensitive to pairing correlations
    - Not sensitive to neutron numbers
  - Local alpha probability: $\bar{A}_0(r)$
    - Sensitive to neutron numbers

- Future perspectives
  - Open shell nuclei, deformed nuclei
  - Rearrangement effect
  - pn pairing