Quantum dynamics of superconducting nanojunctions

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Grenoble Josephson junction team

CNRS – Université Joseph Fourier

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Outline

I. Small Josephson junctions

- Some basic notions

II. Quantum dynamics

- Multilevel coherent oscillations
- Quantum or classical oscillations?

III. Quantum optimal control theory

- Inducing transitions « à la carte »?
- Effect of noise

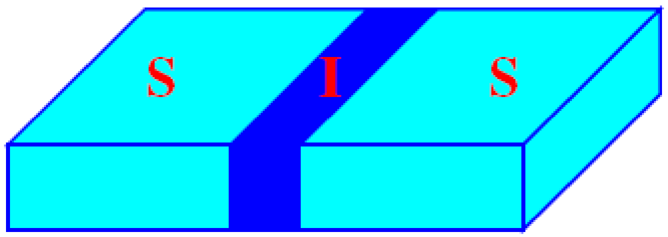
IV. Coupling qubits

- Towards tunable coupling

I. Small Josephson junctions

Some basic notions

Basic building blocks: Josephson junction & SQUID



Josephson relations:

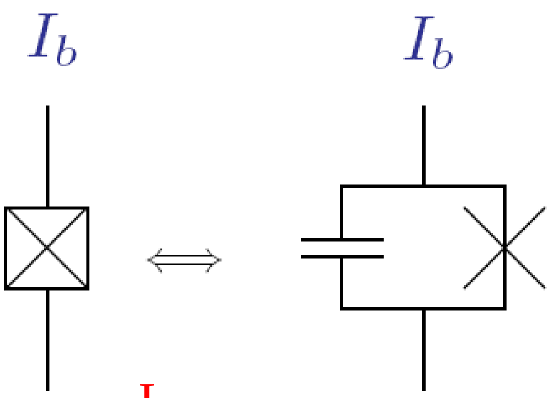
$$I = I_c \sin \phi$$

$$\dot{\phi} = 2eV/\hbar$$

Small Josephson junction: two energy scales

$$E_C = (2e)^2/2C$$

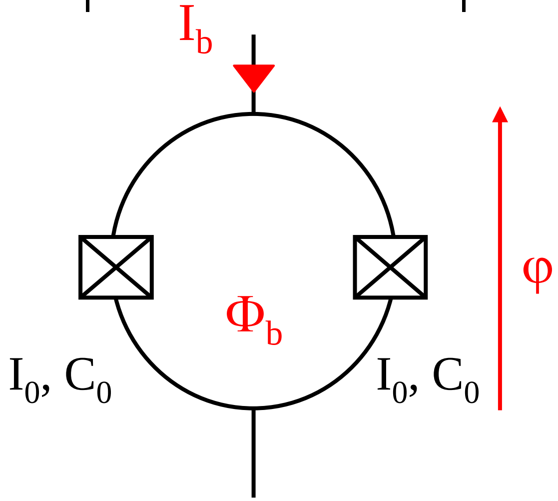
$$E_J = \hbar I_c/(2e)$$



Current conservation: $I_b = I_c \sin \phi + C d^2\phi/dt^2$

Two junctions in parallel: SQUID

$$E_J = E_J(\Phi_b)$$



Hamiltonian

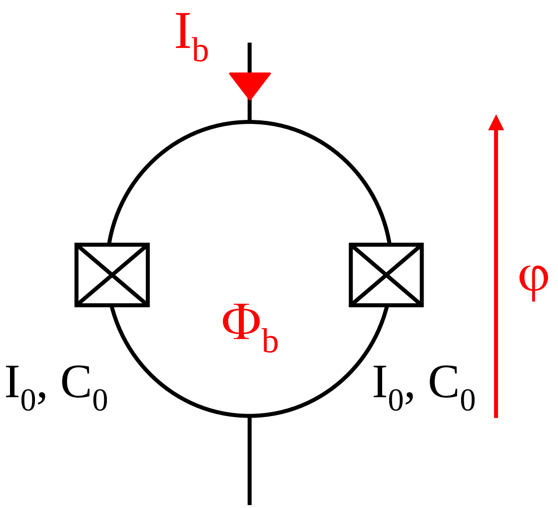
Hamiltonian

$$H = E_c(Q/2e)^2 - E_J \cos\phi - I_b\phi$$

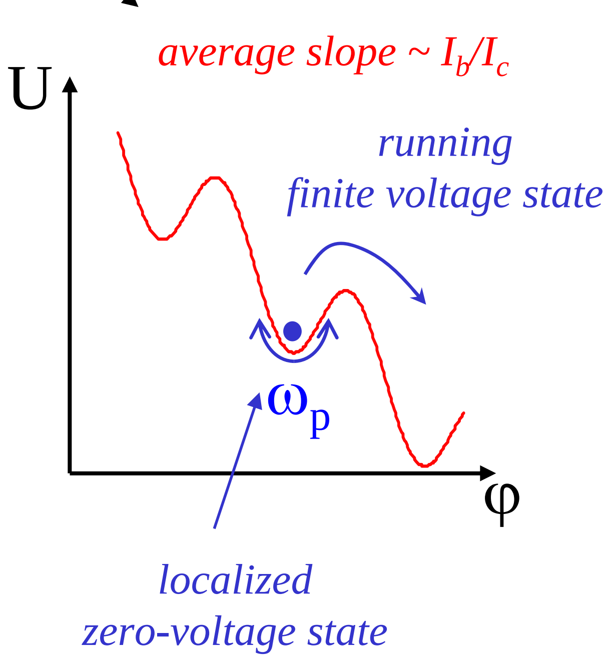
Charging energy   *Tilted washboard potential*

Hamiltonian equations of motion:

current conservation $I_b = I_c \sin\phi + C d^2\phi/dt^2$

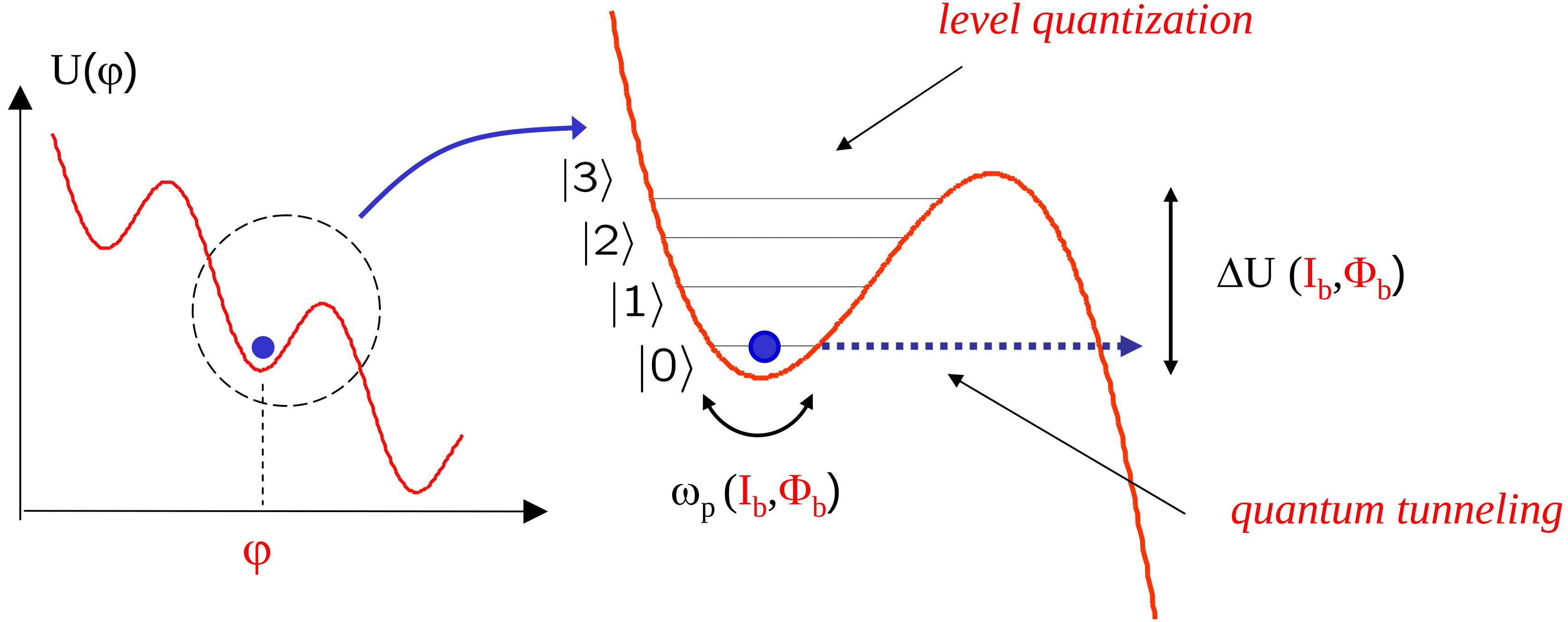


$$\omega_p = \omega_p(I_b, \Phi_b)$$



Current-biased dc SQUID in the quantum limit: anharmonic oscillator

Charge and phase do not commute: $[Q, \varphi] = -2ie$

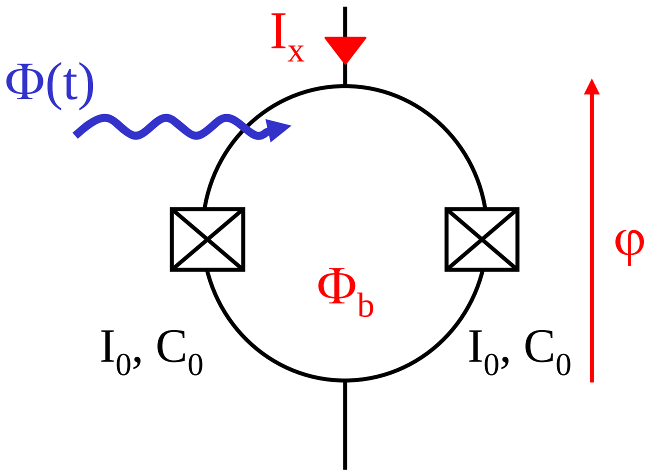


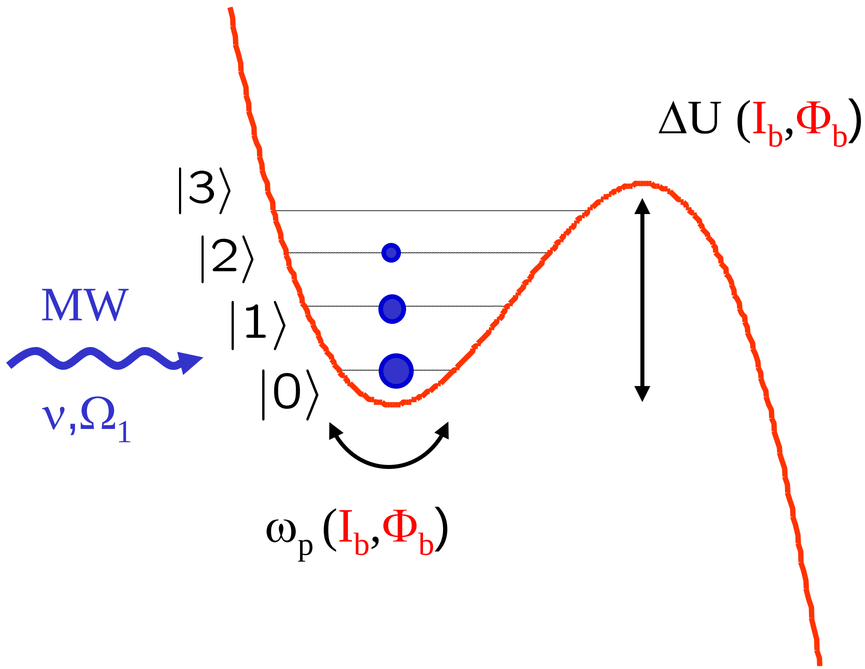
Quantum anharmonic oscillator!

Current-biased dc SQUID: two modes of operation

(O. Buisson, F. Balestro, J.P. Pekola, and FH, PRL 2003)

Deep well with quantised states: quantum *state manipulation & dynamics*





shape of the anharmonic well ⟷ bias point (I_b, Φ_b)

$$\frac{1}{2}\hbar\omega_p\left[\widetilde{P}^2 + \widetilde{X}^2\right] - \hbar\sigma\omega_p\widetilde{X}^3$$

excitation ⟷ MW flux $\Phi(t)$

$$-\hbar\Omega_1 \cos(2\pi\nu t)\sqrt{2}\widetilde{X}$$

Current-biased dc SQUID: two modes of operation

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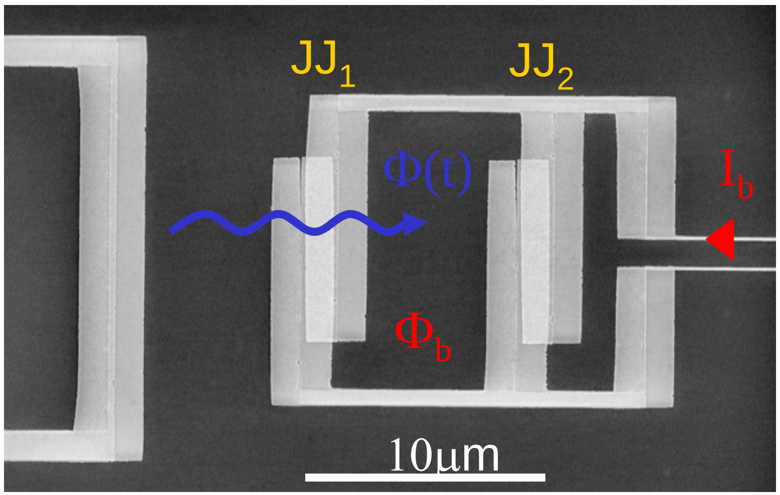

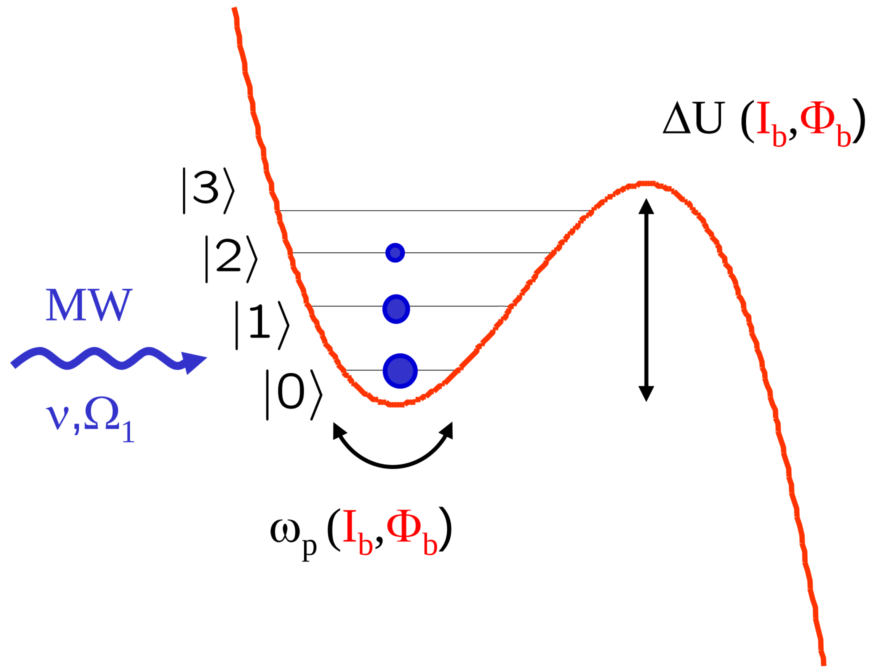

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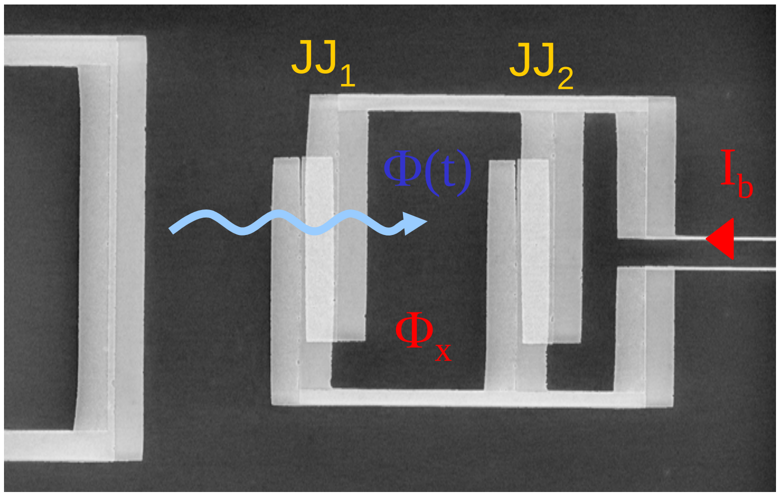
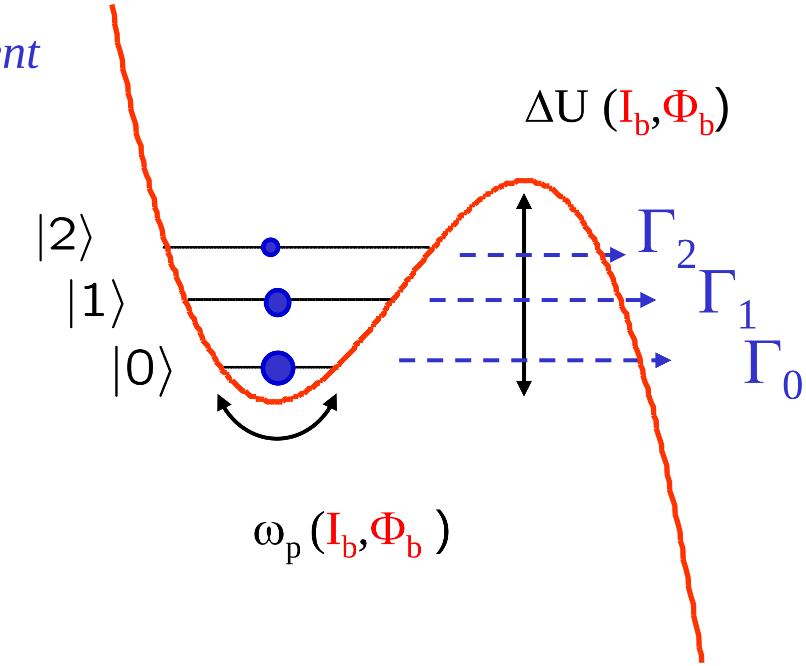
Current-biased dc SQUID: two modes of operation

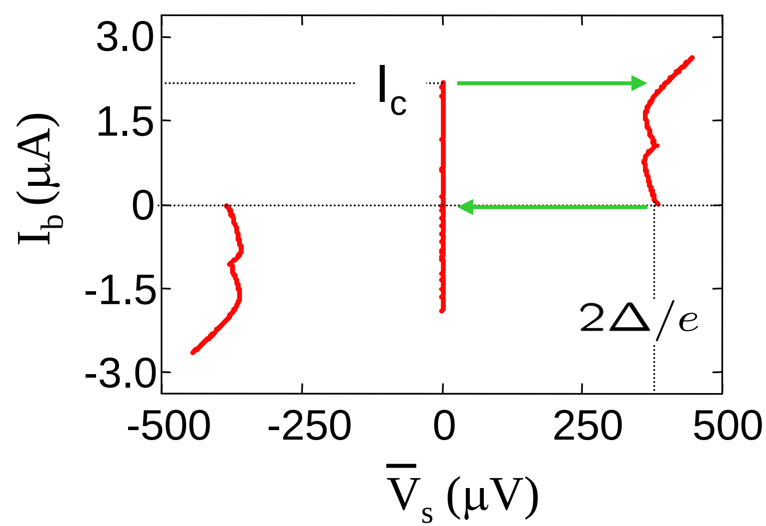
(O. Buisson, F. Balestro, J.P. Pekola, and FH, PRL 2003)

Shallow well with tunnelling: quantum *measurement*





Hysteretic junction: escape leads to voltage



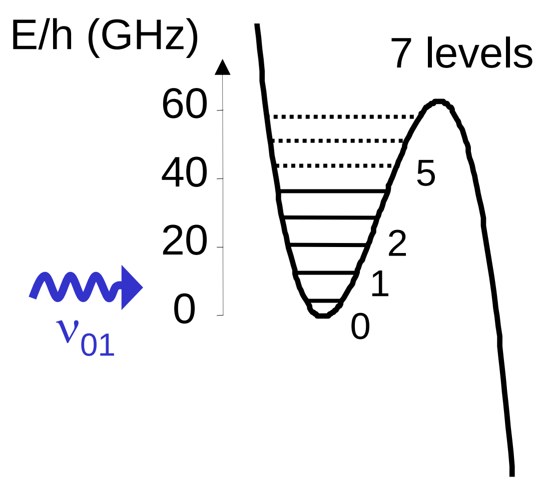
II. Quantum dynamics

Interplay between microwave amplitude and anharmonicity

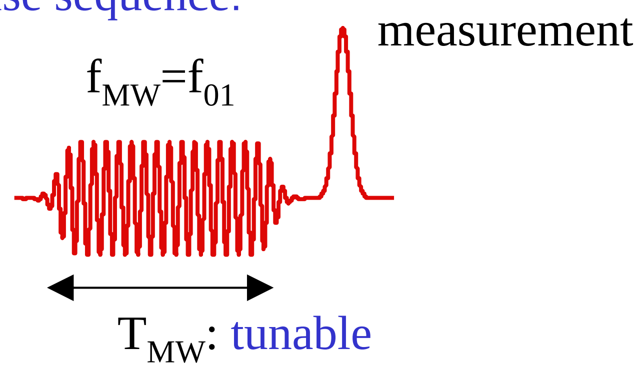
Coherent oscillations in a dc SQUID

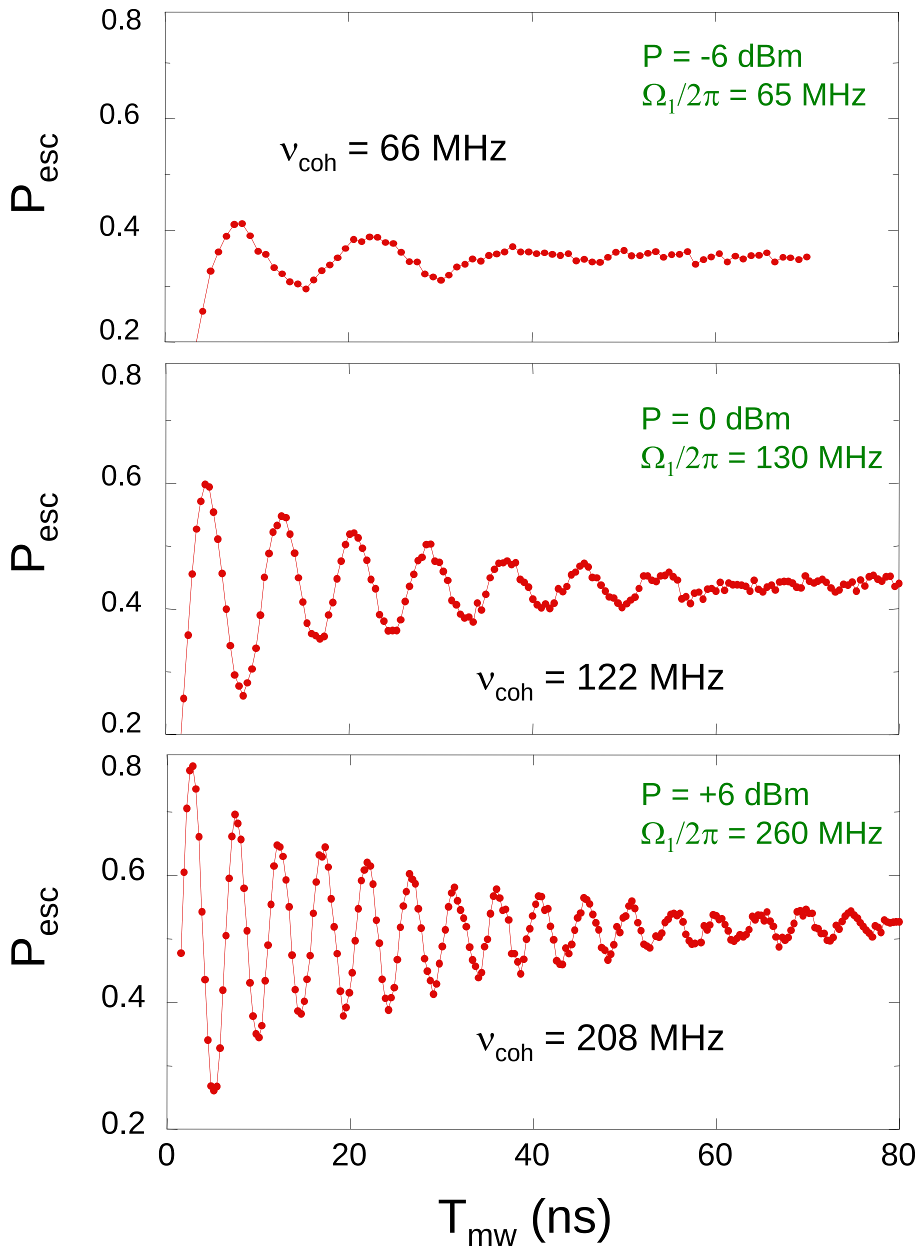
(J. Claudon F. Balestro, FH, and O. Buisson, PRL 2004)

- Anharmonic oscillator:



- Flux-pulse sequence:





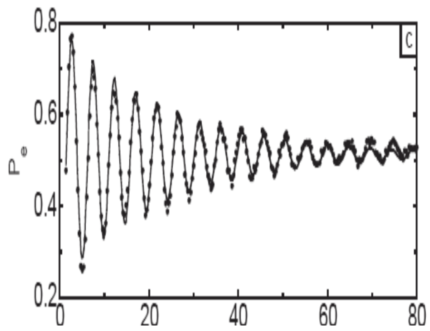
Classical description

(J. Claudon A. Zazunov, FH, and O. Buisson, PRB 2008)

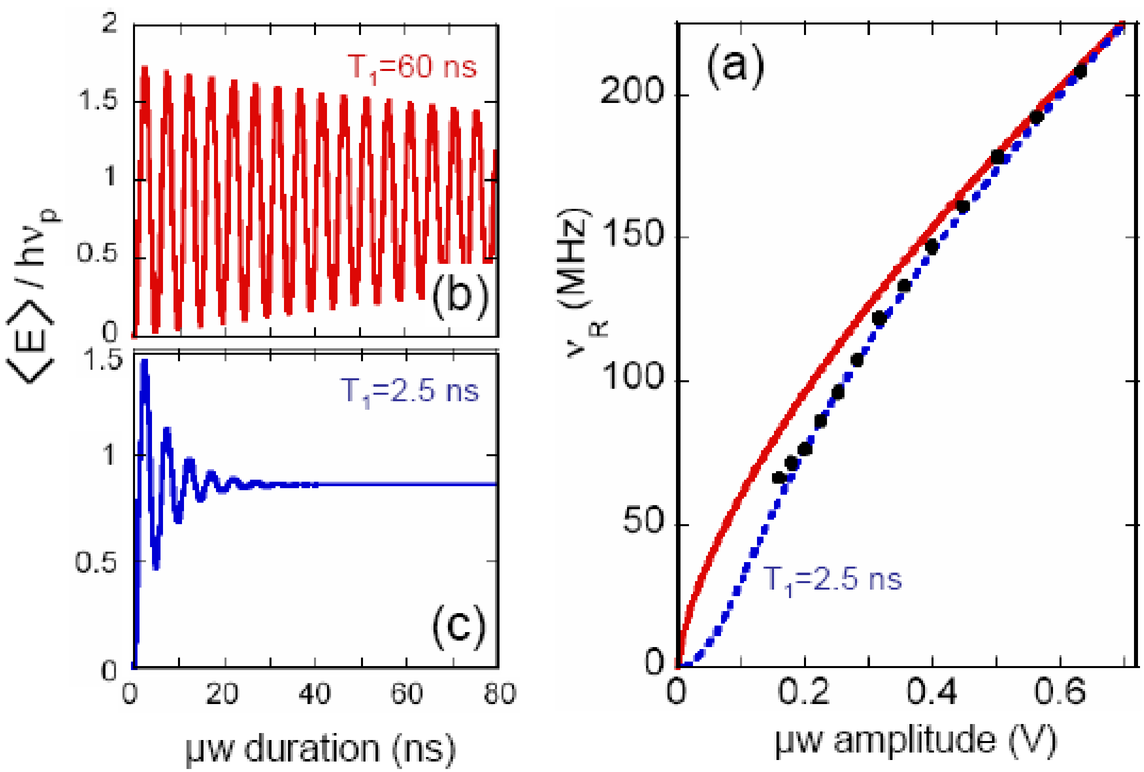
Integrate equations of motion

$$\nu_R = \frac{\sqrt{3}}{4}\nu_p\left(1+2A^2\right)^{1/3}\left(\frac{f_{\mu w}}{m\omega_p^2}\right)^{2/3}$$

$$A = \frac{6a}{m\omega_p^2} = \left[\frac{18}{27}\frac{m\omega_p^2}{\Delta U}\right]^{1/2}$$



(A. Ratchov, PhD-thesis 2005)



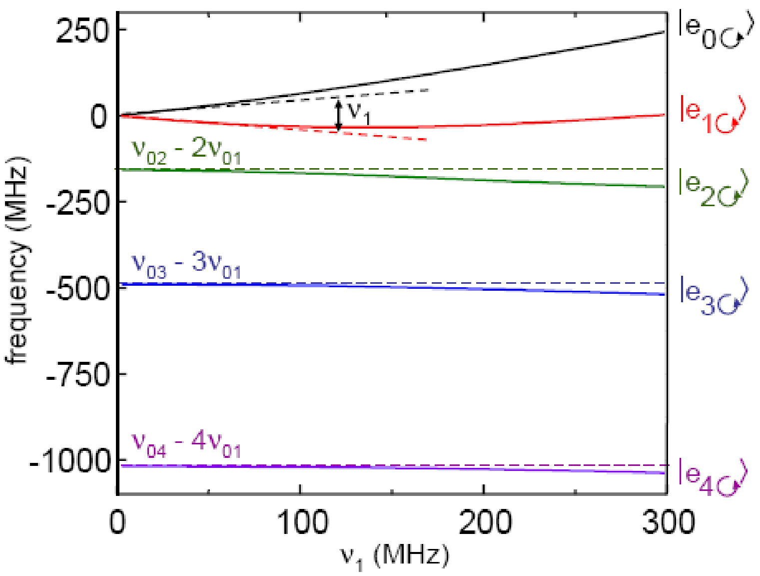
Quantum description

(J. Claudon A. Zazunov, FH, and O. Buisson, PRB 2008)

Hamiltonian & eigenvalues in RWA

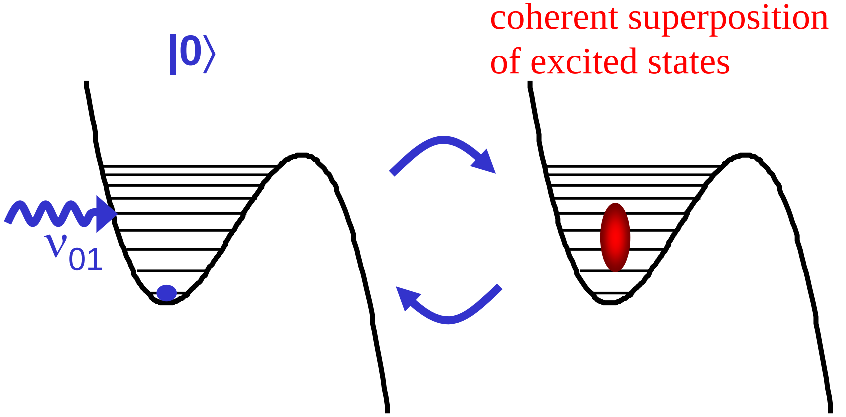
$$\hat{H}_{\circlearrowleft,\mathrm{RWA}} = h \begin{pmatrix} 0 & \frac{\nu_1}{2} & 0 & 0 \\ \frac{\nu_1}{2} & \Delta_1(\nu) & \ddots & 0 \\ 0 & \ddots & \ddots & \sqrt{N-1}\frac{\nu_1}{2} \\ 0 & 0 & \sqrt{N-1}\frac{\nu_1}{2} & \Delta_{N-1}(\nu) \end{pmatrix}$$

$$\Delta_n(\nu) = \nu_{0n} - n\nu$$



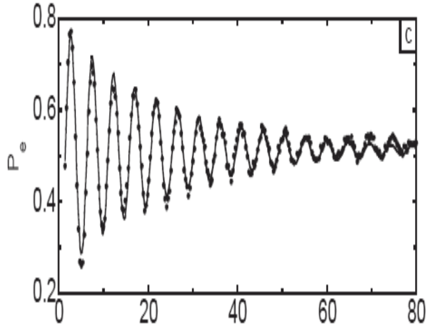
Time-dependent probability

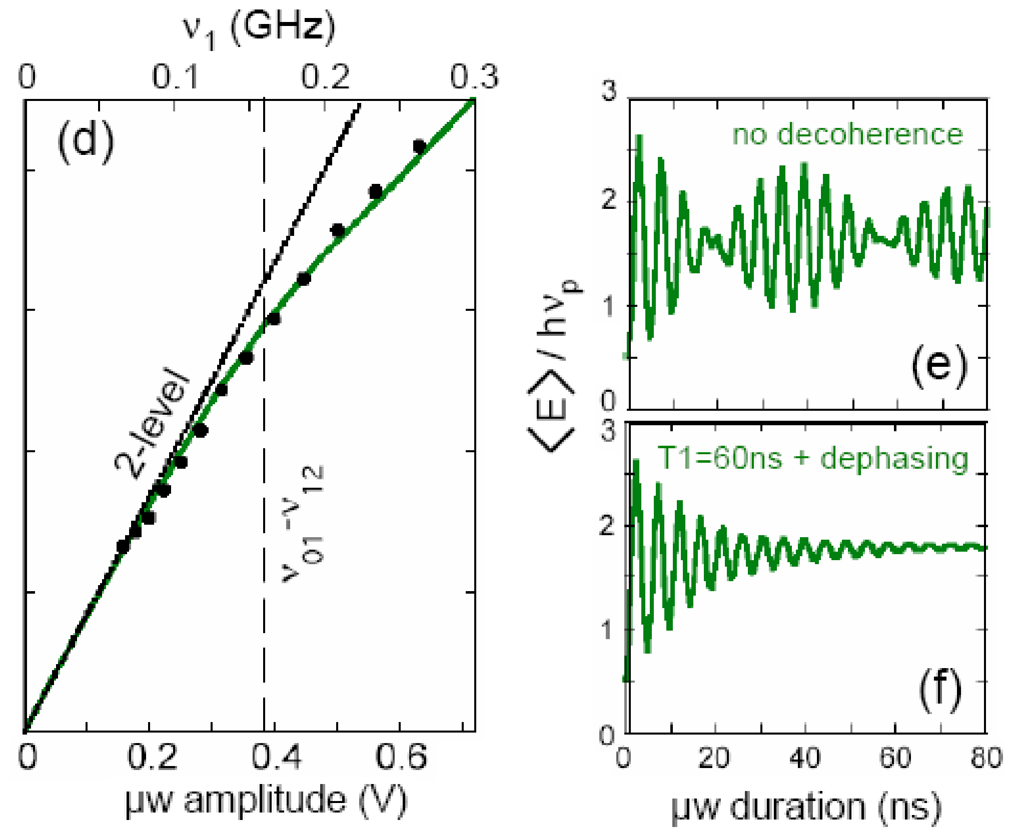
$$p_n(t) = 2\sum_{k,l>k}^{N-1} |\langle n|e_{k\circlearrowleft}\rangle\langle e_{k\circlearrowleft}|0\rangle\langle 0|e_{l\circlearrowleft}\rangle\langle e_{l\circlearrowleft}|n\rangle| \times \cos[2\pi(\lambda_k - \lambda_l)t].$$



Quantum description

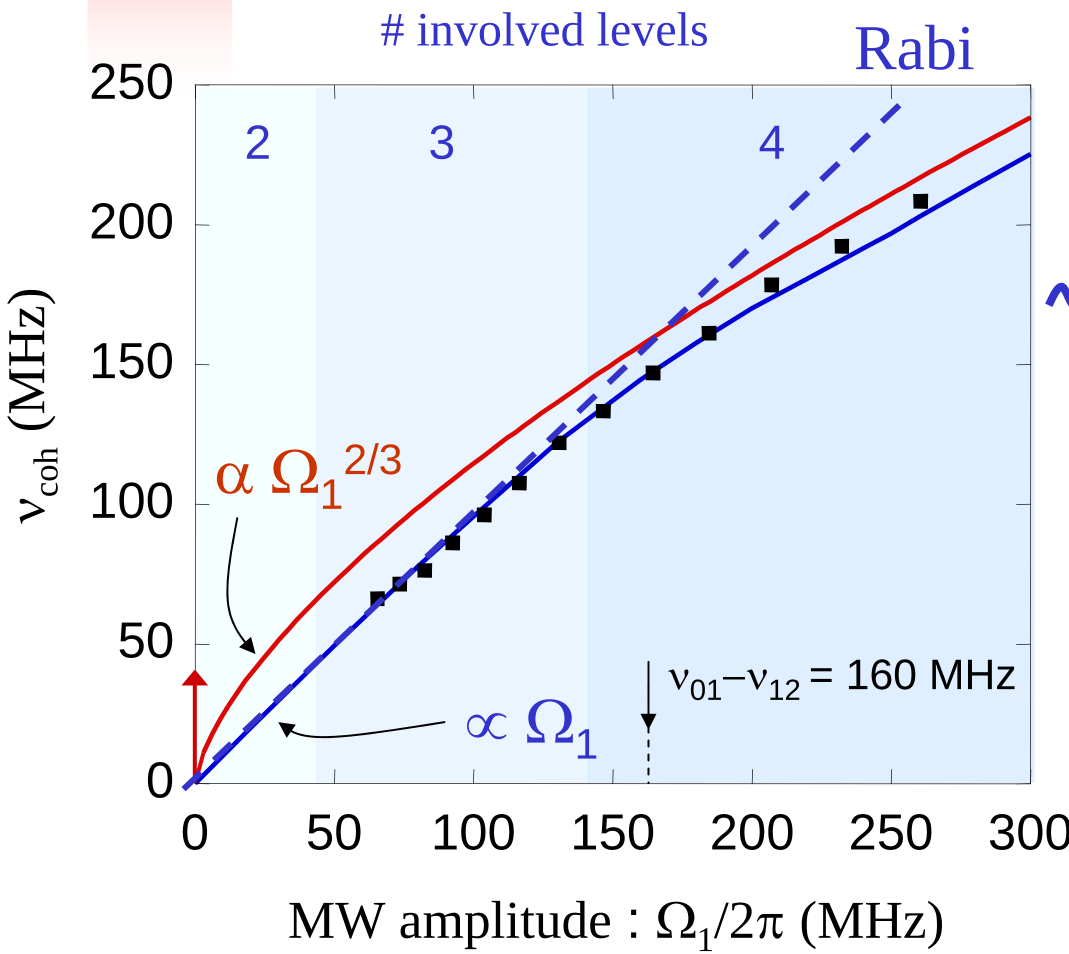
(J. Claudon A. Zazunov, FH, and O. Buisson, PRB 2008)

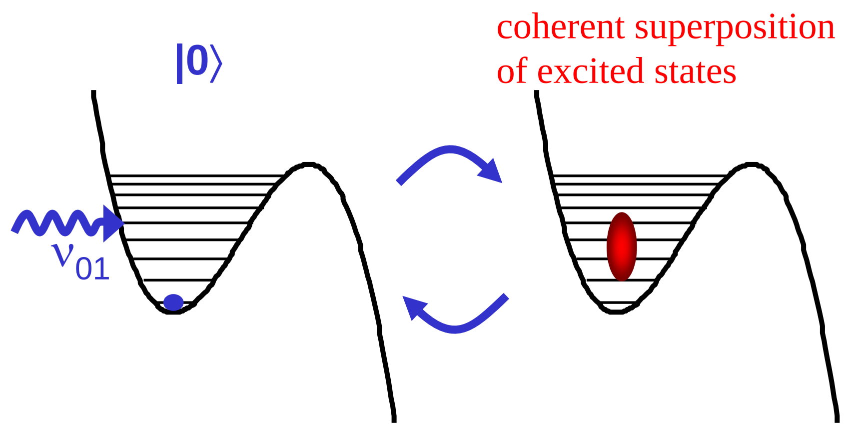




Coherent oscillations in a dc SQUID

(J. Claudon A. Zazunov, FH, and O. Buisson, PRB 2008





quantum theory with $\nu=\nu_{01}$

classical theory with $\nu=\omega_p/2\pi$

(J.E. Marchese et al., cond-mat/0509729; A. Ratchov, PhD-thesis 2005)

- Low excitation power : quantum description
- High excitation power : classical description

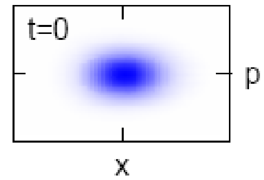
Coherent oscillations in a dc SQUID: Wigner function

Definition of Wigner function

$$W(x,p,t) = \frac{1}{\pi\hbar}\int \mathrm{d}x' \langle x + x' | \hat{\rho}(t) | x - x' \rangle e^{-2ipx'/\hbar}$$

Classical and quantum dynamics of W for Hamiltonian

$$\hat{H}(t) = h\nu_p \left[\frac{\tilde{p}^2 + \tilde{x}^2}{2} - \left(\frac{2}{15} \frac{\delta}{\nu_p} \right)^{\frac{1}{2}} \tilde{x}^3 \right] + \sqrt{2} h\nu_1 \cos(2\pi\nu t)\tilde{x},$$



(J. Claudon A. Zazunov, FH, and O. Buisson, PRB 2008

t/T$_R$	ν_1 = 60 MHz = 0.37 δ	ν_1 = 260 MHz = 1.62 δ	ν_1 = 520 MHz = 3.25 δ
	~ 2-level oscillation	4-level oscillation	7-level oscillation
0.1			
0.15			
0.2			
0.25			
0.5			
0.75			
1			

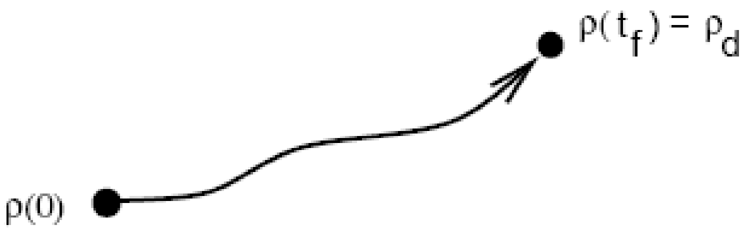
III. Quantum optimal control theory

Inducing transitions « à la carte »

Statement of the problem

P. Pierce *et al.*, Phys .Rev. A **37**, 4950 (1988).

Desired time evolution of quantum system:



We seek a control field :

$$[0, t_f] \ni t \xrightarrow{?} \varepsilon(t)$$

Such that solution of corresponding Liouville equation:

$$i\hbar\partial_t\rho(t) = \left[H_0 + H_{\text{int}}\left\{\varepsilon(t)\right\}, \rho(t)\right]$$

yields the desired one!

(with a reasonable control field)

Mathematical formulation (1)

Let $H = \mathbb{C}^N$ be the Hilbert space and ρ a density operator.

- Hamiltonian: $H = H_0 + H_{\text{int}}\{\varepsilon(t)\}$
- Quantum optimal control problem:

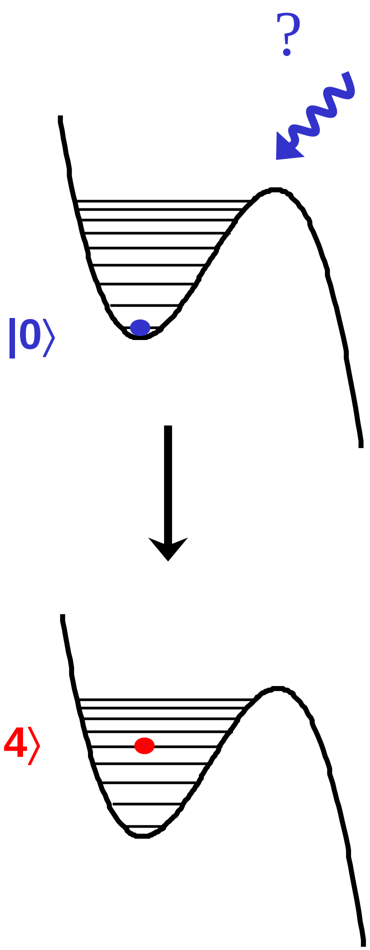
$$\begin{cases} \text{Min } J(\varepsilon) = \frac{1}{2}\|\rho(t_f) - \rho_d\|_F^2 + \frac{\alpha}{2}\int_0^{t_f} \varepsilon^2(t)\, dt \\ i\hbar\partial_t \rho(t) = [H, \rho(t)]\,, \qquad t \in [0, t_f] \\ \rho(0) = \rho_i \qquad \text{(initial conditions)} \end{cases}$$

- Frobenius: $\|A\|_F^2 = \sum_{ij} |a_{ij}|^2 = \sum_{ij} a_{ij} a_{ij}^* = \text{Tr}(AA^\dagger)$

$$\|A - B\|_F^2 = \text{Tr}(AA^\dagger) + \text{Tr}(BB^\dagger) - 2\text{ReTr}(AB^\dagger)$$

Mathematical formulation (2)

Problem: complexity of control field in time domain (frequency content)



Direct $|0\rangle \longrightarrow |4\rangle$

Indirect $|0\rangle \longrightarrow |1\rangle \longrightarrow |2\rangle \longrightarrow |3\rangle \longrightarrow |4\rangle$

$|0\rangle \longrightarrow |6\rangle \longrightarrow |4\rangle$

...

Solution: implementation of filter to restrict control field complexity

$$\left.\frac{\delta J}{\delta \varepsilon(t)}\right|_{\text{filter}} = \mathcal{F}^{-1}\left[g(\omega)\mathcal{F}\left[\frac{\delta J}{\delta \varepsilon(t)}\right]\right]$$

$$g(\omega) = e^{-\gamma(\omega-\omega_0)^2} + e^{-\gamma(\omega+\omega_0)^2}$$

Optimal control for a current-biased SQUID

(H. Jirari, FH and O. Buisson, EPL 2009)

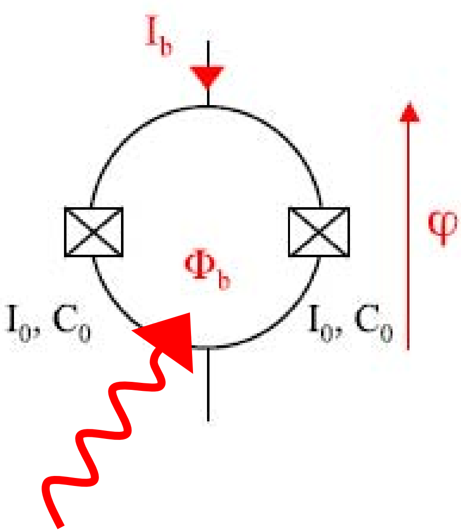
Total Hamiltonian: $\hat{H}_{\mathrm{tot}} = \hat{H}_{\varphi} + \hat{H}_{\mathrm{c}}$

system

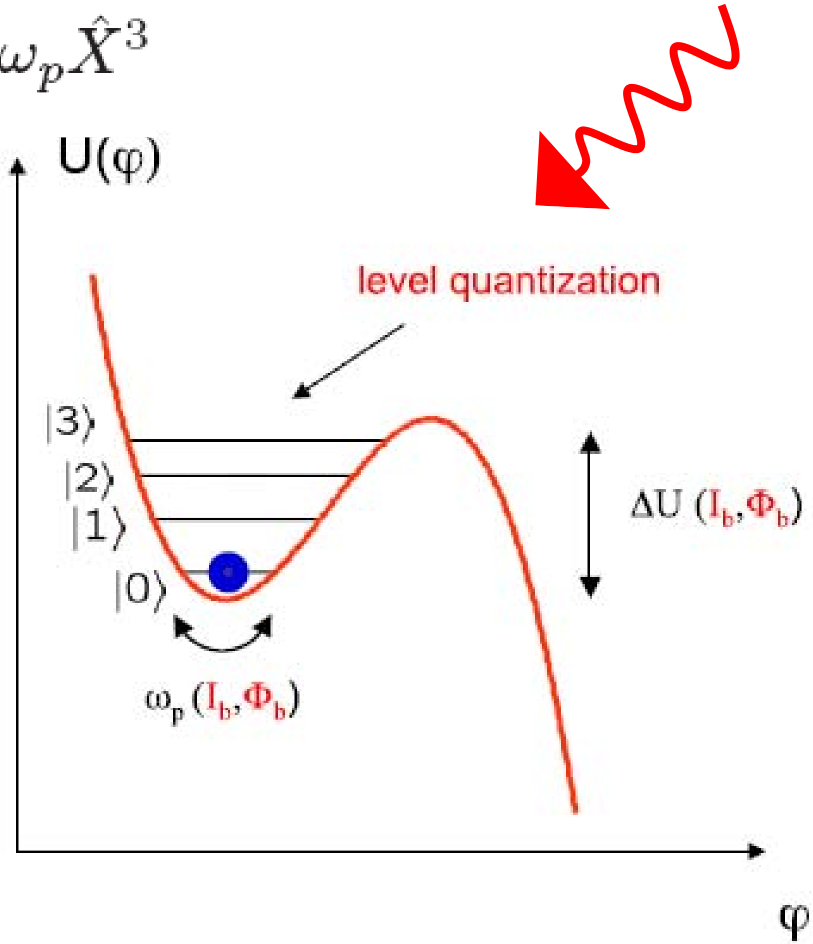
$$\hat{H}_{\varphi} = \frac{1}{2}\hbar\omega_p(\hat{P}^2 + \hat{X}^2) - \sigma\hbar\omega_p\hat{X}^3$$

control

$$H_{\mathrm{c}} = \hbar\omega_p\varepsilon(t)\hat{X}$$



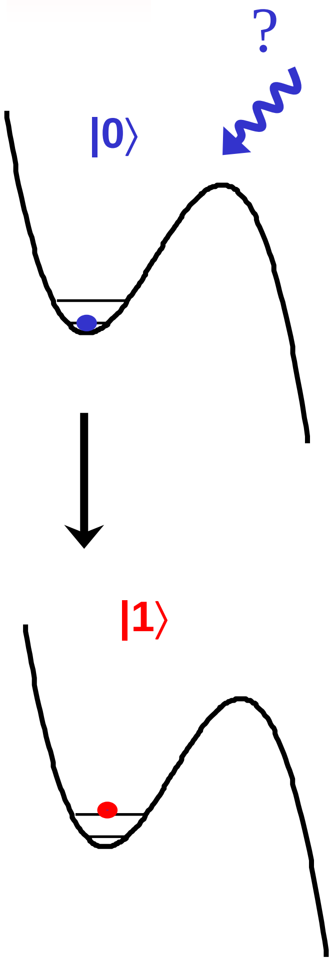
$$\varepsilon(t) = \Phi_b(t)/\Phi_0$$

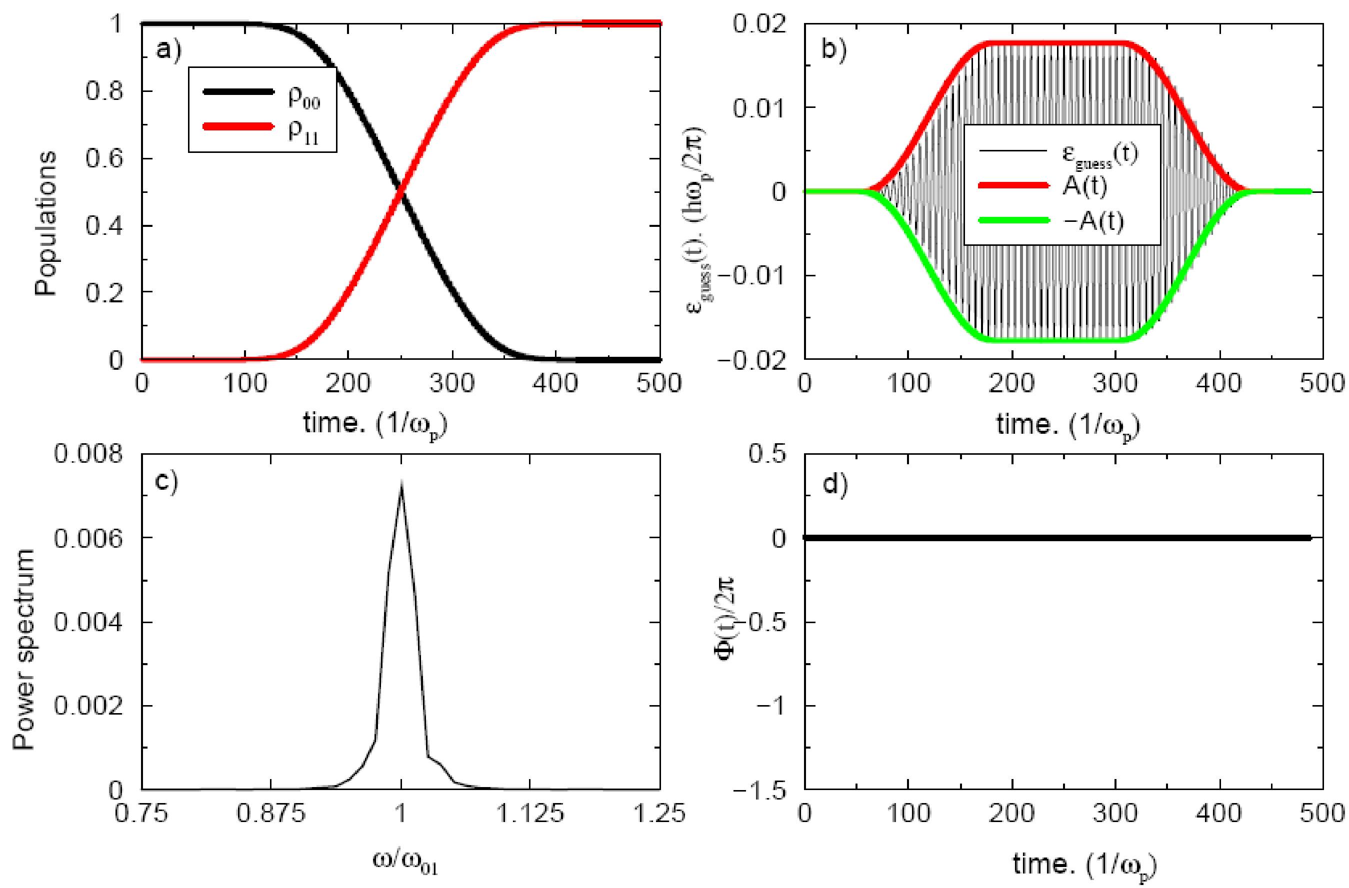


Test for a two-level system: π-pulse

(H. Jirari, FH and O. Buisson, EPL 2009)

$$\varepsilon_{guess}(t) = A(t)\cos\left[\omega_0 t + \phi(t)\right]$$

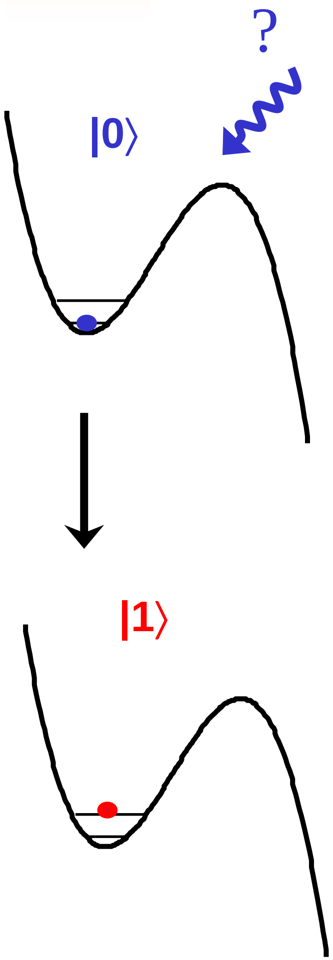


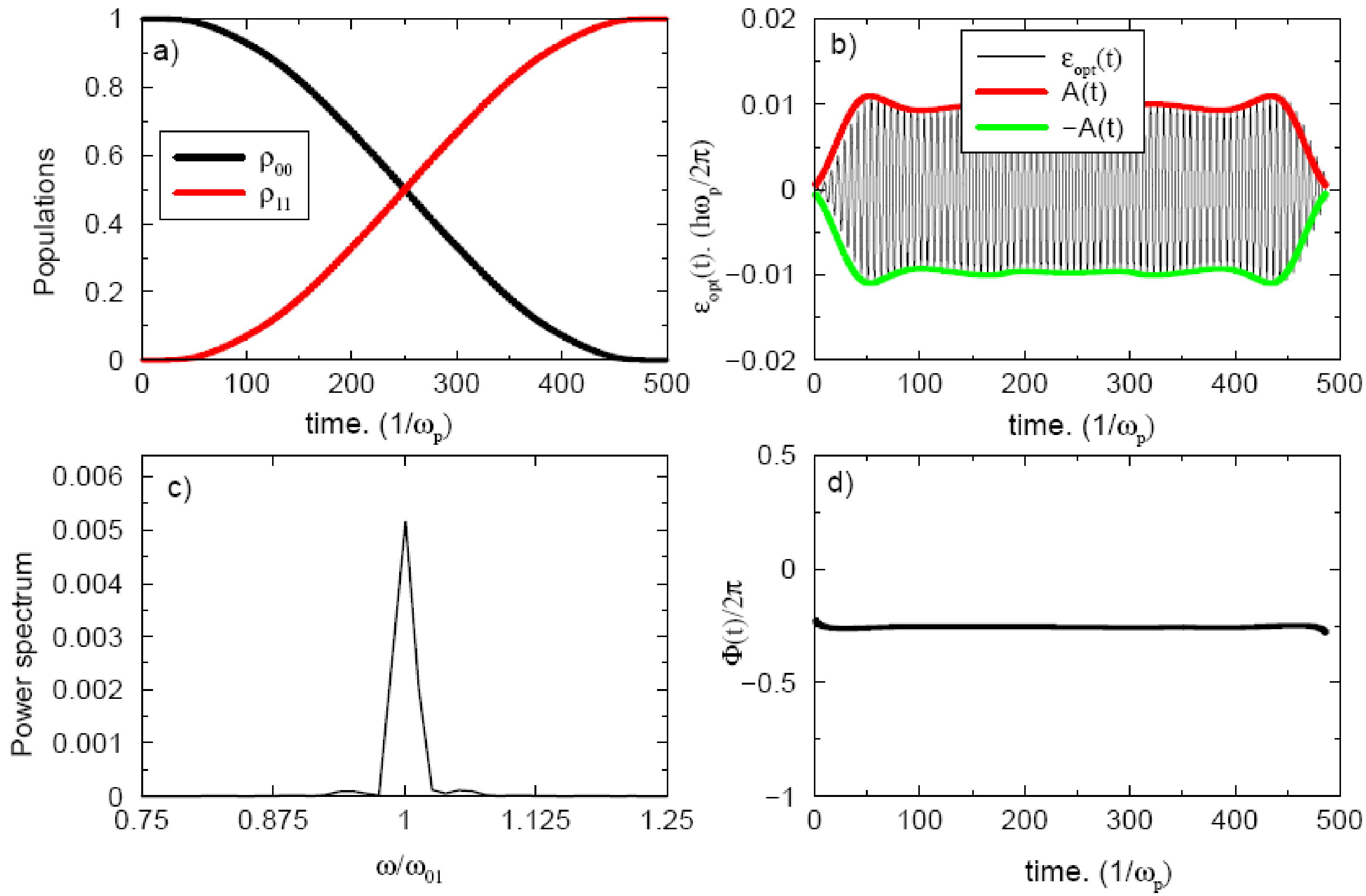


Use π-pulse as a guess for optimal control

(H. Jirari, FH and O. Buisson, EPL 2009)

$$\varepsilon_{\mathrm{opt}}(t) = A(t)\cos\left[\omega_0 t + \phi(t)\right]$$

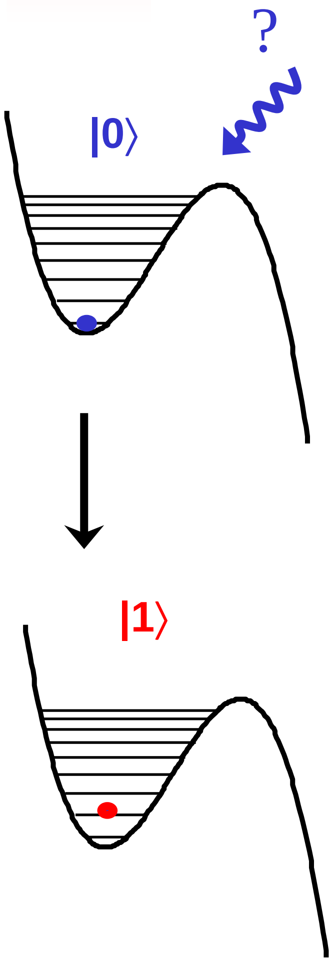


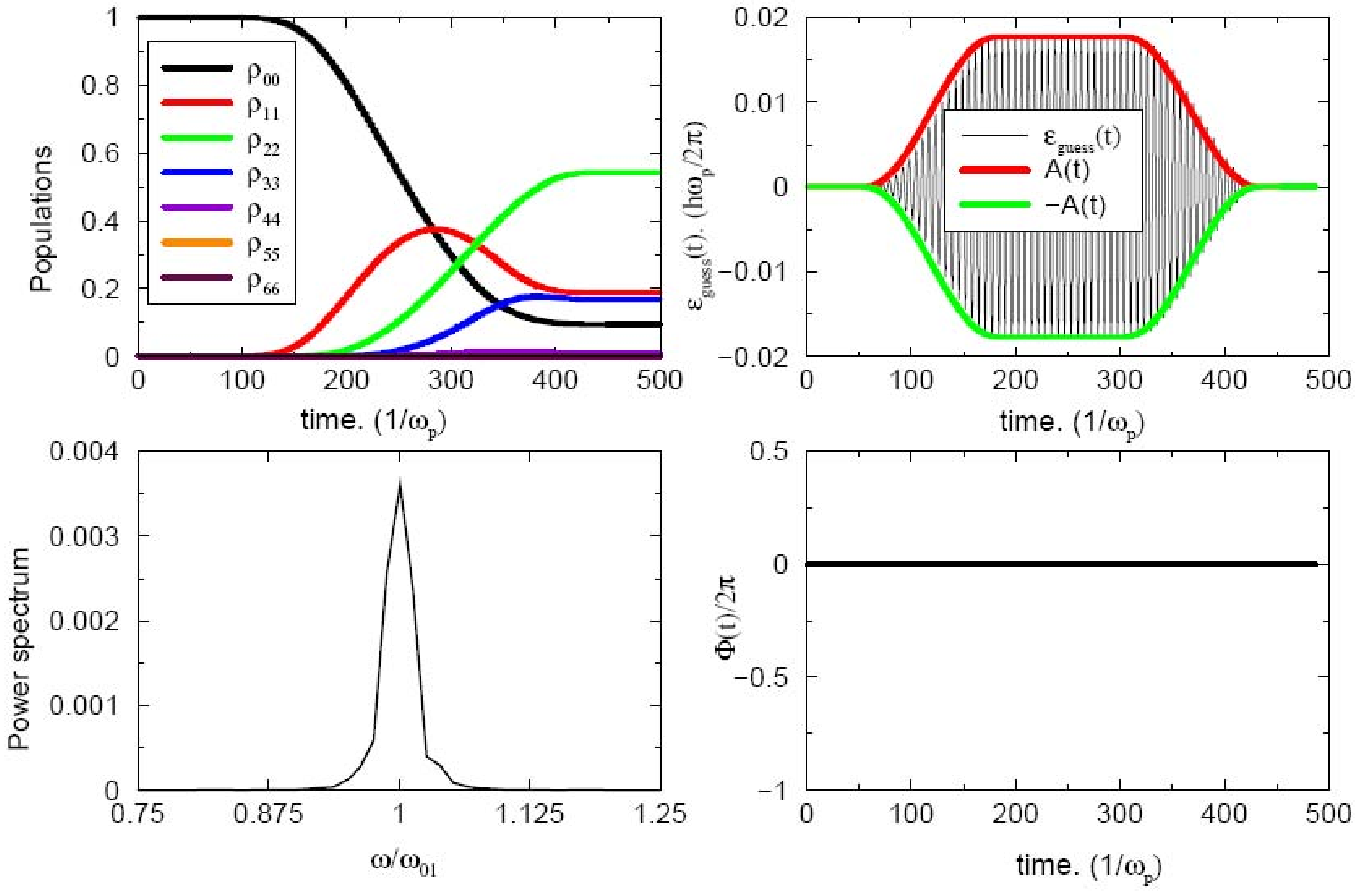


Effect of π-pulse in the presence of other levels

(H. Jirari, FH and O. Buisson, EPL 2009)

$$\varepsilon_{guess}(t) = A(t)\cos\left[\omega_0 t + \phi(t)\right]$$

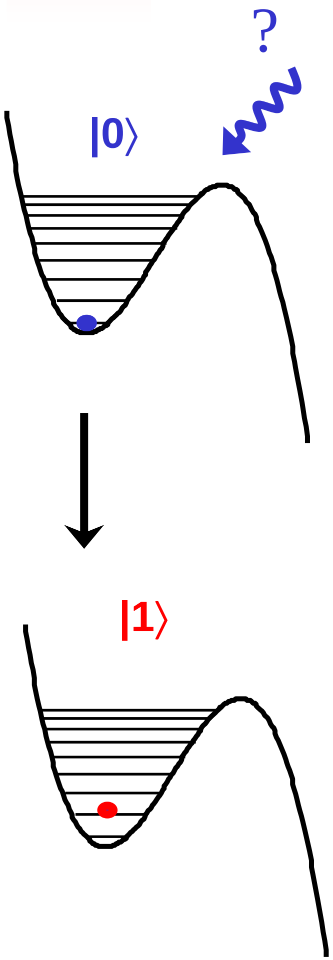


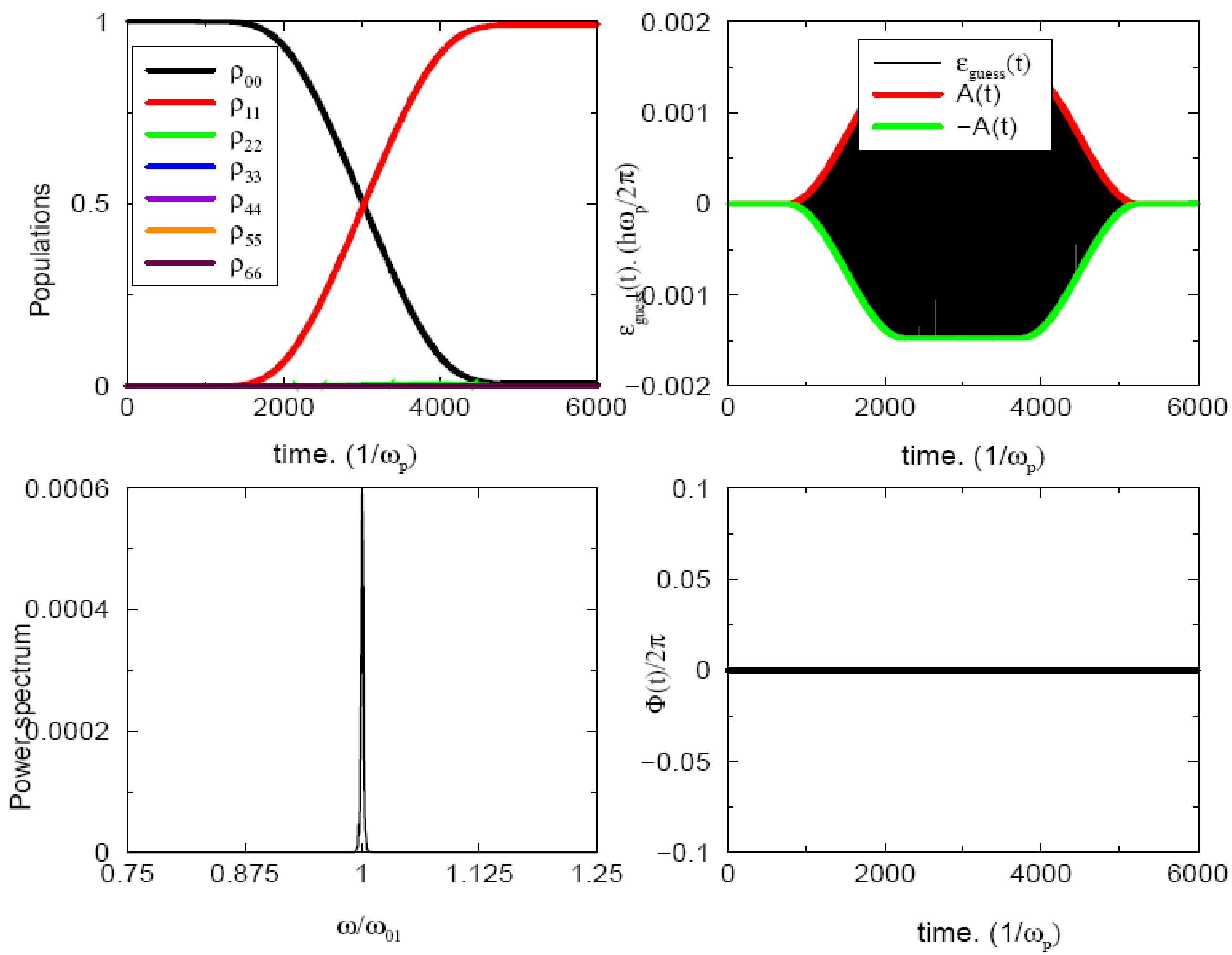


Adjust π-pulse in the presence of other levels

(H. Jirari, FH and O. Buisson, EPL 2009)

$$\varepsilon_{guess}(t) = A(t)\cos\left[\omega_0 t + \phi(t)\right]$$

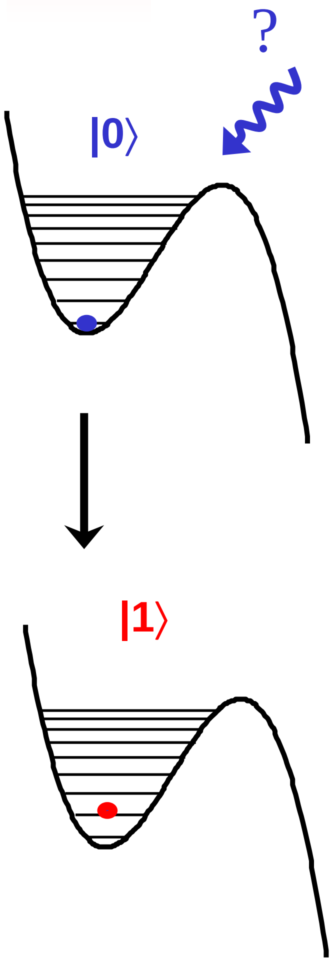


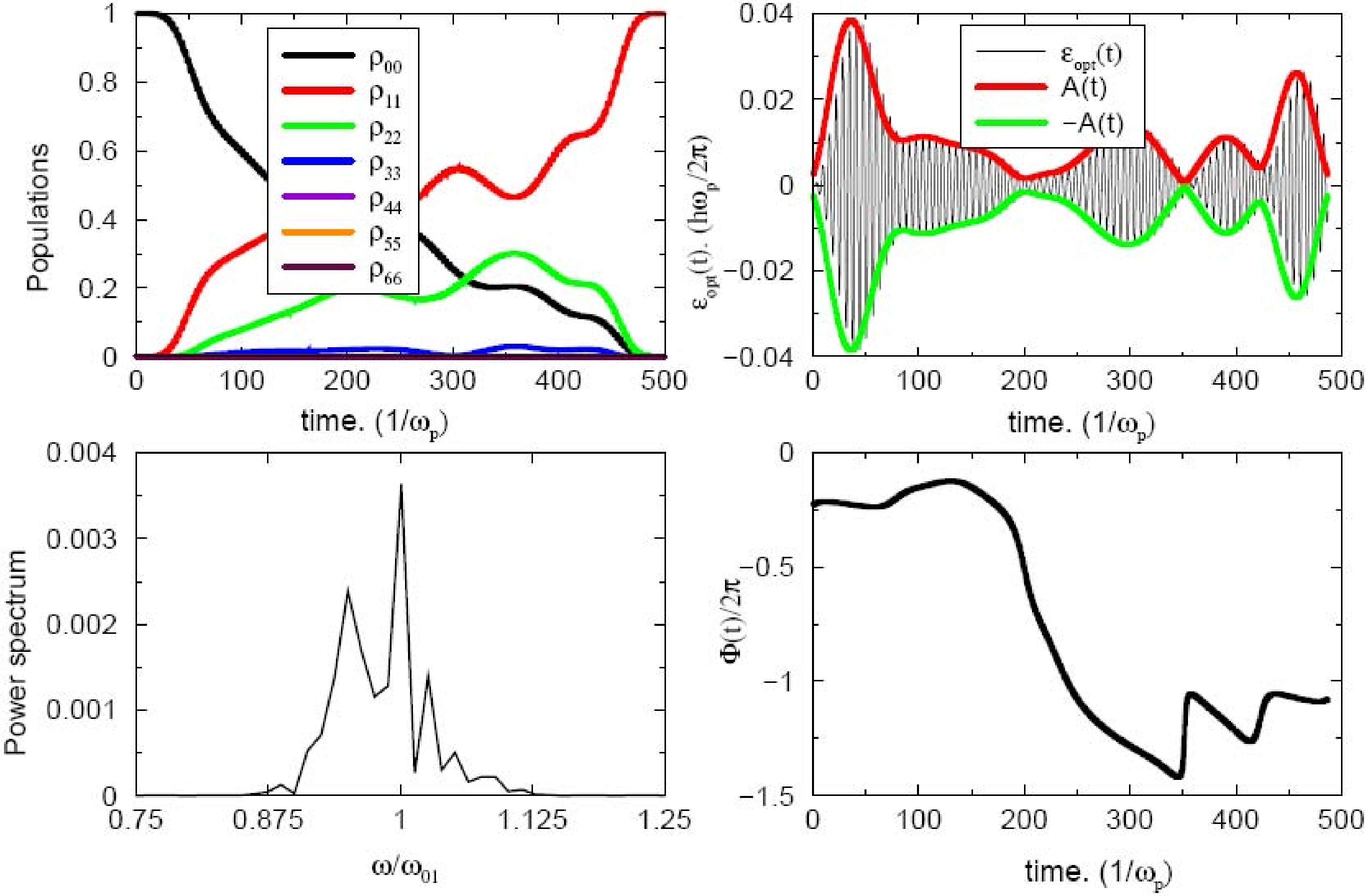


Optimal control in the presence of other levels

(H. Jirari, FH and O. Buisson, EPL 2009)

$$\varepsilon_{\mathrm{opt}}(t) = A(t)\cos\left[\omega_0 t + \phi(t)\right]$$

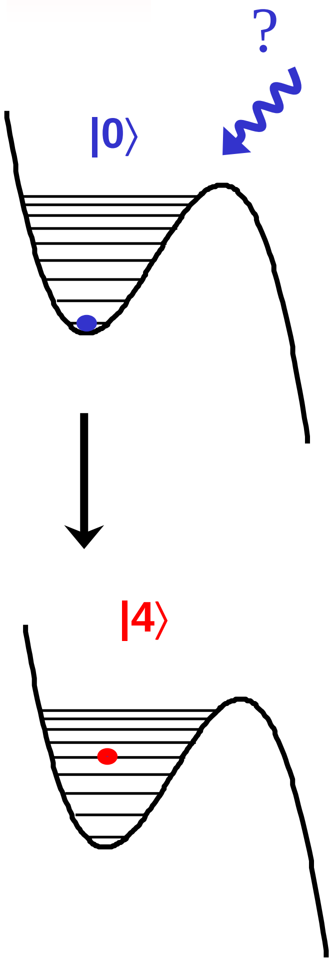


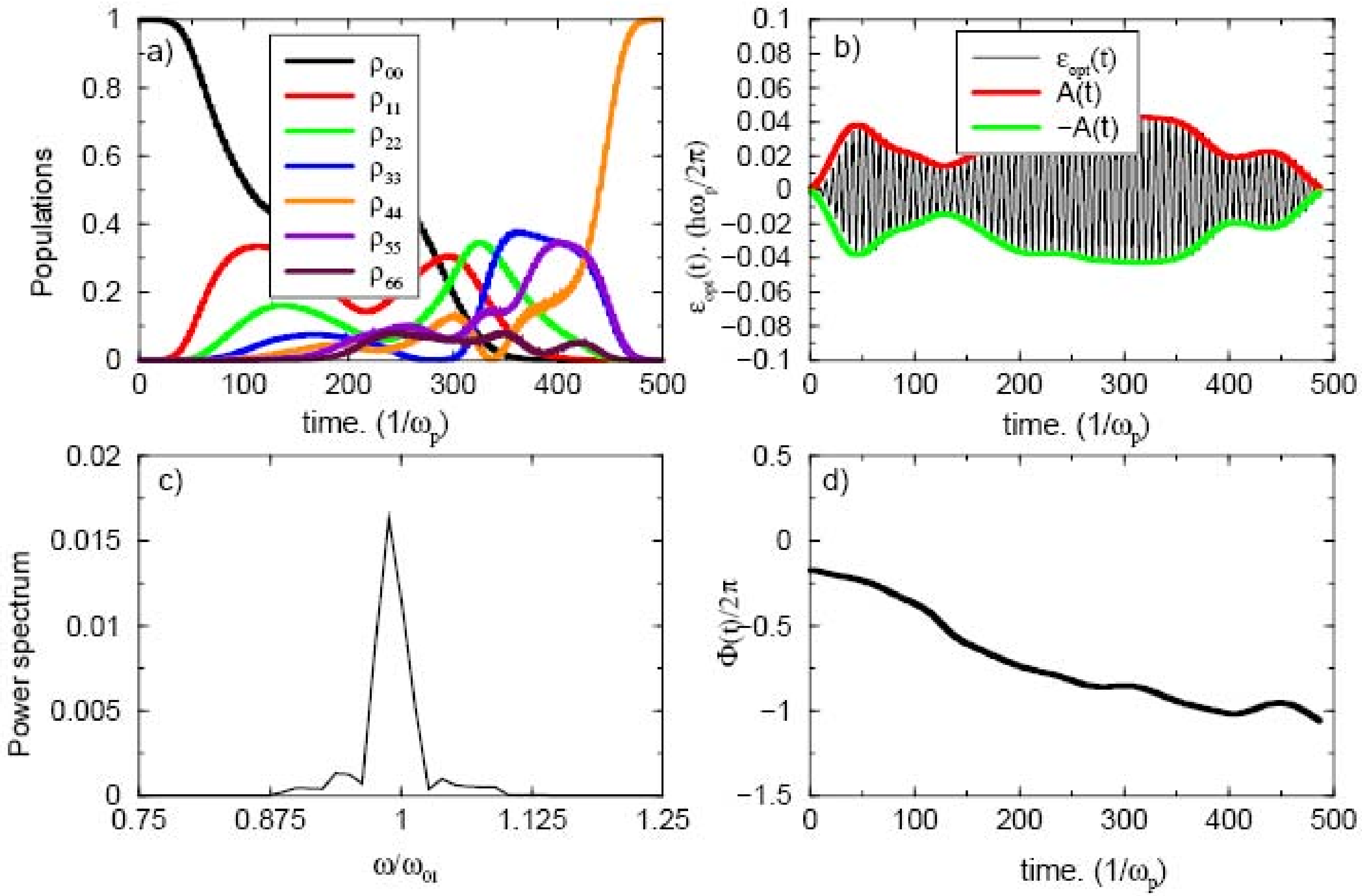


Optimal control for more complicated transitions

(H. Jirari, FH and O. Buisson, EPL 2009)

$$\varepsilon_{\mathrm{opt}}(t) = A(t)\cos\left[\omega_0 t + \phi(t)\right]$$





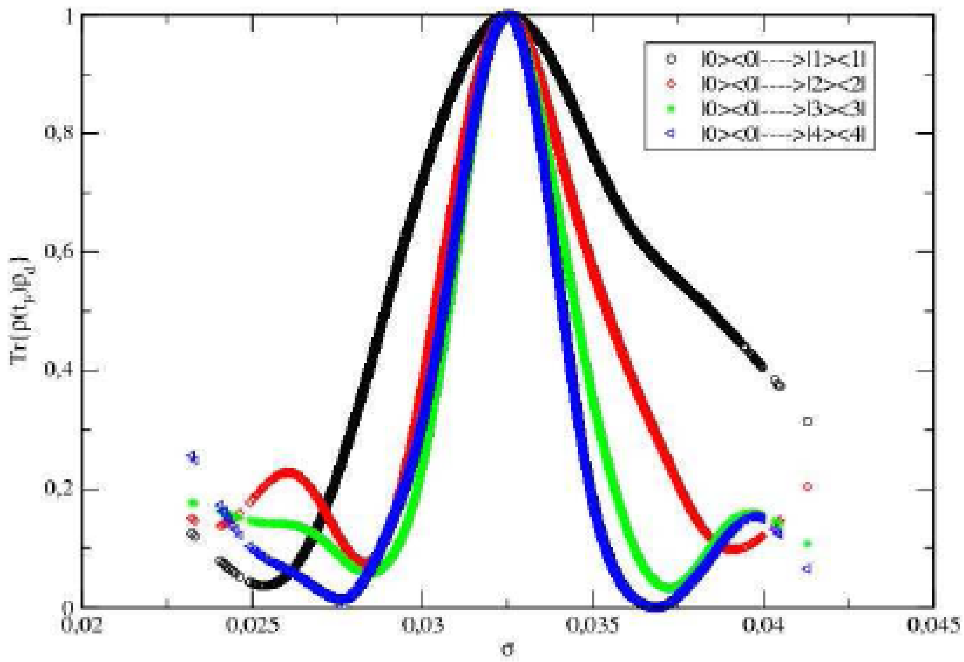
Sensitivity: effect of fluctuating system parameters

(H. Jirari, FH and O. Buisson, EPL 2009)

Low frequency noise: parameters fluctuate from run to run; example fluctuating anharmonicity σ

Strategy: run control field that is optimal for $\sigma = \bar{\sigma} = 0.0325$ *M times, drawing σ from a distribution with standard deviation* $\Delta\sigma = \bar{\sigma}/16$ *(Q-factor = 1000)*

Average fidelity $\overline{\mathrm{F}} = \frac{1}{M}\sum_{i=1}^{M} F_i = \frac{1}{M}\sum_{i=1}^{M} \mathrm{Tr}\{\rho_i(t_F)\rho_d\}$



$$\bar{\mathrm{F}}_{|0\rangle\rightarrow|1\rangle} = 85\%$$

$$\bar{\mathrm{F}}_{|0\rangle\rightarrow|2\rangle} = 73\%$$

$$\bar{\mathrm{F}}_{|0\rangle\rightarrow|3\rangle} = 60\%$$

$$\bar{\mathrm{F}}_{|0\rangle\rightarrow|4\rangle} = 55\%$$

IV. Coupling qubits

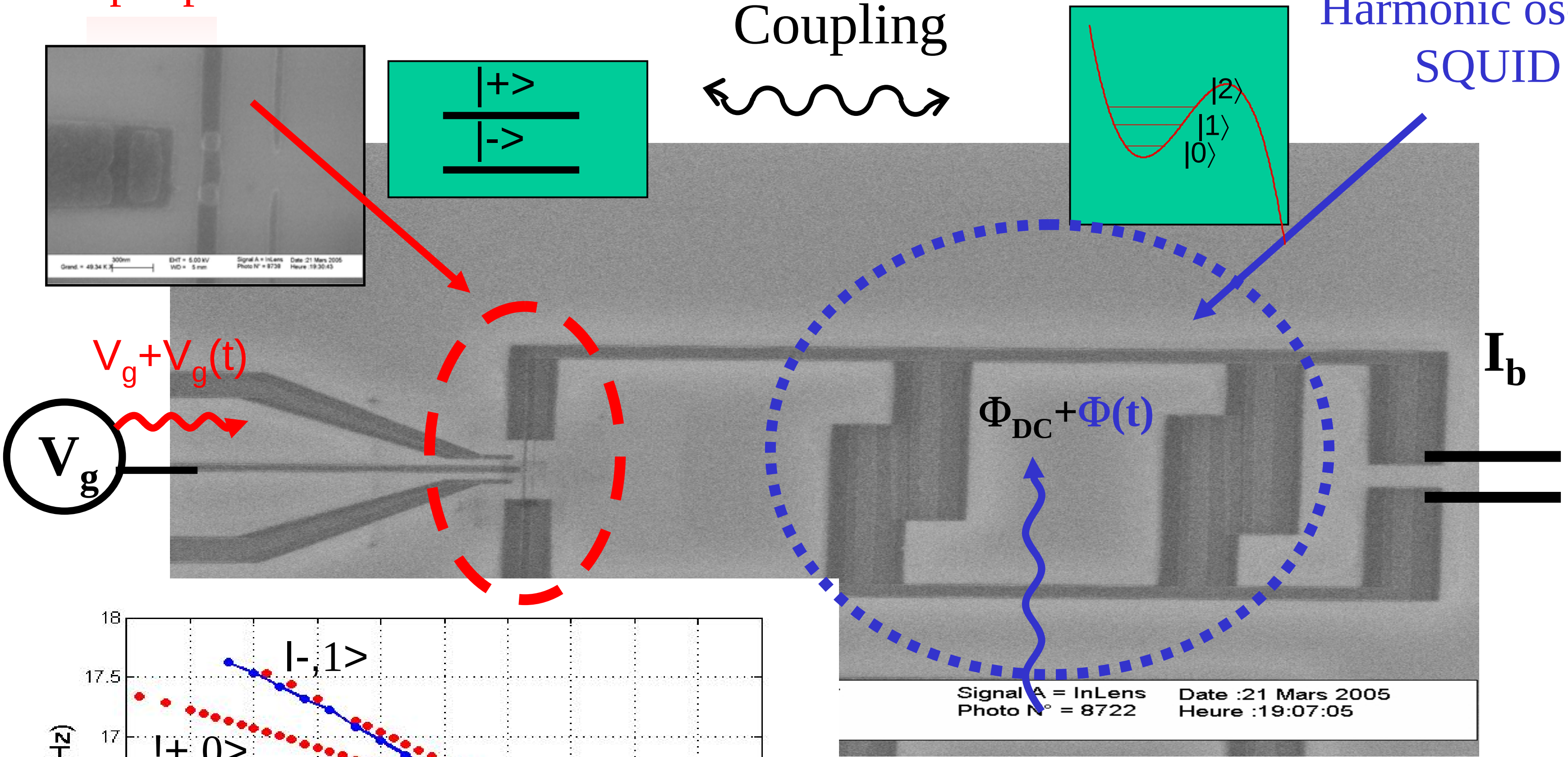
Towards more complicated circuit designs

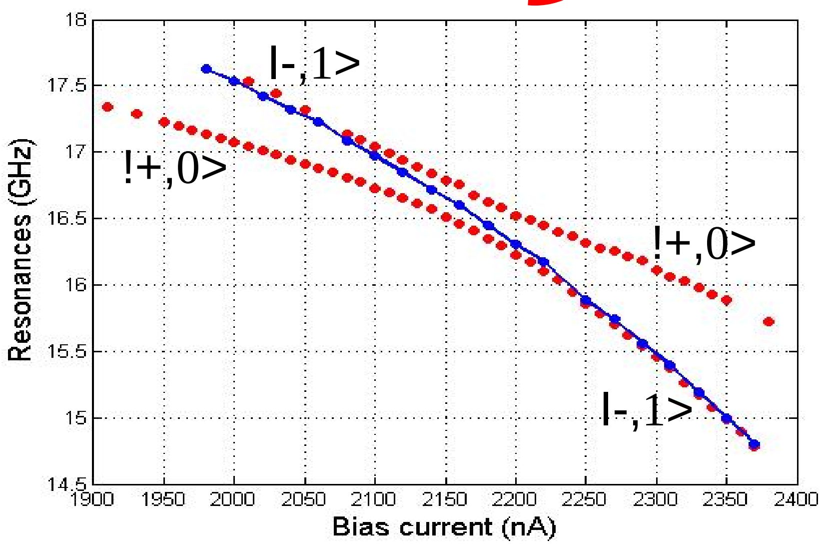
Charge qubit coupled a harmonic oscillator

Charge qubit :
Cooper pair transistor

(A. Fay, W. Guichard, FH, L. Lévy, and O. Buisson, PRL 08)

Harmonic oscillator
SQUID dc



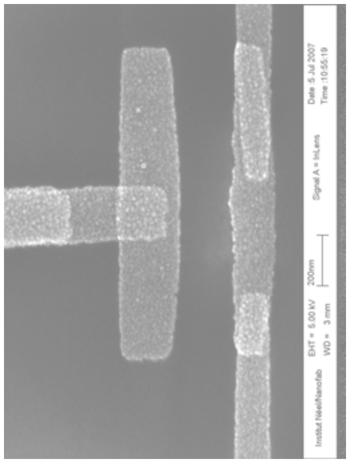


In case of resonance:

→

Entanglement between the Quantum states of the transistor and SQUID (Aurélien Fay)

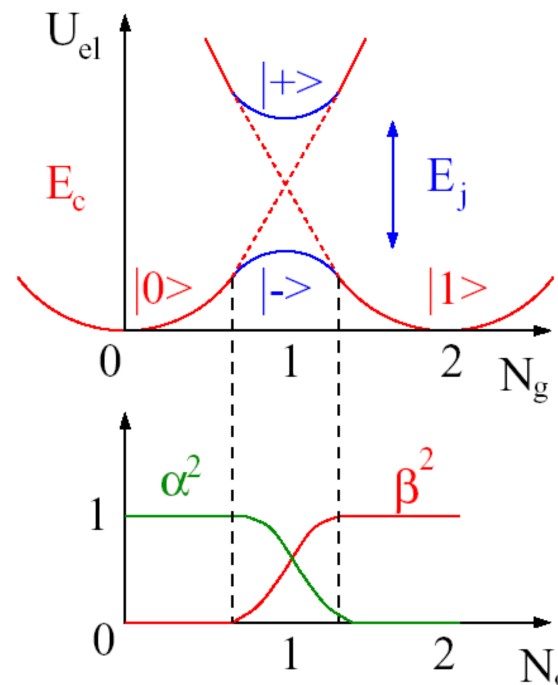
Cooper pair transistor as a charge qubit



$$\hat{H} = \frac{e^2}{2C_\Sigma}(2\hat{n} - N_g)^2 - E_J \cos\phi$$

$$\hat{H} = \sum_n \frac{e^2}{2C_\Sigma}(2n - N_g)^2 |n><n| - \frac{E_J}{2}\left[|n+1><n| + |n-1><n|\right]$$

Regime of interest: $E_J \ll E_C$



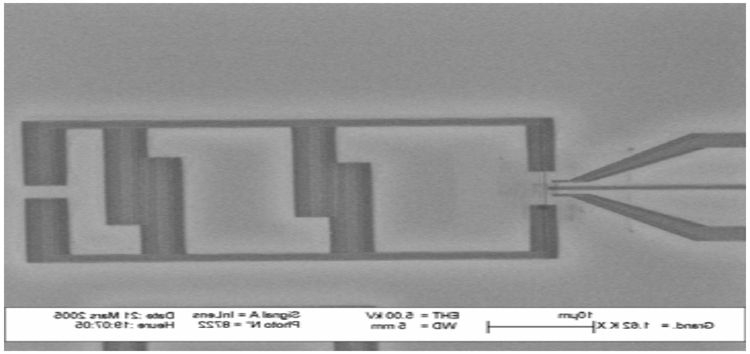
$|+> = \beta\,|0> - \alpha\,|1>$

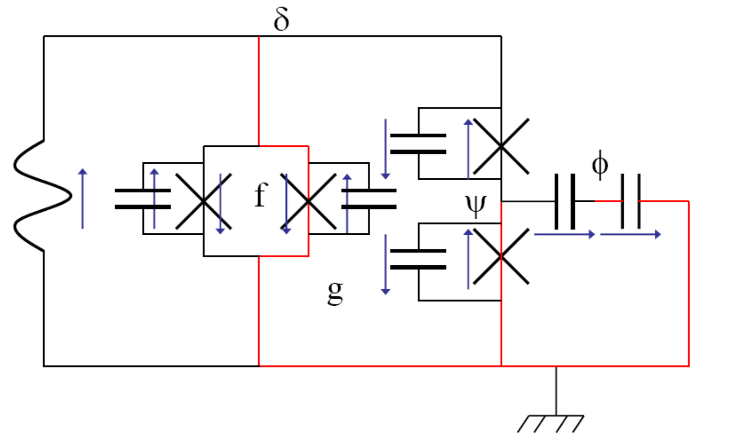
$|-> = \alpha|0> + \beta\,|1>$

Theory of coupled circuit

(A. Fay, W. Guichard, FH, L. Lévy, and O. Buisson, PRL 08)

Coupled circuit





Hamiltonian

$$H = \frac{(Q_\psi + C_g V_g)^2}{2C_\psi} + \frac{Q_\delta^2}{2C_\delta} + \frac{Q_\delta(Q_\psi + C_g V_g)}{C_{\delta\psi}}$$

$$-E_{jS}\cos\delta - E_{jS}\cos(\delta - f) - E_{j1}\cos\psi - E_{j2}\cos(\psi - \delta - g) - I_b\delta,$$

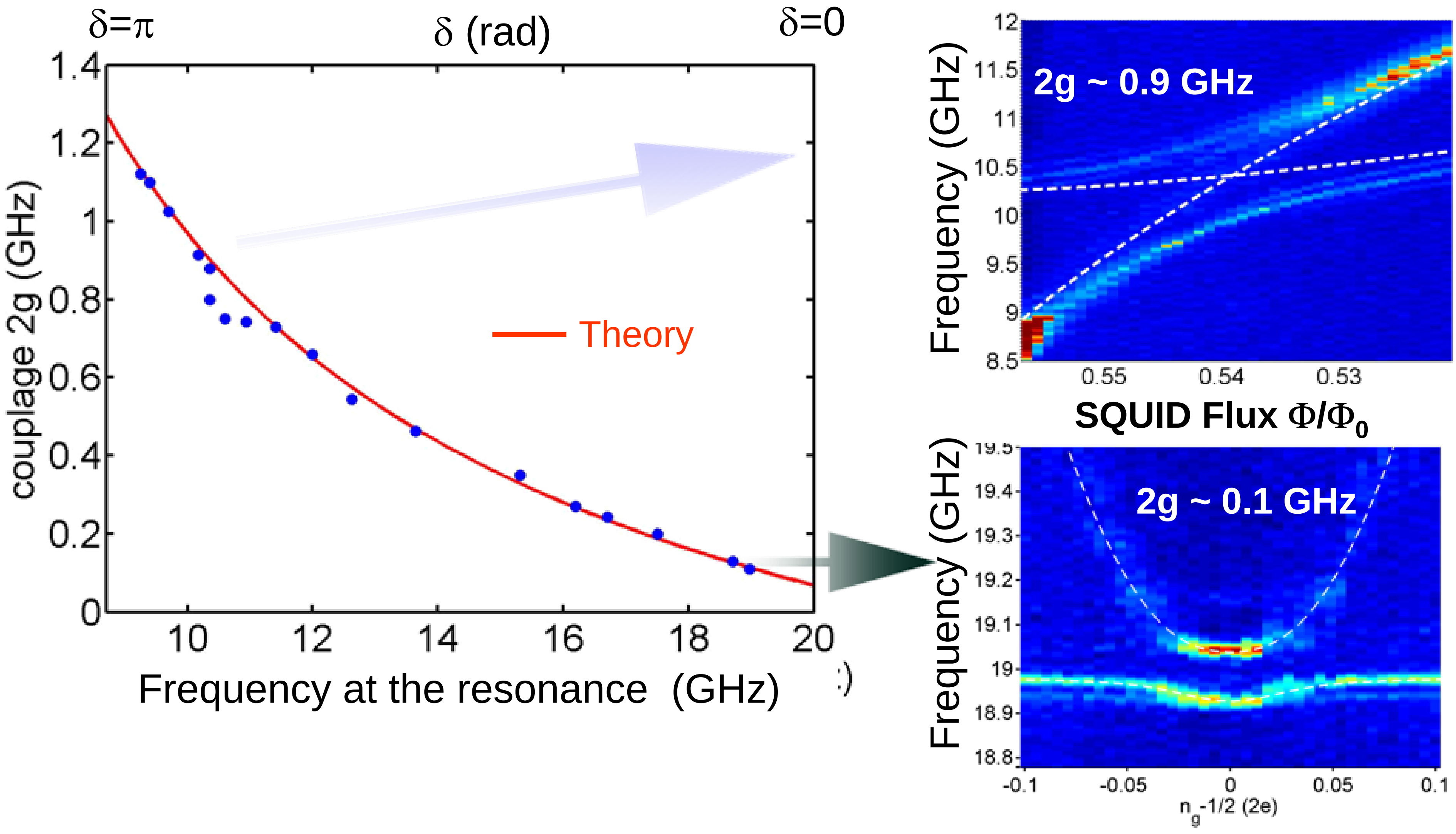
Two-level limit: tunable spin-spin coupling

$$H_{coupl} = (E_{c,c}/4)\sigma_x^{SQUID}\sigma_x^{CPT} - (E_{c,j}/2)\sigma_y^{SQUID}\sigma_y^{CPT}$$

rotating wave approximation $\longrightarrow$ $\frac{1}{2}(ga^\dagger\sigma_- + g^* a\sigma_+)$

Measured coupling strength vs. theory

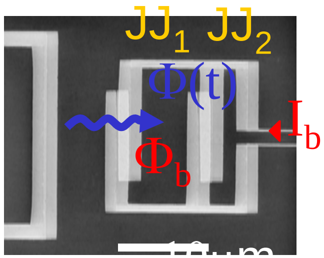
(A. Fay, W. Guichard, FH, L. Lévy, and O. Buisson, PRL 08)



Summary

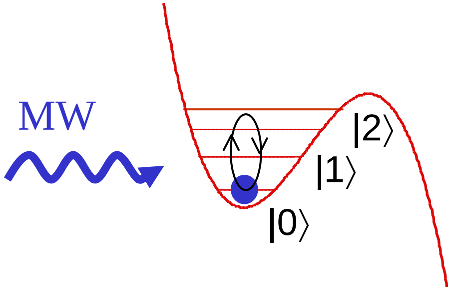
I. Small Josephson junctions

- Some basic notions



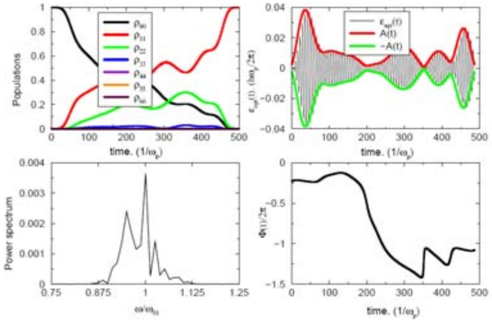
II. Quantum dynamics

- Multilevel coherent oscillations
- Quantum or classical oscillations?



III. Quantum optimal control theory

- Inducing transitions « à la carte »?
- Effect of noise



IV. Coupling qubits

- Towards tunable coupling