Microlocal Analysis approach to Ruelle Pollicott Resonances

Nicolas Roy
(H.U Berlin)

September 9, 2009

Acknowledgments

- Joint work with **F. Faure** and **J. Sjöstrand**
  Published in *Open Math. Journal (2008)*
- After (non-exhaustive alphabetically-ordered list):
  - **Classical dynamical systems** : Baladi-Tsujii, Baladi-Gouëzel, Blank-Keller-Liverani, Gouëzel-Liverani,
  - **Quantum resonances** : Aguilar, Balslev, Combes, Simon, Helffer-Sjöstrand, ...

Plan

1. Correlation functions and mixing
2. Anosov diffeomorphisms
3. Ruelle Resonances
4. Microlocal analysis & Anisotropic Sobolev spaces
5. Quasicompacity and escape function on T^*M

Correlation functions & mixing

Framework

- **Phase space** : (compact) C^∞ manifold M
- **Dynamical system** : C^∞ diffeomorphism

$$f : M \longrightarrow M$$

- **Observables** (test functions): $C^\infty(M)$
- **Additional structures** on M :
  - Riemannian metric g
  - Lebesgue measure μ_{leb} (*usually **not** f-invariant*)
    Nota $dx = d\mu_{leb}$

Correlation functions & mixing

- Explicit solutions vs statistical properties

Correlation functions

- Test functions : $\varphi, \psi \in C^\infty(M)$
- discrete time : $n \in \mathbb{Z}$

$$\forall \varphi, \psi \in C^\infty(M) \rightsquigarrow C_{\varphi,\psi}(n) = \int_M \varphi.\psi \circ f^n dx$$

? Asymptotic of $C_{\varphi,\psi}(n)$ for $n \longrightarrow +\infty$

Correlation functions & mixing

- Explicit solutions vs statistical properties

Correlation functions

- Test functions : $\varphi, \psi \in C^\infty(M)$
- discrete time : $n \in \mathbb{Z}$

$$\forall \varphi, \psi \in C^\infty(M) \rightsquigarrow C_{\varphi,\psi}(n) = \int_M \varphi.\psi \circ f^n dx$$

? Asymptotic of $C_{\varphi,\psi}(n)$ for $n \to +\infty$

Correlation functions & mixing

Def : Lebesgue-Mixing

$\exists$ f-invariant measure μ_{srb} (Sinai-Ruelle-Bowen),
$\forall \varphi, \psi \in C^{\infty}(M)$

$$C_{\varphi,\psi}(n) = \int \varphi.\psi \circ f^n dx \underset{n\to\infty}{\longrightarrow} \left(\int \varphi dx\right)\left(\int \psi d\mu_{srb}\right)$$

? Speed of convergence, asymptotic of $C_{\varphi,\psi}(n)$

! Statistical information available if f sufficiently chaotic

Correlation functions & mixing

Def : *Lebesgue-Mixing*

$\exists$ *f-invariant measure* μ_{srb} (*Sinai-Ruelle-Bowen*),
$\forall \varphi, \psi \in C^{\infty}(M)$

$$C_{\varphi,\psi}(n) = \int \varphi.\psi \circ f^n dx \underset{n\to\infty}{\longrightarrow} \left(\int \varphi dx\right)\left(\int \psi d\mu_{srb}\right)$$

Speed of convergence, asymptotic of $C_{\varphi,\psi}(n)$

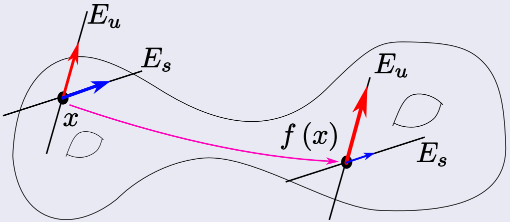
Statistical information available if f sufficiently chaotic

Anosov diffeomorphisms

Def : Anosov Diffeomorphism $f : M \to M$

$\exists$ *decomposition* $TM = E_s \oplus E_u$ *"Stable $\oplus$ Unstable",* f*-invariant,* $\theta < 1$

$$\begin{cases} |df(v)| \leq \theta |v| & \forall v \in E_s \text{ contracting} \\ \left|df^{-1}(v)\right| \leq \theta |v| & \forall v \in E_u \text{ expanding} \end{cases}$$



Anosov diffeomorphisms

Examples

- Hyperbolic torus automorphisms

$$\begin{array}{rcl} f:\mathbb{T}^d & \longrightarrow & \mathbb{T}^d \\ x & \longmapsto & A.x \bmod \mathbb{Z}^d \end{array}$$

with $A \in SL(n,\mathbb{Z})$ and $|\text{eigenvalues}| \neq 1$

- Perturbations , e.g.

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \frac{\varepsilon}{2\pi}\sin(2\pi(2x+y)) \\ 0 \end{pmatrix}$$

Anosov diffeomorphisms

Examples

- Hyperbolic toral automorphisms
- Perturbations

Remarks

- μ_{leb} is preserved for toral automorphism but not for perturbations
- $\exists f$ Anosov $\Rightarrow$ strong restrictions on topology of M
  Conjecture : M is infranil manifold
- Distributions E^u and E^s only Holder continuous w.r.t point $x \in M$

Anosov diffeomorphisms

Examples

- Hyperbolic toral automorphisms
- Perturbations

Remarks

- μ_{leb} is preserved for toral automorphism but not for perturbations
- $\exists f$ Anosov $\Rightarrow$ strong restrictions on topology of M
  Conjecture : M is infranil manifold
- Distributions E^u and E^s only Holder continuous w.r.t point $x \in M$

Anosov diffeomorphisms

Examples

- Hyperbolic toral automorphisms
- Perturbations

Remarks

- μ_{leb} is preserved for toral automorphism but not for perturbations
- $\exists f$ Anosov $\Rightarrow$ strong restrictions on topology of M
  Conjecture : M is infranil manifold
- Distributions E^u and E^s only Holder continuous w.r.t point $x \in M$

Anosov diffeomorphisms

Conjecture

f Anosov on M compact $\Rightarrow$ Lebesgue-mixing

- Proved by Anosov ['67] in the case μ_{leb} =preserved

Theorem [Liverani&al, Baladi&al]

Let f be Anosov.
Then mixing $\Longrightarrow$ exponential mixing

! Understand asymptotic of $C_{\varphi,\psi}(n)$

..... Ruelle-Pollicott resonances .....

Anosov diffeomorphisms

Conjecture

f Anosov on M compact $\Rightarrow$ Lebesgue-mixing

- Proved by Anosov ['67] in the case μ_{leb} =preserved

Theorem [Liverani&al, Baladi&al]

Let f be Anosov.

Then mixing $\Longrightarrow$ exponential mixing

! Understand asymptotic of $C_{\varphi,\psi}(n)$

..... Ruelle-Pollicott resonances .....

Anosov diffeomorphisms

Conjecture

f Anosov on M compact $\Rightarrow$ Lebesgue-mixing

- Proved by Anosov ['67] in the case μ_{leb} =preserved

Theorem [Liverani&al, Baladi&al]

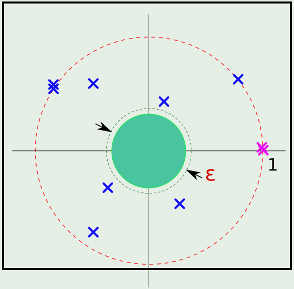
Let f be Anosov.
Then mixing $\Longrightarrow$ exponential mixing

! Understand asymptotic of $C_{\varphi,\psi}(n)$

..... Ruelle-Pollicott resonances .....

Ruelle Resonances

Theorem

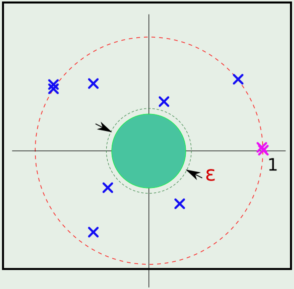


- Resonances : $\lambda_j \in \mathbb{C}$
- $1 = \lambda_0 \geq |\lambda_1| \geq |\lambda_2| \ldots$
- finite number of λ_j outside disc $D(\varepsilon)$
- P_j polynomial in n
- P_j depends on $\partial^k \varphi, \partial^k \psi$ for $k \leq c|\log \varepsilon|$

$$C_{\varphi,\psi}(n) = \sum_{|\lambda_j| > \varepsilon} \lambda_j^n . P_j(n; \varphi, \psi) + O_{\varepsilon,\varphi,\psi}(\varepsilon^n)$$

Ruelle Resonances

Theorem

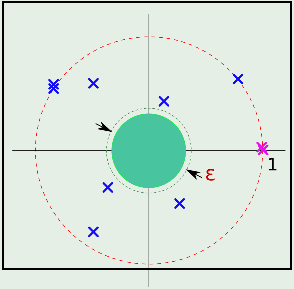


- Resonances : $\lambda_j \in \mathbb{C}$
- $1 = \lambda_0 \geq |\lambda_1| \geq |\lambda_2| \ldots$
- finite number of λ_j outside disc $D(\varepsilon)$
- P_j polynomial in n
- P_j depends on $\partial^k \varphi, \partial^k \psi$ for $k \leq c |\log \varepsilon|$

$$C_{\varphi,\psi}(n) = \sum_{|\lambda_j| > \varepsilon} \lambda_j^n . P_j(n; \varphi, \psi) + O_{\varepsilon,\varphi,\psi}(\varepsilon^n)$$

Ruelle Resonances

Theorem



- Resonances : $\lambda_j \in \mathbb{C}$
- $1 = \lambda_0 \geq |\lambda_1| \geq |\lambda_2| \ldots$
- finite number of λ_j outside disc $D(\varepsilon)$
- P_j polynomial in n
- P_j depends on $\partial^k \varphi, \partial^k \psi$ for $k \leq c |\log \varepsilon|$

$$C_{\varphi,\psi}(n) = \sum_{|\lambda_j| > \varepsilon} \lambda_j^n . P_j(n; \varphi, \psi) + O_{\varepsilon,\varphi,\psi}(\varepsilon^n)$$

Ruelle Resonances

$$C_{\varphi,\psi}(n) = \sum_{|\lambda_j| > \varepsilon} \lambda_j^n . P_j(n; \varphi, \psi) + O_{\varepsilon,\varphi,\psi}(\varepsilon^n)$$

Remarks

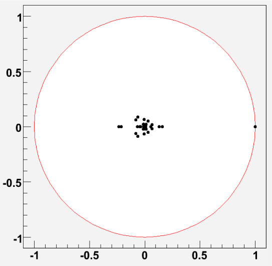
- If λ_0 simple and $|\lambda_{j\neq 0}| < 1 \Rightarrow$ Mixing

$$P_0(n; \varphi, \psi) = \langle \mu_{leb}, \varphi \rangle \langle \mu_{srb}, \psi \rangle$$

- λ_j ←-- spectral decomposition of $\hat{F} : \varphi \mapsto \varphi \circ f$ but on a «complicated» space $\hat{F} : H \to H$

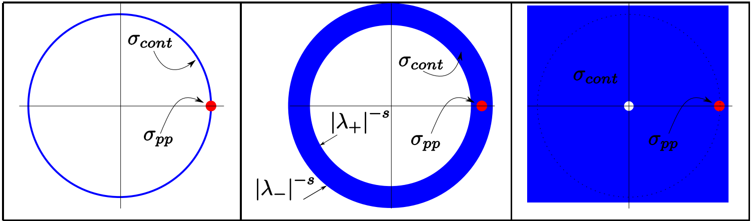
Ruelle Resonances

- $\hat{F} : \varphi \mapsto \varphi \circ f$
- Model $f : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \varepsilon.$Perturb. on $\mathbb{T}^2$
- Resonances (numerics) for $\varepsilon = 0.16$



Ruelle Resonances

- $\hat{F} : \varphi \mapsto \varphi \circ f$
- Model $f : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \varepsilon$.Perturb. on $\mathbb{T}^2$
- Exact computation of spectrum for $\varepsilon = 0$

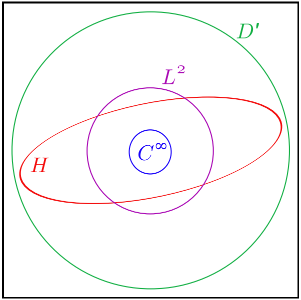


on L^2	on H^s, $s > 0$	on C^∞

Ruelle Resonances

- **Spectral decomposition** of

$$\begin{aligned} \hat{F} : H &\longrightarrow H \\ \varphi &\longmapsto \varphi \circ f \end{aligned}$$



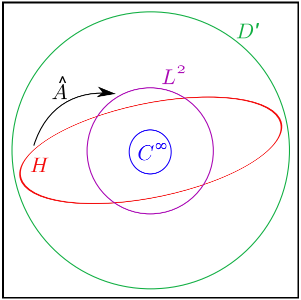
[Faure-R-Sjöstrand '08]
Microlocal approach

- $H = \hat{A}^{-1}\left(L^2\right)$
- $\hat{A}$ pseudodifferential with ***variable order***

Ruelle Resonances

- **Spectral decomposition** of

$$\begin{aligned} \hat{F} : H &\rightarrow H \\ \varphi &\mapsto \varphi \circ f \end{aligned}$$



[Faure-R-Sjöstrand '08]
Microlocal approach

- $H = \hat{A}^{-1}\left(L^2\right)$
- $\hat{A}$ pseudodifferential with ***variable order***

Microlocal Analysis with variable order

Pseudodifferential Operator with **variable order**

$$\hat{A}(\varphi)(x) = \int\int e^{i\xi(x-y)} A(x,\xi)\varphi(y)\,dxd\xi \quad \textit{(local)}$$

- The symbol $A \in C^{\infty}(T^*M)$ satisfies

$$\left|\partial_{\xi}^{\alpha}\partial_{x}^{\beta}A(x,\xi)\right| \leq C_{\alpha,\beta}\left|\xi\right|^{m(x,\xi)-\rho|\alpha|+(1-\rho)|\beta|}$$

  - Type : $\frac{1}{2} < \rho < 1$
  - Order **function** : $m \in C^{\infty}(T^*M)$ (*in fact* $m \in S_r^0$)
- **Symbols** : $A \in S_{\rho}^{m(x,\xi)} \leftrightsquigarrow$ **Operators** : $\hat{A} \in \Psi_{\rho}^{m(x,\xi)}$
- Symbol well-defined up to $S_{\rho}^{m-(2\rho-1)} \rightsquigarrow$ Notion of principal symbol

Microlocal Analysis with variable order

Pseudodifferential Operator with **variable order**

$$\hat{A}(\varphi)(x) = \int\int e^{i\xi(x-y)} A(x,\xi)\varphi(y)\, dx d\xi \quad \textit{(local)}$$

- The symbol $A \in C^\infty(T^*M)$ satisfies

$$\left|\partial_\xi^\alpha \partial_x^\beta A(x,\xi)\right| \leq C_{\alpha,\beta} |\xi|^{m(x,\xi)-\rho|\alpha|+(1-\rho)|\beta|}$$

  - Type : $\frac{1}{2} < \rho < 1$
  - Order **function** : $m \in C^\infty(T^*M)$ (*in fact* $m \in S_r^0$)
- **Symbols** : $A \in S_\rho^{m(x,\xi)} \leftrightsquigarrow$ **Operators** : $\hat{A} \in \Psi_\rho^{m(x,\xi)}$
- Symbol well-defined up to $S_\rho^{m-(2\rho-1)} \rightsquigarrow$ Notion of principal symbol

Microlocal Analysis with variable order

Pseudodifferential Operator with **variable order**

$$\hat{A}(\varphi)(x) = \int\int e^{i\xi(x-y)} A(x,\xi)\,\varphi(y)\,dxd\xi \quad \textit{(local)}$$

- The symbol $A \in C^\infty(T^*M)$ satisfies

$$\left|\partial_\xi^\alpha \partial_x^\beta A(x,\xi)\right| \le C_{\alpha,\beta}\,|\xi|^{m(x,\xi)-\rho|\alpha|+(1-\rho)|\beta|}$$

  - Type : $\frac{1}{2} < \rho < 1$
  - Order **function** : $m \in C^\infty(T^*M)$ (*in fact* $m \in S_r^0$)
- **Symbols** : $A \in S_\rho^{m(x,\xi)} \leftrightsquigarrow$ **Operators** : $\hat{A} \in \Psi_\rho^{m(x,\xi)}$
- Symbol well-defined up to $S_\rho^{m-(2\rho-1)} \rightsquigarrow$ Notion of principal symbol

Microlocal Analysis with variable order

"Theorem"

All standard results extend to the case with variable order.

- ***Composition*** : $\hat{A} \in \Psi_\rho^{m_1(x,\xi)}$ *and* $\hat{B} \in \Psi_\rho^{m_2(x,\xi)} \Rightarrow \hat{A}\hat{B} \in \Psi_\rho^{m_1(x,\xi)+m_2(x,\xi)}$
- ***Continuity*** :
  - $\hat{A} : C^\infty(M) \to C^\infty(M)$ *(Smooth functions)*
  - $\hat{A} : D'(M) \to D'(M)$ *(Distributions)*
- *Notion of* ***ellipticity*** : $|A(x,\xi)| \geq C.|\xi|^{m(x,\xi)}$ *for* $|\xi| \gg 1$

Microlocal Analysis with variable order

"Theorem"

All standard results extend to the case with variable order.

- ***Composition*** : $\hat{A} \in \Psi_{\rho}^{m_1(x,\xi)}$ *and* $\hat{B} \in \Psi_{\rho}^{m_2(x,\xi)} \Rightarrow \hat{A}\hat{B} \in \Psi_{\rho}^{m_1(x,\xi)+m_2(x,\xi)}$
- ***Continuity*** :
  - $\hat{A} : C^{\infty}(M) \rightarrow C^{\infty}(M)$ *(Smooth functions)*
  - $\hat{A} : D'(M) \rightarrow D'(M)$ *(Distributions)*
- *Notion of* ***ellipticity*** : $|A(x,\xi)| \geq C.|\xi|^{m(x,\xi)}$ *for* $|\xi| \gg 1$

Microlocal Analysis with variable order

"Theorem"

All standard results extend to the case with variable order.

- ***Composition*** : $\hat{A} \in \Psi_\rho^{m_1(x,\xi)}$ *and* $\hat{B} \in \Psi_\rho^{m_2(x,\xi)} \Rightarrow \hat{A}\hat{B} \in \Psi_\rho^{m_1(x,\xi)+m_2(x,\xi)}$
- ***Continuity*** :
  - $\hat{A} : C^\infty(M) \to C^\infty(M)$ *(Smooth functions)*
  - $\hat{A} : D'(M) \to D'(M)$ *(Distributions)*
- *Notion of* ***ellipticity*** : $|A(x,\xi)| \geq C.|\xi|^{m(x,\xi)}$ *for* $|\xi| \gg 1$

Microlocal Analysis with variable order

Anisotropic Sobolev spaces

- $\forall$ order function $m(x,\xi)$, define

$$A_m(x,\xi) := \langle \xi \rangle^{m(x,\xi)}, \ \langle \xi \rangle = \sqrt{1+|\xi|^2}.$$

- $A_m \in S_\rho^{m(x,\xi)}$ is elliptic
- Quantize $\rightsquigarrow \hat{A}_m \in \Psi_\rho^{m(x,\xi)}$, invertible on $C^\infty(M)$ and $D'(M)$ and $\hat{A}_m^{-1} \in \Psi_\rho^{-m(x,\xi)}$
- **Anisotropic Sobolev** spaces :

$$H^{m(x,\xi)} = \hat{A}_m^{-1}(L^2)$$

- **Continuity** : if $\hat{B} \in \Psi_\rho^{m(x,\xi)}$ then

$$\hat{B} : H^{s(x,\xi)} \longrightarrow H^{s(x,\xi)-m(x,\xi)}$$

Microlocal Analysis with variable order

Anisotropic Sobolev spaces

- $\forall$ order function $m(x,\xi)$, define

$$A_m(x,\xi) := \langle \xi \rangle^{m(x,\xi)}, \ \langle \xi \rangle = \sqrt{1+|\xi|^2}.$$

- $A_m \in S_\rho^{m(x,\xi)}$ is elliptic
- Quantize $\rightsquigarrow \hat{A}_m \in \Psi_\rho^{m(x,\xi)}$, invertible on $C^\infty(M)$ and $D'(M)$ and $\hat{A}_m^{-1} \in \Psi_\rho^{-m(x,\xi)}$
- **Anisotropic Sobolev** spaces :

$$H^{m(x,\xi)} = \hat{A}_m^{-1}\left(L^2\right)$$

- **Continuity** : if $\hat{B} \in \Psi_\rho^{m(x,\xi)}$ then

$$\hat{B} : H^{s(x,\xi)} \longrightarrow H^{s(x,\xi)-m(x,\xi)}$$

Microlocal Analysis with variable order

Anisotropic Sobolev spaces

- $\forall$ order function $m(x,\xi)$, define

$$A_m(x,\xi) := \langle \xi \rangle^{m(x,\xi)}, \ \langle \xi \rangle = \sqrt{1+|\xi|^2}.$$

- $A_m \in S_\rho^{m(x,\xi)}$ is elliptic
- Quantize $\rightsquigarrow \hat{A}_m \in \Psi_\rho^{m(x,\xi)}$, invertible on $C^\infty(M)$ and $D'(M)$ and $\hat{A}_m^{-1} \in \Psi_\rho^{-m(x,\xi)}$
- Anisotropic Sobolev spaces :

$$H^{m(x,\xi)} = \hat{A}_m^{-1}(L^2)$$

- Continuity : if $\hat{B} \in \Psi_\rho^{m(x,\xi)}$ then

$$\hat{B} : H^{s(x,\xi)} \rightarrow H^{s(x,\xi)-m(x,\xi)}$$

Microlocal Analysis with variable order

Anisotropic Sobolev spaces

- $\forall$ order function $m(x,\xi)$, define

$$A_m(x,\xi) := \langle \xi \rangle^{m(x,\xi)}, \ \langle \xi \rangle = \sqrt{1+|\xi|^2}.$$

- $A_m \in S_\rho^{m(x,\xi)}$ is elliptic
- Quantize $\rightsquigarrow \hat{A}_m \in \Psi_\rho^{m(x,\xi)}$, invertible on $C^\infty(M)$ and $D'(M)$ and $\hat{A}_m^{-1} \in \Psi_\rho^{-m(x,\xi)}$
- **Anisotropic Sobolev** spaces :

$$H^{m(x,\xi)} = \hat{A}_m^{-1}\left(L^2\right)$$

- **Continuity** : if $\hat{B} \in \Psi_\rho^{m(x,\xi)}$ then

$$\hat{B} : H^{s(x,\xi)} \longrightarrow H^{s(x,\xi)-m(x,\xi)}$$

Microlocal Analysis with variable order

Anisotropic Sobolev spaces

- $\forall$ order function $m(x,\xi)$, define

$$A_m(x,\xi) := \langle \xi \rangle^{m(x,\xi)}, \quad \langle \xi \rangle = \sqrt{1+|\xi|^2}.$$

- $A_m \in S_\rho^{m(x,\xi)}$ is elliptic
- Quantize $\rightsquigarrow \hat{A}_m \in \Psi_\rho^{m(x,\xi)}$, invertible on $C^\infty(M)$ and $D'(M)$ and $\hat{A}_m^{-1} \in \Psi_\rho^{-m(x,\xi)}$
- **Anisotropic Sobolev** spaces :

$$H^{m(x,\xi)} = \hat{A}_m^{-1}\left(L^2\right)$$

- **Continuity** : if $\hat{B} \in \Psi_\rho^{m(x,\xi)}$ then

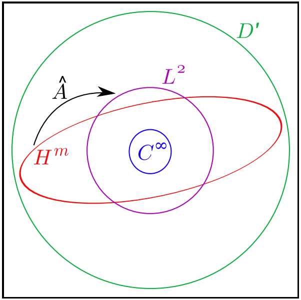
$$\hat{B} : H^{s(x,\xi)} \longrightarrow H^{s(x,\xi)-m(x,\xi)}$$

Quasicompacity of $\hat{F}$

- **Aim** :

$$C_{\varphi,\psi}(n) = \sum_{|\lambda_j| > \varepsilon} \lambda_j^n . P_j(n; \varphi, \psi) + O_{\varepsilon,\varphi,\psi}(\varepsilon^n)$$

- λ_j ←-- spectral decomposition of $\hat{F} : \varphi \mapsto \varphi \circ f$ on $H^{m(x,\xi)}$



- $\hat{F}$ is **quasicompact** on $H^{m(x,\xi)}$
- $H^{m(x,\xi)} = \hat{A}_m^{-1}(L^2)$
- $\hat{A}_m \in \Psi_\rho^{m(x,\xi)}$
- Study $\hat{A}\hat{F}\hat{A}^{-1}$ on L^2

Quasicompacity of $\hat{F}$

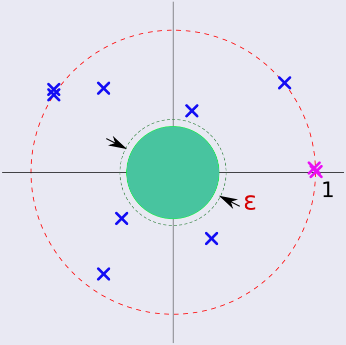
Theorem (Open Math. J. 2008)

$\forall \varepsilon > 0$, $\exists\, m(x,\xi)$, $\exists\, \hat{A} \in \Psi_\rho^{m(x,\xi)}$,

$$\hat{Q} := \hat{A}\hat{F}\hat{A}^{-1} = \hat{r}_\varepsilon + \hat{k}_\varepsilon$$

- $\|\hat{r}_\varepsilon\|_{L^2} \leq \varepsilon$
- $\hat{k}_\varepsilon$ *compact*

$\hat{Q}$ *quasicompact on* L^2
$\rightsquigarrow$ $\hat{F}$ *quasicompact on* $H^{m(x,\xi)}$
$\rightsquigarrow$ *Jordan dec.* $\rightsquigarrow$ *Resonances*



$$C_{\varphi,\psi}(n) = \int \varphi.\hat{F}^n(\psi)\,dx = \sum_{|\lambda_j|>\varepsilon} \lambda_j^n P_j(n;\varphi,\psi) + O(\varepsilon^n)$$

Quasicompacity of $\hat{F}$: idea of proof

- Wanted : $\hat{A} \in \Psi_\rho^{m(x,\xi)}$ with symbol $A(x,\xi) := \langle \xi \rangle^{m(x,\xi)}$
- Work with $\hat{P} := \hat{F}^{-1}\hat{Q} = \left(\hat{F}^{-1}\hat{A}\hat{F}\right)\hat{A}^{-1}$
- Egorov Theorem : $\hat{F}^{-1}\hat{A}\hat{F} \in \Psi_\rho^{m\circ F(x,\xi)}$ with symbol $A \circ F$ where $F : T^*M \to T^*M$ lifts $f^{-1} : M \to M$

$$F(x,\xi) = \left(f^{-1}(x), Df^t(\xi)\right)$$

- The inverse $\hat{A}^{-1} \in \Psi_\rho^{-m(x,\xi)}$ has symbol $\frac{1}{A}$
- Thus : $\hat{P} \in \Psi_\rho^{m\circ F(x,\xi)-m}$ has symbol

$$P = \frac{A \circ F}{A}$$

Quasicompacity of $\hat{F}$: idea of proof

- $\hat{P} \in \Psi_{\rho}^{m \circ F(x,\xi) - m}$ has symbol $P = \dfrac{A \circ F}{A}$
- if $m \circ F(x,\xi) - m \leq 0 \Longrightarrow \hat{P}$ bounded in L^2 and

$$\hat{P} = \hat{R} + \hat{K}$$

with

$$\left\| \hat{R} \right\|_{L^2} \leq \limsup_{(x,\xi) \in T^* M} |P(x,\xi)| + \delta$$

and $\hat{K}$ smoothing

- Need to construct an **escape function** (for F)

$$A \circ F < A$$

Quasicompacity of $\hat{F}$: idea of proof

Escape function : $A \circ F < A$