Exclusive diffractive processes including saturation effects at next-to-leading order

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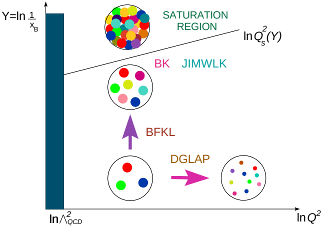
based on works with:
R. Boussarie, A. V. Grabovsky, D. Yu. Ivanov, S. Wallon

References

- *Impact factor for high-energy two and three jets diffractive production*,
  R. Boussarie, A. V. Grabovsky, L. Szymanowski, *S. Wallon.*,
  JHEP **1409** (2014) 026 [arXiv:1405.7676 [hep-ph]]
- *On the one loop $\gamma^{(*)} \to q\bar{q}$ impact factor and the exclusive diffractive cross sections for the production of two or three jets*,
  R. Boussarie, A. V. Grabovsky, L. Szymanowski, *S. Wallon.*,
  JHEP **1611** (2016) 149 [arXiv:1606.00419 [hep-ph]]
- *Next-to-Leading Order Computation of Exclusive Diffractive Light Vector Meson Production in a Saturation Framework*,
  R. Boussarie, A. V. Grabovsky, D. Yu. Ivanov, L. Szymanowski, *S. Wallon.*,
  Phys. Rev. Lett. **119** (2017) 072002 [arXiv:1612.08026 [hep-ph]]
- *Towards a complete next-to-logarithmic description of forward exclusive diffractive dijet electroproduction at HERA: real corrections*,
  R. Boussarie, A. V. Grabovsky, L. Szymanowski, *S. Wallon.*,
  Phys. Rev. D **100** (2019) no.7, 074020 [arXiv:1905.07371 [hep-ph]]

Example: DIS

The various regimes governing the perturbative content of the proton



- "usual" regime: x_B moderate ($x_B \gtrsim .01$):
  Evolution in Q governed by the QCD renormalization group
  (Dokshitser, Gribov, Lipatov, Altarelli, Parisi equation)

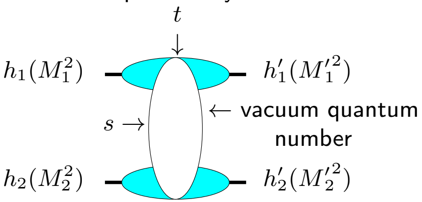
$$\underbrace{\sum_n (\alpha_s \ln Q^2)^n}_{\text{LLQ}} + \underbrace{\alpha_s \sum_n (\alpha_s \ln Q^2)^n}_{\text{NLLQ}} + \cdots$$

- perturbative Regge limit: $s_{\gamma^* p} \to \infty$ i.e. $x_B \sim Q^2/s_{\gamma^* p} \to 0$
  in the perturbative regime (hard scale Q^2)
  (Balitski Fadin Kuraev Lipatov equation)

$$\underbrace{\sum_n (\alpha_s \ln s)^n}_{\text{LLs}} + \underbrace{\alpha_s \sum_n (\alpha_s \ln s)^n}_{\text{NLLs}} + \cdots$$

QCD in the perturbative Regge limit

- One of the important longstanding theoretical questions raised by QCD is its behaviour in the perturbative Regge limit $s \gg -t$
- Based on theoretical grounds, one should identify and test suitable observables in order to test this peculiar dynamics



hard scales: $M_1^2,\, M_2^2 \gg \Lambda^2_{QCD}$ or $M_1'^2,\, M_2'^2 \gg \Lambda^2_{QCD}$ or $t \gg \Lambda^2_{QCD}$
where the $t-$channel exchanged state is the so-called hard Pomeron

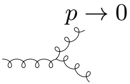
- Inclusive processes: the above picture applies at the level of cross-sections (optical theorem $\Rightarrow t = 0$)
- Diffractive processes: gap in rapidity between two clusters in the detector. The above picture applies at the level of amplitudes

How to test QCD in the perturbative Regge limit?

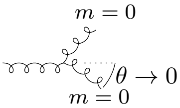
What kind of observable?

- perturbation theory should be applicable:
  selecting external or internal probes with transverse sizes $\ll 1/\Lambda_{QCD}$ (*hard* γ^*, *heavy* meson (J/Ψ, Υ), *energetic* forward jets) or by choosing large t in order to provide the hard scale.

- governed by the *"soft"* perturbative dynamics of QCD

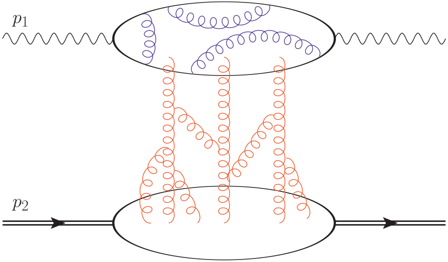


and *not* by its *collinear* dynamics



$\Longrightarrow$ select semi-hard processes with $s \gg p_{T\,i}^2 \gg \Lambda_{QCD}^2$ where $p_{T\,i}^2$ are typical transverse scale, all of the same order.

Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2p^+} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone Sudakov vectors

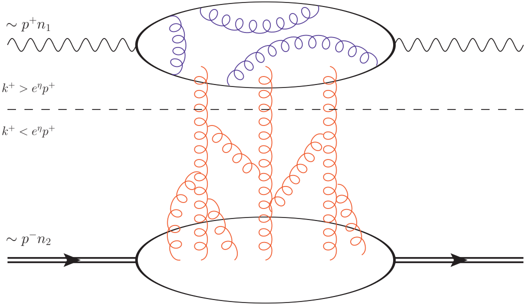
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \to (x^+, x^-, \vec{x}\,)$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation



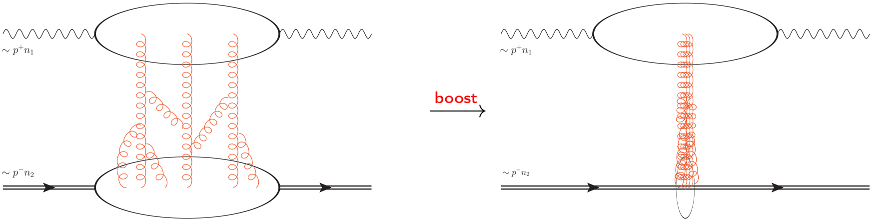
Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}\,) = A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}\,) \quad \text{quantum part}$$
$$+ \; b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k}\,) \quad \text{classical part}$$

$e^\eta \ll 1$

Large longitudinal boost to the projectile frame

Large longitudinal boost: $\Lambda \propto \sqrt{s}$



$$b^\mu(x) \quad \longrightarrow \quad b^-(x)\, n_2^\mu \simeq \delta(x^+)\, \mathbf{B}(\vec{x}\,)\, n_2^\mu$$

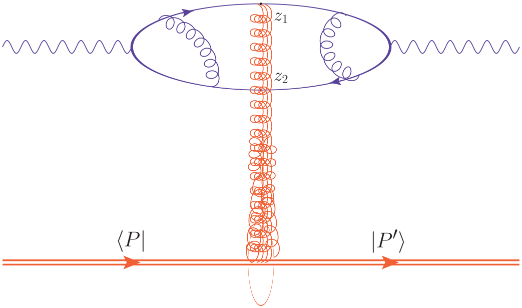
Shockwave approximation

Multiple interactions with the target can be resummed into path-ordered Wilson lines attached to each parton crossing lightcone time 0:

$$\tilde{U}^\eta(\vec{p}) = \int d^{D-2}\vec{z}\; e^{-i(\vec{p}\cdot\vec{z})} U^\eta_{\vec{z}}, \quad U_i^\eta = U^\eta_{\vec{z}_i} = P e^{ig\int b_\eta^-(z_i^+,\vec{z}_i)\, dz_i^+}$$

Factorized picture in the projectile frame

Factorized amplitude



$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^\eta(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation

Evolution for the dipole operator

B-JIMWLK hierarchy of equations
[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta} + \mathcal{U}_{13}^{\eta}\,\mathcal{U}_{32}^{\eta} \right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta}\,\mathcal{U}_{32}^{\eta}}{\partial \eta} = \ldots$$

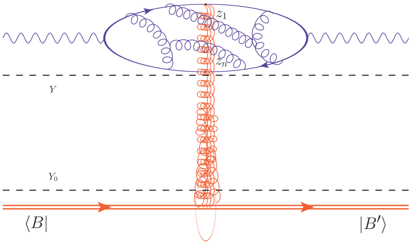
Mean field approximation (large N_C)
$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle \, \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

Non-linear term : saturation

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity Y_0.
- Evaluate the solution at a typical projectile rapidity Y, or at the rapidity of the slowest gluon
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

Exclusive diffraction allows one to probe the $b_\perp$-dependence of the non-perturbative scattering amplitude

Exclusive dijet diffractive production

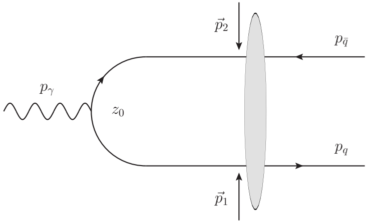
Framework

- Regge-Gribov limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- Otherwise completely general kinematics
- Shockwave (CGC) Wilson line approach
- Transverse dimensional regularization $d = 2 + 2\varepsilon$, longitudinal cutoff

$$|p_g^+| > \alpha p_\gamma^+$$

Exclusive dijet diffractive production

LO diagram



$$\mathcal{A} = \frac{\delta^{ik}}{\sqrt{N_c}} \int d^D z_0 [\bar{u}(p_q, z_0)]_{ij} (-ie_q) \hat{\varepsilon}_\gamma e^{-i(p_\gamma \cdot z_0)} [v(p_{\bar{q}}, z_0)]_{jk} \theta(-z_0^+)$$

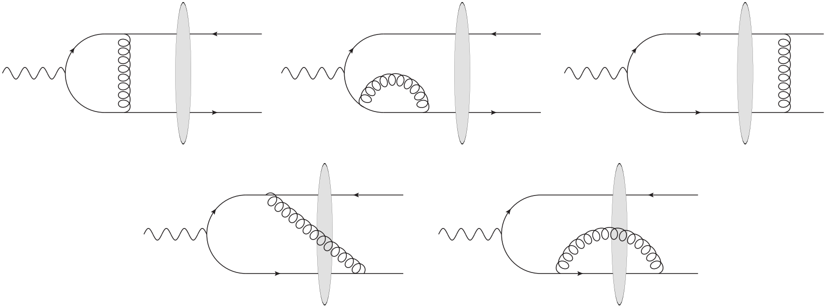
$$= \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d \vec{p}_1 d^d \vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_0(\vec{p}_1, \vec{p}_2\,)$$

$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d \vec{z}_1 d^d \vec{z}_2\, e^{-i(\vec{p}_1 \cdot \vec{z}_1) - i(\vec{p}_2 \cdot \vec{z}_2)} [\tfrac{1}{N_c} \mathrm{Tr}(U^\alpha_{\vec{z}_1} U^{\alpha\dagger}_{\vec{z}_2}) - 1]$$

Exclusive dijet diffractive production

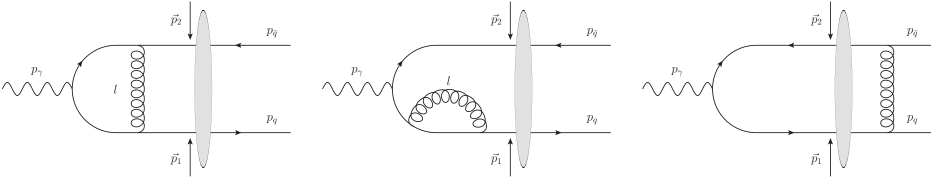
NLO open $q\bar{q}$ production



Diagrams contributing to the NLO correction

Exclusive dijet diffractive production

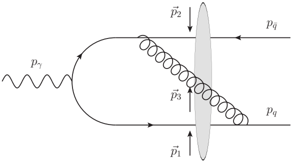
First kind of virtual corrections

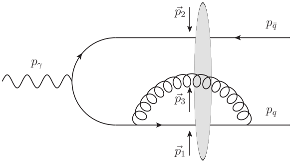


$$\mathcal{A}_{NLO}^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_{V1}(\vec{p}_1, \vec{p}_2) \times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

Exclusive dijet diffractive production

Second kind of virtual corrections





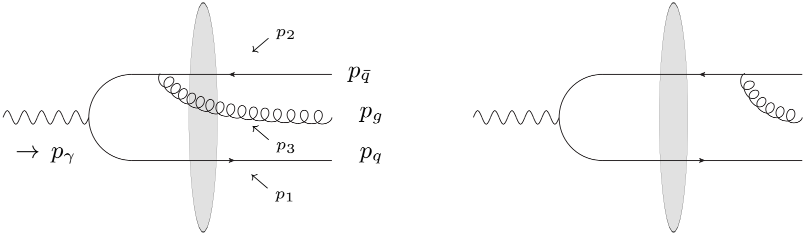
$$\mathcal{A}_{NLO}^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

$$\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$ dipole contribution

$$+ \Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \middle| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \middle| P \right\rangle]$$ double dipole contribution

Exclusive dijet diffractive production

LO open $q\bar{q}g$ production



$$\mathcal{A}_R^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{R1}(\vec{p}_1, \vec{p}_2)\, C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$
$$+ \Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \middle| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \middle| P \right\rangle]$$

$$\mathcal{A}_R^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$
$$\times \Phi_{R1}(\vec{p}_1, \vec{p}_2)\, C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

Exclusive dijet diffractive production

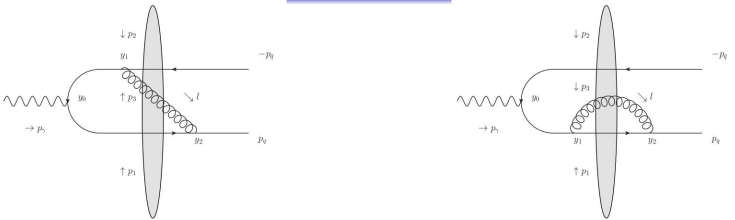
Various type of divergences

Divergences

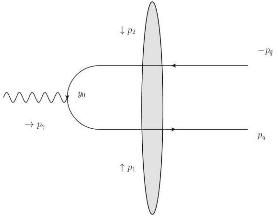
- Rapidity divergence $p_g^+ \to 0$ $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*,\ \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Exclusive dijet diffractive production

Rapidity divergence



Double dipole virtual correction Φ_{V2}



B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Exclusive dijet diffractive production
Rapidity divergence

B-JIMWLK equation for the dipole operator

$$\frac{\partial \tilde{\mathcal{U}}_{12}^{\alpha}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d\vec{k}_1 d^d\vec{k}_2 d^d\vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{\mathcal{U}}_{13}^{\alpha}\tilde{\mathcal{U}}_{32}^{\alpha} + \tilde{\mathcal{U}}_{13}^{\alpha} + \tilde{\mathcal{U}}_{32}^{\alpha} - \tilde{\mathcal{U}}_{12}^{\alpha} \right)$$
$$\times \left[2\frac{(\vec{k}_1 - \vec{p}_1)\cdot(\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}}\Gamma(1-\frac{d}{2})\Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{\left[(\vec{k}_1 - \vec{p}_1)^2\right]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{\left[(\vec{k}_2 - \vec{p}_2)^2\right]^{1-\frac{d}{2}}} \right) \right]$$

η rapidity divide, which separates the upper and the lower impact factors

$$\Phi_0 \tilde{\mathcal{U}}_{12}^{\alpha} \to \Phi_0 \tilde{\mathcal{U}}_{12}^{\eta} + 2\log\left(\frac{e^{\eta}}{\alpha}\right) \mathcal{K}_{BK} \Phi_0 \tilde{\mathcal{W}}_{123}$$

Provides a counterterm to the $\log(\alpha)$ divergence in the virtual double dipole impact factor:

$$\Phi_0 \tilde{\mathcal{U}}_{12}^{\alpha} + \Phi_{V2} \tilde{\mathcal{W}}_{123}^{\alpha} \text{ is finite and independent of } \alpha$$

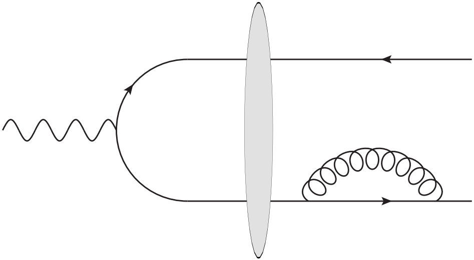
Exclusive dijet diffractive production

Various type of divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\qquad \Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\qquad \Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\qquad \Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+}p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+}p_{\bar{q}}$, $p_g^+ \to 0$ $\qquad \Phi_{R1}\Phi_{R1}^*$

Exclusive dijet diffractive production
UV divergence

Dressing of the external lines



Some null diagrams just contribute to turning UV divergences into IR divergences

$$\Phi = 0 \propto \left(\frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}}\right)$$

Exclusive dijet diffractive production

Various type of divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+}p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+}p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

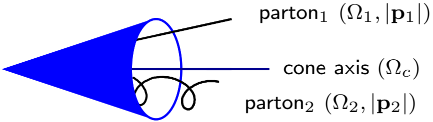
Exclusive dijet diffractive production

Soft and collinear divergence

Cone jet algorithm at NLO (Ellis, Kunszt, Soper)

- Should partons $(|\mathbf{p}_1|, \phi_1, y_1)$ and $(\mathbf{p}_2|, \phi_2, y_2)$ combined in a single jet?
  $|\mathbf{p}_i|$ =transverse energy deposit in the calorimeter cell i of parameter $\Omega = (y_i, \phi_i)$ in $y - \phi$ plane
- define transverse energy of the jet: $p_J = |\mathbf{p}_1| + |\mathbf{p}_2|$
- jet axis:

$$\Omega_c \left\{ \begin{array}{l} y_J = \dfrac{|\mathbf{p}_1|\, y_1 + |\mathbf{p}_2|\, y_2}{p_J} \\[2ex] \phi_J = \dfrac{|\mathbf{p}_1|\, \phi_1 + |\mathbf{p}_2|\, \phi_2}{p_J} \end{array} \right.$$



$\Omega = (y_i, \phi_i)$ in $y - \phi$ plane

If distances $|\Omega_i - \Omega_c|^2 \equiv (y_i - y_c)^2 + (\phi_i - \phi_c)^2 < R^2$ $(i = 1$ and $i = 2)$

$\Longrightarrow$ partons 1 and 2 are in the same cone Ω_c

Applying this (in the small R^2 limit) cancels our soft *and* collinear divergence

Exclusive dijet diffractive production

Various type of divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence

The remaining divergences cancel the standard way:
virtual corrections and real corrections cancel each other

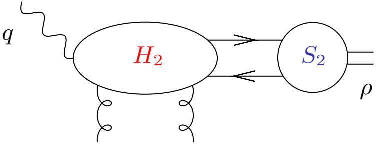
This is done after combining:

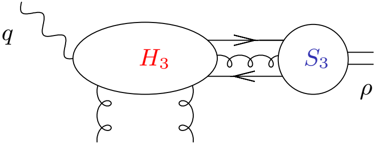
- the (LO + NLO) contribution to $q\bar{q}$ production
- the part of the contribution of the $q\bar{q}g$ production where the gluon is either soft or collinear to the quark or to the antiquark, so that they both form a single jet

Light meson production

Collinear factorization: basic principle

The impact factor is the convolution of a hard part and the vacuum-to-meson matrix element of an operator





$$\int_x (H_2(x))_{ij}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^\alpha(x)\psi_j^\beta(0) \right| 0 \right\rangle \qquad \int_{x_1,x_2} (H_3^\mu(x_1,x_2))_{ij,c}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^\alpha(x_1) A_\mu^c(x_2) \psi_j^\beta(0) \right| 0 \right\rangle$$

H and S are connected by:

- convolution
- summation over spinor and color indices

Once factorization in the t channel is done, now factorize in the s channel with collinear factorization: expand the impact factor in powers of the hard scale

Light meson production
Twist 2

Collinear factorization at twist 2

- Leading twist DA for a longitudinally polarized light vector meson

$$\langle \rho \left| \bar{\psi}(z) \gamma^{\mu} \psi(0) \right| 0 \rangle \to p^{\mu} f_{\rho} \int_0^1 dx e^{ix(p\cdot z)} \varphi_1(x)$$

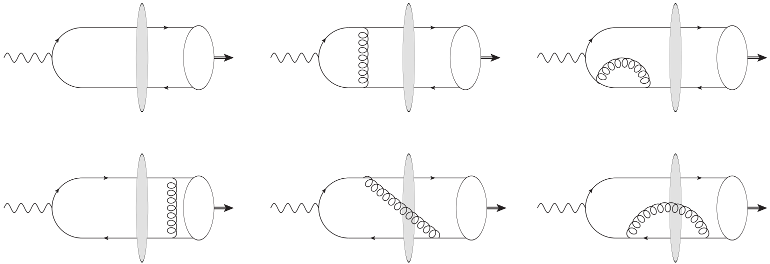
- Leading twist DA for a transversely polarized light vector meson

$$\langle \rho \left| \bar{\psi}(z) \sigma^{\mu\nu} \psi(0) \right| 0 \rangle \to i(p^{\mu} \varepsilon_{\rho}^{\nu} - p^{\nu} \varepsilon_{\rho}^{\mu}) f_{\rho}^{T} \int_0^1 dx e^{ix(p\cdot z)} \varphi_{\perp}(x)$$

The twist 2 DA for a transverse meson is chiral odd, thus $\gamma^* A \to \rho_T A$ starts at twist 3

Light meson production

Exclusive diffractive production of a light neutral vector meson



$$\mathcal{A} = -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_\parallel(x) \int \frac{d^d\vec{p}_1}{(2\pi)^d} \frac{d^d\vec{p}_2}{(2\pi)^d} \frac{d^d\vec{p}_3}{(2\pi)^d}$$
$$\times (2\pi)^{d+1} \delta\left(p_V^+ - p_\gamma^+\right) \delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3\right)$$
$$\times \left[\left(\Phi_0^\beta\left(x, \vec{p}_1, \vec{p}_2\right) + C_F \Phi_{V1}^\beta\left(x, \vec{p}_1, \vec{p}_2\right)\right) \tilde{\mathcal{U}}_{12}^\eta\, (2\pi)^d \delta(\vec{p}_3)\right.$$
$$\left. + \Phi_{V2}^\beta(x, \vec{p}_1, \vec{p}_2, \vec{p}_3)\, \tilde{\mathcal{W}}_{123}^\eta\right]$$

Probes gluon GPDs at low x, as well as twist 2 DAs

Divergences

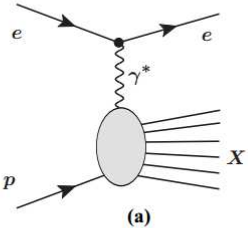
- Rapidity divergence $p_g^+ \to 0$ (spurious gauge pole in axial gauge)
  - Removed via JIMWLK evolution
- UV, soft divergence, collinear divergence
  - Mostly cancel each other, but requires renormalization of the operator in the vacuum-to-meson matrix element $\to$ ERBL evolution equation for the DA

We thus built a finite NLO exclusive diffractive amplitude with saturation effects

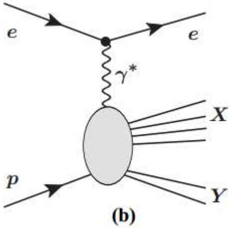
Phenomenology
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA: about 10% of scattering events reveal a rapidity gap



DIS events



DDIS events

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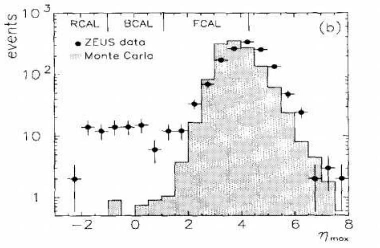
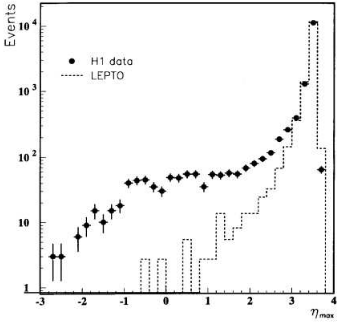


ZEUS, 1993



H1,1994

Phenomenology
Diffractive DIS

Theoretical approaches for DDIS using pQCD

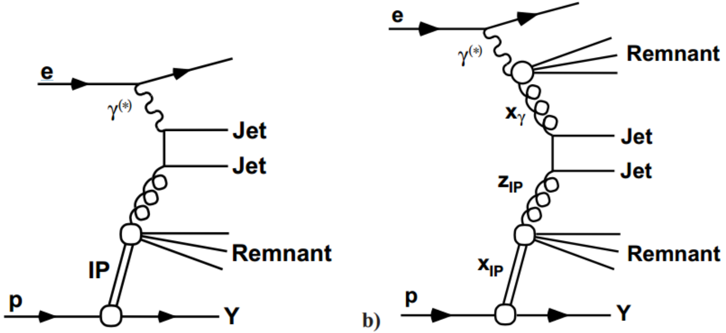
- Collinear factorization approach
  - Relies on QCD factorization theorem, using a hard scale such as the virtuality Q^2 of the incoming photon
  - One needs to introduce a diffractive distribution function for partons *within a pomeron*
- k_T factorization approach for two exchanged gluons
  - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a two-gluon color-singlet state

Phenomenology

Theoretical approaches for DDIS using pQCD

Collinear factorization approach

a)



b)

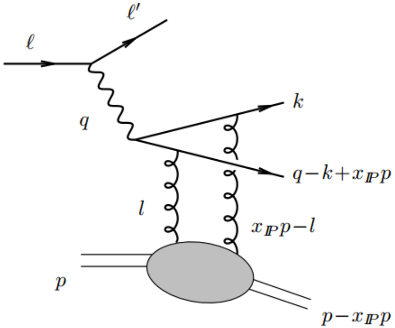
Direct

Resolved

Phenomenology

Theoretical approaches for DDIS using pQCD

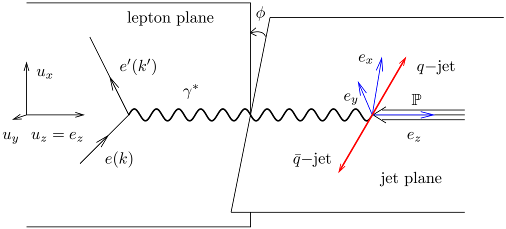
k_T-factorization approach : two gluon exchange



Bartels, Ivanov, Jung, Lotter, Wüsthoff
Braun and Ivanov developed a similar model in collinear factorization

Phenomenology
Azimuthal distribution of the jets

- a ZEUS diffractive exclusive dijet measurements was performed
- the azimuthal distribution of the two jets was obtained



Phenomenology

Theoretical approaches for DDIS using pQCD

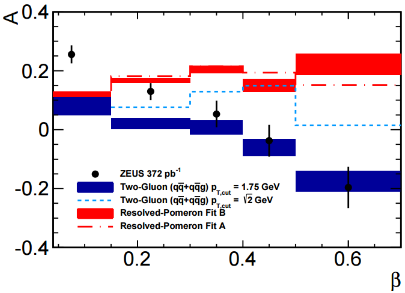
Confrontation of the two approaches with HERA data

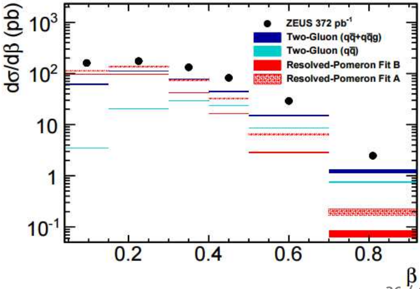
$$\frac{d\sigma_{ep}}{d\beta d\phi} = \frac{1}{\pi}\frac{d\sigma}{d\beta}[1 + A\cos 2\phi] \qquad \phi \in [0, \pi]$$

Bjorken variable normalized to the pomeron momentum: $\beta = \frac{Q^2}{Q^2+M^2_{\mathrm{dijet}}-t} \sim \frac{Q^2}{Q^2+M^2_{\mathrm{dijet}}}$

Collinear factorization approach: $A > 0$

k_T-factorization approach: $A < 0$



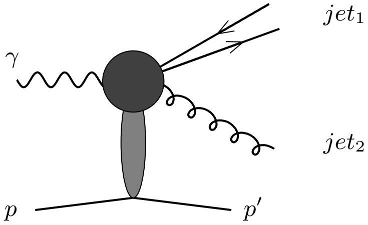


Phenomenology

Towards a NLO CGC approach

large $M^2_{\rm dijet}$:

the dominant contribution comes from the $q\bar{q}$ jet + g jet configuration (dominance of the exchange of a $t-$channel gluon with large $s = M^2_{\rm dijet}$



Phenomenology
Towards a NLO CGC approach

Exclusive k_t jet algorithm for three partons

- Distance between two particles:

$$d_{ij} = 2\min(E_i^2, E_j^2)\frac{1-\cos\theta_{ij}}{M^2} = \min\left(\frac{E_i}{E_j}, \frac{E_j}{E_i}\right)\frac{2p_i \cdot p_j}{M^2}$$

$E_{i,j}$, θ_{ij}: particle's energies and relative angle between them in c.m.f.

- Two particles belong to one jet if $d_{ij} < y_{cut}$
- y_{cut} regularizes both soft and collinear singularities
- ZEUS: $y_{cut} = 0.15 \Rightarrow$ we will rely on a small y_{cut} approximation

Phenomenology

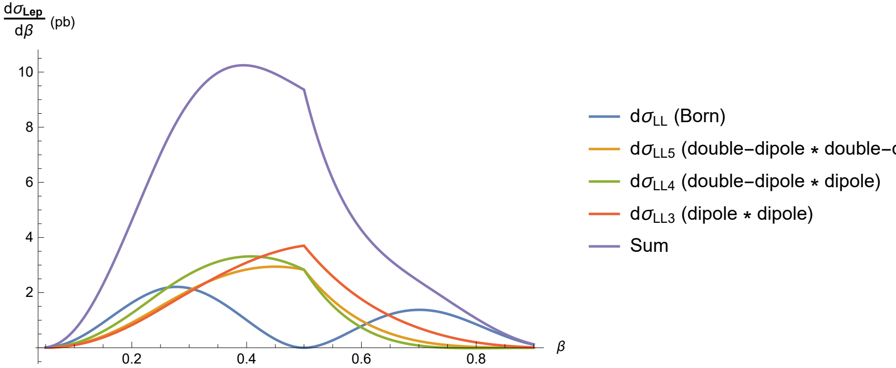
Towards a NLO CGC approach

- ZEUS cuts:
$$5 \text{ GeV} < Q$$
$$5 \text{ GeV} < M_{2jets} < 25 \text{ GeV}$$
$$2 \text{ GeV} < p_{\perp \min}$$
- at Born level, this removes aligned jets configurations (i.e. with a very small longitudinal momentum fraction x)

  $\Rightarrow$ suppression of the leading twist contribution which normally dominates in the Golec-Biernat Wüsthoff saturation model
- the typical hard scale in the impact factor is $\gtrsim p_{\perp \min}^2 > Q_s^2$
- this justifies an expansion in powers of Q_s:
  ZEUS experiment is dominated by the linear BFKL regime
- we restrict ourselves to the dominant contributions:
  - Born cross section
  - real correction with dipole $\times$ dipole and double dipole $\times$ double dipole configurations

Phenomenology
Results

Cross-sections

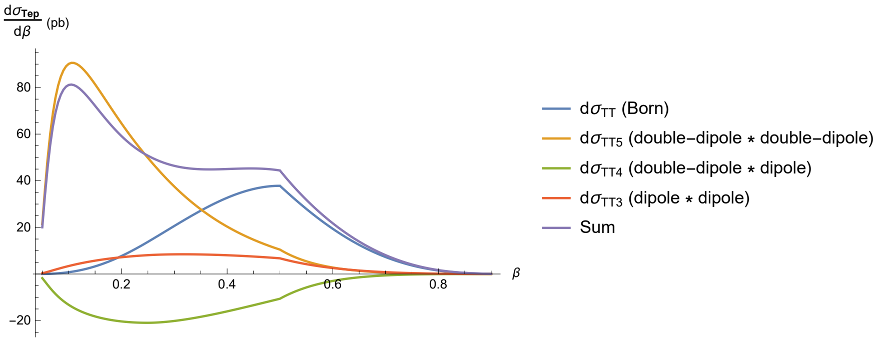


$ep \to ep + 2jets$ cross-section in the case of a longitudinal photon.

Born and gluon dipole contributions.

Phenomenology
Results

Cross-sections

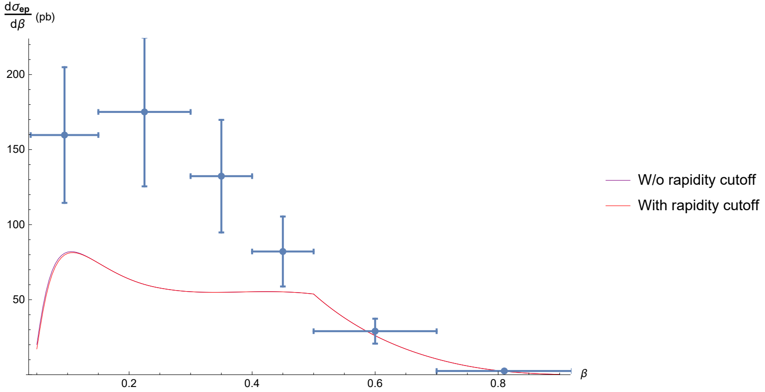


$ep \to ep + 2jets$ cross-section in the case of a transverse photon.

Born and gluon dipole contributions.

Phenomenology
Results

Cross-sections



Born and total gluon dipole contributions to cross section

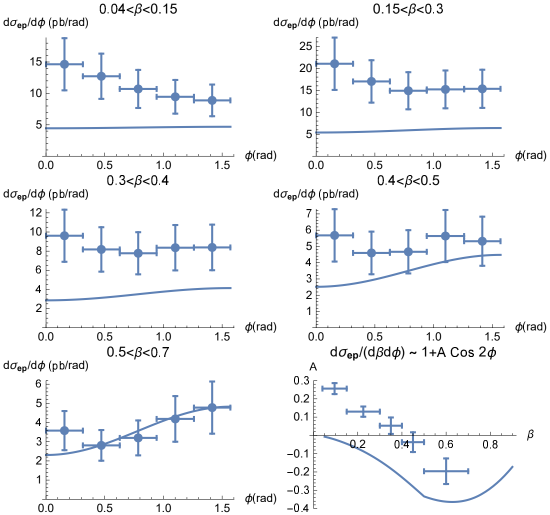
versus

ZEUS experimental data

- large β: good agreement with data
- small β: poor agreement with data, similar to the two gluon model of Bartels et al.

Phenomenology
Results

Azimuthal distribution



First 5 panels:
dependence of the cross-section on ϕ
for each experimental β bin

Good agreement at large β

Last panel:
β dependence of the coefficient A.

$\Rightarrow$ The experimental result for A at large β is puzzling

Summary

- using a small y limit, and for large β, there is a good agreement with a Golec-Biernat Wüsthoff model (in the small Q_s expansion) combined with our NLO impact factor
- within ZEUS kinematical cuts, the linear BFKL regime dominates
- our agreement is a good sign that perturbative Regge-like description are favored with respect to collinear type descriptions
- EIC should give a direct access to the saturated region
- a complete description of ZEUS data, in the whole β-range, requires to go beyond the small y approximation:
  next highly non-trivial step!!

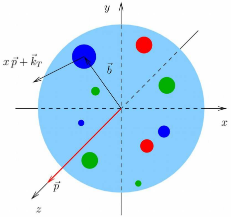
Conclusion

- We provided the full computation of the $\gamma^{(*)} \to Jet\, Jet$ and $\gamma^*_{L,T} \to \rho_L$ impact factors at NLO accuracy
- Our results are perfectly finite, even for photoproduction (at large t for ρ)
- The computation can be adapted for twist 3 $\gamma^{(*)} \to \rho_T$ NLO production in the Wandzura-Wilczek approximation, removing factorization breaking end-point singularities even at NLO for a process which would not factorize in a full collinear factorization scheme
- Exclusive diffractive processes are perfectly suited for precision saturation physics and gluon tomography with $b_\perp$ dependence at the EIC. Dijet production probes the dipole Wigner distribution, ρ meson production probes gluon GPDs at small x.
- At HERA, due to the kinematical cuts, one does not enter the saturation regime through exclusive diffractive dijet production.
- The large β region is well described, while the low β requires to include every NLO contribution.

The ultimate picture

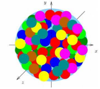
6D/5D

Wigner distributions for hadrons



$W(x, \vec{b}, k_T)$

Experimentally inaccessible directly



perturbative Regge limit

uPDFs (gluons)

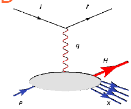
Unintegrated parton distributions

$\int d^3\vec{b}$

$\int d^2k_T \int d\,b_z$

$\int d^2k_T \int Fourier(\vec{b})$

3D



II

TMDs

$f(x, k_T)$

Transverse momentum dependent distributions

Semi-inclusive processes

$b_T \Leftrightarrow \Delta$

$f(x, b_T) \longleftrightarrow H(x, 0, t) \longleftarrow H(x, \xi, t)$

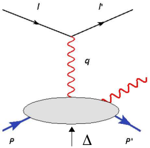
$t=-\Delta^2$

$\xi=0$

Impact parameter distributions

GPDs

generalised parton distributions



exclusive processes

$\int d^2k_T$

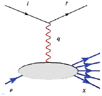
$\int d^2b_T$

$t=0$

$\int dx$

$\int dx\, x^{n-1}$

1D

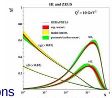


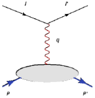
inclusive and semi-inclusive processes

PDFs

$f(x)$

parton distributions



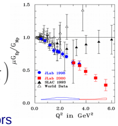


elastic processes

FFs

$G_{E,M}(t)$

form factors



GFFs

generalized form factors

lattices