Forward jet production in proton-nucleus collisions at high energy: from trijet to NLO dijet

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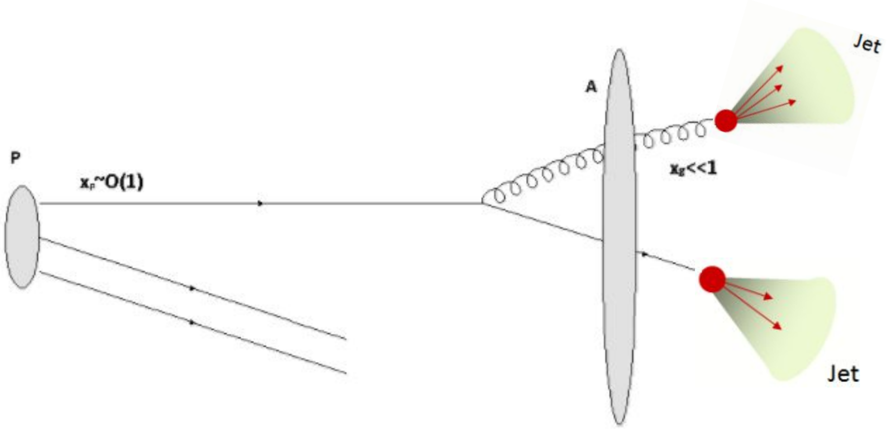
Forward Jet Production

In our approach we adopt the formalism of the light-cone wave function in perturbative QCD, together with the hybrid factorization. The derivation of the forward LO dijet cross-section has been done in hep-ph/0708.0231 (C. Marquet).

The basic setup: a large-x parton from the proton scatters off the small-x gluon distribution in the target nucleus. Large-x parton is most likely a quark.



Quark fragmentation in the presence of a shockwave.

The time evolution of the initial (bare) quark state is given by:

$$\left|q_\lambda^\alpha(q^+,\, \boldsymbol{q})\right\rangle_{\text{in}} \equiv U(0,\, -\infty)\, \left|q_\lambda^\alpha(q^+,\, \boldsymbol{q})\right\rangle$$

Where U denotes a unitary operator:

$$U(t, t_0) = \mathrm{T}\exp\left\{-i\int_{t_0}^{t} dt_1\, H_I(t_1)\right\}$$

The information both on the time evolution and interaction of the bare quark with the target nucleus is given by the “outgoing state”:

$$\left|q_\lambda^\alpha(q^+,\, \boldsymbol{w})\right\rangle_{out} \equiv U(\infty,\, 0)\, \hat{S}\, U(0,\, -\infty)\, \left|q_\lambda^\alpha(q^+,\, \boldsymbol{w})\right\rangle$$

This state will be shown to generate all the possible insertions of the shockwave. More importantly, the outgoing state is directly related to expectation values:

$$\left\langle \hat{O} \right\rangle = \left\langle \langle q|\, U^\dagger\, \hat{S}\, U\, \hat{O}\, U^\dagger\, \hat{S}\, U\, |q\rangle \right\rangle_{cgc}$$

The LO Outgoing State

The production state at leading order is given by

$$\left|q^{\alpha}_{\lambda}(q^+,\, \boldsymbol{w})\right\rangle^{(g)}_{out} \equiv U(\infty,0)\,\hat{S}\,U(0,-\infty)\left|q^{\alpha}_{\lambda}(q^+,\, \boldsymbol{w})\right\rangle = \left|\psi^{\alpha}_{\lambda}(q^+,\, \boldsymbol{w})\right\rangle_{qg} + \left|q^{\alpha}_{\lambda}(q^+,\, \boldsymbol{w})\right\rangle$$

$$\left|\psi^{\alpha}_{\lambda}\right\rangle_{qg} = \left|q^{\gamma}g^{b}\right\rangle\left(-\left\langle q^{\gamma}g^{b}\right|\hat{S}\left|q^{\beta}g^{a}\right\rangle\frac{\left\langle q^{\beta}g^{a}\right|H_{q\to qg}\left|q^{\alpha}\right\rangle}{E_{qg}-E_{q}} + \frac{\left\langle q^{\gamma}g^{b}\right|H_{q\to qg}\left|q^{\beta}\right\rangle}{E_{qg}-E_{q}}\left\langle q^{\beta}\right|\hat{S}\left|q^{\alpha}\right\rangle\right)$$

Where only terms of order g were kept. The following result is obtained for the |qg> contribution:

$$\left|\psi^{\alpha}_{\lambda}(q^+,\, \boldsymbol{w})\right\rangle_{qg} = \int_{\boldsymbol{x},\,\boldsymbol{z}}\int_0^1 d\vartheta\, \frac{ig\phi^{ij}_{\lambda_1\lambda}(\vartheta)\sqrt{q^+}\,\boldsymbol{X}^j}{4\pi^{3/2}\sqrt{\vartheta}\,\boldsymbol{X}^2}\,\delta^{(2)}(\boldsymbol{w}-(1-\vartheta)\boldsymbol{x}-\vartheta\boldsymbol{z})$$
$$\times\left[V^{\gamma\beta}(\boldsymbol{x})\,U^{ba}(\boldsymbol{z})\,t^{a}_{\beta\alpha} - t^{b}_{\gamma\beta}\,V^{\beta\alpha}(\boldsymbol{w})\right]\left|q^{\gamma}_{\lambda_1}((1-\vartheta)q^+,\,\boldsymbol{x})\,g^{b}_{i}(\vartheta q^+,\,\boldsymbol{z})\right\rangle$$

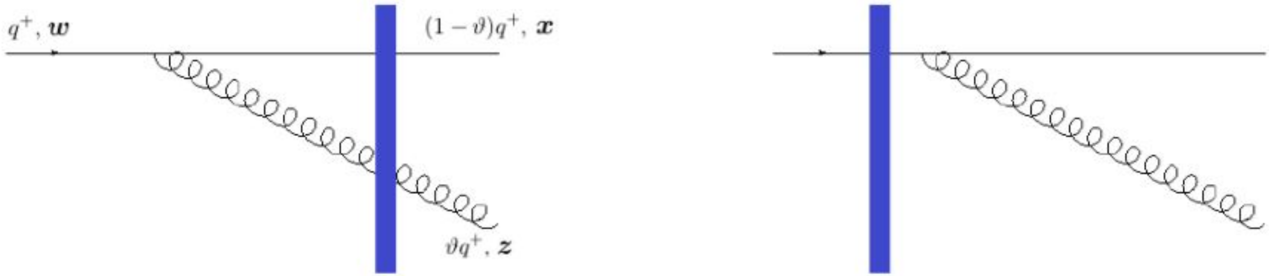
$$\boldsymbol{X} \equiv \boldsymbol{x}-\boldsymbol{z} \qquad \vartheta \equiv k^+/q^+$$

$$\phi^{ij}_{\lambda_1\lambda}(\vartheta) \equiv \chi^{\dagger}_{\lambda_1}\left[(2-\vartheta)\delta^{ij} - i\vartheta\varepsilon^{ij}\sigma^3\right]\chi_{\lambda}$$

$$U(\boldsymbol{x}) = \mathrm{T}\exp\left\{ig\int dx^+\,T^a A^-_a(x^+,\,\boldsymbol{x})\right\}$$

$$V(\boldsymbol{x}) = \mathrm{T}\exp\left\{ig\int dx^+\,t^a A^-_a(x^+,\,\boldsymbol{x})\right\}$$

Diagrammatically (blue bar denotes a shockwave = interaction with the target):



One gluon production at leading order with shockwave before and after the emission.

The LO forward dijet cross-section

From the production state we can pass easily to the quark-gluon dijet cross section:

$$\frac{d\sigma_{\rm LO}^{qA\to qg+X}}{d^3k\, d^3p} \equiv \frac{1}{2N_c\, L}\, {}^{(g)}_{out}\langle q_\lambda^\alpha(q^+,\, \boldsymbol{q})|\, \hat{\mathcal{N}}_q(p)\, \hat{\mathcal{N}}_g(k)\, |q_\lambda^\alpha(q^+,\, \boldsymbol{q})\rangle_{out}^{(g)}$$

$$= \frac{1}{2N_c\, L} \int_{\boldsymbol{w},\, \overline{\boldsymbol{w}}} e^{i(\boldsymbol{w}-\overline{\boldsymbol{w}})\cdot \boldsymbol{q}}\, {}_{qg}\langle \psi_\lambda^\alpha(q^+,\, \overline{\boldsymbol{w}})|\, \hat{\mathcal{N}}_q(p)\, \hat{\mathcal{N}}_g(k)\, |\psi_\lambda^\alpha(q^+,\, \boldsymbol{w})\rangle_{qg}$$

The following number density operators were introduced:

$$\hat{\mathcal{N}}_q(p) \equiv \frac{1}{(2\pi)^3}\, b_\lambda^{\alpha\dagger}(p)\, b_\lambda^\alpha(p) \qquad\qquad \hat{\mathcal{N}}_g(k) \equiv \frac{1}{(2\pi)^3}\, a_i^{a\dagger}(k)\, a_i^a(k)$$

Then the result for the leading-order dijet cross section is given by:

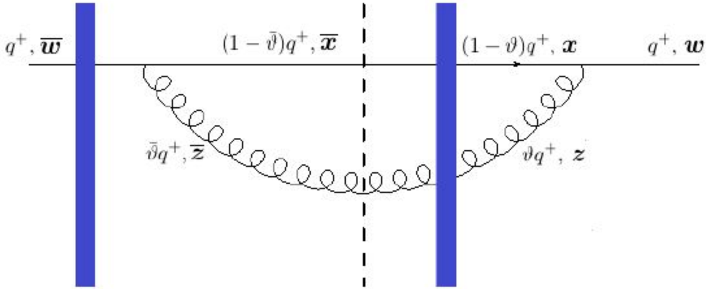
$$\frac{d\sigma_{\rm LO}^{qA\to qg+X}}{dk^+\, d^2\boldsymbol{k}\, dp^+\, d^2\boldsymbol{p}} = \frac{2\alpha_s C_F\left(1+(1-\vartheta)^2\right)}{(2\pi)^6 \vartheta q^+}\, \delta(q^+ - k^+ - p^+)$$

$$\times \int_{\boldsymbol{x},\, \overline{\boldsymbol{x}},\, \boldsymbol{z},\, \overline{\boldsymbol{z}}} \frac{\boldsymbol{X}\cdot\overline{\boldsymbol{X}}}{\boldsymbol{X}^2\, \overline{\boldsymbol{X}}^2}\, \mathrm{e}^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}\cdot(\boldsymbol{z}-\bar{\boldsymbol{z}})}\, \mathbb{S}_{\rm LO}\left(\overline{\boldsymbol{w}},\, \overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \boldsymbol{w},\, \boldsymbol{x},\, \boldsymbol{z}\right)$$

with $\boldsymbol{X} \equiv \boldsymbol{x} - \boldsymbol{z}$, $\overline{\boldsymbol{X}} \equiv \overline{\boldsymbol{x}} - \overline{\boldsymbol{z}}$, $\boldsymbol{w} = (1-\vartheta)\boldsymbol{x} + \vartheta\boldsymbol{z}$ and $\overline{\boldsymbol{w}} = (1-\vartheta)\overline{\boldsymbol{x}} + \vartheta\overline{\boldsymbol{z}}$.

$$\mathbb{S}_{\rm LO}\left(\overline{\boldsymbol{w}},\, \overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \boldsymbol{w},\, \boldsymbol{x},\, \boldsymbol{z}\right) \equiv S_{qgqg}\left(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \boldsymbol{x},\, \boldsymbol{z}\right) - S_{qgq}\left(\overline{\boldsymbol{w}},\, \boldsymbol{x},\, \boldsymbol{z}\right) - S_{qgq}\left(\overline{\boldsymbol{x}},\, \boldsymbol{w},\, \overline{\boldsymbol{z}}\right) + \mathcal{S}\left(\overline{\boldsymbol{w}},\, \boldsymbol{w}\right)$$

There are four different insertions of Wilson lines. For example, below is the relevant diagram which corresponds to $S_{q\bar{q}g}\left(\overline{\boldsymbol{w}},\, \boldsymbol{x},\, \boldsymbol{z}\right)$ (the location of the measurement is denoted by a dashed line).



Where the following combinations of Wilson lines were introduced (in the large Nc limit these combinations represent the quadropole-dipole and dipole-dipole interactions):

$$S^{(1)}_{q\bar{q}gg}\left(\overline{x},\, \overline{z},\, x,\, z\right) \equiv \frac{1}{C_F N_c}\,\mathrm{tr}\left(V^\dagger(\overline{x})\, V(x)\, t^a\, t^c\right)\,\left[U^\dagger(\overline{z})\, U(z)\right]^{ca}$$
$$= \frac{1}{2C_F N_c}\,\left(N_c^2\, \mathcal{Q}(\overline{x},\, x,\, z,\, \overline{z})\, \mathcal{S}(\overline{z},\, z) - \mathcal{S}(\overline{x},\, x)\right) \simeq \mathcal{Q}(\overline{x},\, x,\, z,\, \overline{z})\, \mathcal{S}(\overline{z},\, z)$$

$$S_{q\bar{q}g}\left(\overline{w},\, x,\, z\right) \equiv \frac{1}{C_F N_c}\,\mathrm{tr}\left(V^\dagger(\overline{w})\, t^b\, V(x)\, t^a\right)\, U^{ba}(z)$$
$$= \frac{1}{2C_F N_c}\,\left(N_c^2\, \mathcal{S}(\overline{w},\, z)\, \mathcal{S}(z,\, x) - \mathcal{S}(\overline{w},\, x)\right) \simeq \mathcal{S}(\overline{w},\, z)\, \mathcal{S}(z,\, x)$$

The dipole and quadropole are defined by:

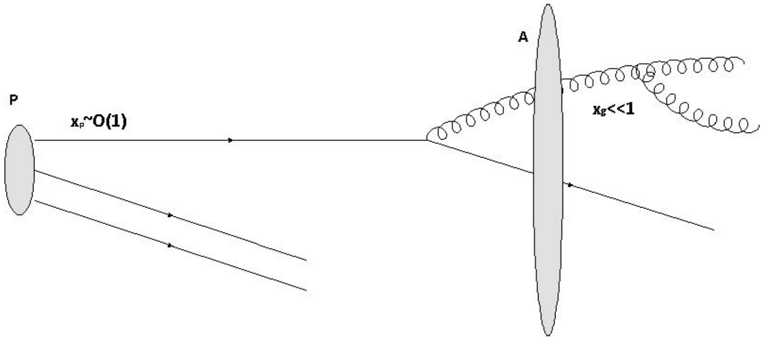
$$\mathcal{S}\left(\overline{\boldsymbol{w}},\, \boldsymbol{w}\right) \equiv \frac{1}{N_c}\,\mathrm{tr}\left[V^\dagger(\overline{\boldsymbol{w}})\, V(\boldsymbol{w})\right] \qquad \mathcal{Q}\left(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z},\, \overline{\boldsymbol{z}}\right) \equiv \frac{1}{N_c}\,\mathrm{tr}\left[V^\dagger(\overline{\boldsymbol{x}})\, V(\boldsymbol{x})\, V^\dagger(\boldsymbol{z})\, V(\overline{\boldsymbol{z}})\right]$$

The Trijet Setup

In the new setup, we have to produce three particles in the final state. There are two configurations of particles:

a) Quark, quark and anti-quark
b) Quark together with two gluons.

Due to the fact that we are using the light-cone gauge, the production of these configurations can happen both instantaneously (via one emission), or in the regular way, via two successive emissions or one emission followed by splitting process.



An example for a contribution with three particles in the final state

The Perturbative Outgoing State

The perturbative expression for the quark outgoing state is:

$$|out\rangle = |in\rangle + |out\rangle^{(1)} + |out\rangle^{(2)} + \cdots$$

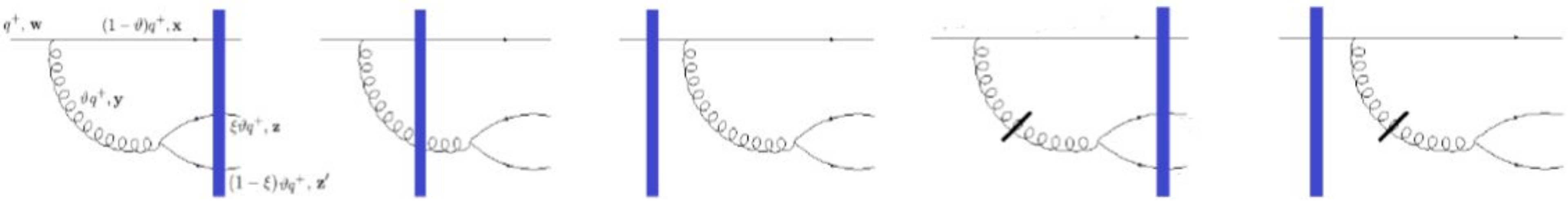
with:

$$|out\rangle^{(1)} = -\sum_{f,j} |f\rangle\langle f|S|j\rangle \frac{\langle j|H_{\text{int}}|in\rangle}{E_j - E_{in}} + \sum_{f,j} |f\rangle \frac{\langle f|H_{\text{int}}|j\rangle}{E_f - E_j} \langle j|S|in\rangle$$

$$\begin{aligned}|out\rangle^{(2)} =& \sum_{f,j,i} |f\rangle\langle f|S|j\rangle \frac{\langle j|H_{\text{int}}|i\rangle \, \langle i|H_{\text{int}}|in\rangle}{(E_j - E_{in})(E_i - E_{in})} + \sum_{f,j,i} |f\rangle \frac{\langle f|H_{\text{int}}|j\rangle \, \langle j|H_{\text{int}}|i\rangle}{(E_f - E_j)(E_f - E_i)} \langle i|S|in\rangle \\ & - \sum_{f,j,i} |f\rangle \frac{\langle f|H_{\text{int}}|j\rangle}{E_f - E_j} \langle j|S|i\rangle \frac{\langle i|H_{\text{int}}|in\rangle}{E_i - E_{in}}\end{aligned}$$

Where i, j and k runs over the relevant bare states, and H_{int} represent the interaction part of the QCD Hamiltonian.

The Quark Quark Anti-quark Outgoing State

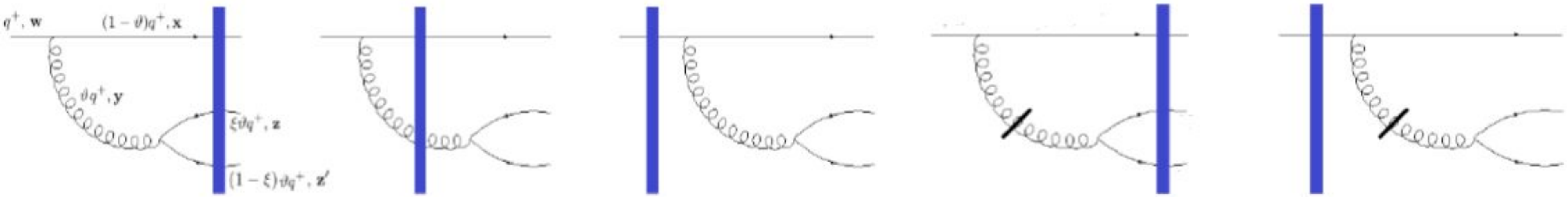


$$|\psi^\alpha\rangle^{inst}_{qq\bar{q}} \equiv |\bar{q}^\rho q^\varrho q^\sigma\rangle \left(\frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \mathsf{H}_{q\to qq\bar{q}} |q^\beta\rangle \langle q^\beta | \hat{S} | q^\alpha\rangle}{E_{qq\bar{q}} - E_q} - \frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \hat{S} |\bar{q}^\epsilon q^\delta q^\beta\rangle \langle \bar{q}^\epsilon q^\delta q^\beta | \mathsf{H}_{q\to qq\bar{q}} |q^\alpha\rangle}{E_{qq\bar{q}} - E_q} \right)$$

$$|\psi^\alpha\rangle^{reg}_{qq\bar{q}} \equiv |\bar{q}^\rho q^\varrho q^\sigma\rangle \left(\frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \hat{S} |\bar{q}^\delta q^\epsilon q^\kappa\rangle \langle \bar{q}^\delta q^\epsilon q^\kappa | \mathsf{H}_{g\to q\bar{q}} |q^\beta g^i\rangle \langle q^\beta g^i | \mathsf{H}_{q\to qg} |q^\alpha\rangle}{(E_{qq\bar{q}} - E_q)(E_{qg} - E_q)} \right.$$
$$\left. + \frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \mathsf{H}_{g\to q\bar{q}} |q^\gamma g^i\rangle \langle q^\gamma g^i | \mathsf{H}_{q\to qg} |q^\beta\rangle \langle q^\beta | \hat{S} |q^\alpha\rangle}{(E_{qq\bar{q}} - E_{qg})(E_{qq\bar{q}} - E_q)} - \frac{\langle \bar{q}^\rho q^\varrho q^\sigma | \mathsf{H}_{g\to q\bar{q}} |q^\gamma g^j\rangle \langle q^\gamma g^j | \hat{S} |q^\beta g^i\rangle \langle q^\beta g^i | \mathsf{H}_{q\to qg} |q^\alpha\rangle}{(E_{qq\bar{q}} - E_{qg})(E_{qg} - E_q)} \right)$$

Note that the result above vanishes under the limit S->1. This property of the results has to be expected since the new particles are produced by the shockwave.

The Quark Quark Anti-quark Outgoing State



After insertion of the matrix elements:

$$
\begin{aligned}
\left|q^{\alpha}_{\lambda}(q^{+},\,\boldsymbol{w})\right\rangle_{qq\bar q} &= -\frac{g^2\,q^+}{(2\pi)^4}\int_{\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'}\int_0^1 d\vartheta\,d\xi \\
&\times\Big\{F^{\lambda_3\lambda_2\lambda_1}_{qq\bar q 1}(\vartheta,\,\xi,\,\mathbf{R},\,\mathbf{Z})\,\Big[V^{\varrho\delta}(\boldsymbol{z}')\,t^a_{\delta\epsilon}\,V^{\dagger\epsilon\rho}(\boldsymbol{z})\,V^{\sigma\beta}(\boldsymbol{x})\,t^a_{\beta\alpha}\,-\,t^a_{\varrho\rho}\,t^a_{\sigma\beta}\,V^{\beta\alpha}(\boldsymbol{w})\Big] \\
&-f^{\lambda_3\lambda_2\lambda_1}_{qq\bar q}(\vartheta,\,\xi,\,\mathbf{R},\,\mathbf{Z})\,\Big[t^b_{\varrho\rho}\,V^{\sigma\beta}(\boldsymbol{x})\,U^{ba}(\boldsymbol{y})\,t^a_{\beta\alpha}\,-\,t^a_{\varrho\rho}\,t^a_{\sigma\beta}\,V^{\beta\alpha}(\boldsymbol{w})\Big]\Big\} \\
&\times\delta^{(2)}\left(\boldsymbol{w}-\boldsymbol{C}\right)\left|q^{\sigma}_{\lambda_1}((1-\vartheta)q^+,\boldsymbol{x})\,q^{\varrho}_{\lambda_2}(\xi\vartheta q^+,\,\boldsymbol{z}')\,\bar q^{\rho}_{\lambda_3}((1-\xi)\vartheta q^+,\,\boldsymbol{z})\right\rangle,
\end{aligned}
$$

$$
F^{\lambda_3\lambda_2\lambda_1}_{qq\bar q}(\vartheta,\,\xi,\,\mathbf{R},\,\mathbf{Z})\equiv\frac{(1-\vartheta)\,\left(\varphi^{il}_{\lambda_2\lambda_3}(\xi)\,\phi^{ij}_{\lambda_1\lambda}(\vartheta)\,\boldsymbol{R}^j\,\boldsymbol{Z}^l\,+\,\xi(1-\xi)\delta_{\lambda_3\lambda_2}\delta_{\lambda_1\lambda}\,\boldsymbol{Z}^2\right)}{\left((1-\vartheta)\boldsymbol{R}^2\,+\,\xi(1-\xi)\boldsymbol{Z}^2\right)\,\boldsymbol{Z}^2}
$$

$$
f^{\lambda_3\lambda_2\lambda_1}_{qq\bar q}(\vartheta,\,\xi,\,\mathbf{R},\,\mathbf{Z})\,\equiv\,\frac{\varphi^{il}_{\lambda_2\lambda_3}(\xi)\,\phi^{ij}_{\lambda_1\lambda}(\vartheta)\,\boldsymbol{R}^j\,\boldsymbol{Z}^l}{\boldsymbol{R}^2\,\boldsymbol{Z}^2}
$$

$$
\varphi^{ij}_{\lambda_2\lambda_1}(\xi)\,\equiv\,\chi^{\dagger}_{\lambda_2}\left[(2\xi-1)\delta^{ij}+i\varepsilon^{ij}\sigma^3\right]\chi_{\lambda_1}
$$

$$
\mathbf{R}=\boldsymbol{x}-\boldsymbol{y}\qquad\qquad \mathbf{Z}=\boldsymbol{z}-\boldsymbol{z}'
$$

$$
\boldsymbol{y}\,\equiv\,\xi\boldsymbol{z}'+(1-\xi)\boldsymbol{z}
$$

$$
\boldsymbol{C}\,\equiv\,(1-\vartheta)\boldsymbol{x}\,+\,\xi\vartheta\boldsymbol{z}\,+\,(1-\xi)\vartheta\boldsymbol{z}'.
$$

From the outgoing state to the trijet cross section

The expression for the forward trijet cross section is composed by two contributions:

$$\frac{d\sigma^{pA\to 3jet+X}}{d^3q_1\, d^3q_2\, d^3q_3} = \int dx_p\, q(x_p,\mu^2)\left(\frac{d\sigma^{qA\to qgg+X}}{d^3q_1\, d^3q_2\, d^3q_3} + \frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\, d^3q_2\, d^3q_3}\right)$$

The two contributions to the two final partonic state:

$$\frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\, d^3q_2\, d^3q_3} \equiv \frac{1}{2N_c\, L}\,{}^{(g^2)}_{out}\!\left\langle q^{\alpha}_{\lambda}(q^+,\boldsymbol{q}=0_\perp)\right|\hat{\mathcal{N}}_q(q_1)\,\hat{\mathcal{N}}_q(q_2)\,\hat{\mathcal{N}}_{\bar{q}}(q_3)\left|q^{\alpha}_{\lambda}(q^+,\boldsymbol{q}=0_\perp)\right\rangle^{(g^2)}_{out}$$

$$\frac{d\sigma^{qA\to qgg+X}}{d^3q_1\, d^3q_2\, d^3q_3} \equiv \frac{1}{2N_c\, L}\,{}^{(g^2)}_{out}\!\left\langle q^{\alpha}_{\lambda}(q^+,\boldsymbol{q}=0_\perp)\right|\hat{\mathcal{N}}_q(q_1)\,\hat{\mathcal{N}}_g(q_2)\,\hat{\mathcal{N}}_g(q_3)\left|q^{\alpha}_{\lambda}(q^+,\boldsymbol{q}=0_\perp)\right\rangle^{(g^2)}_{out}$$

The results for the forward trijet cross section

The contribution to the quark quark anti-quark trijet cross section:

$$\frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\, d^3q_2\, d^3q_3} \equiv \frac{\alpha_s^2 C_F N_f}{2(2\pi)^{10}(q^+)^2}\delta(q^+ - q_1^+ - q_2^+ - q_3^+) \int_{\bar{x},\bar{z},\bar{z}',x,z,z'} e^{-iq_1\cdot(x-\bar{x})-iq_2\cdot(z-\bar{z})-iq_3\cdot(z'-\bar{z}')}$$
$$\times \left[K^1_{qq\bar{q}}(\vartheta,\xi,\overline{\mathbf{X}},\overline{\mathbf{Z}},\mathbf{X},\mathbf{Z})\,\mathbb{S}^1_{qq\bar{q}}(\bar{\mathbf{x}},\bar{\mathbf{z}},\bar{\mathbf{z}}',\mathbf{x},\mathbf{z},\mathbf{z}') + K^2_{qq\bar{q}}(\vartheta,\xi,\overline{\mathbf{X}},\overline{\mathbf{Z}},\mathbf{X},\mathbf{Z})\,\mathbb{S}^2_{qq\bar{q}}(\bar{\mathbf{x}},\bar{\mathbf{z}},\bar{\mathbf{z}}',\mathbf{x},\mathbf{y})\right.$$
$$\left. + h.c. + K^3_{qq\bar{q}}(\vartheta,\xi,\overline{\mathbf{X}},\overline{\mathbf{Z}},\mathbf{X},\mathbf{Z})\,\mathbb{S}_{\rm LO}(\bar{w},\bar{x},\bar{z},w,x,z)\right] + (q_1^+\leftrightarrow q_2^+,\, q_1\leftrightarrow q_2)$$

Where we introduced the following structures:

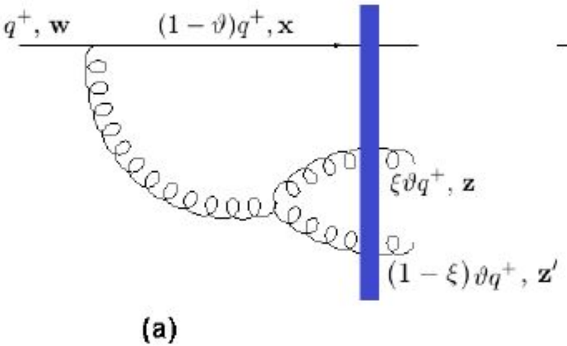
$$\mathbb{S}^1_{qq\bar{q}}(\bar{x},\bar{y},\bar{z},\bar{z}',x,y,z,z') \equiv S_{q\bar{q}qq\bar{q}q}(\bar{x},\bar{z},\bar{z}',x,z,z') - S_{q\bar{q}qqg}(\bar{x},\bar{z},\bar{z}',x,y) - S_{qgq\bar{q}q}(\bar{x},\bar{y},x,z,z') + S_{qgqg}(\bar{x},\bar{y},x,y)$$

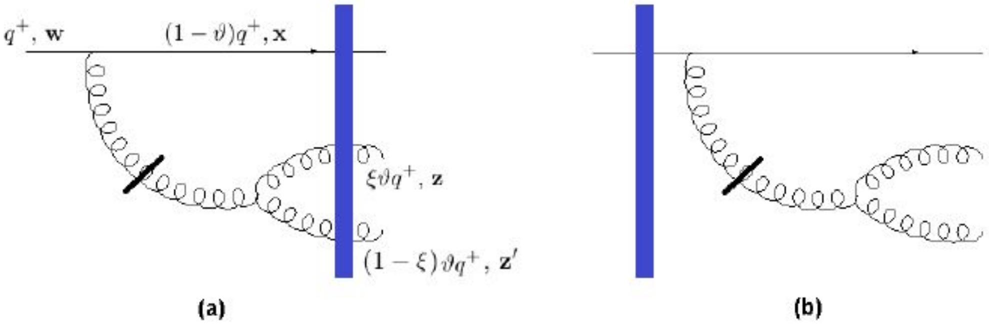
Each of these structures involves contraction of Wilson lines:

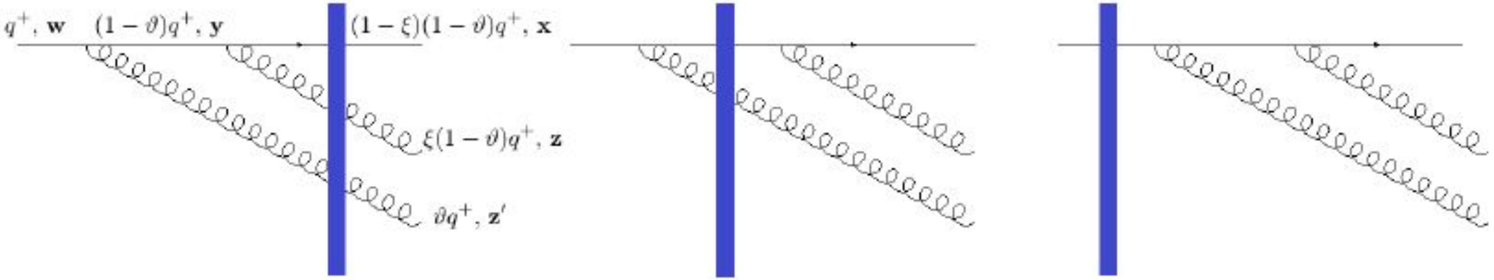
$$S_{q\bar{q}qq\bar{q}q}(\bar{x},\bar{z},\bar{z}',x,z,z') \equiv \frac{2}{C_F N_c}\,\mathrm{tr}\left(V^\dagger(\bar{x})V(x)t^a t^b\right)\mathrm{tr}\left(V(\bar{z}')t^b V^\dagger(\bar{z})V(z)t^a V^\dagger(z')\right)$$
$$= \frac{1}{2C_F N_c}\left(N_c^2\,\mathcal{Q}(\bar{x},x,z',\bar{z}')\mathcal{S}(\bar{z},z) - \mathcal{H}(\bar{x},x,z',\bar{z}',\bar{z},z) - \mathcal{H}(\bar{x},x,\bar{z},z,z',\bar{z}')\right.$$
$$\left. + \mathcal{S}(\bar{x},x)\,\mathcal{Q}(\bar{z},z,z',\bar{z}')\right) \simeq \mathcal{Q}(\bar{x},x,z',\bar{z}')\mathcal{S}(\bar{z},z)$$

At the large N_c limit only the dipole and quadrupole structures remain.

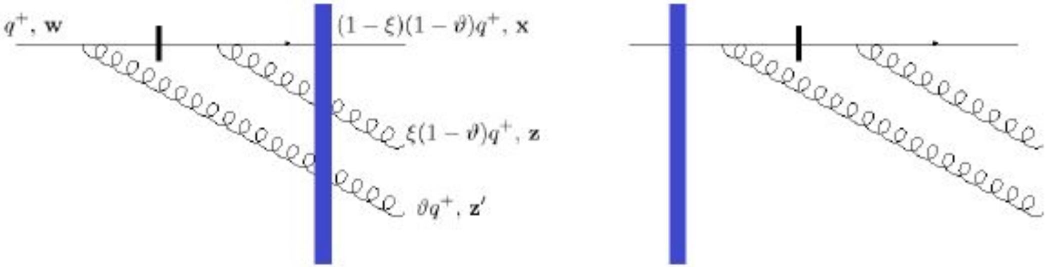
The Diagrams for the Quark and Two Gluons Outgoing States





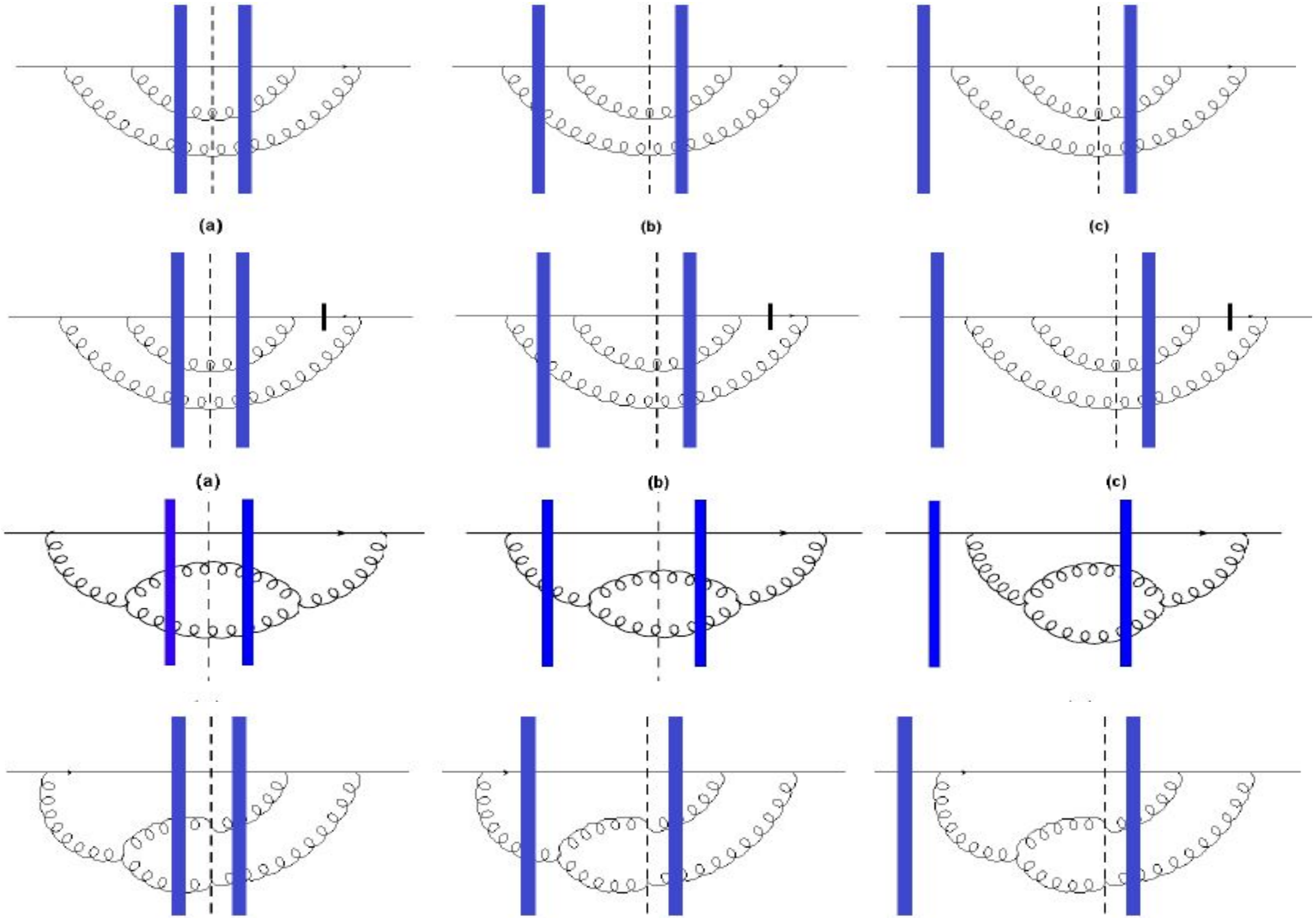

q^+, $\mathbf{w}$
$(1-\vartheta)q^+$, $\mathbf{y}$
$(1-\xi)(1-\vartheta)q^+$, $\mathbf{x}$
$\xi(1-\vartheta)q^+$, $\mathbf{z}$
ϑq^+, $\mathbf{z}'$

(a) (b) (c)


q^+, $\mathbf{w}$
$(1-\xi)(1-\vartheta)q^+$, $\mathbf{x}$
$\xi(1-\vartheta)q^+$, $\mathbf{z}$
ϑq^+, $\mathbf{z}'$

(a) (b)

Gluons Contributions to Trijet Production



The NLO Quark Wave Function

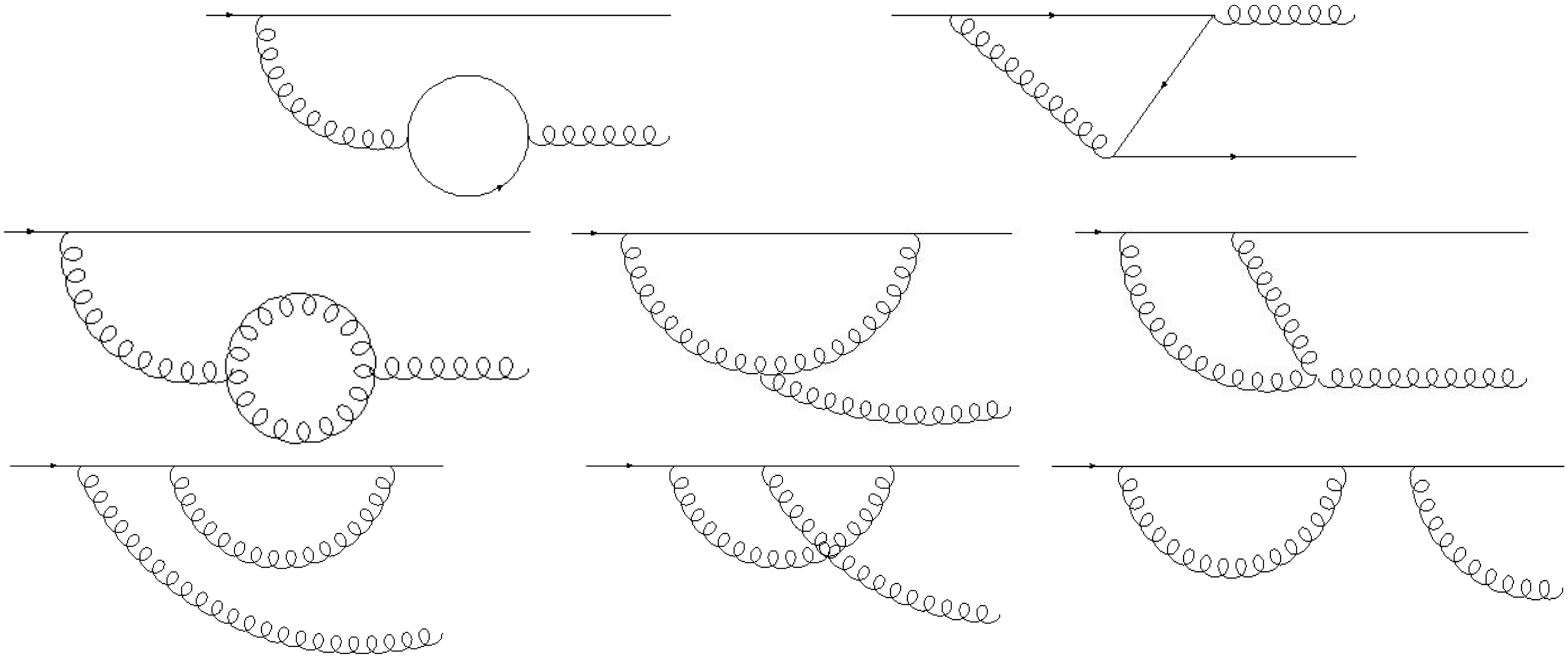
The missing part of the NLO quark outgoing state is the part which involves the production of a quark and a gluon through a loop (virtual) correction.

Each of these diagrams has a dependence on an IR longitudinal momentum cutoff. This dependence must not be a part of the final result for the cross section.

The NLO outgoing quark state has the following structure:

$$\left|q_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{NLO} = \hat{\mathcal{Z}}_{NLO}\left|q_\lambda^\alpha(q^+,\mathrm{w})\right\rangle + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{LO} + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{qg}$$
$$+ \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{qq\bar{q}} + \left|\Phi_\lambda^\alpha(q^+,\mathrm{w})\right\rangle_{qgg}.$$

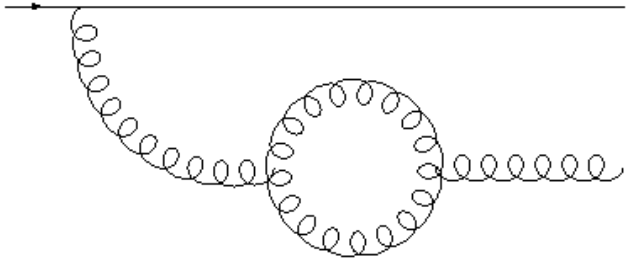
The Diagrams



First calculated also in hep-ph/1611.00497 (T. Lappi and R. Paatelainen)

The structure of virtual contributions

Example of results for the diagrams:



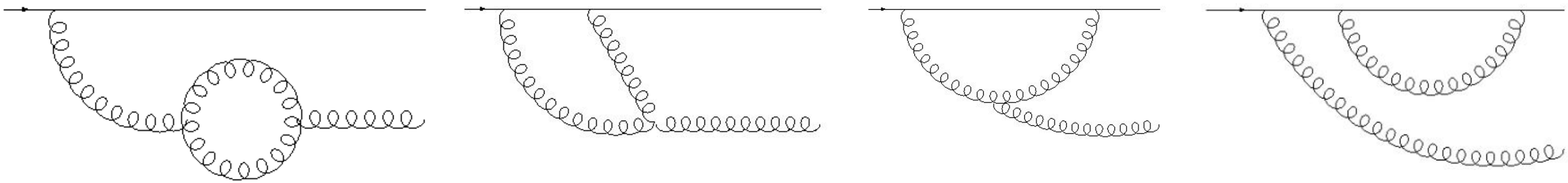
$$|\psi_\lambda^\alpha\rangle^2_{qg} = \int_0^1 d\vartheta \int d^2\tilde{\mathbf{k}} \frac{g^3 N_c t^a_{\beta\alpha} \phi^{ij}_{\lambda_1\lambda}(\vartheta)\tilde{\mathbf{k}}^j \sqrt{q^+}}{4(2\pi)^5\sqrt{2\vartheta}\,\tilde{\mathbf{k}}^2} \left(\left[\frac{11}{3} + 4\ln\left(\frac{\Lambda}{\vartheta q^+}\right)\right]\left[-\frac{2}{\epsilon} + \ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)\right]\right.$$

$$\left. +2\ln^2\left(\frac{\Lambda}{\vartheta(1-\vartheta)q^+}\right) - \frac{67}{9} + \frac{2\pi^2}{3} - \frac{11}{3}\ln(1-\vartheta) - 2\ln^2(1-\vartheta)\right) \left|q^\beta_{\lambda_1}((1-\vartheta)q^+, (1-\vartheta)\mathbf{q} - \tilde{\mathbf{k}})\, g^a_i(\vartheta q^+, \vartheta\mathbf{q} + \tilde{\mathbf{k}})\right\rangle$$

Two types of IR logs are involved: $\ln\left(\frac{\Lambda}{q^+}\right)\ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)$ and $\ln^2\left(\frac{\Lambda}{\vartheta(1-\vartheta)q^+}\right)$

Cancellation of the IR logs

The IR logs cancellation pattern is:



	$\|\psi_\lambda^\alpha\rangle_{qg}^2$	$\|\psi_\lambda^\alpha\rangle_{qg}^3$	$\|\psi_\lambda^\alpha\rangle_{qg}^4$	$\|\psi_\lambda^\alpha\rangle_{qg}^5$
$\ln\left(\frac{\Lambda}{q^+}\right)\ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)$	4	−3	1	−2
$\ln^2\left(\frac{\Lambda}{\vartheta(1-\vartheta)q^+}\right)$	2	−2	1	−1

Results for the NLO WF

By combining all the loop contribution:

$$\left|\psi_\lambda^\alpha(q^+,\boldsymbol{w})\right\rangle_{qg} = -\int_0^1 d\vartheta \int_{\mathbf{x},\mathbf{z}} \frac{ig^3\, N_c\, t^a_{\beta\alpha}\sqrt{q^+}\,\mathbf{X}^j}{4(2\pi)^4\sqrt{2\vartheta}\,\mathbf{X}^2}\left\{\phi^{ij}_{\lambda_1\lambda}(\vartheta)\left(-\beta(\vartheta)\left[\frac{2}{\epsilon}+\ln\left(\frac{\mathbf{X}^2\mu^2_{\overline{MS}}}{4e^{-2\gamma}}\right)\right] + \gamma(\vartheta) + \mathcal{I}(\vartheta)\right) + \kappa^{ij}_{\lambda_1\lambda}(\vartheta)\right\}$$
$$\times\,\delta(\boldsymbol{w}-(1-\vartheta)\boldsymbol{x}-\vartheta\boldsymbol{z})\left|q^\beta_{\lambda_1}((1-\vartheta)q^+,\boldsymbol{x})\,g^a_i(\vartheta q^+,\boldsymbol{z})\right\rangle$$

with:

$$\beta(\vartheta) \equiv \frac{11}{3}N_c - \frac{2}{3}N_f + \Delta_\beta(\vartheta); \qquad \Delta_\beta(\vartheta) \equiv N_c \ln(1-\vartheta)$$

$$\gamma(\vartheta) \equiv \left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c - \frac{10}{9}N_f + \Delta_\gamma(\vartheta) \qquad \Delta_\gamma(\vartheta) \equiv \left[\frac{2}{3}N_f + \left(3Li_2(\vartheta) + \frac{1}{2}\ln\left(e^{\frac{13}{3}}\vartheta^2(1-\vartheta)\right)\right)N_c\right]\ln(1-\vartheta)$$

$$\mathcal{I}(\vartheta) \equiv \left(3 + 4\ln\left(\frac{\Lambda}{\vartheta q^+}\right)\right) N_c \int \frac{d^2\tilde{\boldsymbol{p}}}{\tilde{\boldsymbol{p}}^2}$$

$$\kappa^{ij}_{\lambda_1\lambda}(\vartheta) \equiv \chi^\dagger_{\lambda_1}\left\{\frac{\vartheta(2-\vartheta)}{2}\delta^{ij} + \left[4\vartheta(2-\vartheta) + (2-\vartheta)\ln(1-\vartheta) + \frac{3\vartheta^2}{1-\vartheta}\ln(\vartheta)\right] i\varepsilon^{ij}\sigma^3\right.$$
$$\left.+\vartheta(\vartheta-4)\left(-\frac{2}{\epsilon}+\ln\left(\frac{\tilde{\mathbf{k}}^2}{\mu^2_{\overline{MS}}}\right)\right)(\delta^{ij} - i\varepsilon^{ij}\sigma^3)\right\}\chi_\lambda.$$

Adding the interactions with a shockwave

From the perturbative expansion for the outgoing state at order g^3:

$$
\begin{aligned}
|out\rangle^{(3)} \;=\; - & \left[\frac{\langle q\, g_2|\, H\, |q\, g_3\, g_4\rangle\, \langle q\, g_3\, g_4|\, H\, |q\, g_1\rangle\, \langle q\, g_1|\, H\, |q\rangle}{(E_{qgg}-E_{qg_2})\,(E_{qgg}-E_{qg_1})\,(E_{qg_1}-E_q)}\,(S_F\, S_{A_1}\;-\;S_F)\right. \\
+ & \frac{\langle q\, g_2|\, H\, |q\, g_3\, g_4\rangle\, \langle q\, g_3\, g_4|\, H\, |q\, g_1\rangle\, \langle q\, g_1|\, H\, |q\rangle}{(E_{qgg}-E_{qg_2})\,(E_{qgg}-E_q)\,(E_{qg_1}-E_q)}\,(S_F\, S_{A_3}\, S_{A_4}\;-\;S_F) \\
+ & \left.\frac{\langle q g_2|\, H\, |q\, g_3\, g_4\rangle\, \langle q\, g_3\, g_4|\, H\, |q\, g_1\rangle\, \langle q\, g_1|\, H\, |q\rangle}{(E_{qg_2}-E_q)\,(E_{qgg}-E_q)\,(E_{qg_1}-E_q)}\,(S_F\, S_{A_2}\;-\;S_F)\right]\,|q g_2\rangle\,.
\end{aligned}
$$

Divergences involved in this expansion:

1) *UV divergences*: contribution to the beta function (regularized by dimreg).

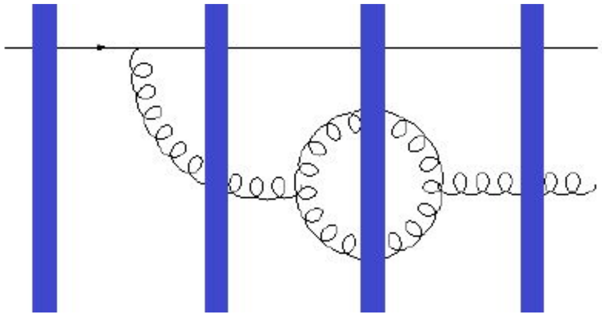
2) *Short distance poles*: when the partons are approaching each other.

3) *Soft and Collinear*: Reabsorbed in DGLAP for the incoming PDF and outgoing fragmentation functions.

The NLO Outgoing Quark State

Dressing the gluon loop with shockwaves:

$$\left|q_\lambda^\alpha(q^+,\boldsymbol{w})\right\rangle_{qg} = -\frac{g^3\sqrt{q^+}}{(2\pi)^6}\int_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}}\int_0^1 d\vartheta\,\frac{\phi^{ij}_{\lambda_1\lambda}(\vartheta)}{\sqrt{\vartheta}}$$
$$\times\left\{\int_{\boldsymbol{z}'}\int_{\frac{\Lambda}{\vartheta q^+}}^{1-\frac{\Lambda}{\vartheta q^+}} d\xi\, F^j(\vartheta,\,\xi,\,\mathbf{R},\,\mathbf{Z})\left[f^{bge}\,f^{acd}\,V^{\gamma\beta}(\boldsymbol{x})\,U^{gd}(\boldsymbol{z})\,U^{ec}(\boldsymbol{z}')\,t^a_{\beta\alpha} - f^{ade}\,t^a_{\gamma\beta}\,V^{\beta\alpha}(\boldsymbol{w})\right]\right.$$
$$\left. - N_c\left(\int_{\boldsymbol{z}'} f^j(\vartheta,\,\mathbf{R},\,\mathbf{Z}) + g^j(\vartheta,\,\mathbf{R})\right)\left[V^{\sigma\beta}(\boldsymbol{x})\,U^{ba}(\boldsymbol{y})\,t^a_{\beta\alpha} - t^b_{\sigma\beta}\,V^{\beta\alpha}(\boldsymbol{w})\right]\right\}$$
$$\times\,\delta^{(2)}(\boldsymbol{w}-\boldsymbol{C})\;\delta^{(2)}\left(\boldsymbol{y}-\xi\boldsymbol{z}-(1-\xi)\boldsymbol{z}'\right)\left|q^\sigma_{\lambda_1}((1-\vartheta)q^+,\boldsymbol{x})\,g^b_i(\vartheta q^+,\,\boldsymbol{y})\right\rangle$$



with:

$$F^j(\vartheta,\,\xi,\,\mathbf{R},\,\mathbf{Z}) \equiv \left(\xi(1-\xi) - 2 + \frac{1}{\xi(1-\xi)}\right)\frac{(1-\vartheta)\,\boldsymbol{R}^j}{\left((1-\vartheta)\boldsymbol{R}^2 + \xi(1-\xi)\boldsymbol{Z}^2\right)\,\boldsymbol{Z}^2}$$

$$f^j(\vartheta,\,\mathbf{R},\,\mathbf{Z}) \equiv -\left(\frac{11}{6} + 2\ln\left(\frac{\Lambda}{\vartheta q^+}\right)\right)\frac{\boldsymbol{R}^j}{\boldsymbol{R}^2\,\boldsymbol{Z}^2}$$

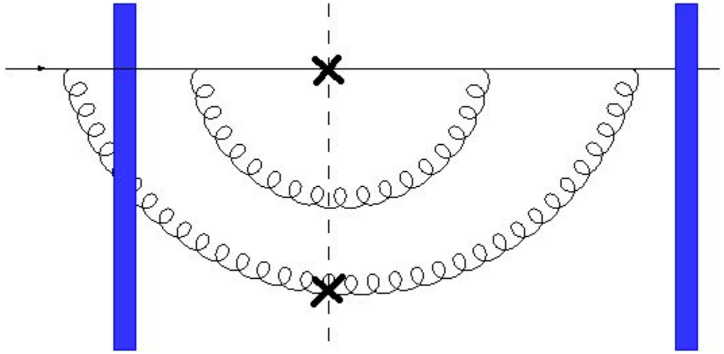
$$g^j(\vartheta,\,\mathbf{R}) \equiv \left(-\left(\frac{11}{6} + 2\ln\left(\frac{\Lambda}{\vartheta q^+}\right)\right)\left(-\frac{2}{\epsilon} + \ln\left(\mu^2_{\overline{MS}}\boldsymbol{R}^2\right)\right) + \frac{67}{18} - \frac{\pi^2}{3} - \ln^2\left(\frac{\Lambda}{\vartheta q^+}\right)\right)\frac{\boldsymbol{R}^j}{\boldsymbol{R}^2}$$

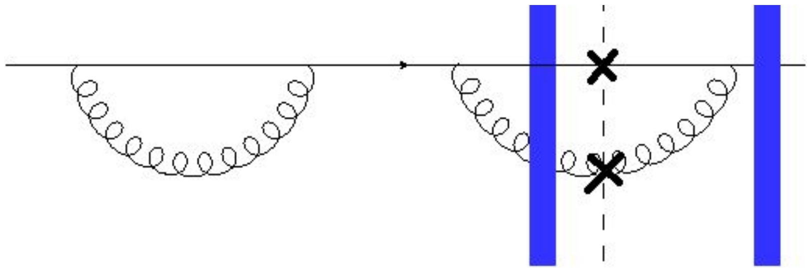
Computing the NLO Dijet Cross-section

The full NLO dijet cross section will involve both g^2 x g^2 and g x g^3 contributions:

$$\frac{d\sigma^{dijet}}{d^3k\, d^3p} \equiv \frac{d\sigma_R^{q\to qqX}}{d^3k\, d^3p} + \frac{d\sigma_R^{q\to q\bar{q}X}}{d^3k\, d^3p} + \frac{d\sigma_R^{q\to ggX}}{d^3k\, d^3p} + \frac{d\sigma_R^{q\to qgX}}{d^3k\, d^3p} + \frac{d\sigma_V^{q\to qgX}}{d^3k\, d^3p}$$

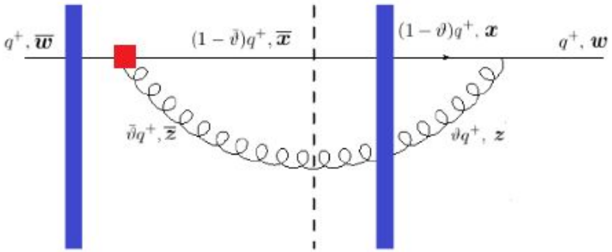
The real contributions are directly related to the results in hep-ph/1809.05526.
Real-Virtual cancellation: analogous to calculation by Chirilli, Xiao, Yuan hep-ph/1112.1061,hep-ph/1203.6139.

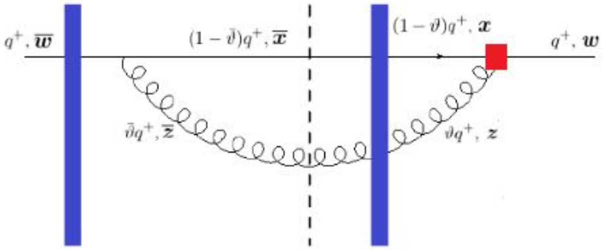




Partial Results for the NLO dijet

The corresponding diagrams for NLO dijet cross section:





Dressing with a shockwave is related to

$$\left|\psi^{\alpha}_{\lambda}(q^+,\, \boldsymbol{w})\right\rangle_{qg} = -\int_0^1 d\vartheta \int_{\mathbf{x},\,\mathbf{z}} \frac{ig^3\, N_c\, t^a_{\beta\alpha}\, \sqrt{q^+}\, \mathbf{X}^j}{4(2\pi)^4\sqrt{2\vartheta}\, \mathbf{X}^2} \left\{ \phi^{ij}_{\lambda_1\lambda}(\vartheta) \left(-\beta(\vartheta) \left[\frac{2}{\epsilon} + \ln\left(\frac{\mathbf{X}^2 \mu^2_{\overline{MS}}}{4e^{-2\gamma}} \right) \right] + \gamma(\vartheta) + \mathcal{I}(\vartheta) \right) + \kappa^{ij}_{\lambda_1\lambda}(\vartheta) \right\}$$
$$\times\, \delta(\boldsymbol{w} - (1-\vartheta)\boldsymbol{x} - \vartheta \boldsymbol{z}) \left[V^{\gamma\beta}(\boldsymbol{x})\, U^{ba}(\boldsymbol{z})\, t^a_{\beta\alpha} - t^b_{\gamma\beta}\, V^{\beta\alpha}(\boldsymbol{w}) \right] \left| q^{\gamma}_{\lambda_1}((1-\vartheta)q^+,\, \boldsymbol{x})\, g^b_i(\vartheta q^+,\, \boldsymbol{z}) \right\rangle$$

Resulting with the following contribution to the beta function / cusp anomalous dimension:

$$\frac{d\sigma_I^{qA\to qg+X}}{dk^+\, d^2\boldsymbol{k}\, dp^+\, d^2\boldsymbol{p}} = \frac{\alpha_s^2}{(2\pi)^6 q^+}\, \delta(q^+ - k^+ - p^+)$$
$$\times \int_{\boldsymbol{x},\, \overline{\boldsymbol{x}},\, \boldsymbol{z},\, \overline{\boldsymbol{z}}} \frac{\boldsymbol{X}\cdot\overline{\boldsymbol{X}}}{\boldsymbol{X}^2\, \overline{\boldsymbol{X}}^2}\, K\left(\vartheta,\, \mathbf{X}^2,\, \overline{\mathbf{X}}^2\right) \mathrm{e}^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}}) - i\boldsymbol{k}\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})}\, \mathbb{S}_{\mathrm{LO}}\left(\overline{\boldsymbol{w}},\, \overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \boldsymbol{w},\, \boldsymbol{x},\, \boldsymbol{z}\right)$$

$$K\left(\vartheta,\, \mathbf{X}^2,\, \overline{\mathbf{X}}^2\right) = \left\{ \frac{1+(1-\vartheta)^2}{\vartheta} \left(\beta(\vartheta) \left[-\frac{2}{\epsilon} + \ln\left(\frac{\mathbf{X}^2 \mu^2_{\overline{MS}}}{4e^{-2\gamma}} \right) \right] + \gamma(\vartheta) + \mathcal{I}(\vartheta) \right) + F(\vartheta) + \vartheta\beta(\vartheta) \right\} + \left(\mathbf{X} \longleftrightarrow \overline{\mathbf{X}}\right)$$

Summary

1) Generalization of the method by C. Marquet (2007) to all orders, for the calculation of the forward particle production in proton-nucleus collisions at high energy, was shown to be possible by adopting the outgoing state approach.

2) We computed the full light-cone wave function of the incoming quark, and partially its corresponding outgoing state.

3) IR divergences has been shown to cancel after combining all the loop contributions (except normalization contribution). Remaining logs will be absorbed to DGLAP and JIMWLK evolution.

4) Partial results for the inclusive forward NLO dijet cross section are available.