Collinear resummations for the high-energy evolution

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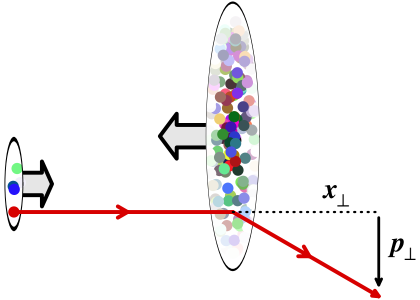
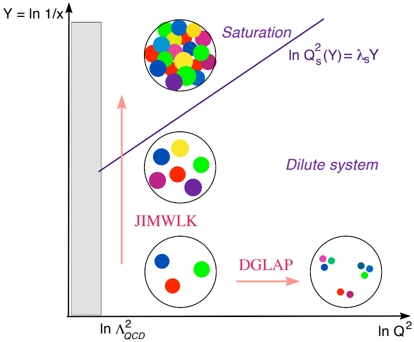


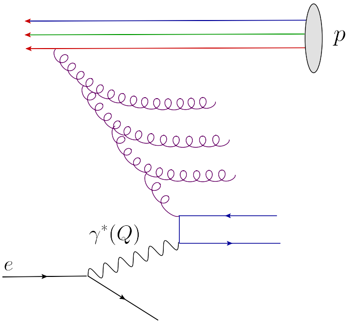


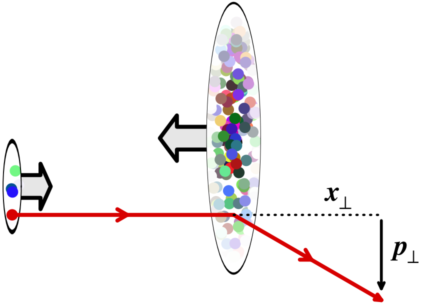
Outline

- About the importance of properly choosing the evolution variables

Motivation : Dilute-dense scattering

- A dilute projectile scatters off the gluon distribution in a dense hadronic target
  - deep inelastic scattering (ep or eA) at small Bjorken $x \ll 1$
  - particle production in pA or pp collisions at forward rapidities





$$x = \frac{Q^2}{2P\cdot q} \simeq \frac{Q^2}{s}$$

$$x = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}$$

Motivation : Dilute-dense scattering

- A dilute projectile scatters off the gluon distribution in a dense hadronic target
  - deep inelastic scattering (*ep* or *eA*) at small Bjorken $x \ll 1$
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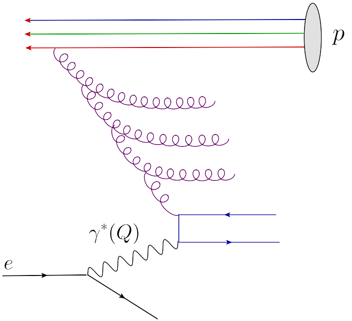
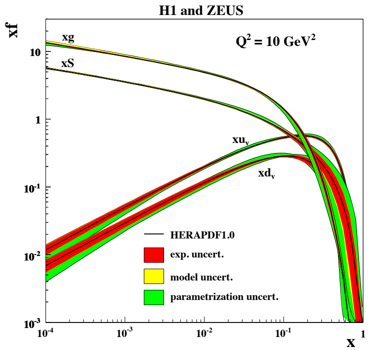
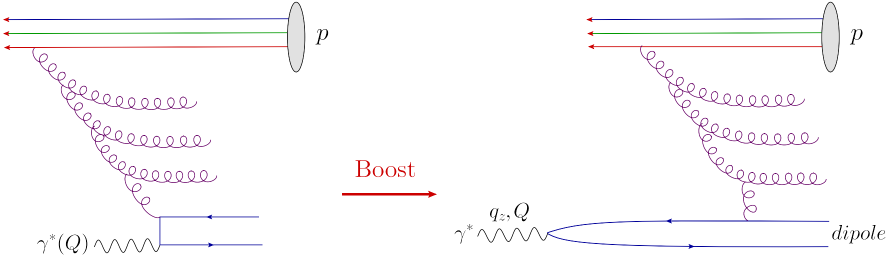




- At small x, the partons in the hadronic target are mostly gluons

The dipole picture

- To study the gluon evolution, it is convenient to use the dipole frame
  - Lorentz boost to a frame where the dipole is energetic : large q^+

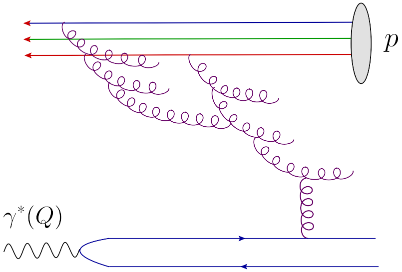


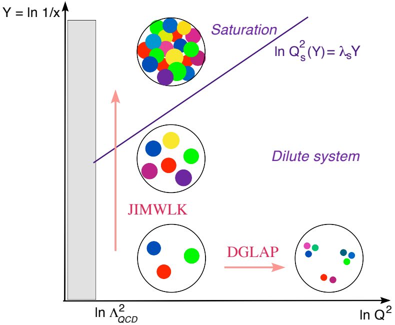
$$x = \frac{Q^2}{2P^- q^+} \ll 1 \iff \Delta x^+ \simeq \frac{2q^+}{Q^2} \gg \frac{1}{P^-}$$

  - the virtual photon fluctuates into a $q\bar{q}$ pair long before the scattering
  - the $q\bar{q}$ color dipole acts as a probe of the gluon distribution

QCD evolution at high energy

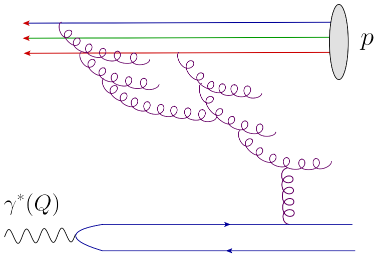
- Large rapidity phase-space $\eta \equiv \ln \frac{1}{x}$ for BFKL evolution
  - probability $\alpha_s \eta$ to emit a gluon with longitudinal fraction $\frac{k^-}{P^-} \geq x$
  - $g \to gg$: rapid rise in the gluon occupation number $n \propto e^{\alpha_s \eta}$
  - when $n \sim \frac{1}{\alpha_s}$, recombination $(gg \to g)$ competes with splitting $(g \to gg)$
- Non-linear evolution leading to gluon saturation : BK/JIMWLK equations

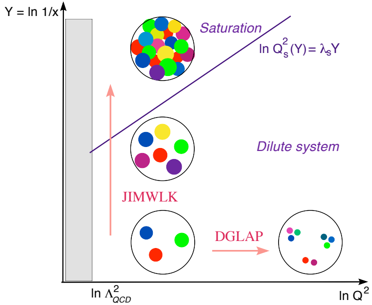




The saturation momentum

- The characteristic transverse momentum scale for the onset of gluon saturation
  - $n(Q^2, \eta) \sim \frac{1}{\alpha_s}$ when $Q^2 \sim Q_s^2(\eta) \simeq Q_0^2\, \mathrm{e}^{\lambda_s \eta}$ with $\lambda_s \simeq 0.2$
  - high energy : $\lambda_s \eta \gtrsim 1 \iff Q_s^2(\eta) \gg Q_0^2$
- Most interesting regime : $Q^2 \gtrsim Q_s^2(\eta)$

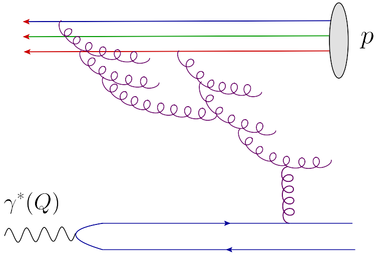


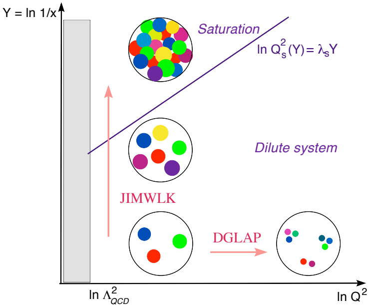


- Simultaneous evolution in energy $(\eta = \ln \frac{1}{x})$ and transverse momentum $(\rho \equiv \ln \frac{Q^2}{Q_0^2})$

A double-logarithmic phase-space

- Without saturation, evolution is controlled by the double-logarithmic approximation
  - DLA : resummation of $(\alpha_s \eta \rho)^n$ with $n \geq 1$
- In presence of saturation, this approximation becomes subtle beyond leading order

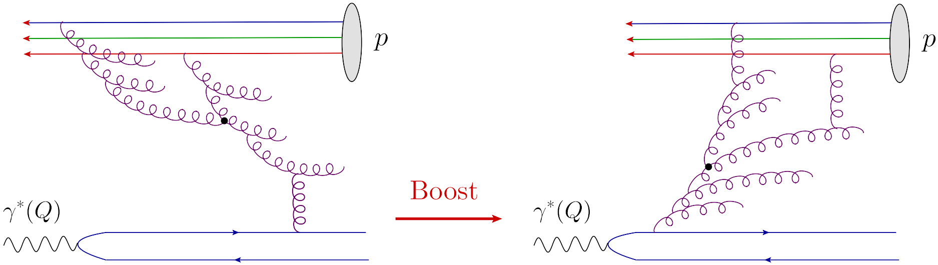




- Gluon evolution in the dense target is complicated by non-linear effects ($gg \to g$)
- So far, explicitly computed only to leading order (LO) : JIMWLK equation

Hadron vs. dipole evolution

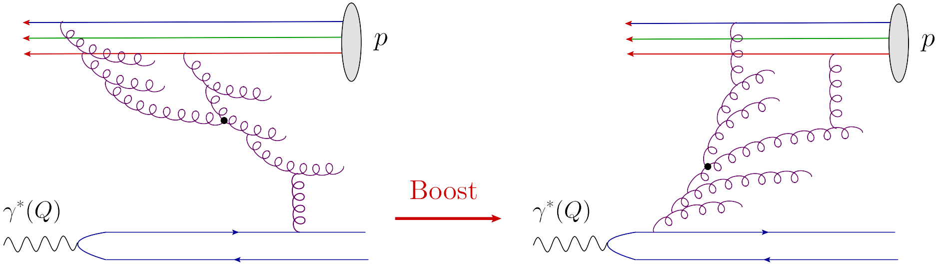
- Via a Lorentz boost, one can transfer the evolution from the hadron to the dipole
  - recombination $(gg \to g)$ gets mapped onto splitting $(g \to gg)$
  - gluon saturation gets mapped onto multiple scattering



- Dipole evolution is simpler : BFKL-like emissions followed by multiple scattering

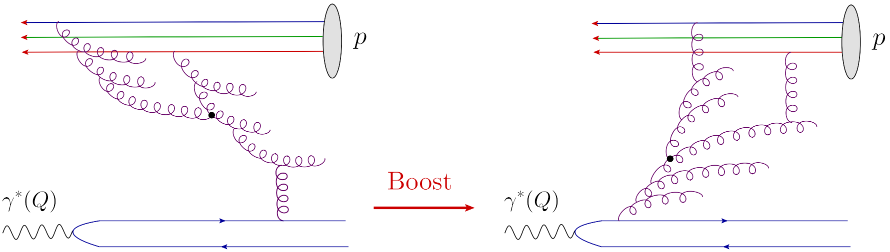
Hadron vs. dipole evolution

- Dipole evolution is currently known to next-to-leading order (NLO)
  - LO : Balitsky hierarchy, 1996 ; Balitsky-Kovchegov equation, 1999 (large N_c)
  - NLO : BK/Balitsky hierarchy/JIMWLK equation (seen as projectile evolution) *(Balitsky and Chirilli, 2008-13 ; Kovner, Lublinsky and Mulian, 2013-16)*



- ... but this comes with a price : different rapidity phase-space for the evolution

Hadron vs. dipole rapidities



- In the original frame, the hadron is an energetic left-mover : large P^-

$$\eta = \ln \frac{P^-}{k^-_{\min}} = \ln \frac{1}{x} \qquad \left(k^-_{\min} = \frac{Q^2}{2q^+} \right)$$

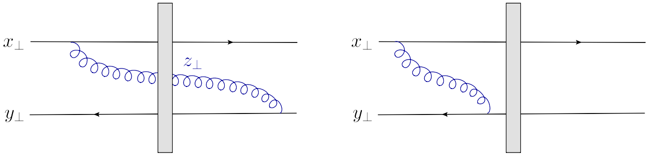
- In the boosted frame, the dipole is an energetic right-mover : large q^+

$$Y \equiv \ln \frac{q^+}{k^+_{\min}} = \ln \frac{1}{x} + \ln \frac{Q^2}{Q_0^2} = \eta + \rho \qquad \left(k^+_{\min} = \frac{Q_0^2}{2P^-} \right)$$

- The difference $Y - \eta = \rho$ starts to matter at NLO : essential in what follows

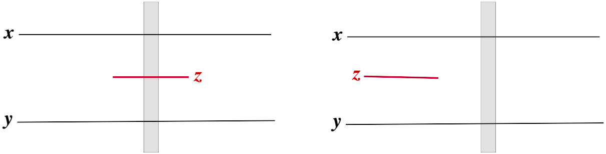
BK equation at LO *(Balitsky, 96 ; Kovchegov, 99)*

- Optical theorem : total DIS cross-section related to dipole forward amplitude T
- $\Delta x^+ = \frac{2q^+}{Q^2} \gg \frac{1}{P^-} \implies$ the dipole scatters off a shockwave
- Multiple scattering can be resummed to all orders in the eikonal approximation
  - transverse coordinates are "good quantum numbers"
  - quark at $\boldsymbol{x}_\perp$, antiquark at $\boldsymbol{y}_\perp$, dipole size $r \equiv |\boldsymbol{x}_\perp - \boldsymbol{y}_\perp| \sim 1/Q$
- One step in the high energy evolution : soft gluon emission $(q^+ \gg k^+ \gg k^+_{\rm min})$



BK equation at LO *(Balitsky, 96 ; Kovchegov, 99)*

- Large N_c : the original dipole splits into two new dipoles



- Both dipoles can scatter $\Longrightarrow$ non-linear completion of the BFKL equation

$$\frac{\partial T_{\boldsymbol{xy}}}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi}\int \mathrm{d}^2\boldsymbol{z}\,\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}\left[T_{\boldsymbol{xz}} + T_{\boldsymbol{zy}} - T_{\boldsymbol{xy}} - T_{\boldsymbol{xz}}T_{\boldsymbol{zy}}\right]$$

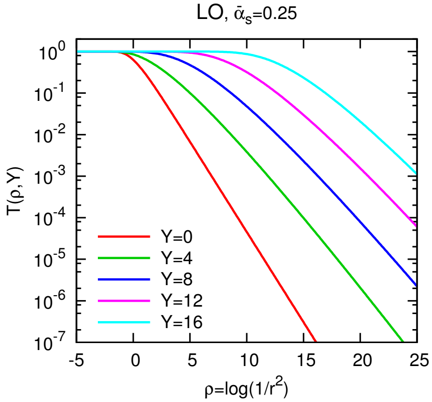
- Dipole kernel : probability for the dipole to emit a soft gluon at $\boldsymbol{z}$ $(\bar{\alpha}_s = \alpha_s N_c/\pi)$

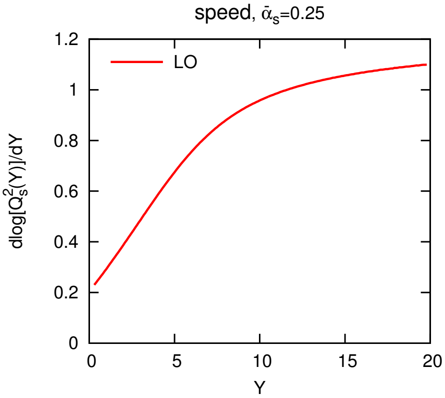
- Unitarity : the "black disk limit" $T = 1$ is a fixed point

- Saturation momentum $Q_s(Y)$: $T(r, Y) = 0.5$ when $r = 1/Q_s(Y)$

The saturation front

- $T(r, Y)$ as a function of $\rho \equiv \ln \frac{1}{r^2 Q_0^2}$ with increasing Y

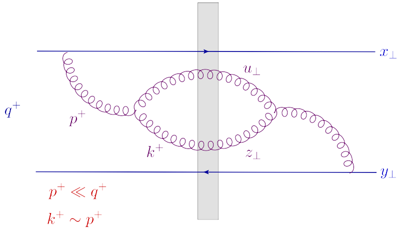


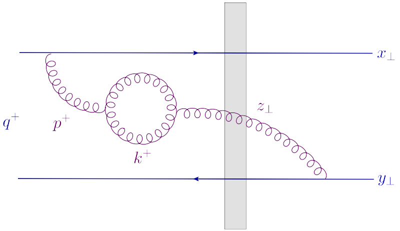


- $Q_s^2(Y) \simeq Q_0^2\, \mathrm{e}^{\lambda_s Y}$ with $\lambda_s \simeq 4.88\bar{\alpha}_s \simeq 1$ for $Y \gtrsim 10$: much too large

BK equation at NLO

- Any effect of $\mathcal{O}(\bar{\alpha}_s^2 Y) \Longrightarrow \mathcal{O}(\bar{\alpha}_s)$ correction to the r.h.s. of BK eq.





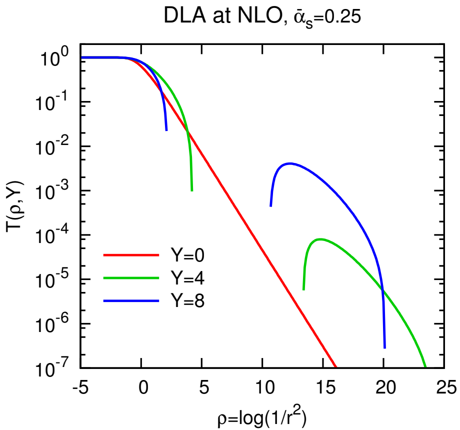
- Two successive, soft, emissions with similar longitudinal momenta : $p^+ \sim k^+ \ll q^+$

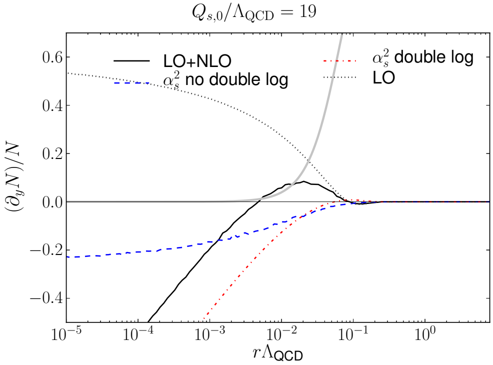
$$\frac{\partial S_{\boldsymbol{xy}}}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \left[1 - \bar{\alpha}_s \ln\frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2} \ln\frac{(\boldsymbol{z}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\right] [S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}]$$

$$+ \bar{\alpha}_s^2 \times \text{“regular”}$$

- The “regular” terms are numerous & complex, but pose no conceptual problems

Unstable numerical solution





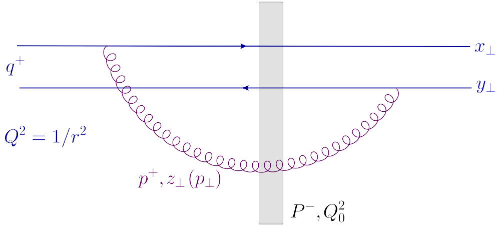
- Left : LO BK + the double collinear logarithm *(E.I. et all, arXiv :1502.05642)*
- Right : full NLO BK *(Lappi, Mäntysaari, arXiv :1502.02400)*
- The main source of instability : the double collinear logarithm

The double (anti)collinear logarithm

- Important only for very large daughter dipoles ...

$$-\frac{1}{2}\ln\frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\ln\frac{(\boldsymbol{y}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\simeq-\frac{1}{2}\ln^2\frac{(\boldsymbol{x}-\boldsymbol{z})^2}{r^2}\quad\text{if}\quad|\boldsymbol{z}-\boldsymbol{x}|\simeq|\boldsymbol{z}-\boldsymbol{y}|\gg r$$

- .... but this is indeed the typical situation : $Q^2 \gg p_\perp^2 \gg Q_0^2$



- In pQCD, double-logs usually occur as "rapidity" $\times$ "collinear", i.e. $\bar{\alpha}_s Y \rho$ (DLA)
- How can a double collinear log, like $\bar{\alpha}_s \rho^2$, be generated ?

The double (anti)collinear logarithm

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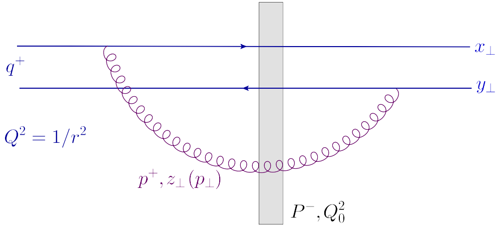
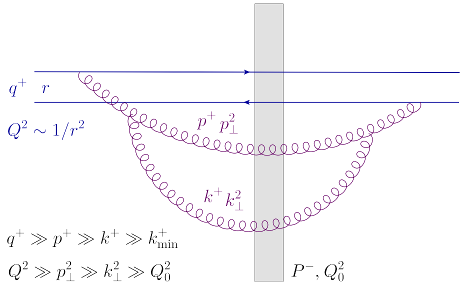


- As a correction to the phase-space for DLA, introduced by time-ordering !

Time ordering

- The dominant, double-logarithmic, corrections require simultaneous ordering
  - in longitudinal momenta : $q^+ \gg p^+ \gg k^+ \cdots \gg k^+_{min} \equiv \frac{Q_0^2}{2P^-}$
  - in transverse momenta : $Q^2 \gg p_\perp^2 \gg k_\perp^2 \cdots \gg Q_0^2$



- ... and in lifetimes

$$\frac{2q^+}{Q^2} \gg \frac{2p^+}{p_\perp^2} \gg \frac{2k^+}{k_\perp^2} \cdots \gg \frac{1}{P^-}$$

- Time-ordering restricts the phase-space

$$\bar{\alpha}_s Y \rho \to \bar{\alpha}_s (Y-\rho)\rho$$

- ... but it is ignored at LO

- The correct phase-space for DLA is rather $\bar{\alpha}_s \eta \rho = \bar{\alpha}_s Y \rho - \bar{\alpha}_s \rho^2$

Collinear resummations in Y

- The lack of time-ordering at LO generates a tower of ρ^2-enhanced corrections
  - leading series : powers of $\alpha\rho^2$; the subleading one : powers of $\alpha^2\rho^2$, etc
- The leading series can be resummed by enforcing time-ordering in the LO BK eq.
- Two strategies for "collinear improvement" (equivalent to DLA, but not beyond)
  - same kernel as at LO, but non-local in Y *(G. Beuf, arXiv :1401.0313)*
  - a local equation in Y, but with all-order resummed kernel & initial condition *(E.I., Madrigal, Mueller, Soyez, Triantafyllopoulos, arXiv :1502.05642)*
- The results look promising ... at a first sight ☺
  - the resummed equations are stable
  - they physical predictions look appealing (e.g. good fits to DIS)

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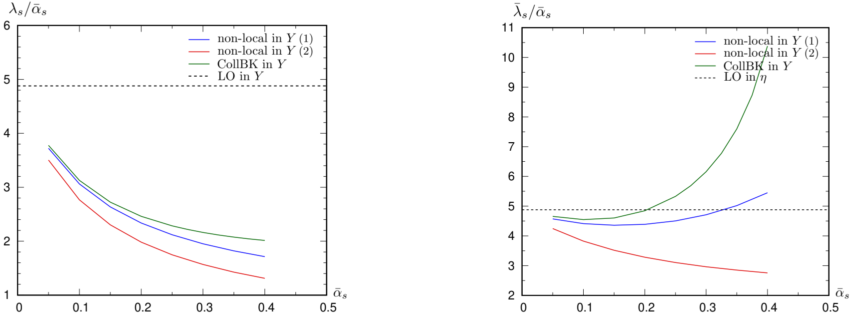
  - the resummed equations are stable
  - they physical predictions look appealing (e.g. good fits to DIS)
- ... but they become disappointing after a closer inspection ☹ *(Ducloué, E.I., Madrigal, Mueller, Soyez, Triantafyllopoulos, 1902.06637)*

Resummations in Y : Scheme dependence

- The "physical" interpretation of the results has been discussed in terms of Y
- After translating to $\eta = Y - \rho = \ln \frac{1}{x}$: strong scheme dependence



  - 3 different prescriptions : one local in Y ("CollBK") & two non-local
  - leading order result (in either Y or η) : $\lambda_s^{(0)} = 4.88\bar{\alpha}_s$
- Subleading ρ^2–enhanced corrections which are not under control

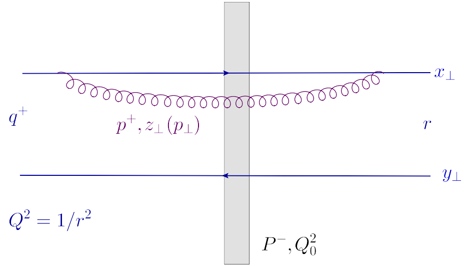
NLO BK evolution in η

(Ducloué, E.I., Madrigal, Mueller, Soyez, and Triantafyllopoulos, arXiv :1902.06637)

- Violation of time-ordering by the evolution in Y leads to endless complications
- Why not order the evolution directly in terms of η, i.e. of gluon lifetimes ?
  - ordering in $\tau_k = \frac{2k^+}{k_\perp^2} = \frac{1}{k^-}$ $\Longleftrightarrow$ ordering in $\eta = \ln \frac{P^-}{k^-}$: target rapidity
- Target evolution in the presence of saturation is hopelessly complicated ... ☹
  - ... but we propose something else :
- Compute dipole evolution in Y, order by order in pQCD, and then make the change of variables $Y = \eta + \rho$
- We deduced NLO BK equation in η starting with the known equation in Y ☺

NLO BK evolution in η

- NLO BK equation in η is better behaved than the original equation in Y
  - the double anti–collinear log disappears (time-ordering is now automatic !)
  - replaced by a double collinear log : very small daughter dipole
  - violations of k^+-ordering by atypical configurations : less problematic



- Ordering in lifetimes :

$$\frac{2q^+}{Q^2} \gg \frac{2p^+}{p_\perp^2} \gg \frac{1}{P^-}$$

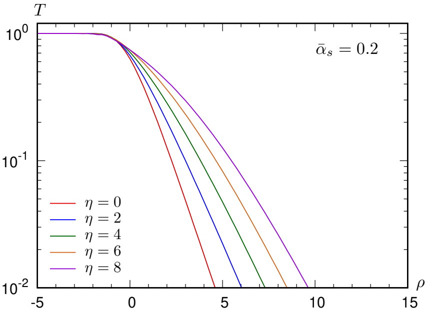
- ... also implies ordering in k^+ :

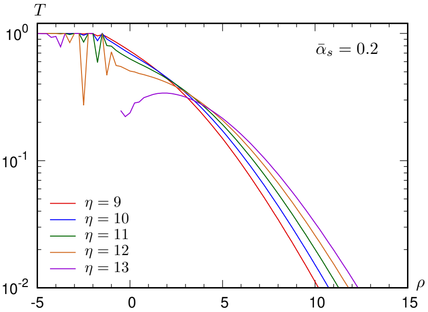
$$q^+ \gg p^+ \gg k^+_{min} = \frac{Q_0^2}{2P^-}$$

- ... unless $p_\perp^2 \gg Q^2$ (or $|\boldsymbol{x} - \boldsymbol{z}| \ll r$)

NLO BK evolution in η

- Numerical solutions to "NLO BK in η" (LO BK + the double collinear log)





- Although disfavoured by the typical evolution, the collinear double-logs do still entail a (mild) instability
  - the instability develops only for sufficiently large η
  - it first appears for relatively large dipole sizes, close to $1/Q_s$
- Fluctuations leading to large dipoles which then fragment into smaller ones

Collinear resummation in η

- The leading series of double collinear logs can be resummed via a "rapidity shift" $\Longrightarrow$ non-local evolution in η

$$\frac{\partial S_{\boldsymbol{xy}}(\eta)}{\partial \eta} = \frac{\bar{\alpha}_s}{2\pi}\int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\left[S_{\boldsymbol{xz}}(\eta-\delta_{\boldsymbol{xz}})S_{\boldsymbol{zy}}(\eta-\delta_{\boldsymbol{zy}}) - S_{\boldsymbol{xy}}(\eta)\right]$$

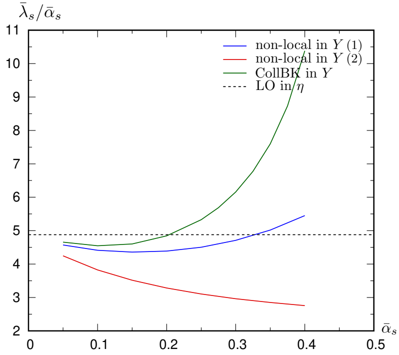
  - rapidity shift if one daughter dipole is much smaller than its parent :

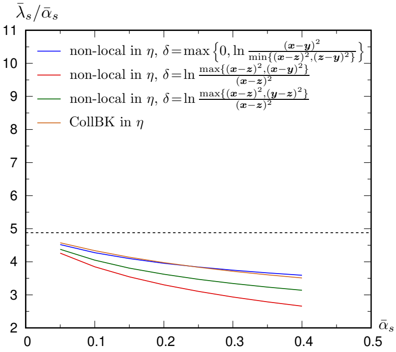
$$\delta_{\boldsymbol{xz}} \equiv \Theta\left(r^2-(\boldsymbol{x}-\boldsymbol{z})^2\right)\,\ln\frac{(\boldsymbol{x}-\boldsymbol{z})^2}{r^2}$$

- As before, this prescription is not unique beyond double-log accuracy ...
- ... but the associated scheme dependence is reasonably small
- Extension to full NLO accuracy possible (for a given prescription)
- Initial value problem : $S(\eta_0, r) = S_0(r)$

Resummed BK evolution in η : fixed coupling

- $\bar{\lambda}_s \equiv \frac{\mathrm{d} \ln Q_s^2}{\mathrm{d}\eta}$: the speed of the saturation front in η

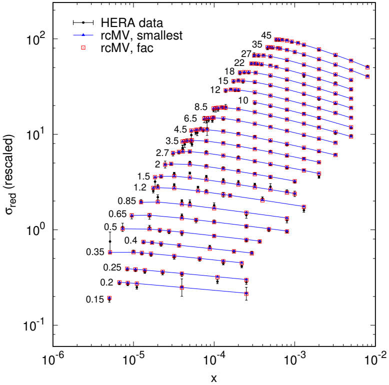


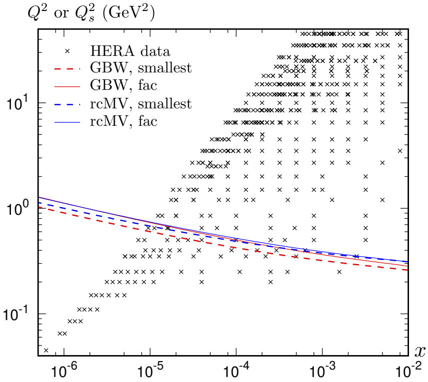


- Left : resummations in Y : strong scheme dependence, no clear pattern
- Right : resummations in η : weak scheme dependence $\sim \mathcal{O}(\alpha_s^2)$
  - consistent with the expected perturbative accuracy of the resummed equation

A fit to DIS at HERA

- Excellent fit to the HERA data at small x : $x_{\mathrm{Bj}} \leq 0.01,\ Q^2 \leq 50\ \mathrm{GeV}^2$
  - 4 free parameters, all encoded in the initial condition $S(\eta_0, r)$
  - partial resummation of the NLO single logs (DGLAP)

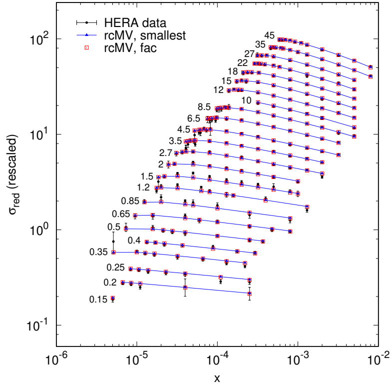




- Right : the saturation scale given by the fit on top of the data points

A fit to DIS at HERA

- Excellent fit to the HERA data at small x : $x_{\mathrm{Bj}} \leq 0.01,\ Q^2 \leq 50\ \mathrm{GeV}^2$
  - 4 free parameters, all encoded in the initial condition $S(\eta_0, r)$
  - partial resummation of the NLO single logs (DGLAP) : essential



init cdt.	RC schm	double logs	single logs	χ^2/npts for Q^2_{max} 50	100	200	400
GBW	small	yes	no	2.05	2.17	2.27	2.24
GBW	small	no	yes	1.26	1.26	1.35	1.46
GBW	small	yes	yes	1.18	1.21	1.31	1.39
GBW	fac	yes	no	1.65	1.75	1.94	2.01
GBW	fac	no	yes	1.19	1.23	1.37	1.51
GBW	fac	yes	yes	1.14	1.17	1.25	1.32
rcMV	small	yes	no	1.72	1.86	1.93	1.92
rcMV	small	no	yes	1.07	1.08	1.04	1.03
rcMV	small	yes	yes	1.03	1.04	1.01	1.00
rcMV	fac	yes	no	1.31	1.34	1.35	1.33
rcMV	fac	no	yes	0.98	0.98	0.95	0.95
rcMV	fac	yes	yes	1.01	1.03	1.01	1.00

Table 2: Evolution of the fit quality when increasing Q^2_{max} (in GeV^2).

- Right : the quality of the best fit remains constant up to $Q^2_{\mathrm{max}} = 400\ \mathrm{GeV}^2$

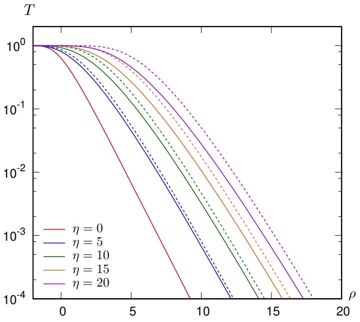
Conclusions

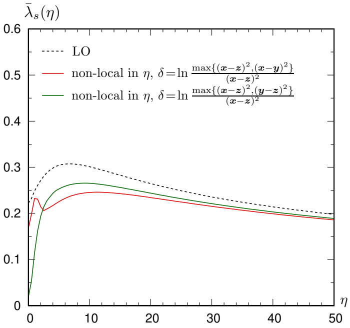
- pQCD for the high-energy evolution is simpler for the dilute projectile
- However, its results are not suitable for physics studies :
  - large radiative corrections associated with violations of time ordering
  - instabilities
  - standard resummations (DLA) don't work
- The pQCD expansion can be rephrased in terms the rapidity of the dense target via a simple change of variables
- By itself, this change of variables solves most of the problems
- Weak remaining instability, that can be dealt with via standard resummations
  - weak scheme dependence, predictive power
  - can be promoted to full NLO accuracy
  - encouraging applications to DIS


THANK YOU !

Resummed BK evolution in η : running coupling

- Recall : phenomenology requires $\bar{\lambda}_s \simeq 0.20 \div 0.25$
- The main reduction comes from the use of a running coupling
  - below : $\bar{\alpha}_s(r_{\min})$ where $r_{\min} = \min\{|\boldsymbol{x}-\boldsymbol{y}|, |\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}|\}$

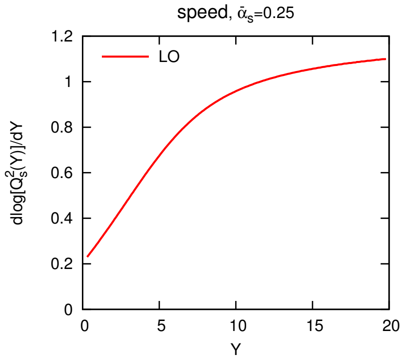


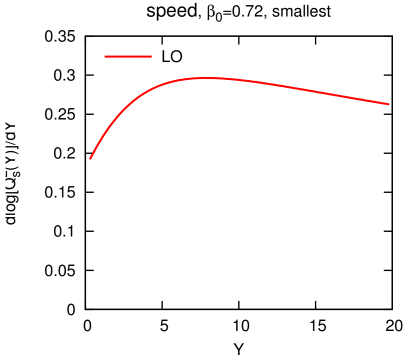


- Left : saturation fronts in η : collBK (full lines) vs. LO BK (dashed)
- Right : saturation exponent : $\bar{\lambda}_s \simeq 0.2$ at large η ☺

LO BK with running coupling : rcBK

- Saturation exponent : $\lambda_s \simeq 4.88\bar{\alpha}_s \simeq 1$ for $Y \gtrsim 5$: much too large
  - phenomenology requires a much smaller valuer $\lambda_s \simeq 0.2 \div 0.3$
- Including running coupling dramatically slows down the evolution





- Rather successful phenomenology based on rcBK