Initial fluctuations and anisotropies in heavy-ion collisions

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based on:

Albacete, Guerrero-Rodriguez, CM, JHEP 1901 (2019) 073
Giacalone, Guerrero-Rodriguez, Luzum, CM and Ollitrault, PRC 100 (2019) 024905
Gelis, Giacalone, Guerrero-Rodriguez, CM and Ollitrault, arXiv:1907.10948

Outline

- The fluid paradigm in heavy-ion collisions

  final-state momentum anisotropies from
  initial-state spatial anisotropies

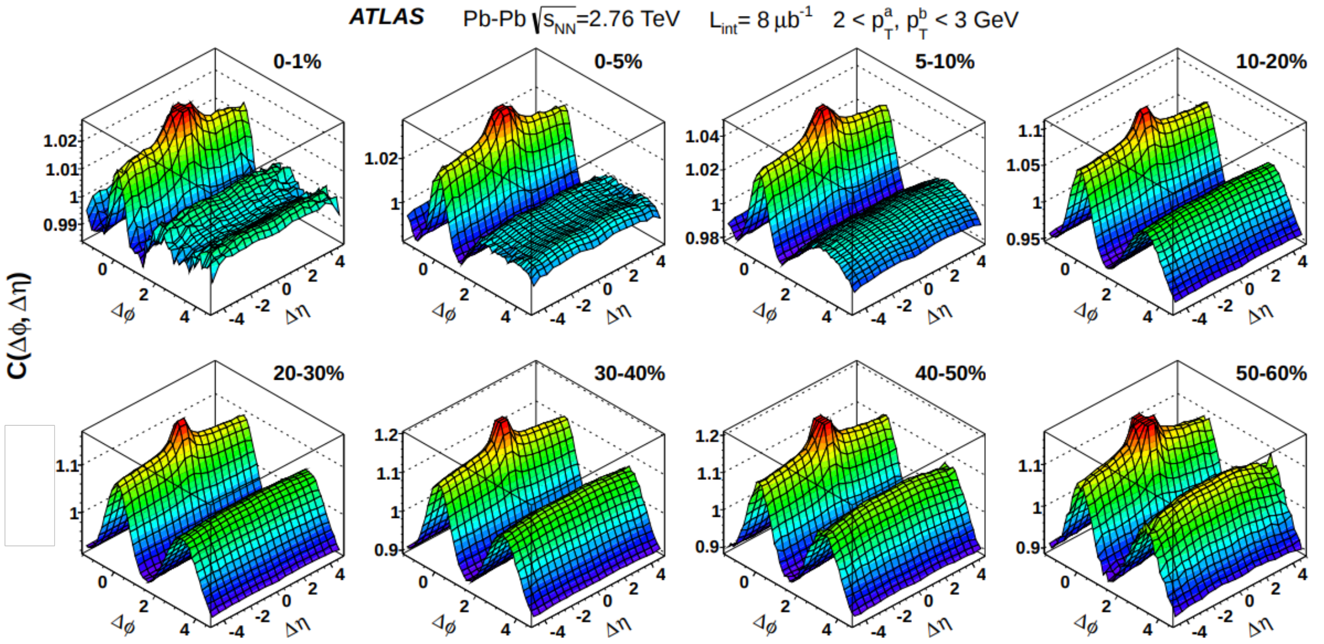
- Initial eccentricities from the Glasma

  how to calculate the initial spatial anisotropies:
  from the energy momentum tensor and its fluctuations

- A new picture of initial-state fluctuations

  ab-initio description free from the Monte Carlo Glauber Ansatz

Flow in heavy-ion collisions

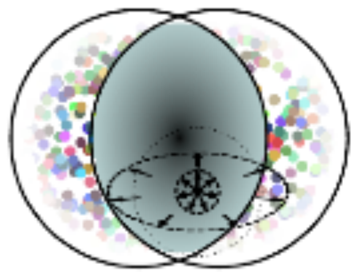


Two ingredients needed for flow

flow is an **initial spatial anisotropy** turned into a momentum anisotropy by the **hydrodynamic expansion** of the medium

final-state harmonic ⟵ $$v_n = \kappa_n \varepsilon_n$$ ⟶ initial-state harmonic

v_2 has two components: a geometric one and one due to fluctuations (the geometric component vanishes in central collisions)



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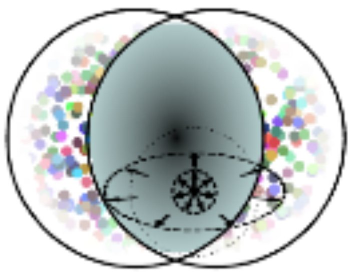
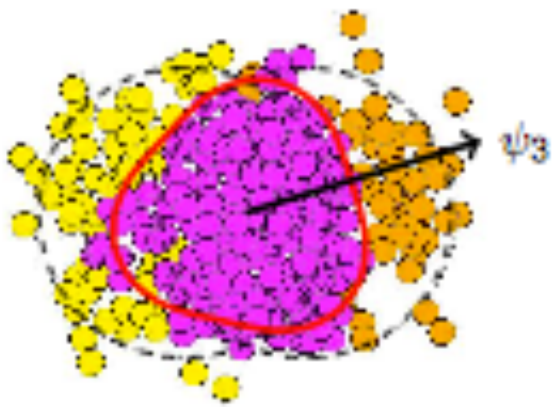




v_3 is only due to fluctuations

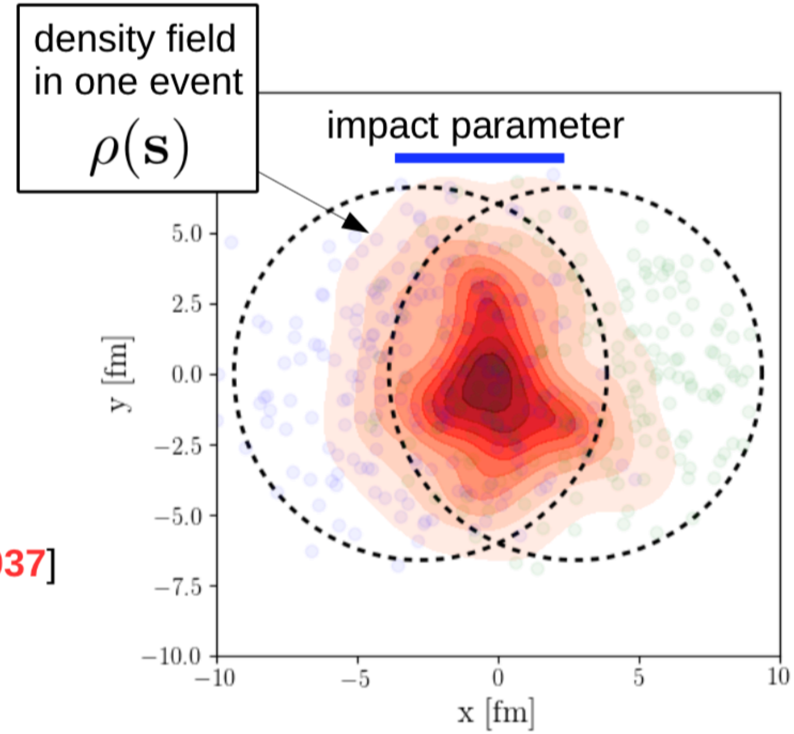
The eccentricity harmonics

How do we calculate the initial anisotropy?

[Teaney, Yan **1010.1876**]

$$\varepsilon_n \equiv \frac{\int_{\mathbf{s}} \mathbf{s}^n \rho(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}|^n \rho(\mathbf{s})}$$

$$\boxed{\mathbf{s} = x + iy}$$



Origin of anisotropy:

n=2
elliptic flow ⟶ **geometry + fluctuations**
[PHOBOS Collaboration **nucl-ex/0610037**]

n=3
triangular flow ⟶ **fluctuations only**
[Alver, Roland **1003.0194**]

The theoretical input is a model for $\rho(\mathbf{s})$ and its fluctuations.

Relevant averaged quantities

- averaged quantities relevant for experiments:

geometry + fluctuations ←

$$\sqrt{\langle v_2^2 \rangle} = v_2\{2\} = \kappa_2 \varepsilon_2\{2\}$$

$$\sqrt[4]{2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle} = v_2\{4\} = \kappa_2 \varepsilon_2\{4\}$$

→ geometry only

fluctuations only ←

$$\sqrt{\langle v_3^2 \rangle} = v_3\{2\} = \kappa_3 \varepsilon_3\{2\}$$

Relevant averaged quantities

- averaged quantities relevant for experiments:

geometry + fluctuations $\longleftarrow$ $$\sqrt{\langle v_2^2 \rangle} = v_2\{2\} = \kappa_2 \varepsilon_2\{2\}$$

$$\sqrt[4]{2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle} = v_2\{4\} = \kappa_2 \varepsilon_2\{4\}$$ $\longrightarrow$ geometry only

fluctuations only $\longleftarrow$ $$\sqrt{\langle v_3^2 \rangle} = v_3\{2\} = \kappa_3 \varepsilon_3\{2\}$$

- a 10-year old prescription to compute *ρ(s)*:

- start with the 'Glauber Monte Carlo Ansatz'
(sample nucleons according to the Woods-Saxon distribution)

- propose some mechanism (more or less motivated by physics) to convert nucleons/partons into a smooth map of energy density (e.g. through interactions)

instead, can we describe experimental data using the
correlation functions of the primordial energy density as (only) input ?

Our strategy

we follow Blaizot, Bronjowski, Ollitrault (2014)

$$\rho(\mathbf{s}) = \boxed{\langle\rho(\mathbf{s})\rangle} + \delta\rho(\mathbf{s}), \quad \langle\rho(\mathbf{s})\rangle \gg \delta\rho(\mathbf{s})$$

Connected 2-point function: (i.e. fluctuations)

$$\boxed{S(\mathbf{s}_1,\mathbf{s}_2) \equiv \langle\delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2)\rangle = \langle\rho(\mathbf{s}_1)\rho(\mathbf{s}_2)\rangle - \langle\rho(\mathbf{s}_1)\rangle\langle\rho(\mathbf{s}_2)\rangle}$$

Perturbative expansion of the anisotropy: $\varepsilon_n \equiv \dfrac{\int_{\mathbf{s}} \mathbf{s}^n \rho(\mathbf{s})}{\int_{\mathbf{s}} |\mathbf{s}|^n \rho(\mathbf{s})}$

To first nontrivial order:

$$\boxed{\langle\varepsilon_2\varepsilon_2^*\rangle = \varepsilon_2\{2\}^2 = \sigma^2 + \bar{\varepsilon}_2^2}$$

fluctuations $\sigma^2 = \dfrac{\int_{\mathbf{s}_1,\mathbf{s}_2} (\mathbf{s}_1)^2 (\mathbf{s}_2^*)^2 S(\mathbf{s}_1,\mathbf{s}_2)}{\left(\int_{\mathbf{s}} |\mathbf{s}|^2 \langle\rho(\mathbf{s})\rangle\right)^2}$

geometry $\bar{\varepsilon}_2 = \dfrac{\int_{\mathbf{s}} \mathbf{s}^2 \langle\rho(\mathbf{s})\rangle}{\int_{\mathbf{s}} |\mathbf{s}|^2 \langle\rho(\mathbf{s})\rangle}$ ~ $\varepsilon_2\{4\}$

$$\boxed{\langle\varepsilon_3\varepsilon_3^*\rangle = \varepsilon_3\{2\}^2 = \frac{\int_{\mathbf{s}_1,\mathbf{s}_2} (\mathbf{s}_1)^3 (\mathbf{s}_2^*)^3 S(\mathbf{s}_1,\mathbf{s}_2)}{\left(\int_{\mathbf{s}} |\mathbf{s}|^3 \langle\rho(\mathbf{s})\rangle\right)^2}}$$

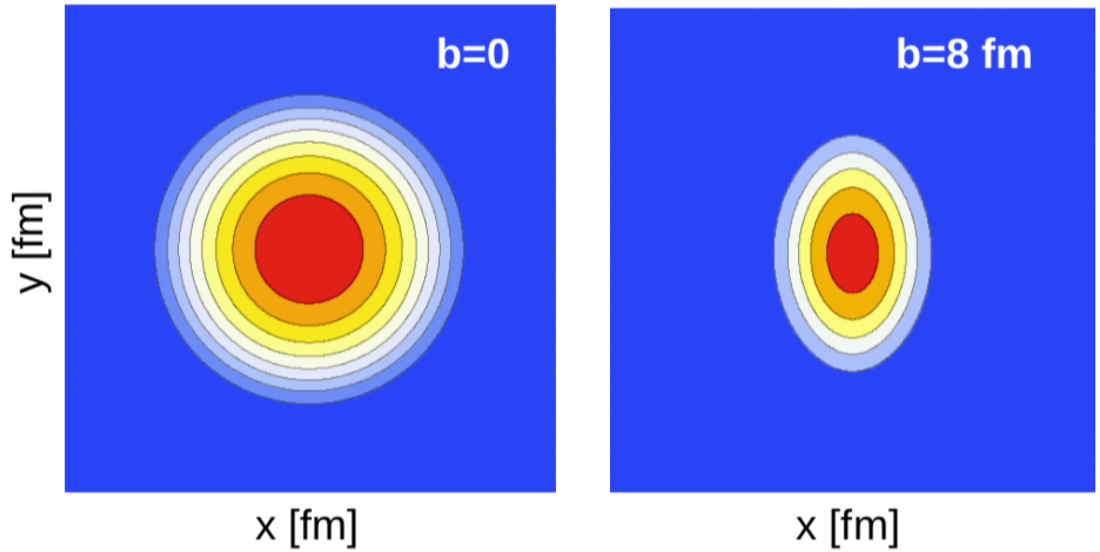
fluctuations only!

r.m.s. spatial anisotropy can be mapped to experimental data
we need the 1-point and 2-point correlators

What is needed ?

- $\boxed{\langle \rho(\mathbf{s}) \rangle}$

The average density.



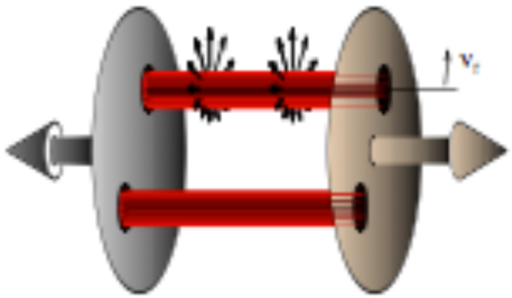
- **The fluctuations** around the average.

$$\boxed{S(\mathbf{s}_1, \mathbf{s}_2) = \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2) \rangle - \langle \rho(\mathbf{s}_1) \rangle \langle \rho(\mathbf{s}_2) \rangle}$$

in particular the integral $\xi(\mathbf{s}) = \int_{\mathbf{r}} S\left(\mathbf{s} + \frac{\mathbf{r}}{2}, \mathbf{s} - \frac{\mathbf{r}}{2}\right)$

$\longrightarrow$ compute the 1-point and 2-point energy correlators

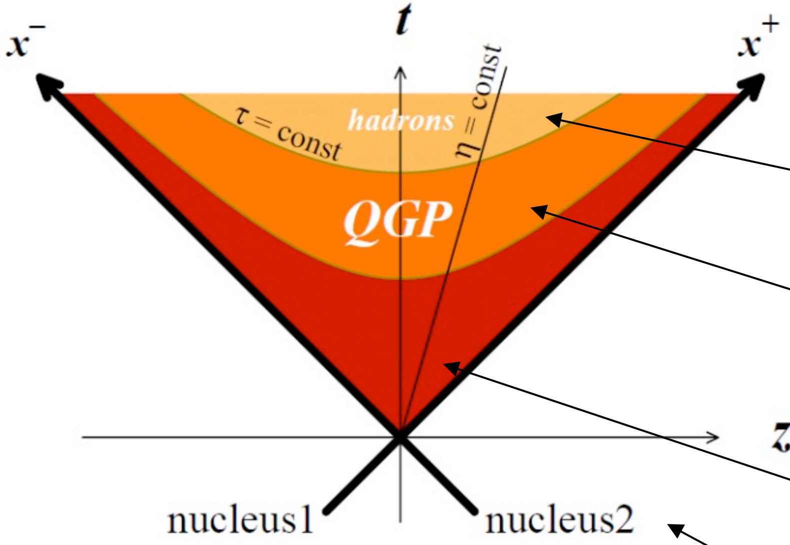
Initial eccentricities from the Glasma



Albacete, Guerrero-Rodriguez, CM, JHEP 1901 (2019) 073

What is the Glasma ?

the initial strong color field created after the collision



space-time picture of heavy-ion collisions

5. **Individual hadrons**

   freeze out

4. **Hadron gas**

   cooling with expansion

3. **Quark Gluon Plasma**

   thermalization, expansion

2. **Pre-equilibrium state** ← **Glasma**

   collision

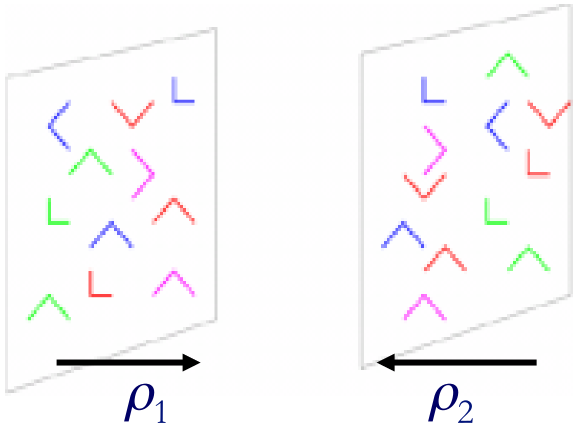
1. **Nuclei (initial condition)** ← **CGC**

$$v_n = \kappa_n \varepsilon_n$$

CGC = Color Glass Condensate

The collision of two CGCs

- the initial condition for the time evolution in heavy-ion collisions



before the collision:

$$J^\mu = \delta^{\mu +}\delta(x^-)\rho_1(x_\perp) + \delta^{\mu -}\delta(x^+)\rho_2(x_\perp)$$

$$\rho_1 \sim 1/g \qquad \rho_2 \sim 1/g$$

the distributions of ρ contain the small-x evolution of the nuclear wave functions

$$|\Phi_{x_1}[\rho_1]|^2 \qquad |\Phi_{x_2}[\rho_2]|^2$$

$\rho(x_\perp) = -\nabla^2\alpha(x_\perp)$ denotes the color charge which generates the field

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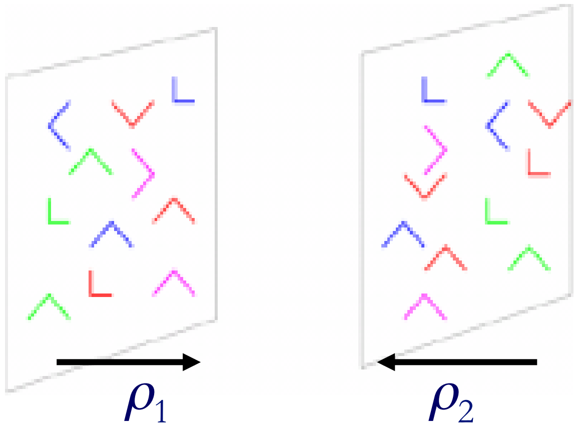

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- after the collision

the gluon field is a complicated function of the two classical color sources

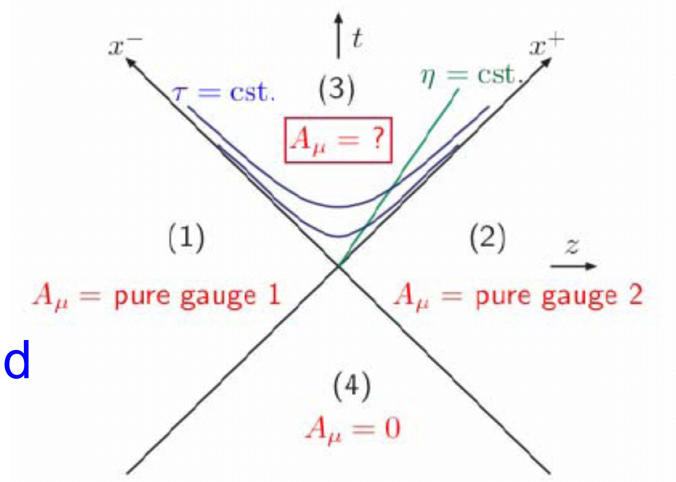
the field decays, once it is no longer strong (classical)
a particle description is again appropriate

"strong-field" QCD factorization

- solve Yang-Mills equations

$$[D_\mu, F^{\mu\nu}] = J^\nu \quad \longrightarrow \quad \mathcal{A}_\mu[\rho_1, \rho_2]$$

this is done numerically (it can be done analytically in the p+A case)



- express observables in terms of the field

determine $O[\mathcal{A}_\mu]$, in general a non-linear function of the sources

e.g. for this talk $$T^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^{\lambda\sigma} F_{\lambda\sigma} - F^{\mu\lambda} F^{\nu}_{\lambda}$$

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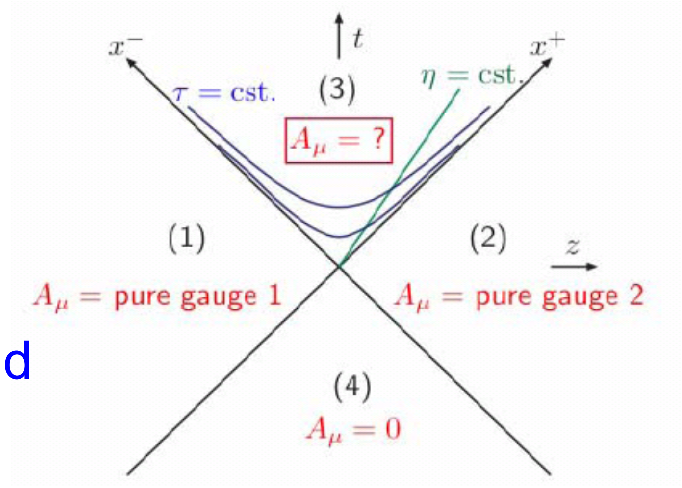

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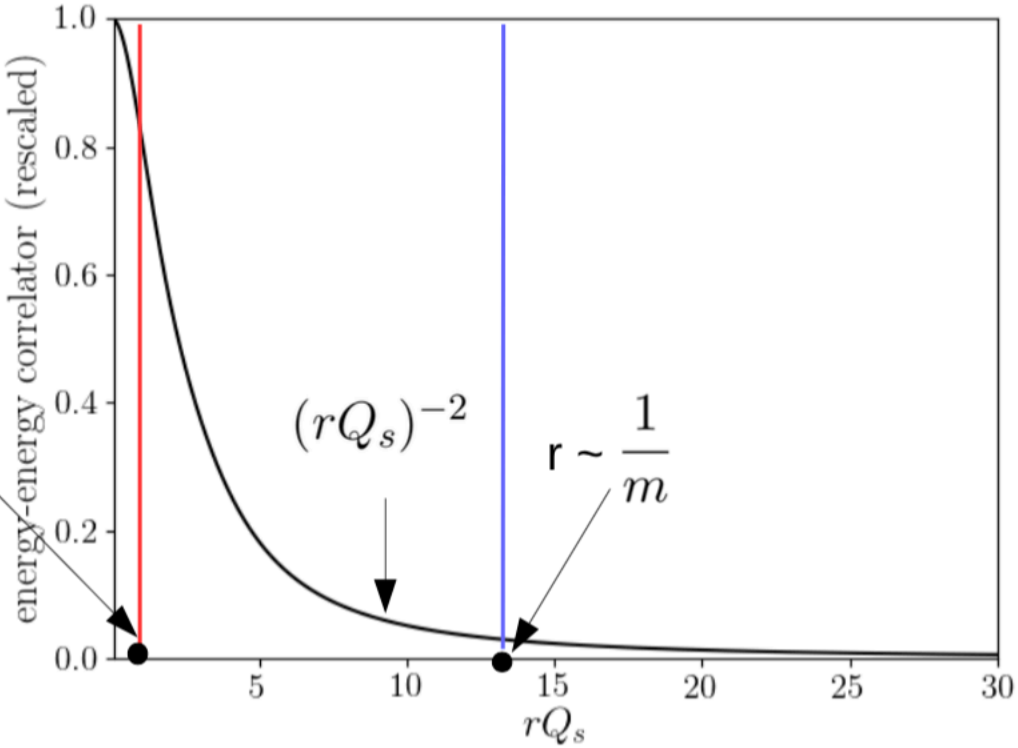
- perform the averages over the color charge densities

$$\langle O \rangle = \int D\rho_1 D\rho_2 |\Phi_{x_1}[\rho_1]|^2 |\Phi_{x_2}[\rho_2]|^2 O[\mathcal{A}_\mu]$$

we shall use the MV model for $|\Phi_{x_1}[\rho_1]|^2$ and $|\Phi_{x_2}[\rho_2]|^2$

$\longrightarrow$ each nucleus is characterized by its saturation scale $Q_s^2(\mathbf{s}) \propto T(\mathbf{s})$

Relevant features of $S(s_1,s_2)$



r ~ $\frac{1}{Q_s}$

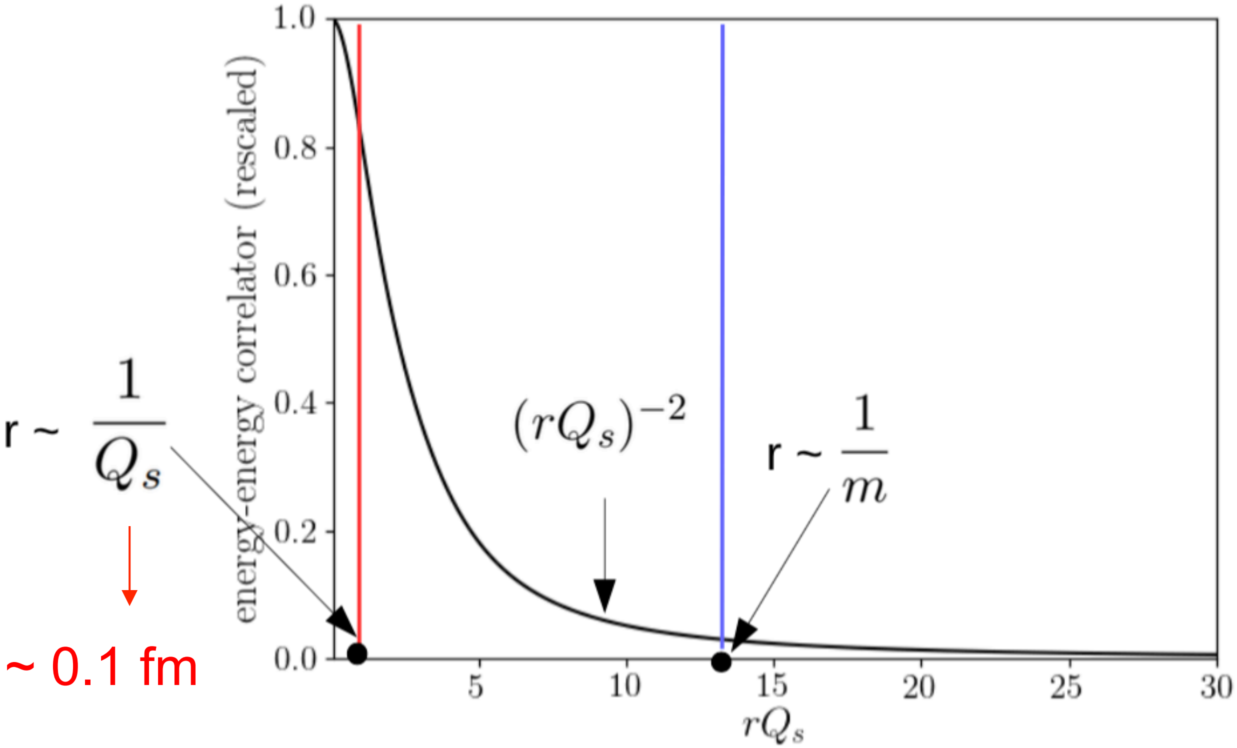
~ 0.1 fm

system size
~ 10 fm

$$\frac{1}{Q_s} \ll \frac{1}{m} \ll R$$

scale of color neutrality
~ 1 fm

Relevant features of $S(s_1,s_2)$



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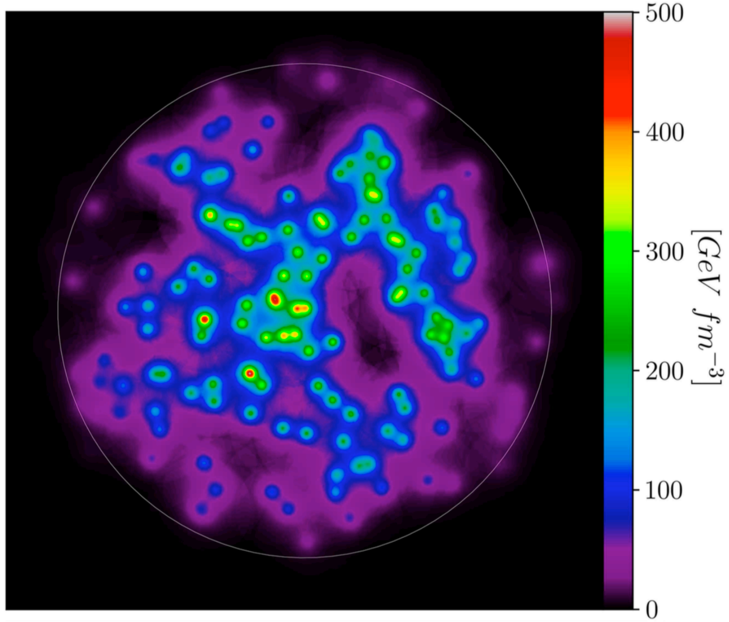
$$\frac{1}{Q_s} \ll \frac{1}{m} \ll R$$

scale of color
neutrality
~ 1 fm

$$\langle \rho(\mathbf{s}) \rangle = \frac{4}{3g^2} Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s})$$

$$\xi(\mathbf{s}) = \frac{16\pi}{9g^4} Q_A^2(\mathbf{s}) Q_B^2(\mathbf{s}) \left(Q_A^2(\mathbf{s}) \ln \left(1 + \frac{Q_B^2(\mathbf{s})}{m^2} \right) + Q_B^2(\mathbf{s}) \ln \left(1 + \frac{Q_A^2(\mathbf{s})}{m^2} \right) \right)$$

Results and data comparisons



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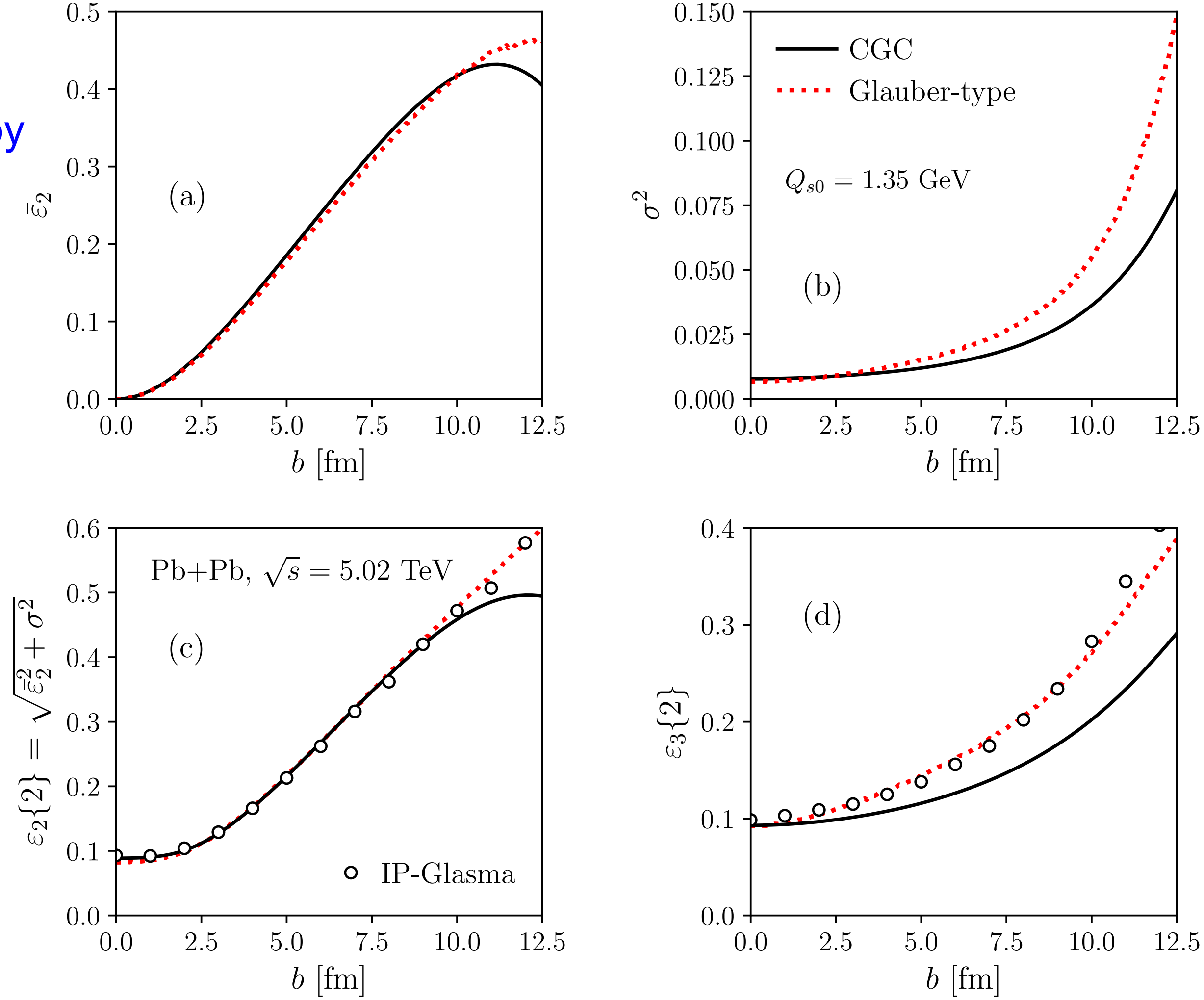
Comparison to MC Glauber models

mean anisotropy

ε_2 fluctuations



rms ε_2

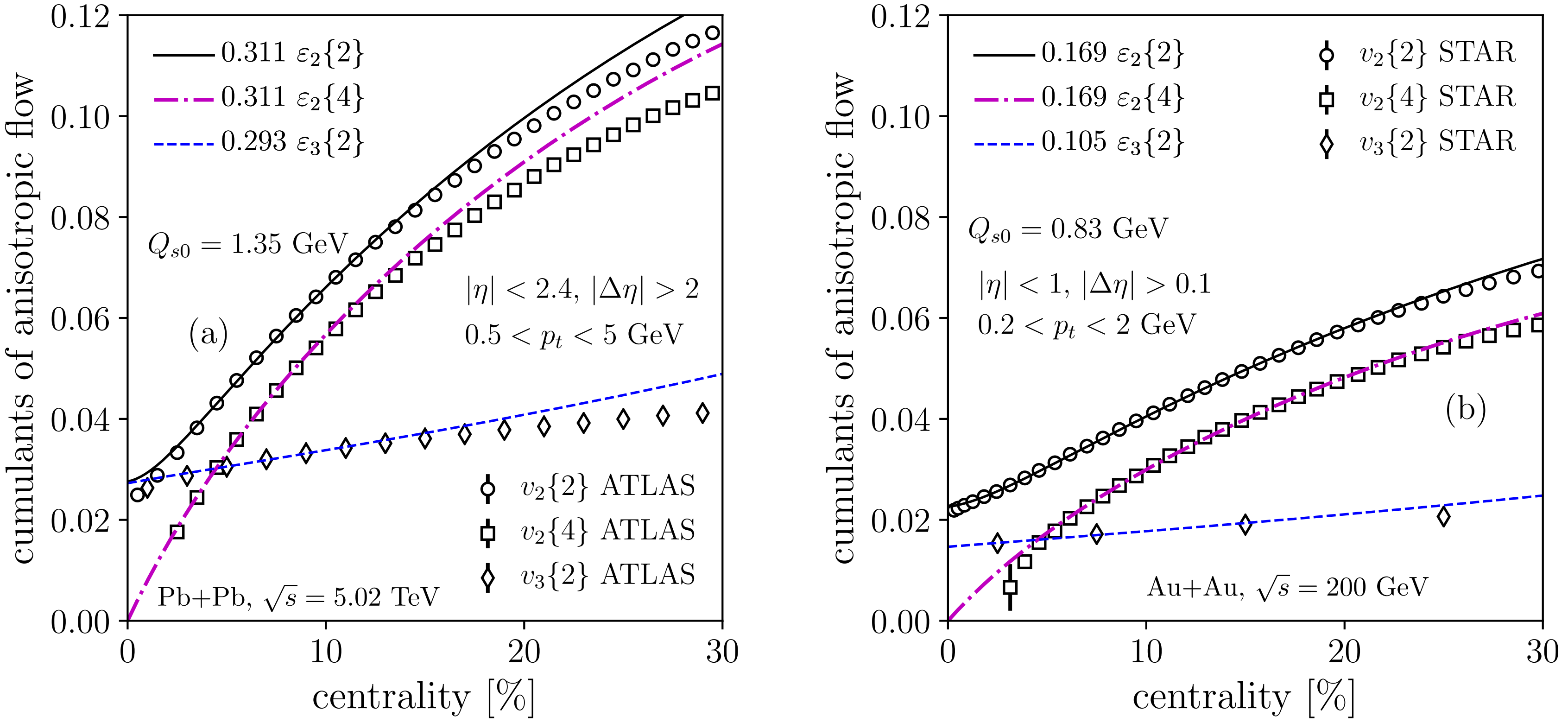
rms ε_3

IP Glasma ~ MC Glauber, its fluctuations are dominated by the nucleon position sampling

we only have fluctuations of the local color charge

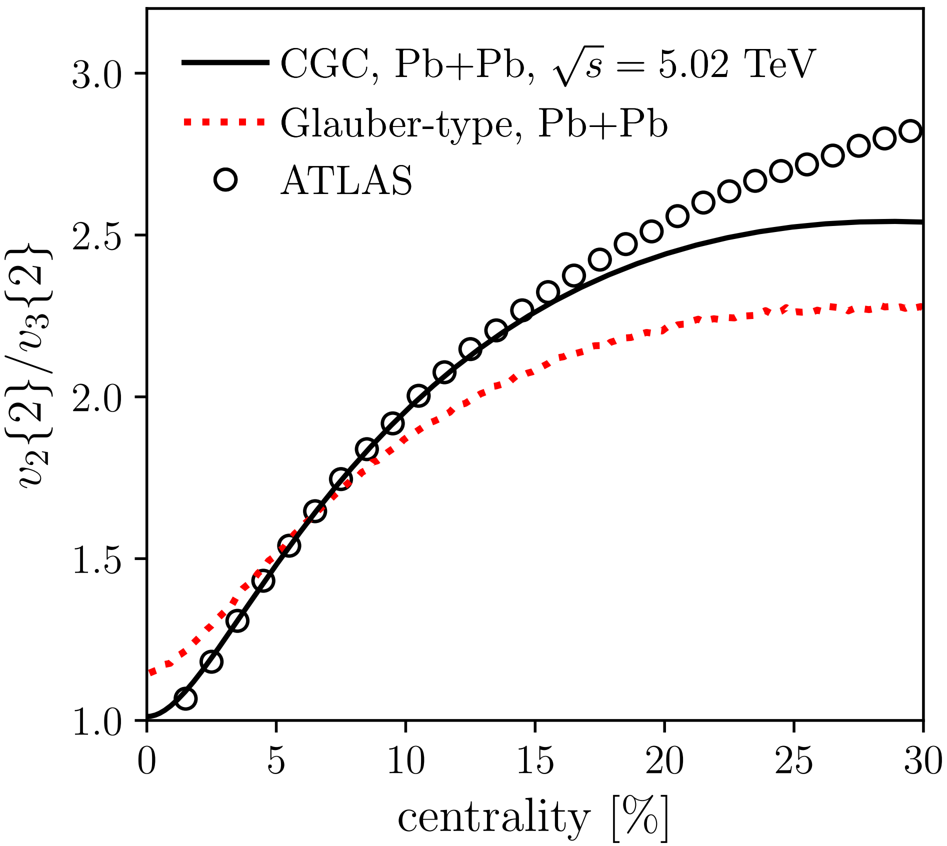
Comparison to data

- $v_2\{4\}$ simply probes the average geometry, we use it to fix κ_2
- then, with a reasonable Qs value, we can reproduce the fluctuations in $v_2\{2\}$



- the response coefficients are compatible with state-of-the-art hydro simulations
- Qs(LHC) > Qs(RHIC) explains more eccentricity fluctuations at RHIC

Triangular flow

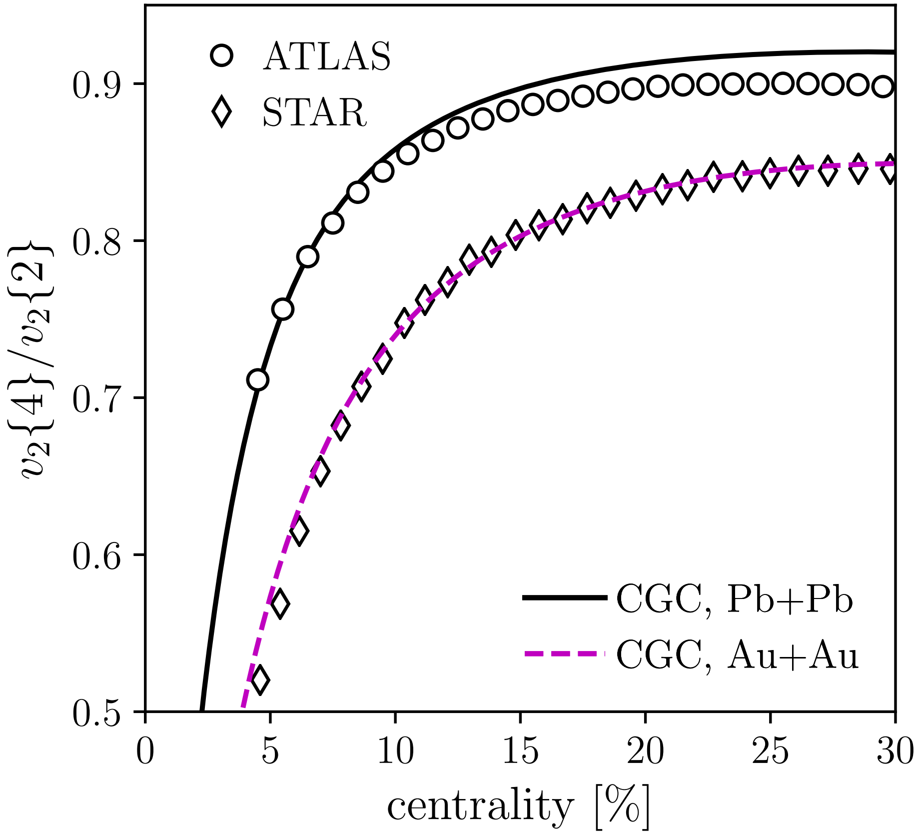


we solve a long-standing problem of hydro-to-data comparisons:
the ratio $v_2\{2\}$ / $v_3\{2\}$ grows quickly with centrality

Energy dependence

fluctuations produce the splitting between the $v_2\{4\}$ and $v_2\{2\}$

data indicate that they are larger at RHIC energies



Qs(LHC) ~ 1.3 GeV

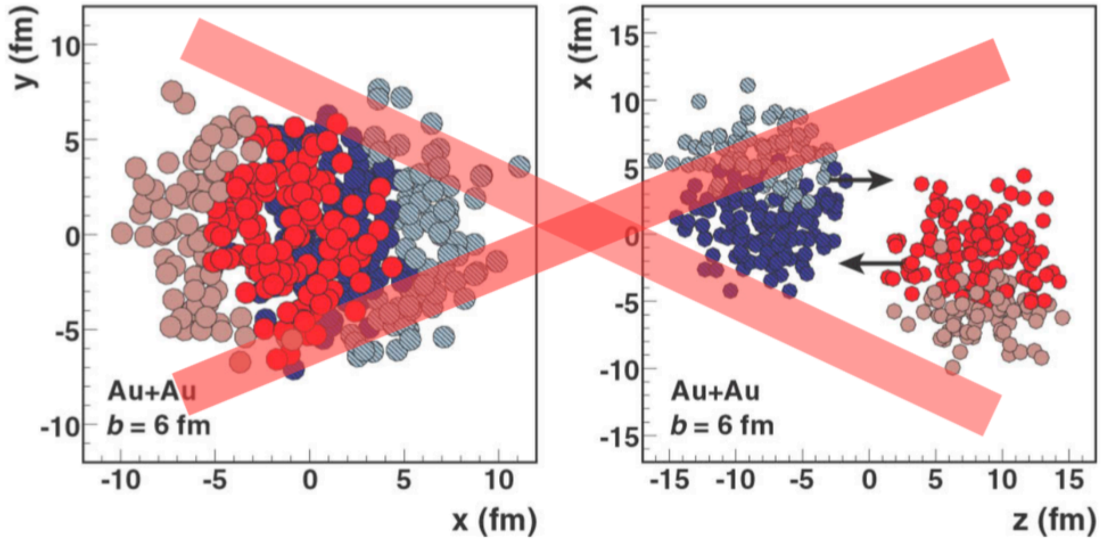
Qs(RHIC) ~ 0.8 GeV

this is compatible with what we expect from QCD evolution towards high energies

$$\frac{Q_s(x_1)}{Q_s(x_2)} = \left(\frac{\sqrt{s_2}}{\sqrt{s_1}}\right)^{\sim 0.28}$$

by contrast, MC Glauber-type calculations do not make any specific predictions for the $v_2\{4\}$ / $v_2\{2\}$ ratio

Conclusions



ab-initio description free from the Monte Carlo Glauber Ansatz:

- No random sampling of nucleons
- No ad hoc prescriptions about the deposition of energy
- Non perturbative physics only through the mass parameter