

Spin-3/2 Dark Matter in simple s or t channel models

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Based on:

1. M. O. Khojali, A. Goyal, M. Kumar and A. S. Cornell, Eur. Phys. J. C **77**, no. 1, 25 (2017); arXiv:1608.08958 [hep-ph].
2. M. O. Khojali, A. Goyal, M. Kumar and A. S. Cornell, Eur. Phys. J. C **78**, no. 11, 920 (2018); arXiv:1705.05149 [hep-ph].



The amount of CDM has been estimated to be
 $\Omega_{DM}\chi^2 = 0.1198 \pm 0.0015$

The nature of DM particles and their properties is the subject matter of intense investigation.

Detection of DM signatures involve:

- missing energy \cancel{E}_T accompanied by a single or two jet events
- direct detection experiments, which measure the nuclear-recoil energy and its spectrum in DM-nucleon elastic scattering



- indirect detection experiments looking for signals of DM annihilation into SM particles in the cosmic rays
...

Null results from the direct detection experiments have provided the most stringent upper bounds on the spin-independent DM-nucleon elastic scattering cross-section over a wide range of DM masses.



EFT in which the DM-SM interactions are mediated by scalar-heavy particles which are not accessible at the LHC energy have been analysed in detail and the limits from direct and indirect searches examined.

In these models the interaction between the DM and SM particles is mediated by spin-0 and spin-1 particles in the s -channel, whereas in the t -channel models, the mediator can be a scalar, a fermion or a vector particle which will typically also carry colour or lepton number

Here we consider a spin-3/2 DM particle as an alternative to the conventional scalar, vector or spin-1/2 CDM particles.



Spin-3/2 CDM has been studied in EFT models and constraints from the relic density, direct and indirect observations obtained.

Spin-3/2, 7.1 keV WDM was considered to provide an explanation from the anomalous 3.1 keV X-ray line observed by the XMM Newton.

Furthermore spin-3/2 DM with a Higgs portal has also been investigated.

Recall that spin-3/2 particles exist in several models BSM *viz* in models of supergravity, spin-3/2 fermions also exist in KK models in string theory and in models of composite fermions



The Model

The SM is extended by including a spin-3/2 particle χ which is a SM singlet that interacts with the SM particles through:

- the exchange of a vector particle Z' in the s -channel, done by extending the SM gauge symmetry by adding new $U(1)'$ symmetry which is spontaneously broken such that the mediator obtains a mass $m_{Z'}$.

We also invoke a discrete symmetry \mathbb{Z}_2 under which the spin-3/2 DM particle is odd whereas all other SM particles including the vector mediator are even.



- OR the exchange of a scalar (S) or a vector (V^μ) which carries a baryonic (colour) or lepton index. In general the mediator couples to right-handed up-type quarks (or leptons), right-handed down-type quarks (or leptons) or left-handed quark (or lepton) doublets.

We consider here the right-handed up-type quark case for simplicity, where the other cases are similar.



The spin-3/2 free Lagrangian is given by:

$$\mathcal{L} = \bar{\chi}_\mu(p) \Lambda^{\mu\nu} \chi_\nu(p),$$

with

$$\Lambda^{\mu\nu} = (i\not{\partial} - m_\chi)g^{\mu\nu} - i(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu) + i\gamma^\mu\not{\partial}\gamma^\nu + m_\chi\gamma^\mu\gamma^\nu,$$

where χ_μ satisfies $\Lambda^{\mu\nu}\chi_\nu = 0$ and being on mass-shell.

We have

$$(\not{p} - m_\chi)\chi_\mu(p) = \partial^\mu\chi_\mu(p) = \gamma^\mu\chi_\mu(p) = 0.$$



The spin sum for spin-3/2 fermions

$$S_{\mu\nu}^+(p) = \sum_{i=-3/2}^{3/2} u_{\mu}^i(p) \bar{u}_{\nu}^i(p), \text{ and } S_{\mu\nu}^-(p) = \sum_{i=-3/2}^{3/2} v_{\mu}^i(p) \bar{v}_{\nu}^i(p),$$

are given by:

$$S_{\mu\nu}^{\pm}(p) = -(\not{p} \pm m_{\chi}) \left[g_{\mu\nu} - \frac{1}{3} \gamma_{\mu} \gamma_{\nu} - \frac{2}{3m_{\chi}^2} \not{p}_{\mu} \not{p}_{\nu} \mp \frac{1}{3m_{\chi}} (\gamma_{\mu} p_{\nu} - \gamma_{\nu} p_{\mu}) \right]$$

In view of the non-renormalisable nature of interacting spin-3/2 theories, we can only write generic interactions which respect the SM gauge symmetry between the singlet, χ , and the SM particles.



The s -channel

The interaction of the spin-1 mediator with the SM-fermions and the DM χ_α is not restricted by MFV to be either vector or axial vector and can be written in terms of the vector and axial-vector couplings as

$$\mathcal{L}_{f,Z'} \supset \bar{\chi}_\alpha \gamma^\mu (g_\chi^V - \gamma^5 g_\chi^A) \chi_\beta Z'_\mu g^{\alpha\beta} + \sum_{f=q,l,\nu} \bar{f} \gamma^\mu (g_f^V - \gamma^5 g_f^A) f Z'_\mu.$$

In general this interaction will induce FCNCs which are strongly constrained by low energy phenomenology.

The constraints can be avoided by imposing MFV structure on the couplings or by restricting to one generation.



The decay width $\Gamma(Z' \rightarrow f\bar{f} + \chi_\alpha\bar{\chi}_\alpha)$ is given by

$$\begin{aligned}
&= \sum_f \frac{N_f}{12\pi} m_{Z'} \sqrt{1 - \frac{4m_f^2}{m_{Z'}^2}} \times \left[\left((g_f^V)^2 + (g_f^A)^2 \right) \right. \\
&\quad \left. + \frac{2m_f^2}{m_{Z'}^2} \left((g_f^V)^2 - 2(g_f^A)^2 \right) \right] + \frac{m_{Z'}}{108\pi} \left(\frac{m_\chi^2}{m_{Z'}^2} \right) \sqrt{1 - \frac{4m_\chi^2}{m_{Z'}^2}} \left[(g_\chi^V)^2 \right. \\
&\quad \times \left(36 - 2\frac{m_{Z'}^2}{m_\chi^2} - 2\frac{m_{Z'}^4}{m_\chi^4} + \frac{m_{Z'}^6}{m_\chi^6} \right) + (g_\chi^A)^2 \\
&\quad \times \left(-40 + 26\frac{m_{Z'}^2}{m_\chi^2} - 8\frac{m_{Z'}^4}{m_\chi^4} + \frac{m_{Z'}^6}{m_\chi^6} \right) \left. \right].
\end{aligned}$$



There are several interesting consequences on the DM mass and couplings arising from the decay width expressions.

If the DM mass $m_\chi > \frac{m_{Z'}}{2}$, the only decay channel available to the mediator Z' is into SM fermions. Since $\Gamma_{Z'} < m_{Z'}$, for the case of vector coupling g_f^V we get

$$\frac{8m_{Z'}}{12\pi} (g_f^V)^2 < m_{Z'} \quad \Rightarrow \quad (g_f^V)^2 < \frac{3\pi}{2},$$

similarly, for the axial coupling.

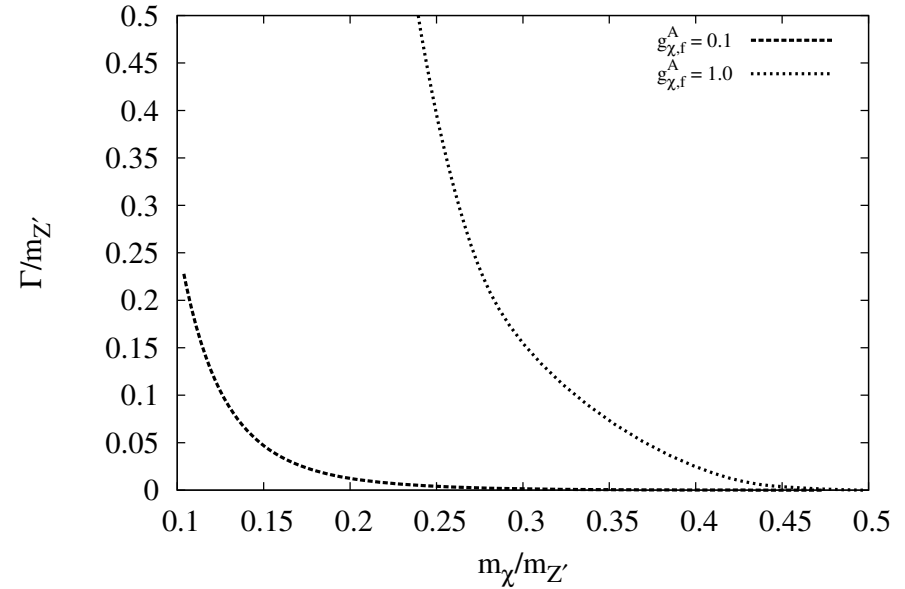
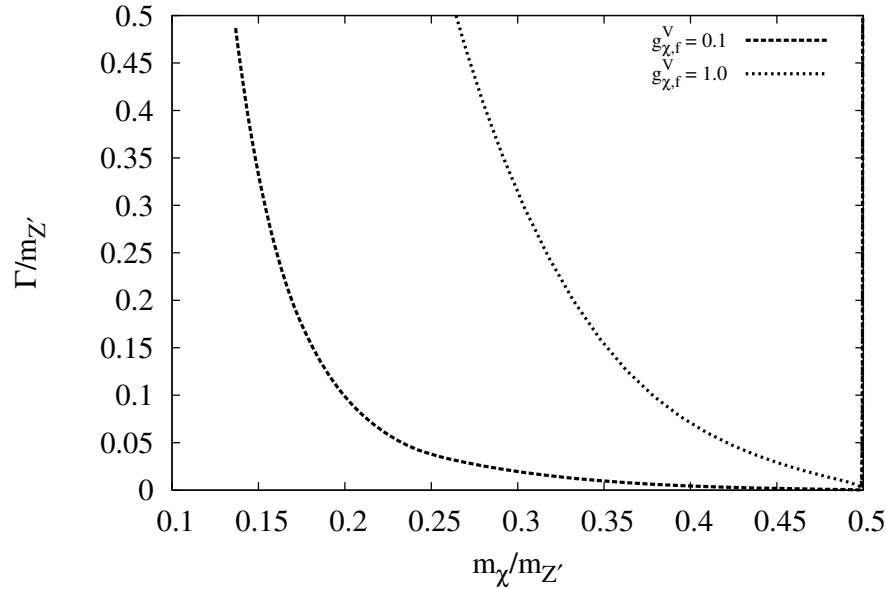


If DM mass $m_\chi < m_{Z'}/2$, allowing the mediator to decay into DM pairs, there is a minimum limit on the DM mass, given by roughly

$$\frac{1}{108\pi} \left(\frac{m_{Z'}}{m_\chi} \right)^4 (g_\chi^{V,A})^2 < 1$$

In what follows we consider the vector and axial-vector couplings separately and for simplicity use universal couplings $g_\chi^V = g_f^V$ and $g_\chi^A = g_f^A$ and restrict to one generation of SM fermions.





The mediator Z' decay width as a function of m_χ , in the left panel for vector couplings, and in the right panel for axial-vector.

There exists a minimum m_χ mass for a given coupling. A mass less than this limit gives unphysical values of the decay width. This feature is peculiar to the spin-3/2 nature of the DM.



Constraints on the s -channel

Relic Density

In the early universe, the DM particles χ 's were kept in thermal equilibrium with the rest of the plasma through the creation and annihilation of χ 's. The cross-section of annihilation process $\chi_\alpha \bar{\chi}_\alpha \rightarrow f \bar{f}$ proceeds through Z' and the spin averaged cross-section in the non-relativistic limit of the lab frame can be expanded in powers of v^2 as

$$\sigma|v| = a + bv^2 + \mathcal{O}(v^4)$$



The relic density contributed by the DM particles χ can be obtained by numerically solving the Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma|v|\rangle[n_\chi^2 - (n_\chi^{eq})^2],$$

where

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi\rho}{3M_{pl}^2}},$$

is the Hubble rate, $M_{pl} = 1.22 \times 10^{19}$ GeV is the Planck mass and $\langle\sigma|v|\rangle$ is the thermally averaged χ -annihilation cross-section, $\langle\sigma(\chi\bar{\chi} \rightarrow f\bar{f})v\rangle$.



The Boltzmann equation is solved numerically to yield

$$\Omega_{DM}\chi^2 \simeq \frac{2 \times 1.07 \times 10^{19} X_F}{M_{pl} \sqrt{g_*} (a + \frac{3b}{X_F})},$$

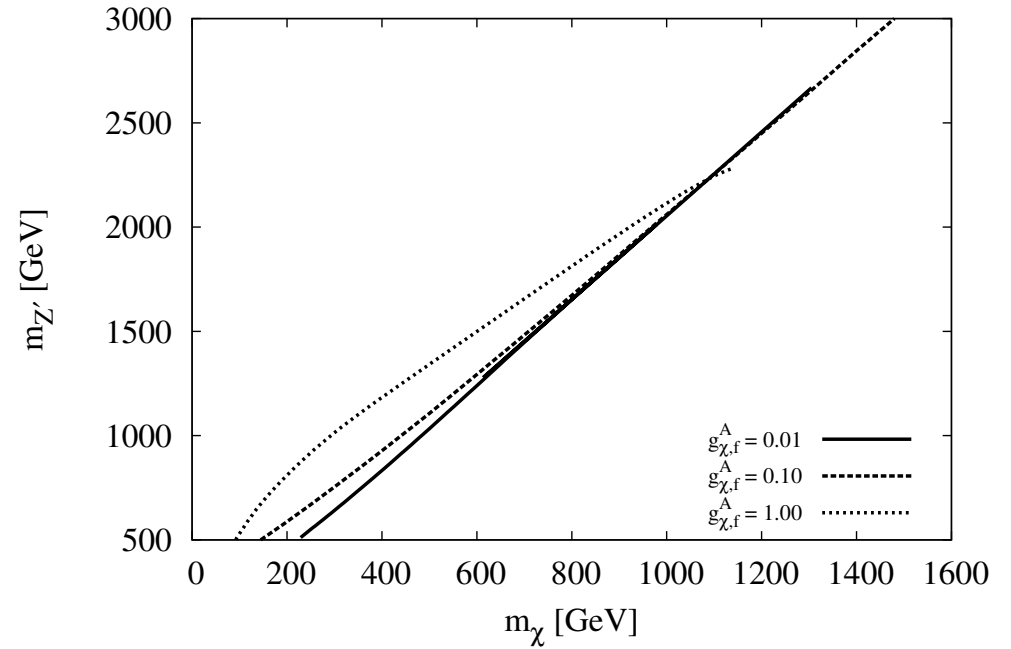
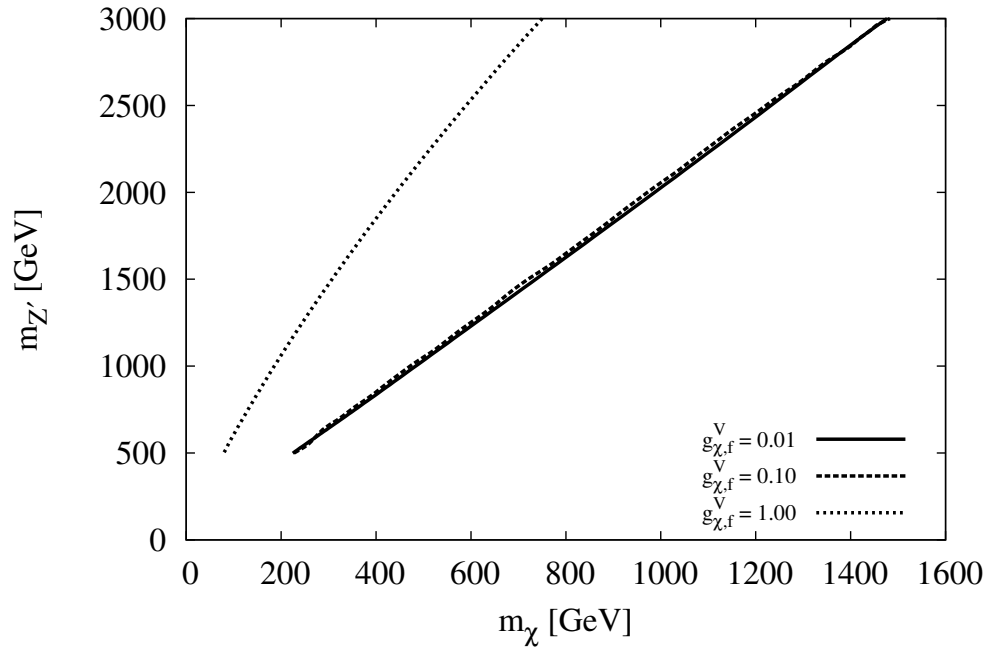
g_* is the number of degrees of freedom at T_F and is taken to be 92 for $m_b < T_F < m_{Z'}$, $X_F = m_\chi/T_F$.

The freeze-out temperature is obtained by solving

$$X_F = \ln \left[c(c+2) \sqrt{\frac{45}{8}} \frac{g M_{pl} m_\chi (a + \frac{6b}{X_F})}{2\pi^3 \sqrt{g_*} \sqrt{X_F}} \right],$$

where c is order of a unit and it taken to be 1/2. g for spin-3/2 DM is 4. The expressions for a and b are given in the paper.





The contour graphs between the mass of the mediator and DM mass, by assuming that the DM saturates the observed DM density. The left panel for the vector couplings. The right panel for the axial-vector couplings.



Direct Detection

Constraints from DM detection experiments can be obtained from the elastic DM-nucleon cross-section.

Owing to the presence of both vector and axial-vector couplings, the DM-nucleon scattering has both spin-independent and spin-dependent components.

The spin-independent and spin-dependent sub-dominant cross-sections are given by

$$\sigma_{\chi N}^{SI} = \frac{\mu^2 f_N^2}{\pi m_{Z'}^4} = \frac{9\mu^2 (g_q^V g_\chi^V)^2}{\pi m_{Z'}^4}$$



$$\sigma_{\chi N}^{SI} \simeq 1.4 \times 10^{-37} (g_{\chi}^V g_q^V) \left(\frac{\mu}{1\text{GeV}} \right)^2 \left(\frac{300\text{GeV}}{m_{Z'}} \right)^4 \text{cm}^2,$$

and

$$\begin{aligned} \sigma_{\chi N}^{SD} &= \frac{5\mu^2}{3\pi m_{Z'}^4} a_N^2 = \frac{5\mu^2 (g_q^A g_{\chi}^A)^2}{3\pi m_{Z'}^4} (\Delta u^N + \Delta d^N + \Delta s^N)^2 \\ &\simeq 4.7 \times 10^{-39} (g_{\chi}^A g_q^A) \left(\frac{\mu}{1\text{GeV}} \right)^2 \left(\frac{300\text{GeV}}{m_{Z'}} \right)^4 \text{cm}^2, \end{aligned}$$

where $\mu = \mu_N = (m_p + m_n)/2 \simeq 0.939$ GeV is the nucleon-mass for direct detection



$$f_p = g_\chi^V (2g_u^V + g_d^V), \quad f_n = g_\chi^V (2g_d^V + g_u^V)$$

$$\text{and } a_{p,n} = \sum_{q=u,d,s} g_\chi^A \Delta q^{p,n} g_q^A.$$

The coefficients $\Delta q^{p,n}$ depend on the light quark contributions to the nucleon spin:

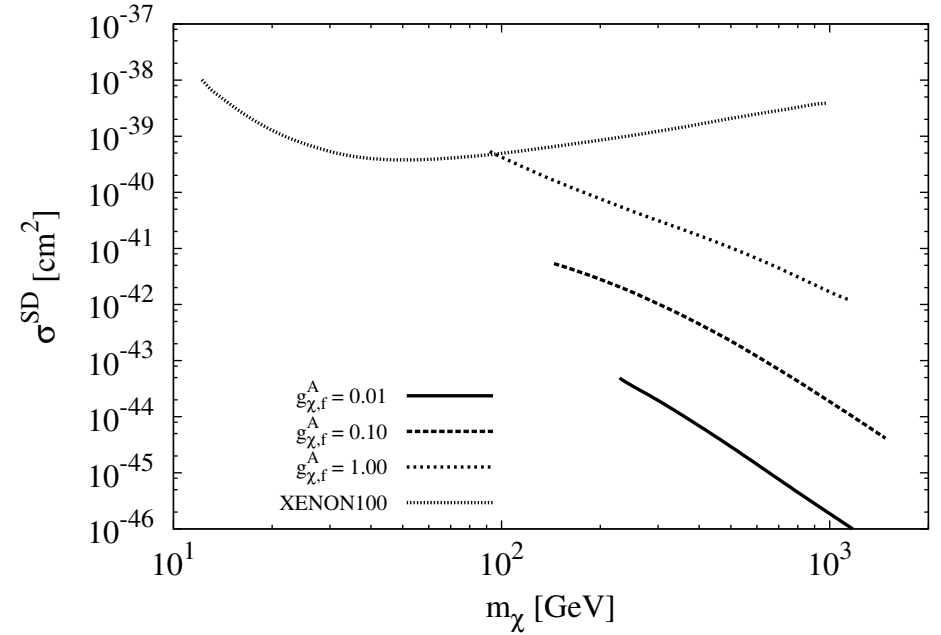
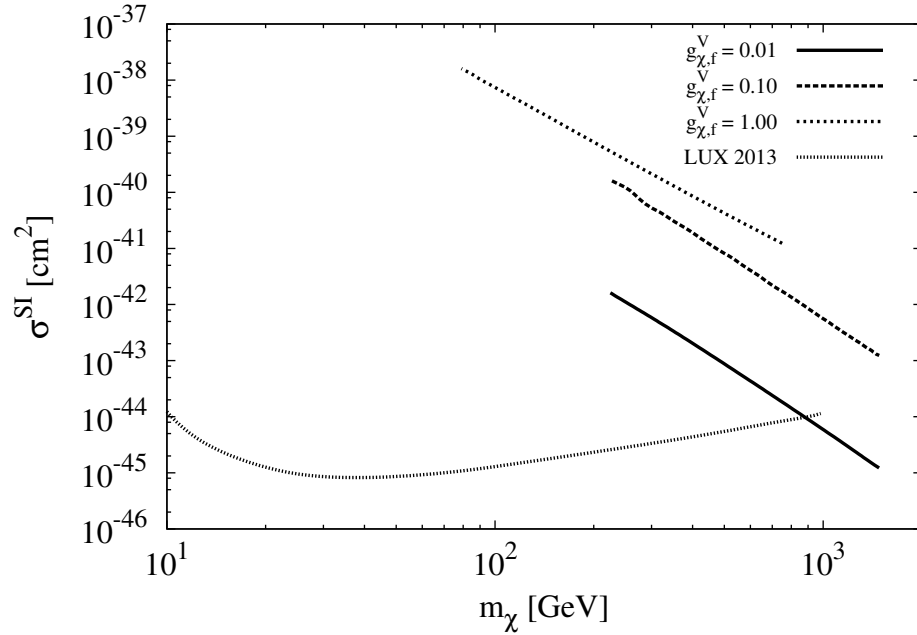
$$\Delta u^p = \Delta d^n = 0.84 \pm 0.02,$$

$$\Delta d^p = \Delta u^n = -0.43 \pm 0.02,$$

$$\Delta s^p = \Delta s^n = -0.09 \pm 0.02.$$

The cross-section-axial vector terms proportional to $g_q^V g_\chi^V$ are suppressed by the momentum transfer or the DM vel.





The predictions for the spin-independent and spin-dependent cross-sections as a function of DM mass m_χ .

The corresponding experimental bounds from LUX and XENONIT are also displayed for three representative values of vector and axial-vector couplings. The mediator mass $m_{Z'}$ is set to give the observed relic density for all values of m_χ .

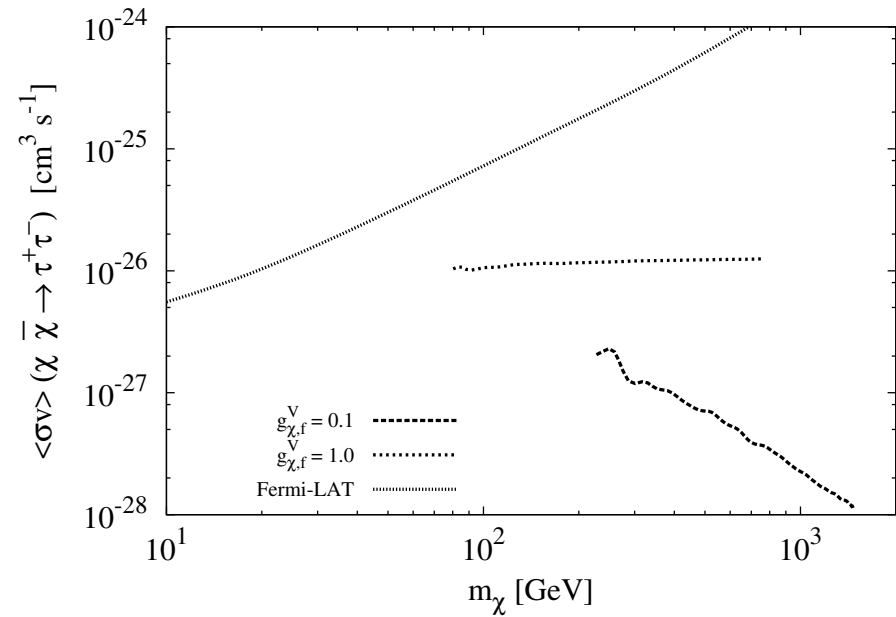
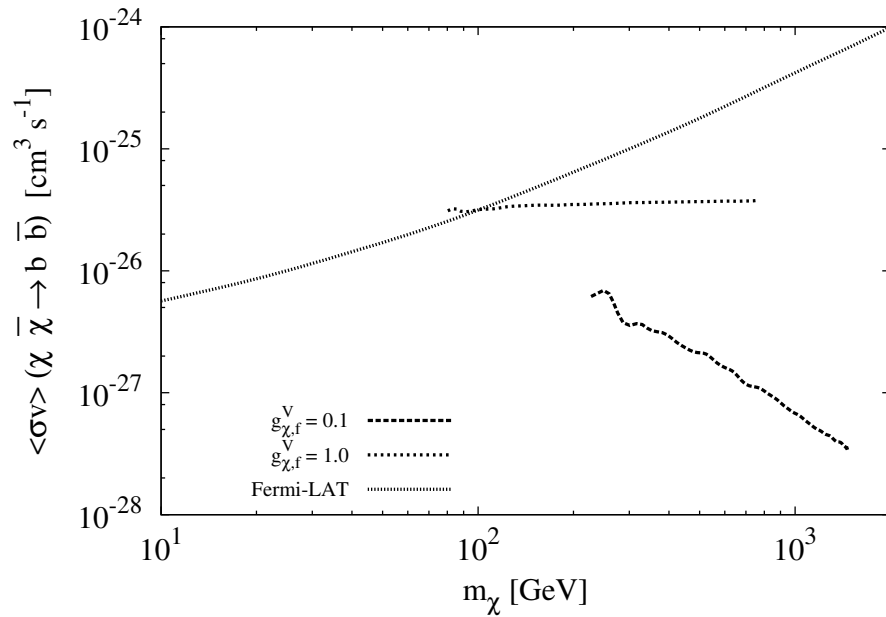


Indirect Detection

DM annihilation in the universe would result in cosmic ray fluxes which can be observed by dedicated detectors.

The Fermi LAT collaboration has produced constraints on the DM annihilation cross-section into a few final state *viz* e^+e^- , $\mu^+\mu^-$, $\tau^+\tau^-$, $b\bar{b}$, $u\bar{u}$, W^+W^- , etc.





The prediction for the DM annihilation into $b\bar{b}$ and $\tau\bar{\tau}$ for two representative values of the vector as well as axial-vector couplings as a function of m_χ .

The predictions shown here for different couplings and DM mass m_χ are consistent with the observed relic density.

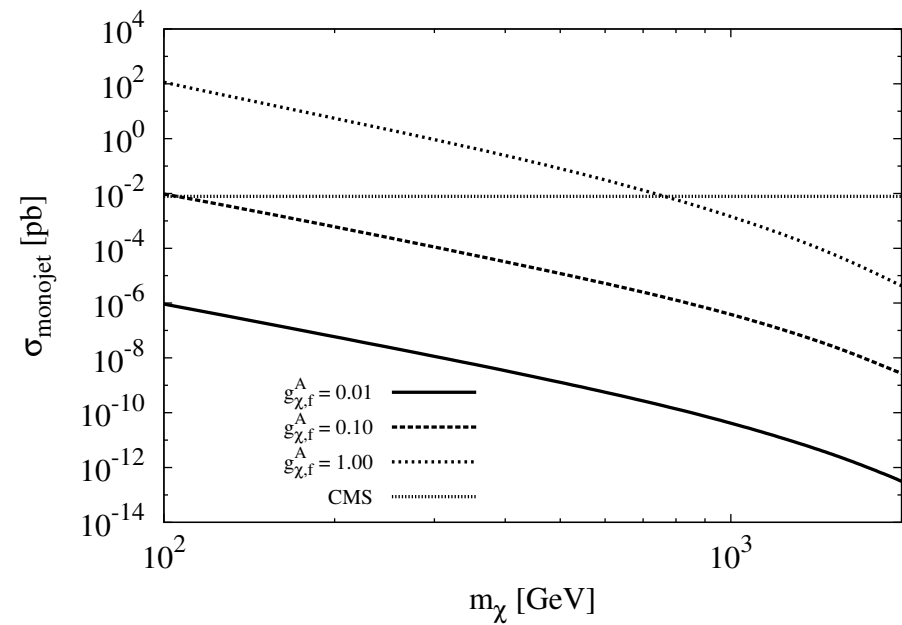
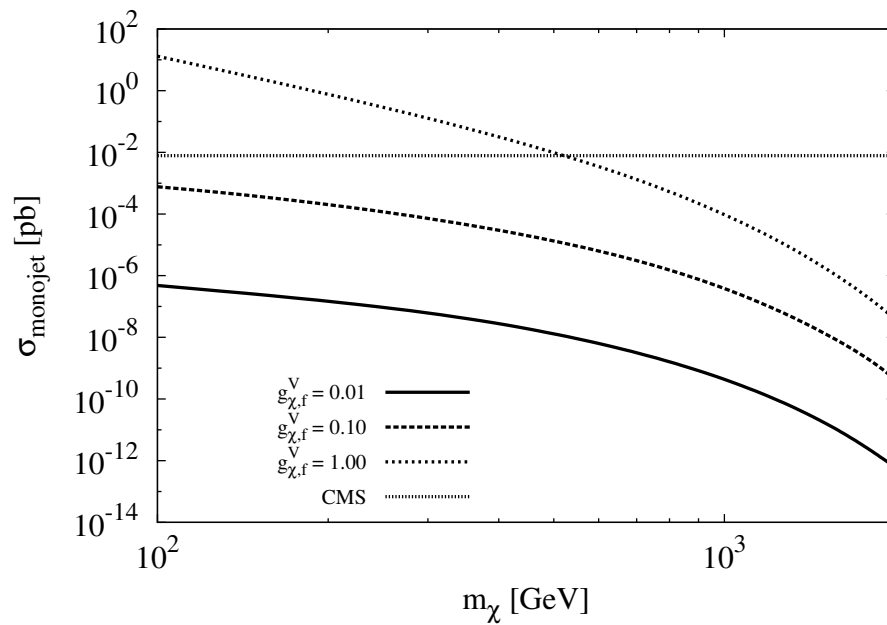


Monojet Searches

Monojet searches at the LHC with missing transverse energy \cancel{E}_T have been used by CMS at 8 TeV, based on an integrated luminosity of 19.7 fb^{-1} to put constraints on the interaction of quarks and DM particles.

The parameter space $(m_\chi, m_{Z'})$ for spin-3/2 DM consistent with the observed DM density for benchmark couplings used here, these were considered to obtain bounds by requiring $\cancel{E}_T > 450 \text{ GeV}$ for which the CMS results exclude new contributions to monojet cross-sections exceeding 7.8 fb at 95% CL.





The resulting monojet cross-section for the vector and axial-vector couplings find that for the vector coupling the results are in agreement with the bounds from the direct detection experiments.

In the case of axial couplings, the monojet search puts stronger constraints on the parameters in comparison to the constraints from direct and indirect searches albeit for $g_{\chi,f}^A \sim 1$.



The t -channel

Looking at the vector and scalar mediator cases separately:

(a) Scalar mediator S :

For the scalar mediator case we can write the SM gauge invariant interaction as:

$$\mathcal{L}_{int} \supset -\frac{(g_\chi^S)^i}{\Lambda} \bar{\chi}_\mu g^{\mu\nu} u_R^i D_\nu S_i^* + \text{h.c.}$$

In this case we do not have a dimension-4 interaction term because the χ_μ , which on mass-shell, satisfies $\gamma^\mu \chi_\mu = 0$, and thus it is impossible to construct a Lorentz-invariant dimension-4 interaction term involving χ_μ , S and the Dirac field u_R .



- (b) Vector mediator V_μ : In this case we can write dimension-4 as well as dimension-5 interaction terms, namely

$$\mathcal{L}_{int} \supset i \left(c_\chi^V\right)^i \bar{\chi}_\mu u_R^i (V_i^\mu)^* + \text{h.c.}$$

and

$$\mathcal{L}_{int} \supset i \frac{(g_\chi^V)^i}{\Lambda} \bar{\chi}_\mu g^{\mu\alpha} \gamma^\beta u_R^i V_{\alpha\beta}^{*i} + \text{h.c.} .$$



For all calculations we set $\Lambda = 1$ TeV, and the interaction Lagrangian for the scalar and vector can be written as:

$$\mathcal{L}_{scalar} = (D_\mu S_i)^\dagger (D^\mu S_i) - m_{S_i}^2 S_i^\dagger S_i,$$

$$\mathcal{L}_{vector} = -\frac{1}{4} V_{\mu\nu}^{\dagger i} V_i^{\mu\nu} + m_V^2 V_{\mu i}^\dagger V^{\mu i} + i g_s V_{\mu i}^\dagger t^a V_\mu^i G_a^{\mu\nu},$$

where $V_{\mu\nu}^i = D_\mu V_\nu^i - D_\nu V_\mu^i$.

The covariant derivative is given by

$$D_\mu = \partial_\mu + i g_s t_a G_\mu^a + i g \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu + i g' \frac{1}{2} Y B_\mu,$$

where g_s is the QCD strong coupling constant.



Unlike the s -channel mediator, where a single mediator is required, in the t -channel model we require a different mediator for each generation.

In general, the interaction given in Lagrangians induce FCNCs, which are strongly constrained by low energy phenomenology.

The FCNC constraints can be avoided by imposing a MFV structure on the Yukawa couplings.

The parameter space will consist of the DM candidates mass m_χ , the vector (scalar) couplings $\left((c_\chi^V)^i, (g_\chi^V)^i\right), (g_\chi^S)^i$ and the mediator masses m_S^i (m_V^i), for each generation.



For simplicity we will set the couplings and mediator masses for all the generations to be equal.

If the mediator mass is in the kinematically accessible region of the LHC, the decay of the mediator and the ensuing signal will become important.

The decay width of the scalar and vector mediators $\Gamma(S^i / V^i \rightarrow \chi \bar{u}_i)$, dropping the generation index, are given by:



$$\begin{aligned}
\Gamma(S \rightarrow \chi \bar{u}) &= \frac{(g_\chi^S)^2 m_S^5}{96 \pi \Lambda^2 m_\chi^2} \left[1 - \left(\frac{m_\chi}{m_S} + \frac{m_u}{m_S} \right)^2 \right] \\
&\times \left[1 - \left(\frac{m_\chi}{m_S} - \frac{m_u}{m_S} \right)^2 \right] \times \left[1 - \frac{m_\chi^2}{m_S^2} - \frac{m_u^2}{m_S^2} \right] \\
&\times \lambda^{1/2} \left(1, \frac{m_\chi^2}{m_S^2}, \frac{m_u^2}{m_S^2} \right) \\
&\simeq \frac{(g_\chi^S)^2 m_S^5}{96 \pi \Lambda^2 m_\chi^2} \left(1 - \frac{m_\chi^2}{m_S^2} \right)^4,
\end{aligned}$$

since $m_S^i, m_\chi \gg m_u$ is true for all quarks, except the top quark, and $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$;



$$\begin{aligned}
\Gamma(V \rightarrow \chi \bar{u}) &= \frac{(c_\chi^V)^2 m_V}{288 \pi} \left(1 - \frac{m_\chi^2}{m_V^2} - \frac{m_u^2}{m_V^2} \right) \left[5 + \frac{m_V^2}{m_\chi^2} + \frac{m_\chi^2}{4 m_V^2} \right. \\
&\quad \left. - \frac{m_u^2}{m_\chi^2} - \frac{m_u^2}{2 m_V^2} + \frac{m_u^4}{4 m_V^2 m_\chi^2} \right] \lambda^{1/2} \left(1, \frac{m_\chi^2}{m_V^2}, \frac{m_u^2}{m_V^2} \right) \\
&\simeq \frac{(c_\chi^V)^2 m_V}{288 \pi} \left(1 - \frac{m_\chi^2}{m_V^2} \right)^2 \left(5 + \frac{m_V^2}{m_\chi^2} + \frac{m_\chi^2}{4 m_V^2} \right),
\end{aligned}$$

for the dimension-4 Lagrangian

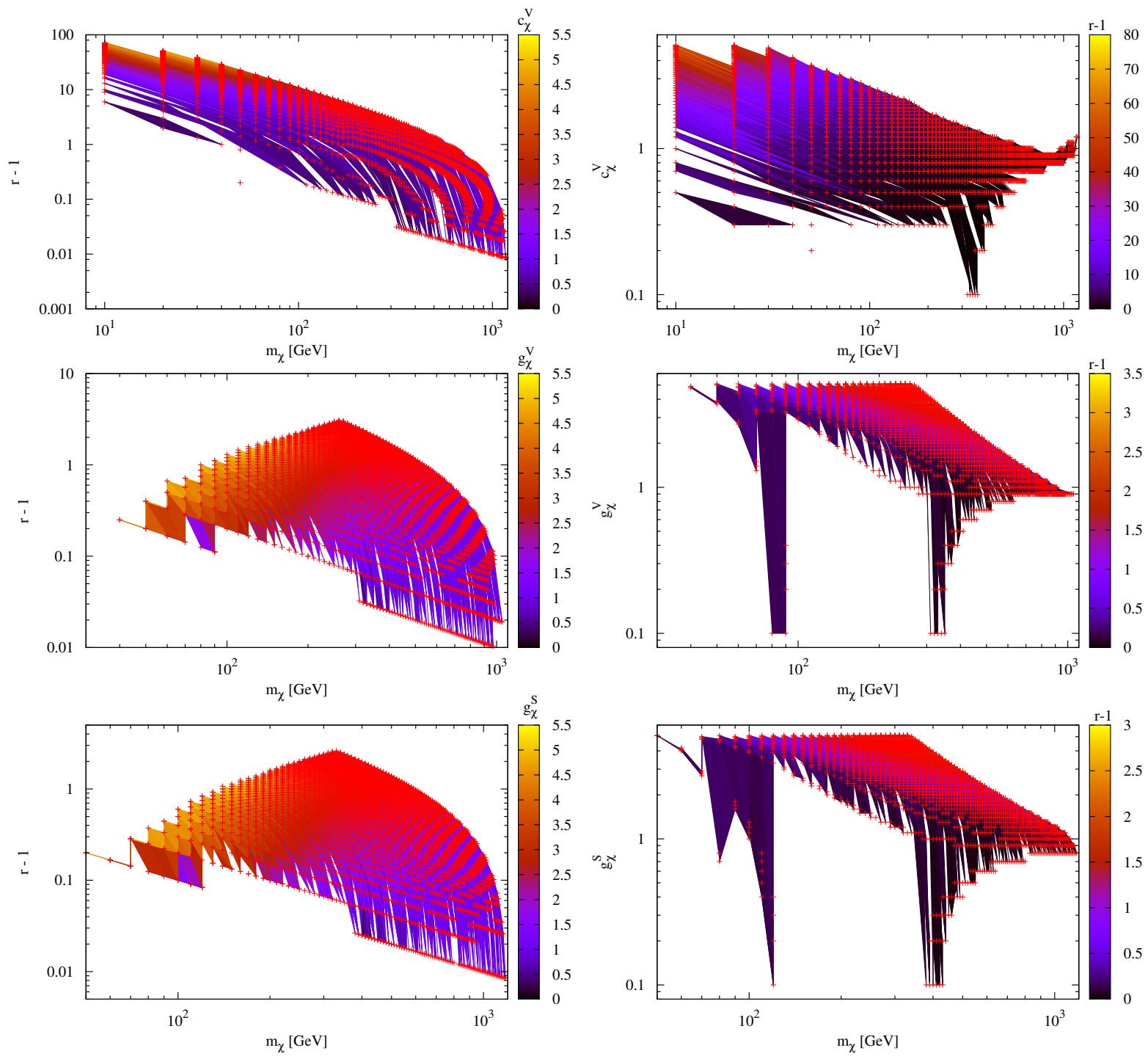


and

$$\begin{aligned}\Gamma(V \rightarrow \chi \bar{u}) = & \frac{(g_\chi^V)^2 m_V^5}{288 \pi \Lambda^2 m_\chi^2} \left[\frac{m_\chi^2}{m_V^2} + \frac{m_\chi^4}{m_V^4} - \frac{3 m_\chi^6}{m_V^6} \right. \\ & \left. + \left(1 - \frac{m_u^2}{m_V^2} \right)^3 + \frac{5 m_\chi^4 m_u^2}{m_V^6} - \frac{m_\chi^2 m_u^4}{m_V^6} \right] \\ & \times \lambda^{1/2} \left(1, \frac{m_\chi^2}{m_V^2}, \frac{m_u^2}{m_V^2} \right) \simeq \frac{(g_\chi^V)^2 m_V^5}{288 \pi \Lambda^2 m_\chi^2} \\ & \times \left[1 + \frac{m_\chi^2}{m_V^2} + \frac{m_\chi^4}{m_V^4} - \frac{3 m_\chi^6}{m_V^6} \right] \left(1 - \frac{m_\chi^2}{m_V^2} \right),\end{aligned}$$

for the dimension-5 interaction Lagrangian.





Contour plots in the allowed DM mass m_χ and the mass splitting ratio $r - 1$ (with $r = m_S(m_V)/m_\chi$) in the left panels, and in the DM mass m_χ and the couplings in the right panels.

We have assumed that the DM χ saturates the observed relic density, where the top panels are for the dimension-4 interaction term for the vector mediator case. The middle and the bottom panels are for dimension-5 vector and scalar mediator cases respectively.

In the left panels the colour gradient corresponds to the Yukawa couplings required to give the observed relic density, which in the right panels, the colour gradient corresponds to the mass splitting consistent with the observed relic density.



Constraints on the t -channel model

In this section we examine the constraints on the model parameters m_χ, m_S, m_V and the coupling constants from the relic density, direct and indirect observations.

Relic density

In the early Universe the DM relic density is determined by the dominant DM annihilation processes $\chi \bar{\chi} \rightarrow u \bar{u}$, mediated by the t -channel exchange of scalar/vector mediators.



Since the mediators in this model carry colour and charge, co-annihilation processes like $\chi S(V) \rightarrow u g$ and $S S^* (V V^*) \rightarrow g g$ (even though exponentially suppressed when mass splitting $(m_{S/V} - m_\chi) > T_f$), will be important if the DM mass gets close to the mediator mass.

The co-annihilation processes $\chi S(V) \rightarrow u g$ are mediated by t -channel exchanges of mediators, as well as by s -channel exchanges of gluons and through the four-point interaction involving the DM, mediator, u -quark and the gluon vertex.

These processes will reduce the Yukawa couplings needed to generate the required thermal relic abundance.



Self annihilation mediator processes $S S^* (V V^*) \rightarrow g g$ are generated by purely gauge interactions, and are independent of the Yukawa couplings, having the potential to suppress the relic density below the observed value.

At freeze-out the DM and mediator particles are non-relativistic. In the non-degenerate parameter space the channel

$\chi \bar{\chi} \xrightarrow{S/V} u \bar{u}$ cross-section can be easily evaluated, and in the limit $m_\chi, m_S, m_V \gg m_u$ are given by

$$\langle \sigma(\chi \bar{\chi} \xrightarrow{S} u \bar{u}) | v \rangle \simeq \frac{(g_\chi^S)^4 m_\chi^2}{768 \pi \Lambda^4} \frac{1}{(1 + r^2)^2},$$



$$\langle \sigma(\chi \bar{\chi} \xrightarrow{V} u \bar{u}) | v \rangle \simeq \frac{(c_\chi^V)^4}{1536 \pi m_\chi^2} \frac{1}{(1 + r^2)^2} \left[5 - \frac{4}{r^2} + \frac{2}{r^4} \right],$$

and

$$\langle \sigma(\chi \bar{\chi} \xrightarrow{V} u \bar{u}) | v \rangle \simeq \frac{(g_\chi^V)^4 m_\chi^2}{768 \pi \Lambda^4} \frac{1}{(1 + r^2)^2} \left[5 + \frac{1}{24} \frac{1}{(1 + r^2)^2} \right],$$

for the scalar-mediator and the vector-mediator dimension-4 and dimension-5 interaction Lagrangians, with the mass ratio $r = m_S(m_V)/m_\chi$.



The thermal relic density of χ 's is obtained by solving the Boltzmann equation:

$$\frac{d\eta_\chi}{dt} + 3H\eta_\chi = -\langle\sigma|v\rangle (\eta_\chi^2 - (\eta_\chi^{\text{eq}})^2),$$

where H is the Hubble constant, $\langle\sigma|v\rangle$ is the thermally averaged χ -annihilation cross-section, and

$$\eta_\chi^{\text{eq}} = 4 \left(\frac{m_\chi T}{2\pi} \right)^{3/2} \exp \left(-\frac{m_\chi}{T} \right).$$

The freeze out occurs when the χ 's are non-relativistic, and then $\langle\sigma|v\rangle$ can be written as:

$$\sigma|v| = a + b v^2 + \mathcal{O}(v^4).$$



The Boltzmann equation can be solved numerically to yield:

$$\Omega_{\text{DM}} h^2 \simeq \frac{2 \times 1.07 \times 10^9 X_F}{M_{pl} \sqrt{g_*} (a + \frac{3b}{X_F})},$$

where g_* is the number of degrees of freedom at freeze-out temperature T_F , $X_F = m_\chi/T_F$ is obtained by solving

$$X_F = \ln \left[c(c+2) \sqrt{\frac{45}{8}} \frac{g M_{pl} m_\chi (a + \frac{6b}{X_F})}{2\pi^3 \sqrt{g_*(X_F)} \sqrt{X_F}} \right],$$

where c is taken to be $1/2$. $g_*(X_F)$ is taken to be 92 for our estimate, and $g = 4$.



The annihilation cross-section for the co-annihilation processes $\chi S(V) \rightarrow u g$ in this limit are given by

$$\langle \sigma(\chi S \rightarrow u g) | v \rangle \simeq \frac{(g_\chi^S)^2 g_s^2}{64 \pi \Lambda^2} \frac{(1 + r)}{r^3} \left[1 + \frac{14}{9} r + \frac{13}{27} r^2 \right],$$

$$\begin{aligned} \langle \sigma(\chi V \rightarrow u g) | v \rangle \simeq & \frac{(c_\chi^V)^2 g_s^2}{165888 \pi m_\chi^2} \frac{1}{r^6 (1 + r)} \left[1164 + 5628 r \right. \\ & + 11403 r^2 + 12568 r^3 + 8242 r^4 \\ & \left. + 2452 r^5 + 319 r^6 \right], \end{aligned}$$



and

$$\langle \sigma(\chi V \rightarrow u g) | v \rangle \simeq \frac{(g_\chi^V)^2 g_s^2}{497664 \pi \Lambda^2} \frac{1}{r^5(1+r)} \left[372 + 2724 r + 6537 r^2 + 8742 r^3 + 7072 r^4 + 5222 r^5 + 307 r^6 \right].$$

We have checked the relic abundance in the non-degenerate parameter space for some representative values of the parameters, and found them to be in agreement with the numerical calculations done by `micrOMEGAS`.



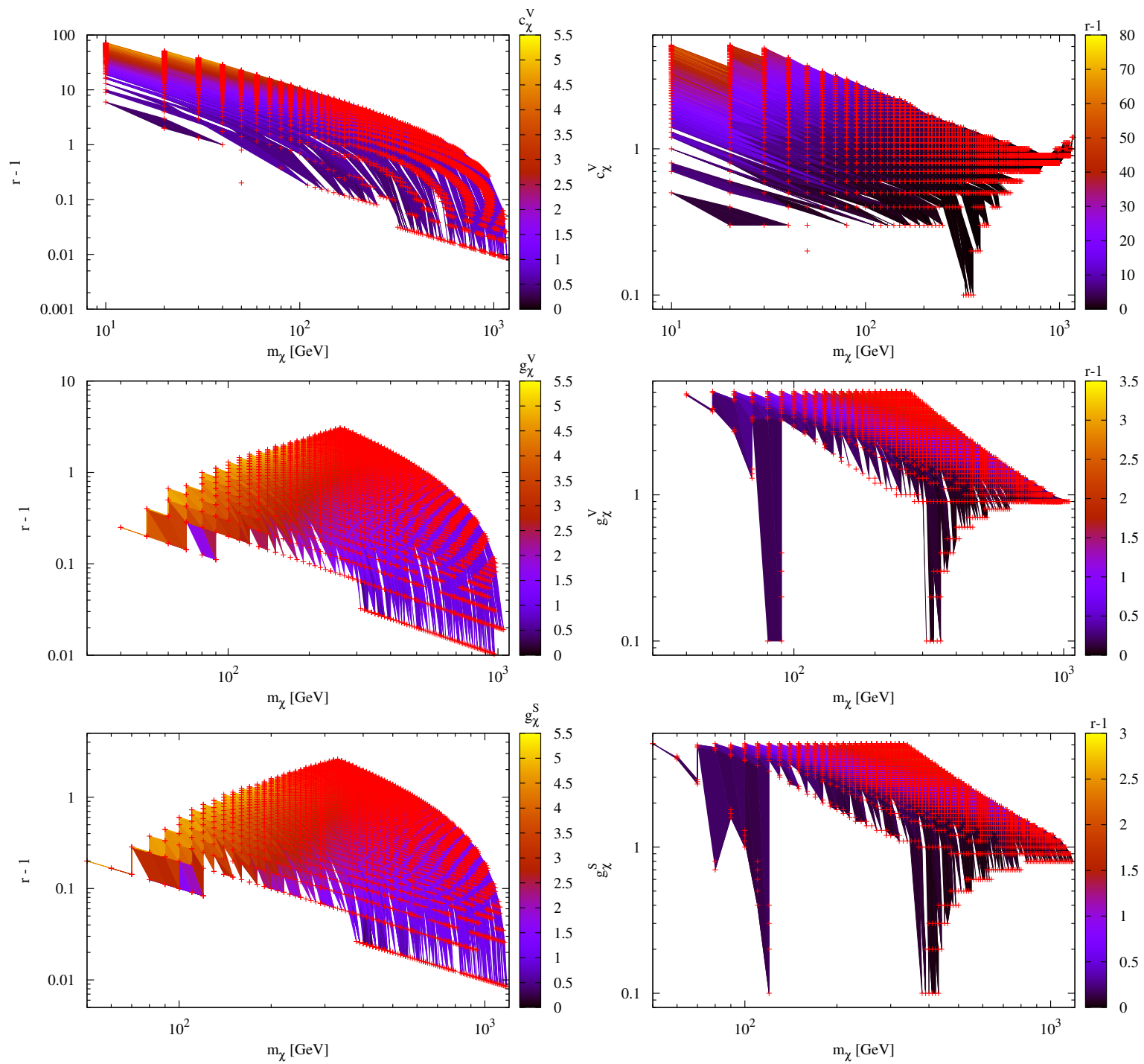
The necessary model files for micrOMEGAS were built using FeynRules

In the parameter space in which co-annihilation is not important, comparatively large Yukawa couplings are required to obtain the required relic density.

In the co-annihilation region on the other hand, we find the couplings to be reduced for almost all DM masses both for the scalar and vector mediator cases.

We find that with the increase in DM masses, the co-annihilation channels take over the DM self annihilation processes and the co-annihilation channels involving gauge interactions alone are able to depress the relic density below the observed value.





We see from the left-hand panels that there are two regions in the DM mass, one around $80 \text{ GeV} \leq m_\chi \leq 100 \text{ GeV}$ and another one around $300 \text{ GeV} \leq m_\chi \leq 400 \text{ GeV}$, where the co-annihilation processes result in a sharp drop in the couplings required for the requisite relic density.



Direct detection

Elastic nucleon-DM scattering have provided the strongest bounds on DM mass and interactions in a large number of conventional DM models.

In the t -channel spin-3/2 DM model the cross-sections at zero momentum transfer can be easily calculated

The dominant contribution to the spin independent DM-nucleon scattering cross-section is estimated by noting there are two scales present in the scattering.

The TeV cut-off scale Λ and the QCD scale ~ 100 MeV, which represents the typical energy and momentum of quarks bound inside the non-relativistic nucleons.



In the leading order, neglecting the quark momenta, the DM-nucleon scattering amplitude for the dimension-5 vector interaction, for example, proceeds through the s -channel exchange and is roughly given by

$$M \simeq \frac{1}{2} \frac{(g_\chi^V)^2 m_\chi^2}{\Lambda^2} \bar{u}^\alpha(p_2) \gamma^\mu P_R u^\beta(p_1) \frac{\left(g_{\alpha\beta} - \frac{p_\alpha p_\beta}{m_V^2}\right)}{s - m_V^2} \times \bar{v}(p_3) \gamma_\mu P_R v(p_4),$$

where p_1 and p_2 are the momenta of the incoming and outgoing DM particles, p_3 and p_4 are the corresponding incoming and outgoing quark momenta and $p = p_1 + p_3$.



In the non-relativistic approximation

$$M \simeq \frac{1}{8} \frac{(g_{\chi}^V)^2 m_{\chi}^2}{\Lambda^2} \frac{1}{m_{\chi}^2 - m_V^2} \times \left[\bar{u}^{\alpha}(p_2) \gamma^{\mu} u_{\alpha}(p_1) \bar{v}(p_3) \gamma_{\mu} v(p_4) \right].$$

The DM-nucleon scattering cross-section is then

$$\sigma^{\text{SI}} \simeq \frac{1}{64 \pi} \left(\frac{g_{\chi}^V}{\Lambda} \right)^4 \frac{\mu^2}{(1 - r^2)^2} f_N,$$



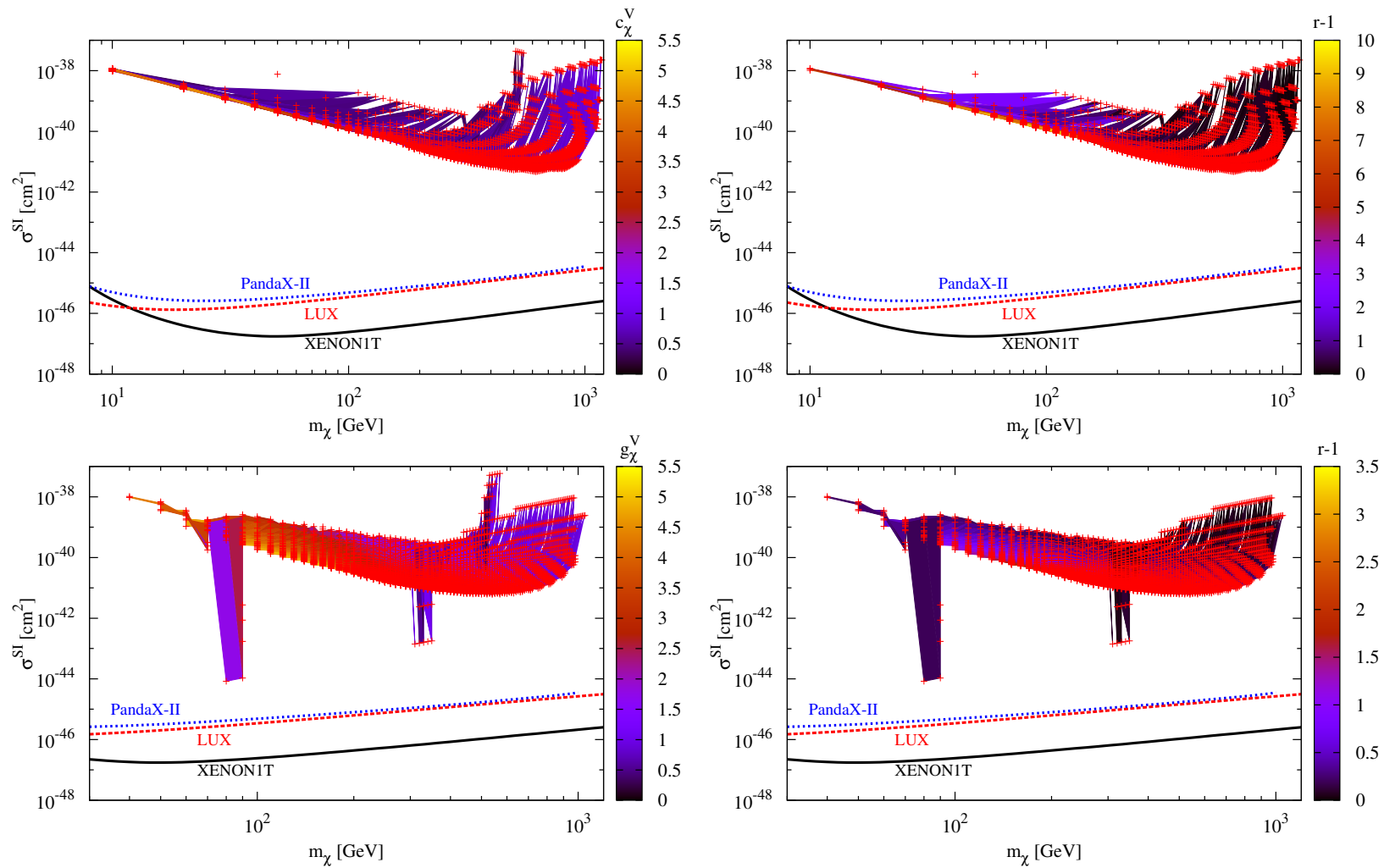
and similarly for the dimension-4 vector mediated interaction, we get

$$\sigma^{\text{SI}} \simeq \frac{1}{64 \pi} \frac{(c_\chi^V)^2 \mu^2}{m_\chi^4 (1 - r^2)^2} f_N,$$

where $\mu = \frac{m_\chi m_N}{m_\chi + m_N}$, $f_N = 4$ for protons and 1 for the neutrons, and we have dropped the terms proportional to the quark mass and momenta in comparison to the leading term.

The elastic nucleon-DM cross section for the case of scalar mediator is suppressed by terms proportional to quark momenta and has not been considered here.





The predictions for the spin-independent DM-proton scattering cross-sections, σ^{SI} , for the vector mediator case. In the left panels the colour gradient corresponds to the coupling, and in right panels to the mass splitting $r - 1$.



In the left panels we observe that for every DM mass and mass splitting the Yukawa coupling is obtained such that the parameters conform to the observed relic density, whereas in the right panels the required mass splitting is obtained for a given Yukawa coupling.

We find that for any DM mass the scattering cross-section generally increases as the degenerate parameter region is approached.

This happens because of a resonant enhancement of σ^{SI} near $r = 1$.

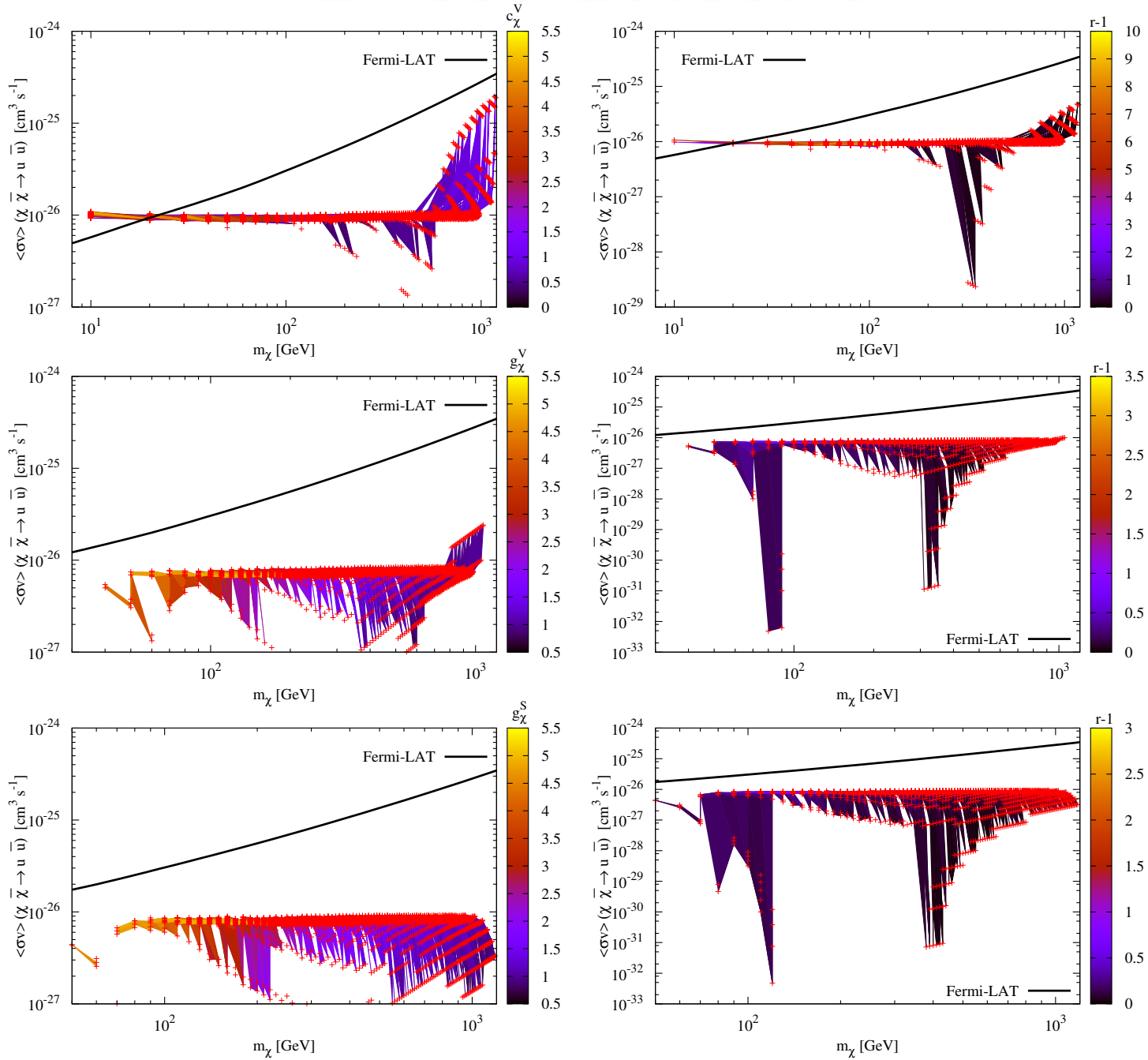


For the case of dimension-5 vector interactions (bottom panels), we see a drop of several orders of magnitude in the scattering cross-section around the same DM mass regions where the co-annihilation results have a sharp drop in the couplings.

We have also shown the current upper limits from LUX, PandaX-II and the projected upper limit for the XENON1T experiment.



Indirect detection



The predictions shown here are for the DM mass, mediator mass and the couplings consistent with the observed relic density. We have also shown the bounds from the 95% CL upper limits on the thermally-averaged cross-section for DM particles annihilating into $u\bar{u}$ Fermi-LAT observations.

As expected in the parameter region where co-annihilation is important ($r \simeq 1$) the $\chi\bar{\chi}$ annihilation cross-section in the $u\bar{u}$ channel is greatly suppressed.

Even in the region away from resonance ($r \gg 1$) the Fermi-LAT data does not provide strong bounds on the mass and coupling parameters in the entire range consistent with $\Omega_{\text{DM}}h^2 = 0.12$.



Collider bounds

The t -channel mediator model considered here has scalar and vector mediators which carry colour, $SU(2)_L$ and $U(1)$ charges.

They can thus be singly produced in association with DM particles, or pair produced if they are light enough at the LHC. These processes will contribute to the monojet and dijet signals with missing energy, with distinct signatures that can be searched for in dedicated searches.

For the monojet events $q g \rightarrow q \chi \bar{\chi}$ are the dominant processes, in comparison to $q \bar{q} \rightarrow g \chi \bar{\chi}$, because of the large parton distribution probability of the gluon, as compared to quark and antiquark in the proton.



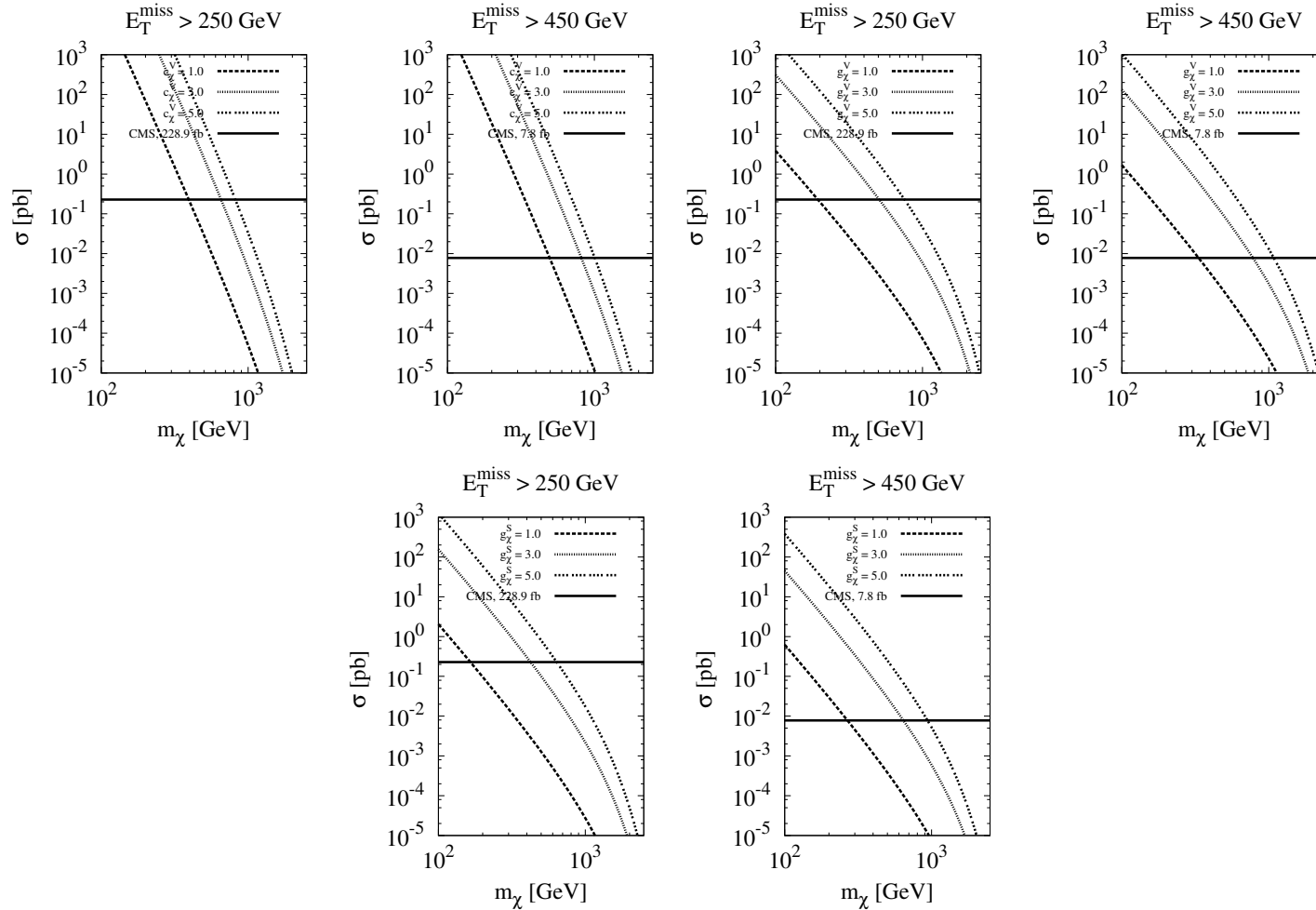
In the present study we confine ourselves to constraints arising from the monojet signals using the parameters space $(m_\chi, m_{S/V})$ for different values of the couplings $(g_\chi^S)^i / (g_\chi^V)^i / (c_\chi^V)^i$ consistent with the observed relic density.

The cross-section for monojet events is obtained by generating parton level events for the process $pp \rightarrow \chi \bar{\chi} j$ using MadGraph, where the model file for the Lagrangian is obtained from FeynRules, and we use CTEQ611 parton distribution function for five flavour quarks in the initial state.



We employ the usual cuts, and the cross-sections are calculated to put bounds on the parameters of the model by requiring (i) $E_T^{miss} > 250 \text{ GeV}$ and (ii) $E_T^{miss} > 450 \text{ GeV}$, for which the CMS result excludes new contributions to the monojet cross-section for the scalar and vector mediators as shown





The monojet cross-section in [pb] at the LHC with missing energy for two case (i) $E_T^{\text{miss}} > 250$ GeV and (ii) $E_T^{\text{miss}} > 450$ GeV.



The cross-sections are obtained for all masses and couplings consistent with the observed relic density.

(a) and (b) correspond to the dimension-4 and dimension-5 vector interactions terms respectively and (c) for the dimension-5 interaction term for the scalar mediator.

The monojet cross-section from 8 TeV CMS collaboration data based on an integrated luminosity 19.7 fb^{-1} is shown by solid black line.



We find that the collider bounds are much weaker compared to the bounds from the direct detection experiments for the vector mediator case.

The scalar mediator case is interesting in this case as the collider bound rules out low mass DM particles.

The bounds from the monojet + missing energy cross-section puts a lower limit on the DM particle mass, where the limit depends on the coupling, and increases with the coupling.



Summary

We considered spin-3/2 DM interacting with the SM fermions through a vector mediator in the s -channel.

Assuming MFV, we used universal vector and axial-vector couplings and restricted ourselves to one generation.

The main observations are the following:

- In view of the spin-3/2 nature of the DM, in addition to the restriction on the coupling arising from the decay width, there also exists a minimum value of the DM mass for a given coupling and mediator mass.



- For the vector coupling almost the entire parameters space $(m_\chi, m_{Z'})$ consistent with the observed relic density is ruled out by direct detection through nucleon-DM elastic scattering bounds given by LUX 2013. Even for small vector coupling $g_{\chi,f}^V \sim 0.01$ there is only a narrow window allowed in the parameter space *viz* $m_\chi > 1 \text{ TeV}$, $m_{Z'} \simeq 2 \text{ TeV}$.
- The case of vector mediator with pure axial-vector coupling in contrast is different. The parameter space consistent with the observed relic density is also allowed by the direct (XENON100 neutron) and indirect observations.



- For the benchmark couplings considered here, there are no strong bounds on vector coupling from the monojet searches at the LHC and the results are in broad agreement with the direct detection experiments.
- The case of pure axial coupling is however different. Here the monojet search puts stronger constraints obtained from the XENON100 neutron observations.

For a spin-3/2 DM particle interacting with the SM fermions through the exchange of a scalar or a vector mediator in the t -channel we found that by invoking MFV we restricted ourselves to the coupling of DM candidates with SM singlet right-handed quarks with universal coupling.



The thermal relic DM abundance has been computed by taking into account the co-annihilation processes.

Co-annihilation has the effect of reducing the Yukawa couplings needed to generate the required DM density.

The co-annihilation effects are more pronounced in the large m_χ regime, where mediator self annihilation into gauge bosons has the potential to suppress the relic density below the observed value.

Similar behaviour was observed in t -channel model for spin-1/2 and scalar DM particles.



Our main observations are the following:

- (a) The direct detection experiments, through DM-nucleon elastic scattering data, provide the most stringent bounds for the case of a vector mediator.

In this case the entire parameter space allowed by the relic density is already ruled out by the LUX data. This result is consistent with the earlier studies.

The co-annihilation is unable to ameliorate this.



- (b) There are no strong bounds from the the direct detection experiments on the scalar mediated interactions due to the velocity suppression of σ^{SI} .

In contrast, in the EFT framework, both the scalar as well as vector interactions give rise to dominant spin-independent nucleon-DM scattering cross-section and direct detection rules out scalar interaction for spin-3/2 DM particles of mass lying between 10 GeV and 1 TeV.

- (c) The current constraints from indirect searches like, Fermi-LAT data, are not sensitive enough to put any meaningful constraints.



- (d) Monojet searches at the LHC do not provide strong bounds at the vector couplings in comparison to the bounds from direct detection experiments.

However, for the case of the scalar mediator, where we do not get any strong bounds from the direct detection experiment, collider bounds put a lower limit on the DM mass which is $m_\chi \geq 300$ GeV.

This limit rises with the increase in coupling.

Finally, it may be mentioned that bounds from direct detection experiments can, however, be evaded by foregoing the universal coupling between DM mediators and quarks, and letting the DM particles interact with only one gen., say with the third gen. quarks (top-philic DM).

