

Contribution Prospectives 2020

Cosmology with Gravitational Waves – Part I

Principal author: D.A. Steer

Co-authors: E. Chassande-Mottin, M. Barsuglia, S. Babak, V. Vennin, D. Langlois, K. Noui, F. Nitti, E. Kiritsis, E. Porter, M. Khlopov, E. Huguet,(Theory group at APC)

Virgo groups at APC, ARTEMIS, ILM, IPHC, IP2I, LAL, LAPP and LKB

Abstract

Gravitational waves, which were detected directly for the first time by the LIGO-Virgo scientific collaboration, can be used to probe early and late-time cosmology. In this letter we present different aspects of research in the areas of cosmology which we believe will be of crucial importance in the next 10 years: 1) measurements of cosmological parameters with GWs; 2) early universe cosmology with GWs. This letter is supplemented by the companion “Testing General Relativity and modified gravity with Gravitational Waves”.

Overview

The Standard Cosmological model, also referred as Λ CDM, is a highly successful model of the universe, whose predictions have passed many precise observational tests in both the early and late-time universe. Despite that, there remain a number of open questions. One, for instance, pertains to the expansion rate of the universe today — namely the Hubble constant H_0 — which is a fundamental parameter of the Λ CDM. Indeed, today there is a 4.2σ discrepancy between the value of $H_0 = 66.93 \pm 0.62$, $\text{kmMpc}^{-1}\text{s}^{-1}$ inferred by the Planck collaboration from Cosmic Microwave Background (CMB) and the value of $H_0 = 73.5 \pm 1.4 \text{ kmMpc}^{-1}\text{s}^{-1}$ measured from Type Ia Supernovae. A recent improvement of the latter estimation method (using extended HST observations of Cepheids in the LMC) leads to the even larger discrepancy of 4.4σ [1].

Another concern is the “dark energy” (DE) component, contributing some 70% to the energy budget of the universe, whose origin and nature is yet totally unknown. Alternative theories of gravity at cosmological scales have been developed in part both in order to solve the H_0 tension and the DE problem.

Direct detections of GWs with the LIGO and Virgo interferometers, and in the future with LISA and Einstein telescope, as well as measurements of the stochastic background of GWs (SGWB), provide a fascinating new way with which to probe what lies beyond Λ CDM, and provide new constraints on modified theories of gravity. In the next 10 years, furthermore, it is very likely that GWs data will be cross-correlated with other cosmological probes such as CMB (through for e.g. Simons Array, Simons Observatory, LiteBIRD) and Large Scale Structure

(through for e.g. Euclid and LSST) to yield a new and detailed picture of the universe beyond Λ CDM.

The scientific potential of GWs to constrain cosmology was demonstrated by the first detection of GWs by the LIGO and Virgo collaborations. Beyond showing the existence of binary black hole (BBH) systems which coalesce within the age of the Universe, unveiling a population of stellar mass black holes (BHs) with masses larger than 20 solar masses, and showing a link between short gamma-ray bursts and kilonovae with binary neutron star (BNS) mergers (to mention a few results), the LVC has shown how GWs can play crucial role in cosmology by providing a new independent channel for measuring H_0 . The inclusion of such GW-based H_0 measurement can significantly reduce the parameter space volume when testing beyond Λ CDM models based on a time-varying dark energy equation of state [2].

The first GW estimation of H_0 was from the BNS event GW170817 and the subsequent identification of its host galaxy. The estimate is $H_0 = 70.0^{+12.0}_{-8.0} \text{ kmMpc}^{-1}\text{s}^{-1}$. While this current GW-based measurement is not accurate enough to solve the H_0 discrepancy, forecasts for H_0 predict a constraint at 2% accuracy with about 100 BNS detections with electromagnetic (EM) counterparts and at 10% with 100 BBH detections with no EM counterpart.

Furthermore, from the same BNS event, the speed of GWs c_T was shown to be equal to that of EM waves, c , with $(c_T - c)/c \lesssim 10^{-15}$. This has led to significant constraints on — and even the elimination of — certain modified gravity theories. The APC theory group is recognised world-wide for its significant contributions to the development of generalised scalar-tensor theories of modified gravity, which contain an additional scalar field relative to General Relativity (GR). These include Horndeski theory, beyond-Horndeski theories, DHOST theories, and also a more general Effective Field Theory (EFT) approach which describes a large class of these modified gravity models. This synergy between theorists and the members gravitation group in APC (which is very active in both LIGO-Virgo and LISA) is developing rapidly, and we expect it to build significantly in the next 10 years.

In the next years, the discovery potential of GW observations will extend to early universe cosmology. GWs propagate freely in the early universe, immediately after they are generated, carrying with them unique information about the state of the universe at epochs and energy scales unreachable by other available observational probes, mostly based on electromagnetic emission. Processes generating GWs in the early universe lead to stochastic GW backgrounds (SGWBs), and are typically based on theories beyond the Standard Model of particle physics (BSM). They can occur within a broad range of energy scales, from the QCD scale up to the inflationary one. Therefore, SGWB detection can provide information on high energy physics, in a complementary way to the Large Hadron Collider or future particle colliders. The potential of GW detection to improve our knowledge of the early universe is in principle comparable to the one of the Cosmic Microwave Background at its dawn, which marked the beginning of modern cosmology. Still, the GW signal must have sufficient amplitude, and be in the right frequency range to be captured by current and future GW detectors.

In the next 10 years we expect that much of our efforts will be focused on 1) theoretical developments in modified gravity/dark energy; 2) measurements of cosmological parameters with GWs; 3) Early universe cosmology with GWs; 4) Tests of general relativity with GWs.

Measurements of cosmological parameters with GWs

Cosmological parameters, such as the Hubble constant H_0 mentioned in the introduction, can be measured in different ways with GWs. The most direct method, already applied by the LIGO-Virgo collaborations, is when a “standard siren” is observed — that is, a binary neutron star (BNS) with an electromagnetic (EM) counterpart. The redshift z to the source can be obtained from the absorption lines in the spectrum of the host galaxy or of the EM counterpart directly. The GW detection gives the luminosity distance d_L ; and from these two $H_0 = cz/d_L$. With more sources, the error on H_0 is expected to decrease, and forecasts (see e.g. [?]) for H_0 predict a constraint at 2% accuracy with about 100 BNS detections with EM counterparts.

The rate of observable BNS mergers is very uncertain, with a confidence interval spanning an order of magnitude. Extrapolating from the preliminary results obtained during the first part of the on-going LIGO-Virgo run O3, the 100 detected BNS milestone could be achieved before the end of the decade when the network composed of LIGO, Virgo, the Japanese detector KAGRA and LIGO-India will reach their design sensitivity. The 3rd generation of GW detectors will undoubtedly largely surpass the 100 BNS bar. Einstein telescope is expected to detect of order 10^5 BNS merger/year.

It is more difficult to predict which fraction of those events will be accompanied by an observed EM counterpart. So far, this has been the case for a single event only. There is however a range of ideas for removing the need for an EM counterpart and allow the measurement of both d_L and z from the sole GW signal.

For instance, one can use the so-called “statistical method”: from the GW detection one can not only to determine d_L but also the expected localisation of the event in the sky. This can then be correlated with galaxy catalogues, and the redshift distribution of the potential host galaxies thus identified includes the true value if the catalog is complete. This method has been demonstrated with the best-localized GW events so far, GW170814 and GW170817 [3, 4]. The large statistics provided by the BBH mergers allows to obtain a competitive accuracy by cumulatively applying this method to those numerous events. Forecasts for H_0 predict a constraint at 10% with ~ 100 BBH detections which could be reached within the next 5 years.

There are other GW-only methods accessible to the 3rd generation detectors:

- If the equation of state of neutron stars is known by 2030 (which is possible), the observation of tidal effects [5] or of the post-merger signal [6] just before or after the final BNS merger allows to break the mass-redshift degeneracy and thus a direct z measurement from the GW signal.
- One can also exploit the narrowness of the distribution of masses of the underlying neutron-star population [7].
- The delays between images of strongly lensed GW signals and their EM counterparts offers a promising way to measure H_0 very accurately, not limited by systematics [8].

The combination of those methods allows a consistency check for the GW-based measurements.

We plan to contribute to future measurements of H_0 with ground-based detectors of the 2nd and 3rd generations. As outlined above, this contains many different aspects. We expect, amongst the other points, to contribute to (i) understanding correctly the luminosity distance uncertainty (which is a fundamental component for the H_0 inference); (ii) developing GW likelihood approximant which take into account the correlations in the parameter space given

a detector network and is also valid for longer-duration signals, like the ones expected with the Einstein Telescope. We expect (iii) to invest significant work into the expected contributions of the LISA satellite to measurements of cosmological parameters. LISA will be able to detect events at higher red-shifts, and as a result can probe not only H_0 but also other cosmological parameters such as $\Omega_{M,R}$ [9]. This work will be carried out in the context of the LISA cosmology working group, of which the leader is Chiara Caprini of APC, and who has considerable expertise in GW standard sirens. We also expect (iv) to start cross-correlating GW data with other cosmological probes such as CMB (through for e.g. PolarBear, Simons Array, Qubic...) and Large Scale Structure (through for e.g. Euclid).

Early universe cosmology with GWs

As mentioned in the introduction, the discovery potential of GW observations will soon extend to *early universe cosmology*, and we expect this to be an area of intense research activity. As opposed to other observational probes, mostly based on electromagnetic emission, GWs propagate freely in the early universe immediately after they are generated, carrying with them unique information about the state of the universe at these epochs and energy scales. Processes generating GWs in the early universe lead to *primordial stochastic GW backgrounds* (SGWBs); examples are from certain models of inflation, phase transitions, primordial black holes, reheating instabilities and cosmic strings, see [10] for a review. Of crucial importance will be to study the properties of this SGWB $\Omega_{GW}(f)$, including its spectral shape, possible anisotropies, as well as non-gaussianities and parity dependence.

EW phase transition and GWs

The Laser Interferometer Space Antenna (LISA) is a space-based GW interferometer that will be launched by the European Space Agency (ESA) in the early 2030s [11]. It is composed of a triangular constellation of three satellites, containing masses in free fall, linked by lasers with which the interferometric measurement is made. The length of the interferometer arms is 2.5 billion km setting the instrument sensitivity in the frequency range 10^{-4} to 0.1 Hz. In the context of GW sources operating in the early universe, this corresponds to the electroweak (EW) scale and beyond. LISA can therefore test the GW emission by processes connected to the EW symmetry breaking and beyond, up to about 10^4 TeV. Such a detection would shed light on Beyond Standard model scenarios, in a complementary way to future particle colliders.

In the course of its adiabatic expansion, the universe might have undergone several phase transitions (PTs) as the temperature decreases. A variety of processes related to primordial PTs that can lead to the production of a SGWB, for instance topological defects such as cosmic strings (see below), or processes related to the collision of bubbles in first order phase transitions. Indeed, towards the end of a *first order* PT when the true vacuum bubbles collide and convert the entire universe to the symmetry-broken phase, the spherical symmetry of the bubble walls as well as and of the bulk fluid velocity configuration surrounding them is broken. This generates a non-zero tensor anisotropic stress which actively sources GWs. Depending on the characteristics of the first order PT, the GW signal can be picked up at GW detectors.

Most research to date has focused on GW generation from the bulk fluid motions driven by the expanding bubbles. Friction, due to the coupling between field driving the PT and the cosmological fluid, causes transfer of the scalar field kinetic energy to bulk kinetic energy of the

fluid. The first numerical simulations of the full system of the scalar field and the surrounding fluid, were carried out recently [12, 13, 14], leading to the crucial observation that the expansion of the bubbles generates sound waves in the surrounding fluid: these act as a powerful source of GWs even after the merging of the bubbles is completed and the scalar field has everywhere settled into the true vacuum.

A number of crucial questions remain which will be tackled in the next years. For instance, sound waves remain present in the fluid until they either are damped by kinetic viscosity, or develop shocks. If the characteristic time-scale on which shocks arise is larger than the Hubble time (low root mean square fluid velocity), no shocks are expected and the sound waves continue to source GWs until they are dissipated by the fluid viscosity. If, instead, shocks appear within one Hubble time, the modes of the flow are likely to convert from being compressional to turbulent. Plasma turbulence and the associated magnetic field are also independent sources of GWs.

We therefore expect to work on questions such as (i) what is the the rate at which vorticity and shocks develops in the plasma, (ii) we plan to extend the recent numerical simulations of weakly first order phase transitions to stronger phase transitions where vorticity is expected to become more important; (iii) to model analytically the SGWB from turbulence: as yet, no consensus has been reached on this question and simulations with a larger dynamical range are required in order to bridge the gap between analytical predictions and numerical simulations. Also, (iv) a definite prediction of the SGWB frequency shape as a function of the PT parameters (essentially its temperature, strength and duration, the bubble wall velocity, the friction) is still lacking.

Primordial Black Holes and GWs

Primordial Black Holes (PBH) are expected to form from rare large density perturbations produced during inflation, when they re-enter the cosmological horizon and collapse into black holes (BHs). For the scales probed in the CMB, the amplitude of the fluctuations is too small to yield a substantial abundance of PBHs. At smaller scales however, where the amplitude of the fluctuations is less constrained, inhomogeneities produced during inflation could be large enough, and PBHs thus open up a new observational window.

There has been renewed and ever increasing interest in PBHs since the LIGO/VIRGO collaboration reported the first detection of gravitational waves associated to black-hole mergers in 2015 [15]. They may indeed explain the existence of progenitors for these events. PBHs may also solve a number of problems currently encountered in astrophysics and cosmology, such as explaining the seeding of the supermassive black holes in galactic nuclei [16], the generation of the large-scale structure [17, 18] (either individually through the “seed” effect or collectively through the “Poisson” effect), the minimum radius and the large mass-to-light ratios of ultra-faint dwarf galaxies [19], the generation of correlations between the soft X-ray and infrared backgrounds [20], and the missing dwarf satellite problem (see Ref. [19] for other hints in favour of the existence of PBHs).

Tight constraints on the abundance of PBHs have been placed in various mass ranges (see *e.g.* Refs. [21, 22] for reviews), from the gravitational lensing, production of gravitational waves by merging, or Hawking evaporation they should induce. This leaves two mass windows open for PBHs to constitute an appreciable fraction, and possibly all, of dark matter, around $M \sim 10^{-12}M_{\odot}$ and $M \sim 10 - 100M_{\odot}$. Interestingly, the second window precisely falls within the LIGO/VIRGO detection band.

Various observational perspectives should confirm (or exclude) the presence of PBHs in our universe. For instance, a straightforward way to distinguish between stellar and primordial origins for the BHs is to detect a merger involving a black hole with a mass smaller than the Chandrasekhar limit of $1.4 M_{\odot}$, which is within the reach of the upcoming runs of LIGO/VIRGO. Another way is to measure the spin and mass distribution of the BHs (since PBHs should form with negligible spins [23] contrary to astrophysical ones [24], and due to the existence of universal conditions on the mass distributions of PBHs [25]), which should soon be better constrained with improved statistics. The PBH scenario can also be tested with the stochastic gravitational wave background associated with PBH binaries. In the future, the LISA project [26], which has been selected as a L3 mission of ESA’s “Cosmic Vision” program, will vastly increase the sensitivity and frequency coverage of currently running experiments and will give access to such backgrounds. This will be complemented by the pulsar timing arrays constraints from the SKA project [27]. Finally, laser interferometers could also detect the bursts of gravitational waves coming from hyperbolic encounters of PBHs in dense clusters [28].

The detection of a stochastic gravitational background associated with PBHs would have very important consequences for cosmology. It would indeed add PBHs to the list of cosmological probes at our disposal, to reconstruct the dynamics of the expansion of the universe. Since they can merge at any time in the cosmic history, they can give access to periods that are difficult to reconstruct otherwise, such as the dark ages. Their possible contribution to the dark matter is also very appealing, as it would allow us to solve this long-standing issue of cosmology in a “minimum way” (black holes are already known to exist, and no exotic particle or modification to GR is required). Finally, since PBHs form from large cosmological perturbations produced during inflation at scales smaller than the ones probed in the CMB, they would give invaluable constraints on the physics of inflation. Indeed, the CMB only gives access to a limited range of scales, and the time frame during which these scales are generated during inflation is therefore limited as well, and cannot encompass more than ~ 7 e -folds (over the ~ 60 e -folds elapsed between the generation of these scales and the end of inflation). PBHs have the potential to unveil the missing part of the inflationary potential, hence to allow us to better understand the nature of the inflaton field(s) and its connection to the rest of the standard model degrees of freedom.

Cosmic strings and GWs

Another cosmological source of GWs as well as ultra-high-energy particles are cosmic strings (or alternatively, cosmologically stretched fundamental strings of String Theory, formed for instance at the end of brane inflation) [29]. The energy per unit length of a string μ , is of order η^2 , where η is the energy scale of the phase transition in which strings were formed. In the simplest cases, the string tension is also of order μ , and strings are relativistic objects that typically move at a considerable fraction of the speed of light. The combination of a high energy scale and a relativistic speed clearly indicates that strings are a natural source of GWs.

A network of strings formed in the early Universe emits GWs (as well as possibly other radiation, such as gamma rays) throughout the history of the Universe, generating a SGWB from the superposition of many uncorrelated sources. The crucial dimensionless combination is $G\mu$ (where $G = 1/M_p^2$ is Newton’s constant, and $M_p = 1.22 \times 10^{19}$ GeV the Planck mass), which is related to the energy scale η through

$$G\mu \sim 10^{-6} \left(\frac{\eta}{10^{16} \text{ GeV}} \right)^2, \quad (1)$$

and which parametrizes the gravitational interactions of the string. The SGWB generated from cosmic strings ranges over many decades of frequency and can be constrained by pulsar timing arrays, by LISA, and also by LIGO-Virgo. The LIGO-Virgo constraint is $G\mu \lesssim 10^{-9}$ depending on the cosmic string model [30], whereas the LISA satellite will be able to put a very tight constraint on $G\mu$: $G\mu \lesssim 10^{-17}$ [31]. These results, however, depend on the model to describe the evolution of the cosmic string network.

Future research is expected to focus on (i) model-independent constraints on cosmic-strings in the context of the LIGO-Virgo-collaboration: (ii) theoretical work on modelling cosmic string network. Indeed, for some years it has been hotly debated as to what is the correct model of cosmic strings, and in particular what is the size of the closed *loops* of cosmic strings which are the main source of both bursts of GW as well as the SGWB. Recently it seems that this question may have been answered: and the answer involves loops decaying not only into GR but also particles. For this reason, we expect to develop new models but also as a further subject work on (iii) constraints on cosmic strings from their particle emission (using, for instance, Fermi-LAT data). Finally (iv), an interesting question is how one can probe cosmic strings through the spectral shape of the SGWB spectrum as well as its anisotropies.

References

- [1] A. G. Riess, S. Casertano, W. Yuan, L. M. Macri and D. Scolnic, *Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics beyond Λ CDM*, *Astrophys. J.* **876** (2019) 85, [1903.07603].
- [2] E. Di Valentino, D. E. Holz, A. Melchiorri and F. Renzi, *The cosmological impact of future constraints on H_0 from gravitational-wave standard sirens*, *Phys. Rev.* **D98** (2018) 083523, [1806.07463].
- [3] DES, LIGO SCIENTIFIC, VIRGO collaboration, M. Soares-Santos et al., *First Measurement of the Hubble Constant from a Dark Standard Siren using the Dark Energy Survey Galaxies and the LIGO/Virgo Binary-Black-hole Merger GW170814*, *Astrophys. J.* **876** (2019) L7, [1901.01540].
- [4] LIGO SCIENTIFIC, VIRGO collaboration, M. Fishbach et al., *A Standard Siren Measurement of the Hubble Constant from GW170817 without the Electromagnetic Counterpart*, *Astrophys. J.* **871** (2019) L13, [1807.05667].
- [5] C. Messenger and J. Read, *Measuring a cosmological distance-redshift relationship using only gravitational wave observations of binary neutron star coalescences*, *Phys. Rev. Lett.* **108** (2012) 091101, [1107.5725].
- [6] C. Messenger, K. Takami, S. Gossan, L. Rezzolla and B. S. Sathyaprakash, *Source redshifts from gravitational-wave observations of binary neutron star mergers*, *Phys. Rev. X* **4** (Oct, 2014) 041004.
- [7] S. R. Taylor, J. R. Gair and I. Mandel, *Hubble without the Hubble: Cosmology using advanced gravitational-wave detectors alone*, *Phys. Rev.* **D85** (2012) 023535, [1108.5161].

- [8] K. Liao, X.-L. Fan, X.-H. Ding, M. Biesiada and Z.-H. Zhu, *Precision cosmology from future lensed gravitational wave and electromagnetic signals*, *Nature Commun.* **8** (2017) 1148, [1703.04151].
- [9] C. Caprini and N. Tamanini, *Constraining early and interacting dark energy with gravitational wave standard sirens: the potential of the eLISA mission*, *JCAP* **1610** (2016) 006, [1607.08755].
- [10] C. Caprini and D. G. Figueroa, *Cosmological Backgrounds of Gravitational Waves*, *Class. Quant. Grav.* **35** (2018) 163001, [1801.04268].
- [11] H. Audley et al., *Laser Interferometer Space Antenna*, 1702.00786.
- [12] M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, *Gravitational waves from the sound of a first order phase transition*, *Phys. Rev. Lett.* **112** (2014) 041301, [1304.2433].
- [13] M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, *Numerical simulations of acoustically generated gravitational waves at a first order phase transition*, *Phys. Rev.* **D92** (2015) 123009, [1504.03291].
- [14] M. Hindmarsh, S. J. Huber, K. Rummukainen and D. J. Weir, *Shape of the acoustic gravitational wave power spectrum from a first order phase transition*, *Phys. Rev.* **D96** (2017) 103520, [1704.05871].
- [15] LIGO SCIENTIFIC, VIRGO collaboration, B. P. Abbott et al., *Observation of Gravitational Waves from a Binary Black Hole Merger*, *Phys. Rev. Lett.* **116** (2016) 061102, [1602.03837].
- [16] R. Bean and J. Magueijo, *Could supermassive black holes be quintessential primordial black holes?*, *Phys. Rev.* **D66** (2002) 063505, [astro-ph/0204486].
- [17] P. Meszaros, *Primeval black holes and galaxy formation*, *Astron. Astrophys.* **38** (1975) 5–13.
- [18] B. Carr and J. Silk, *Primordial Black Holes as Generators of Cosmic Structures*, *Mon. Not. Roy. Astron. Soc.* **478** (2018) 3756–3775, [1801.00672].
- [19] S. Clesse and J. García-Bellido, *Seven Hints for Primordial Black Hole Dark Matter*, *Phys. Dark Univ.* **22** (2018) 137–146, [1711.10458].
- [20] A. Kashlinsky, *LIGO gravitational wave detection, primordial black holes and the near-IR cosmic infrared background anisotropies*, *Astrophys. J.* **823** (2016) L25, [1605.04023].
- [21] B. J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, *New cosmological constraints on primordial black holes*, *Phys. Rev.* **D81** (2010) 104019, [0912.5297].
- [22] B. Carr, M. Raidal, T. Tenkanen, V. Vaskonen and H. Veermäe, *Primordial black hole constraints for extended mass functions*, *Phys. Rev.* **D96** (2017) 023514, [1705.05567].
- [23] T. Chiba and S. Yokoyama, *Spin Distribution of Primordial Black Holes*, *PTEP* **2017** (2017) 083E01, [1704.06573].

- [24] T. Kinugawa, H. Nakano and T. Nakamura, *Gravitational wave quasinormal mode from Population III massive black hole binaries in various models of population synthesis*, *PTEP* **2016** (2016) 103E01, [1606.00362].
- [25] B. Kocsis, T. Suyama, T. Tanaka and S. Yokoyama, *Hidden universality in the merger rate distribution in the primordial black hole scenario*, *Astrophys. J.* **854** (2018) 41, [1709.09007].
- [26] N. Bartolo et al., *Science with the space-based interferometer LISA. IV: Probing inflation with gravitational waves*, *JCAP* **1612** (2016) 026, [1610.06481].
- [27] W. Zhao, Y. Zhang, X.-P. You and Z.-H. Zhu, *Constraints of relic gravitational waves by pulsar timing arrays: Forecasts for the FAST and SKA projects*, *Phys. Rev.* **D87** (2013) 124012, [1303.6718].
- [28] J. Garcia-Bellido and S. Nesseris, *Gravitational wave bursts from Primordial Black Hole hyperbolic encounters*, *Phys. Dark Univ.* **18** (2017) 123–126, [1706.02111].
- [29] T. Vachaspati, L. Pogosian and D. Steer, *Cosmic Strings*, *Scholarpedia* **10** (2015) 31682, [1506.04039].
- [30] LIGO SCIENTIFIC, VIRGO collaboration, B. P. Abbott et al., *Constraints on cosmic strings using data from the first Advanced LIGO observing run*, *Phys. Rev.* **D97** (2018) 102002, [1712.01168].
- [31] P. Auclair et al., *Probing the gravitational wave background from cosmic strings with LISA*, 1909.00819.