Prospectives Nationales 2020-2030 : Astro-particles Physics (GT04)

Stochastic Gravitational Wave Background

Authors:
Nelson Christensen (Artemis)
Tania Regimbau (LAPP)

Abstract:
A stochastic background of gravitational waves can be created by the superposition of a large number of independent sources. The physical processes occurring at the earliest moments of the universe certainly created a stochastic background that exists, at some level, today. This is analogous to the cosmic microwave background, which is an electromagnetic record of the early universe. The recent observations of gravitational waves by the Advanced LIGO and Advanced Virgo detectors imply that there is also a stochastic background that has been created by binary black hole and binary neutron star mergers over the history of the universe. Whether the stochastic background is observed directly, or upper limits placed on it in specific frequency bands, important astrophysical and cosmological statements about it can be made. Here we summarize the current state of research of the stochastic background, from the sources of these gravitational waves, to the current methods used to observe them. We present our recommendations for detection strategies for the future.

Introduction

A consequence of Einstein's general theory of relativity are gravitational waves, perturbations to spacetime that travel away from their source at the speed of light. A stochastic gravitational-wave background (SGWB) signal is formed from the superposition of a wide variety of independent and unresolved sources from different stages of the evolution of the Universe.

Stochastic searches by LIGO and Virgo targets the SGWB of both cosmological or astrophysical origin. Potential cosmological sources include the amplification of vacuum fluctuations following inflation [1, 2, 3], phase transitions in the early universe [4, 5], cosmic (super)strings [6, 7, 8, 9], and can also arise in pre-Big Bang models [10, 11]. Astrophysical contributions to the stochastic background consist of an incoherent superposition of sources that are unresolved or too weak to be detected individually. The detection of a cosmological background would be a landmark discovery of enormous importance to the larger physics and astronomy community. The detection of an astrophysical background would also be of great interest as it would give important constraints on the star formation history and the evolution of the mass distributions with redshift. The most promising contribution for terrestrial detectors comes from the population of compact binary such as binary neutron stars [12] or binary black holes [13]. The implication from Advanced LIGO/Virgo's first and second observing runs is that the stochastic gravitational-wave background from binary black holes and binary neutron stars is consistent with optimistic predictions, and is potentially observable with advanced detectors [12, 13, 14]. Third generation ground based gravitational wave detectors, such as the Einstein Telescrope [15], may be able to provide important information about a cosmologically produced stochastic background [16, 17].

General relativity allows only for two gravitational-wave polarizations – the tensor plus and cross modes. Alternative theories, such as scalar-tensor theories [18, 19], $f(R)$ gravity [20, 21], bimetric [22] and massive [23] gravity theories, generically predict up to four additional vector and scalar polarization states. The direct measurement of gravitational-wave polarizations may therefore serve as a powerful phenomenological test of gravity.

The methods used to search for the stochastic background make minimal assumptions on the morphology of the signal, relying instead on excess coherence in the cross-correlated data streams from multiple detectors, as opposed to the independent and uncorrelated individual detector noise. The standard coss correlation statistics assumes that the signal is isotropic and its sensitivity decreases with the separation and relative orientation of the detectors. For that reason Virgo in Europe contributes little compared to the two LIGO detectors in the US. However this is not the case of the search for an anisotropic stochastic background for which a GW radiometer is used to map the angular power distribution in a pixel or spherical harmonic basis by applying appropriate time varying delays between detectors. The GW radiometer can also be applied to identify persistent gravitational waves in specific directions in the sky and then can be used to quickly identify any low level gravitational-wave signal, including pulsars or phenomena not yet conceived.

The spectrum of a SGWB is usually described by the dimensionless quantity $\Omega_{gw}(f)$ which is the gravitational-wave energy density per unit logarithmic frequency, divided by the critical energy density ρ_c ($\rho_c = 3c^2 H_0^2/8\pi G$, where H_0 is the present value of the Hubble constant) to close the universe,

$$\Omega_{gw}(f) = \frac{f}{\rho_c}\frac{d\rho_{gw}}{df} \ . \tag{1}$$

Many theoretical models of the SGWB in the observation band of LIGO and Virgo are characterized by a power-law spectrum which assumes that the fractional energy density in gravitational waves has the form

$$\Omega_{gw}(f) = \Omega_\alpha \left(\frac{f}{f_{ref}}\right)^\alpha \ , \tag{2}$$

where α is the spectral index and f_{ref} is a reference frequency. Cosmologically produced SGWBs are typically approximated by a power law in the LIGO-Virgo frequency band, $\alpha = 0$, while $\alpha = 3$ is characteristic of some astrophysical models (and also a flat strain power spectral density spectrum). A SGWB from binary black holes in Advanced LIGO and Advanced Virgo's most sensitive frequency band (10 Hz - 100 Hz) would have $\alpha = 2/3$.

Results from Advanced LIGO Observing Runs O1 and O2

Advanced LIGO's first observing run O1 went from September 2015 to January 2016, while its second observing run O2 went from November 2016 to August 2017. Advanced Virgo participated in O2 for the month of August 2017. Together LIGO and Virgo detected gravitational waves from 10 binary black hole mergers and a binary neutron star merger [24]. The data from the two Advanced LIGO detectors, LIGO Hanford and LIGO Livingston, were used for the search for a SGWB. Data quality cuts removed problematic times and frequencies from the analysis. In total for O1, 30 days of coincident data were analyzed, while for O2 the data amounted to 99 days. No SGWB was detected.

Combined O1 and O2 Isotropic Results

Assuming that the frequency dependence of the energy density of the SGWB is flat, namely $\alpha = 0$, the constraint on the energy density is $\Omega(f) < 6.0 \times 10^{-8}$ with 95% confidence within the 20 Hz - 86 Hz frequency band [25]. This is a factor of 2.8 better than the upper limit set by using just the O1 data [26]. For a spectral index of $\alpha = 2/3$ the constraint on the energy density is $\Omega(f) < 4.8 \times 10^{-8}$, while for $\alpha = 3$ it is $\Omega(f) < 7.9 \times 10^{-9}$ [25] (both with with 95% confidence, and a reference frequency of $f_{ref} = 25$ Hz when $\alpha \neq 0$). A prior that is flat in Ω_{gw} has been used. The O1 and O1 + O2 results have been used to limit cosmic string parameters [25, 27], similar to what was done with initial LIGO and initial Virgo [28, 29].

The dramatic improvement in the upper limit on the SGWB energy density was important, but not the most important SGWB outcome of observing runs O1 and O2. The observation of the gravitational waves from stellar mass binary black hole mergers [24, 30, 31, 32] and a binary neutron star merger [33] implies that these events are far more numerous in the universe than originally expected. In fact, it is likely that the SGWB produced from these type of events will be at the level of $\Omega_{gw} \sim 10^{-9}$ in the observing band of Advanced LIGO and Advanced Virgo [12, 13, 25, 26].

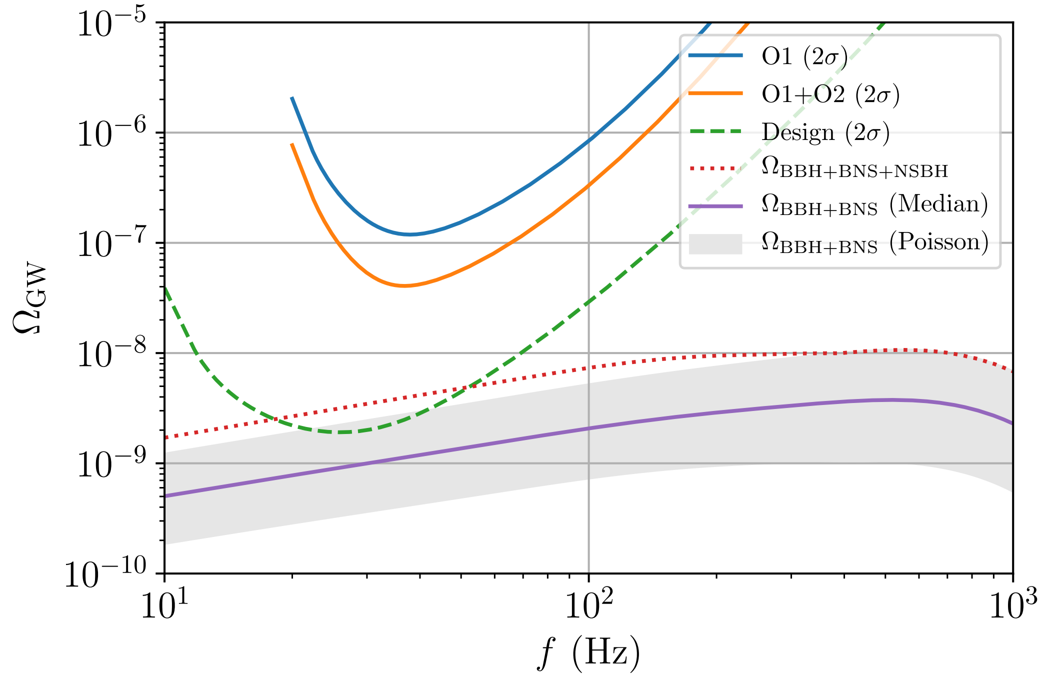

Figure 1: The Advanced LIGO SGWB sensitivity curves for O1 [26], combined O1+O2 [25], and design sensitivity [36, 37]. The purple line is the median total SGWB, combining binary black holes (BBH) and binary neutron stars (BNS); this uses the model presented in [12] with updated mass distributions and rates from [24, 34]; the gray box is the Poisson error region. The dotted gray line is the sum of the upper limit for the combined BBH and BNS backgrounds with the upper limit on the neutron star - black hole binary (NSBH) background.

Figure 1 displays the prediction of the astrophysical SGWB from binary black holes and binary neutron stars, along with the statistical Poisson uncertainties derived from the local binary merger rate. Also included is the estimate of the contribution from the addition neutron star - black hole binaries. The same binary formation and evolution scenario is used to compute the SGWB from from neutron star - black hole binaries as in [12], but an update was made for the mass distributions and rates so as to be consistent with the most recent results given in [24, 34]. For the neutron star - black hole binaries, the same evolution with redshift was used as for the binary neutron stars.

The SGWB energy density limits achieved by LIGO and Virgo have been impressive, gaining nine orders of magnitude since the first upper limit by LIGO in 2004 [35]. In the next few years Advanced LIGO and Advanced Virgo could very well detect a SGWB created by binary black hole and binary neutron star mergers. This would provide important information about how these systems have formed over the history of the universe, and would be a dramatic accomplishment in astrophysics.

Tests of General Relativity with the Stochastic Gravitational-Wave Background

LIGO and Virgo have used the recent observation of gravitational waves from binary black hole and binary neutron star coalescences to test general relativity [32, 38, 39, 40]. The LIGO-Virgo SGWB search has also be extended in order to test general relativity [41]. There is not necessarily a reason to expect extra polarizations of gravitational waves, nor extra polarizations in the SGWB (the consequences would be huge even if the chances are, a priori, not large); however, LIGO and Virgo have the ability to search for these modes, and will do so. With general relativity there are only two possible polarizations for gravitational waves, namely the two tensor modes. Alternative theories of gravity can also generate gravitational waves with scalar or vector polarizations [42].

Since there are six possible polarization modes, Advanced LIGO (with only two detectors, that are essentially co-aligned with respect to each other) cannot identify the polarization of short duration gravitational wave signals [32, 42, 43], such as those that have been recently observed [30, 31, 32]. A minimum of six detectors would be necessary to resolve the polarization content (scalar, vector and tensor) of a short duration gravitational wave [42]. A search for long duration gravitational waves, such as those from rotating neutron stars or the SGWB by the two Advanced LIGO detectors, can directly measure the polarizations of the gravitational waves [41, 43, 44, 45]. A detection of a SGWB by Advanced LIGO and Advanced Virgo would allow for a verification of general relativity that is not possible with short duration gravitational wave searches.

The LIGO-Virgo search for a SGWB has been expanded to a search for 6 polarizations: two tensor modes, two vector modes, and two scalar modes [41], and applied to the Advanced LIGO Observing Run O1 and O2 data [25, 46]. The addition of Advanced Virgo to the network does not improve detection prospects (because of its longer distance displacement from the LIGO detectors), however it will improve the ability to estimate the parameters of a SGWB of mixed polarizations. The eventual inclusion of KAGRA [47] and LIGO-India [48] will further expand the ability to resolve different polarizations of the SGWB, and further test general relativity.

There was no evidence found for a SGWB of tensor, vector or scalar polarizations in the Advanced LIGO O1 and O2 data, and upper limits were set [25, 46]. In the coming years it will be important for LIGO and Virgo (and soon KAGRA) to continue to test general relativity and look for a non-tensor SGWB. This has important consequences in fundamental physics and cosmology.

Correlated Magnetic Noise

A search for the SGWB uses a cross-correlation between the data from two detectors. Inherent in such an analysis is the assumption that the noise in one detector is statistically independent from the noise in the other detector. Correlated noise would introduce an inherent bias in the analysis. It is for this reason that the data from two separated detectors is used. At one time initial LIGO had two co-located detectors at the LIGO Hanford site. An attempt was made to measure the SGWB with these two detectors, but correlated noise at low frequencies contaminated the measurement, and a clean analysis could only be made above 460 Hz [49].

The LIGO, Virgo and KAGRA detector sites are thousands of kilometers from one another, and the simple assumption is that the noise in the detectors at these sites is independent from one another. However, this assumption has been demonstrated to be false for magnetic noise. The Earth's surface and the ionosphere act like mirrors and form a spherical cavity for extremely low frequency electromagnetic waves. The Schumann resonances are a result of this spherical cavity, and resonances are observed at 8, 14, 20, 26, ... Hz [50]. Most of these frequencies fall in the important SGWB detection band (10 Hz to 100 Hz) for Advanced LIGO and Advanced Virgo. The resonances are driven by the 100 or so lightning strikes per second around the world. The resonances result in magnetic fields of order 0.5 - 1.0 pT $\mathrm{Hz}^{1/2}$ on the Earth's surface [50]. In the time domain, 10 pT bursts appear above a 1 pT background at a rate of $\approx$ 0.5 Hz [51].

This magnetic field noise correlation has been observed between magnetometers at the LIGO, Virgo and KAGRA sites [52, 53]. Magnetic fields can couple into the gravitational wave detectors and create noise in the detectors' output strain channel. It has been determined that the correlated magnetic field noise did not affect the SGWB upper limits measured by initial LIGO and Virgo. For the observing runs O1 and O2 it has been demonstrated that the upper limits on the SGWB were not contaminated by correlated magnetic noise [25]. However, it is possible that correlated magnetic noise could contaminate the future results of Advanced LIGO and Advanced Virgo [54]. If that is the case, then methods must be taken to try and monitor the magnetic fields and subtract their effects. This could be done, for example, via Wiener filtering [53, 54, 55]. Low noise magnetometers are now installed at the LIGO and Virgo sites in order to monitor this correlated magnetic noise, and to be used if Wiener filtering is necessary for the SGWB searches. In addition to long term magnetic noise correlations, short duration magnetic transients, produced from lightning strikes around the world, are seen to be coincidently visible at the detector sites and could affect the search for short duration gravitational wave events [56].

As LIGO and Virgo continue to improve their limits on the energy density of the SGWB they will need to ensure that coherent magnetic field noise is not contaminating the measurement. For Advanced Virgo and Advanced LIGO, as well as third generation gravitational wave detectors, it will be necessary to actively monitor the magnetic noise environment of the detector sites, and make careful measurements of the magnetic field transfer functions of the gravitational wave detectors.

Third Generation Gravitaitonal Wave Detectors

Proposed third generation ground based gravitational wave detectors, such as the Einstein Telescope [15], will have a detection sensitivity approximately 10 times better than Advanced LIGO and Advanced Virgo. Consequently, their sensitivity to a SGWB will be better as well. What is even more promising is that the sensitivity of the Einstein Telescope will be sufficient to observe essentially every stellar mass binary black hole merger in the universe, and most binary neutron star mergers. As such, this foreground from compact binaries can be removed, allowing for a search for a cosmologically produced SGWB at the level of $\Omega_{GW} \simeq 10^{-13}$ [17]. This is too high for the detection of the standard slow roll inflation whose upper limit of about $\Omega_{GW} \simeq 10^{-15}$ is given by Bicep2 and Planck [57, 58, 59]. Enhanced contributions to the SGWB are predicted by some theories of inflations, for example amplifying the background at higher frequencies, including the LIGO-Virgo observational band [60, 61]. Non-detections also set important constraints on cosmological theories. Many important cosmological theories will be addressed by the SGWB measurements by third generation detectors, such as axion inflation [2], the effects of equations of state during inflation [3], pre Big Bang models [11], and many others [62].

The detection of a primordial SGWB could be one of the most important measurements in the history of physics. This would allow for the observation of the universe at its earliest moments. The development of third generation gravitational wave detectors should be a priority in France and world-wide.

Summary and Recommendations for the Future

Advanced LIGO and Advanced Virgo are approaching sensitivities where the detection of a SGWB produced by compact binary systems is possible. The observation of such a SGWB would provide information about the formation of these systems over the history of the universe, and would be a significant astrophysical result.

The continued improvement in the sensitivities of Virgo and LIGO is important for SGWB searches. *We recommend upgrades to the detector network. The Advanced Virgo+ project is consequently of great importance.*

The possibility to observe a cosmologically produced SGWB could happen with third generation gravitational wave detectors, such as the Einstein Telescope.*We recommend that the technology development for Einstein Telescope be a high priority in Europe and in France..*

References

[1] E. W. Kolb & M. S. Turner, *The Early Universe*. Westview Press, 1994.

[2] N. Barnaby, E. Pajer, and M. Peloso, "Gauge Field Production in Axion Inflation: Consequences for Monodromy, non-Gaussianity in the CMB, and Gravitational Waves at Interferometers," *Phys. Rev. D*, vol. 85, p. 023525, 2012.

[3] L. A. Boyle and A. Buonanno, "Relating gravitational wave constraints from primordial nucleosynthesis, pulsar timing, laser interferometers, and the cmb: Implications for the early universe," *Phys. Rev. D*, vol. 78, p. 043531, Aug 2008.

[4] A. A. Starobinskii, "Spectrum of relic gravitational radiation and the early state of the universe," *JETP Lett.*, vol. 30, 1979.

[5] R. Bar-Kana, "Limits on Direct Detection of Gravitational Waves," *Phys. Rev. D*, vol. 50, 1994.

[6] T. W. B. Kibble, "Topology of cosmic domains and strings," *J. Phys.*, vol. A9, 1976.

[7] T. Damour & A. Vilenkin, "Gravitational radiation from cosmic (super)strings: bursts, stochastic background, and observational windows," *Phys. Rev. D*, vol. 71, 2005.

[8] S. Olmez, V. Mandic, and X. Siemens, "Gravitational-Wave Stochastic Background from Kinks and Cusps on Cosmic Strings," *Phys. Rev. D*, vol. 81, p. 104028, 2010.

[9] S. Olmez, V. Mandic, and X. Siemens, "Anisotropies in the Gravitational-Wave Stochastic Background," *J. Cosmol. Astropart. Phys.*, vol. 2012, p. 009, 2011.

[10] A. Buonanno, "Spectrum of relic gravitational waves in string cosmology," *Phys. Rev. D*, vol. 55, 1997.

[11] V. Mandic & A. Buonanno, "Accessibility of the Pre-Big-Bang Models to LIGO," *Phys. Rev. D*, vol. 73, 2006.

[12] B. P. Abbott *et al.*, "GW170817: Implications for the Stochastic Gravitational-Wave Background from Compact Binary Coalescences," *Phys. Rev. Lett.*, vol. 120, no. 9, p. 091101, 2018.

[13] B. P. Abbott *et al.*, "GW150914: Implications for the Stochastic Gravitational-Wave Background from Binary Black Holes," *Phys. Rev. Lett.*, vol. 116, p. 131102, Mar 2016.

[14] B. P. Abbott *et al.*, "All-sky search for long-duration gravitational-wave transients in the second advanced ligo observing run," *Phys. Rev. D*, vol. 99, p. 104033, May 2019.

[15] M. Punturo *et al.*, "The einstein telescope: a third-generation gravitational wave observatory," *Classical and Quantum Gravity*, vol. 27, p. 194002, sep 2010.

[16] T. Regimbau, D. Meacher, and M. Coughlin, "Second Einstein Telescope mock science challenge: Detection of the gravitational-wave stochastic background from compact binary coalescences," PRD, vol. 89, p. 084046, Apr. 2014.

[17] T. Regimbau, M. Evans, N. Christensen, E. Katsavounidis, B. Sathyaprakash, and S. Vitale, “Digging deeper: Observing primordial gravitational waves below the binary-black-hole-produced stochastic background,” *Phys. Rev. Lett.*, vol. 118, p. 151105, Apr 2017.

[18] C. Brans and R. Dicke, “Mach’s principle and a relativistic theory of gravitation,” *Phys.Rev.*, vol. 124, pp. 925–935, 1961.

[19] Y. Fujii and K. Maeda, *The Scalar-Tensor Theory of Gravitation*. Cambridge Monograph on Mathematical Physics, Cambridge: Cambridge University Press, 2002.

[20] T. P. Sotiriou and V. Faraoni, “f(R) Theories Of Gravity,” *Rev.Mod.Phys.*, vol. 82, pp. 451–497, 2010.

[21] A. D. Felice and S. Tsujikawa, “f(R) Theories,” *Living Rev. Relativity*, vol. 13, no. 3, 2010.

[22] M. Visser, “Mass for the graviton,” *General Relativity and Gravitation*, vol. 30, no. 12, pp. 1717–1728, 1998.

[23] C. de Rham, G. Gabadadze, and A. J. Tolley, “Resummation of massive gravity,” *Phys. Rev. Lett.*, vol. 106, p. 231101, Jun 2011.

[24] B. P. Abbott *et al.*, “GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs,” *Phys. Rev.*, vol. X9, no. 3, p. 031040, 2019.

[25] The LIGO Scientific Collaboration, the Virgo Collaboration, B. P. Abbott, R. Abbott, T. D. Abbott, S. Abraham, F. Acernese, K. Ackley, C. Adams, and V. B. Adya, “A search for the isotropic stochastic background using data from Advanced LIGO’s second observing run,” *arXiv e-prints*, p. arXiv:1903.02886, Mar 2019.

[26] B. P. Abbott *et al.*, “Upper Limits on the Stochastic Gravitational-Wave Background from Advanced LIGO’s First Observing Run,” *Phys. Rev. Lett.*, vol. 118, no. 12, p. 121101, 2017.

[27] B. P. Abbott *et al.*, “Constraints on cosmic strings using data from the first Advanced LIGO observing run,” *Phys. Rev.*, vol. D97, no. 10, p. 102002, 2018.

[28] B. Abbott *et al.*, “Searching for Stochastic Gravitational Waves with LIGO,” *Nature*, vol. 460, p. 990, 2009.

[29] J. Aasi *et al.*, “Constraints on cosmic strings from the LIGO-Virgo gravitational-wave detectors,” *Phys.Rev.Lett.*, vol. 112, p. 131101, 2014.

[30] B. P. Abbott *et al.*, “Observation of gravitational waves from a binary black hole merger,” *Phys. Rev. Lett.*, vol. 116, p. 061102, Feb 2016.

[31] B. P. Abbott *et al.*, “GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence,” *Phys. Rev. Lett.*, vol. 116, p. 241103, Jun 2016.

[32] B. P. Abbott *et al.*, "Binary Black Hole Mergers in the First Advanced LIGO Observing Run," *Phys. Rev. X*, vol. 6, p. 041015, Oct 2016.

[33] B. P. Abbott *et al.*, "Gw170817: Observation of gravitational waves from a binary neutron star inspiral," *Phys. Rev. Lett.*, vol. 119, p. 161101, Oct 2017.

[34] B. P. Abbott *et al.*, "Binary Black Hole Population Properties Inferred from the First and Second Observing Runs of Advanced LIGO and Advanced Virgo," *Astrophys. J.*, vol. 882, no. 2, p. L24, 2019.

[35] B. Abbott *et al.*, "Analysis of first LIGO science data for stochastic gravitational waves," *Phys. Rev. D*, vol. 69, p. 122004, 2004.

[36] E. Thrane and J. D. Romano, "Sensitivity curves for searches for gravitational-wave backgrounds," *Phys. Rev. D*, vol. 88, p. 124032, Dec 2013.

[37] L. Barsotti, P. Fritschel, M. Evans, and S. Gras, "Updated Advanced LIGO sensitivity design curve," tech. rep., LIGO Laboratory, https://dcc.ligo.org/T1800044-v5/public, 2018.

[38] B. P. Abbott *et al.*, "Tests of General Relativity with GW150914," *Phys. Rev. Lett.*, vol. 116, p. 221101, May 2016.

[39] B. P. Abbott *et al.*, "Tests of General Relativity with the Binary Black Hole Signals from the LIGO-Virgo Catalog GWTC-1," 2019.

[40] B. P. Abbott *et al.*, "Tests of General Relativity with GW170817," *Phys. Rev. Lett.*, vol. 123, no. 1, p. 011102, 2019.

[41] T. Callister, A. S. Biscoveanu, N. Christensen, M. Isi, A. Matas, O. Minazzoli, T. Regimbau, M. Sakellariadou, J. Tasson, and E. Thrane, "Polarization-Based Tests of Gravity with the Stochastic Gravitational-Wave Background," *Physical Review X*, vol. 7, p. 041058, Oct. 2017.

[42] C. M. Will, "The confrontation between general relativity and experiment," *Living Reviews in Relativity*, vol. 17, no. 1, p. 4, 2014.

[43] J. D. Romano and N. J. Cornish, "Detection methods for stochastic gravitational-wave backgrounds: a unified treatment," *Living Rev. Rel.*, vol. 20, no. 1, p. 2, 2017.

[44] E. Berti *et al.*, "Testing general relativity with present and future astrophysical observations," *Classical and Quantum Gravity*, vol. 32, no. 24, p. 243001, 2015.

[45] M. Isi, M. Pitkin, and A. J. Weinstein, "Probing Dynamical Gravity with the Polarization of Continuous Gravitational Waves," *Phys. Rev.*, vol. D96, no. 4, p. 042001, 2017.

[46] B. P. Abbott *et al.*, "Search for tensor, vector, and scalar polarizations in the stochastic gravitational-wave background," *Phys. Rev. Lett.*, vol. 120, p. 201102, May 2018.

[47] Y. Aso, Y. Michimura, K. Somiya, M. Ando, O. Miyakawa, T. Sekiguchi, D. Tatsumi, and H. Yamamoto, "Interferometer design of the KAGRA gravitational wave detector," *Phys. Rev. D*, vol. 88, p. 043007, Aug 2013.

[48] C. S. Unnikrishnan, "IndIGO and LIGO-India: Scope and Plans for Gravitational Wave Research and Precision Metrology in India," *International Journal of Modern Physics D*, vol. 22, no. 01, p. 1341010, 2013.

[49] J. Aasi *et al.*, "Searching for stochastic gravitational waves using data from the two colocated LIGO Hanford detectors," *prd*, vol. 91, p. 022003, 2015.

[50] D.D. Sentman, "Handbook of atmospheric electrodynamics," vol. 1, p. 267, 1995.

[51] M. Füllekrug, "Schumann resonances in magnetic field components," *Journal of Atmospheric and Terrestrial Physics*, vol. 57, no. 5, pp. 479 – 484, 1995.

[52] E. Thrane, N. Christensen, and R. Schofield, "Correlated magnetic noise in global networks of gravitational-wave interferometers: observations and implications," *Phys. Rev. D*, vol. 87, p. 123009, 2013.

[53] M. W. Coughlin *et al.*, "Measurement and subtraction of Schumann resonances at gravitational-wave interferometers," *Phys. Rev. D*, vol. 97, p. 102007, May 2018.

[54] E. Thrane, N. Christensen, R. M. S. Schofield, and A. Effler, "Correlated noise in networks of gravitational-wave detectors: subtraction and mitigation," *Phys. Rev. D*, vol. 90, p. 023013, 2014.

[55] M. W. Coughlin *et al.*, "Subtraction of correlated noise in global networks of gravitational-wave interferometers," *Classical and Quantum Gravity*, vol. 33, no. 22, p. 224003, 2016.

[56] I. Kowalska-Leszczynska *et al.*, "Globally coherent short duration magnetic field transients and their effect on ground based gravitational-wave detectors," *Classical and Quantum Gravity*, vol. 34, no. 7, p. 074002, 2017.

[57] P. A. R. Ade *et al.*, "Planck 2015 results. XIII. Cosmological parameters," *Astron. Astrophys.*, vol. 594, p. A13, 2016.

[58] P. A. R. Ade *et al.*, "Detection of b-mode polarization at degree angular scales by bicep2," *Phys. Rev. Lett.*, vol. 112, p. 241101, Jun 2014.

[59] P. A. R. Ade *et al.*, "Joint Analysis of BICEP2/$KeckArray$ and $Planck$ Data," *Phys. Rev. Lett.*, vol. 114, p. 101301, 2015.

[60] Y.-S. Piao and Y.-Z. Zhang, "Phantom inflation and primordial perturbation spectrum," *Phys. Rev. D*, vol. 70, p. 063513, Sep 2004.

[61] D. Baumann, H. Lee, and G. L. Pimentel, "High-scale inflation and the tensor tilt," *Journal of High Energy Physics*, vol. 2016, p. 101, Jan 2016.

[62] N. Christensen, "Stochastic gravitational wave backgrounds," *Reports on Progress in Physics*, vol. 82, no. 1, p. 016903, 2019.