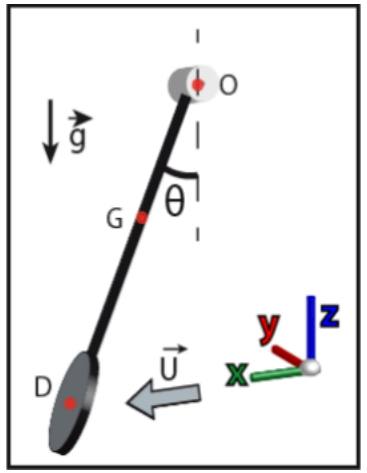
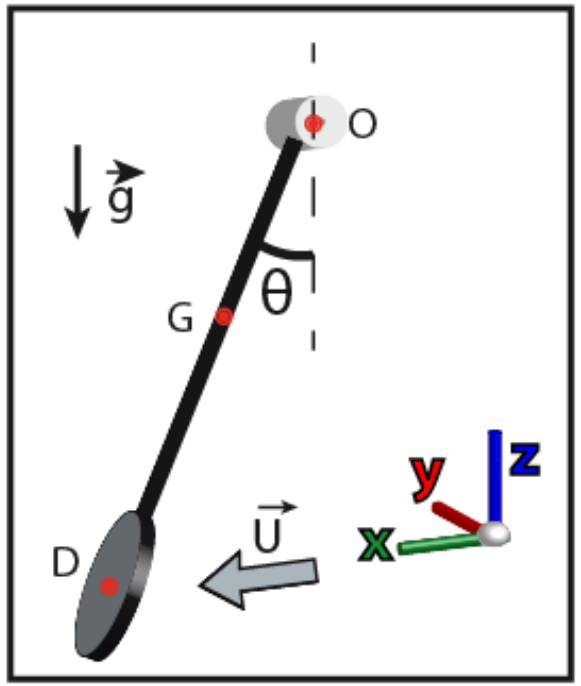


BISTABILITY OF A PENDULUM IN A FLOW: CASE OF A BALANCED PENDULUM

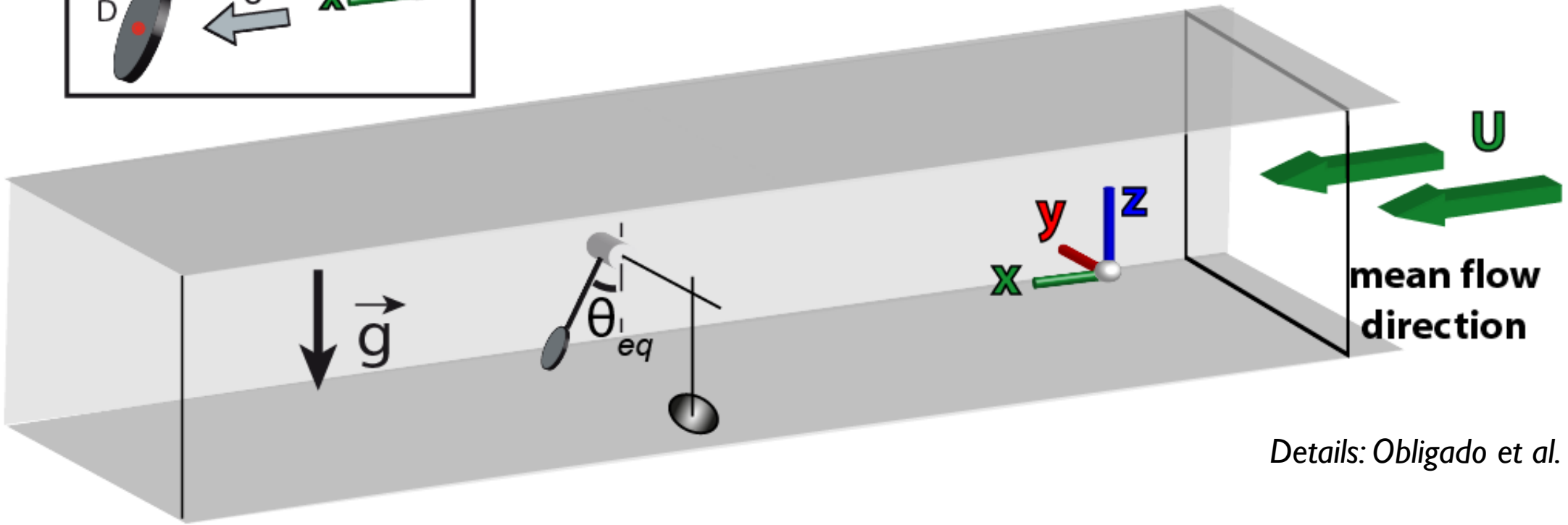
ARIANE GAYOUT, NICOLAS PLIHON, MICKAËL BOURGOIN (LP ENS DE LYON)

ÁRMANN GYLFASON (REYKJAVIK UNIVERSITY)



HÁSKÓLINN Í REYKJAVÍK
REYKJAVIK UNIVERSITY

- Laminar wind tunnel experiment
- Pitot measurements for flow velocity
- Angular (θ) measurements via potentiometer



Details: Obligado et al. 2013

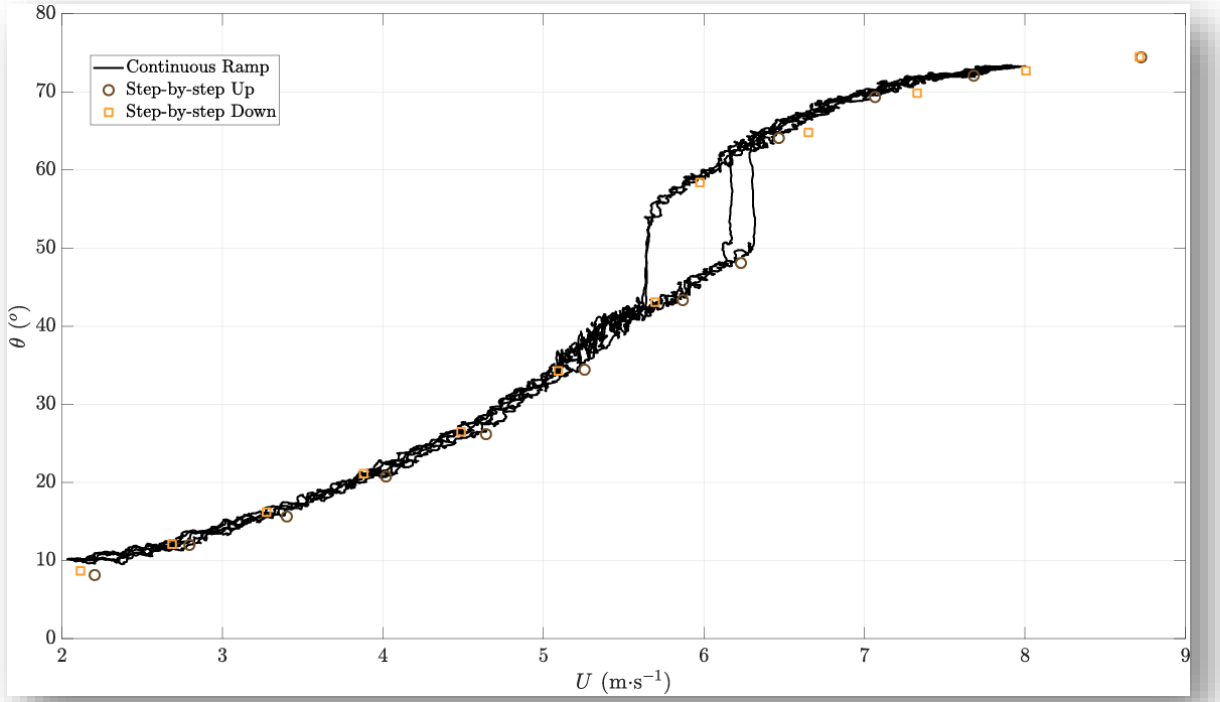
PENDULUM IN A FLOW

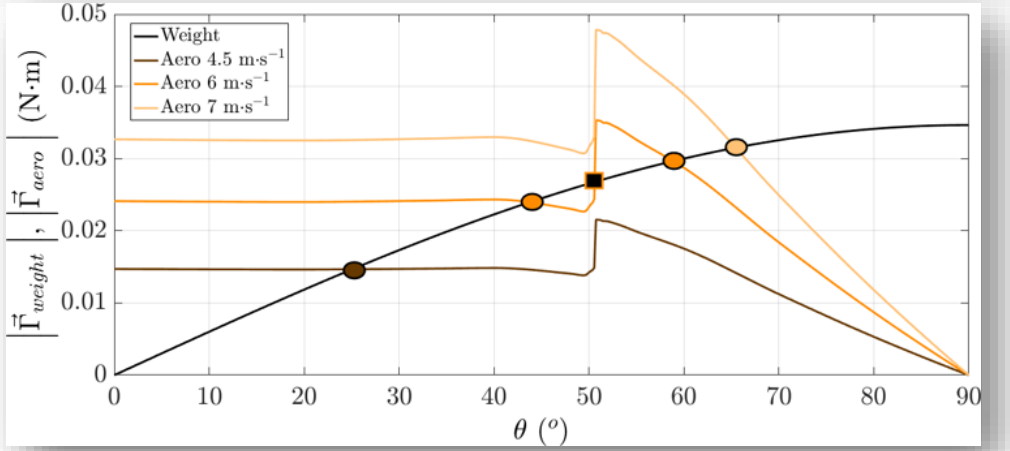
https://youtu.be/2xjSyT1LAgc

PENDULUM IN A FLOW

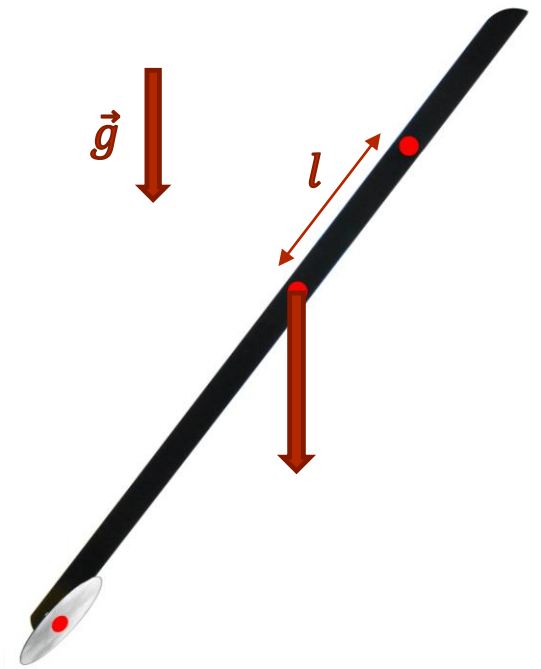
$$\Gamma_{weight} = mg\overline{OG}\sin(\theta_{eq}) = \frac{1}{2}\rho S\overline{OD}U^2 C_N(\theta_{eq}) = \Gamma_{aero}$$

l L





PENDULUM WITH GRAVITY



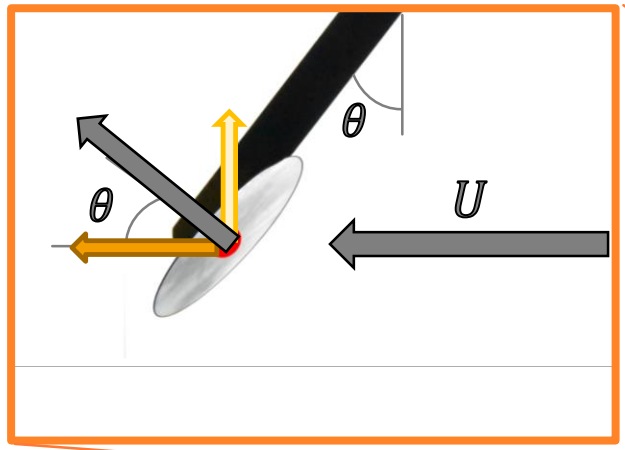
- Dynamical equation at small $\dot{\theta}$:

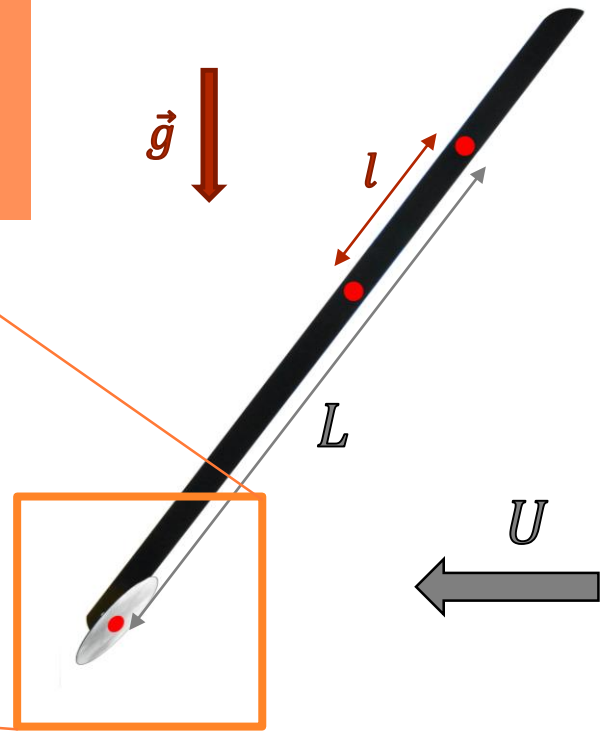
$$J\ddot{\theta} = -mgl\sin(\theta)$$

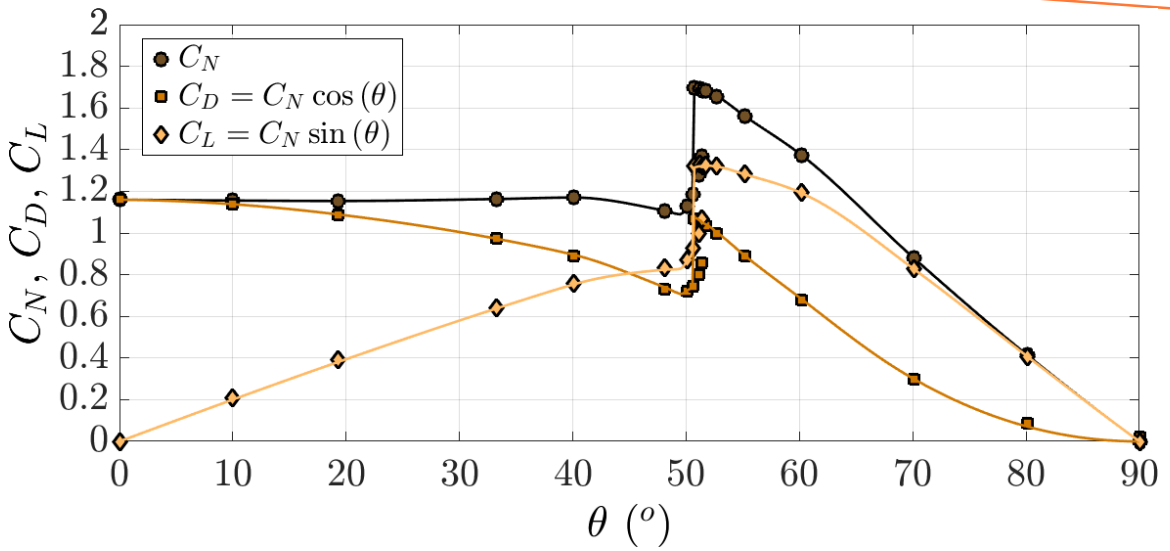
PENDULUM IN A FLOW WITH GRAVITY

- Dynamical equation at small $\dot{\theta}$:

$$J\ddot{\theta} = -mgl\sin(\theta) + \frac{1}{2}\rho S L U^2 C_N(\theta)$$







PENDULUM IN A FLOW WITH GRAVITY

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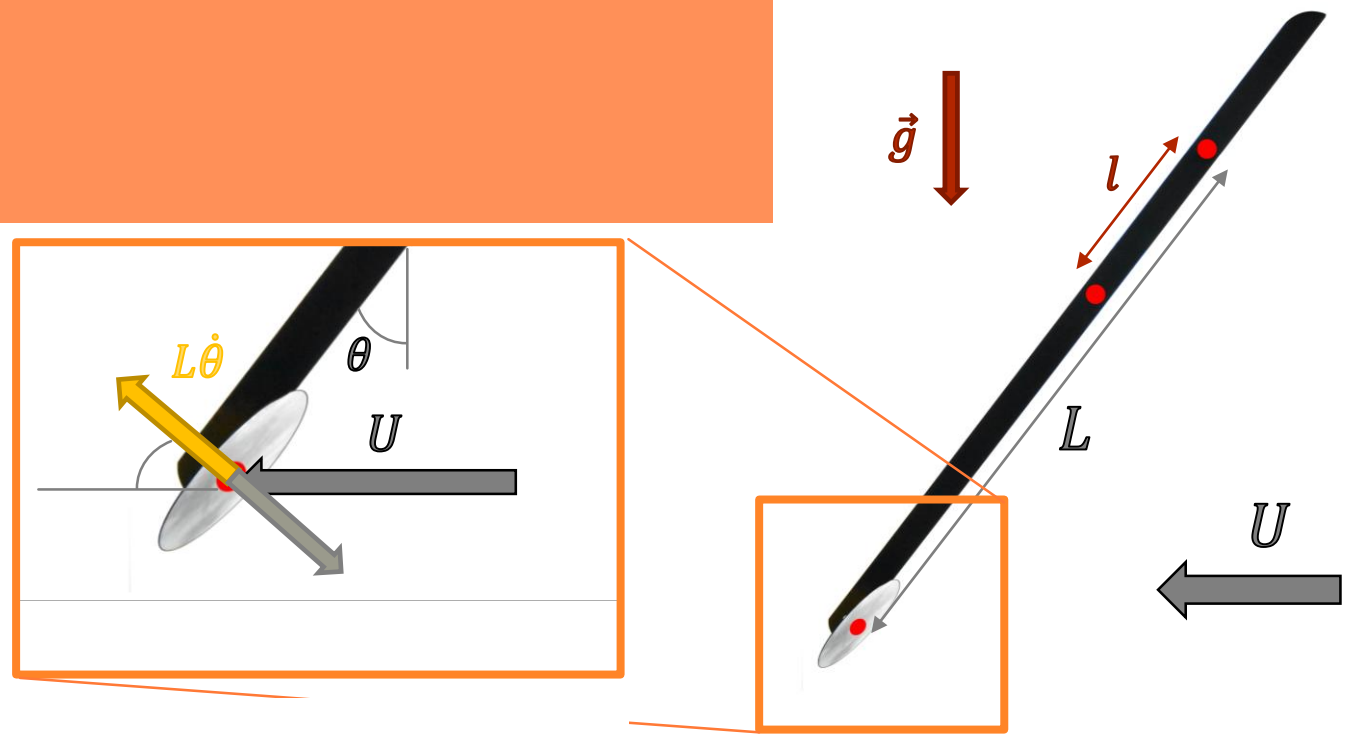
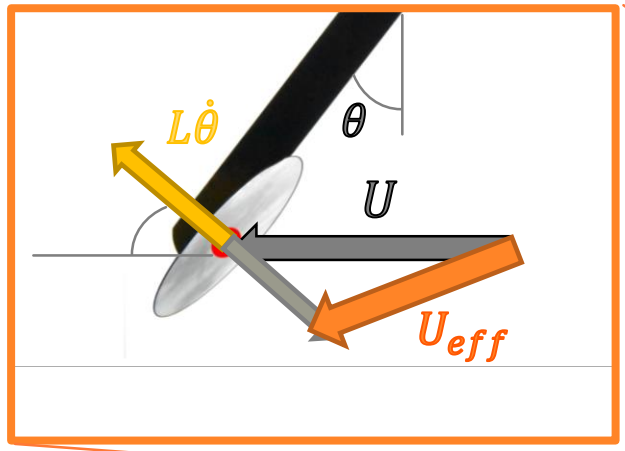
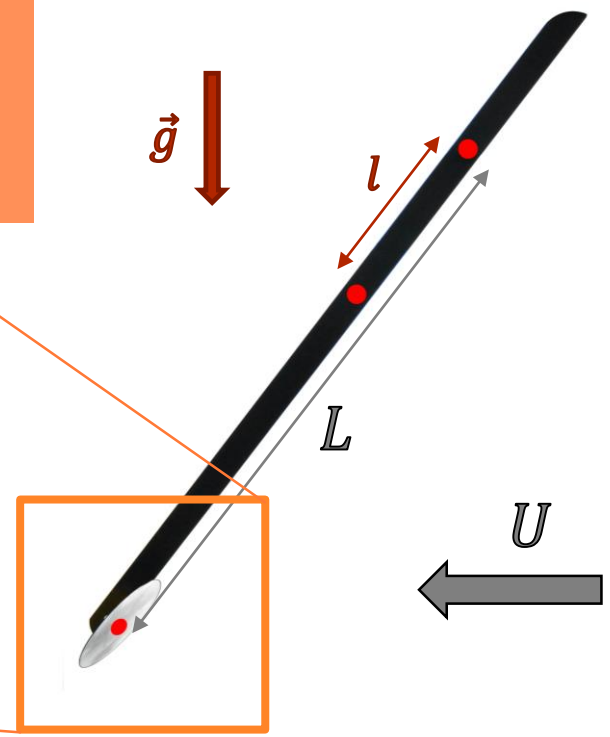
$$J\ddot{\theta} = -mgl\sin(\theta) + \frac{1}{2}\rho S L U^2 C_N(\theta)$$

- Correction in U:



PENDULUM IN A FLOW WITH GRAVITY





- Dynamical equation at small $\dot{\theta}$:

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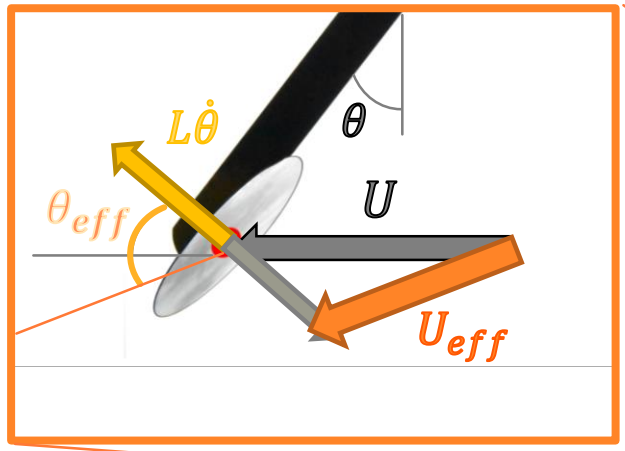
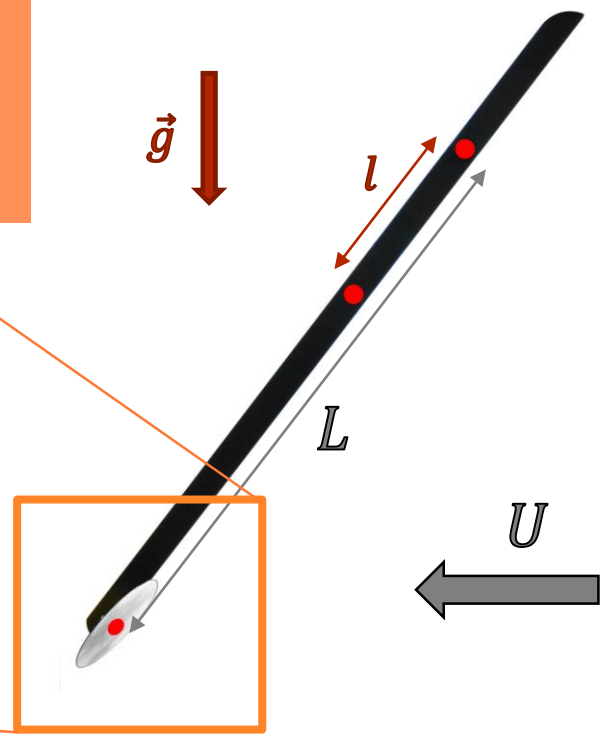
PENDULUM IN A FLOW WITH GRAVITY





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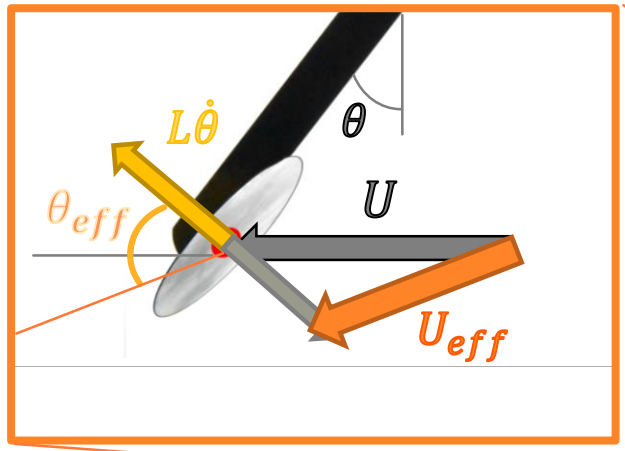
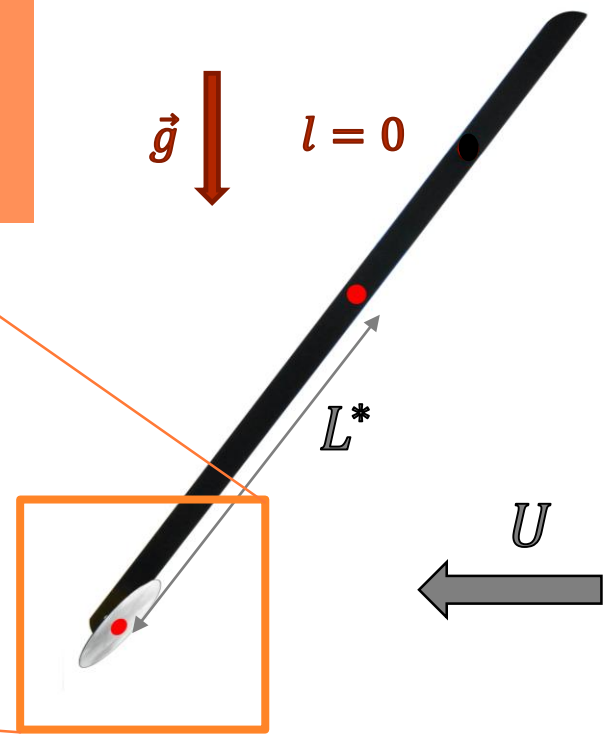
PENDULUM IN A FLOW WITHOUT GRAVITY





- Dynamical equation at small $\dot{\theta}$:

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$$J^*\ddot{\theta} = -\cancel{mgl}\sin(\theta) + \frac{1}{2}\rho S L^* (U^2 - 2UL^*\cos(\theta)\dot{\theta} + L^{*2}\dot{\theta}^2)\, C_N(\theta)$$

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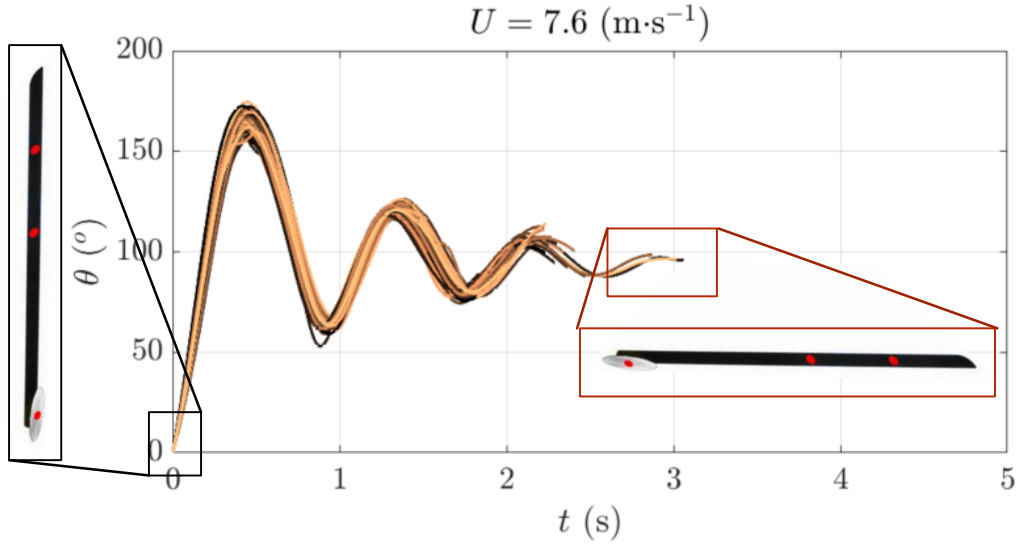
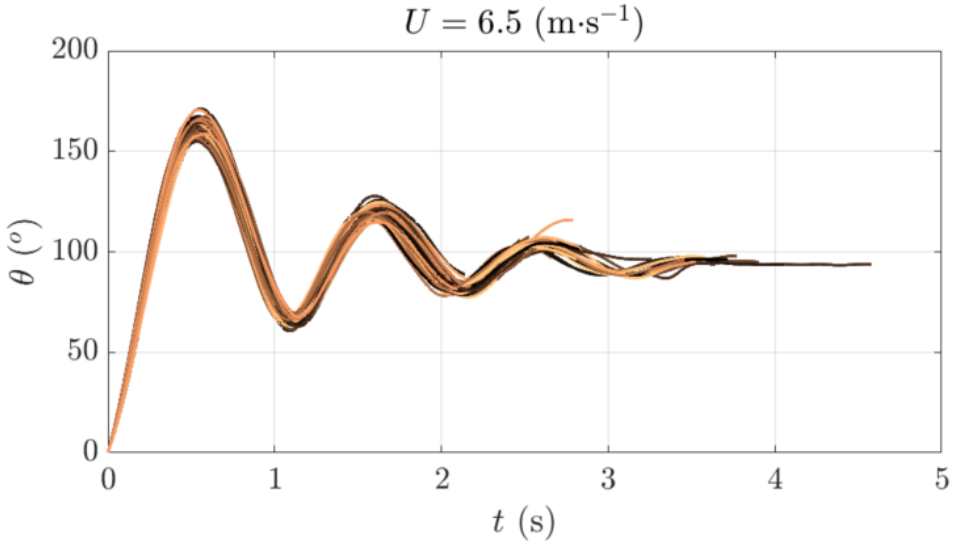
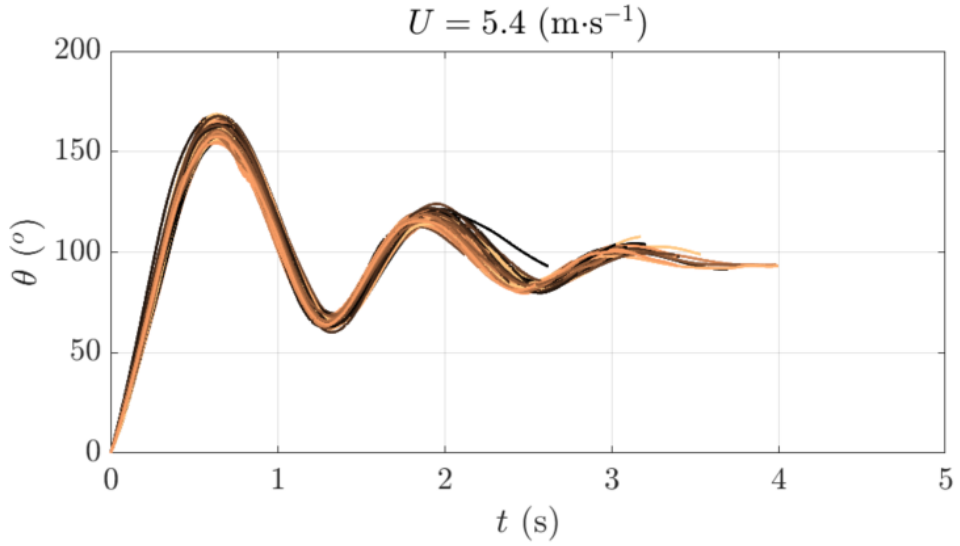
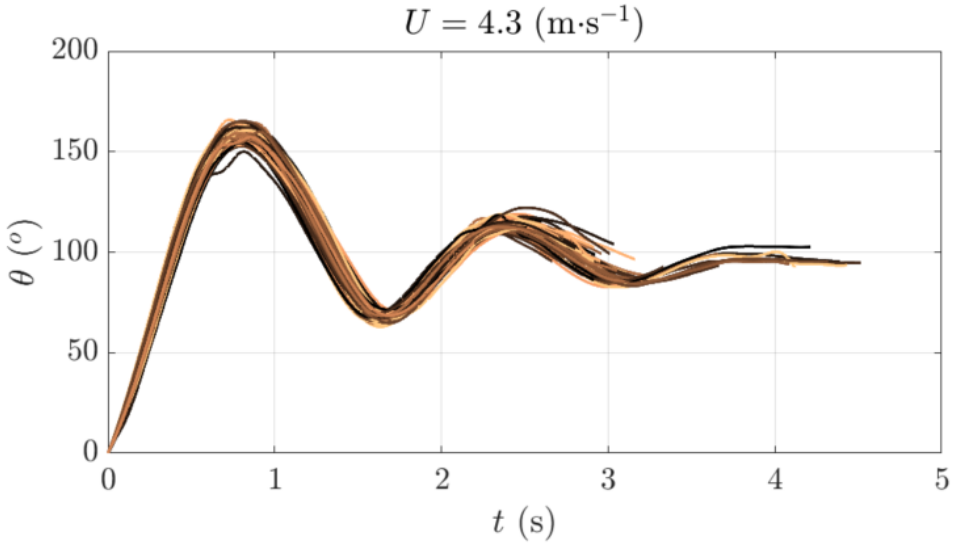
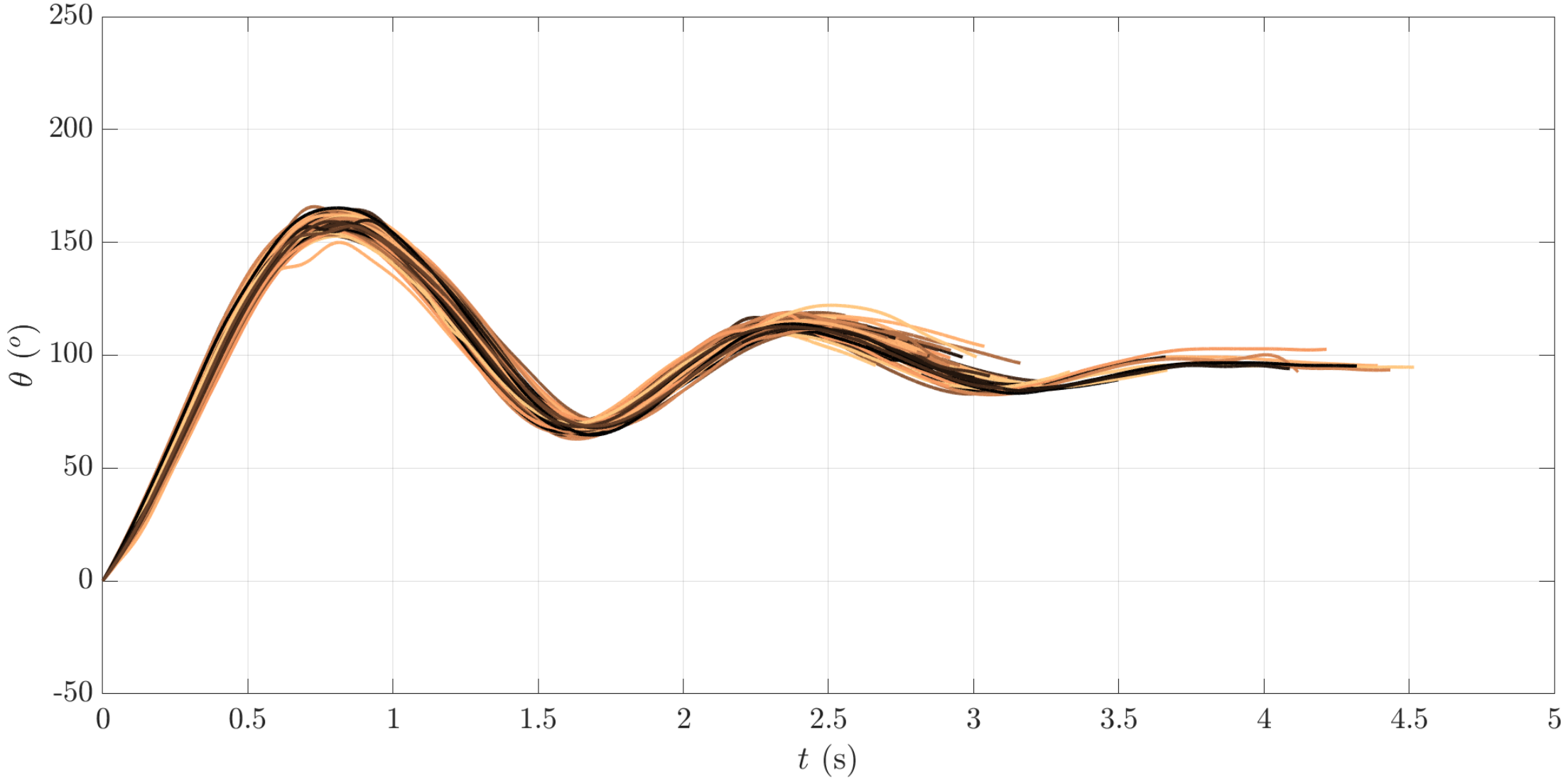




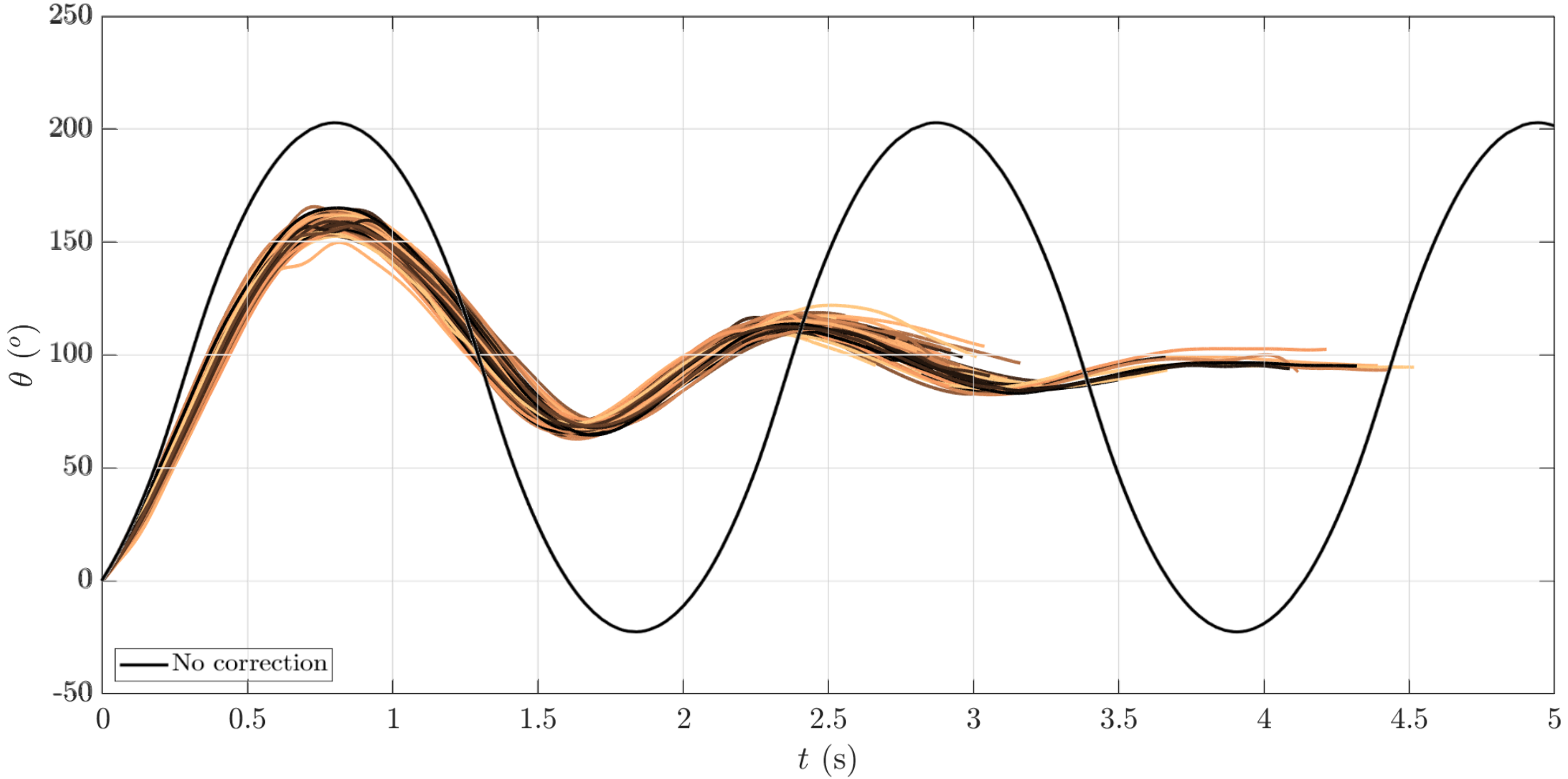




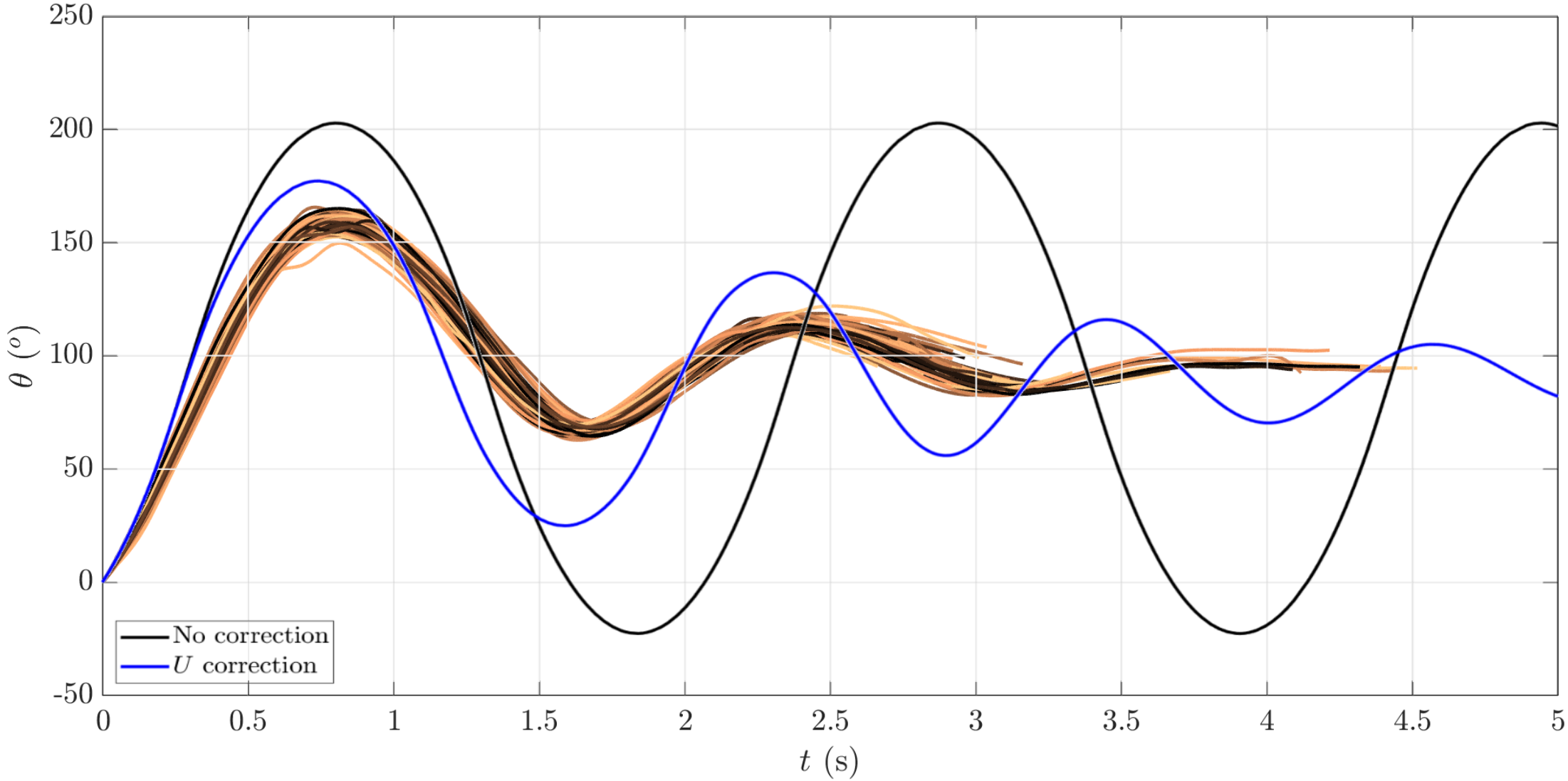
EXPERIMENTAL TRAJECTORIES



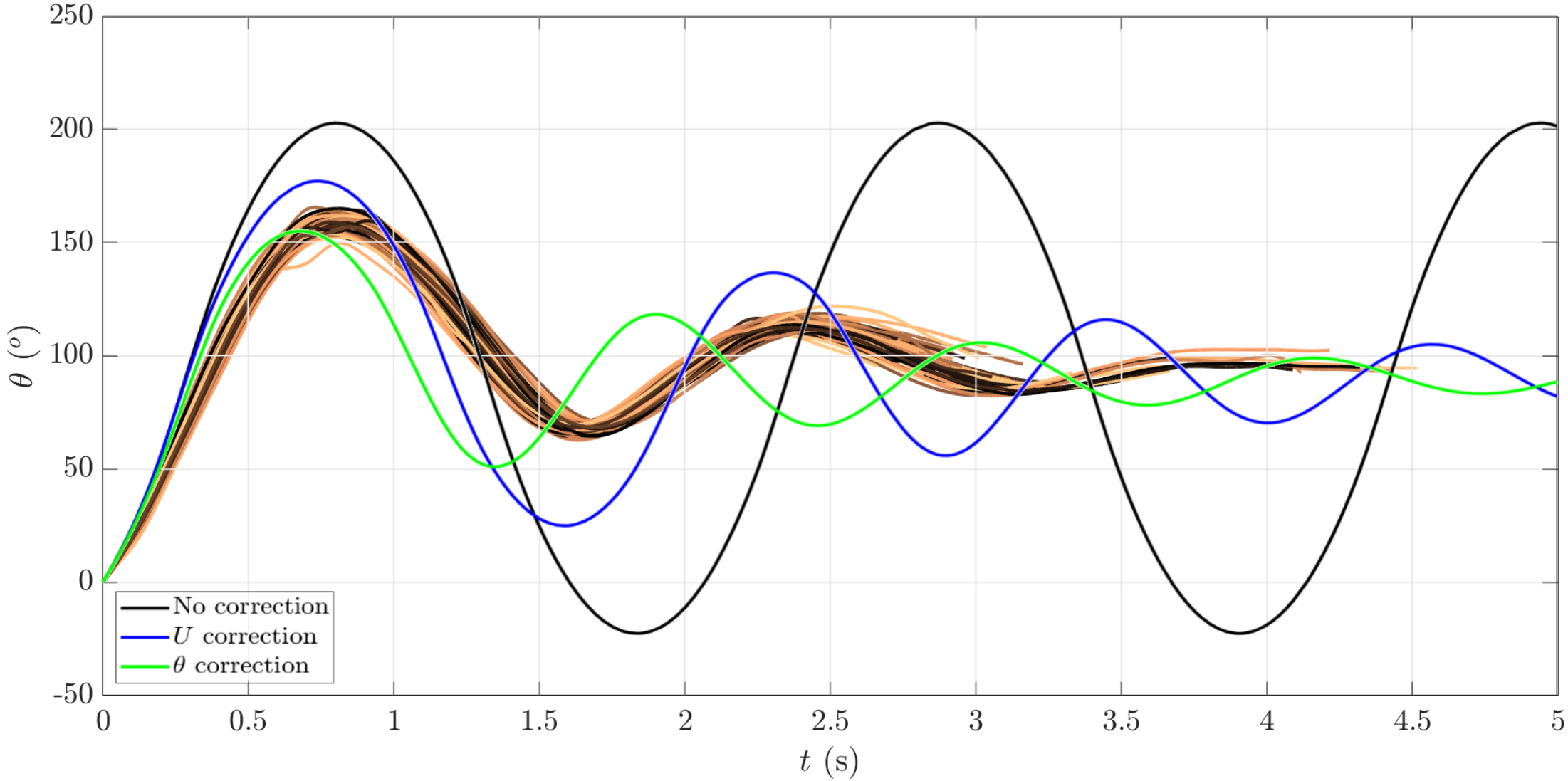
EXPERIMENT VS THEORY



EXPERIMENT VS THEORY

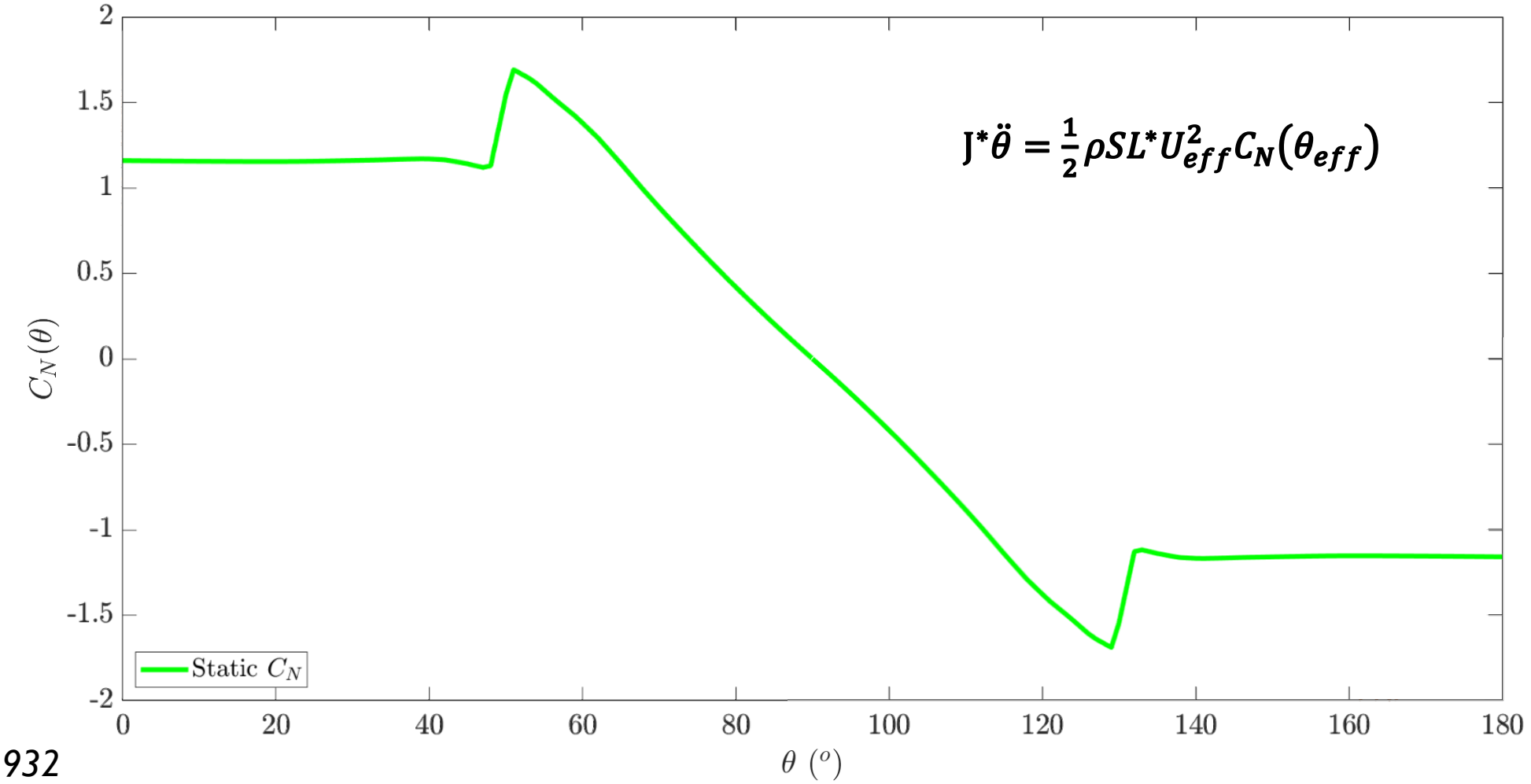


EXPERIMENT VS THEORY



EXPERIMENT VS THEORY

WHAT ABOUT THE C_N COEFFICIENT?



Flachsbart, 1932

WHAT ABOUT THE C_N COEFFICIENT?

$$J^*\ddot{\theta} = \frac{1}{2}\rho S L^* (U^2 - 2UL^*\cos(\theta)\dot{\theta} + L^{*2}\dot{\theta}^2)\, C_N \left(\theta + \tan^{-1}\left(\frac{L^*\dot{\theta}\sin\theta}{U - L^*\dot{\theta}\cos\theta}\right)\right)$$

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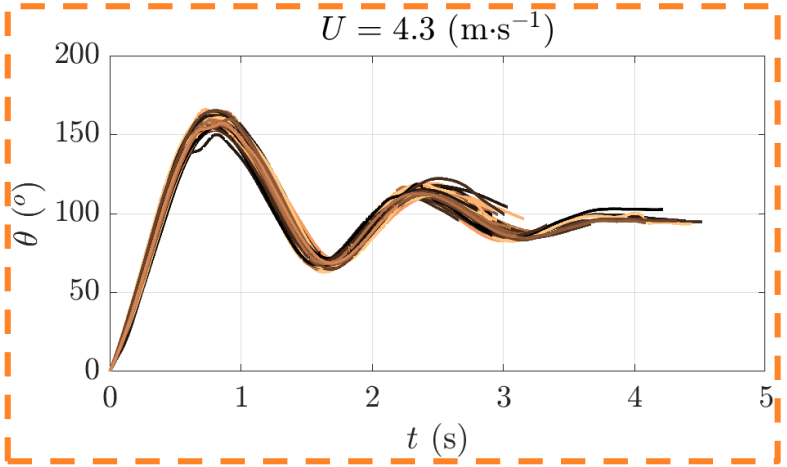

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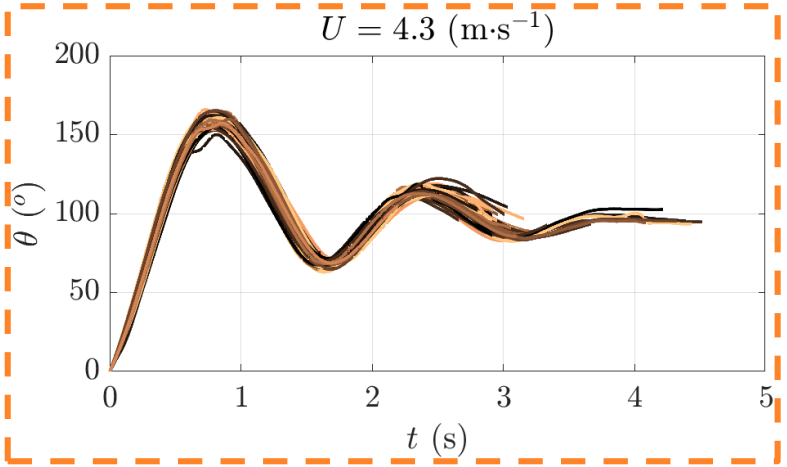

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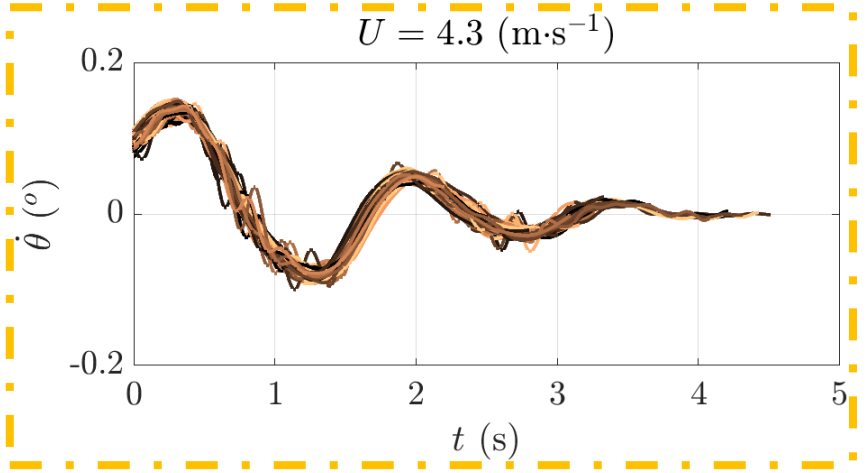

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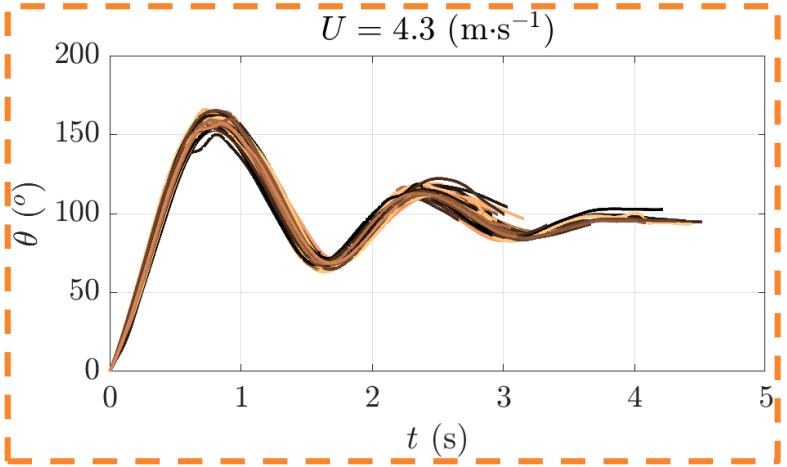

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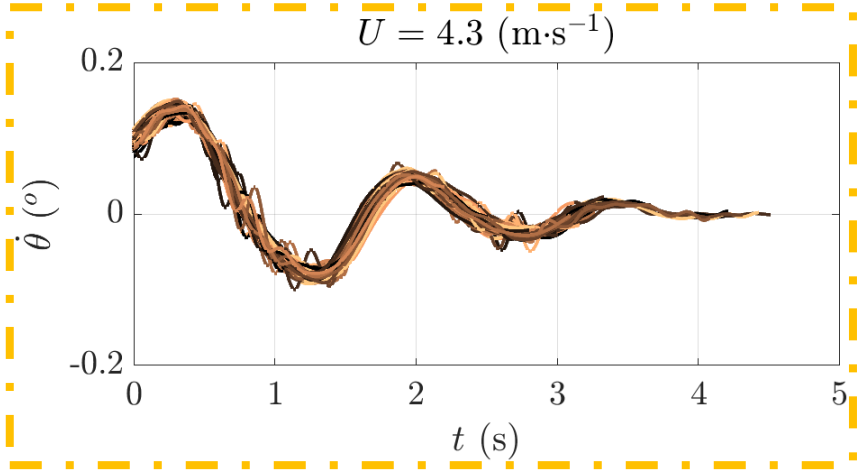

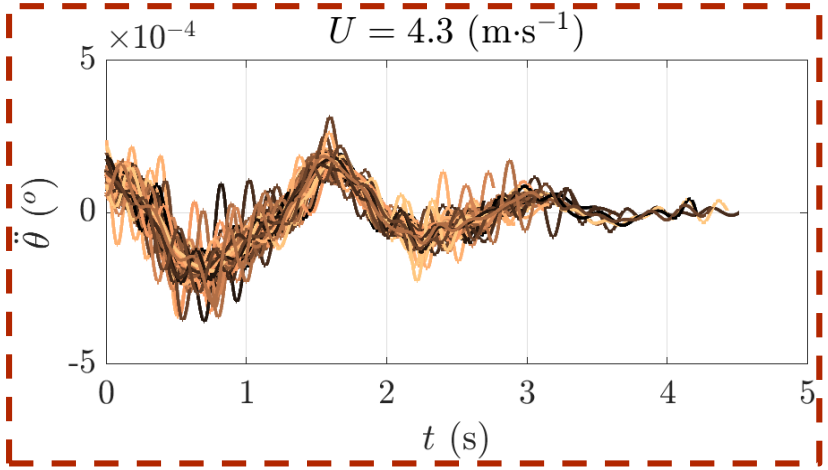

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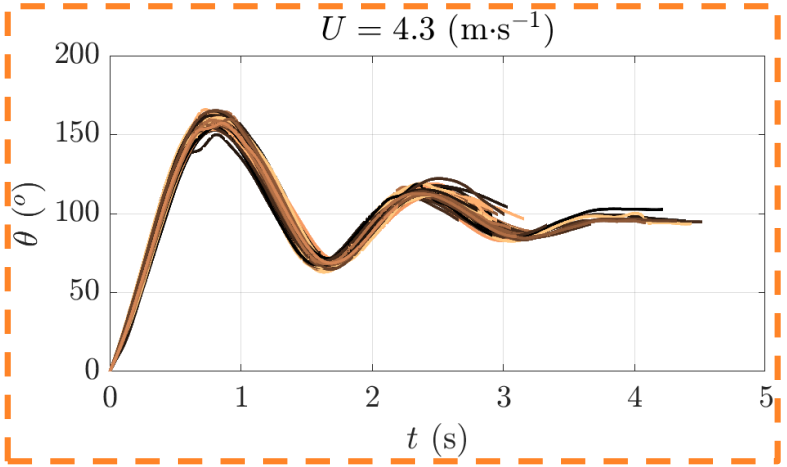

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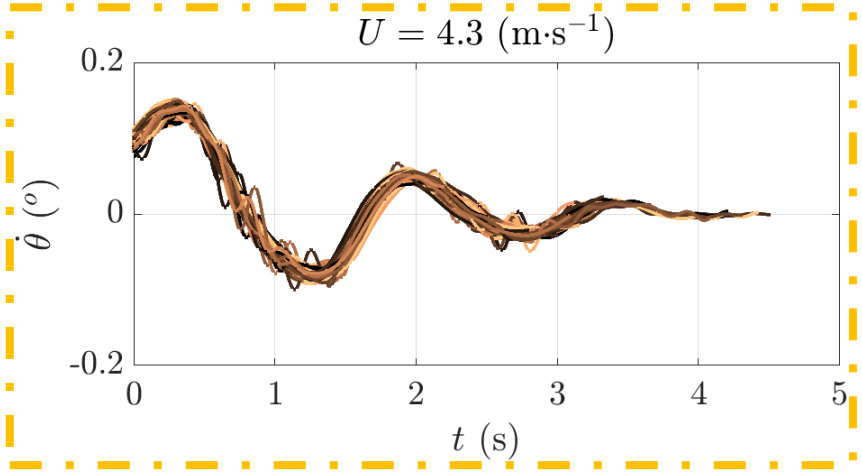
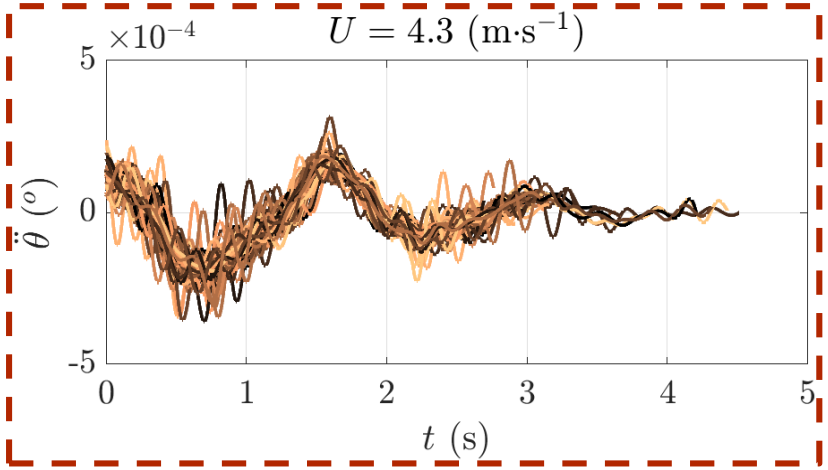
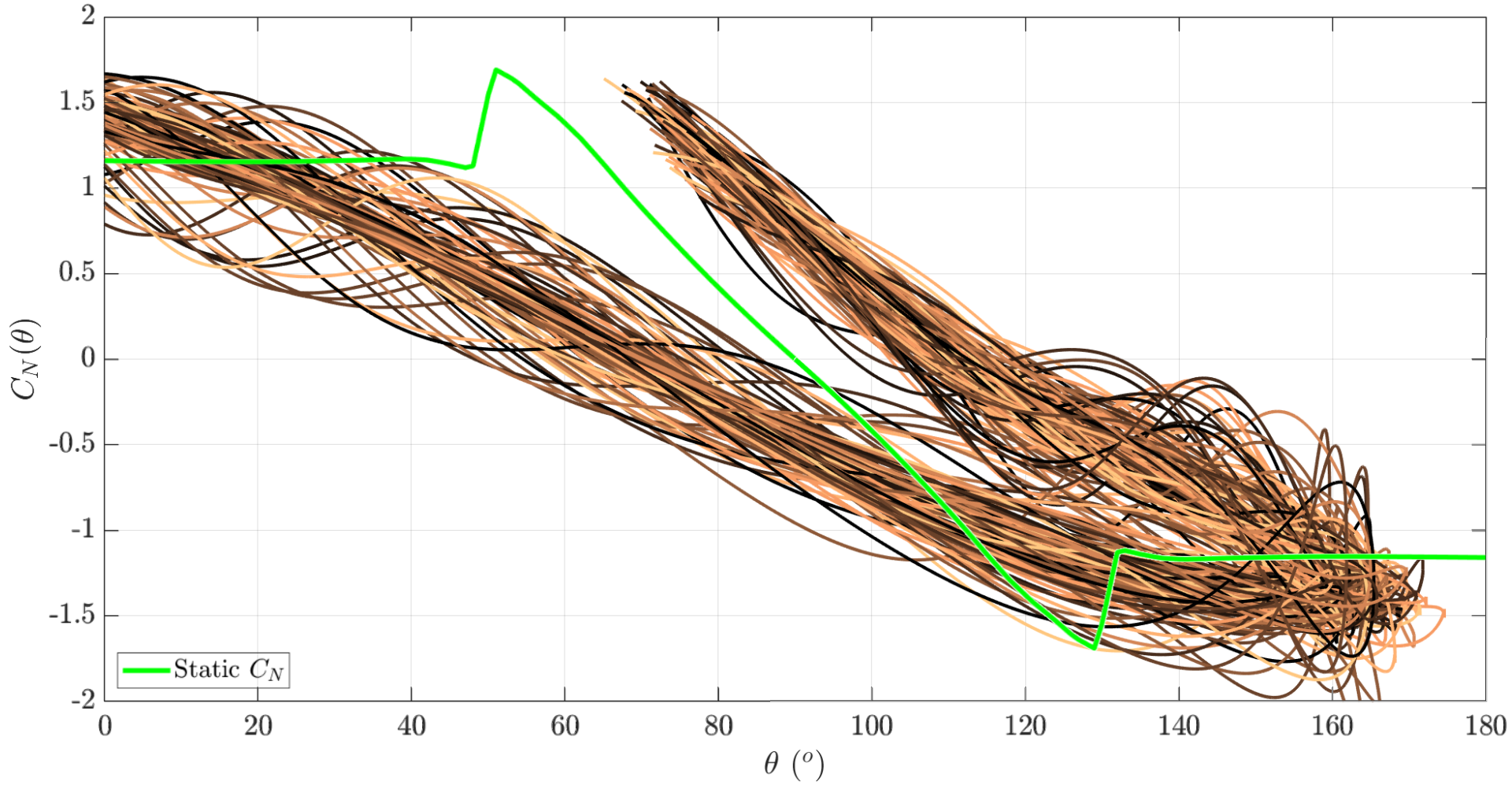




$$C_N \left(\theta + \tan^{-1}\left(\frac{L^*\dot{\theta}\sin\theta}{U - L^*\dot{\theta}\cos\theta}\right)\right) = \frac{J^*\ddot{\theta}}{\frac{1}{2}\rho S L^* (U^2 - 2UL^*\cos(\theta)\dot{\theta} + L^{*2}\dot{\theta}^2)}$$

A DYNAMICAL C_N COEFFICIENT



DYNAMICAL TRANSLATION INTO FUNDAMENTAL EQUATION

$$J^*\ddot{\theta} = \frac{1}{2}\rho S L^* U_{eff}^2 C_N^*\left(\theta_{eff}, \dot{\theta}, \ddot{\theta}, U, t\right)$$

2 ways of taking into account C_N^* in the equation at 1st order:

DYNAMICAL TRANSLATION INTO FUNDAMENTAL EQUATION

$$J^*\ddot{\theta} = \frac{1}{2}\rho S L^* U_{eff}^2 C_N^*(\theta_{eff}, \dot{\theta}, \ddot{\theta}, U, t)$$

2 ways of taking into account C_N^* in the equation at 1st order:

- Added mass ⇔ $\ddot{\theta}$ correction:

$$J_{eff}^*\ddot{\theta} = \frac{1}{2}\rho S L^*(U^2 - 2UL^*\cos(\theta)\dot{\theta} + L^{*2}\dot{\theta}^2)\, C_N\left(\theta + \tan^{-1}\left(\frac{L^*\dot{\theta}\sin\theta}{U - L^*\dot{\theta}\cos\theta}\right)\right)$$

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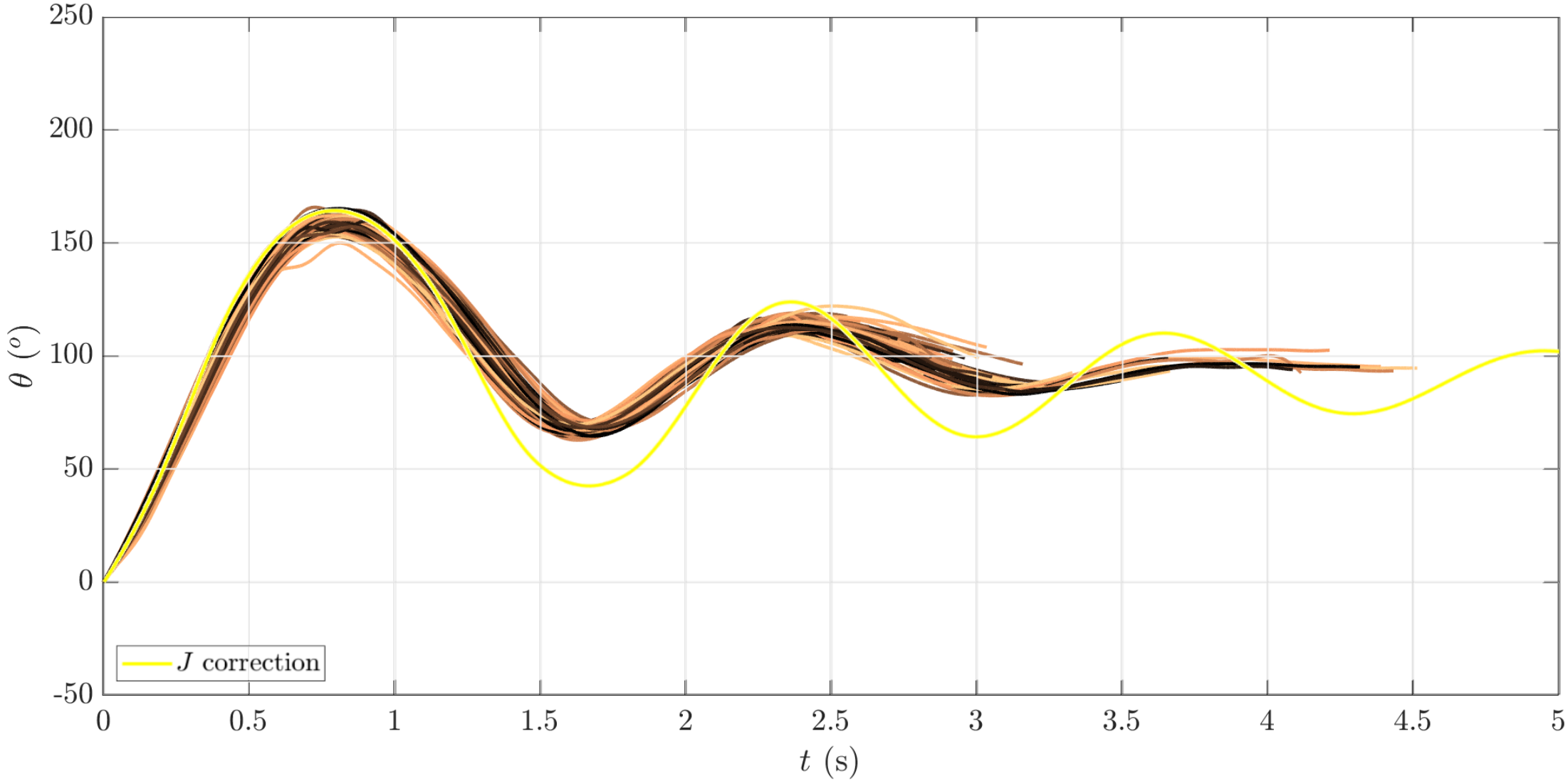
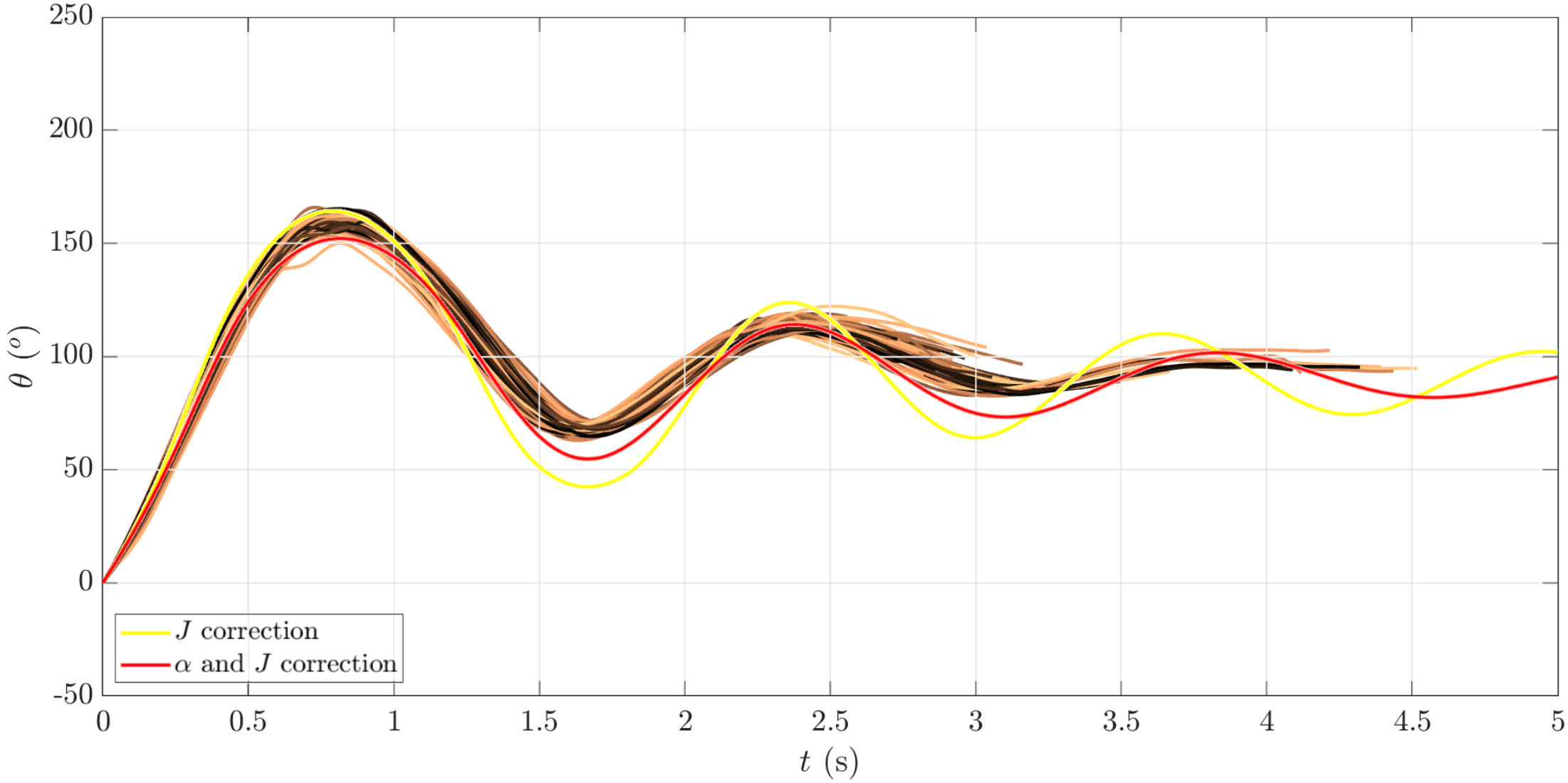
- Added damping ⇔ $\dot{\theta}$ correction :

$$J^*\ddot{\theta} = -\alpha_U J^*\dot{\theta} + \frac{1}{2}\rho S L^*(U^2 - 2UL^*\cos(\theta)\dot{\theta} + L^{*2}\dot{\theta}^2)\, C_N\left(\theta + \tan^{-1}\left(\frac{L^*\dot{\theta}\sin\theta}{U - L^*\dot{\theta}\cos\theta}\right)\right)$$



CORRECTION EFFECTS ON THE MODEL



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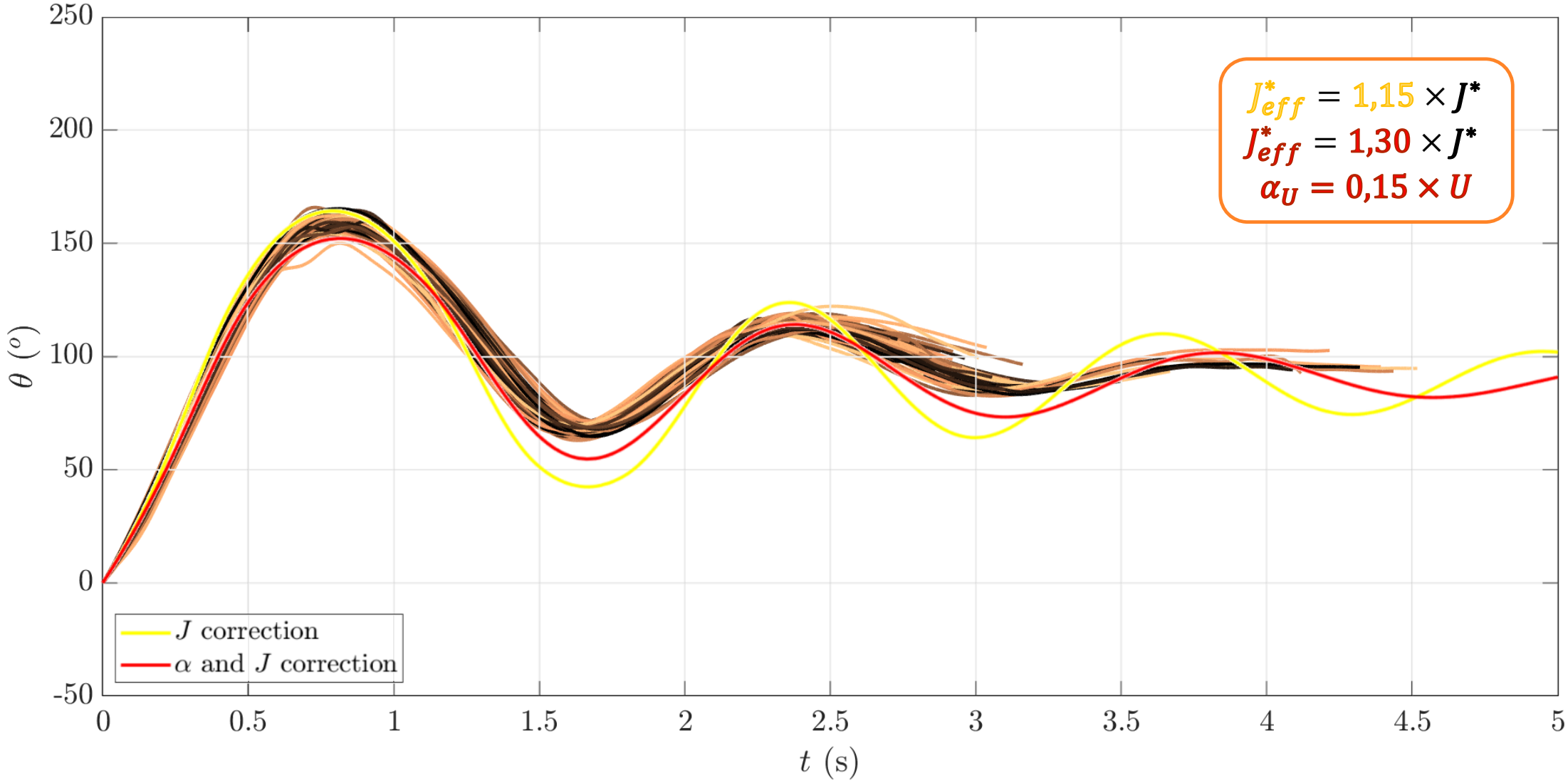
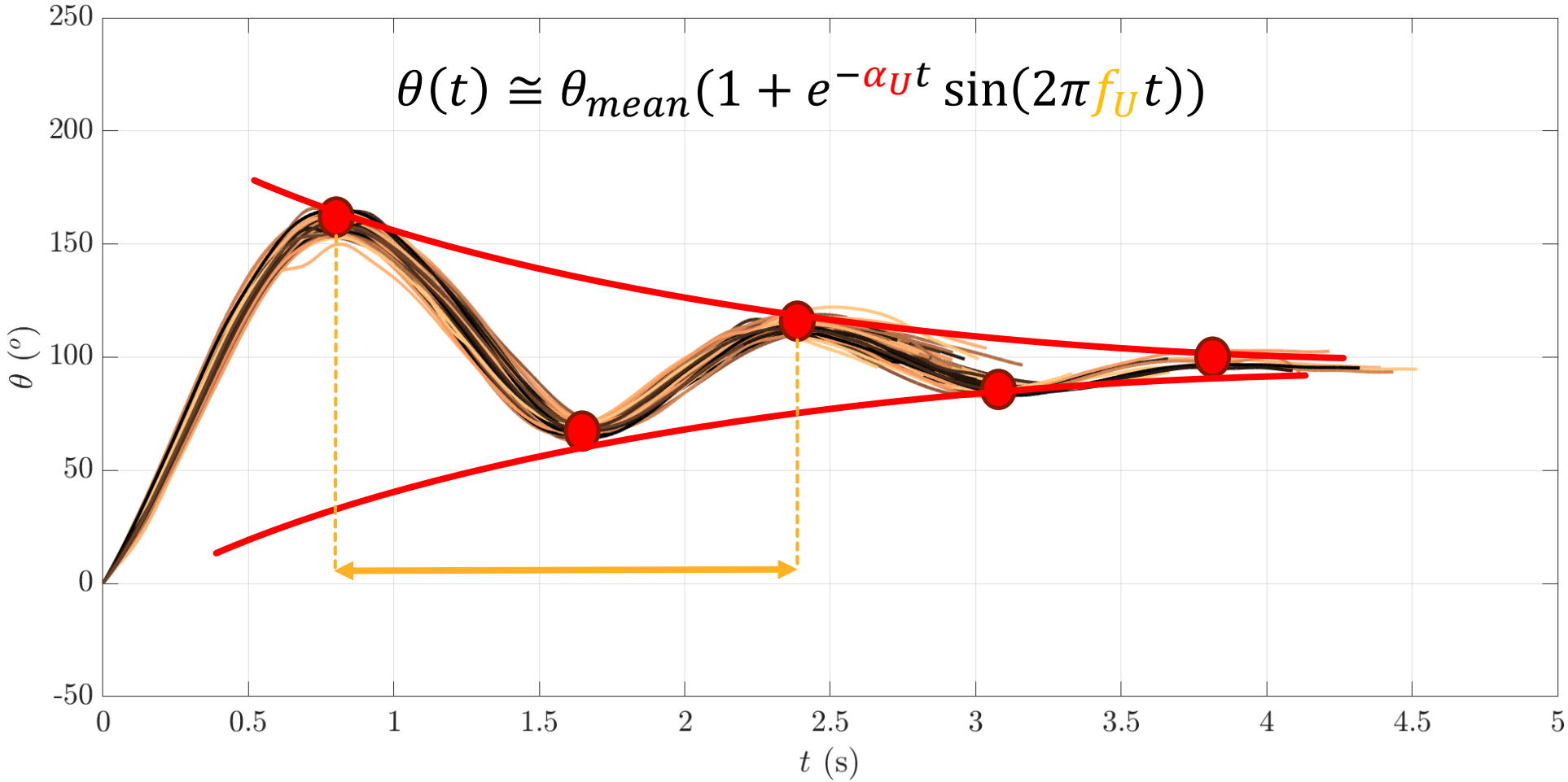
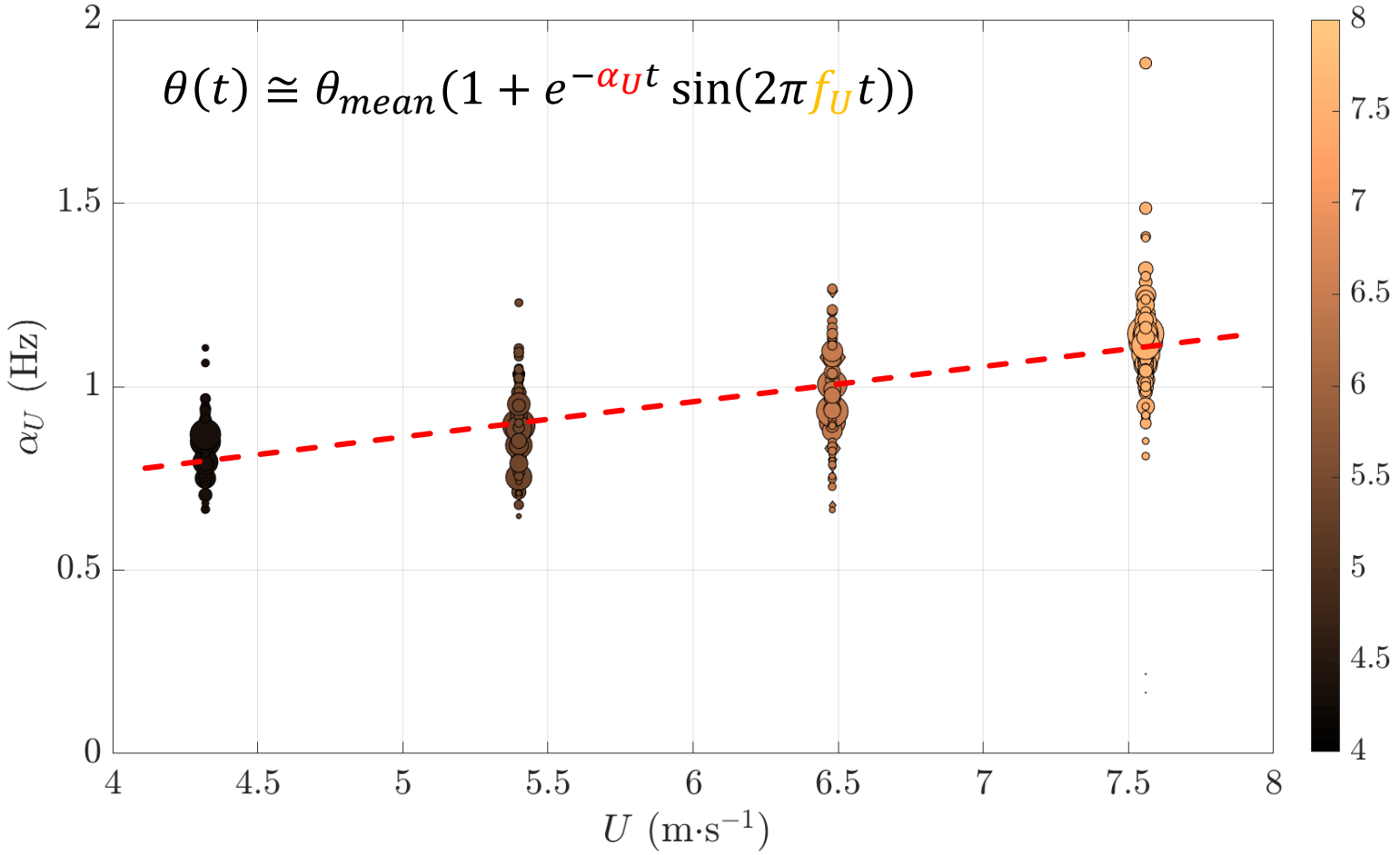


CORRECTION EFFECTS ON THE MODEL

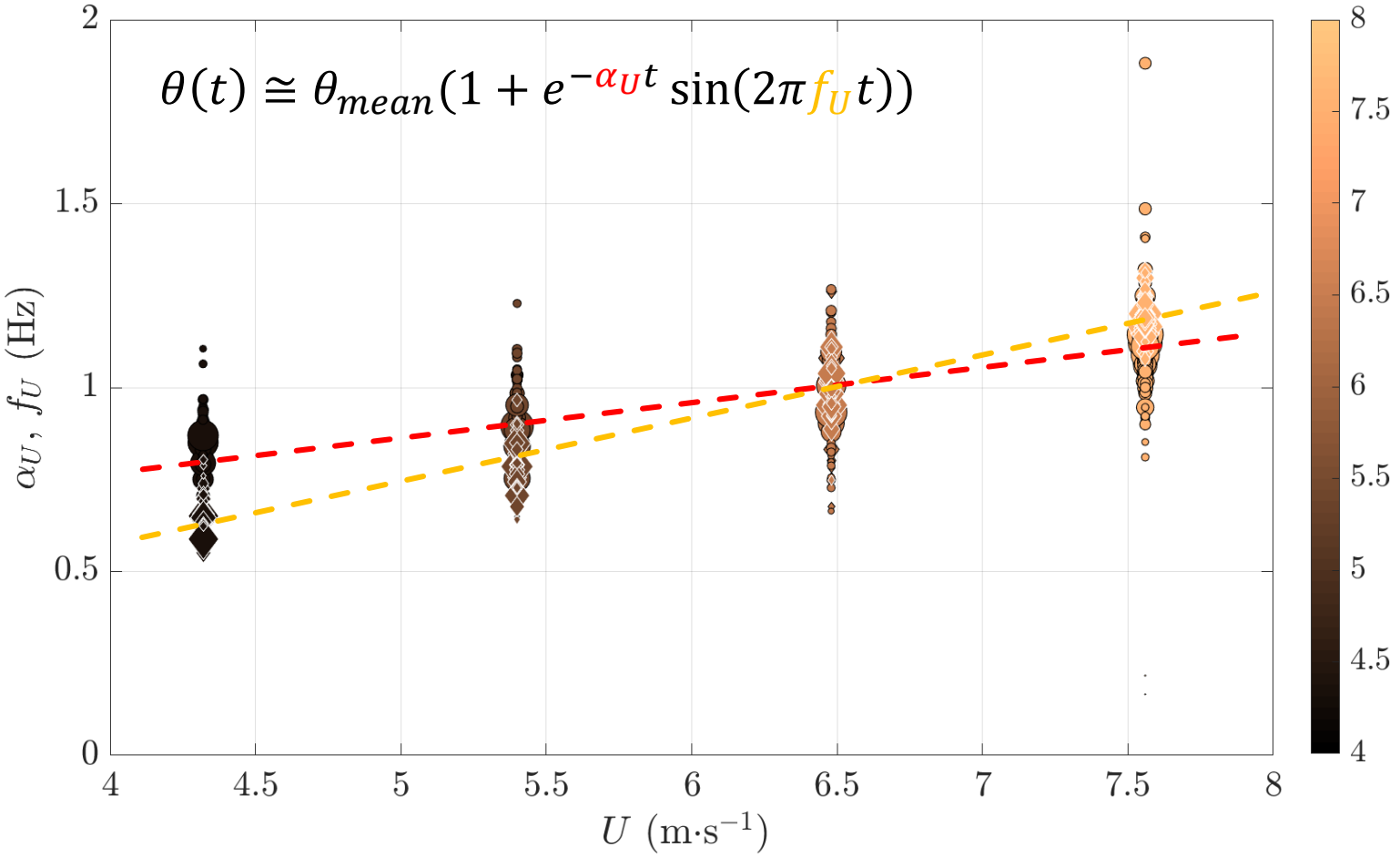
ESTIMATING THE CORRECTIONS



UNDERSTANDING THE DAMPING

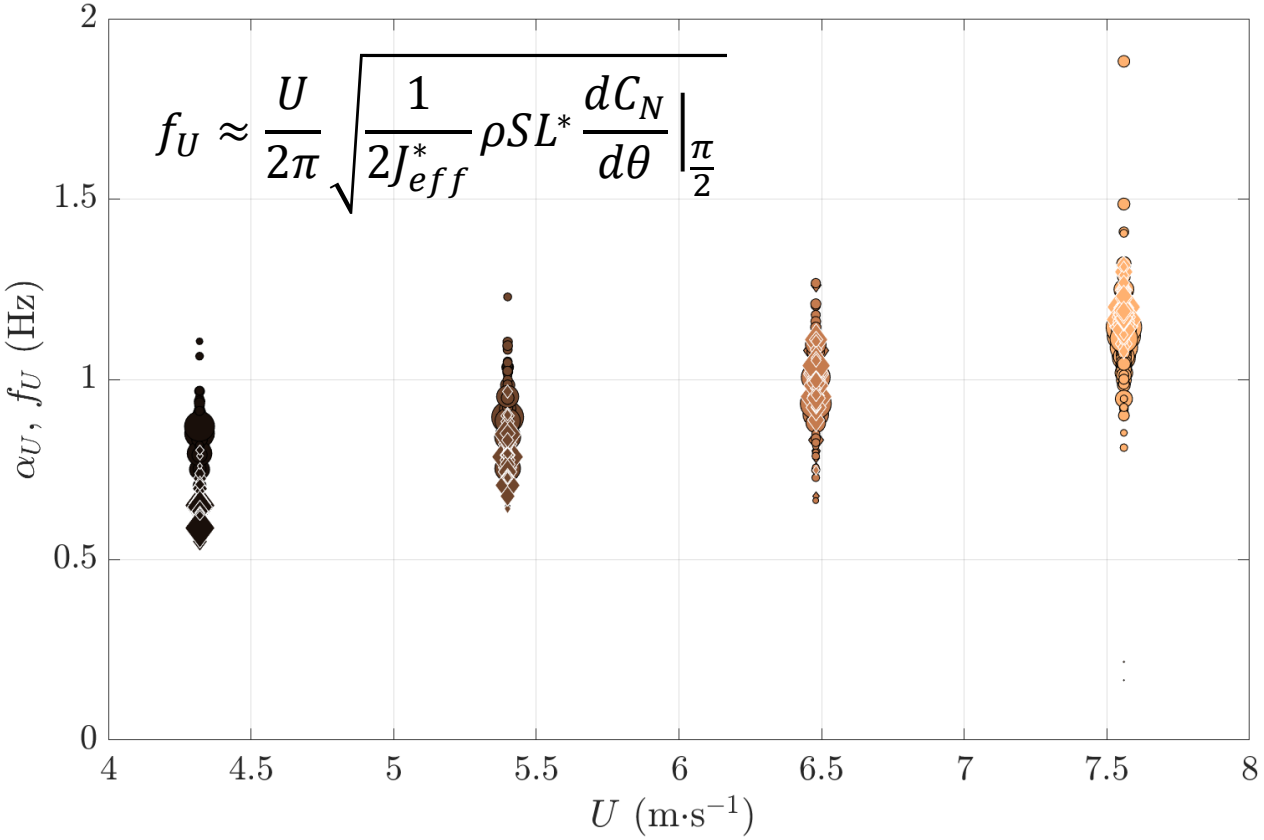


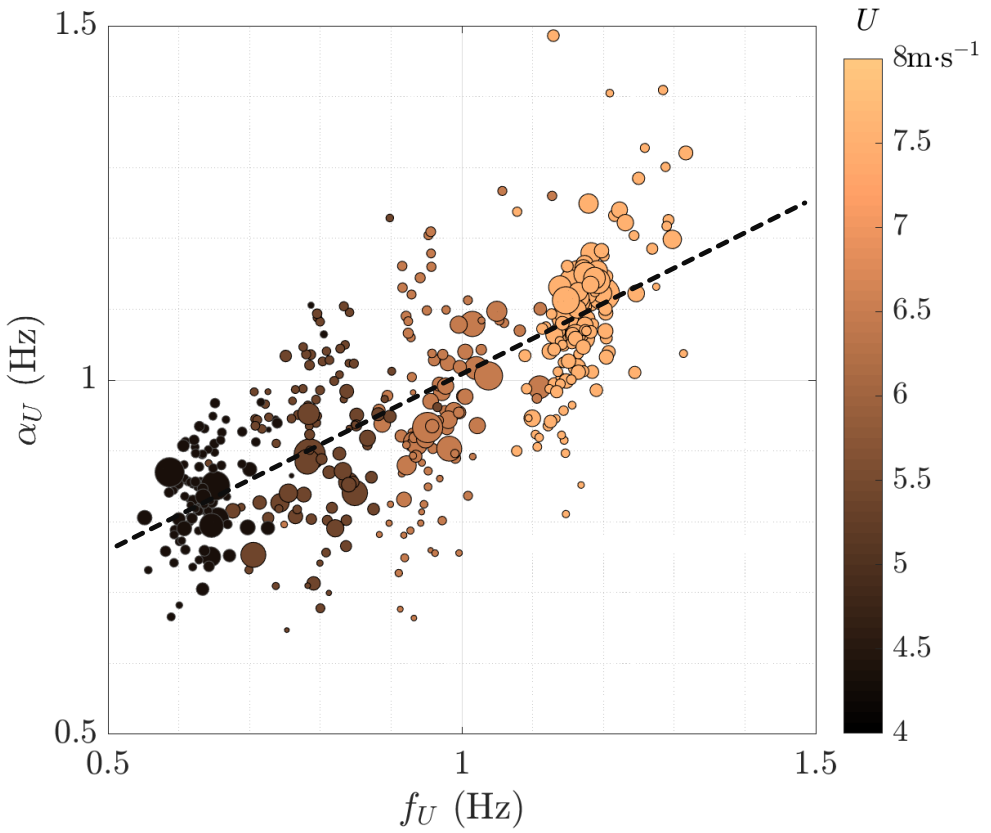
UNDERSTANDING WHERE THE DAMPING COMES FROM



UNDERSTANDING WHERE THE DAMPING COMES FROM

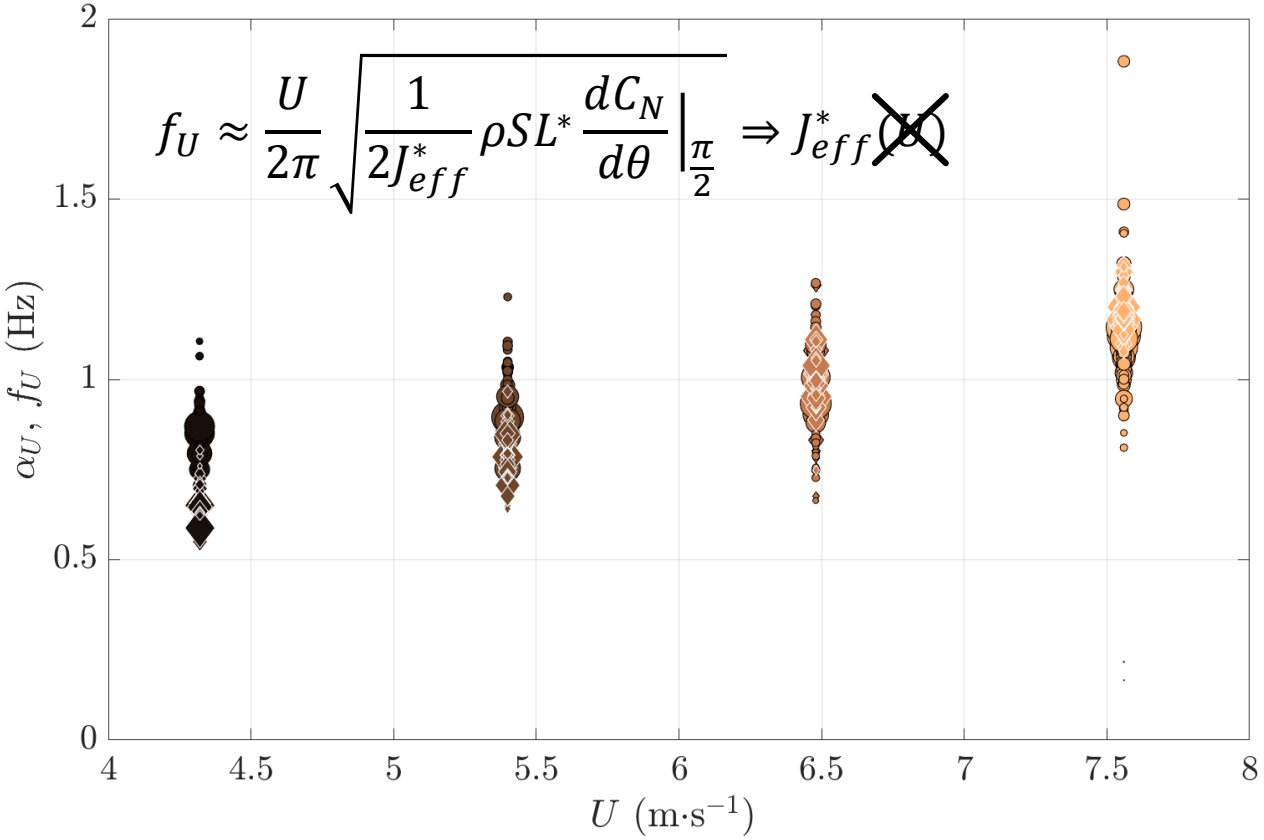
$$f_U \approx \frac{U}{2\pi}\sqrt{\frac{1}{2J^*_{eff}}\rho S L^* \frac{dC_N}{d\theta}\Big|_{\frac{\pi}{2}}}$$

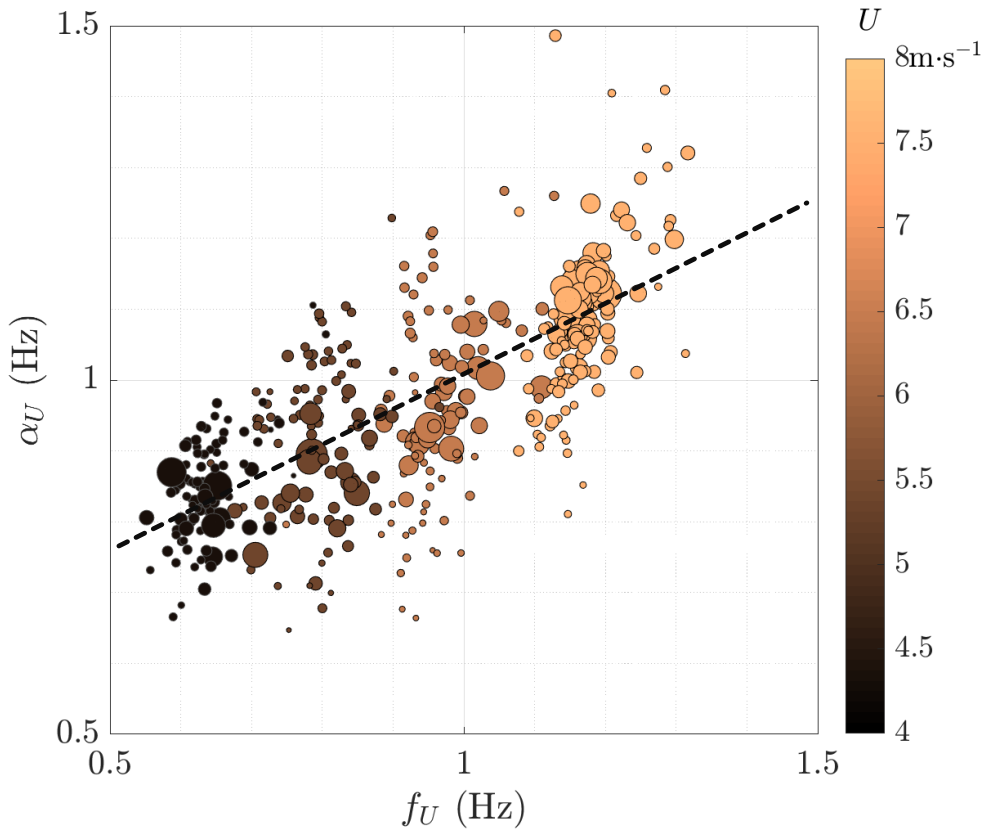




UNDERSTANDING WHERE THE DAMPING COMES FROM

$$f_U \approx \frac{U}{2\pi}\sqrt{\frac{1}{2J^*_{eff}}\rho S L^* \frac{dC_N}{d\theta}\Big|_{\frac{\pi}{2}}} \Rightarrow J^*_{eff}(U)$$



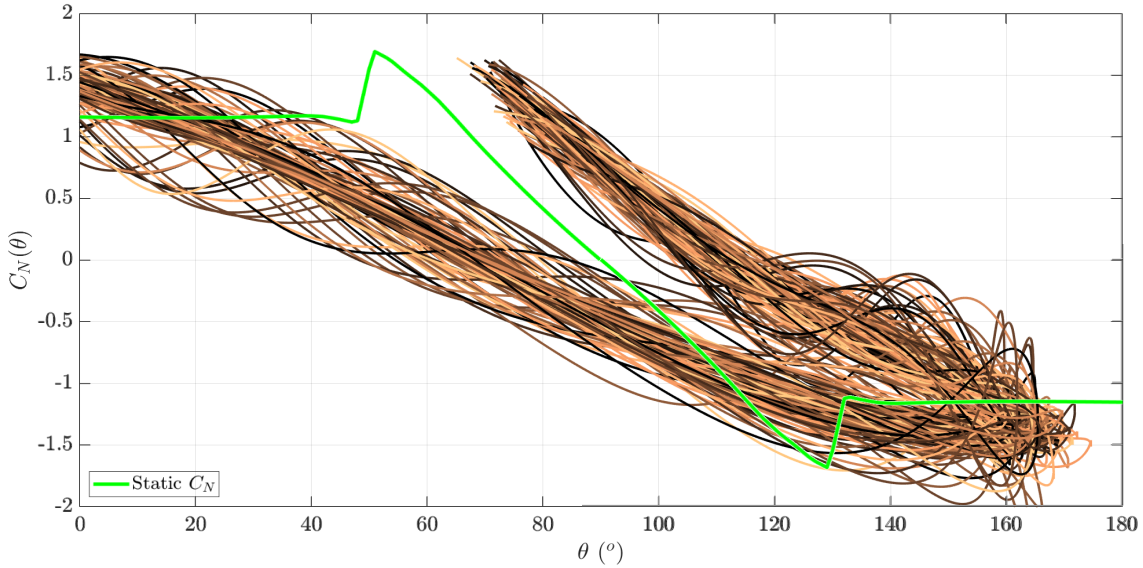


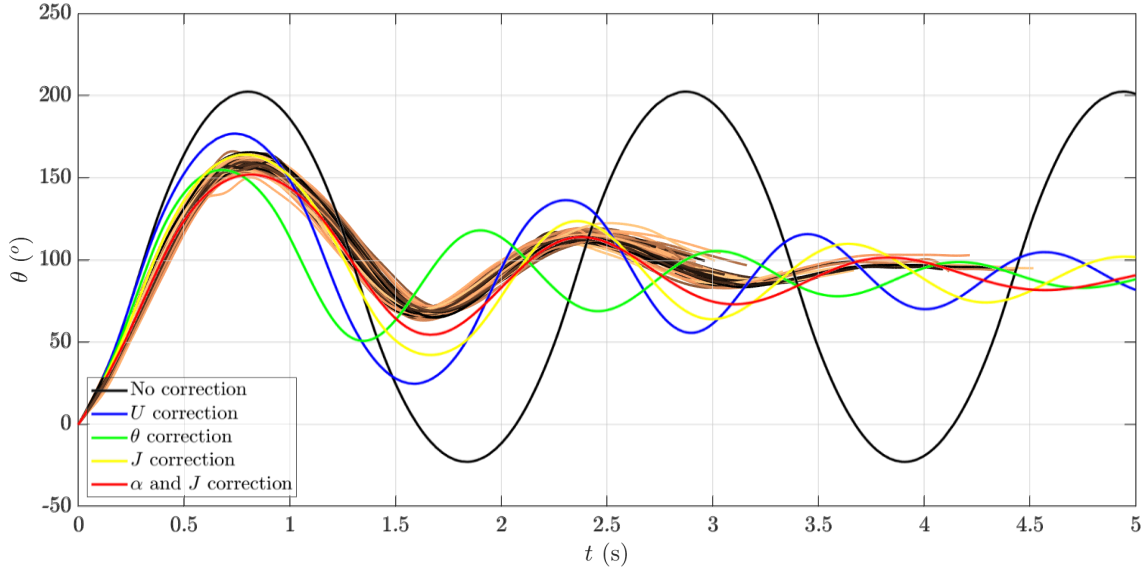
CONCLUSION

Dynamical C_N coefficient

Non stationnary corrections

$$J^*\ddot{\theta} = \frac{1}{2}\rho S L^* U_{eff}^2 C_N^*(\theta_{eff}, \dot{\theta}, \ddot{\theta}, U, t)$$





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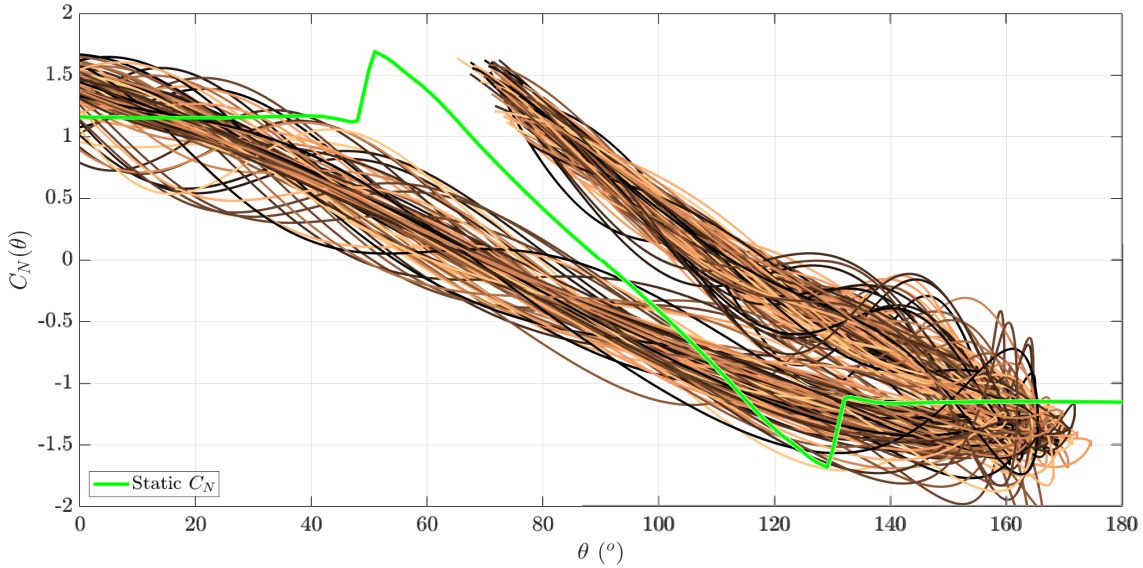

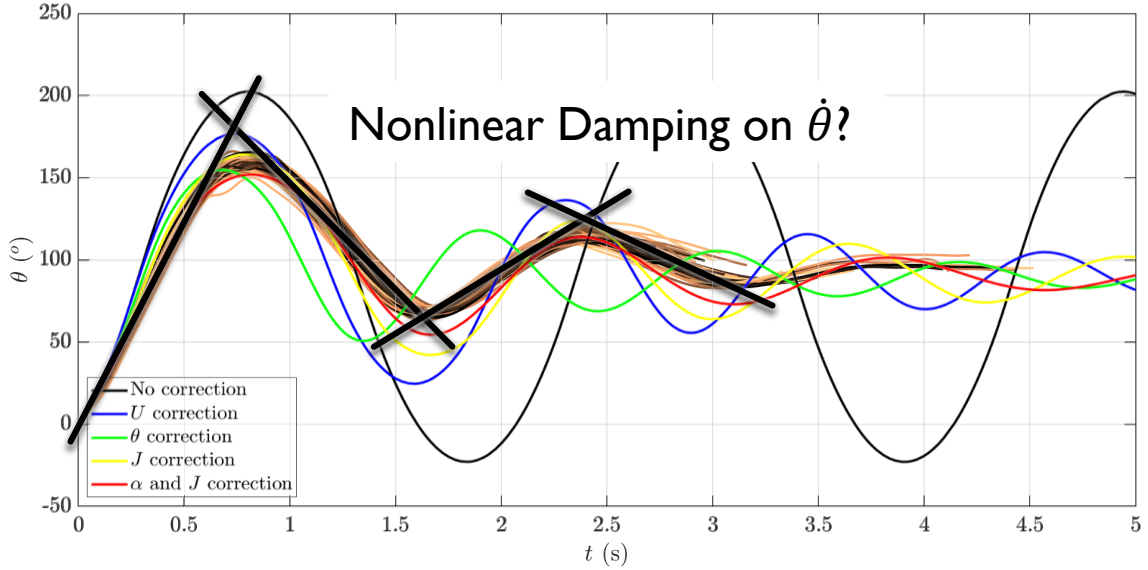

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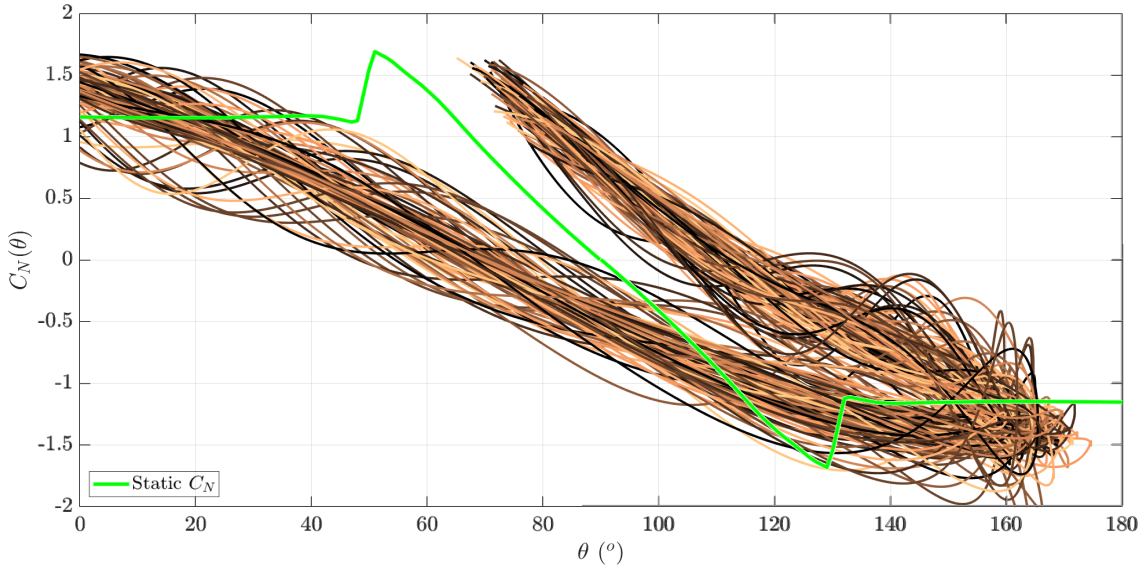

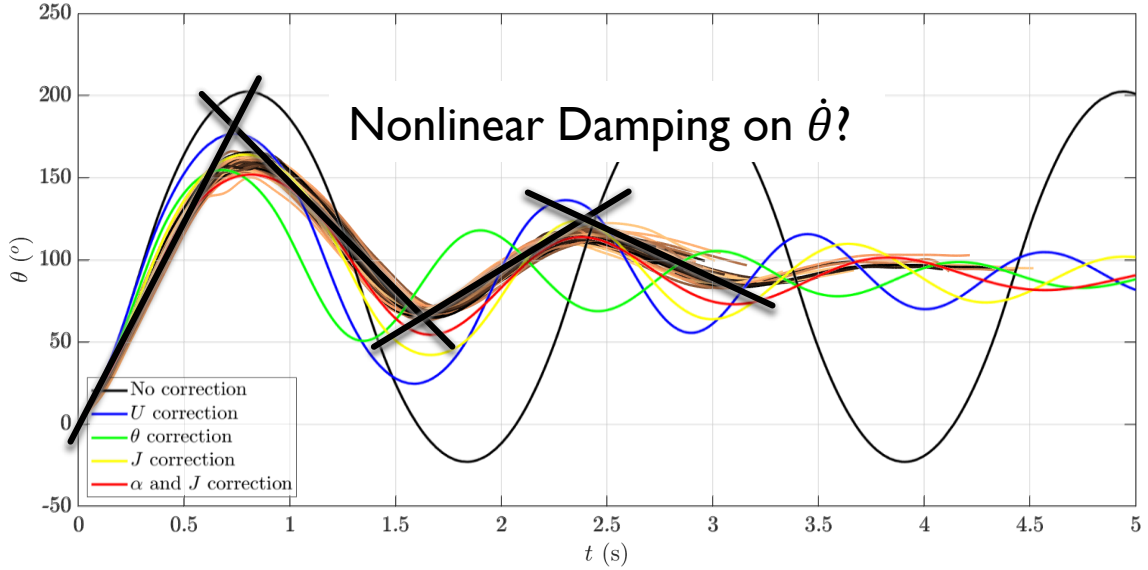

"With Great Amplitude Comes Great Nonlinearity"

THANK YOU FOR YOUR ATTENTION

ANY QUESTIONS?