

The muon magnetic moment: a precision test shaking the Standard Model

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- April 7 2021: announcement of the first result of the Fermilab experiment measuring the muon magnetic anomaly
- Comparison with the theoretical prediction within the Standard Model shows an excess at the level of 4.2σ , larger than the previous 3.7σ with respect to the Brookhaven experiment
- In this talk, after a general introduction and some information on the experiment, I will review the status of the hadronic vacuum polarization contribution using a dispersion relation based on the measured cross sections for $e^+ e^- \rightarrow \text{hadrons}$

Work done with Andreas Hoecker, Bogdan Malaescu, Zhiqing Zhang

The electron g-2 early history

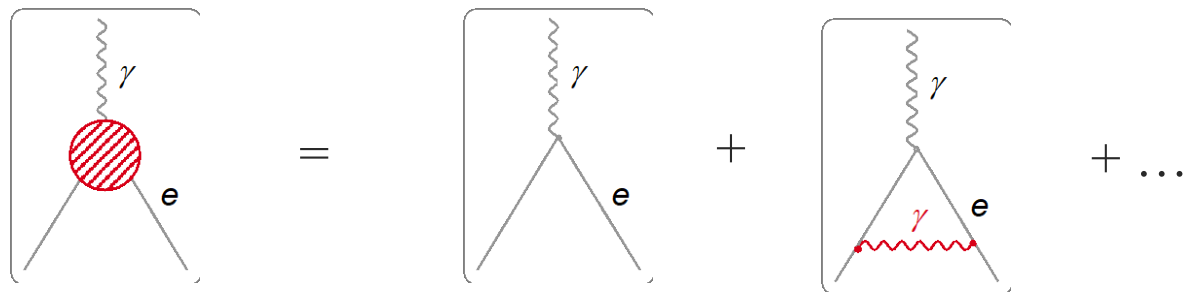
- Dirac's relativistic theory of the electron (1928) naturally accounted for quantized spin, and described elementary spin-1/2 particles (and their anti-particles). Classical limit \Rightarrow Pauli equation with a magnetic moment:

$$\vec{\mu} = -g_e \frac{e}{2m_e} \vec{S} \quad \text{with } |g_e| = 2 \text{ is the gyromagnetic factor}$$

- Dirac's prediction was confirmed by Kinsler & Houston in 1934 through studying the Zeeman effect in neon, but a deviation from $g_e = 2$ was found by Nafe, Nels & Rabi in 1947 and established with increased precision by Kusch & Foley

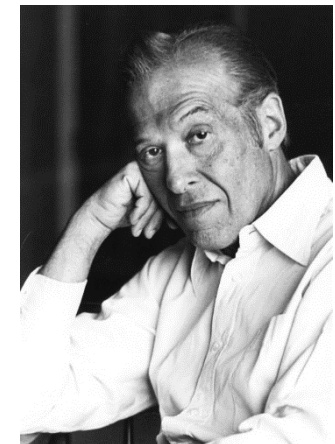
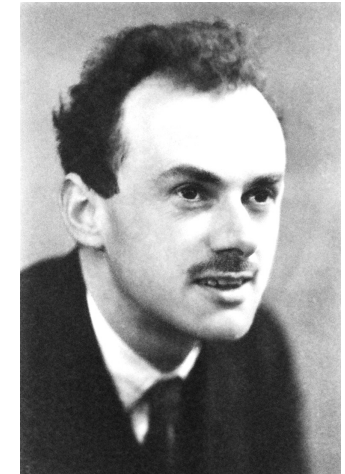
magnetic anomaly $a = (g-2) / 2 \sim 10^{-3}$

- Development of quantum electrodynamics (Dyson, Feynman, Schwinger, Tomonaga): Dirac's $g = 2$ corresponds to the lowest order QED graph



$$a_e^{\text{QED}} = \frac{\alpha}{2\pi} + \dots = 0.001\,161\,...$$

- The power of precision to discover new physics (here QED quantum fluctuations)



Why measure the muon $g-2$?

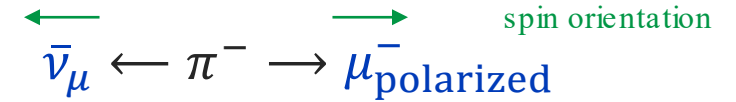
- 3 families of fermions (leptons and quarks) with universal coupling strengths to electroweak interactions
- The 3 charged leptons $l \equiv (e, \mu, \tau)$ differ only by their own leptonic quantum numbers and their masses
 $m_e = 0.511 \text{ MeV}$ $m_\mu = 105.7 \text{ MeV}$ $m_\tau = 1776.9 \text{ MeV}$
- e stable, μ and τ are unstable and decay through the weak interaction with lifetimes $2.2 \mu\text{s}$ and 390 fs
- sensitivity of a_l to new physics at energy scale Λ goes like m_l^2 / Λ^2
- Muon more sensitive by large factor $(m_\mu/m_e)^2 \sim 43000$, but measurement limited by short lifetime
- Measurement for τ lepton not practical at the moment

Key ingredients for measurement: polarized muons and muon spin analysis through decay electrons, both following from maximum P violation in weak interaction

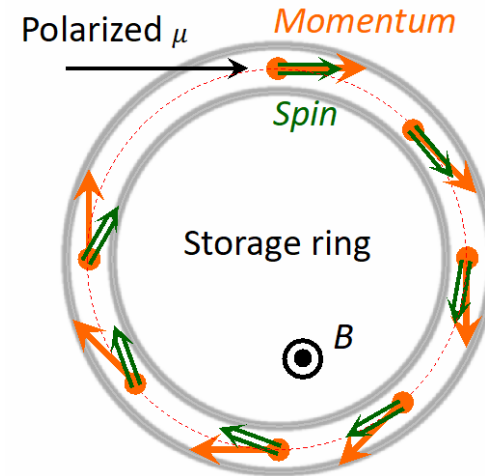
- Muons produced at accelerators in pion decays are polarized
- Angle of energetic decay electrons is correlated with muon spin

Principle of muon g-2 measurement (CERN 1960-80)

1. Parity violation polarizes muons in pion decay



2. Anomalous frequency proportional to a_μ

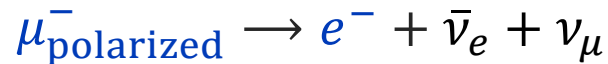


- Very uniform magnetic field
- Focusing with electrostatic quadrupoles

3. Magic γ to cancel $\beta \times E$ effect:

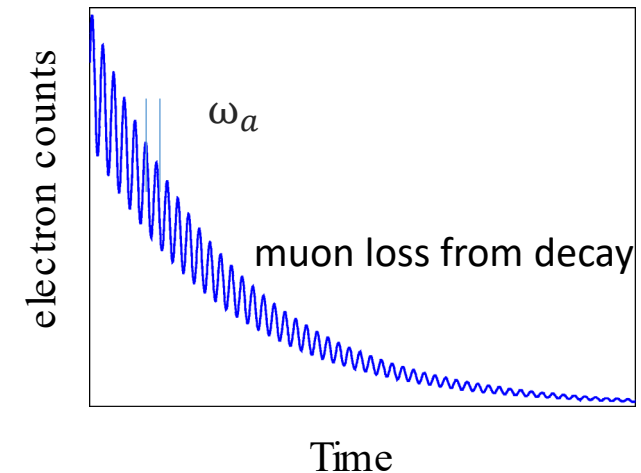
$$\vec{\omega}_a = \frac{e}{m_\mu c} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] \approx \frac{e}{m_\mu c} a_\mu \vec{B} \quad P_\mu = 3.09 \text{ GeV}/c$$

4. Again parity violation in muon decay



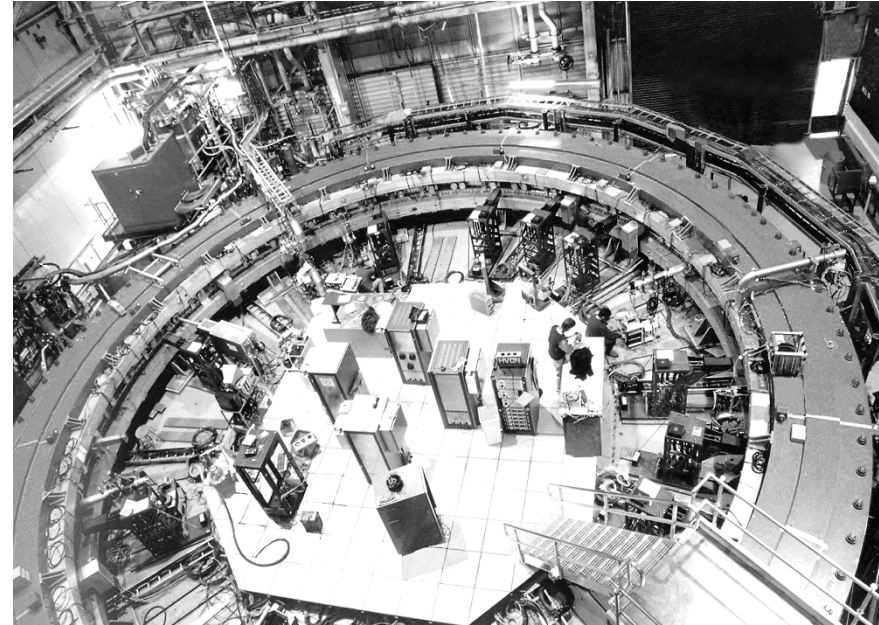
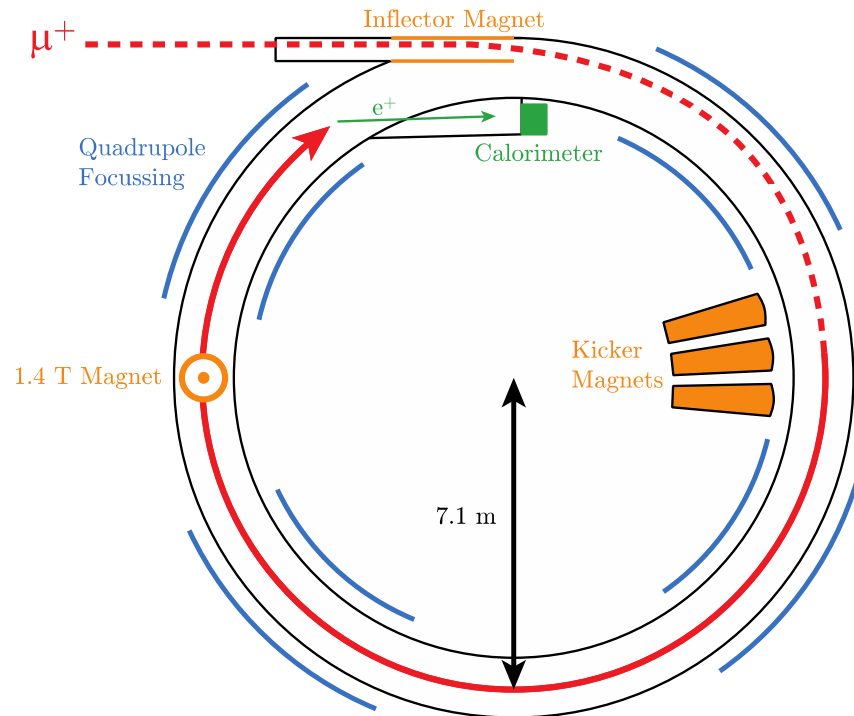
fast electron emitted in direction opposite to muon spin

Double miracle by virtue of P violation !



Muon g-2 measurement (Brookhaven 1990-2006)

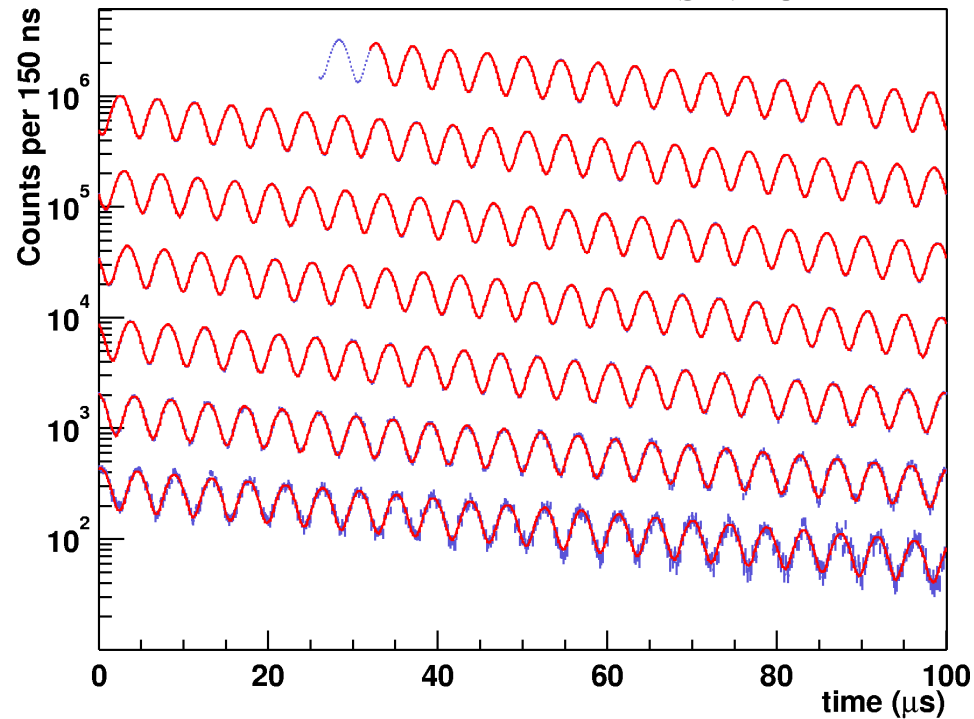
- A 24 GeV proton beam (AGS) incident on a target produces large number of pions that decay to muons
- The 3.1 GeV muon beam (relativistically enhanced lifetime of 64 μs) is injected into a 7.1 m radius ring with 1.4 T vertical magnetic field, which produces cyclotron motion matching the ring radius
- Electrostatic focusing of the beam is provided by a series of quadrupole lenses around the ring.



- Decay electrons (correlated with μ spin precession) counted vs. time in calorimeters inside ring ($\rightarrow \omega_a$)
- Precise measurement of ω_a and B allows to extract a_μ

Muon g-2 measurement (Brookhaven 1990-2006)

E821 (g -2), hep-ex/0202024



Observed positron rate in successive 100 μs periods
 ~ 150 polarisation rotations during measurement period

$$\omega_a \approx \frac{e}{m_\mu c} a_\mu B$$

obtained from time-dependent fit

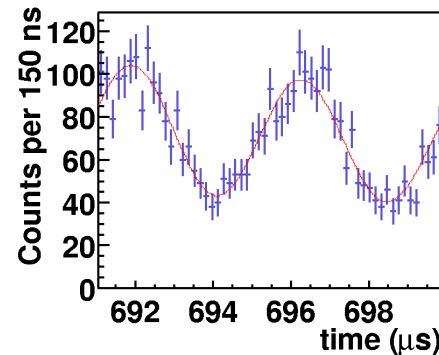
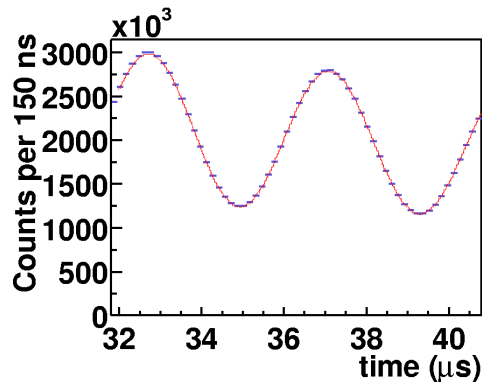
$$N(t) = N_0 e^{-t/\gamma\tau} [1 - A \cdot \sin(\omega_a t - \phi)]$$

In blue: fit parameters

B field measured with Hall probes with RMN frequency as reference
 $\Rightarrow a_\mu$ obtained as ratio of 2 frequencies (double blind analysis)

Total systematic uncertainty on ω_a : 0.2–0.3 ppm,
 with largest contributors:

- pileup (\sim in-time arrival of two low-E electrons)
- muon losses
- coherent betatron oscillation (muon loss and CBO amplitude [frequency: 0.48 MHz, compared to ω_a : 0.23 MHz] are part of fit)
- calorimeter gain changes



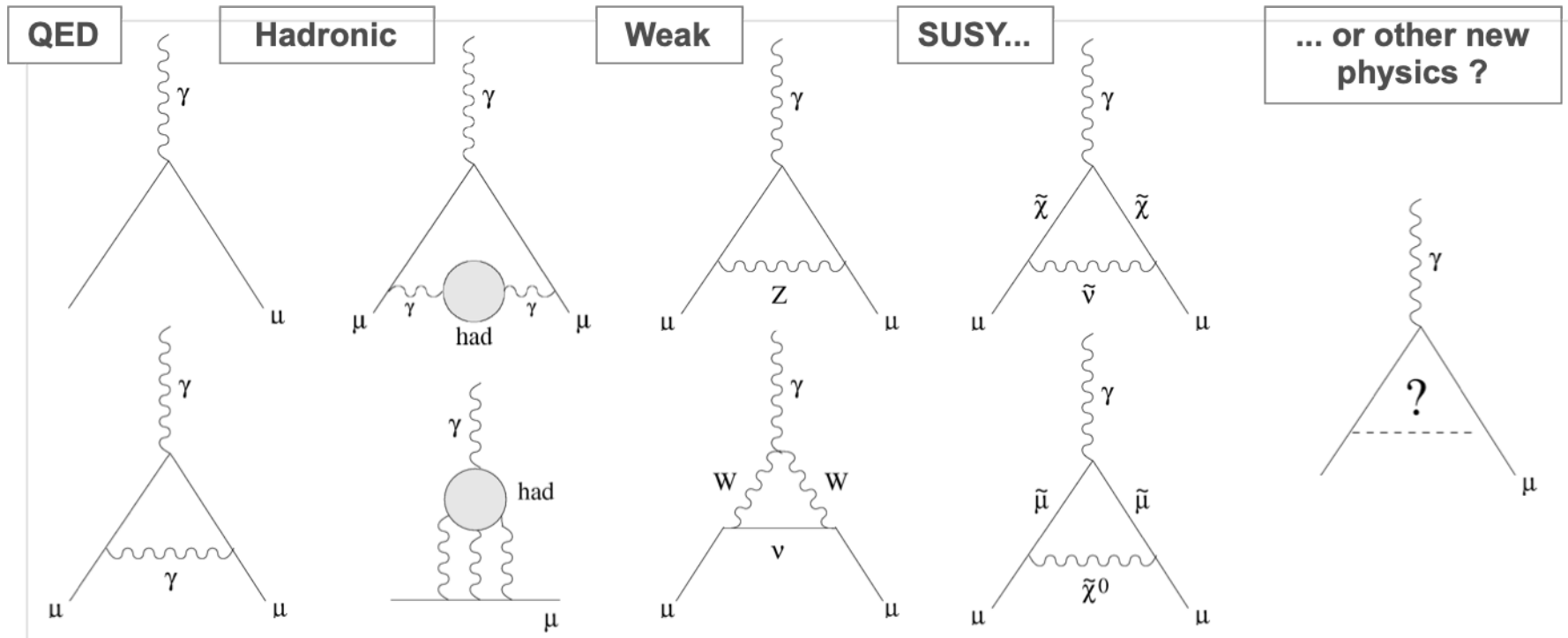
$$a_\mu = 11\,659\,209.1 \underset{\text{stat}}{(5.4)} \underset{\text{syst}}{(3.3)} \cdot 10^{-10}$$

Theoretical prediction for a_μ

$$a_\mu^{\text{th}} = a_\mu^{\text{SM}} + a_\mu^{\text{BSM}}$$

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{Weak}}$$

a_μ^{BSM}



Theoretical prediction for a_μ : QED

Known to 5 loops, good convergence, diagrams with internal electron loops enhanced:

$$a_\mu^{\text{QED}} = \frac{\alpha}{2\pi} + A_2 \left(\frac{\alpha}{\pi}\right)^2 + A_3 \left(\frac{\alpha}{\pi}\right)^3 + A_4 \left(\frac{\alpha}{\pi}\right)^4 + A_5 \left(\frac{\alpha}{\pi}\right)^5$$

A_2 A_3 known analytically, A_4 A_5 obtained with Monte Carlo techniques, partially checked analytically for A_4

Aoyama, Hayakawa, Kinoshita, Nio (2012-2019)

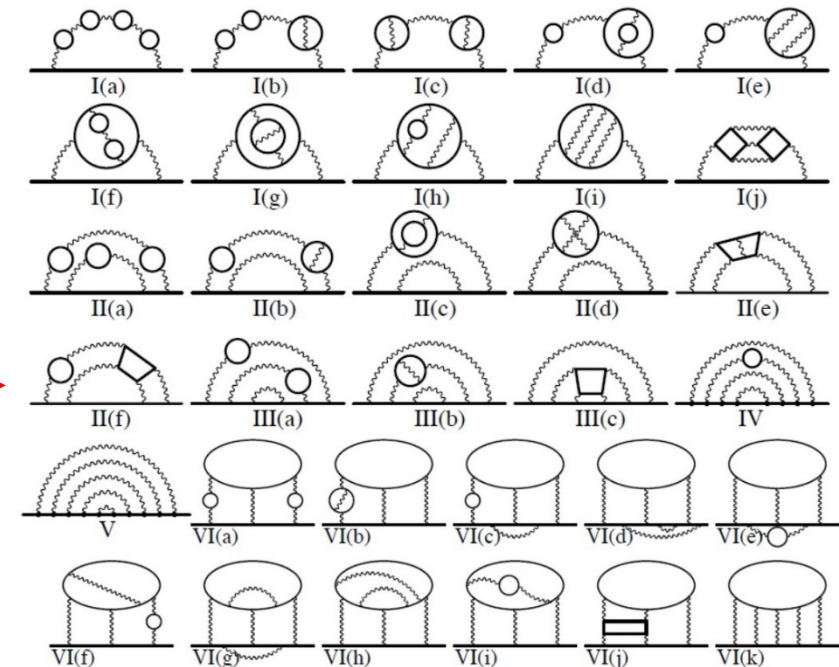
$\alpha = 137.035\,999\,046\,(27)$ from Cs recoil measurement (Mueller et al.)

$$\begin{aligned}
 a_\mu^{\text{QED}} &= 116\,140\,973.321\,(23) \\
 &+ 413\,217.626\,(7) \\
 &+ 30\,141.902\,(33) \\
 &+ 381.004\,(17) \\
 &+ 5.078\,(6) \\
 &= 116\,584\,718.931\,(104)
 \end{aligned}
 \quad (\times 10^{-11})$$

uncertainty dominated by estimate on α^6 term

α
 α^2
 α^3
 α^4
 α^5

12672 diagrams



Theoretical prediction for a_μ : EW, hadronic light-by-light

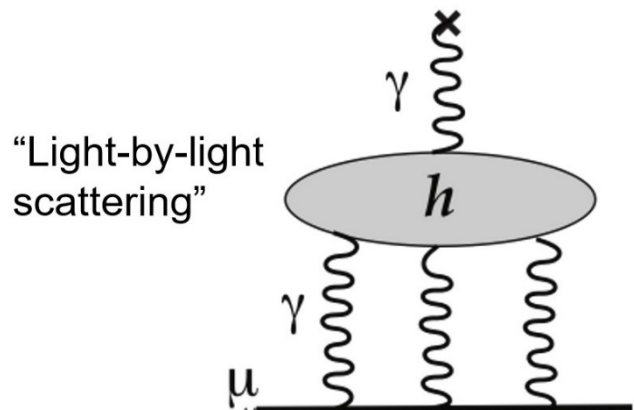
- **EW**: one-loop + two-loop involving W, Z bosons (little sensitivity to Higgs boson mass)

$$a_\mu^{\text{EW}} = 153.6 (1.0) \times 10^{-11}$$

shows level of sensitivity of a_μ to physics at large mass scales $\sim O(0.1 \text{ TeV})$

Precision at low energies \Leftrightarrow high energy frontier

- Hadronic light-by-light: α^3 contribution not computable by analytical QCD; so far only estimated by phenomenological models using intermediate particles; new approach partly using experimental data (2017); also first results from QCD lattice simulations (2019)



small contribution

$$a_\mu^{\text{HLbL}} = 94 (19) \times 10^{-11}$$

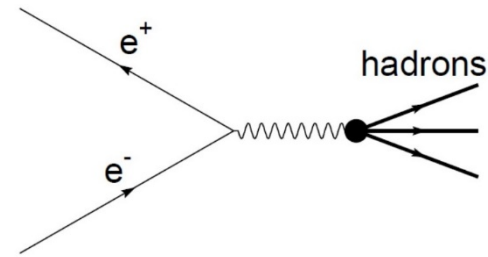
Theoretical prediction for a_μ : Hadronic Vacuum Polarization

Dominant uncertainty for the theoretical prediction from HVP part which cannot be calculated from QCD (low mass scale), but one can use experimental data on $e^+e^- \rightarrow$ hadrons cross section

Born: $\sigma^{(0)}(s) = \sigma(s)(\alpha/\alpha(s))^2$

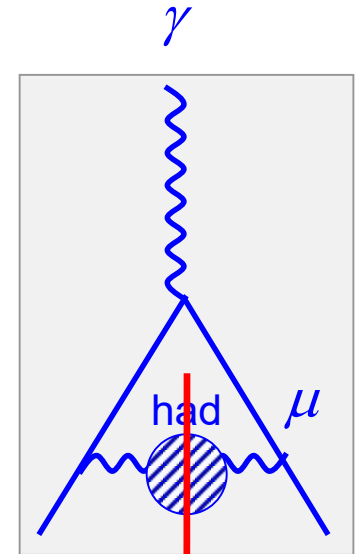
$$12\pi \operatorname{Im}\Pi_\gamma(s) = \frac{\sigma^0 [e^+e^- \rightarrow \text{hadrons} (\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

$$\operatorname{Im}[\text{diagram}] \propto |\text{diagram} \text{ hadrons}|^2$$

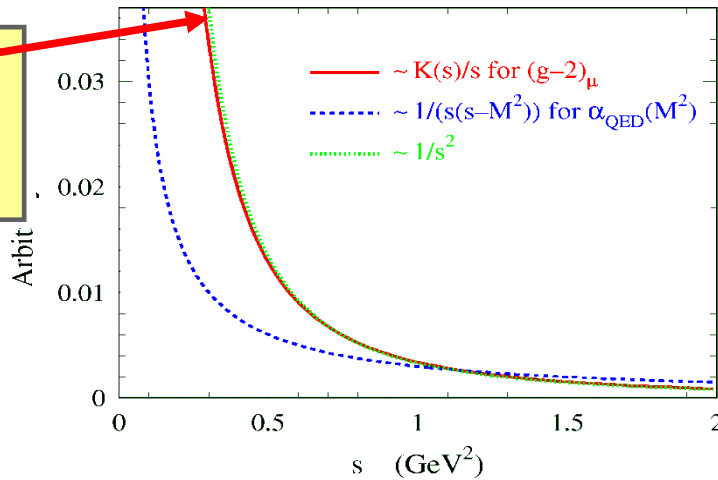


- unitarity
- analyticity

\Rightarrow dispersion relation



$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$



Precise $\sigma(e^+e^- \rightarrow \text{hadrons})$ measurements at low energy are necessary

Bouchiat-Michel (1961)
Brodsky-de Rafael (1968)

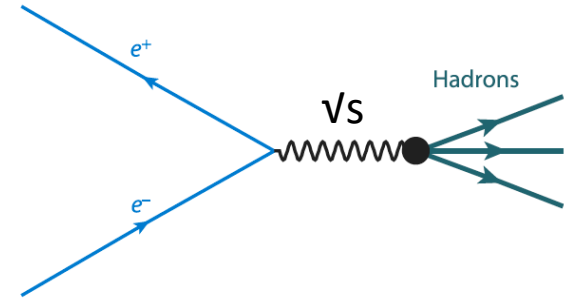
Hadronic Vacuum Polarization (DHMZ group)

- HVP has been for long and still now the largest contribution to the uncertainty of the SM a_μ prediction
- Limited by the accuracy of e^+e^- experimental data
- DHMZ group (MD, Andreas Hoecker, Bogdan Malaescu, Zhiqing Zhang) involved since 1997
- Result used as reference for the Brookhaven experiment: comparison revealed a deficit in the prediction at $\sim 2-3 \sigma$ level, hence our motivation to continue this effort toward a more precise prediction
- Main contributions to **data treatment**
 - Compilation of existing data for e^+e^- annihilation to obtain R as a sum of exclusive processes
 - Robust combination techniques taking into account all correlated uncertainties as function of energy, between exclusive channels, and between experiments
 - Correct for unmeasured processes using isospin constraints
 - Determine energy regions where perturbative QCD calculations are safe (experience with τ at LEP)
- Launched a dedicated program of **e^+e^- cross section measurements using the BABAR detector** (Stanford) to get more precise data (2001-2014) with the new Initial State Radiation (ISR) method. A new phase is still underway.
- Same data and techniques used for the running of α (energy) from $\alpha(0)$ to $\alpha(M_Z)$ \Rightarrow prediction for M_{Higgs}

Measurements of $\sigma(e^+e^- \rightarrow \text{hadrons})$

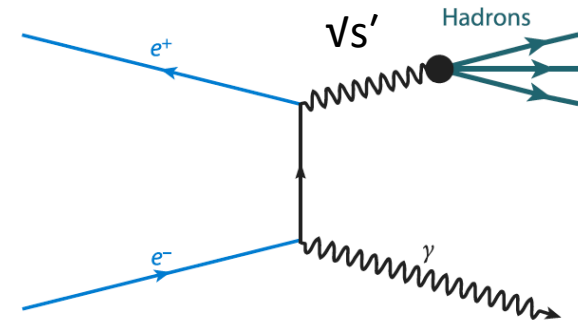
1. The scan method: e.g. CMD-2/3, SND at Novosibirsk

- Advantages:
 - Well defined \sqrt{s}
 - Good energy resolution $\sim 10^{-3}\sqrt{s}$
- Disadvantages:
 - Energy gap between two scans
 - Low luminosity at low energies
 - Limited \sqrt{s} range of a given experiment



2. The ISR approach: e.g. BaBar, KLOE, BES, CLEOc

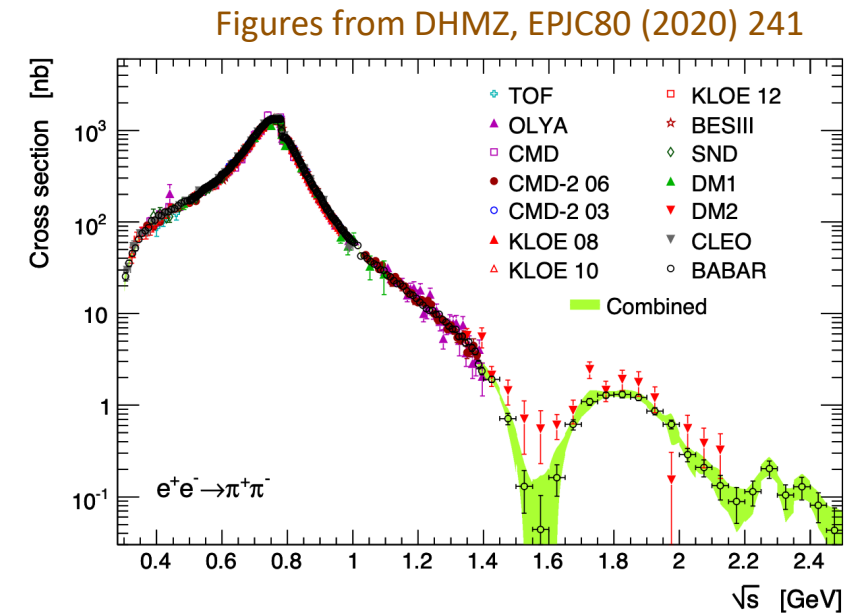
- Advantages:
 - Continuous cross section measurement over a broad energy range down to threshold
 - large acceptance for hadrons if ISR detected at large angle
 - $\sigma(e^+e^- \rightarrow \text{hadrons})$ may be measured over $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ thus reducing some syst uncertainties
- Disadvantages:
 - Requires high luminosity to compensate higher order in α



$$s' = (1-x)/s$$
$$x = 2E_\gamma/\sqrt{s}$$

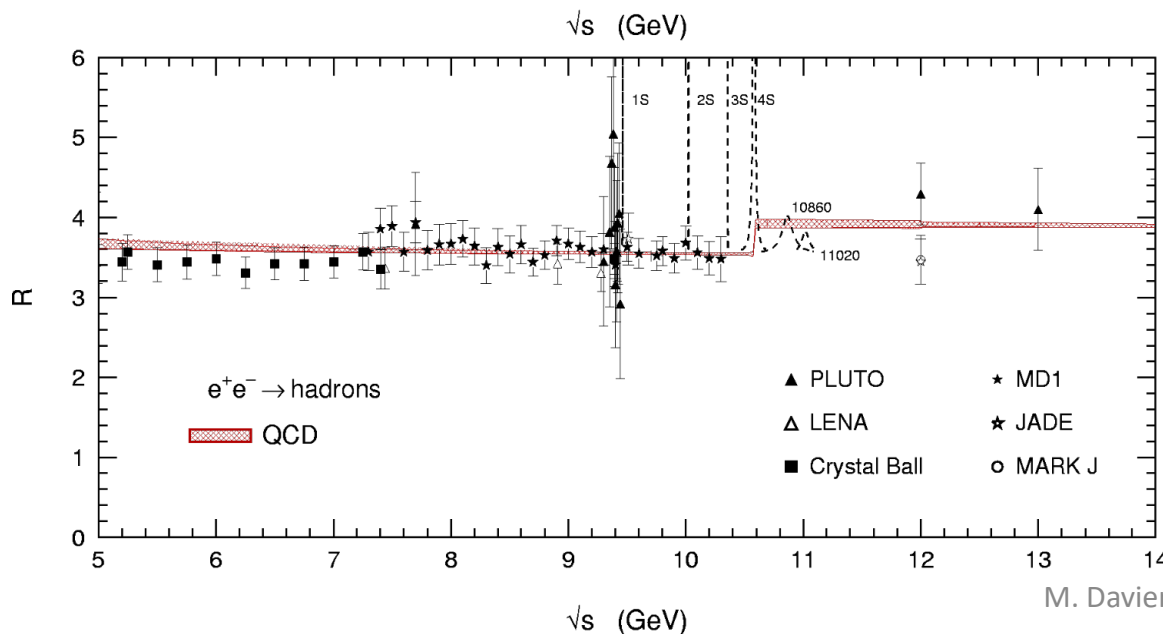
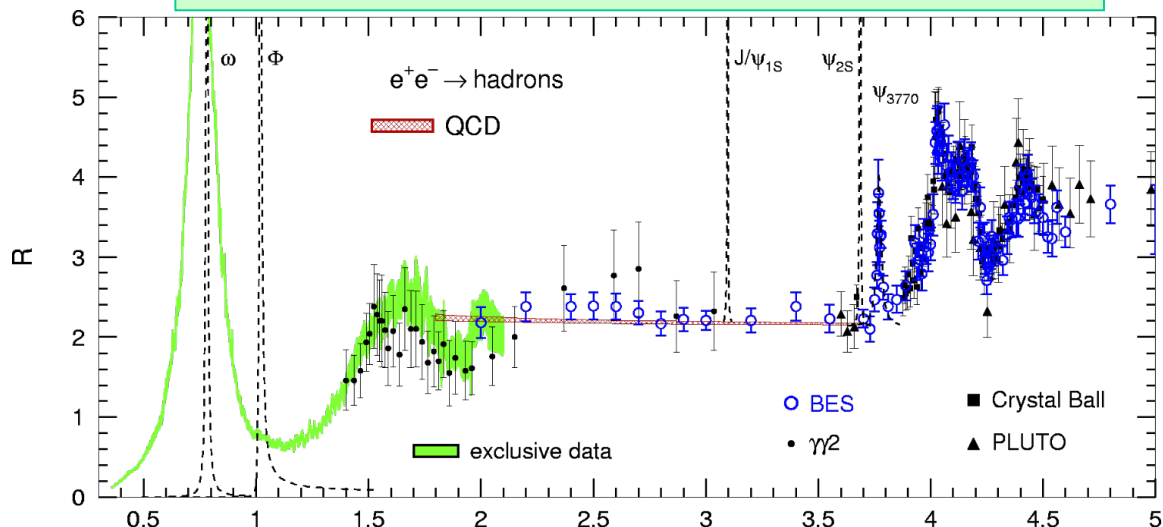
Combining cross section data (HVPTools)

- Combine experimental spectra with arbitrary point spacing / binning
- Properly propagate uncertainties and correlations
 - *Between measurements (data points/bins) of a given experiment* (covariance matrices and/or detailed split of uncertainties)
 - *Between experiments* (common systematic uncertainties, e.g. VP)
 - *Between different channels*, e.g. luminosity, radiative corrections, some efficiencies
- Linear/quadratic splines to interpolate between the points/bins of each experiment
- Fluctuate data points taking into account correlations and re-do the splines for each (pseudo-)experiment
- Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys
- Pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections
 - Integral evaluation

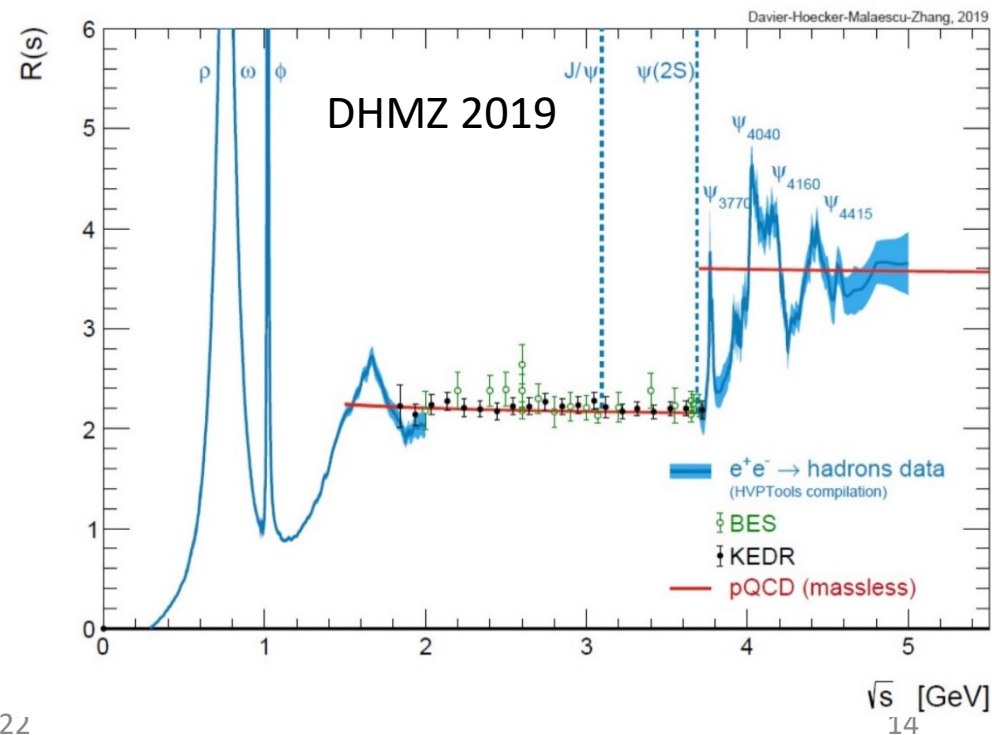


Different energy regions for R(s)

Davier-Hoecker-Zhang, RMP 78 (2006) 1043



- [$\pi^0\gamma$ threshold-1.8GeV]
 - sum about 22 \rightarrow 37 exclusive channels
 - estimate unmeasured channels (isospin relations ($< 0.1\%$))
- [1.8-3.7] GeV
 - good agreement between data and 4-loop QCD calculation
 - J/ψ , $\psi(2s)$: Breit-Wigner integral
- [3.7-5] GeV use data
- $>5\text{GeV}$ use 4-loop pQCD calculation

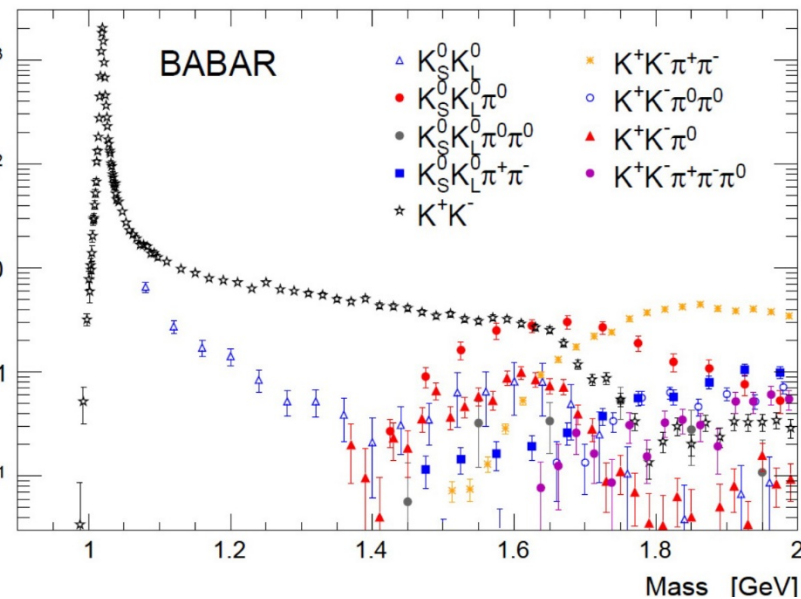
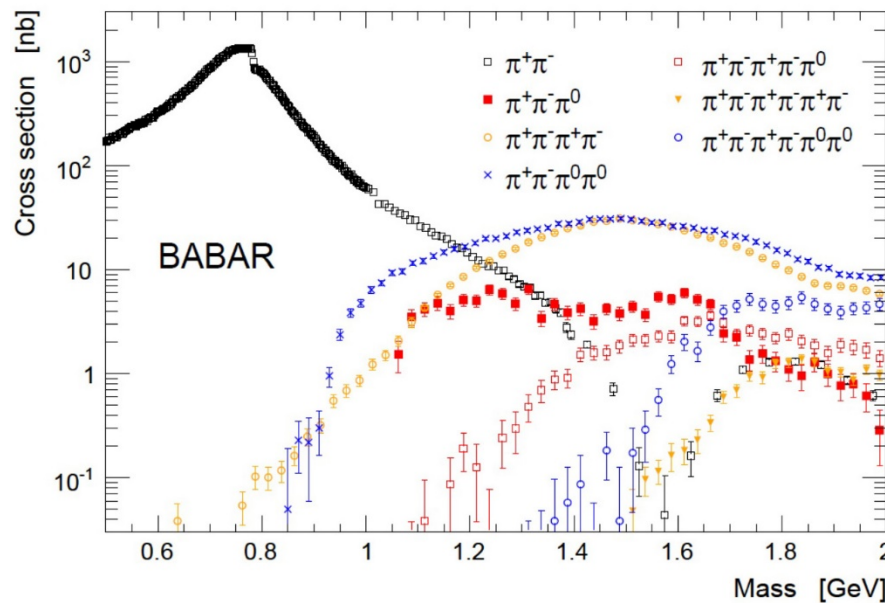
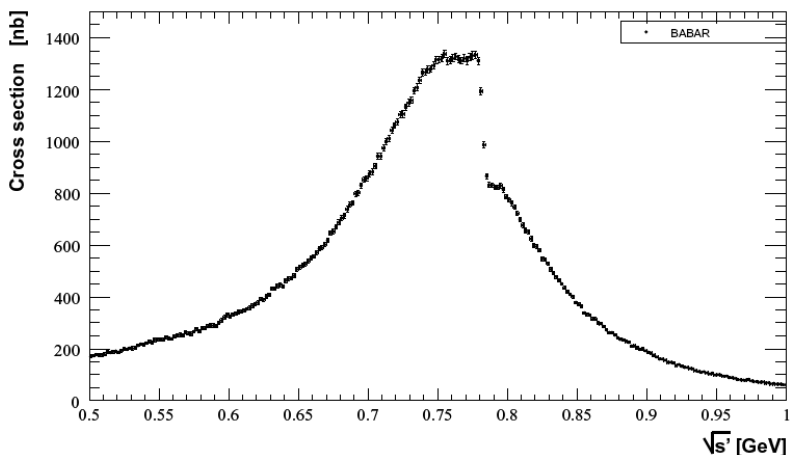
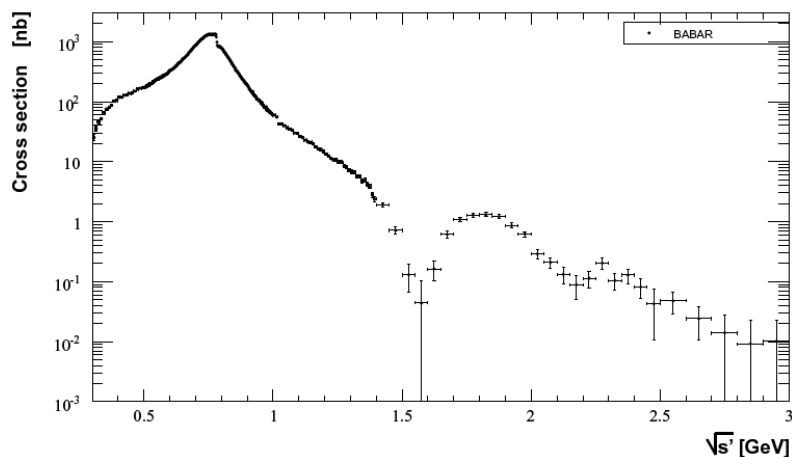


The BaBar complete set of exclusive measurements

$$e^+e^- \rightarrow \pi^+\pi^-$$

BABAR (PRL 2009, PRD 2012)

73% of HVP contribution to a_μ

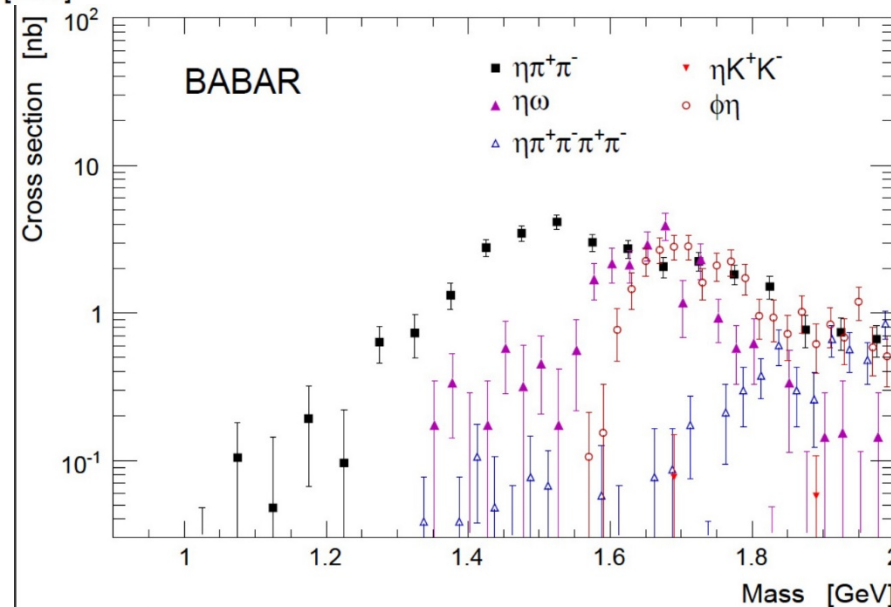


Bare cross sections including FSR

many papers (2004-2021)

~ 40 processes

complete reconstruction of $R < 2$ GeV



Consistency between experimental data

- Latest dispersive evaluations rely on a rather complete set of measurements of $e^+e^- \rightarrow \text{hadrons}$ up to 6π , $\eta 4\pi$, $KK2\pi$ in all charge configurations, and a few more higher-multiplicity processes
- missing channels in the range [1.5-1.8] GeV are estimated to contribute $< 0.1\%$ using isospin symmetry
- discrepancies exist in the K^+K^- channel on the $\phi(1020)$ (CMD-3 vs. CMD-2, SND, BABAR), taken into account
- **A more significant discrepancy occurs in the $\pi^+\pi^-$ channel between the 2 most precise results (BABAR and KLOE)**
- **Taking into account the BABAR/KLOE disagreement in the combination, all experiments are in agreement within an enlarged combination uncertainty (0.7%), already a remarkable result given different experimental conditions: ISR (10.6 GeV BABAR, ~ 4 GeV BES CLEOc, 1.02 GeV KLOE), direct scan (CMD-2, SND)**

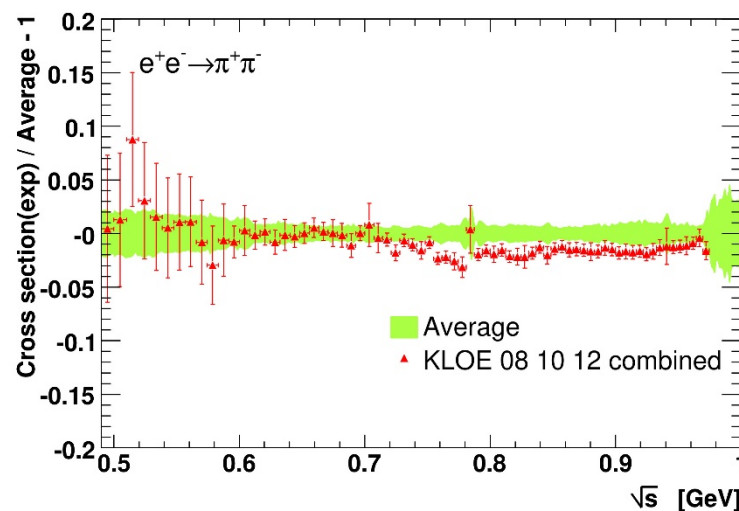
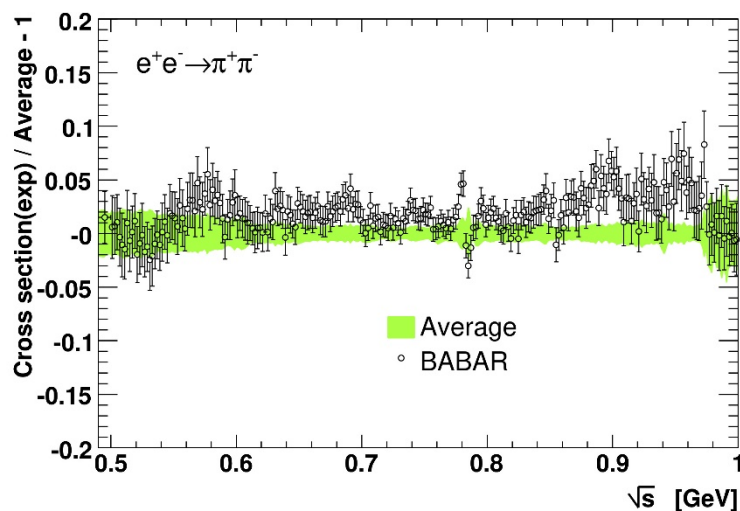
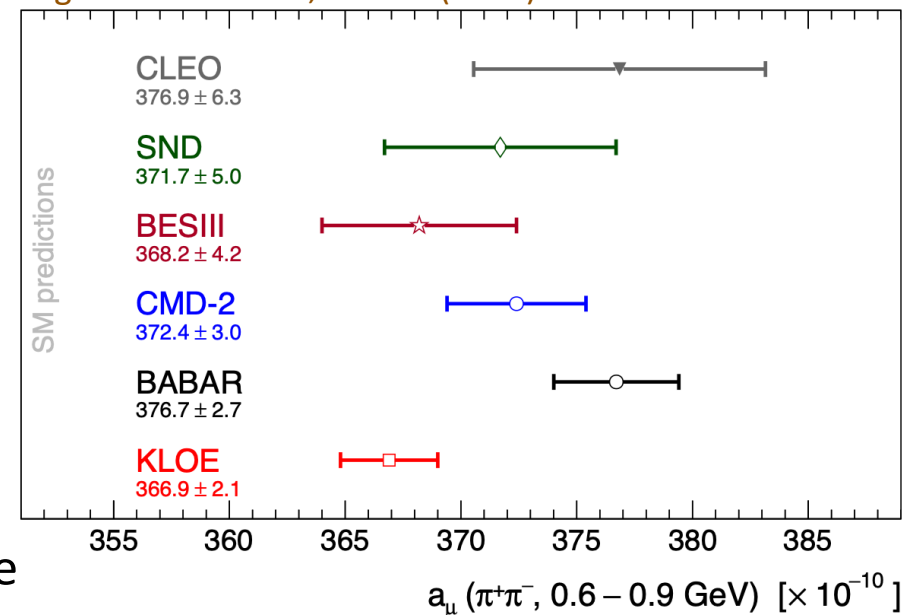


Figure from DHMZ, EPJC80 (2020) 241



- Additional systematic error added because of BABAR-KLOE difference \Rightarrow degrades uncertainty by 30%

All contributions (DHMZ19)

Channel	$a_\mu^{\text{had, LO}} [10^{-10}]$	$\Delta\alpha(m_Z^2) [10^{-4}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$	$34.49 \pm 0.06 \pm 0.20 \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$\eta\pi^+\pi^-\pi^0$ (non- ω , ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \rightarrow \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega(\pi\pi)^0$ ($\omega \rightarrow \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non- 3π , $\pi\gamma$, $\eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
$K_S K_L$	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
ϕ (non- $K\bar{K}$, 3π , $\pi\gamma$, $\eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\bar{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
$K\bar{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
$K\bar{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$	$-0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta K\bar{K}$ (non- ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega K\bar{K}$ ($\omega \rightarrow \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega 3\pi$ ($\omega \rightarrow \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07	7.09 ± 0.08
$\psi(2S)$ (BW integral)	1.57 ± 0.03	2.50 ± 0.04
R data [3.7 – 5.0] GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
$R_{\text{QCD}} [1.8 - 3.7 \text{ GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$	$24.27 \pm 0.18 \pm 0.28_{\text{dual}}$
$R_{\text{QCD}} [5.0 - 9.3 \text{ GeV}]_{udsc}$	6.86 ± 0.04	34.89 ± 0.17
$R_{\text{QCD}} [9.3 - 12.0 \text{ GeV}]_{udscb}$	1.21 ± 0.01	15.56 ± 0.04
$R_{\text{QCD}} [12.0 - 40.0 \text{ GeV}]_{udscb}$	1.64 ± 0.00	77.94 ± 0.12
$R_{\text{QCD}} [> 40.0 \text{ GeV}]_{udscb}$	0.16 ± 0.00	42.70 ± 0.06
$R_{\text{QCD}} [> 40.0 \text{ GeV}]_t$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$	$275.42 \pm 0.15 \pm 0.72 \pm 0.23 \pm 0.09_\psi \pm 0.55_{\text{QCD}}$

40 exclusive channels
(<1.8 GeV) evaluated

Estimation for missing
modes based on isospin
constraints becomes
negligible (0.016%)

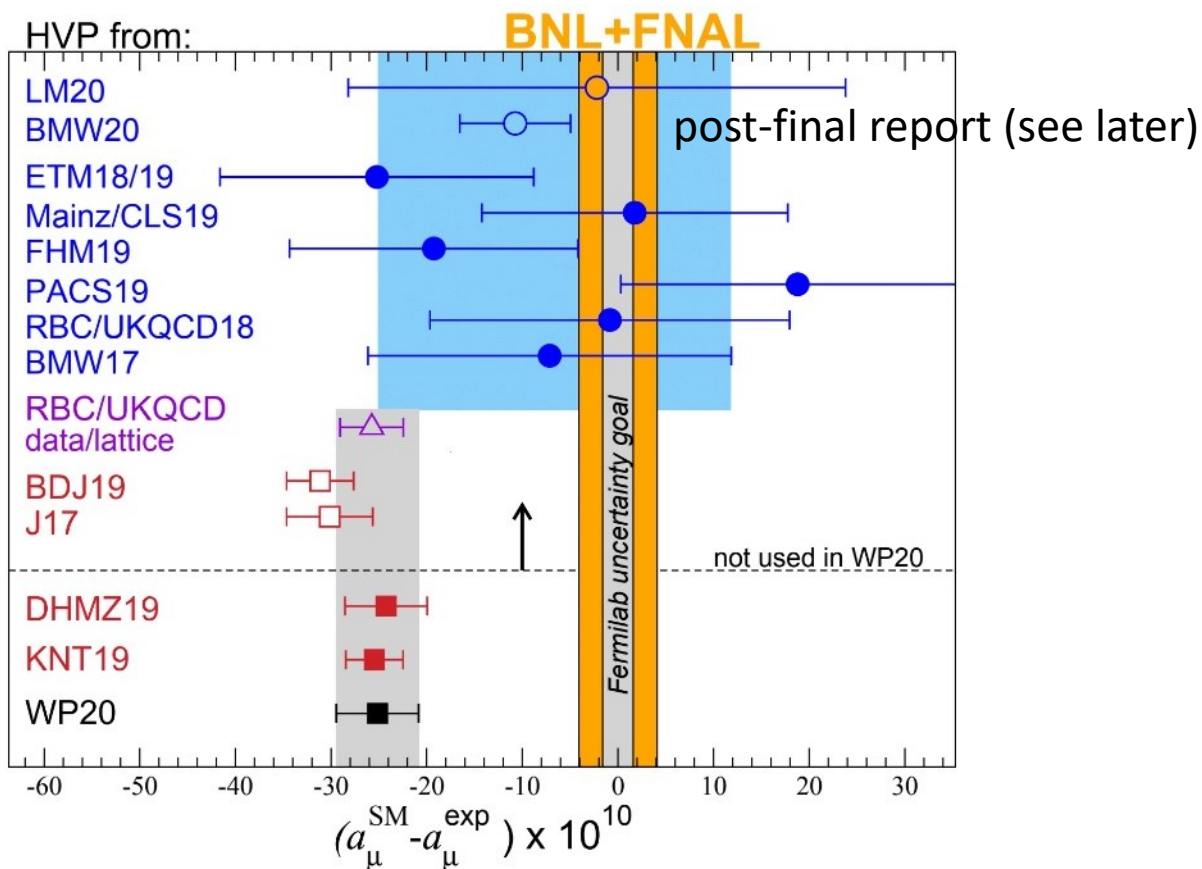
Table taken from
DHMZ, EPJC80
(2020) 241

The g-2 theory initiative (2017-2020)

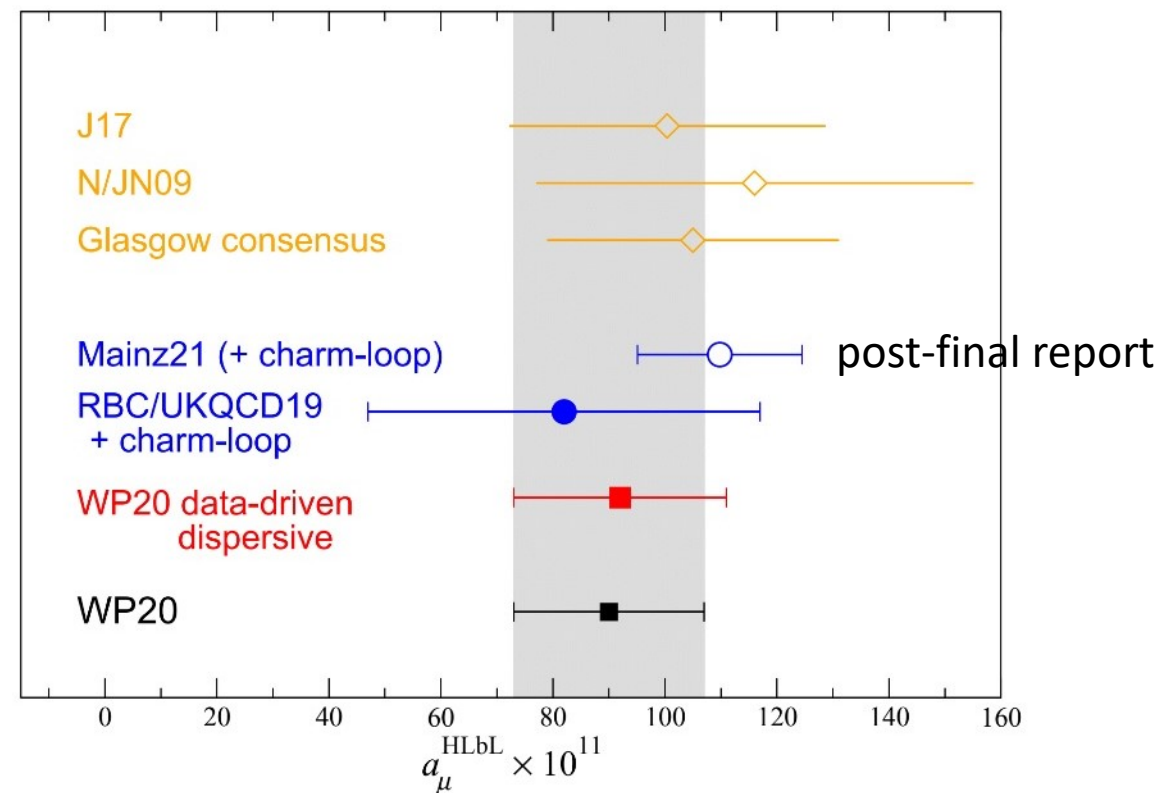
- By 2012, prediction using more precise e+e- data confirmed the discrepancy with the Brookhaven measurement, reaching $\sim 3.5 \sigma$
- In view of forthcoming results from the new g-2 direct experiment at Fermilab, a concerted effort was organized to try to produce the most reliable prediction ahead of time (**blind to the new result**)
- Organized 6 workshops followed by ~ 130 physicists (many lattice QCD theorists)
- Progress in hadronic LbL calculations with phenomenological and lattice methods, uncertainty reduced
- For HVP
 - lattice groups very active, but could not produce a reliable and competitive result
 - the dispersive approach based on data was adopted: results of 2 groups used (DHMZ and KNT) with the DHMZ conservative approach of estimating uncertainties prevailing
- Comprehensive report (166 pages) ready early 2020 and published in Physics Reports, well before the Fermilab release

The g-2 theory initiative prediction (WP2020)

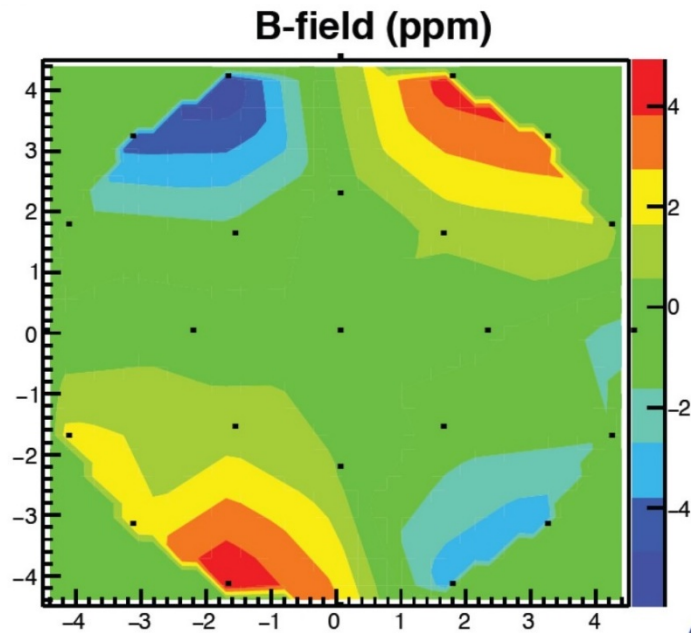
HVP



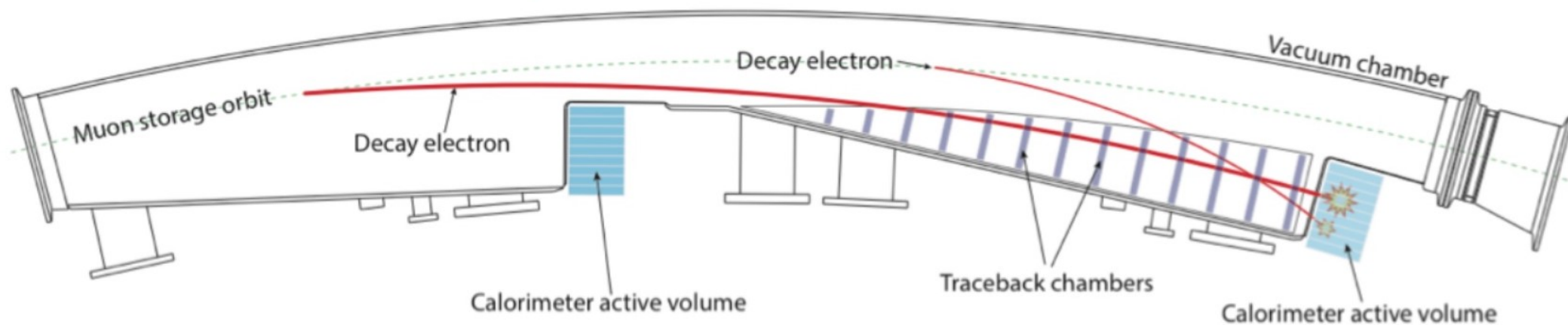
HLbL



The muon g-2 Fermilab experiment: a few features



- Large SC magnet shipped from BNL to Fermilab
- B-field uniformity after careful magnet shimming
- Checked every 3 days with special trolley with probes
- Large number of fixed probes to interpolate shifts
- Real-time reconstruction of muon beam position/shape to obtain B-field as seen by the muons
- Possible using tracking system of electron detectors
- Calorimeters with PbF2 crystals read-out by SiPM's (reduce pile-up)



The muon g-2 Fermilab experiment: correcting systematic effects

- Large number of systematic studies to establish corrections and to estimate uncertainties
- Beam distortions/oscillations
- Muon losses
- E-field residual effect
- Different methods for ω_a determination
- B-field (ω_p)
- Several groups for each topics
- Double unblinding for ω_a and ω_p with secret offsets for clock frequencies
- precision dominated by statistics
- Guarantees progress for future analyses (so far only 6% of total data)

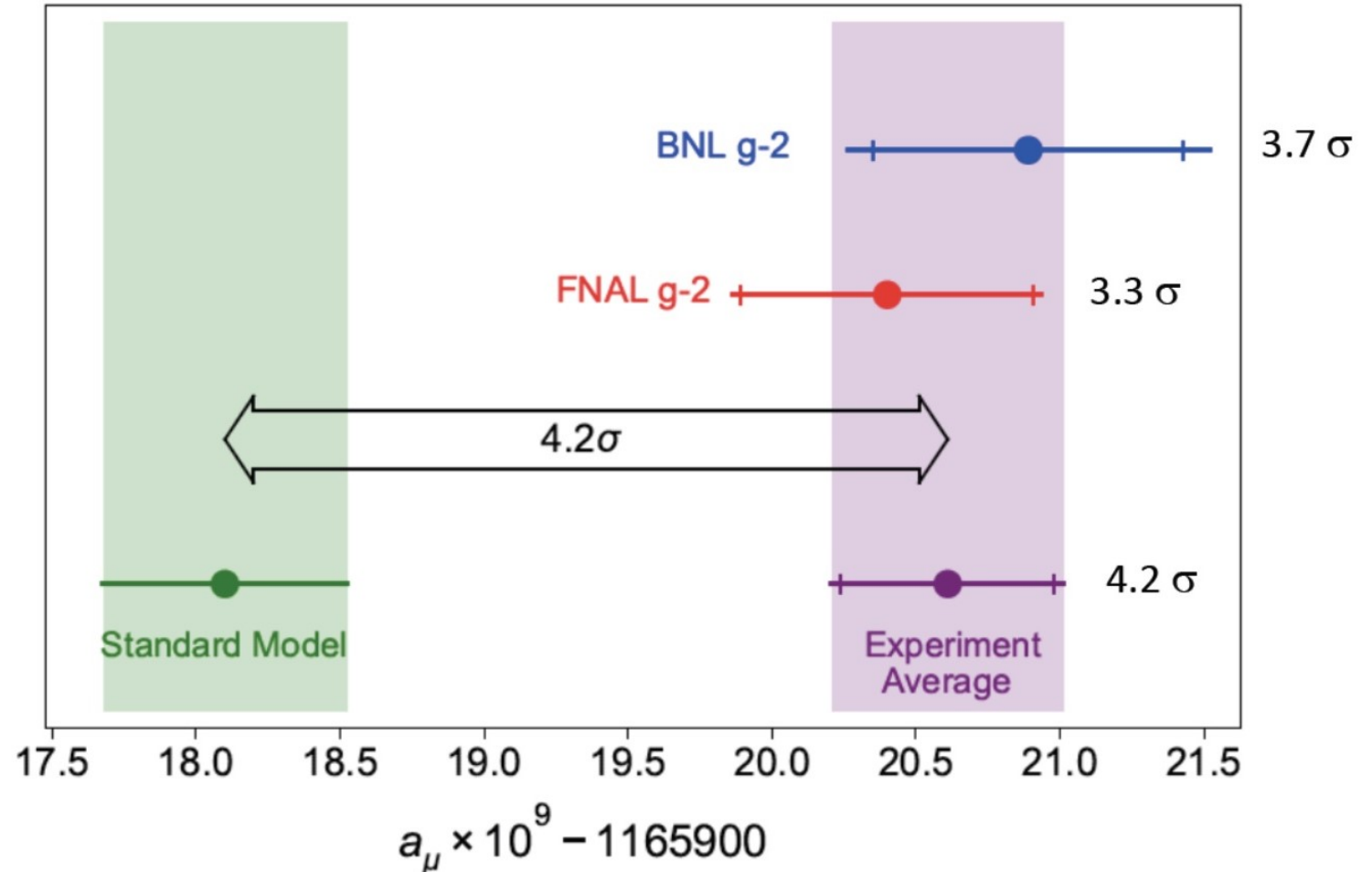
Quantity	Correction Terms (ppb)	Uncertainty (ppb)
ω_a (statistical)	–	434
ω_a (systematic)	–	56
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{calib} \langle \omega'_p(x, y, \phi) \times M(x, y, \phi) \rangle$	–	56
B_q	-17	92
B_k	-27	37
$\mu'_p(34.7^\circ)/\mu_e$	–	10
m_μ/m_e	–	22
$g_e/2$	–	0
Total	–	462

434 ppb stat \oplus 157 ppb syst error



The muon g-2 Fermilab experiment: the result

$$a_\mu(\text{Fermilab}) = 116\,592\,040(54) \times 10^{-11}$$

- Agreement with Brookhaven value
- Precision comparable
- Excess / SM prediction increased to 4.2σ
- Caution about significance:
 - statistics-dominated measurement
 - prediction uncertainty limited by systematic effects (not Gaussian)
- Nevertheless, large discrepancy (the largest so far between measurement and SM anywhere)

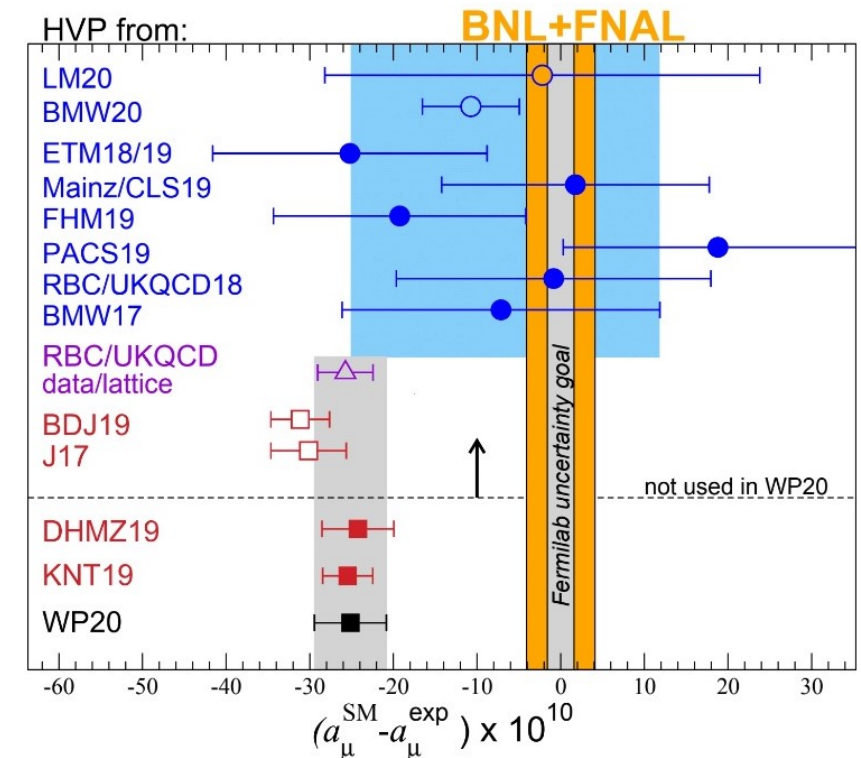


60 years of muon g-2 measurements and theory predictions

Experiment	Beam	Measurement	$\delta a_\mu/a_\mu$	Required th. terms
Columbia-Nevis (57)	μ^+	$g=2.00\pm 0.10$		$g=2$
Columbia-Nevis (59)	μ^+	0.001 13(+16)(-12)	12.4%	α/π
CERN 1 (61)	μ^+	0.001 145(22)	1.9%	α/π
CERN 1 (62)	μ^+	0.001 162(5)	0.43%	$(\alpha/\pi)^2$
CERN 2 (68)	μ^+	0.001 166 16(31)	265 ppm	$(\alpha/\pi)^3$
CERN 3 (75)	μ^\pm	0.001 165 895(27)	23 ppm	$(\alpha/\pi)^3 + \text{had}$
CERN 3 (79)	μ^\pm	0.001 165 911(11)	7.3 ppm	$(\alpha/\pi)^3 + \text{had}$
BNL E821 (00)	μ^+	0.001 165 919 1(59)	5 ppm	$(\alpha/\pi)^3 + \text{had}$
BNL E821 (01)	μ^+	0.001 165 920 2(16)	1.3 ppm	$(\alpha/\pi)^4 + \text{had} + \text{weak}$
BNL E821 (02)	μ^+	0.001 165 920 3(8)	0.7 ppm	$(\alpha/\pi)^4 + \text{had} + \text{weak} + ?$
BNL E821 (04)	μ^-	0.001 165 921 4(8)(3)	0.7 ppm	$(\alpha/\pi)^4 + \text{had} + \text{weak} + ?$
 FNAL Run1 (21)	μ^+	0.001 165 920 40(54)	0.46 ppm	$(\alpha/\pi)^4 + \text{had} + \text{weak} + ?$ 

A controversy on HVP

- BMW lattice collaboration preprint posted on arxiv early in 2020
- Statistics x10 compared to other groups (huge computing power)
- Claimed systematic uncertainty (dominant) also much smaller
- Central value much closer to the g-2 measurement
- Result scrutinized during one year (special workshop organized)
- Criticism expressed (precision), but no fundamental flaw discovered so far
- Small changes made in 2nd and 3rd versions
- Paper finally published in Nature with aggressive publicity
- New method at this level of precision; lack of maturity/dispersive approach
 - QCD solved numerically on a discretized space-time of finite volume (up to 11 fm³) and small spacing (impressive, massive computing)
 - Extrapolation to the continuum is one of the issues concerning systematic biases and error estimate
- Needs confirmation by other lattice collaborations (4 groups); slow progress
- Clear discrepancy between cross section for e+e- → hadrons and BMW result
- DHMZ is collaborating with BMW to localize the energy region where the differences with data-driven results occur
- BMW result impact on the EW fit investigated: Hoferichter-Crivellin, Keshavarzi et al, Malaescu-Schott 2021



What new physics could produce this excess?

- Presently the confrontation theory/experiment indicates a missing contribution in the Standard Model at more than 4σ
- The excess, $\Delta a_\mu = 251(59) \times 10^{-11}$, is comparable to the electroweak contribution of W and Z bosons (mass ~ 100 GeV) $\Delta a_\mu^{\text{EW}} = 153.6(1.0) \times 10^{-11}$
- Depending on possible enhancements due to the specific new interaction, masses for the new particles could be in the 0.1-1 TeV range
- Exactly what was expected for **minimal supersymmetry** (SUSY), enhancement given here by a $\tan\beta$ factor
- This simple scenario is almost ruled out by negative searches of SUSY-particles at LHC. Not completely?
- Another possibility is a relatively **low mass scalar boson or a dark photon** interacting weakly, but this is also largely ruled out by direct searches
- Other, more contrived, models are considered.... BSM theorists are active...

Summary and perspectives

- New measurement of the muon magnetic anomaly released at Fermilab
- Result in agreement with previous Brookhaven experiment
- A large effort was devoted to produce a reliable and conservative theoretical prediction within the Standard Model
- The Hadronic Vacuum Polarization contribution plays a very important role in the value and accuracy of the prediction
- The DHMZ group at Orsay has more than 20 years of experience using the mature dispersive approach based on experimental data on e^+e^- cross sections measured with innovative methods
- Presently the confrontation theory/experiment indicates a missing contribution in the Standard Model at 4.2σ
- This conclusion is challenged by an alternative approach using QCD on a lattice which needs confirmation by other groups before concluding. For the moment one should stay with the well-tested standard approach
- Prospects for improving the direct measurement at Fermilab look good (reduction of uncertainty by a factor of 4 over the next 4 years)
- New precise e^+e^- measurements underway (CMD-3, BaBar independent/blinded, BESIII, BelleII)
- >2027: a new experiment is under preparation at JPARC in Japan using a completely different approach, thus allowing to crosscheck the traditional method

Backup slides

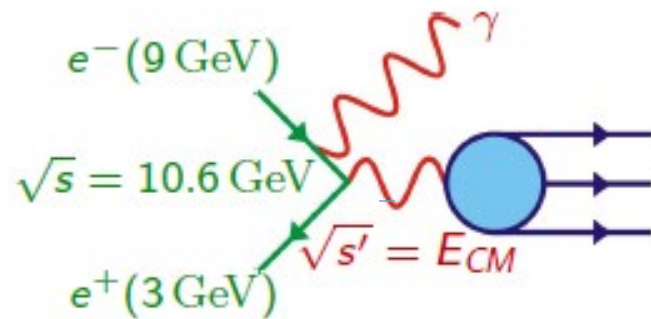
List of DHMZ publications

1. ADH 1998, [Eur.Phys.J.C 2 \(1998\) 123](#) [366 citations*]
2. DH 1998, [Phys.Lett.B 419 \(1998\) 419](#) [223 citations]
3. DH 1998, [Phys.Lett.B 435 \(1998\) 427](#) [294 citations]
4. DEHZ 2003, [Eur.Phys.J.C 27 \(2003\) 497](#) [432 citations]
5. DEHZ 2003, [Eur.Phys.J.C 31 \(2003\) 503](#) [498 citations]
6. DHMZ+ 2010, [Eur.Phys.J.C 66 \(2010\) 127](#) [161 citations]
7. DHMYZ 2010, [Eur.Phys.J.C 66 \(2010\) 1](#) [209 citations]
8. DHMZ 2011, [Eur.Phys.J.C 71 \(2011\) 1515](#) [918 citations]
9. DHMZ 2017, [Eur.Phys.J.C 77 \(2017\) 827](#) [394 citations]
10. DHMZ 2019, [Eur.Phys.J.C 80 \(2020\) 241](#) [342 citations]
11. Theory initiative WP 2020, [Phys.Rept. 887 \(2020\) 1](#) [570 citations]
→ Total number of citations: ~4400

* Status of March 2022

The ISR method at BABAR

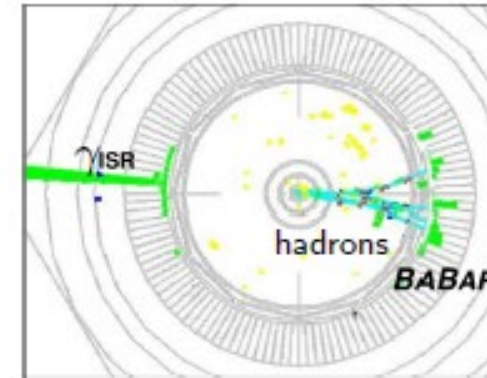
BABAR, operating on the high-luminosity asymmetric PEP II e+e- collider, was designed to study CP violation in the B-antiB system and led to the validation of the Cabibbo-Kobayashi-Maskawa matrix. The ISR program was a powerful by-product



$$x = 2E_{\gamma}^* / \sqrt{s}$$

$$s' = s(1 - x)$$

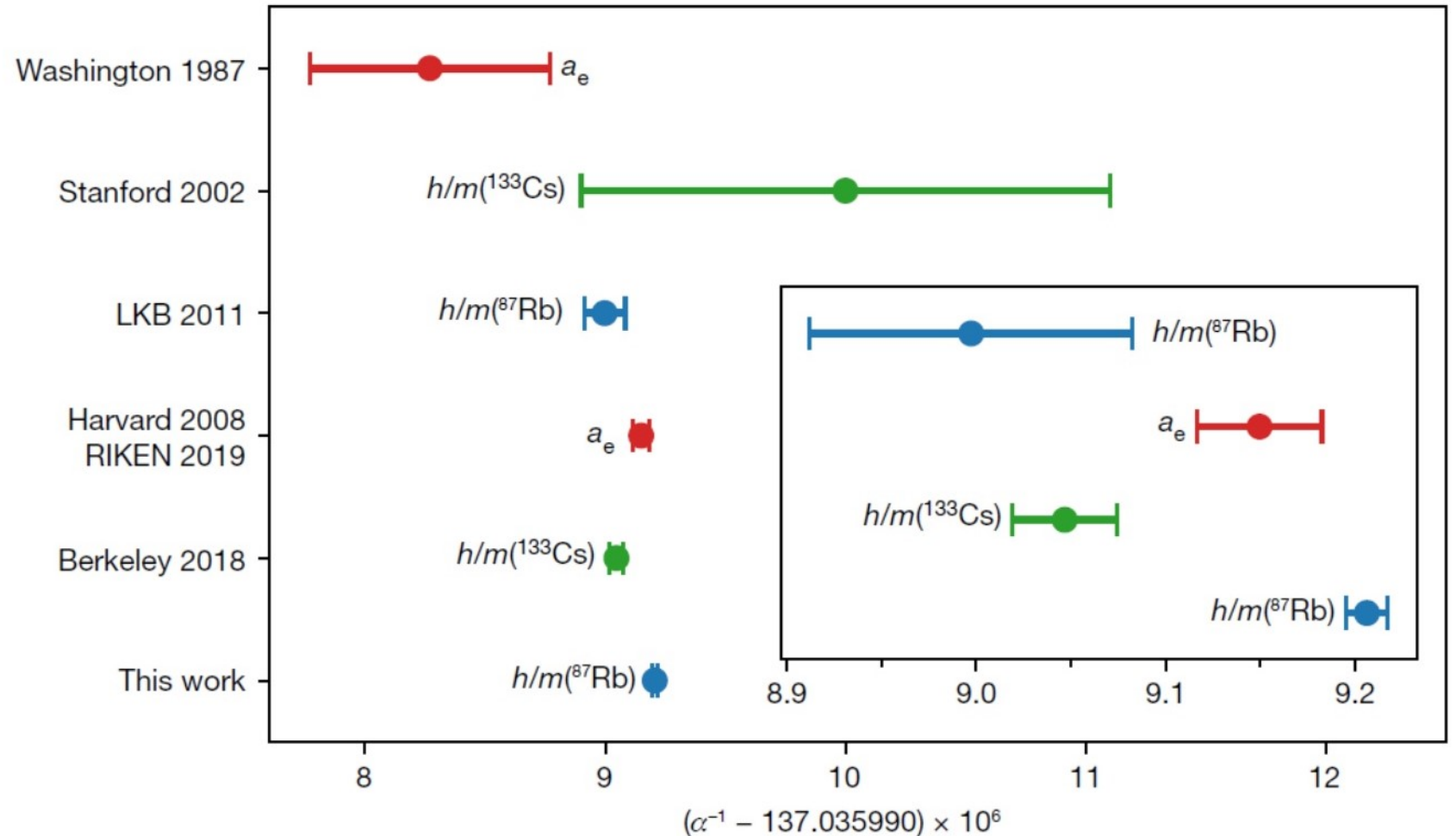
$$(M_{\text{hadrons}}^2)$$



- High energy ($E_{\gamma}^* > 3 \text{ GeV}$) detected at large angle
- Event topology: ISR photon back-to-back to hadrons \rightarrow high acceptance
- Final state can be hadronic or leptonic (QED)
 $\rightarrow \mu^+ \mu^- \gamma(\gamma)$ to get ISR luminosity
- Continuous measurement from threshold to 3-5 GeV
 \rightarrow reduces systematic uncertainties compared to multiple data sets with different colliders and detectors

Status on electron anomalous magnetic moment

- a_e completely dominated by QED
- Very precise measurements from Gabrielse's group at Harvard
- situation confused
- LKB latest α determination (disagrees with previous result from Berkeley)



Role of BABAR/KLOE discrepancy in the WP-DISP/BMW21 difference

- Comparing $10^{10} \times a_{\mu}^{\text{LO had}}$ results:

BMW21	$707.5 \pm 2.3 \pm 5.0$ (5.5)	0.78%	--
DHMZ19 all	$694.0 \pm 1.0 \pm 2.5 \pm 0.7 \pm 2.8$ (4.0)	0.58%	2.0 σ
	stat syst QCD BABAR-KLOE		
DHMZ19 –KLOE	$696.8 \pm (3.1)$	0.44%	1.7 σ
DHMZ19 –BABAR	$691.2 \pm (3.1)$	0.44%	2.6 σ
WP20 all	$693.1 \pm 2.8 \pm 0.7 \pm 2.8$ (4.0)	0.58%	2.1 σ
(merging DHMZ-KNT-CHKS)	exp QCD BABAR-KLOE		

- BABAR/KLOE discrepancy results in a 30% loss in precision for HVP
- It is also substantial (5.6) compared to the BMW21-WP20 difference (14.4)
- It is mandatory that all the new e+e- analyses (BABAR, BelleII, BESIII, CMD-3) be done at a precision level < 0.5% with a demonstrably strict BLIND approach.
- KLOE pushing up the discrepancy with BNL/Fermilab
- Removing KLOE does not change the significance of the deviation (22.3+-5.3 instead of 25.1+-5.9, 4.2 σ)