## Brisure et restauration de symétries

## Michael Bender

Institut de Physique des 2 Infinis de Lyon
CNRS/IN2P3 \& Université de Lyon \& Université Lyon 1
F-69622 Villeurbanne, France

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Symmetries of the nuclear Hamiltonian

- Rotational
- Space inversion
- Global gauge space
- Translational \& Galilean invariance
- Time translation
- Time reversal
$\Rightarrow$ angular momentum
$\Rightarrow$ parity
$\Rightarrow$ proton and neutron number
$\Rightarrow$ separation of center-of-mass and internal motion
$\Rightarrow$ energy conservation

Approximate symmetry of the nuclear Hamiltonian

- Rotation in isospin space $\quad \Rightarrow$ isospin

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$$
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$$

Consequences of symmetries

- Quantum numbers of states
- Selection rules for electromagnetic/weak/strong transitions between states
- Correlations (nucleons do not move independently)


## But many nuclear phenomena are interpreted as if symmetries are broke CחrS



Möller, Sierk, Bengtsson, Sagawa, Ichikawa, ADNDT98 (2012) 149


- deformation (collective rotational bands!)
- shape coexistence


## But many nuclear phenomena are interpreted as if symmetries are broke CחrS



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- deformation (collective rotational bands!)
- shape coexistence
- fission

- clusterisation
- pairing (nuclear superfluidity)

Scamps \& Simenel, Nature 564 (2018) 382

Symmetry-conserving approaches

- conventional shell-model
- (most) conventional ab-initio approaches
- ...

Symmetry-breaking approaches

- Self-consistent mean-field models (either EDF-based or valence space with schematic interactions)
- Cluster models
- ...

Symmetry-breaking \& restoration

- Projected symmetry-breaking states
- Monte-Carlo shell model
- ...

Technically, " projection" means inserting a numerical discretization of an operator that extracts the targeted components when calculating matrix elements.
"Special case" of a Generator Coordinate Method (non-orthogonal CI)

Typical situations:

- localised finite system (broken translational symmetry)
- axial deformation (partially broken rotational symmetry)
- non-axial deformation (broken rotational symmetry)
- states with finite angular momentum (broken time-reversal)
- octupole deformation (broken parity + broken rotational symmetry)
- HFB-type pairing (broken global gauge symmetry)

Some rarely addressed situations

- absence of more than one plane symmetry
- broken signature (angular momentum not in the direction of a major axis )
- proton-neutron mixing (broken "axiality" in isospin)
- lowest projected state usually has more broken symmetries than mean-field minimum
- additional correlation energy varies quickly for transitional nuclei




Bender, Bonche, Duguet, Heenen, PRC 69 (2004) 064303


Bender, Bertsch, Heenen, PRCC 73 (2006) 034322; PRC 78 (2008) 054312


- In odd- $A$ nuclei, bands get easily mixed.

Heenen, Bally, Bender, Ryssens, EPJ Web of Conf 131 (2016) 02001;
Bally, Bender, Heenen, to be published
seniority- 2 states in ${ }^{46} \mathrm{Ca}$


- Characteristic pattern emerges when 2qp states are mixed in
- Mixed with collective bands.


## Non-standard deformation modes might only show up when breaking \& restoring symmetries C@IS

## Clustering

Dominant configuration in the ground-state of ${ }^{12} \mathrm{C}$


Dominant configurations in the Hoyle state


Parity, angular-momentum \& center-of-massprojected Fermionic Molecular Dynamics
Chernykh et al, PRL98 (2007) 032501; Neff, JPhys Conf Ser 403 (2012) 012028


Parity \& angular-momentum projected axial relativistic mean field
Marevic, Ebran, E. Khan, T. Niksic, Vretenar, PRC 99, 034317 (2019)

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Exotic deformations

borrowed from a talk by J. Dudek @ IPNL (2017)

- physically broken by the Coulomb interaction, the difference of proton and neutron masses, and small contributions of the strong interaction
- spuriously broken by mean-field methods, with some severe consequences for calculated properties of nuclei close to the $N=Z$ line.
- spurious breaking can be removed by diagonalizing the symmetry-breaking Hamiltonian in a basis of isospin-projected mean-field states.
- isospin projection mixes proton and neutron single-particle states.
- controlling the physical isospin breaking has relevance for tests of the standard model through the analysis of superallowed $0^{+} \rightarrow 0^{+}$Fermi $\beta$ decay


Example: Isospin correction to Fermi decays


Konieczka, P. Baczyk, Satuła, arXiv:1909.09350

## Need for specific functionals

$\mathrm{N} \& \mathrm{Z}$ projecction of an HFB state of ${ }^{18} \mathrm{O}$


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$\mathrm{N} \& \mathrm{Z}$ projecction of an HFB state of ${ }^{18} \mathrm{O}$


- energy as a function of deformation
- general EDF


Lacroix, Duguet, Bender, PRC 79 (2009) 044318
Bender, Duguet, Lacroix, PRC 79 (2009) 044319
Duguet, Bender, Bennaceur, Lacroix, Lesinski, PRC 79 (2009) 044320

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Bender, Duguet, Lacroix, PRC 79 (2009) 044319
Duguet, Bender, Bennaceur, Lacroix, Lesinski, PRC 79 (2009) 044320

J-projetion of a HF state of ${ }^{25} \mathrm{Mg}$


- energy decomposition of one pathological state as a function of discretisation of the projector
- Left: density-dependent EDF
- right: non-density-dependent EDF from generating operator
- predictive well-defined EDFs turn out to be difficult to construct; necessary operator structure has not yet been identified.

triaxial HF, axial J projection
Bounseng Bounthong, thèse, Strasbourg, 2016

triaxial HF, axial J projection Bounseng Bounthong, thèse, Strasbourg, 2016





Bally, Sánchez-Fernández, Rodríguez, PRC 100 (2019) 044308

## Multi-Reference In-Medium

 Similarity Renormalization Group

Access to $0 \nu \beta \beta$ matrix element.

Yao, Engel, Wang, Jiao, Hergert, PRCC 98, 054311 (2018); Yao, Bally, Engel, Wirth, Rodríguez, Hergert, arXiv:1908.05424

Many-body perturbation theory combined with
symmetry-breaking+restoration


## Schematic pairing Hamiltonian

Ripoche, Lacroix, Gambacurta, Ebran, Duguet,
PRC95, 014326 (2017)
Also particle-number restored Bogoliubov Coupled-Cluster,

Qiu, Henderson, Duguet, Scuseri, PRC 99 (2019) 044301

## Bogoliubov MBPT




Huge reduction of CPU time!

Tichai, Arthuis .Duguet, Hergert, . Somà, Roth, PLB 786 (2018) 195

Why breaking \& restoring symmetries?

- Symmetry breaking configurations simplify the interpretation of numerous nuclear phenomena
- Breaking \& restoring symmetries instead of using symmetry-conserving schemes can enormously reduce numerical cost because of much quicker convergence with the number of (much richer) configurations to be considered.

Necessary developments concerning EDF methods

- breaking additional symmetries to describe exotic shapes and/or exotic rotational phenomena
- combined restoration of many broken symmetries
- Construct EDF that is well-defined for symmetry restoration

Transfer of "breaking \& restoring symmetries" to other frameworks

- valence-space symmetry-restored GCM with shell-model Hamiltonians.
- ab-initio calculations of singly- doubly-open shell nuclei (Green's functions, Coupled-Cluster, Many-Body Perturbation Theory, Multi-Reference In-Medium-Similarity- Renormalization Group, No-core shell model, ...)
- Time-dependent approaches (EDF-based and others)

Advancing all areas will require reinforcement with young motivated scientists.

