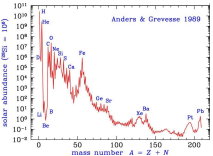
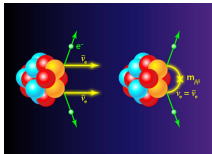
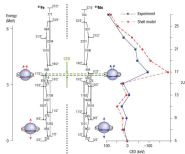


# Nuclear Shell Model: structure of exotic nuclei, weak processes and astrophysical issues

F. Nowacki, K. Sieja (IPHC), N. Smirnova (CENBG), P. Van Isacker (GANIL)

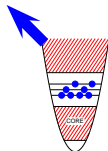
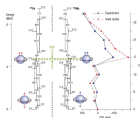


Prospectives nationales 2020-2030  
Physique Nucléaire et Astrophysique nucléaire

# New Frontiers for Shell Model calculations

## Fundamental interactions and collective excitations

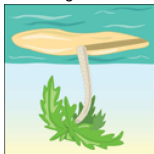
- Deformation, Superdeformation, Dipole/M1 resonances
- Superfluidity, Symmetries
- Isospin symmetry breaking



- define effective interaction
- $\mathcal{H}_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$
- build and diagonalize energy matrix

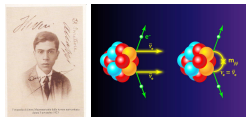
## Nuclear structure far from stability

- New magic numbers
- Vanishing of shell closures



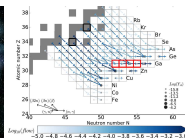
## Weak processes

- $\beta$  decay  $\iff$  fundamental interactions
  - $\beta\beta$  decay  $\iff$  nature of neutrinos
- $$[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 \langle m_{\nu} \rangle^2$$



## Astrophysics and nucleosynthesis

- rp process
- r process



# Shell Model: Giant Computations

- exponential growth of basis dimensions:

$$|\Phi_\alpha\rangle = \prod_{i=nljm\tau} a_i^\dagger |0\rangle = a_{i_1}^\dagger \dots a_{i_A}^\dagger |0\rangle$$

$$D \sim \begin{pmatrix} d_\pi \\ \rho \end{pmatrix} \cdot \begin{pmatrix} d_\nu \\ n \end{pmatrix}$$

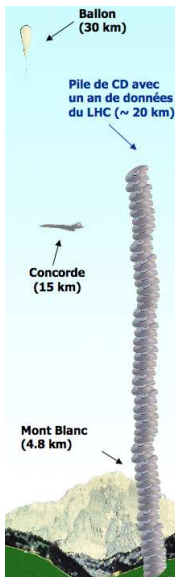
In *pf* shell :

$^{56}\text{Ni}$  **1,087,455,228**

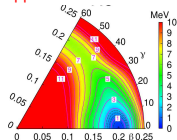
In *pf-sdg* space :

$^{78}\text{Ni}$  **210,046,691,518**

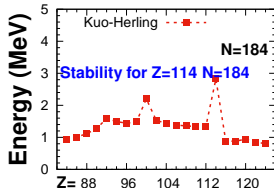
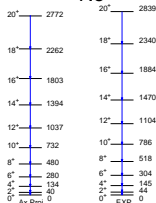
- Actual limits in giant diagonalizations:  
**0.2  $10^{12}$**  for  $^{114}\text{Sn}$  core excitations
- Largest matrices up to now contain up to  $\sim 10^{14}$  non-zero matrix elements.
- This would require more than 1,000,000 CD-ROM's to store the information for a single matrix !
- They cannot be stored on hard disk and are computed on the fly.



New opportunities: BMF-SM



$^{254}\text{No}$



B. Bounthong, PhD Thesis,

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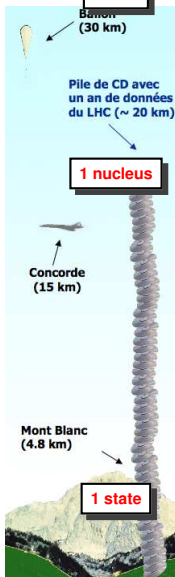
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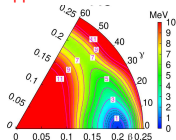
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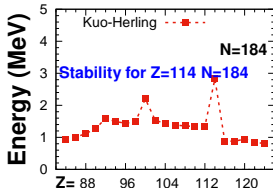
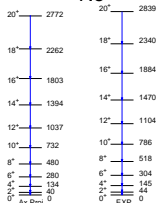
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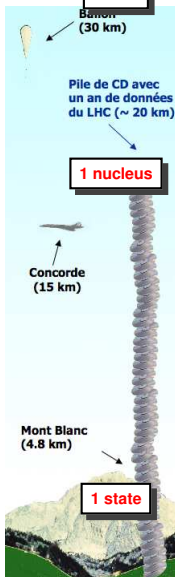
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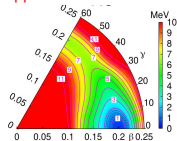
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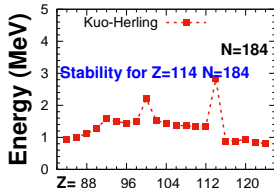


$^{254}\text{No}$



D. Dao post-doc (2020-2021)

IN2P3/IPHC

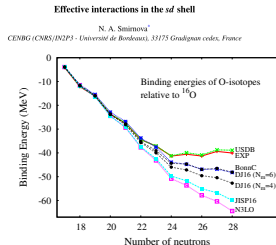


B. Bounthong, PhD Thesis,

# Structure and decay of exotic nuclei

- Nowadays, LSSM calculations in extended model spaces comprising a few oscillator shells to deal with the changing shell structure and the onset of deformation in very neutron-rich nuclei
- Development of accurate description of isospin-symmetry breaking using charge-dependent hamiltonians
- Construction of fully microscopic interactions for valence-space calculations as a path towards regions where no experimental data are available
- Development of numerical techniques and state-of-the-art computations
- Search for additional guidelines and shortcuts using symmetry based approaches

PHYSICAL REVIEW C **100**, 054329 (2019)

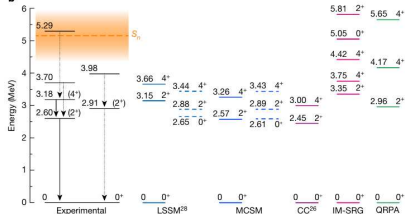


## ARTICLE

<https://doi.org/10.1103/PhysRevC.100.054329>

### $^{78}\text{Ni}$ revealed as a doubly magic stronghold against nuclear deformation

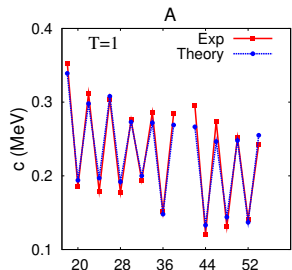
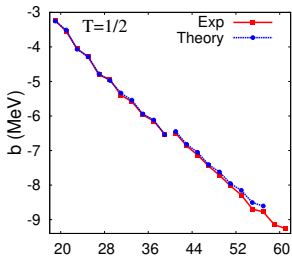
R. Taniuchi et al., NATURE **569**, 53-58 (2019)



# Weak-interaction processes and physics beyond the Standard Model

## Isospin symmetry breaking in the Shell-Model context

- Accurate description of Isospin violation and associated phenomena through development of Isospin Non Conserving Shell Model hamiltonians in extended valence spaces:  $((sd), (pf), (s_{1/2}d_{3/2}f_{7/2}p_{3/2}))$  and beyond
- $\beta - p$  and  $\beta - p\gamma$  decay studies and extraction of Isospin mixing in the IAS
- Development of Isospin Non Conserving Shell Model hamiltonian in the  $(sd - pf)$  valence space and numerous applications
- Improvement of MED and TED description within a band
- Interpretation for b and c coefficients staggering
- support to forthcoming experimental studies



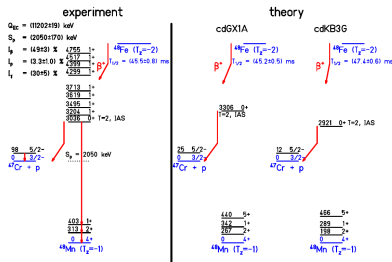
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ISOSPIN MIXING FROM  $\beta$ -DELAYED PROTON EMISSION

PHYSICAL REVIEW C 95, 054301 (2017)



$$^{48}\text{Mn}: S_{exp} = 2.7(9) \cdot 10^{-3}, \alpha_{exp} = 1.4(5)\%$$

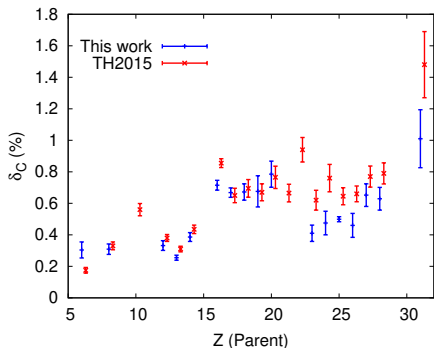


# Weak-interaction processes and physics beyond the Standard Model

## Superallowed $\beta$ $0^+ \rightarrow 0^+$ as a test of fundamental interactions

- Large scale calculations for all emitters below  $A \leq 40$  including nuclei in the vicinity of  $^{40}\text{Ca}$
- Use of Isospin Non Conserving hamiltonians and Woods-Saxon wave functions for untruncated  $sd$  and  $pf$  calculations
- New approach of radii determination without closure approximation
- Use of new effective interactions developed in Strasbourg
- Lanczos Structure Function Method for  $\delta_C$
- New ( $sd - pf$ ) interaction
- Effective Fermi Operator
- HF wave functions
- New emitters such as  $^{58}\text{Zn}$

$$\begin{aligned} \mathcal{F}t &= (1 + \delta_R)(1 + \delta_{NS} - \delta_C)ft \\ &= \frac{K}{M_{F0}^2 G_F^2 |V_{ud}|^2 (1 + \Delta_R)} \end{aligned}$$



# Weak-interaction processes and physics beyond the Standard Model

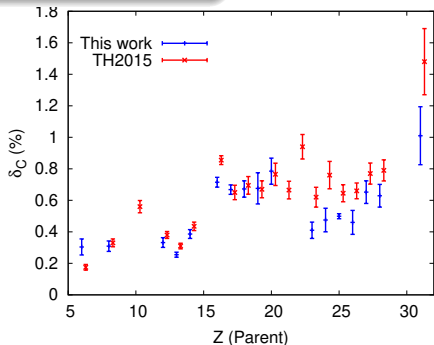
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Nadezda Smirnova  
Isospin-symmetry breaking:  
exotic nuclei, fundamental  
interactions and astrophysics  
(2017-2022)

IN2P3/CENBG/IPHC/GANIL

$$\frac{1 + \delta_{NS} - \delta_C}{K} ft$$
$$\frac{1}{|f_{ud}|^2 (1 + \Delta_R)}$$

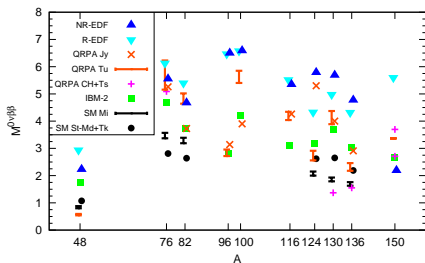


# Weak-interaction processes and physics beyond the Standard Model

## Reliable nuclear matrix elements needed to plan and fully exploit impressive experiments looking for neutrinoless $\beta\beta$ decay

- Matrix elements differences between present calculations, factor 2-3 besides additional “quenching” ?
- $^{48}\text{Ca}$  and  $^{76}\text{Ge}$  matrix elements in larger configuration space increase  $\approx 30\%$ , missing correlations introduced in IBM, EDF
- First Ab-initio calculations of  $\beta$  decays do not need additional “quenching”, Ab-initio  $^{48}\text{Ca}$  matrix elements in progress
- $2\nu\beta\beta$  decay,  $\mu$ -capture/ $\nu$ -nucleus scattering and double Gamow-Teller transitions can give insight on  $0\nu\beta\beta$  matrix elements

$$[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 |f(m_i, U_{ei})|^2$$



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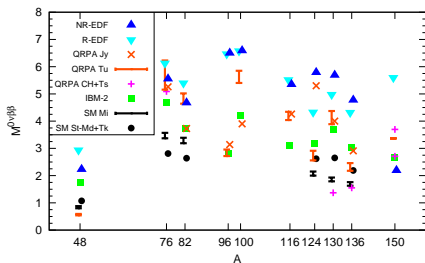
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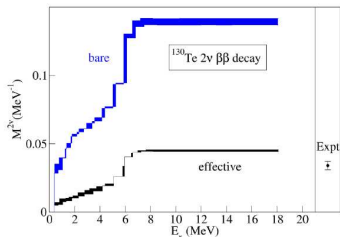
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PHYSICAL REVIEW C **100**, 014316 (2019)

Renormalization of the Gamow-Teller operator within the realistic shell model

L. Coraggio,<sup>1</sup> L. De Angelis,<sup>1</sup> T. Fukui,<sup>1</sup> A. Gargano,<sup>1</sup> N. Itaco,<sup>2,1</sup> and F. Nowacki<sup>3,4,2</sup>



Renormalisation of the  $(\beta\beta)_{2\nu}$  operator by MBPT

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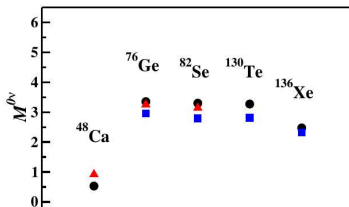
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The calculation of the neutrinoless double- $\beta$  decay matrix element within the realistic shell model

L. Coraggio,<sup>1</sup> A. Gargano,<sup>1</sup> N. Itaco,<sup>2,1</sup> R. Mancino,<sup>2,1</sup> and F. Nowacki<sup>3,4,2</sup>



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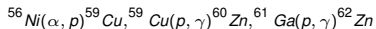
Collaboration IPHC - INFN/Université de Naples

# Stellar rp and r-processes and nucleosynthesis

## Proton capture reaction rates calculations for rp process (X-ray bursts) or novae

$$N_A \langle \sigma v \rangle = 1.54 * 10^{15} (\mu T_9)^{-3/2} \omega \gamma \exp\left(\frac{-11.605 E_r}{T_9}\right) \text{cm}^3 \cdot \text{s}^{-1} \cdot \text{mol}^{-1}$$

- Strong impact reactions (Cybert et al. AAJ, 2016):



- Theoretical determination of unknown quantities:

Resonance energies (with Isospin breaking)

Widths with respect to proton and gamma emission

- Several reactions in *sd* shell

*pf* shell nuclei around  $^{40}\text{Ca}$

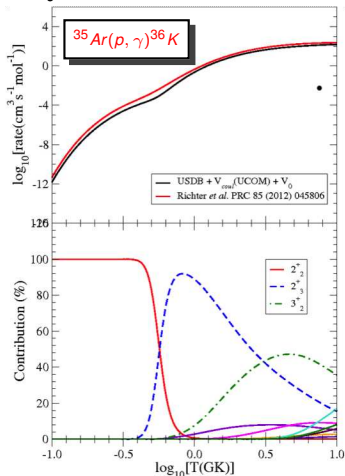
Thomas-Ehrman shift in *sd* shell

$(\alpha, \gamma)$ ,  $(\alpha, p)$ ,  $(p, \alpha)$  capture/emission modeling

$^{22}\text{Mg}(\alpha, p)^{25}\text{Al}$  and other reactions

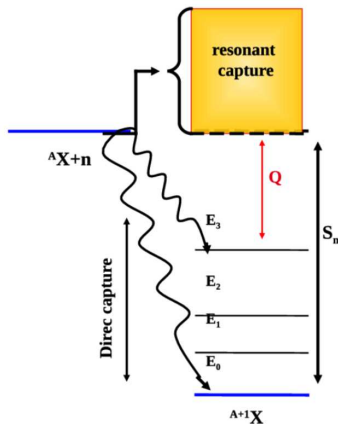
- *p* – *sd* – *pf* valence space for non-natural parity states

Electron capture rates around  $^{40}\text{Ca}$



## Radiative Neutron Capture: Theoretical Models and Applications

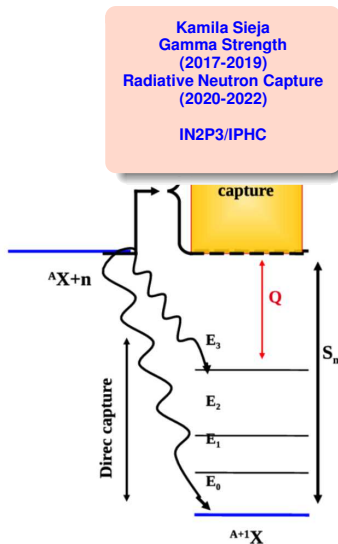
- SM can provide reliable spectroscopic factors and help testing usual theoretical assumptions in cases no experimental data is known → work in progress
- Spectroscopy of neutron-rich nuclei around  $^{78}\text{Ni}$  is still of interest for nuclear models
- $E1/M1$  RSF and PSF can be microscopically obtained within the SM
- Shell effects survive at higher excitation energies and are visible in  $M1$  dipole strength functions
- $M1$  upbend has a significant impact on neutron capture cross sections in exotic nuclei :  $X10$
- develop/constraint/improve global microscopic models (HFB, QRPA) on a SM basis for all kind of applications (astrophysics, nuclear data, reactors etc ...)





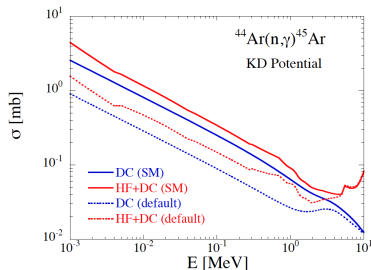
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K. Sieja, work in progress

## SM Studies

- High predictive power, accurate and detailed information (structure near and far from stability, nuclear and electroweak processes)
- success and robustness of the approach encourage further developments and applications
- Potential powerful description of simultaneous low-lying phenomena but need of precise dedicated local studies
- Intense support for future experimental programs and developments (but manpower insufficient ...)
- Existing cross fertilizing collaborations in several domains: ab-initio studies, isospin symmetry breaking, astrophysics,  $\beta\beta$  decay ... and others to develop !

# Ab-initio challenge

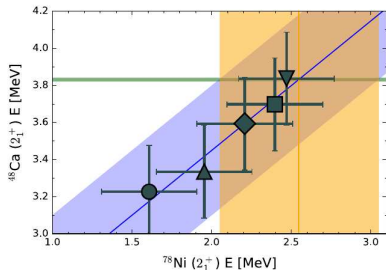


FIG. 2. Correlation between the energies of the  $2_1^+$  excited state in  $^{48}\text{Ca}$  and  $^{78}\text{Ni}$ , obtained from the interactions NNLO<sub>sat</sub> (circle), “2.0/2.0 (PWA)” (square), “2.0/2.0 (EM)” (diamond), “2.2/2.0 (EM)” (triangle up), and “1.8/2.0 (EM)” (triangle down). The error bars estimate uncertainties from enlarging the model space from  $N = 12$  to  $N = 14$ . The thin horizontal line marks the known energy of the  $2_1^+$  state in  $^{48}\text{Ca}$ .

[Phys. Rev. Lett. 117, 172501 \(2016\)](#)

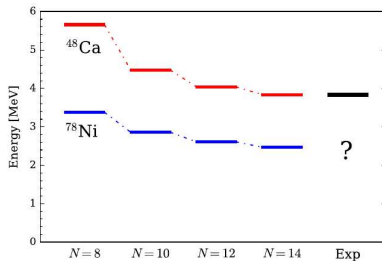
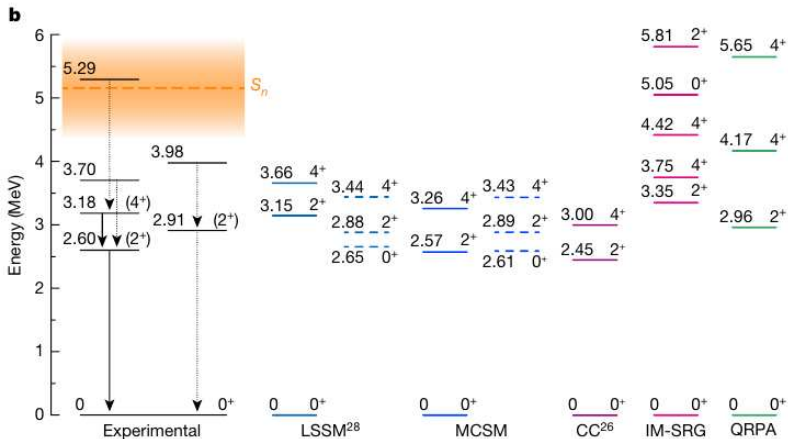


FIG. 3. Convergence of the first  $2_1^+$  excited state of  $^{48}\text{Ca}$  and  $^{78}\text{Ni}$  with increasing model-space size and compared to the data for the interaction 1.8/2.0 (EM) of Ref. [33].

# Ab-initio challenge

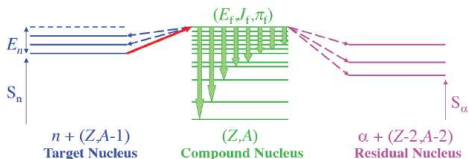


R. Taniuchi et al., NATURE 569, 53-58 (2019)



# Stellar rp and r-processes and nucleosynthesis

## Resonant Capture



$$\sigma_{(n,\gamma)}^{\mu\nu}(E_i, n) = \frac{\pi \hbar^2}{2M_{i,n} E_{i,n}} \frac{1}{(2J_i^\mu + 1)(2J_n + 1)} \sum_{J,\pi} (2J + 1) \frac{T_n^\mu T_\gamma^\nu}{T_n^\mu + T_\gamma^\nu}$$

for  $E_n \sim \text{keV}$   $T_n^\mu \gg T_\gamma^\nu \rightarrow \sigma^{\mu\nu} \sim T_\gamma^\nu$

$E_{i,n}, M_{i,n}$  - center-of-mass energy, reduced mass of the system

$J_n = 1/2$  - neutron spin

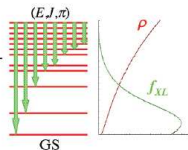
$T_n^\mu = T_n(E, J, \pi; E_i^\mu, J_i^\mu, \pi_i^\mu)$   $T_\gamma^\nu = T_\gamma(E, J, \pi; E_m^\nu, J_m^\nu, \pi_m^\nu)$  - transmission coefficients

For a given multipolarity

$$T_{\chi L}(E, J, \pi, E^\nu, J^\nu, \pi^\nu) = 2\pi E_\gamma^{2L+1} f_{\chi L}(E, E_\gamma)$$

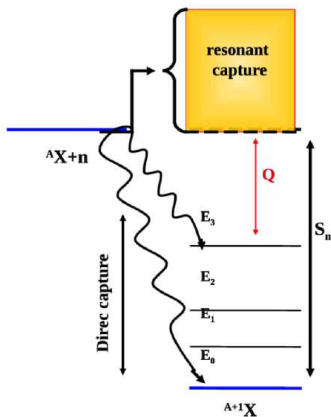
• Test, using SM, the key ingredients of Hauser-Feshbach calculations:

- description of  $\gamma$  emission spectra
- Brink-Axel hypothesis



# Stellar rp and r-processes and nucleosynthesis

## Direct Capture



*Xi. Yu and S. Goriely, Phys. Rev. C86 (2012) 045801*

$$\sigma^{DC}(E) = \sum_{f=0}^X S_f \sigma_{dis}(E) + \langle S \rangle \int_{E_x}^{S_n} \sum_{J_f, \pi_f} \rho(E_f, J_f, \pi_f) \times \sigma_f^{cont} dE_f$$

If no experimental data available:

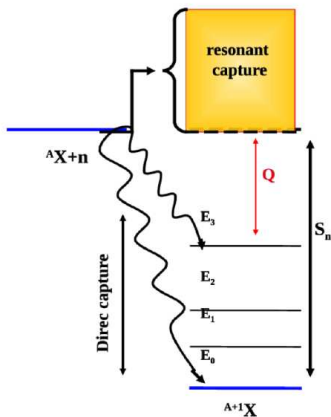
- use combinatorial model for the level density with  $\langle S \rangle = 0.5$

- ☛ The key ingredients: low-energy levels and spectroscopic factors
- ☛ Validate theoretical approximations (HFB) in exotic nuclei using SM predictions



# Stellar rp and r-processes and nucleosynthesis

## Direct Capture



*Xi. Yu and S. Goriely, Phys. Rev. C86 (2012) 045801*

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