The Boundaries of KKLT

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De Sitter Constructions in String Theory
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Saclay (virtually)

Motivation

- Before summer, thought constructions ~settled
- Curious why so difficult, why disagreement
- Part of problem “many moving parts”
  - No one has full 10d Lagrangian
- But will argue in this talk part of problem a relevant field mostly left out
  - Field is **radion** associated with throat
  - Will show how it can help resolve some controversies
- Will argue:
  - Construction clever and in principle should work
  - Some inconsistencies can be resolved
  - But there is indeed an instability unless $\sqrt{g_s M} <\sim 7$
    - Bena, Dudas, Grana, Luest, Blumenhagen, Klawer Schlechter
  - Independently found similar field and issue in 5d eft

Outline

- KKLT as a 5d theory
- Goldberger-Wise Mechanism for stabilizing 2 branes,
  - GW field, and radion
- How it resolves (at least) one issue with uplift
  - Review "bent branes"/KR
  - Consistency with apparently differet 4d geometries on UV, IR boundaries (CY and conifold end)
- Also sheds insight into "conifold destabilization"
- 5d EFT radion IS conifold deformation parameter
  - Potential of the right form
- Destabilization for too small g_sM^2 is destabilization of 5d eft; leads to runaway of IR brane
  - Independent from volume modulus metastability
  - Another light field that has mostly been ignored
- Some interesting supersymmetry breaking implications

5d EFT for KKLT

- 5d Klebanov-Strassler Geometry
- AdS_5 but with "running N_{eff}"
  - N_{UV}=MK; N_{IR}=M
- CY space serves as UV boundary
- Conifold deformation ends space on IR boundary
  - (negative tension)
- Hierarchy from $e^{-2 \Pi K/Mgs}$

Boundary conditions

- Original construction Minkowski

condensate or antibrane are included) as a stringy correction associated with wrapped seven branes. The charge is determined topologically whereas in the original construction the induced curvature/tension could be determined through the BPS condition where the first is from

$$-\mu_3 \int_{R^4\times\Sigma} C_{(4)} \wedge \frac{p_1(\mathcal{R})}{48} = \frac{\mu_7}{96}(2\pi\alpha')^2 \int_{R^4\times\Sigma} C_{(4)} \wedge \mathrm{Tr}\left(\mathcal{R}_{(2)} \wedge \mathcal{R}_{(2)}\right) . \qquad (2.1)$$

and the second from the DBI action

$$-\frac{\mu_7}{96}(2\pi\alpha')^2 \int_{R^4\times\Sigma} d^4x\sqrt{-g}\,\mathrm{Tr}(\mathcal{R}_{(2)} \wedge *\mathcal{R}_{(2)}) . \qquad (2.2)$$

- With gaugino condensate, AdS$_4$ UV boundary
- Consistent construction would “slice” with AdS$_4$
- IR boundary would have to be consistent for time-independent solution

“Bent Brane”/KR Review

5D space is considered. As in the original setup [24], the action we consider is just that of 5d gravity with a negative cosmological constant $\Lambda^{5d} = -3/L^2$ coupled to a brane of tension λ:

$$L = \int d^5x\sqrt{g}\left[-\frac{1}{4}R - \Lambda^{5d}\right] - \lambda \int d^4x\, dr\, \sqrt{|\det g_{ij}|}\,\delta(r), \tag{3.1}$$

where g_{ij} is the metric induced on the brane by the ambient metric $g_{\mu\nu}$. Note we retain the notation of [25] so that Λ here refers to the 5d cc and H^2 for four dimensions is given below in terms of c or alternatively the brane tension mismatch.

We use the ansatz for the solution to be a warped product with warp factor $A(r)$,

$$ds^2 = e^{2A(r)}\bar{g}_{ij}dx^i dx^j - dr^2\ , \tag{3.2}$$

allowing for the 4d metric to be Minkowski, de Sitter or anti-de Sitter with the 4d cosmological constant Λ being zero, positive or negative respectively following the conventions of [27].

The 4d cosmological constant is given by the detuning $M = \frac{\lambda L}{3}$ of the brane tension: $\Lambda_{dS} = \frac{1}{L^2}(M^2 - 1)$, $\Lambda_{AdS} = \frac{1}{L^2}(1 - M^2)$.

From a 4d point of view, our toy model of KKLT appears as if it should readily uplift

Uplift

- Add antiD3brane
- Net energy density adds to zero (really a little positive)
- But without backreaction slicing would be inconsistent in bulk
  - AdS_4 in UV, a different AdS_4 in IR!
- How to resolve?
- Clearly need a backreaction of some sort
  - Associated with stabilized geometry
  - Can absorb and transfer energy
- In 5d parlance, Goldberger-Wise Mechanism

GW Review

bulk action

$$S_b = \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G}\left(G^{AB}\partial_A\Phi\partial_B\Phi - m^2\Phi^2\right), \tag{3.3}$$

where G_{AB} with $A, B = \mu, \phi$ is the 5d warped metric. In addition there are boundary terms which are assumed to set the UV and IR values to v_h and v_v respectively. Matching at the boundaries sets the boundary conditions and yields the potential for the GW field. With $\epsilon = m^2/4k^2$ assumed to be small the general solution is

$$\Phi(\phi) = e^{2\sigma}[Ae^{\nu\sigma} + Be^{-\nu\sigma}], \tag{3.4}$$

with $\nu = \sqrt{4 + m^2/k^2} = \sqrt{4+\epsilon}$ where $\sigma(\phi) = kr_c|\phi|$ and

$$A = v_v e^{-(2+\nu)kr_c\pi} - v_h e^{-2\nu kr_c\pi}, B = v_h(1 + e^{-2\nu kr_c\pi}) - v_v e^{-(2+\nu)kr_c\pi}, \tag{3.5}$$

Notice that in the GW solution the unsuppressed v_h in the B term determines the boundary values and the A term, which vanishes at the leading order minimum, determines the derivative across the boundaries.

The minimum of the potential is at

$$kr_c = \left(\frac{4}{\pi}\right)\frac{k^2}{m^2}\ln\left[\frac{v_h}{v_v}\right]. \tag{3.6}$$

Dual Interpretation

- GW field an almost marginal operator
- Conformal dimension slightly deviating from 4
- Net potential

$$V = \lambda_1 \phi^4 + \lambda_2 \phi^{4-\epsilon}$$

- Can be running coupling for example
- Gets strong in IR

Need a GW field, also a radion

Following GW, it is readily seen that the potential takes the form

$$V_\phi \approx 4ke^{-kr\pi}\left(v_v - v_h e^{-\epsilon k\pi r}\right)^2 (1+\frac{\epsilon}{4}) - k\epsilon v_h e^{-(4+\epsilon)k\pi r}\left(2v_v - v_h e^{-\epsilon k\pi r}\right) \tag{3.7}$$

The radion will shift to accommodate the uncancelled $e^{-4k\pi r_c}\epsilon v_v^2$ at 3.6 and there will be a stabilized radion with bulk mass squared of order $\epsilon^2 e^{-2k\pi r_c} v_v^2$ with the depth of the potential ϵ suppressed. The true potential arises from the matching at the IR brane [52, 54] in which case the mass squared can be suppressed by a single factor of ϵ.

$$\int_{\epsilon^-}^{\epsilon^+} d\phi \Phi^\dagger 2\frac{1}{r_c^2}\partial_\phi\left(e^{-4\sigma}\partial_\phi\Phi\right) + \int d\phi e^{-4\sigma}\delta T\frac{\delta(\phi-\pi)}{r_c}.$$

$$\delta\Phi\partial_r\Phi = (4+\epsilon)kv_v\delta v_v = 2r_c\delta T$$

Change in radion: $\delta v_v/(\epsilon k\pi v_v)$

Really, point is near minimum we can redefine radion so it is localized in IR

Can identify GW field

$$V = \lambda_1 \phi^4 + \lambda_2 \phi^{4-\epsilon}$$

From a dual perspective, you really have a running coupling
Explicit breaking conformal invariance from running
Spontaneous breaking at IR brane position

We can explicitly identify running in KKLT

in the IR. Ref. [19] identifies the GW field, B and argues its slowly varying potential in the radial direction is a result of the kinetic term for a field originating in the 10d theory from the flux of the NS 2-form potential B_2 on the S^2 cycle of the $T^{1,1}$. They explicitly construct a potential consistent with "running N_{eff}" and describe how with this field they can stabilize a geometry that consists of the CY region, a conifold region with constant warp factor, and the warped deform conifold. This is in the spirit of the dual interpretation of the GW mechanism,

Potential Other Problem/ Conifold Instability

- Can now identify radion in KKLT too
- But first let's consider the "conifold instability"
- See Luest, Grana talks as well
- S: Conifold deformation parameter

$$\sum_{a=1}^{4} \omega_a^4 = S\,. \qquad (3.10)$$

The deformation parameter S is the complex structure modulus whose absolute value corresponds to the size of the 3-sphere at the tip of the cone.

$$\int_A \Omega_3 = S\ , \qquad (3.11)$$

Potential Instability: Add antibrane

- Add antibrane potential
- Note general form helps to identify maximum perturbation

The antibrane contributes a perturbation

$$V_{\overline{D3}} = \frac{\pi^{1/2}}{\kappa_{10}} \frac{1}{(Im\rho)^3} \frac{2^{1/3}}{I(\tau)} \frac{|S|^{4/3}}{g_s(\alpha' M)^2} \, . \tag{3.18}$$

We follow [56] and define $c'' = \frac{2^{1/3}}{I(0)} \approx 1.75$. For p anti-D3 branes the potential is multiplied by p, and this is taken care by simply replacing $c'' \to c''p$.

Potential for S

The supersymmetric potential for this field induced by the Klebanov-Strassler geometry is

$$V_{KS} = \frac{\pi^{3/2}}{\kappa_{10}} \frac{g_s}{(Im\rho)^3} \left[c \log \frac{\Lambda_0^3}{|S|} + c' \frac{g_s(\alpha' M)^2}{|S|^{4/3}} \right]^{-1} \left| \frac{M}{2\pi i} \log \frac{\Lambda_0^3}{S} + i \frac{K}{g_s} \right|^2 , \qquad (3.12)$$

where g_s is the stabilized vev of the dilaton, $Im\rho = (\mathrm{Vol}_6)^{3/2}$, c as we argue below is not relevant here (and is in any case suppressed in the small S region), whereas the constant c', multiplying the term coming solely from the warp factor, denotes an order one coefficient, whose approximate numerical value was determined in [46] to be $c' \approx 1.18$.

The potential for the S field is essentially the potential above that we had for a GW field, but takes a slightly different form than that above due to supersymmetry, namely

$$V = S^{4/3} \left(\lambda_1 - \lambda_2 \log \frac{\Lambda_0^3}{S} \right)^2 = \lambda_1^2 S^{4/3} \left(1 - \frac{\lambda_2}{\lambda_1} \log \frac{\Lambda_0^3}{S} \right)^2 \approx \lambda_1^2 S^{4/3} \left(5 - 6 \left(\frac{S}{\Lambda_0^3} \right)^{\epsilon} + 2 \left(\frac{S}{\Lambda_0^3} \right)^{2\epsilon} \right) \qquad (3.13)$$

where rewriting the potential in this GW form breaks down near the "IR brane" where $(2\pi M/K) \log S/\Lambda_0^3$ gets big. Really the original form is enough to see that we have weakly explicitly broken scale invariance. Here $\lambda_1 = K g_s$ and $\lambda_2 = (M/2\pi)$, and $\epsilon = \lambda_2/\lambda_1$. The minimum occurs at $S_{KS} = \Lambda_0^3 e^{-2\pi k/M g_s} = \Lambda_0^3 e^{-\lambda_1/\lambda_2} = \Lambda_0^3 e^{-1/\epsilon}$. Here the $S^{4/3}$ dependence comes from the Kahler potential whereas the remaining dependence is from the superpotential. The nonrernormalization theorems in the supersymmetric potential guarantee the full potential is always proportional to the leading order potential.

Massive Scalar: S~Φ3

We can also expand about S_{KS} to find

$$V \approx S^{4/3}\lambda_2^2(\frac{S - S_{KS}}{S_{KS}})^2$$

mass is not suppressed by ϵ. When we use

$$m_S^2 \equiv \frac{1}{M_{pl}^2} G^{S\bar{S}} \partial_{\bar{S}} \partial_S V \Big|_{S=s_{\rm KS}} . \qquad (3.15)$$

we find the S mass squared is suppressed by $1/g_s M^2$. In terms of the properly normalized field ϕ (see below), the mass squared scales (over the exponential suppression) as $1/(g_s M^2)^2$, which is how all KK masses associated with the IR region of the conifold throat would scale as well

Eq. 3.16 as,

$$\phi = \frac{3 M_{pl}\sqrt{c'}}{\pi^{1/2}\,\|\Omega\|\, V_w^{1/2}} \alpha' \sqrt{g_s} M S^{1/3} = \frac{3\sqrt{g_s} M \sqrt{c'}}{8\pi^4 \alpha' \,\|\Omega\|} S^{1/3} \,, \qquad (3.16)$$

where $c'' \approx 1.75$ so that the parameter S is indeed related to ϕ^3, and is the parameter determining the warping in the throat. This ϕ is precisely the radion of GW and has the correct potential to both determine the length of the throat and the warping in the IR as well as to respond to perturbations to generate a consistent geometry. The radion mass squared, as with the values of KK mass squared. is suppressed by a factor $1/(g_s M)^2$ in units of the confinement scale, where the confinement scale is suppressed relative to the warped string scale by $1/(\sqrt{g_s} M)$.

Radion/Conifold Deformation Parameter

- We identify based on its effect on metric
- And its potential
- $S \sim \Phi^3$ where Φ is the radion in GW potential
- This means checking for stability of RS type geometry is checking for stability wrt conifold deformation parameter!
- Exactly what Bena, Dudas, Grana, Luest; Blumenhagen, Klaewer Shlechter did

Aside

- Bena et al called state modulus that becomes nonnormalizable in decompactification limit

$$G_{S\bar{s}} = \frac{1}{\pi \|\Omega\|^2 V_w} \left(c \log \frac{\Lambda_0^3}{|S|} + c' \frac{g_s(\alpha' M)^2}{|S|^{4/3}} \right)$$

- Radion is in fact IR localized state that survives decompactification limit
- Just c’ term; gives correct behavior of potentials
- Important later for supersymmetry

Runaway radion if too big a perturbation

The general form of the potential (we factor out $\lambda_1^2 \pi g_s / c'$ is

$$V = S^{4/3}\left(1 + \epsilon \log \frac{S}{\Lambda_0^3}\right) + \delta S^{4/3} \qquad (3.19)$$

The barrier disappears when $\delta/\epsilon^2 = 9/16$.

We see that the perturbation from the antibrane (yielding the δ type perturbation above) yields the potential proportional to the above with $\delta = c"c'/g_s \pi \lambda_1^2$ and $|\epsilon| = M g_s / 2\pi K$. By writing it this way we keep ϵ and δ as small parameters. This gives precisely the stability condition found in [56], namely

$$\sqrt{g_s} M > M_{\min} \qquad \text{with} \qquad M_{\min} = \frac{8}{3}\sqrt{\pi c' c''} \approx 6.8\sqrt{p}\,. \qquad (3.20)$$

- Two important consequences **if bound satisfied**
  - Hierarchy limited $(KM/g_s M^2) <\sim 5$ for known manifolds
  - Cosmological phase transition won’t complete
    - Cremenelli, Nicolis, Rattazzi//Hassanain, March-Russell, Schellvinger

Real potential instability

- Need largish $g_s M^2$
- But then hierarchy problematic
  - $K/Mg_s \sim KM/M^2 g_s$
  - KM bounded in a given geometry
- Another problem
  - Cosmological phase transition for RS like geometries
  - High temperature AdS/Schwarschild
  - Need to evolve to RS
  - Upper bound on $M^2 \sim 21$ for this geometry
  - Inconsistent

Implication

- KKLT clever way to avoid issues of stabilizing moduli and exploiting supersymmetry to control solution
- But for known manifolds, the net flux isn’t big enough to trust the approximations
- Even if better ones found the issue of the cosmological phase transition has to be addressed

On the other hand

- New manifolds, new ideas on cosmology quite possible
- In any case a very interesting class of constructions for susy breaking
- Naturally (almost) sequestered
- With warping allows for interesting hierarchies
  - Usually anomaly-mediation all masses~β $m_{3/2}$
  - Here can be warped in addition
    - If susy breaking leaves preserved essential global symmetries

Supersymmetry Breaking

- Mostly Anomaly-Mediation/sequestered
- Also volume modulus
- $F \sim F_\chi/(\sigma\ a) \sim \beta\ g^2\ F_\chi$
  - Of size of anomaly mediation
- Also radion develops F term
- Allows for uplift as discussed earlier

Radion F term

The superpotential in string units takes the form

$$W = \left(S\frac{M}{2\pi i}(\log\frac{\Lambda_0^3}{S} + 1) + \frac{1}{g_s}KS \right) - iT_{antibrane}\frac{S}{M/2\pi} \qquad (4.1)$$

where we have treated the antibrane energy as a contribution to the superpotential, even though we know ultimately the theory breaks supersymmetry (through a diferent sector). So far as the radion F term goes, the important point is that due to the cross term, this superpotential is linear in the antibrane tension. Near S_c (equivalently ϕ_c, this leads to $F_\phi \approx T_{antibrane}/(M/2\pi)$, since we found ϕ_c assuming no antibrane tension. Also note that the original logarithmic superpotential can be expanded as $(\phi - \phi_c)$. As before, without the

Conclude

- Controversy actually interesting
- Where there’s smoke there’s fire
- Interesting physics buried in (some of) the disagreements
- In particular role of radion cannot be neglected
- Helps for consistency
- Also demonstrates potential instability
- Not necessarily an impossible barrier, but we still don’t have completely perturbative trustworthy string construction for de Sitter space