Two Routes to de Sitter in Holography

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de Sitter Constructions in String Theory - IPhT, December 11, 2019

1704.05075 with C.Charmousis and E.Kiritsis
1711.08462, 1807.09794 with J.K.Ghosh, E.Kiritsis, L.Witkowski
1904.02727 with A.Amariti, C.Charmousis, D. Forcella and E.Kiritsis

Introduction

Typical de Sitter constrution in string theory: warped compactification

$$ds_{10}^2 = e^{2A(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n \quad + \quad \text{Fluxes, etc}$$

1. Compact 6d manifold (finite 4d M_p) $\Rightarrow$ low-energy 4d EFT;
2. Effective 4d potential V_{eff} $\Rightarrow$ stabilize scalars;
3. $V_{eff} > 0$ at the stabilized minimum;

1 + 2 + 3 $\Rightarrow$ 4d de Sitter metric is solution for effective 4d theory.

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I will illustrate this in a simple bottom-up model based on holography.

- Gravity side: non-compact asymmetric braneworld
- Dual field theory: no *dynamical* 4d gravity in the UV;
- Emergent 4d gravity coupled to observed fields;
- No *local* 4d EFT description
- Positive curvature because of either:
  1. External sources turned on;
  2. Excited state above the vacuum;
- Mechanism to control the cosmological constant *(self-tuning)*

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Top down
sting th.

KKLT-like

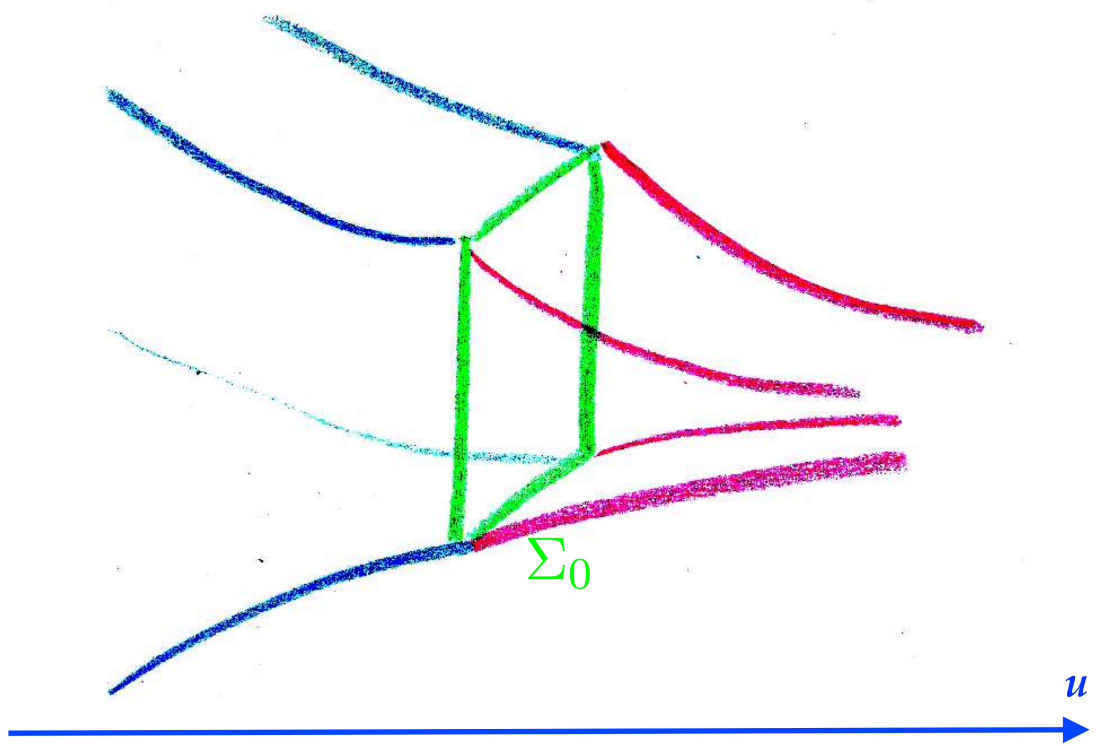
Outline

- Braneworld Setup
- Self-tuning Minkowski vacua.
- dS #1: Stabilized de Sitter brane in an RG flow geometry
- dS #2: de Sitter geometry on a moving brane.

The model

$$S = M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]$$

$$+ M^3 \int_{\Sigma_0} d^4\sigma \sqrt{-\gamma} \left[-W_B(\varphi) - \frac{1}{2} Z(\varphi) \gamma^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + U(\varphi) R^{(\gamma)} \right]$$



The model

5d Bulk action

$$S = \boxed{M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]}$$

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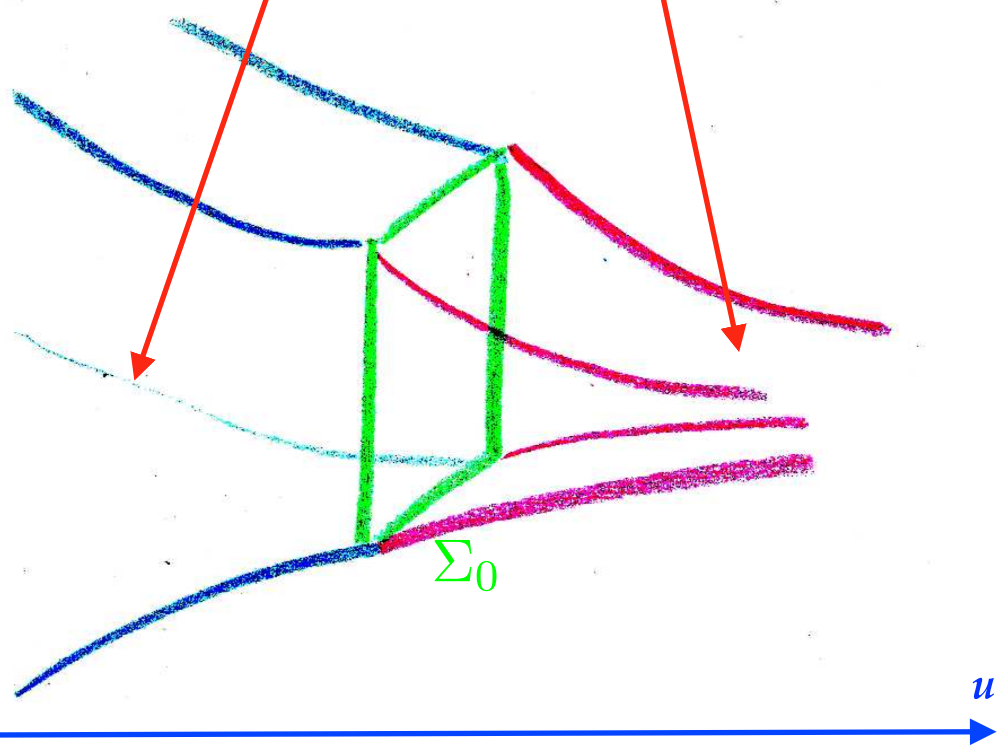

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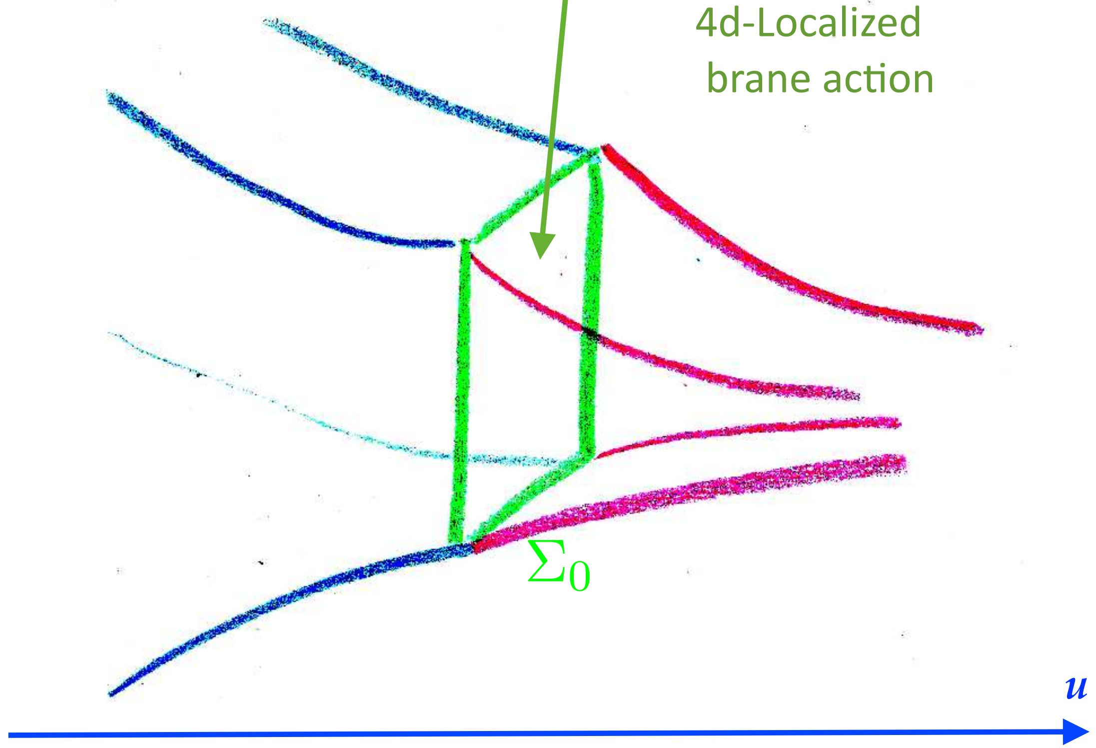
4d-Localized brane action



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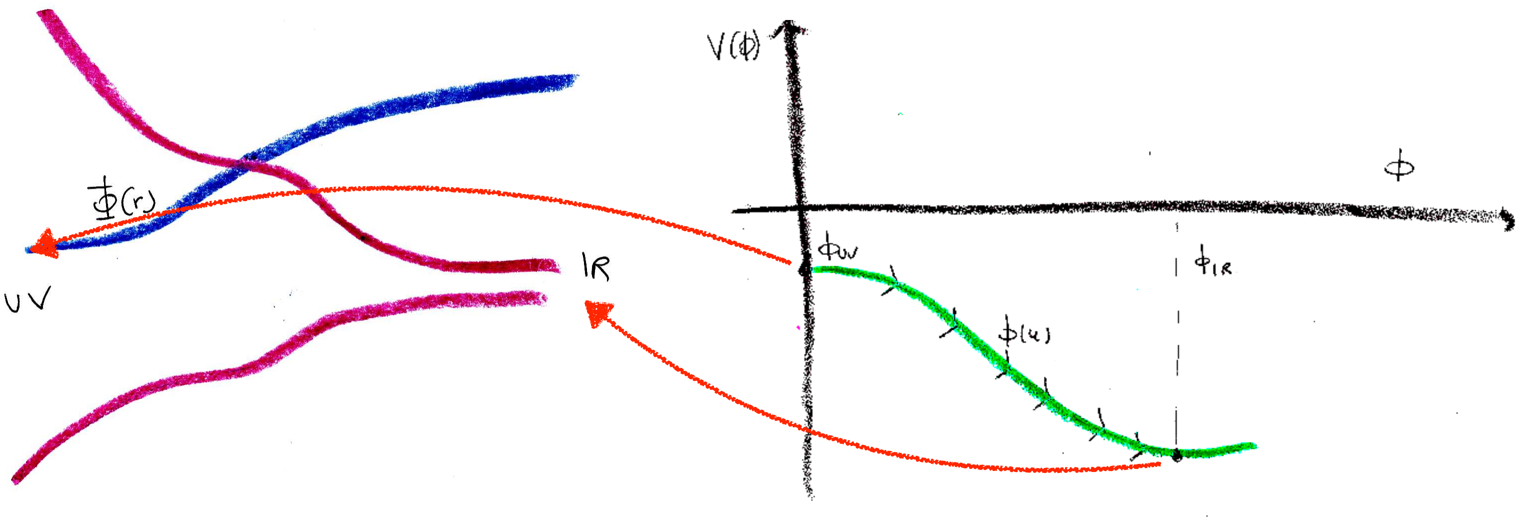
Holographic dual interpretation:

- Strongly interacting large N QFT (the bulk) coupled to 4d degrees of freedom (the brane).
- Coupling to 4d fields induce localized terms for the bulk fields at low energy.

Minkowski vacuum solution

Bulk geometry: holographic RG flow

$$ds^2 = du^2 + e^{2A(u)}\eta_{\mu\nu}dx^\mu dx^\nu, \qquad \varphi = \varphi(u)$$

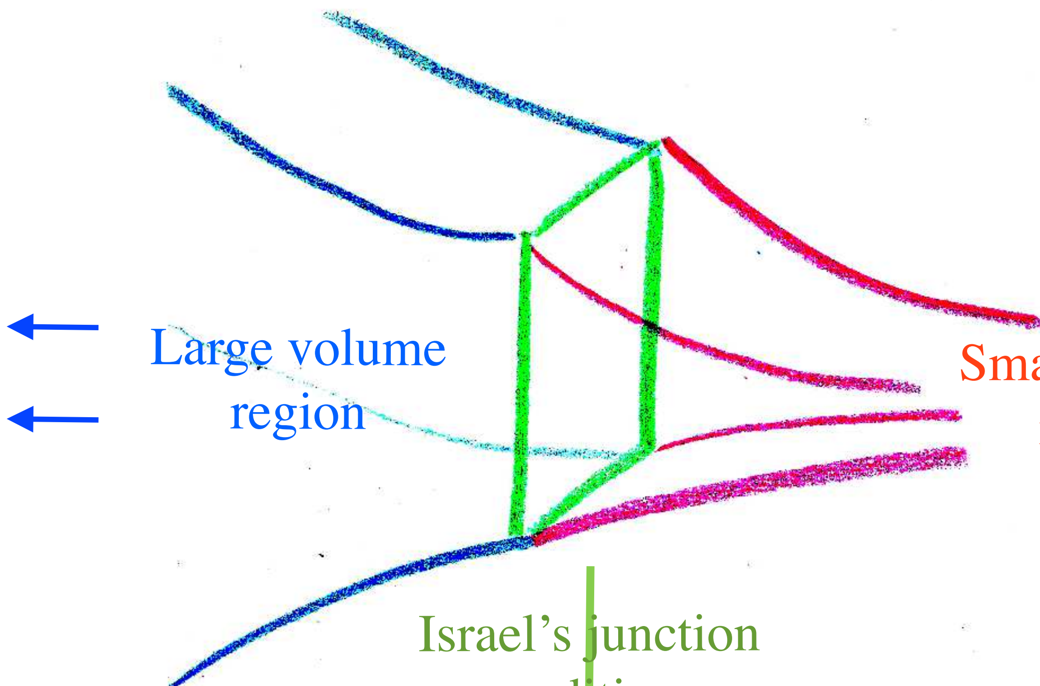


Supported by negative bulk potential with one or more AdS extrema.

Minkowski vacuum solution

Two solutions joined at the brane:

$$ds^2 = du^2 + e^{2A(u)}\eta_{\mu\nu}dx^\mu dx^\nu, \qquad \varphi = \varphi(u)$$



$$A_{UV}(u), \varphi_{UV}(u) \qquad\qquad A_{IR}(u), \varphi_{IR}(u)$$

$$e^{A_{UV}} \to +\infty,\ \varphi_{UV} \to 0 \qquad\qquad e^{A_{IR}} \to 0,\ \varphi_{IR} \to \varphi_* \text{ or } +\infty$$

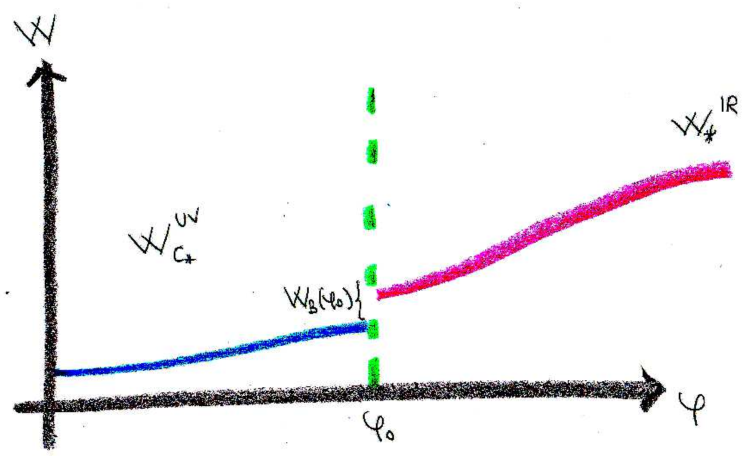
First order formalism

Solutions are conveniently characterized by scalar function $W(\varphi)$

$$W = -6\dot{A}, \quad W' = \dot{\varphi}, \quad -\frac{1}{3}W^2 + \frac{1}{2}\left(W'\right)^2 = V$$

Junction conditions in φ-space:

$$[W_{UV} - W_{IR}]_{\varphi_0} = W_B(\varphi_0), \qquad \left[W'_{UV} - W'_{IR}\right]_{\varphi_0} = W'_B(\varphi_0)$$



IR Regularity + junction conditions $\Rightarrow$ isolated solution(s) for generic brane vacuum energy (*self-tuning of CC*)

Emergent gravity on the brane

Do gravitational interactions between brane sources look 4d?

Emergent gravity on the brane

Do gravitational interactions between brane sources look 4d?

- Volume is infinite in the UV $\Rightarrow$ no low energy 4d gravity.
- Localized Einstein term $\Rightarrow$ existence of a 4d-like graviton resonance (Dvali,Gabadadze,Porrati, '00) at "short" distances.

$$S = M^3 \int du\, d^4x\, \sqrt{g} R_5 + \ldots + M^3 \int_{u=u_0} d^4x\, \sqrt{\gamma} U(\varphi_0) R_4$$

$$M_p \simeq M^3 U(\varphi_0)$$

- Localized EH term will be generated generically when SUSY is broken (that can be at a high scale, and generating also a CC is not an issue).

Where to find de Sitter

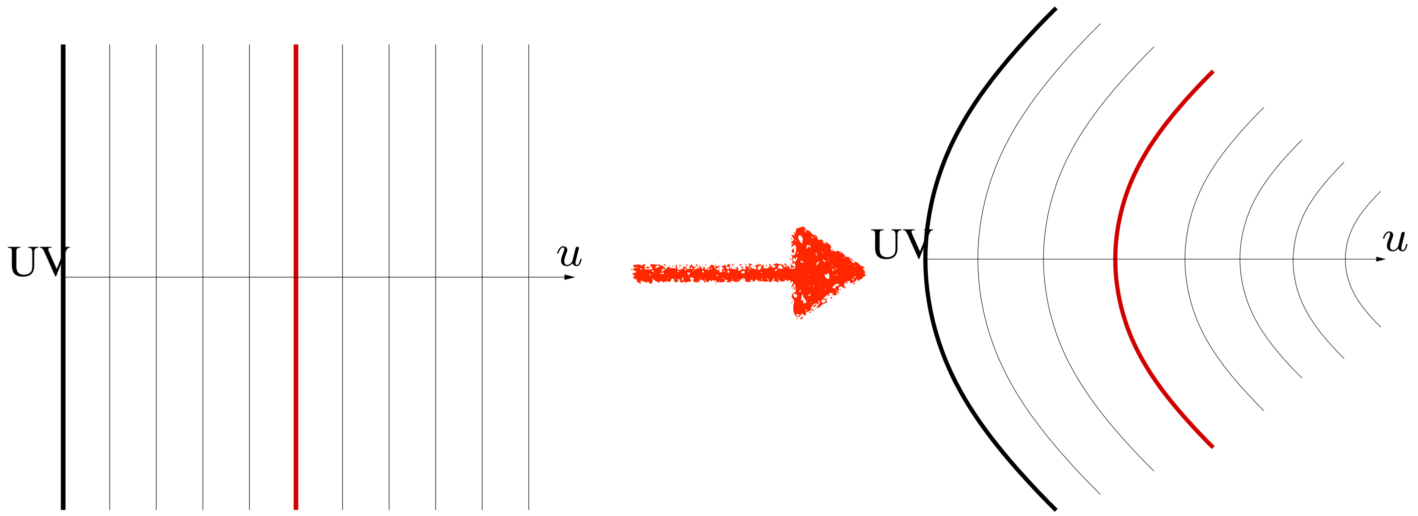
Generically, isolated stabilized 4d Minkowski-brane solutions exist.
What about 4d curved-brane solurtions?

Where to find de Sitter

Generically, isolated stabilized 4d Minkowski-brane solutions exist.
What about 4d curved-brane solurtions?

- Generically, no stabilzed curved-brane with the same boundary conditions as the flat vacuum solution.
- Two options:
  1. Change the boundary theory and turn on metric source on the boundary *(Forced holography)*
  2. Depart from vacuum state and look at time-dependent excited states *(Brane cosmology)*

Option 1: Stabilized de Sitter 4d brane



Need two ingredients:

1. Bulk: Holographic RG flows of QFTs on curved spacetimes
2. Brane: Solve junction conditions for a curved brane

Holographic RG flows on curved manifolds

Ghosh, Kiritsis, FN, Witkowski, 1711.08462

Turn on metric sources in the UV dual QFT:

- In the gravity dual theory, any solution looks asymptotically:

$$ds^2 \simeq du^2 + e^{-2u}\zeta_{\mu\nu}(x)dx^\mu dx^\nu + \text{subleading} \qquad u\to-\infty$$

$$\varphi(u) = \varphi_- e^{\Delta_- u} + \text{subleading} \qquad u\to-\infty$$

- $\zeta_{\mu\nu}$: metric of space-time on which UV CFT is defined. (a particular kind of deformation (coupling) in the UV theory). Associated operator: $T_{\mu\nu}$.
- φ_-: Source of relevant operator driving the flow
- Δ_- = $4-$(dimension of relevant op. in the UV QFT)

Holographic RG flows on curved manifolds

For the full bulk solution, take the ansatz:

$$ds^2 = du^2 + e^{2A(u)}\zeta_{\mu\nu}dx^\mu dx^\nu, \qquad \varphi = \varphi(u)$$

with $\zeta_{\mu\nu}$ an Einstein metric:

$$R^{(\zeta)}_{\mu\nu} = \frac{R}{4}\zeta_{\mu\nu} \qquad R = \text{scalar curvature in the dual QFT in the UV}$$

- Study the space of IR-regular solutions as a functions of R.
- Geometry controlled by dimensionless parameter:

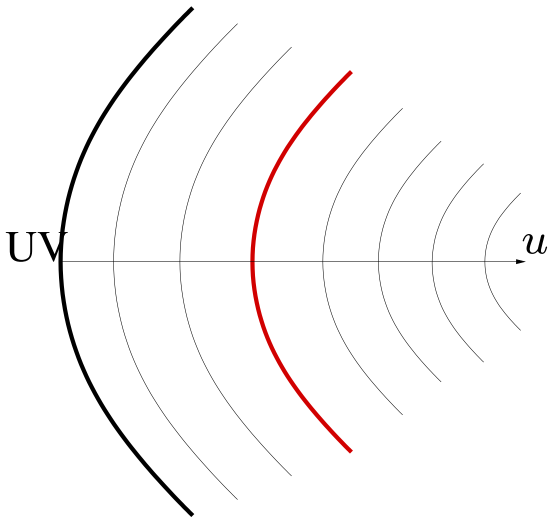
$$\mathcal{R} \equiv \frac{R}{\varphi_-^{2/\Delta_-}}$$

Stabilized de Sitter brane

Ghosh, Kiritsis, FN, Witkowski, 1807.09794

Introduce 3 superpotentials $W(\varphi), S(\varphi), T(\varphi)$

$$W = -2(d-1)\dot{A}, \quad S = \dot{\varphi}, \quad T = e^{-2A}R$$



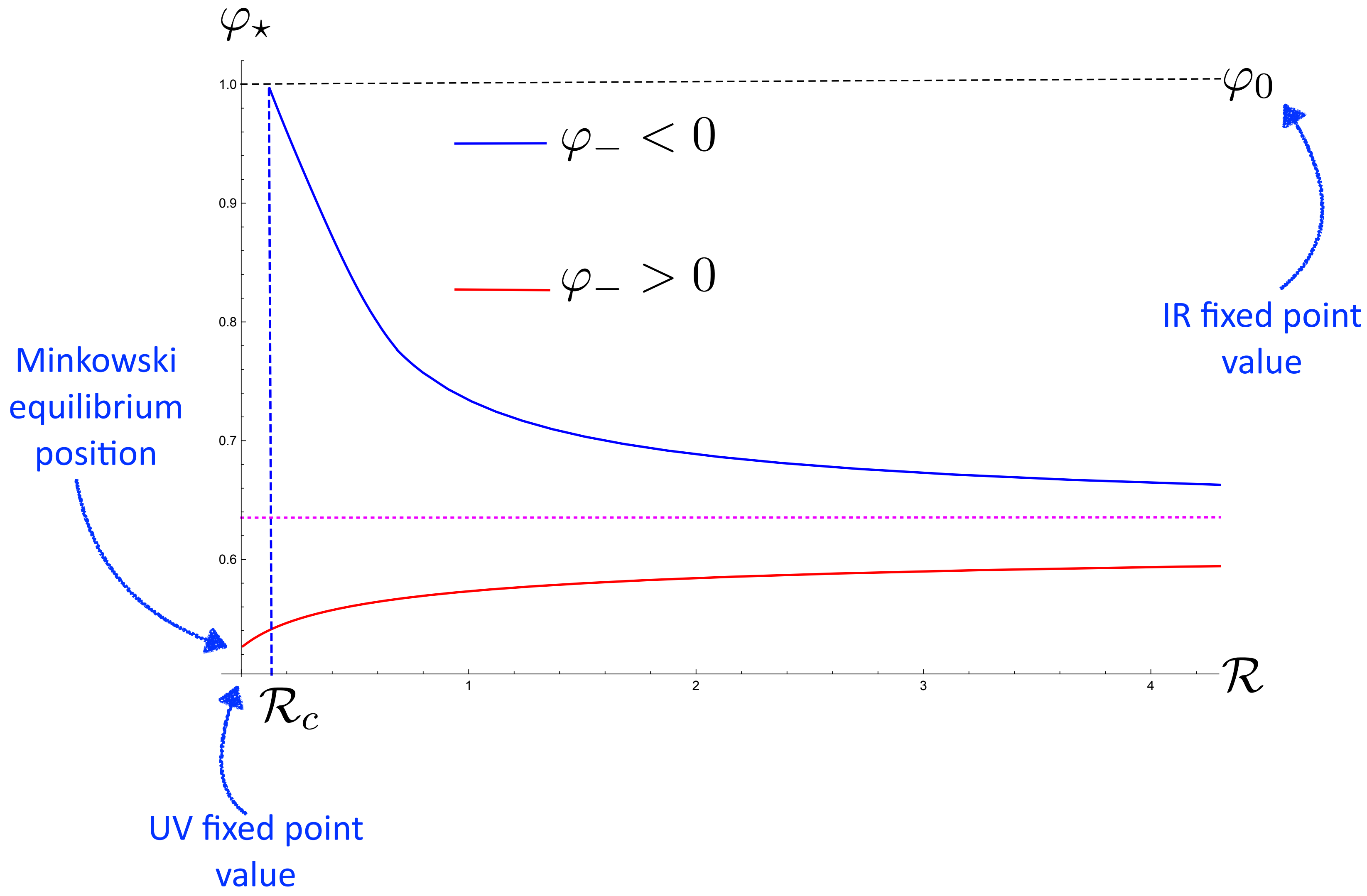
$$[W_{UV} - W_{IR}]_{\varphi_*} = [W_B + U\,T/2]_{\varphi_*}\,,$$

$$[S_{UV} - S_{IR}]_{\varphi_*} = \left[W_B' - U_B' T\right]_{\varphi_*}$$

- IR Regularity + Junction eqs $\Rightarrow$ Stabilized de Sitter brane at φ_* ($\neq$ Minkowski value φ_0)
- Equivalently: use flat boundary metric but turn on time-dependent scalar field source $\varphi_-(t) \sim t^{-\Delta_-}$.

Example

Take quartic bulk potential $V(\varphi)$ and exponential $W_B(\varphi)$ and $U(\varphi)$:



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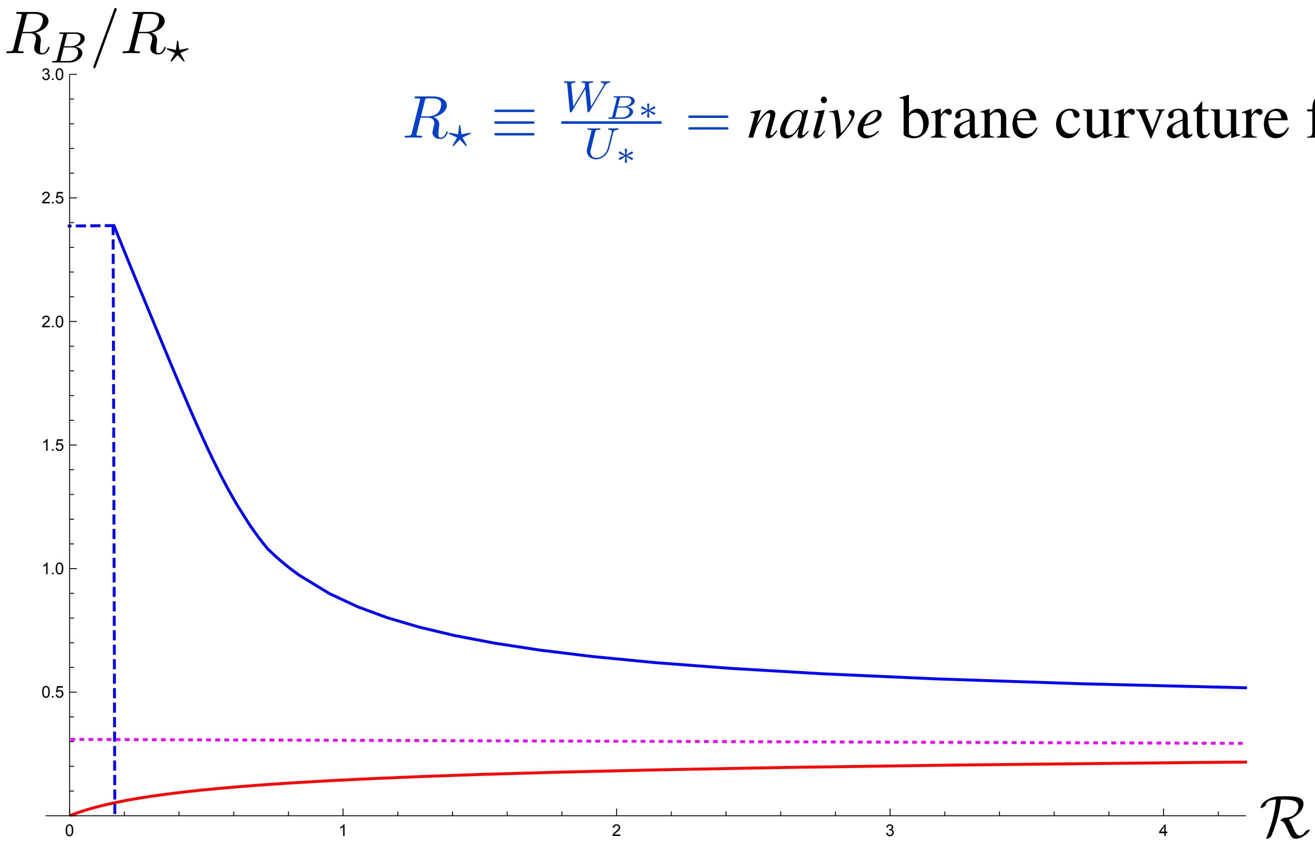

$R_\star \equiv \frac{W_{B*}}{U_*}$ = *naive* brane curvature from 4d

$$\mathcal{R} = R/|\varphi_-|^{2/\Delta_-}$$

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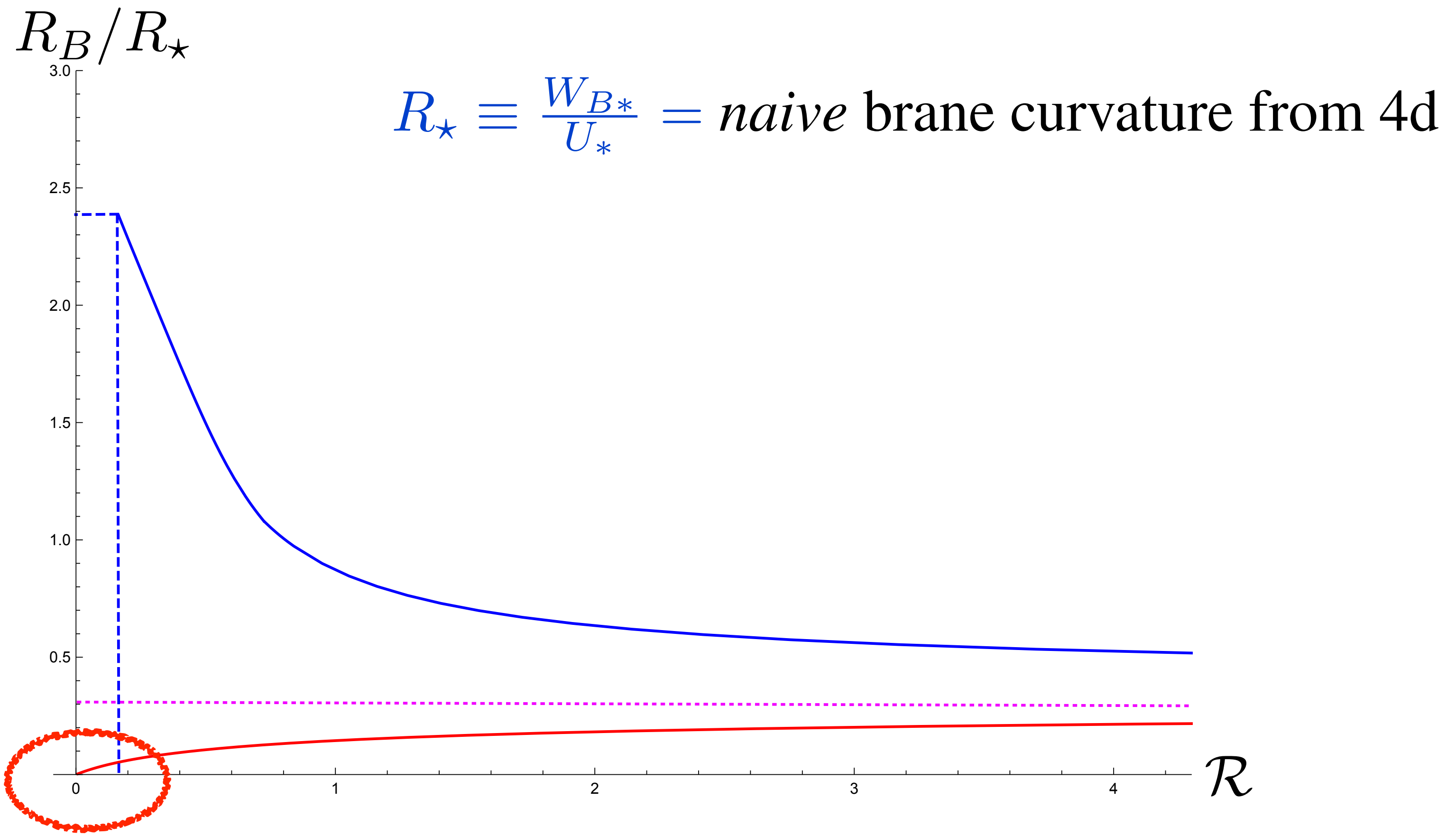

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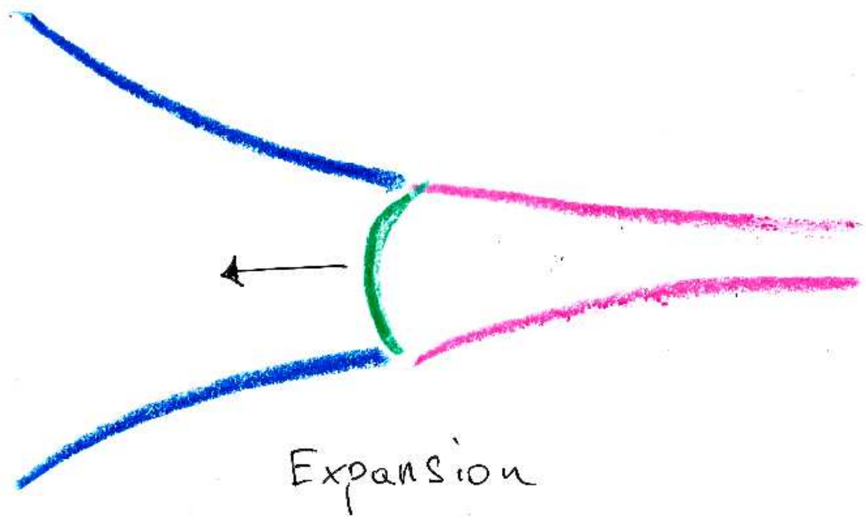
Option 2: Cosmological de Sitter brane

A brane moving with a non-zero velocity in warped geometry experiences a FRW induced metric (brane cosmology)



Can the 4d induced metric be de Sitter *without* sources for boundary metric?

Realize de Sitter as an excited state in the same theory with Minkowski vacuum

Amariti, Charmousis, Forcella, Kiritsis, FN '19

(I Can't Get No) Backreaction

Jagger,Richards 1965

To get a qualitative grip: look at the system in the probe limit:

- Bulk is the same as the (static) vacuum

$$ds^2 = du^2 + e^{2A(u)}\left(-dt^2 + d\vec{x}^2\right), \quad \varphi = \varphi(u),$$

- Brane position is time-dependent $u = u(t)$, neglect backreaction on the bulk.

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$$ds^2_{brane} = -(e^{2A} - \dot{u}^2)dt^2 + e^{2A(u(t))}d\vec{x}^2 \rightarrow -d\tau^2 + e^{2A(u(\tau))}d\vec{x}^2$$

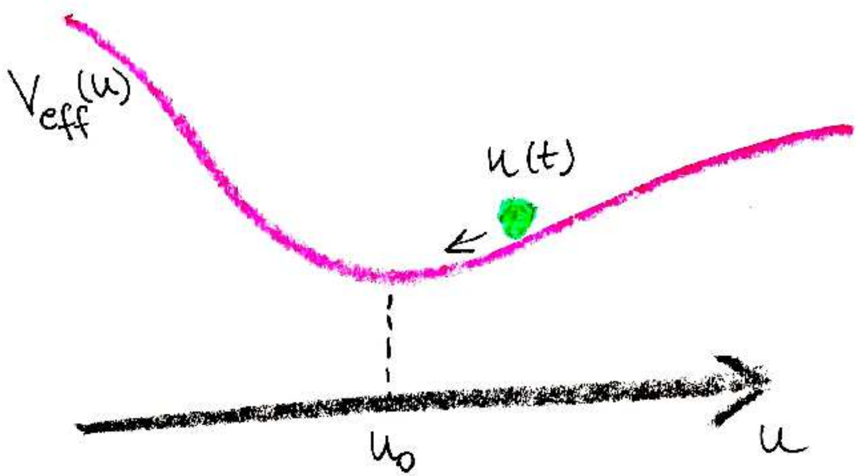
All is needed is bulk scale factor $A(u)$ plus trajectory $u(\tau)$.

Con: Not generically applicable (probe condition may fail)

Pro: $u(\tau)$ exactly solvable after $A(u)$ is given

Recovering self-tuning

- Brane trajectory $u(t)$ described by a classical Lagrangian system with "energy" E an integral of the motion.
- Non-relativistic limit $\dot{u} \ll e^{2A}$ $\Rightarrow$ Point particle in a potential



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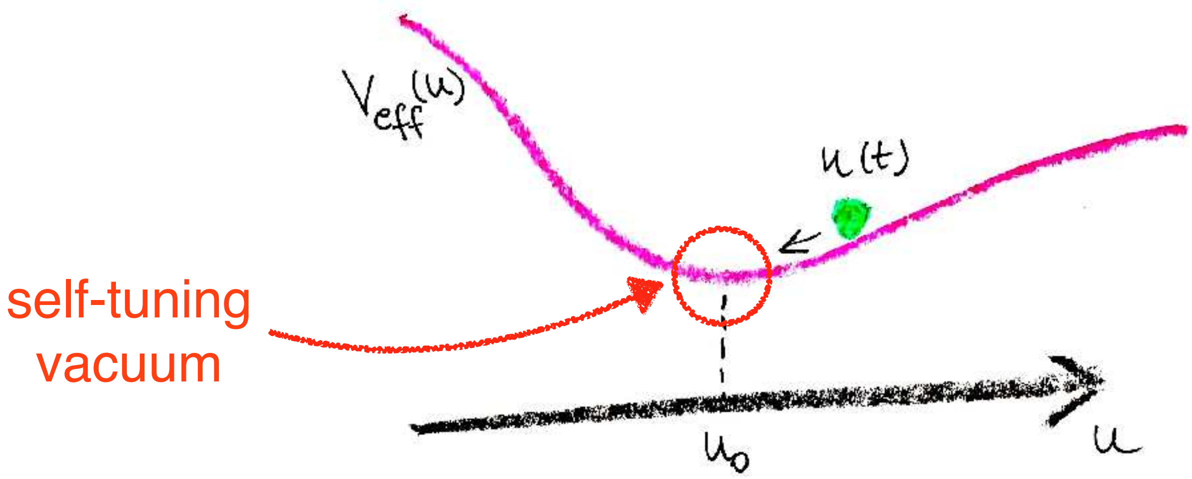

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UV regime

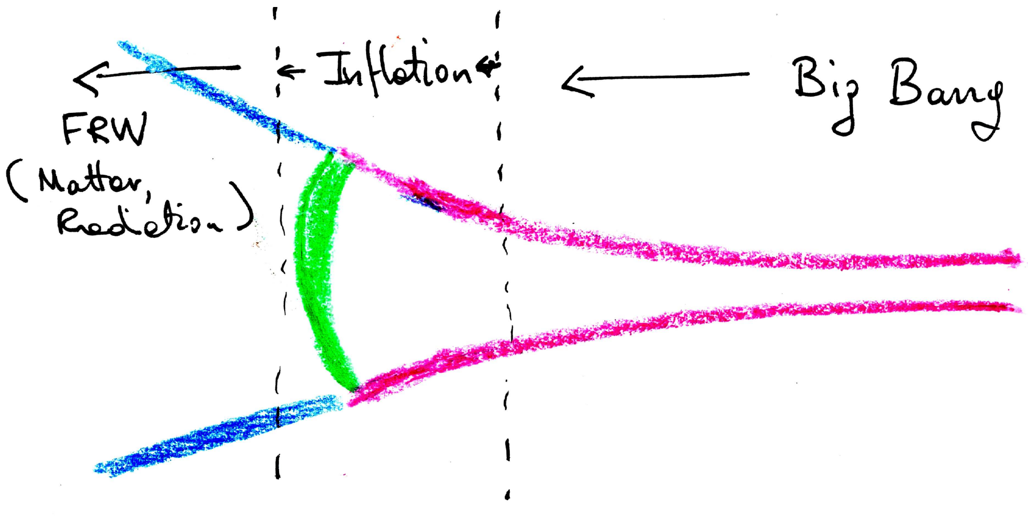
- Scalar approaching UV fixed point at $\varphi = 0$:

$$\varphi \simeq 0 \quad W, W_B, U_B, Z_B \;\rightarrow\; \text{constants}.$$

$$u(\tau) \simeq \tau \ell H_{eff} \quad \Rightarrow \quad a(\tau) \simeq \exp\left[-\tau H_{eff}\right]$$

- Solution approaches a de Sitter brane with $H_{eff} = \sqrt{\frac{W_B}{U_B}}\Big|_{\varphi=0}$.
- Same H as one would get from the 4d induced action alone

Intermediate inflation period



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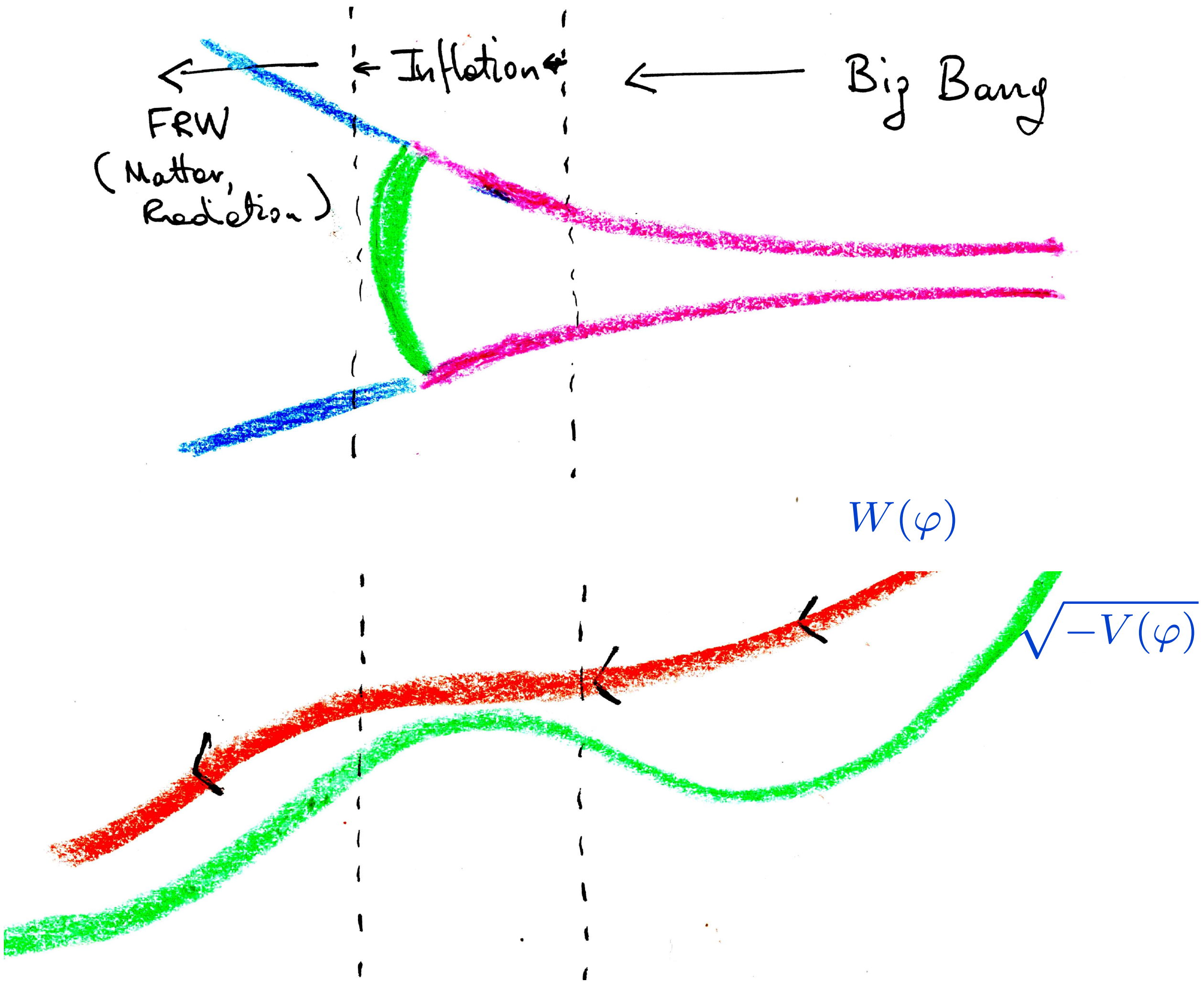
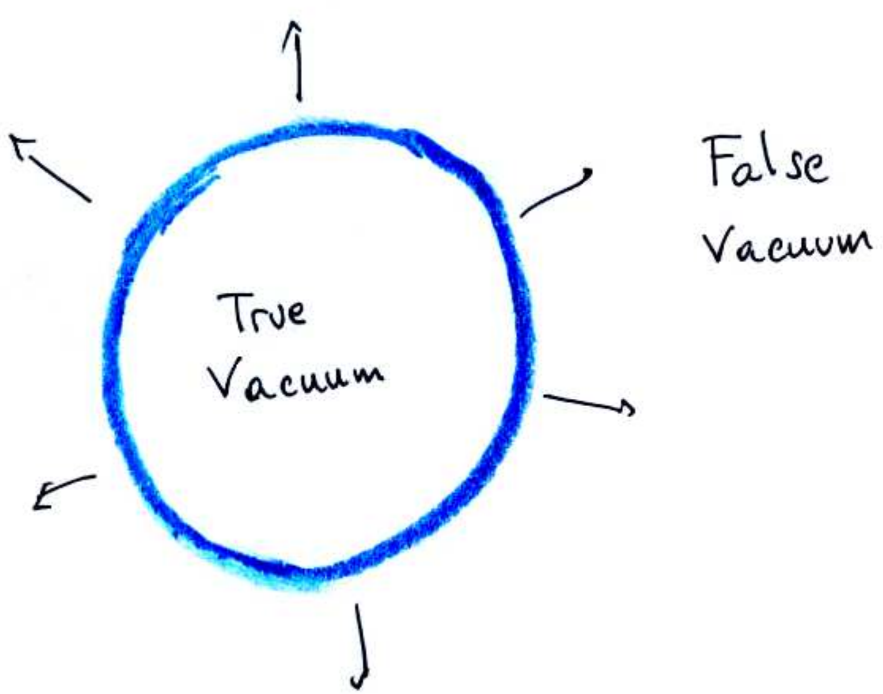


A period of inflation can be realized around intermediate extrema of the bulk potential.

Bubble-wall de Sitter

Related ideas by Danielsson *et al.* '18 -'19:
$dS_4 =$ wall on a vacuum bubble in $AdS_5 \to AdS_5$ vacuum decay.

- Vacuum decay by brane nucleation (infinitely thin, cannot be realized with scalars and a potential).
- Spatial sections are spheres.
- Universe starts *big*



Conclusion and outook

- Alternative realizations of dS which are not *vacua*
  - External sources
  - Excited state
- Can we realize any of this from top-down?
- Some constraints evaporate
  - No finite volume;
  - No worries about constant vacuum energy term;
  - Can one get scales right?

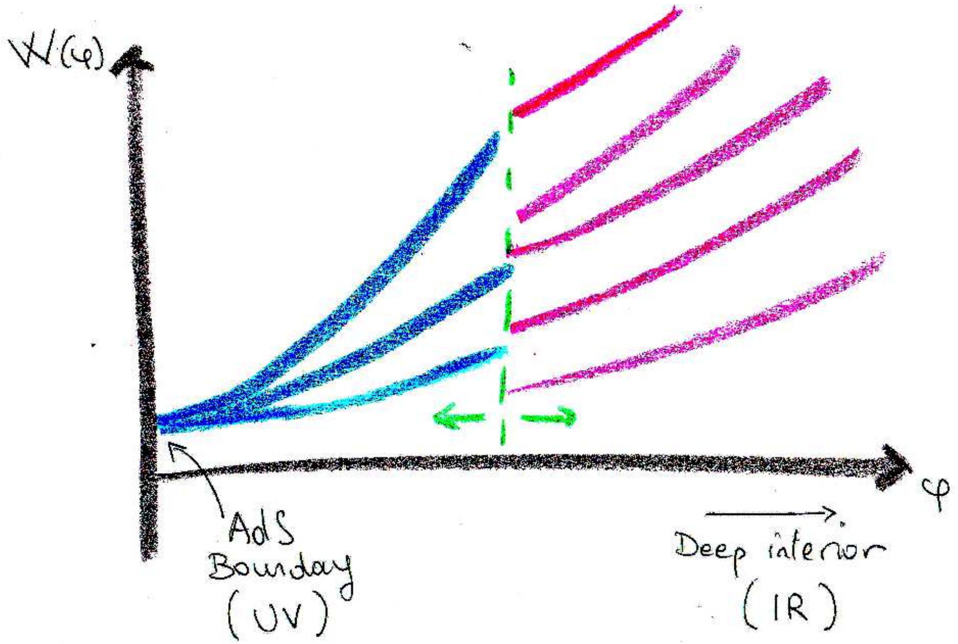
Even if you don’t like bottom-up,

have some courtesy, have some sympathy, and some taste

Self-tuning

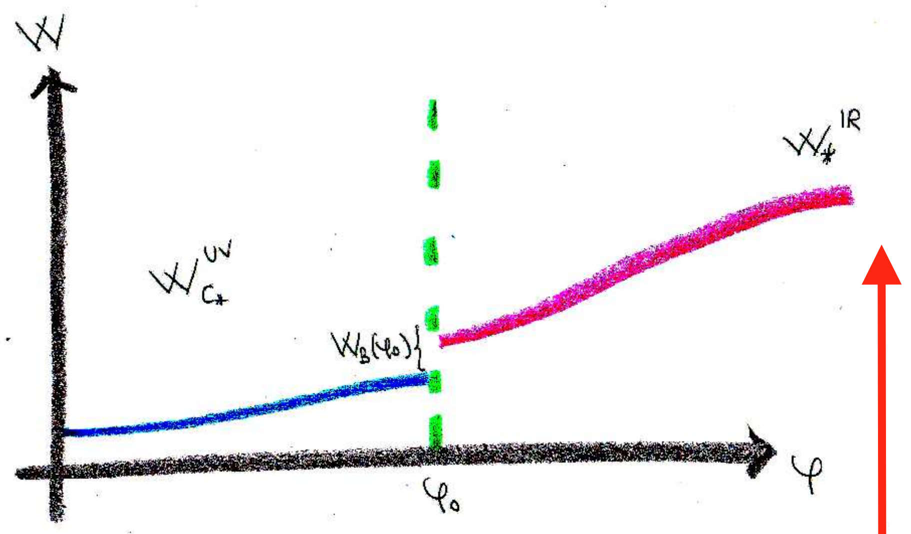
$$-\frac{1}{3}W^2 + \frac{1}{2}\left(W'\right)^2 = V$$

$$\left[W^{UV} - W^{IR}\right]_{\varphi_0} = W_B(\varphi_0), \qquad \left[\frac{dW^{UV}}{d\varphi} - \frac{dW^{IR}}{d\varphi}\right]_{\varphi_0} = \frac{dW_B}{d\varphi}(\varphi_0)$$



one-parameter family of solutions on each side.

Equilibrium solution



- Regularity fixes the IR solution
- Israel's junction conditions fix both UV solution and the brane position.
- For generic brane vacuum energy $\sim \Lambda^4$, UV geometry and brane position adjust so that the brane is flat and the UV glues to the regular IR (*self-tuning*).

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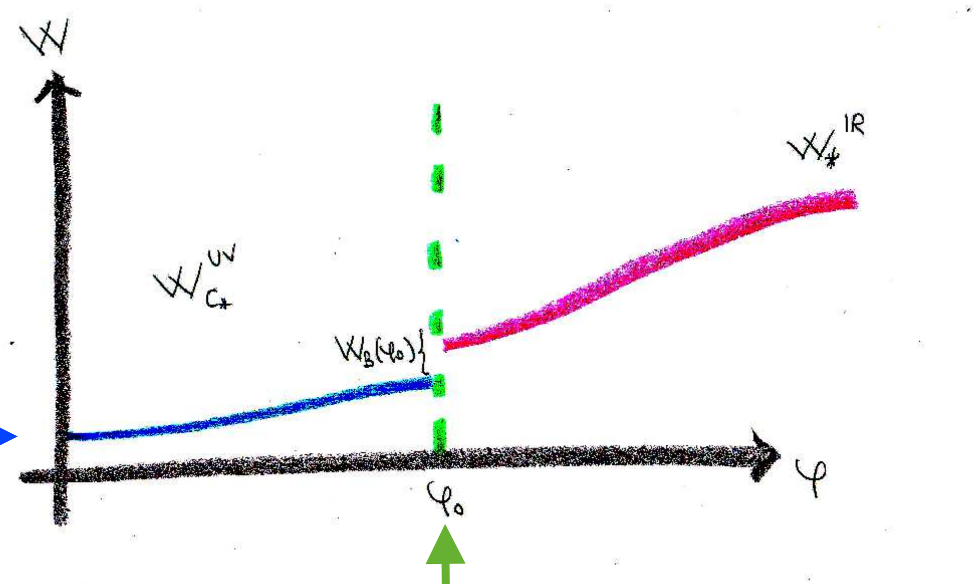

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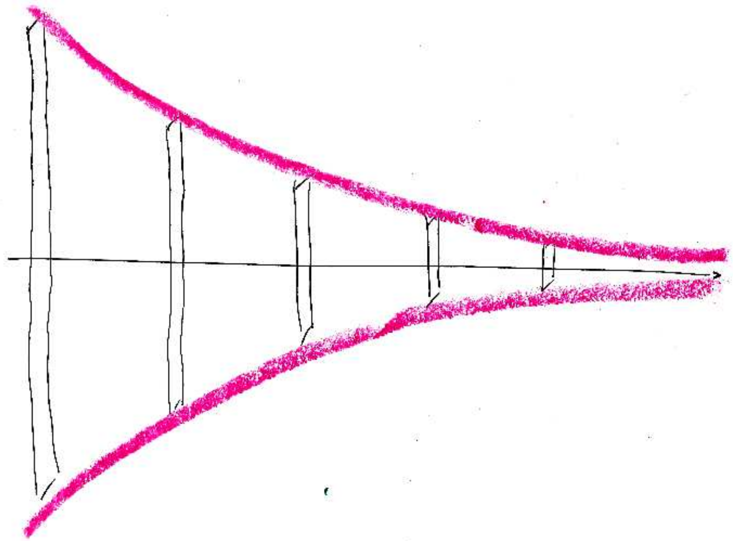
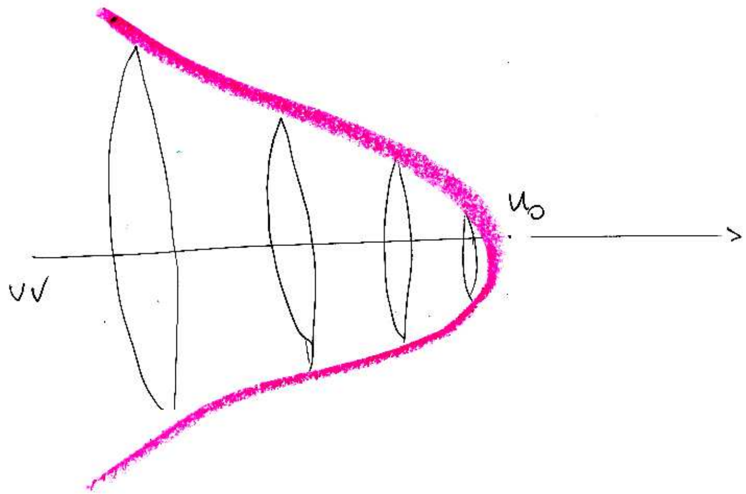
RG flows on $(d)S_4$

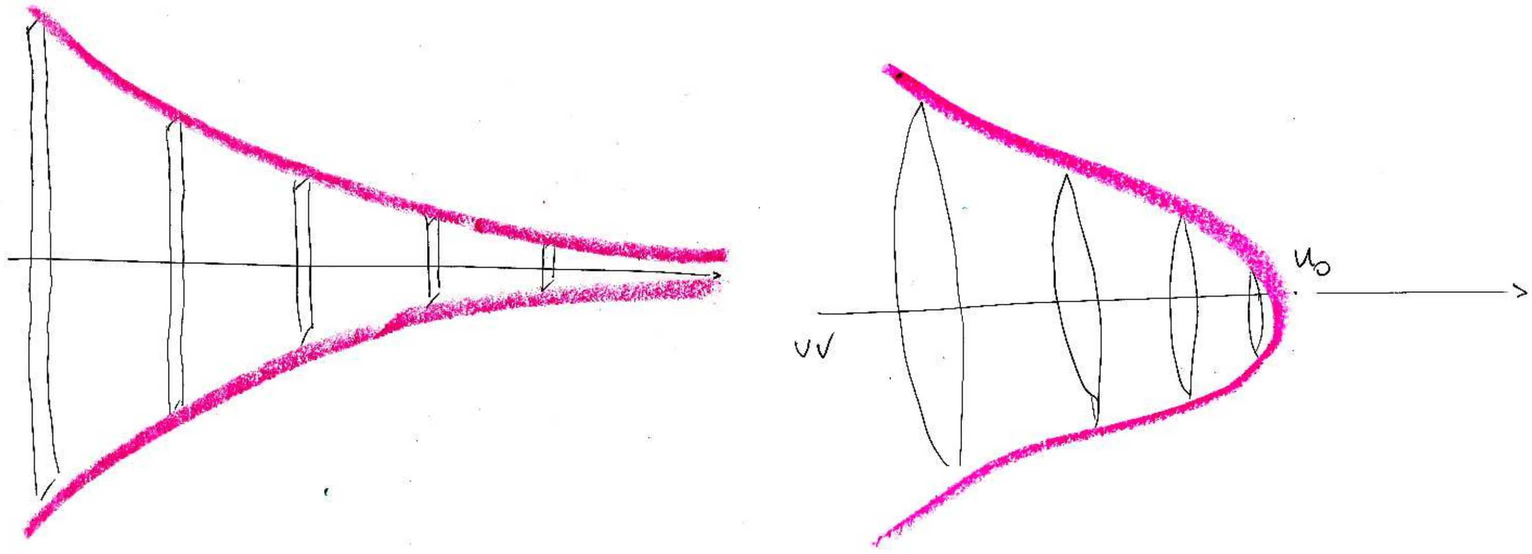


$R = 0$



$R > 0$

RG flows on $(d)S_4$



$R = 0$ $R > 0$

- Curvature effect subleading in the UV, but dominates in the IR
- $R \neq 0$: spacetimes ends at finite u_0, with $e^{A(u)} \sim (u_0 - u)$
- Scalar field does not reach IR fixed-point: $\varphi(u_0)$ determined by $\mathcal{R}$ through IR regularity

First order formalims

Introduce 3 superpotentials $W(\varphi), S(\varphi), T(\varphi)$

$$W = -2(d-1)\dot{A}, \quad S = \dot{\varphi}, \quad T = e^{-2A} R^{uv}$$

Einstein equations:

$$S^2 - SW' = -\frac{2}{d}T, \quad \frac{d}{2(d-1)}W^2 - S^2 - 2T = -2V,$$

$$SS' - \frac{d}{2(d-1)}SW = -V'$$

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- System is second order: Two integration constants $C, \mathcal{R}$
- $R = 0, \Rightarrow S = W', T = 0$, back to flat superpotential eq.

UV Expansion

Close to UV maximum of V expand solution in powers of φ:

$$W_-(\varphi) = 2(d-1) + \frac{\Delta_-}{2}\varphi^2 + \ldots + \frac{\mathcal{R}}{d}\varphi^{2/\Delta_-} + \ldots + C\varphi^{d/\Delta_-} + \ldots \qquad \varphi \to 0$$

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Integrating:

$$\varphi \simeq \varphi_- e^{\Delta_- u} + \ldots, \qquad A(u) = -u + \ldots + \frac{\mathcal{R}\varphi_-^{2/\Delta_-}}{4d(d-1)} e^{2u} + \ldots \qquad u \to -\infty$$

$$\mathcal{R}\varphi_-^{2/\Delta_-} = R^{uv}$$

Given $\mathcal{R}$ and C: 1-one parameter family of solutions parametrized by either φ_- or R^{uv}.

Geometry in the IR

- Original IR fixed point φ_* unreachable: $W \to +\infty$ at $\varphi_0 < \varphi_*$. Solution ends here.

$$\varphi \to \varphi_0 : \qquad W \simeq \frac{W_0}{\sqrt{\varphi_0 - \varphi}}$$

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- Pick endpoint φ_0: imposing regularity fixes $W_0 = cnst\sqrt{-V'(\varphi_0)}$ such that space ends regularly:

$$ds^2 \simeq du^2 + (u_0 - u)^2 d\Omega_4^2 \qquad _{u \to u_0}$$

- No regular solution arriving from "wrong side" of V

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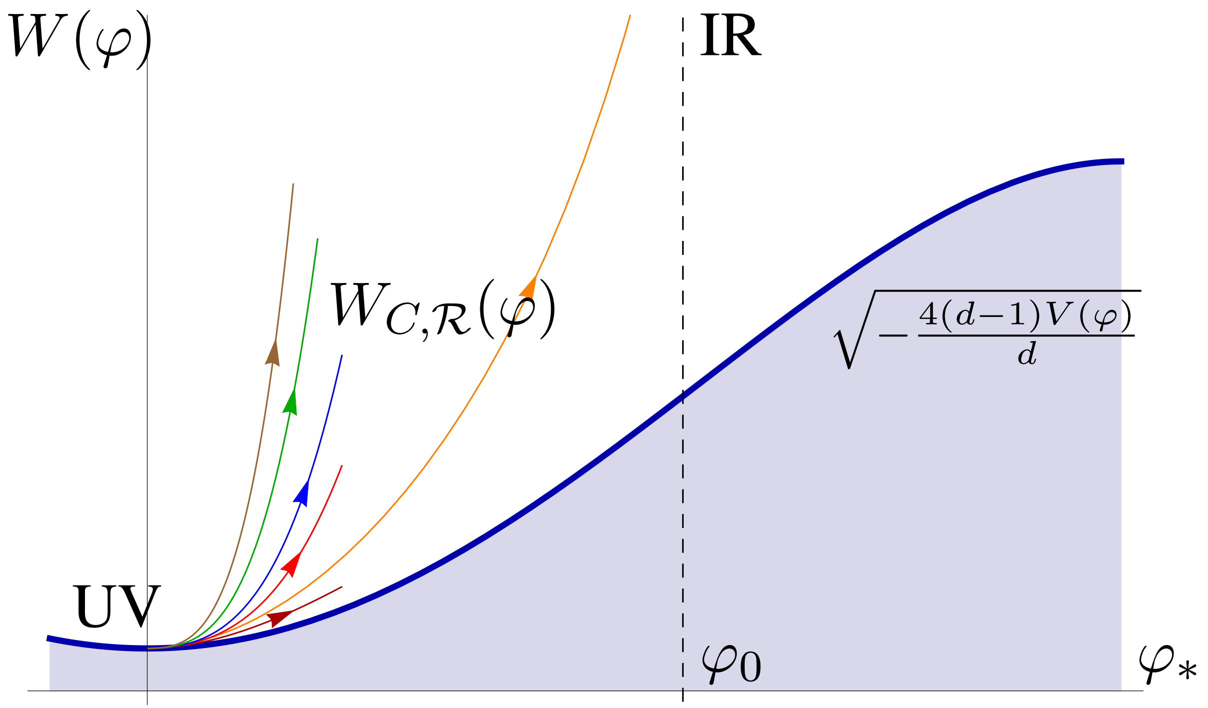
$$ds^2 \simeq du^2 + (u_0 - u)^2 d\Omega_4^2 \qquad u \to u_0$$

- No regular solution arriving from "wrong side" of V
- Choice of φ_0 fixes $\mathcal{R}, C$
- Equivalent: choose $\mathcal{R}$ as independent, determine endpoint.

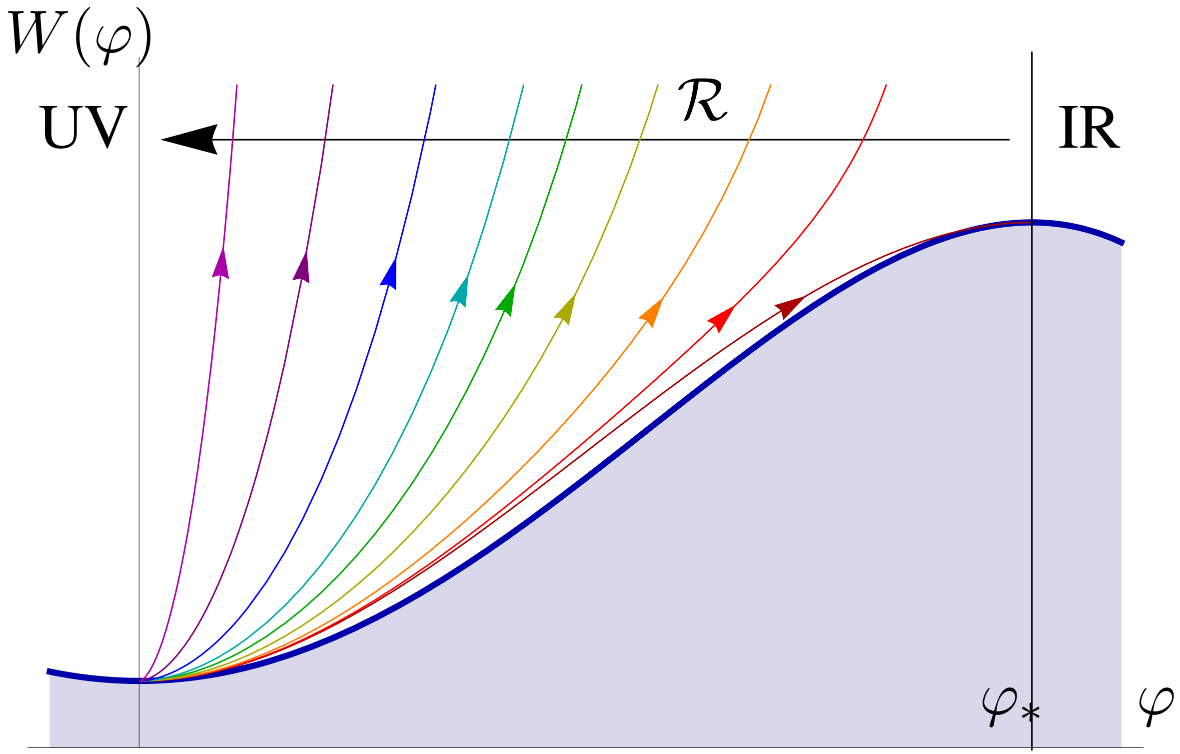
Regular Curved RG flows



One-parameter family of regular solutions parametrized by $\mathcal{R}$

$$C = C(\mathcal{R}), \quad \varphi_0 = \varphi_0(\mathcal{R}) \qquad < O >= cnst\ C(\mathcal{R})\varphi_-^{\Delta_+/\Delta_-}$$

UV and IR limits



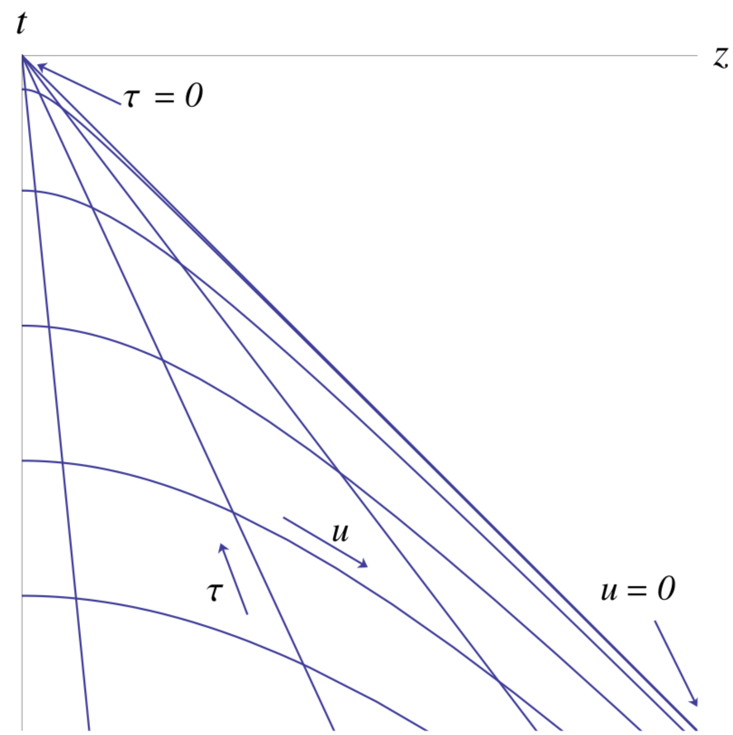
$\mathcal{R} \to 0 \qquad \varphi_0 \to \varphi_* \quad$ (IR limit)

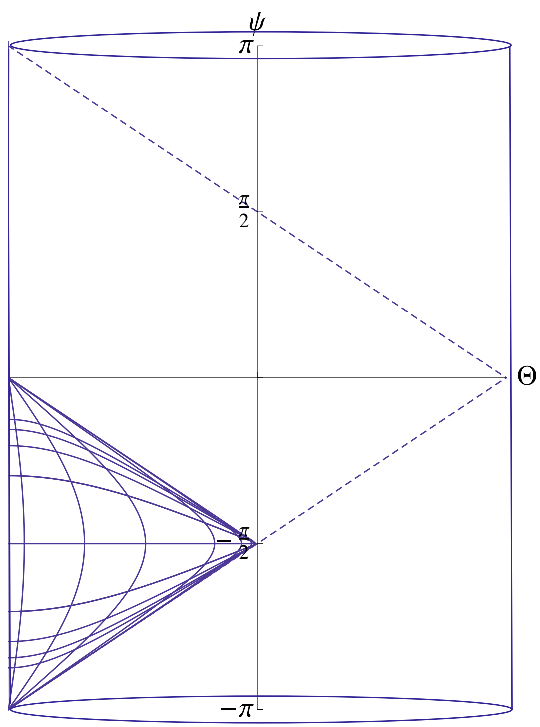
$\mathcal{R} \to +\infty \qquad \varphi_0 \to 0 \quad$ (UV limit)

Fixed-point solution

UV limit: solutions approaches $(d)S_d$ slicing of $(E)AdS_{d+1}$.

$$ds^2 = du^2 + \sinh^2(u_0 - u)d\Omega_4^2 \qquad R^{uv} = 4d(d-1)e^{-2u_0}$$





dS_d cosmological patch covers 1/4 Poincarè patch of AdS_{d+1}