Constraints on dS from Higher Dimensions

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De Sitter Constructions in String Theory
IPhT, CEA/Saclay

Outline

- Big Picture
- Why 10-d?
- Some Constraints on dS from Higher Dimensions
- Some Examples of dS

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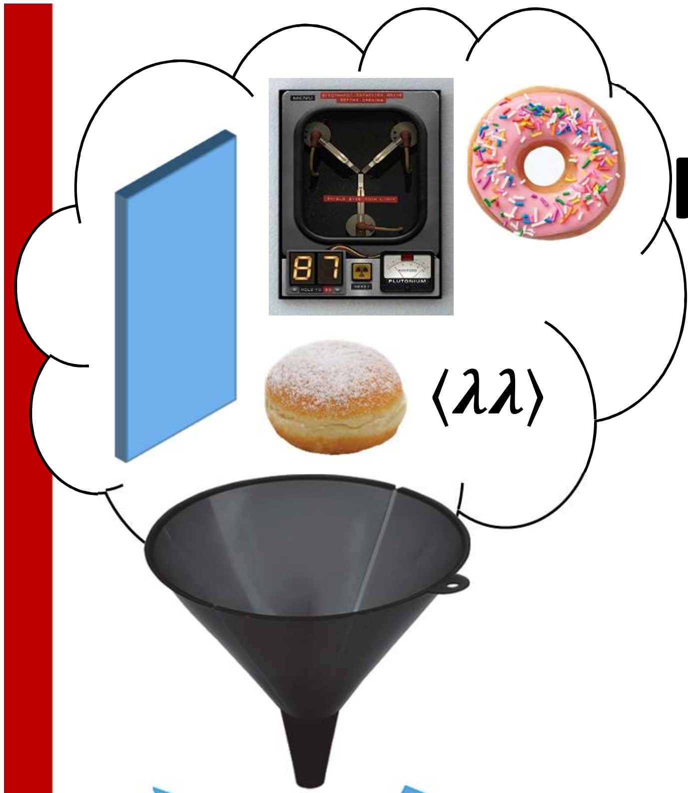
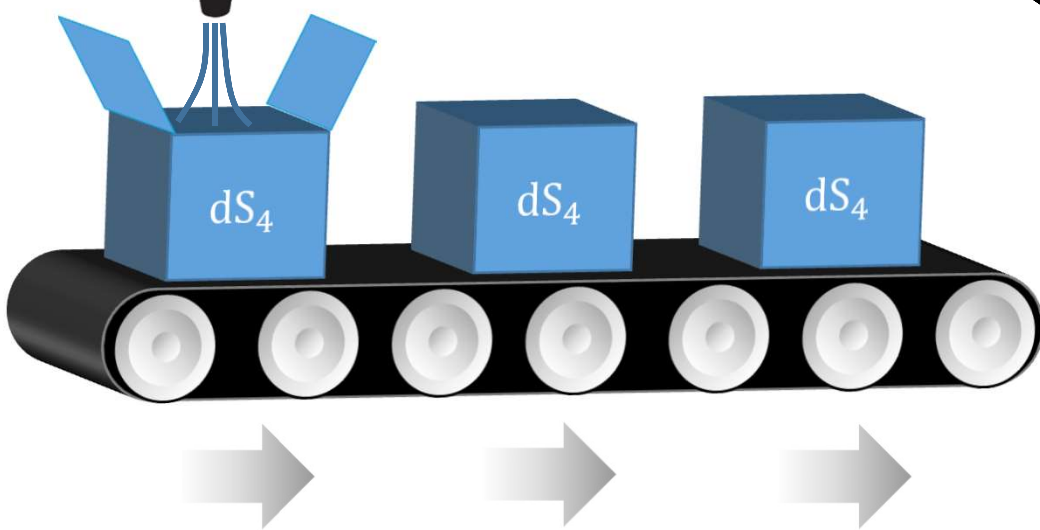
The dS Factory?





Checklist for dS:

- ❑ Backreaction of ingredients
- ❑ Stability
- ❑ Viable Range of Parameters
- ❑ Correct set of moduli
- ❑ **Higher Dimensional Consistency**

This Talk

Quality Control

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Why 10-d?

- "10-d Einstein Equations are identical to the 4-d effective potential minimization conditions Why not just work with 4-d effective potential?"

$$\begin{cases} V\big|_{\min} = \Lambda \\ \partial_{\rho_i} V\big|_{\min} = 0 \end{cases}$$

   - Backreaction: 4-d EFT can often hide backreaction effects.

A short selection…

Bena, Grana, Halmagyi, '09; Dymarsky, '11; Bena, Giecold, Grana, Halmagyi, Massai, '11;
Bena, Grana, Kuperstein, Massai, '14; Michel, Mintun, Polchinski, Puhm, Saad, '14;
Cohen-Maldonado, Diaz, Van Riet, Vercnocke, '15; Armas, Nguyen, Niarchos, Obers, Van Riet, '18;

Moritz, Retolaza, Westphal, '17, '19
Hamada, Hebecker, Shiu, Soler, '18
Gautason, Van Hemelryck, Van Riet, '18
Bena, Dudas, Graña, Lüst, '18
Carta, Moritz, Westphal, '19; Gautason, Van Hemelryck, Van Riet, 19; Hamada, Hebecker, Shiu, Soler, '19
Bena, Graña, Kovensky, Retolaza, '19
Bena, Buchel, Lüst, '19

+ everyone at this conference…?

Why 10-d?

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- Backreaction: 4-d EFT can often hide backreaction effects.

- Moduli:
  → Need to be sure 4d degrees of freedom are correct.
  Ex: Volume Modulus in Warped (GKP) Backgrounds

$$ds_{10}^2 = e^{2A_0(y)}\,\hat{\eta}_{\mu\nu}\,dx^\mu dx^\nu + e^{-2A_0(y)}\,\tilde{g}_{mn}\,dy^m dy^n$$

GKP, '01

10-d EOM: $G_{\mu m} - \kappa_{10}^2\,T_{\mu m} = -2\,(\partial_\mu L)\,(\partial_m e^{-4A_0}) = 0 \;\Rightarrow$ Either: $\begin{cases} \text{modulus is trivial} & \partial_\mu L = 0 \\ \text{warp factor is trivial} & \partial_m e^{-4A_0} = 0 \end{cases}$

Correct form for the volume modulus is $c(x) \sim L^4$:

$$ds_{10}^2 = e^{2\Omega(x)}\left[e^{-4A_0(y)} + c(x)\right]^{-1/2}\hat{\eta}_{\mu\nu}\,dx^\mu dx^\nu + \left[e^{-4A_0(y)} + c(x)\right]^{1/2}\tilde{g}_{mn}\,dy^m dy^n$$
$$-\,2\left[e^{-4A_0(y)} + c(x)\right]^{-1/2} e^{2\Omega}\,(\partial_\mu c)\,(\partial_m K(y))\,dx^\mu dy^m$$

$$e^{-2\Omega(x)} = c(x) + \frac{1}{\tilde{V}}\int\sqrt{\tilde{g}}\,e^{-4A_0}$$

Giddings, Maharana, '05
Frey, Torroba, Underwood, Douglas, '08
Koerber, Martucci, '07; Martucci, '09, '14

Why 10-d?

- "10-d Einstein Equations are identical to the 4-d effective potential minimization conditions Why not just work with 4-d effective potential?" $\begin{cases} V\big|_{\min} = \Lambda \\ \partial_{\rho_i} V\big|_{\min} = 0 \end{cases}$

  - Backreaction: 4-d EFT can often hide backreaction effects.

  - Moduli:
    → Need to be sure 4d degrees of freedom are correct.
    → 4-d EFT must know dependence on correct 4d degrees of freedom.

For the correct metric:

$$ds_{10}^2 = e^{2\Omega(x)}\left[e^{-4A_0(y)} + c(x)\right]^{-1/2} \hat{\eta}_{\mu\nu}\, dx^\mu dx^\nu + \left[e^{-4A_0(y)} + c(x)\right]^{1/2} \tilde{g}_{mn}\, dy^m dy^n$$
$$- 2\left[e^{-4A_0(y)} + c(x)\right]^{-1/2} e^{2\Omega} \left(\partial_\mu c\right) \left(\partial_m K(y)\right) dx^\mu dy^m$$

$$e^{-2\Omega(x)} = c(x) + \frac{1}{\tilde{V}} \int \sqrt{\tilde{g}}\, e^{-4A_0}$$

Giddings, Maharana, '05
Frey, Torroba, Underwood, Douglas, '08
Koerber, Martucci, '07; Martucci, '09, '14

$\overline{D3}$ tension$\sim 2\, T_3\, e^{4A}\, e^{4\Omega} = \frac{T_3}{\left(c + \frac{\tilde{V}_W}{\tilde{V}}\right)^2} \frac{1}{e^{-4A_0} + c}$

$$\text{reproduces} \sim \begin{cases} \dfrac{T_3}{c^3} & \text{for weak warping} \\[2ex] \dfrac{T_3 e^{-4A_0}}{c^2} & \text{for strong warping} \end{cases}$$

Why 10-d?

- "10-d Einstein Equations are identical to the 4-d effective potential minimization conditions Why not just work with 4-d effective potential?" $\begin{cases} V\Big|_{\min} = \Lambda \\ \partial_{\rho_i} V\Big|_{\min} = 0 \end{cases}$

  - Backreaction: 4-d EFT can often hide backreaction effects.
  - Moduli:
    → Need to be sure 4d degrees of freedom are correct.
    → 4-d EFT must know dependence on correct 4d degrees of freedom.
    → 10-d analysis insensitive to precise moduli dependence → *"On-shell"*

Gautason, Van Hemelryck, Van Riet, '18

Dimensional Reduction of Action

$$S_{10} = \int \sqrt{g_{10}} \left(R_{10} - \frac{1}{2} F_p^2 \right)$$

$$V_{eff}(\sigma_i) \sim +\left|F_p\right|^2$$

"off-shell":
good if you know full moduli dependence
could be misleading if you don't!

10-d Trace-Reversed

$$R_{MN} = T_{MN} - \frac{1}{8} g_{MN} T_L^L$$

$$V_{eff}\Big|_{\min}$$

"on-shell":
gives value of cc at minimum
can't tell you about stability
doesn't need moduli dependence

Why 10-d?

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  - Model-building:
    10-d analysis can point towards new sources & mechanisms to getting dS.

    → Negative Curvature
    Douglas, Kallosh, '10

    → Gaugino Condensation (4-fermion)
    Hamada, Hebecker, Shiu, Soler, '18; Kallosh, '19 Carta, Moritz, Westphal, '19;
    Gautason, Van Hemelryck, Van Riet, 19;
    Hamada, Hebecker, Shiu, Soler, '19

    → Quantum Corrections
    Dasgupta, Gwyn, McDonough, Mia, Tatar, '14
    Dasgupta, Emelin, Faruk, Tatar, '18, '19 x 2,

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Some Constraints on dS

- Gibbons/Maldacena-Nunez (GMN)

Gibbons ‘84
Malacena, Nunez ‘00

$$ds_D^2 = \boldsymbol{\Omega^2(y)}\left(\hat{g}_{\mu\nu}\, dx^\mu dx^\nu + \tilde{g}_{mn}\, dy^m dy^n\right)$$

$$\hat{R}_4\,\tilde{\mathcal{V}} = \frac{1}{D-2}\int\sqrt{\tilde{g}}\;\tilde{\nabla}^2\Omega^{D-2} - \frac{1}{D-2}\int\sqrt{\tilde{g}}\;\Omega^D\;\tilde{T}^{\text{matt}}, \qquad \tilde{T}^{\text{matt}} = 4\,T_m^m - (D-6)\,T_\mu^\mu$$

Total derivative?

Takeaways

1. To get dS, matter must satisfy $\sim\int\Omega^D\,\tilde{T}^{\text{matt}} < 0 \;\;\Rightarrow\;\; \int\Omega^D\left[(D-6)T_\mu^\mu - 4\,T_m^m\right] > 0$

   $\int\Omega^D\left[T_\mu^\mu - T_m^m\right] > 0$

   <u>Op-planes</u>

   $(T^{Op})_\mu^\mu - (T^{Op})_m^m \sim |\mathrm{T_{Op}}| > 0$ <u>O3-planes can evade this constraint.</u> (GKP has O3)

2. Negative Curvature needed?

   $$\hat{R}_4\tilde{\mathcal{V}} = -2\tilde{R}_p - \hat{T}_4 + \text{warping}$$

   Douglas, Kallosh ‘10

<u>Advantages</u>:
- Not specific to a background
- Simple!

<u>Disadvantages</u>:
- Assumes Einstein Equations
- Integrated Constraint: no singularities or boundaries in internal dims

Some Constraints on dS

- GKP Tadpole Constraint

GKP, '01
de Alwis, '03

$$ds_D^2 = e^{2A(y)}\,\hat{g}_{\mu\nu}(x)\,dx^\mu dx^\nu + e^{-2\,A(y)}\tilde{g}_{mn}\,dy^m dy^n$$

$$\hat{R}_4 = -\int \sqrt{\tilde{g}}\left[\frac{e^{2A}}{\mathrm{Im}\,\tau}|G_3^-|^2 + e^{-6}\;|d\Phi_-|^2 + \frac{e^{2A}}{2\pi}\,\Delta\right], \qquad \Delta = \frac{1}{4}\left(T_m^m - T_\mu^\mu\; - 4\mu_3\rho_3\right)$$

$$G_3^- = G_3 - i\,\tilde{\star}\;G_3, \qquad \Phi_- = e^{4A} - \alpha$$

Takeaways

1. To get dS, matter must satisfy $\int e^{2A}\left[T_\mu^\mu - \;T_m^m + 4\,\mu_3\rho_3\right] > 0$

(Compare to (GMN) $\int \Omega^D\left[T_\mu^\mu\; - T_m^m\right] > 0$)

O3-planes no longer evade this constraint.

Advantages:

- Also uses $\tilde{F}_5$ Bianchi Identity
- Simple constraint on matter

Disadvantages:

- Assumes EEs & IIB SUGRA
- Integrated Constraint: no singularities or boundaries in internal dims

Some Constraints on dS

- Raychaudhuri Null Energy Condition

Das, Haque, Underwood, '19

$$ds_D^2 = \Omega^2(z)\,(-dt^2 + a^2(t)\,d\vec{x}^2 + \bar{g}_{mn}\,dz^m dz^n)$$
$$= \Omega^2(\chi, y^m)\,(-dt^2 + a^2(t)\,d\vec{x}^2 + d\chi^2 + f^2(\chi)\,\tilde{g}_{mn}\,dy^m dy^n)$$

Similar to "warped throat" form

$$ds^2 = e^{2A}\,(-dt^2 + a^2(t)\,d\vec{x}^2) + e^{-2}\quad(dr^2 + f^2(r)\,\tilde{g}_{mn} dy^m dy^n)$$

(We will relax this structure later)

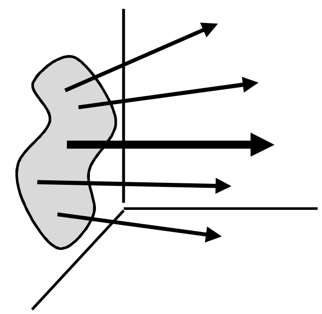
Some Constraints on dS

- Raychaudhuri Null Energy Condition

Das, Haque, Underwood, '19

$$ds_D^2 = \Omega^2(z)\,(-dt^2 + a^2(t)\,d\vec{x}^2 + \bar{g}_{mn}\,dz^m dz^n)$$
$$= \Omega^2(\chi, y^m)\,(-dt^2 + a^2(t)\,d\vec{x}^2 + d\chi^2 + f^2(\chi)\,\tilde{g}_{mn}\,dy^m dy^n)$$

Null, affine vector: $N^M = \Omega^{-2}\underset{t,\ \vec{x},\ \chi,\ \vec{y}}{\left(1, \vec{0}, 1, \vec{0}\right)}$



Expansion θ:
Rate of change of area
perpendicular to bundle

<u>Raychaudhuri equation for null vectors N^M</u>

$$\frac{d\theta}{d\lambda} = -\frac{1}{D-2}\,\theta^2 - \sigma_{MN}\sigma^{MN} - R_{MN}N^M N^N$$

Expansion $\theta = \frac{1}{\sqrt{-g_D}}\,\partial_A\left(\sqrt{-g_D}\,N^A\right) = \Omega^{-2}\left[3\,\mathrm{H} + \partial_\chi \log\left(\Omega^{D-2} f^{n-1}(\chi)\right)\right]$

Shear Tensor $\sigma_{MN} = \frac{1}{2}\,(\nabla_M N_N + \nabla_N N_M) - \frac{1}{D-2}\,\hat{h}_{MN}\,\theta$

Transverse metric

$$3\left(\dot{H} + H^2\right) = \bar{R}_{\chi\chi} + (D-2)\left[\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega\right] - \Omega^4\,R_{MN}\,N^M N^N$$

Let's examine the consequences of this expression.

Some Constraints on dS

- Raychaudhuri Null Energy Condition

Das, Haque, Underwood, '19

$$ds_D^2 = \Omega^2(\chi, y^m)\,(-dt^2 + a^2(t)\,d\vec{x}^2 + \underbrace{d\chi^2 + f^2(\chi)\,\tilde{g}_{mn}\,dy^m dy^n}_{\bar{g}_{mn}})$$

Let's examine the consequences of this expression:

$$\underbrace{3\,(\cancel{\dot{H}} + H^2)}_{\text{Must be }(+)\text{ for dS}} = \underbrace{\bar{R}_{\chi\chi} + (D-2)\left[\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega\right] - \Omega^4\,R_{MN}\,N^M N^N}_{\text{Must also be }(+)\text{ for dS}}$$

Takeaways

- $\dot{H} = 0$ for dS: LHS is positive $\Rightarrow$ RHS is positive

Some Constraints on dS

- Raychaudhuri Null Energy Condition

Das, Haque, Underwood, '19

$$ds_D^2 = \Omega^2(\chi, y^m)\,(-dt^2 + a^2(t)\,d\vec{x}^2 + \underbrace{d\chi^2 + f^2(\chi)\,\tilde{g}_{mn}\,dy^m dy^n}_{\bar{g}_{mn}})$$

Let's examine the consequences of this expression:

$$3\left(\cancel{\dot{H}} + H^2\right) = \bar{R}_{\chi\chi} + (D-2)\left[\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega\right] - \Omega^4\, R_{MN}\, N^M N^N$$

Negative curvature doesn't help

Trivial Warping?

Null Energy Condition
$R_{MN}\, N^M N^N = T_{MN} N^M N^N \geq 0$

Takeaways

- $\dot{H} = 0$ for dS: LHS is positive $\Rightarrow$ RHS is positive
- **No dS if**
  - Curvature is non-positive $\bar{R}_{\chi\chi} \leq 0$
  - Warping $\Omega \sim \text{const}$
  - Null Convergence Condition/NEC satisfied $R_{MN}N^M N^N \geq 0$

Must violate **at least** one of these conditions to get dS

Some Constraints on dS

- Raychaudhuri Null Energy Condition

Das, Haque, Underwood, '19

$$ds_D^2 = \Omega^2(\chi, y^m)\,(-dt^2 + a^2(t)\,d\vec{x}^2 + \underbrace{d\chi^2 + f^2(\chi)\,\tilde{g}_{mn}\,dy^m dy^n}_{\bar{g}_{mn}})$$

Let's examine the consequences of this expression:

$$\underbrace{3\,(\cancel{\dot{H}} + H^2)}_{\text{Independent of } \{\chi, y^m\}} = \underbrace{\bar{R}_{\chi\chi} + (D-2)\left[\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega\right] - \Omega^4\, R_{MN}\, N^M N^N}_{\text{Must also be independent of } \{\chi, y^m\}}$$

Takeaways

- $\dot{H} = 0$ for dS: LHS is positive $\Rightarrow$ RHS is positive
- **No dS if**
  - Curvature is non-positive $\bar{R}_{\chi\chi} \leq 0$
  - Warping $\Omega \sim \text{const}$
  - Null Convergence Condition/NEC satisfied $R_{MN}N^M N^N \geq 0$

  Must violate **at least** one of these conditions to get dS
- LHS is independent of $\{\chi, y^m\}$, so RHS must be independent of $\{\chi, y^m\}$

  → If source of dS is NEC violation, it must violate NEC homogeneously pointwise throughout the extra dimensions.

Douglas, Kallosh, '10

Some Constraints on dS

- Raychaudhuri Null Energy Condition - Generalized   Das, Haque, Underwood, '19

$$ds_D^2 = \Omega^2(\chi, y^m)\,(-dt^2 + a^2(t)\,d\vec{x}^2 + \tilde{g}_{mn}\,dy^m dy^n)$$

Null, affine vector: $N^M = \Omega^{-2}(\underset{t,}{1}, \underset{\vec{x},}{\vec{0}}, \underset{\vec{y}}{\tilde{n}^m})$ $\tilde{n}^m$ spacelike affine unit vector

Pick your favorite direction!

$$3\,(\cancel{\dot{H}} + H^2) = \tilde{R}_{mn}\tilde{n}^m\tilde{n}^n + (D-2)\,\tilde{n}^m\tilde{n}^n\big[\partial_n \log\Omega\,\partial_m \log\Omega - \tilde{\nabla}_m\partial_n \log\Omega\big] - \Omega^4\,R_{MN}\,N^M N^N$$

$$3\,(\dot{H} + H^2) = \bar{R}_{\chi\chi} + (D-2)\left[(\partial_\chi \log\Omega)^2 - \partial_\chi^2 \log\Omega\right] - \Omega^4\,R_{MN}\,N^M N^N$$

Previous Case

Takeaways

- $\dot{H} = 0$ for dS: LHS is positive $\Rightarrow$ RHS is positive
- **No dS if**
  - Curvature is non-positive $\bar{R}_{\chi\chi} \le 0$
  - Warping $\Omega \sim \mathrm{const}$
  - Null Convergence Condition/NEC satisfied $R_{MN}N^M N^N \ge 0$

  Must violate **at least** one of these conditions to get dS
- LHS is independent of $\{\chi, y^m\}$, so RHS must be independent of $\{\chi, y^m\}$

  If source of dS is NEC violation, it must violate NEC homogeneously pointwise throughout the extra dimensions.   Douglas, Kallosh, '10

Some Constraints on dS

- Raychaudhuri Null Energy Condition - Generalized   Das, Haque, Underwood, '19

$$ds_D^2 = \Omega^2(\chi, y^m)\left(-dt^2 + a^2(t)\, d\vec{x}^2 + \tilde{g}_{mn}\, dy^m dy^n\right)$$

Null, affine vector: $$N^M = \Omega^{-2}\underset{t,\vec{x},\vec{y}}{\left(1, \vec{0}, \tilde{n}^m\right)}$$ $\tilde{n}^m$ spacelike affine unit vector

Pick your favorite direction!

$$3\left(\cancel{\dot{H}} + H^2\right) = \tilde{R}_{mn}\tilde{n}^m\tilde{n}^n + (D-2)\,\tilde{n}^m\tilde{n}^n\left[\partial_n \log\Omega\, \partial_m \log\Omega - \tilde{\nabla}_m \partial_n \log\Omega\right] - \Omega^4\, R_{MN}\, N^M N^N$$

Advantages:

- Not specific to a particular background
- Does not assume Einstein Equations
- **Multiple conditions: one for each choice of direction $\tilde{n}^m$ in extra dimensions**

Disadvantages:

- Not so simple anymore?

Some Constraints on dS

- Comparing to GMN & GKP Energy Conditions

Does this source evade the dS constraint?	GMN $\tilde{T} \sim T^m_m - T^\mu_\mu < 0?$	GKP $\Delta \sim T^m_m - T^\mu_\mu - 4\mu_3\rho_3 < 0?$	Raychaud. NEC $T_{MN}N^M N^N < 0?$
p-form Fluxes $T^p_{MN} = 2\,p\,F_{Ma_2\ldots a_p}F_N^{\ a_2\ldots a_p} - g_{MN}\,F^2$	NO	NO	NO
Op-planes $T^{Op}_{\mu\nu} = \lvert T_{Op}\rvert\, g_{\mu\nu}\,\delta(\Sigma)$ $T^{Op}_{mn} = \lvert T_{Op}\rvert\, \Pi^\Sigma_{mn}\,\delta(\Sigma)$	YES	NO	NO (pointwise)
D-dim CC $T^\Lambda_{MN} = -\Lambda_\mathrm{D}\,\mathrm{g}_\mathrm{MN}$	YES $\tilde{T} \sim -8\,\Lambda_D$	YES $\Delta \sim -8\,\Lambda_D$	NO $T^\Lambda_{MN}N^M N^N = 0$

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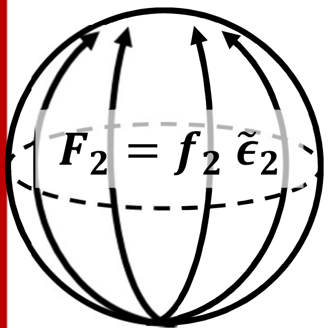
Some Examples of dS

Lesson:

- **Positive**, not negative, curvature is a way to get bulk solutions with dS_4

Let's examine some existing dS solutions through this lens…

- Freund-Rubin + Bulk CC



Ingredients:

- Positive curvature S^2
- 2-form flux along sphere
- 6-D CC

$$\overbrace{ds_6^2 = L^{-2}\,\hat{g}_{\mu\nu}\,dx^\mu dx^\nu}^{\mathrm{dS}_4} + \overbrace{L^2(d\chi^2 + \sin^2\chi\;d\phi^2)}^{S^2}$$

$$S_6 = \int d^6x\,\sqrt{-g_6}\left[M_6^4\,R_6 - \Lambda_6 - \frac{1}{4}\,F_2^2\right]$$

$$3H^2 = \underbrace{\bar{R}_{\chi\chi}}_{\text{Positive curvature}} + \underbrace{(D-2)\left[\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega\right]}_{\text{Trivial Warping}} - \underbrace{\Omega^4\,T_{MN}\,N^M N^N}_{\text{Null Energy Condition}}$$

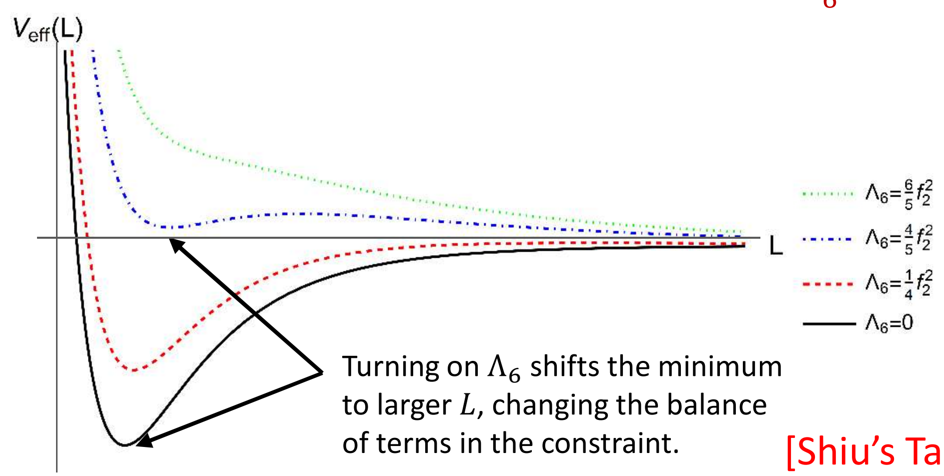
$$R_{MN}\,N^M N^N = T_{MN} N^M N^N = \frac{1}{2\,M_6^4}\frac{f_2^2}{L^6} \geq 0$$

$$3H^2 = \frac{1}{2}\frac{\tilde{R}_2}{L^4} - \frac{1}{2\,M_6^4}\frac{f_2^2}{L^6}$$

Puzzle:

We know that $\Lambda_6 = 0$ gives AdS.

But Λ_6 doesn't appear in the constraint equation. How does it "uplift" in this perspective?



[Shiu's Talk?]

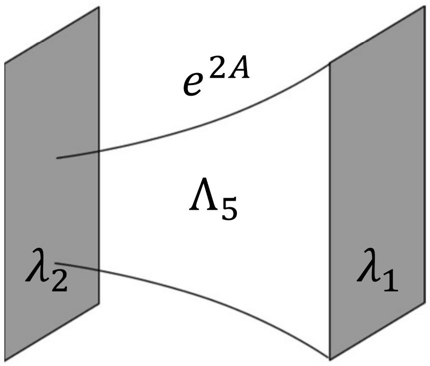
Some Examples of dS

Lesson:

- "Uplift" of bulk solutions to dS_4 with local objects (e.g. branes) requires **global backreaction** on warp factor**.**

Let's examine some existing dS solutions through this lens…

- dS in RS (Randall-Sundrum)



Ingredients:

- Warping
- Localized branes
- 5-D CC (negative)

$$ds^2_{RS} = e^{2A(z)}\hat{g}_{\mu\nu}\,dx^\mu dx^\nu + dz^2$$
$$= e^{2A(\chi)}\left(\hat{g}_{\mu\nu}\,dx^\mu dx^\nu + d\chi^2\right)$$

- λ_1, λ_2 localized, $T_{MN}N^M N^N \sim \delta(z)$
- Bulk CC saturates NEC $T^{\Lambda}_{MN}N^M N^N = 0$

$$3H^2 = \bar{R}_{\chi\chi} + (D-2)\left[\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega\right] - \Omega^4\, T_{MN}\, N^M N^N$$

Zero curvature

- RS warp factor, $e^{A(\chi)} = \chi/L$

Using RS warp factor, get 4d Minkowski!

$$\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega = 0 \quad\Rightarrow\quad 3\,H^2 = 0$$

"Tuned"

$$\lambda_1 = -\lambda_2 = L\, M_5\, |\Lambda_5|$$

- De-tuned warp factor, $e^{A(z)} = \sqrt{\Lambda_4}\, L \sinh\left(\frac{z_* - z}{L}\right)$

[Randall's Talk]

$$\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega = \Lambda_4 \quad\Rightarrow\quad 3\,H^2 = 3\,\Lambda_4$$

dS comes from warping!

"De-Tuned"

$$\lambda_1 = 6\frac{M_5^3}{L}\coth\frac{z_*}{L}$$
$$\lambda_2 = -6\frac{M_5^3}{L}\coth\frac{z_* - z_0}{L}$$

Kaloper, '99; DeWolfe et al, '99; Kim, Kim, '99; Tye, Wasserman, '00; Karch, Randall, '00

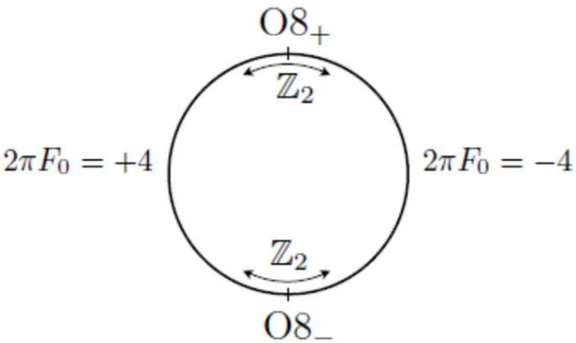
Some Examples of dS

Let's examine some existing dS solutions through this lens…

Lesson:

- "Uplift" of bulk solutions to dS_4 requires **global backreaction** on warp factor.

- "Classical" dS Solutions* in Massive IIA with $O8_{\pm}$



$$ds_{10}^2 = e^{2W(z)}\hat{g}_{\mu\nu}\, dx^\mu dx^\nu + e^{-2W(z)}\left(dz^2 + e^{2\lambda(z)}\, ds_{M_5}^2\right)$$

Ingredients:

- Warping
- Negative curvature M_5
- Localized branes $O8_{\pm}$
- Romans Mass F_0
- Dilaton ϕ

Cordova et al, '18
Cordova et al, '19

$N^M = \Omega^{-2}\left(1, \vec{0}, \tilde{n}^m\right)$ $\tilde{n}^m$ spacelike affine unit vector

$$3\, H^2 = \tilde{R}_{mn}\tilde{n}^m\tilde{n}^n + (D-2)\,\tilde{n}^m\tilde{n}^n\left[\partial_n \log\Omega\, \partial_m \log\Omega - \tilde{\nabla}_m\partial_n \log\Omega\right] - \Omega^4\, T_{MN}\, N^M N^N$$

Numerical Solutions (near $O8_+$ @ $z=0$):

$$e^{-4W} = c_1 + \frac{F_0}{\sqrt{c_2}}\, z - 2\, c_1^2\, \Lambda_4\, z^2 + \mathcal{O}(z^3)$$

Etc… for $\phi(z), \lambda(z)$

- Warp factor gains explicit dependence on Λ_4
- Dependence on Λ_4 not necessarily localized

*Still some uncertainty about boundary conditions of fields at location of $O8_-$
Cribiori, Junghans, '19

KKLT

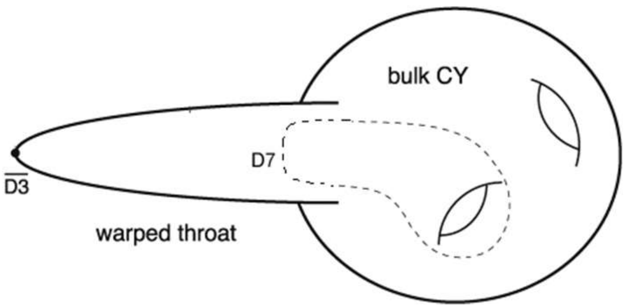


Figure adapted from Baumann et al, '08

KKLT, '03

Ingredients:
- Warping
- Ricci flat $\tilde{R}_6$
- Fluxes $\left(G_3, \tilde{F}_5\right)$
- Localized branes O3, $\overline{\text{D3}}$, D7
- Gaugino cond. on D7

$$ds_{10}^2 = e^{2A(y)}\,\hat{g}_{\mu\nu}\,dx^\mu dx^\nu + e^{-2\ \ (y)}\,\tilde{g}_{mn} dy^m dy^n$$

GKP, '01
KKLT, '03

- Deep in throat ($\sim$AdS throat)

$$ds_{10}^2 = \frac{r^2}{L^2}\,\hat{g}_{\mu\nu}\,dx^\mu dx^\nu + \frac{L^2}{r^2}\left(dr^2 + r^2 dS_5^2\right)$$
$$= \frac{L^2}{\chi^2}\left(\hat{g}_{\mu\nu}\,dx^\mu dx^\nu + d\chi^2 + \chi^2 dS_5^2\right)$$

$$\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega = 0$$

$$3H^2 = \bar{R}_{\chi\chi} + (D-2)\left[\left(\partial_\chi \log\Omega\right)^2 - \partial_\chi^2 \log\Omega\right] - \Omega^4\, T_{MN}\, N^M N^N$$

Ricci-flat

No warp factor contribution in warped throat

Homogeneous NEC Violation?
- $\langle\lambda\lambda\rangle$: ***localized*** on/near D7-branes
- $\overline{D3}$: ***localized*** in warped throat (~~NEC~~)
- $G_3 = G_3^{flux} + \delta G_3$ [Shiu's Talk] (~~NEC~~)

Questions:
- Where will **homogeneous** (not localized) violation of NEC come from?
- If warp factor plays a role in this constraint, then expect **global backreaction** on warp factor $e^A \sim e^{A_0 + H\,r}$? (c.f. dS in RS)

Carta, Moritz, Westphal, '19
Gautason, Van Hemelryck, Van Riet, 19
Hamada, Hebecker, Shiu, Soler, '19

KKLT

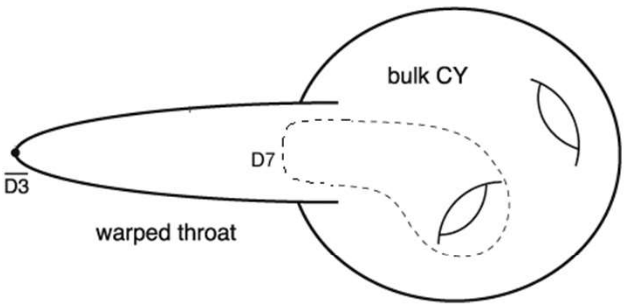


Figure adapted from Baumann et al, '08

KKLT, '03

Ingredients:

- Warping
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$$ds_{10}^2 = e^{2A(y)}\,\hat{g}_{\mu\nu}\,dx^\mu dx^\nu + e^{-2A(y)}\,\tilde{g}_{mn} dy^m dy^n$$

GKP, '01
KKLT, '03

- More generally…

$$3\,H^2 = \tilde{R}_{mn}\tilde{n}^m\tilde{n}^n + 2\,e^{4A}\,\tilde{\nabla}^2 A\; - 8\,\tilde{n}^m\tilde{n}^n\,(\partial_n A)\,(\partial_m A)\; - e^{4A}\,T_{MN}\,N^M N^N$$

Ricci-flat

Fixed (at high scale?) by fluxes and O-planes, does not contribute to dS_4?

Homogeneous NEC Violation?

- $\langle\lambda\lambda\rangle$: ***localized*** on/near D7-branes
- $\overline{D3}$: ***localized*** in warped throat
- "Non-local" contributions from gauginos?

Questions:

- Where will **homogeneous** (not localized) violation of NEC come from?
- If warp factor plays a role in this constraint, then expect **global backreaction** on warp factor $e^A \sim e^{A_0 + H\,r}$? (c.f. dS in RS)

Summary

- Several different constraints on dS from higher dimensions
  - GMN, GKP
  - Raychaudhuri NEC: Non-positive curvature + trivial warping + NEC = no dS

$$3H^2 = \bar{R}_{\chi\chi} + (D-2)\left[\left(\partial_\chi \log \Omega\right)^2 - \partial_\chi^2 \log \Omega\right] - \Omega^4\, T_{MN}\, N^M N^N$$

- Raychaudhuri NEC has advantages:
  - Not specific to particular background
  - Local, not integrated: Must satisfy at every point
  - Does not assume Einstein Equations
  - Multiple Conditions: one for each choice of internal null vector
  - Strong: Matter which "passes" other dS constraints doesn't pass Raychaud. NEC

- Application of Raychaudhuri NEC to dS solutions gives some

Take-Home Lessons:
  - **Positive**, not negative, curvature is a way to get bulk solutions with dS_4
  - "Uplift" of bulk solutions to dS_4 with local objects (e.g. branes) requires **global backreaction** of warp factor.

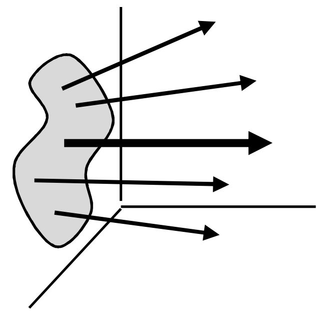
- Would be interesting to see how this works for KKLT?
  - Where will **homogeneous** (not localized) violation of NEC come from?
  - If warp factor plays a role in the constraint, then expect **global backreaction** on warp factor.

Extra Slides

Apparent Horizons

Oppositely oriented affine null rays N_+^M (outgoing) N_-^M (ingoing)

Expansion $\theta_\pm = \frac{1}{\sqrt{-g_D}}\,\partial_A\left(\sqrt{-g_D}\,N_\pm^A\right)$



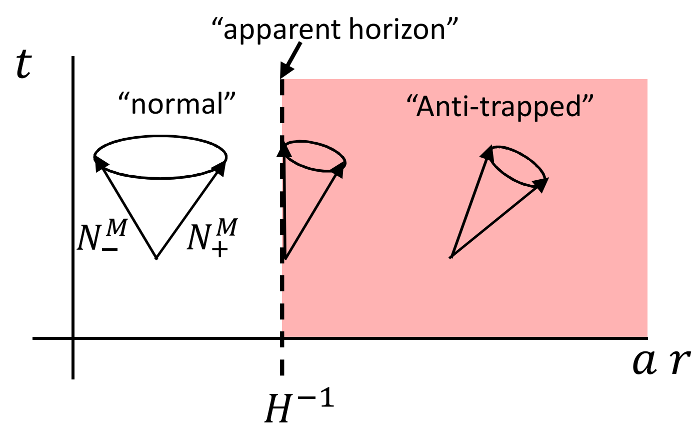
Expansion θ:
Rate of change of area perpendicular to bundle

Ex:
FRW Cosmology $ds^2 = -dt^2 + a^2(t)\,(dr^2 + r^2 d\Omega_2^2)$

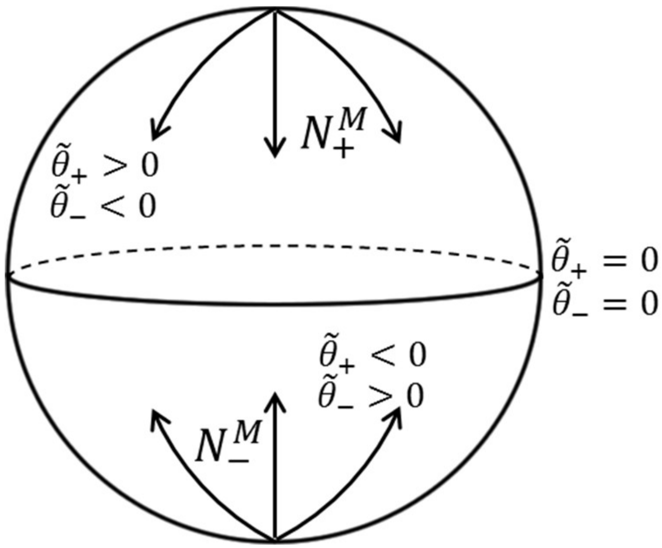
$$N_\pm^M = \left(\frac{1}{a}, \pm\frac{1}{a^2}, 0, 0\right)$$

$$\rightarrow \quad \theta_+ = \frac{2}{a}\left(H + \frac{1}{ar}\right), \qquad \theta_- = \frac{2}{a}\left(H - \frac{1}{ar}\right)$$

- For $ar < H^{-1}$, $\theta_+ \rightarrow \text{positive}$, $\theta_- \rightarrow \text{negative}$ "normal region"
- For $ar > H^{-1}$, $\theta_+ \rightarrow \text{positive}$, $\theta_- \rightarrow \text{positive}$ "anti-trapped region"
- For $ar = H^{-1}$, $\theta_+ \rightarrow \text{positive}$, $\theta_- \rightarrow \text{zero}$ "apparent horizon"



Expansions in Compact Space



- Simple Example: an S^2

$$ds^2 = -dt^2 + R^2(d\chi^2 + \sin^2\chi \; d\alpha^2)$$

$$N_\pm^M = \left(\underset{t,}{1}, \underset{\chi,}{\pm\frac{1}{R}}, \underset{\alpha}{0}\right)$$

Expansions $\boldsymbol{\tilde{\theta}_\pm = \pm\frac{\cot\chi}{R}}$

- Diverge at the poles $\chi \to 0, \pi$
- $\tilde{\theta}_+$ positive for $0 < \chi < \frac{\pi}{2}$, negative for $\frac{\pi}{2} < \chi < \pi$ (and visa-versa for θ_-)
- $\tilde{\theta}_\pm = 0$ on equator

Integral of the expansion vanishes:

$$\int \sqrt{g_2}\, \tilde{\theta}_\pm \, d\chi \, d\alpha = 0$$

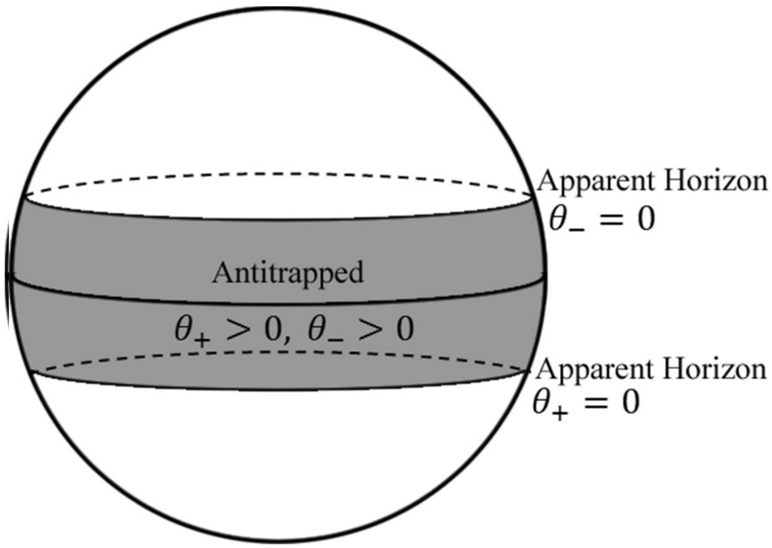
Equal amounts of positive and negative expansion.

$$\sqrt{g_2}\, \tilde{\theta}_\pm = \partial_m\left(\sqrt{g_2}\, N_\pm^m\right)$$

Expansion is a **total derivative** on compact space

Expansions in Compact Space

Das, Haque, Underwood, '19



Since $H\ R \ll 1$, expect anti-trapped region to be quite "thin"

Extend this example: FLRW $\times\ S^2$

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 + R^2(d\chi^2 + \sin^2\chi\ d\alpha^2)$$

$$N_\pm^M = \underset{t,\ \vec{x},\ \ \chi,\ \ \ \alpha}{\left(1, 0, \pm\frac{1}{R}, 0\right)}$$

Expansions $\boldsymbol{\theta_\pm = 3H \pm \frac{\cot\chi}{R}}$

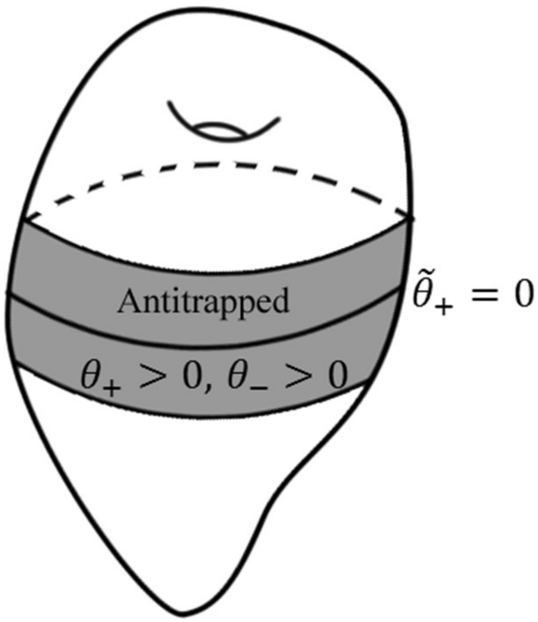
- Both θ_+ & θ_- are positive in a narrow band below the equator $\pi/2 < \chi < \pi/2 + \cot^{-1}(3HR)$
- Similarly, both θ_+ & θ_- are positive in a narrow band above the equator as well.
  $\rightarrow$ ***Anti-trapped region?***
- The boundary to these regions has $\theta_+ = 0$ and $\theta_- > 0$ (and visa-versa)
  $\rightarrow$ ***Apparent horizons?***

These apparent horizons are also (inner past) trapping horizons when

$$\mathcal{L}_+\theta_- > 0 \ \ \Rightarrow \ \ 3\dot{H} + \frac{(1 + (3HR)^2)}{R^2} > 0$$

Horizons in Compact Space

Das, Haque, Underwood, ‘19



- True more generally – any direct product of FLRW with a compact space will develop an anti-trapped band

$$ds^2 = \Omega^2(y)(-dt^2 + a^2(t)\, d\vec{x}^2 + \tilde{g}_{mn}(y)\, dy^m dy^n$$

- Anti-trapped region arises from *shear*
  → null rays at fixed co-moving coordinates $\vec{x}$ expand due to the expansion of the universe
  → but expansion $\theta_\pm$ is a scalar, cannot be decomposed into “parallel” and “shear” components.

- Null rays can traverse from one end of compact space to the other in finite (affine) time
  → What is the physical significance of the anti-trapped regions and apparent horizons?